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## Complex power, power factor, power factor correction

For sinusoidal signals the power factor is defined by

$$
\mathrm{pf}=\cos \theta
$$

where $\theta$ is the phase angle of the voltage or current relative to some reference. For power circuits, the generator (or line) voltage is usually taken as the reference since the loads are usually connected in parallel.

For phasors we can define complex (also called the reactive) power as:
$C P=v \times i^{*}=$ real power $\pm j$ reactive power $=P \pm j Q$
where
real power or $\mathrm{P}=\mathrm{v} \times \mathrm{i} \times \cos \theta$
and
reactive power or $\mathrm{Q}=\mathrm{v} \times \mathrm{i} \times \sin \theta$
The relationship between P and Q determines the power factor as shown below.
inductive load
capacitive load



Complex power relationships for inductive and capacitive loads


Relationship between voltage and current in complex loads


Relationship between real and imaginary power in a complex load. Note that the diagram is drawn for an inductive circuit. It would be reversed for a capacitive load.

Example:
A 30 kW heater and a 150 kVA induction motor with a power factor of 0.6 lagging are connected in series in an industrial plant. The plant input voltage is 4000 volts.
(a) Determine the total plant current and the plant power factor.
(b) Correct the plant power factor to 0.9 lagging using a lossless capacitor. Determine C .

Although the problem sounds complicated the actual circuit and solution are fairly simple. Drawing the equivalent circuit for the plant


The first thing to realize is that a lagging power factor is inductive. Almost ALL motors are inductive so you could have guessed this immediately. The power factor of 0.6 corresponds to $\cos \theta=0.6$, or $\theta=53^{\circ}$. The complex power of the motor is then given by
$\mathrm{CP}_{\text {motor }}=150 \cos \theta+\mathrm{j} 150 \sin \theta=90+\mathrm{j} 120 \mathrm{kVA}$
The complex power of the heater (only real) is
$\mathrm{CP}_{\text {heater }}=30+\mathrm{j} 0$
The total complex power of the plant is simply the sum of the complex powers of the heater and the motor since powers always add.
$\mathrm{CP}_{\text {plant }}=(90+\mathrm{j} 120)+(30+\mathrm{j} 0)=120+\mathrm{j} 120=169.7 \angle 45^{\circ}$
The input current to the plant is then
$\mathrm{I}=\frac{169.7}{4}=42.43$ amperes
and the plant power factor is
$\mathrm{pf}=\cos 45^{\circ}=0.707$
For the second part of the problem consider the capacitor shown in the following circuit:


The new power factor is to be 0.9 which means that $\theta$ can be determined from the definition of power factor
$\theta=\operatorname{Cos}^{-1}(0.9)=25.84^{\circ}$
A capacitor does not dissipate heat and, consequently, cannot change the real power of the circuit. It can only change the imaginary power. To find this required change in the imaginary power (the real power remains the same) use the definition of the power factor angle and complex power
$\tan \theta=0.484=\frac{\mathrm{Q}}{\mathrm{P}}=\frac{\mathrm{VI} \sin \theta}{\mathrm{VI} \cos \theta}$
The plant power for a pf of 0.9 must then be
$120+\mathrm{j} 120 \tan \theta=120+\mathrm{j} 120(0.484)=120+\mathrm{j} 58.1$
The capacitor C must be responsible for the change in Q since all other factors are the same.
Summing the imaginary parts of the power we can write
$\mathrm{j} 58.1=\mathrm{jQ}$ capacitor +j 120
$j Q_{\text {capacitor }}=-j 62 \mathrm{kVAr}$
Knowing the complex power of the capacitor we can find the current I through it, i.e. $-\mathrm{j} 62000=v \mathrm{v}^{*}=\left(4000 \angle 0^{\circ}\right) \mathrm{i}^{*}$

Solving for i , the current through the capacitor is $\mathrm{i}=+\mathrm{j} 15.5$ amperes.
Applying Ohm's Law to the capacitor V=IZ
$\mathrm{Z}=\frac{1}{\mathrm{j} \omega \mathrm{C}}=\frac{\mathrm{V}}{\mathrm{I}}=\frac{4000 \angle 0^{\circ} \text { volts }}{\mathrm{j} 15.5 \mathrm{amperes}}$
Assuming $\mathrm{f}=60 \mathrm{~Hz}$, or $\omega=2 \pi(60)=377$, and solving for C
$\mathrm{C}=\frac{15.5}{(377) 4000} \approx 1.5 \times 10^{-5}=15 \mu \mathrm{~F}$

## Ideal Transformers

Transformers are devices used to translate AC voltages and current. Transformers have two pairs of terminals: an input and an output. Even though the transformer changes the voltage and current the ac power at the input must equal the ac power at the output. Transformers are usually specified by their voltage transformation ratios


With respect to the above drawing the input voltage and current are $\mathrm{V}_{1}$ and $\mathrm{I}_{1}$. The output voltage and current are $\mathrm{V}_{2}$ and $\mathrm{I}_{2}$. The number of turns on the input is $\mathrm{N}_{1}$ and the number of turns on the transformer output is $\mathrm{N}_{2}$. The voltage transformation ratio between the output and input is
$\frac{\mathrm{E}_{2}}{\mathrm{E}_{1}}=\frac{\mathrm{N}_{2}}{\mathrm{~N}_{1}}$

## Example:

An ideal transformer is rated at 100 kVA (these units will be explained in Section 14.10 on complex power) with a secondary voltage of 100 volts (rms). Determine the primary (input) current and voltage if
$\frac{\mathrm{N}_{2}}{\mathrm{~N}_{1}}=\frac{1}{50}$
and the transformer is operating at its power rating.
Using the relationship between the voltage and turns ratios
$\frac{\mathrm{E}_{2}}{\mathrm{E}_{1}}=\frac{100}{\mathrm{E}_{1}}=\frac{\mathrm{N}_{2}}{\mathrm{~N}_{1}}=\frac{1}{50}$
and solving for $E_{1}$, we get $E_{1}=5000$ volts. Using the fact that the transformer is ideal (the input power equals the output power) we have the relationship
$\mathrm{V}_{1} \mathrm{I}_{1}=\mathrm{V}_{2} \mathrm{I}_{2}$
which, upon substitution, becomes
$(5000$ volts $) \mathrm{I}_{1}=(100,000$ Volt-amperes $)$
and, solving for $\mathrm{I}_{1}$,
$\mathrm{I}_{1}=20$ amperes.

Example:
Questions 1-5 relate to the single-phase electrical distribution network shown below.


Instantaneous time expressions/Equivalent root mean square phasor expression
$\mathrm{e}_{\mathrm{S}}=622.2 \cos (120 \pi \mathrm{t})$ volts $\leftrightarrow \mathrm{Es}=440 \angle 0^{\circ}$ volts
$\mathrm{i}_{\mathrm{S}}=195.1 \cos (120 \pi \mathrm{t}-0.227)$ amperes $\leftrightarrow \mathrm{I}_{\mathrm{S}}=138 \angle-13^{\circ}$ Amperes
Load A is a bank of single-phase induction motors with the bank having an efficiency of $85 \%$, a power factor of 0.75 lagging, and an output load of 30.0 horsepower.

Load B is a bank of over-excited single-phase synchronous motors drawing 22.0 kilovolt-amperes with the bank load current having a leading phase displacement with respect to the line voltage of 25.8 electrical degrees.

Load C is a lighting load of 13.2 kilowatts at unity power factor.
Load D is a proposed single-phase synchronous condenser to correct the source power factor to unity and thereby gain a more favorable energy rate from the utility providing the electrical service.

1. The root-mean-square magnitude of the load current $\mathrm{I}_{\mathrm{A}}$ is most nearly?

The trick in this answer is that the power for load A must be converted to electrical power:
$\mathrm{P}_{\mathrm{A}}=30 \mathrm{hp} \times 746 \mathrm{watts} / \mathrm{hp}=22.38 \times 10^{3}$ watts $=22.38 \mathrm{~kW}$
and, correcting for the $85 \%$ efficiency, $\mathrm{P}_{\mathrm{A}}=22.38 / .85=26.33 \mathrm{~kW}$ of real power.
The complex power is given by
$|\mathrm{CP}|=\frac{\mathrm{P}_{\mathrm{A}}}{\cos \theta}=\frac{26.33 \mathrm{~kW}}{0.75}=35.107 \mathrm{kVA}$
Since complex power $\mathrm{CP}=\mathrm{vi}$ *,
$\mathrm{CP}=35.107 \angle \operatorname{Cos}^{-1}(0.75) \mathrm{kVA}=35.107 \angle 41.41^{\circ}=\left(440 \angle 0^{\circ}\right.$ volts $) \mathrm{i}^{*}$
and, solving for the current,
$\mathrm{i}=\left(\frac{35.107 \angle 41.41^{\circ} \mathrm{kVA}}{440 \mathrm{~V} \angle 0^{\circ} \text { volts }}\right)^{*} \cong 80 \angle-41.41^{\circ}$ Amperes
2. The phase displacement of the load current $\mathrm{I}_{\mathrm{A}}$ with respect to the line voltage is most nearly?

This was already solved for in questions 1 . Load A is given with a power factor of 0.75 lagging. This means that the current must have a negative angle with respect to the reference voltage as shown above. The question is simply to find the phase displacement, or magnitude of the angle, which is easily gotten from the power factor:
$\cos \theta=0.75$ or $\theta=\operatorname{Cos}^{-1}(0.75)=41.4^{\circ}$
3. The power drawn by the bank of synchronous motors is most nearly?

This is answered from the definition of power: $\mathrm{P}=\mathrm{v} \times \mathrm{i} \times \cos \theta$. Specifying 22.0 kilovolt-amperes is a fancy way of referring to the complex power. ALWAYS CHECK THE UNITS. The question is asking for the real power which is given as.
$\mathrm{P}=\mathrm{CP} \times \cos \theta=(22,000)\left(\cos 25.8^{\circ}\right)=19,800$ Watts
4. The power factor associated with the system as viewed by the source prior to installation of load D is most nearly
(a) 0.227 , capacitive in nature
(b) 0.227 , inductive in nature
(c) 0.314 , inductive in nature
(d) 0.974 , capacitive in nature
(e) 0.974 , inductive in nature

The power factor being referred to is the cosine of the complex power
$\widehat{\mathrm{CP}}=\hat{\mathrm{e}}_{S} \widehat{\mathrm{I}}_{\mathrm{S}}^{*}=\left(440 \angle 0^{\circ}\right.$ volts $)\left(138 \angle-13^{\circ} \text { amperes }\right)^{*}=60720 \angle+13^{\circ}$ volt-amperes
which is
power factor $=\cos \left(+13^{\circ}\right)=0.974$
The angle for the power triangle is $+13^{\circ}$; thus, the angle between the voltage (our reference) and the current is $-13^{\circ}$. This is a lagging relationship, i.e. the current lags the voltage and the overall circuit is inductive. The correct answer is (e).

IMPORTANT NOTE: Load B shows one of the very few times when a motor looks like a capacitor. Specifically, Load B is a bank of over-excited single-phase synchronous motors drawing 22.0 kilovolt-amperes with the bank load current having a leading phase displacement with respect to the line voltage of 25.8 electrical degrees. This means that the load is capacitive.
5. The capacitance associated with the synchronous condenser that will give a unity power factor for the system is most nearly
(a) $480 \mu \mathrm{~F}$
(b) $318 \mu \mathrm{~F}$
(c) $187 \mu \mathrm{~F}$
(d) $131 \mu \mathrm{~F}$
(e) $93.4 \mu \mathrm{~F}$

The capacitor C must cancel out the imaginary part Q of the complex power CP . The power triangle gives
$\sin 13^{\circ}=\frac{\mathrm{Q}}{60720 \mathrm{VA}}$
and, solving for Q ,
$\mathrm{Q}=13659 \mathrm{VAr}$

Knowing the reactive power of the capacitor we can find the current i through it, i.e.
$-j 13659=v i^{*}=\left(440 \angle 0^{\circ}\right) i^{*}$
Solving for i , the current through the capacitor is $\mathrm{i}=+\mathrm{j} 31.04$ amperes.
Applying Ohm's Law to the capacitor
V=IZ
$\mathrm{Z}=\frac{1}{j \omega \mathrm{C}}=\frac{\mathrm{V}}{\mathrm{I}}=\frac{440 \angle 0^{\circ} \text { volts }}{\mathrm{j} 31.04 \text { amperes }}$
Assuming $\mathrm{f}=60 \mathrm{~Hz}$,or $\omega=2 \pi(60)=377$, and solving for C
$\mathrm{C}=\frac{31.04}{(377) 440} \approx 1.87 \times 10^{-4}=187 \mu \mathrm{~F}$
The nearest answer is (c).

## CIRCUITS 23

The power supplied by the current source is closest to
(a) 0 watts
(b) 5 watts
(c) 10 watts
(d) 15 watts
(e) 20 watts


We proceed by calculating the voltage drop across the load impedance
$\mathrm{V}_{\mathrm{L}}=\mathrm{I}_{\mathrm{L}} \mathrm{Z}=\left(2 \angle 45^{\circ}\right)(2+\mathrm{j} 3 \Omega)=\left(2 \angle 45^{\circ}\right)\left(3.61 \angle 56.31^{\circ}\right)=7.22 \angle 101.31^{\circ}$
The voltage V across the current source is then given as
$\mathrm{V}=5 \angle 40^{\circ}+\mathrm{V}_{\mathrm{L}}=5 \angle 40^{\circ}+7.22 \angle 101.31^{\circ}=3.83+\mathrm{j} 3.21-1.416+\mathrm{j} 7.07$
$\mathrm{V}=2.41+\mathrm{j} 10.28=10.56 \angle 76.8^{\circ}$
The power is then given by
$\mathrm{CP}=\mathrm{vi}^{*}=\left(10.56 \angle 76.8^{\circ}\right)\left(2 \angle-45^{\circ}\right)=21.1 \angle 31.7^{\circ}=18.0+\mathrm{j} 11.12$ watts
The real power supplied by the current source is 18 watts; the nearest correct answer is (e).

## CIRCUITS 27

440 volts, 60 Hz
20 kW
p.f. $=0.8$ lagging

Given a 440 volt line supplying 20 kilowatts at a power factor of 0.8 lagging, the line current is closest to:
(a) 20 amps
(b) 40 amps
(c) 60 amps
(d) 80 amps
(e) 100 amps

Since the power factor of the load is specified to be 0.8 lagging the complex power must obey the diagram shown below. imaginary


Since the p.f. is $0.8, \theta=\operatorname{Cos}^{-1}(0.8)=36.8^{\circ}$. The complex power in the load is then $\mathrm{CP}=20 \mathrm{~kW} / \cos \theta=20 \mathrm{~kW} / \cos \left(36.8^{\circ}\right)=25 \mathrm{kVA}$. The complex line current is then given by Ohm's Law
$\mathrm{i}=\frac{\mathrm{CP}}{\mathrm{V}}=\frac{25 \times 10^{3}}{440 \text { volts } \mathrm{rms}}=57 \mathrm{amps} \mathrm{rms}$
The correct answer is (c).

CIRCUITS 28

400 volts, 60 Hz


20 kW
p.f. $=0.8$ lagging

The value of C in the above circuit so the load presented to the power line has unity power factor is closest to
(a) 100 microfarads
(b) 200 microfarads
(c) 250 microfarads
(d) 300 microfarads
(e) 320 microfarads

Since the power factor of the load is specified to be 0.8 lagging the complex power must obey the diagram shown below. imaginary


Since the p.f. is $0.8, \theta=\operatorname{Cos}^{-1}(0.8)=36.8^{\circ}$. The reactive power in the load is then
$P_{\text {REACTIVE }}=20 \mathrm{~kW} \tan \theta=20 \mathrm{~kW} \tan \left(36.8^{\circ}\right)=14.96 \mathrm{kVA}$. Since the real part of the power cannot be changed by the addition of the capacitor C , the reactive power for the overall circuit must change to 0 for p.f. $=1$. This means that we want $\mathrm{P}_{\text {CAPACITOR }}=-j 14.96 \mathrm{kVA}$. The complex power of the capacitor is purely reactive and is given by
$\widehat{\mathrm{P}}_{\text {CAPACITOR }}=\widehat{\mathrm{v}}^{*}=\widehat{\mathrm{v}}\left(\frac{\widehat{\mathrm{v}}}{\frac{1}{j \omega \mathrm{C}}}\right)^{*}=-\mathrm{j} \omega \mathrm{C} \widehat{\mathrm{v}} \widehat{\mathrm{v}}^{*}$
Solving for C
$\mathrm{C}=\frac{\widehat{\mathrm{P}}_{\text {CAPACITOR }}}{-\mathrm{j} \omega \widehat{\mathrm{v}}^{*}}=\frac{-\mathrm{j} 14.96 \mathrm{kVAR}}{-\mathrm{j}(377)(400)^{2}}=0.248 \times 10^{-9}=248 \mu \mathrm{~F}$
The correct answer is (c).

## CIRCUITS 24

If 20 kW is supplied to a load of 30 amps at a power factor of 0.7 lagging, what is the reactive part of the load?
(a) $-22.7 \Omega$
(b) $+22.7 \Omega$
(c) $+6.4 \Omega$
(d) $-6.4 \Omega$
(e) $0 \Omega$

This problem is most easily solved by considering the power triangle
imaginary


The components of the complex power are given by
$\mathrm{CP}=\frac{\mathrm{P}_{\text {REAL }}}{\operatorname{Cos} \theta}=\frac{20 \mathrm{~kW}}{0.7}=28.571 \mathrm{kVA}$
and
$\mathrm{Q}=20 \mathrm{~kW} \tan \theta=20.4 \mathrm{kVAr}$
Writing the expression for the complex power and using Ohm's Law
$\mathrm{CP}=28.571 \angle \operatorname{Cos}^{-1}(0.7)=28.571 \angle 45.573^{\circ}=\mathrm{vi}^{*}=(\mathrm{iZ}) \mathrm{i}^{*}$
Solving for Z
$\mathrm{Z}=\frac{28.571 \angle 45.573^{\circ}}{\text { ii }^{*}}=\frac{28.571 \angle 45.573^{\circ}}{(30)^{2}}=31.75 \angle 45.573^{\circ}=22.23+\mathrm{j} 22.67 \Omega$
The reactive part of the load impedance is $+\mathrm{j} 22.67 \Omega$ and the nearest correct answer is (b).

