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Please inform me of your opinion of the relative emphasis of the review material by simply making comments on this page and sending it to me at:

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Your information will be used to revise next year's exam review course. I may be reached for questions about the review material at:

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The morning session (also known as the A.M. session) has 120 multiple-choice questions, each with four possible answers lettered (A) to (D). Responses must be recorded with a number 2 pencil on special answer sheets. No credit is given for answers recorded in ink.

Each problem in the morning session is worth one point. The total score possible in the morning is 120 points. Guessing is valid; no points are subtracted for incorrect answers.

Morning FE Exam Subjects

subject	number of questions
chemistry	11
computers	7
dynamics	9
electrical circuits	12
engineering economics	5
ethics	5
fluid mechanics	8
material science and structure of matter	8
mathematics	24
mechanics of materials	8
statics	12
thermodynamics	11

Afternoon FE Exam Subjects
(General Exam)

subject	number of questions
chemistry	5
computers	3
dynamics	5
electrical circuits	6
engineering economics	3
ethics	3
fluid mechanics	4
material science and structure of matter	3
mathematics	12
mechanics of materials	4
statics	6
thermodynamics	6

There are six different versions of the afternoon session (also known as the P.M. session), five of which correspond to a specific engineering discipline: chemical, civil, electrical, industrial, and mechanical engineering.

Each version of the afternoon session consists of 60 questions. All questions are mandatory. Questions in each subject may be grouped into related problem sets containing between two and ten questions each.

The sixth version of the afternoon examination is a general examination suitable for anyone, but in particular, for engineers whose specialties are not one of the other five disciplines. Though the subjects in the general afternoon examination correspond to the morning subjects, the questions are more complex — hence their double weighting.

Questions on the afternoon examination are intended to cover concepts learned in the last two years of a four-year degree program. Unlike morning questions, these questions may deal with more than one basic concept per question.

The numbers of questions for each subject in the general afternoon session examination are given in the above table

The numbers of questions for each subject in the, discipline-specific afternoon session examination are listed on the following pages. The discipline specific afternoon examinations cover substantially different bodies of knowledge than the morning examination. Formulas and tables of data needed to solve questions in these examinations will be included in either the NCEES FE Reference Handbook or in the body of the question statement itself.

Each afternoon question consists of a problem statement followed by multiple-choice questions. Four answer choices lettered (A) through (D) are given, from which you must choose the best answer.

- Each question in the afternoon is worth two points, making the total possible score 120 points.
- The scores from the morning and afternoon sessions are added together to determine your total score. No points are subtracted for guessing or incorrect answers. Both sessions are given equal weight

CHEMICAL ENGINEERING

subject	number of questions
chemical reaction engineering	6
chemical thermodynamics	6
computer and numerical methods	3
heat transfer	6
mass transfer	6
material/energy balances	9
pollution prevention (waste minimization)	3
process control	3
process design and economics evaluation	6
process equipment design	3
process safety	3
transport phenomenon	6

INDUSTRIAL ENGINEERING

subject	number of questions
computer computations and modeling	3
design of industrial experiments	3
engineering economics	3
engineering statistics	3
facility design and location	3
industrial cost analysis	3
industrial ergonomics	3
industrial management	3
information system design	3
manufacturing processes	3
manufacturing systems design	3
material handling system design	3
mathematical optimization and modeling	3
production planning and scheduling	3
productivity measurement and management	3
queuing theory and modeling	3
simulation	3
statistical quality control	3
total quality management	3
work performance and methods	3

ELECTRICAL ENGINEERING

subject	number of questions
analog electronic circuits	6
communications theory	6
computer and numerical methods	3
computer hardware engineering	3
computer software engineering	3
control systems theory and analysis	6
digital systems	6
electromagnetic theory and applications	6
instrumentation	3
network analysis	6
power systems	3
signal processing	3
solid state electronics and drives	6

CIVIL ENGINEERING

subject	number of questions
computers and numerical methods	6
construction management	3
environmental engineering	6
hydraulics and hydrologic systems	6
legal and professional aspects	3
soil mechanics and foundations	6
structural analysis (frames, trusses, etc.)	6
structural design (concrete, steel, etc.)	6
surveying	6
transportation facilities	6
water purification and treatment	6

MECHANICAL ENGINEERING

subject	number of questions
automatic controls	3
computer (numerical methods, automation, etc.)	3
dynamic systems (vibrations, kinematics, etc.)	6
energy conversion and power plants	3
fans, pumps, and compressors	3
fluid mechanics	6
heat transfer	6
material behavior/processing	3
measurement and instrumentation	6
mechanical design	6
refrigeration and HVAC	3
stress analysis	6
thermodynamics	6

<u>Morning:</u>

computers	7
electrical circuits	12

total of 19/120, about 10%

<u>General Afternoon:</u>

computers	3
electrical circuits	6

total of 9/60, again about 10%

I did a count of the various sample exams and came up with the following topical distribution.

Morning general examination:

Laplace transform	1
power triangle	1
impedance diagram (phasors)	1
transients	2
electromagnetic fields	1
DC circuits	1
computers	2

Afternoon general examination:

transients	3
computers	1

Afternoon, EE specific examination:

op-amps	4
transistors (BJT & FET)	4
control	3
communications	2
E&M	2
Digital filters	3
Solid State	3
Phasors	2
Three-phase power	2
digital (mostly counters)	3
differential equations	1
computer	1

Current may be defined by a derivative, i.e. the rate at which charge is moved:

$$i = \frac{dq}{dt}$$

or in the form of an integral as the total charge moved:

$$q = \int_{t_0}^{t} i(t)\, dt$$

where i is in units of coulombs/second , or amperes.

Voltage

Voltage is a measure of the work required to move a unit charge through an electric field (usually inside an electrical circuit element such as a resistor). It requires one joule of energy to move one coloumb of charge through a potential difference of one volt.

$$w = \int v\, dq$$

where w is in joules, v is in volts and dq is in coulombs.

Resisitivity and Resistance

The resistance of ordinary wire can be calculated provided one knows the resistivity of the wire as

$$R = \frac{\rho l}{A}$$

where l is the length of the wire and A is its cross-sectional area. The resistivity ρ is a property of the material and a function of temperature as given by

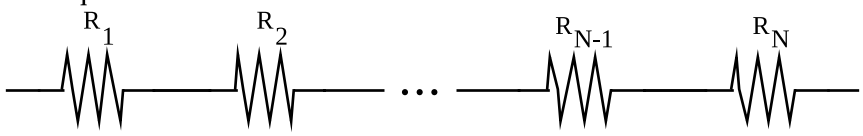
$$\rho = \rho_0\left(1 + \alpha\Delta t\right)$$

where $\rho_0 = 1.7241\times10^{-6}$ Ω-cm^2/cm at 20°C, α=0.00382/°C for hard-drawn copper of the type most commonly used for electrical wiring, and ΔT is the temperature difference between the temperature of the desired resistance and that at which ρ_0 is specified, in this case $\Delta T = T_{specified} - 20°C$.

Be careful of the units of ρ. Very commonly ρ is given in units of Ω-cmil/foot. This requires that A be given in circular mils, i.e. the area of a 0.001 inch diameter circle. The formula for converting between actual diameter and circular mils is:

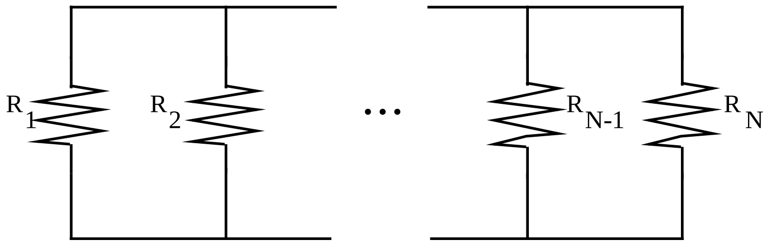
$$A_{cmils} = \left(\frac{d_{inches}}{0.001}\right)^2$$

The equivalent value of N series resistors is:



$$R_{eq} = \sum_{j=1}^{N} R_j$$

The equivalent value of N parallel resistors is:

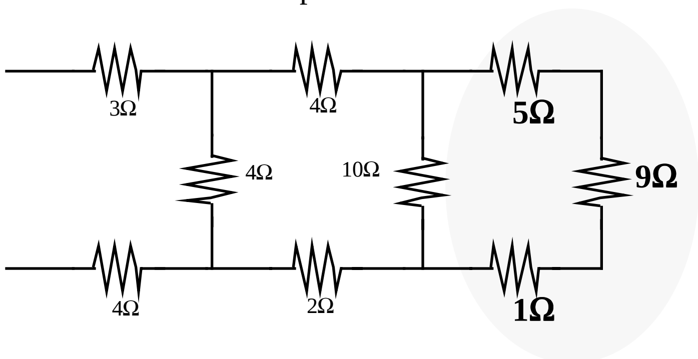


$$R_{eq} = \frac{1}{\sum_{j=1}^{N} \frac{1}{R_j}}$$

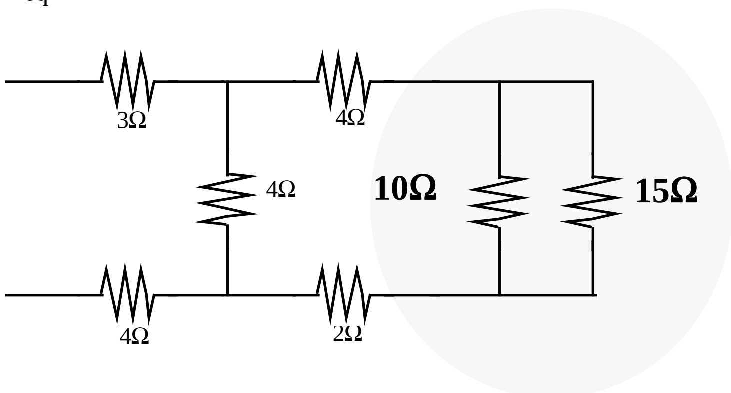
For the important case of two resistors in parallel:

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

Equivalent resistance for a complex network:

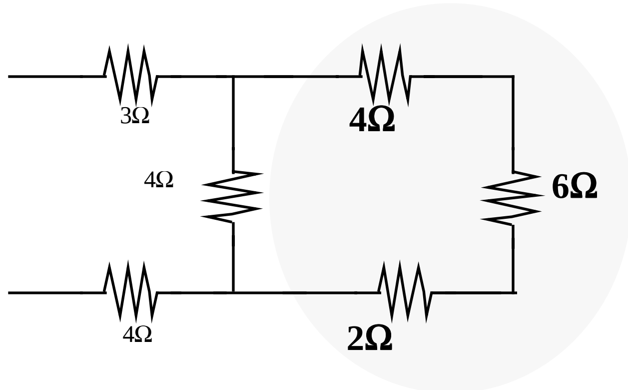


Looks like three resistors in series:
$R_{eq} = 5\Omega + 1\Omega + 9\Omega = 15\Omega$

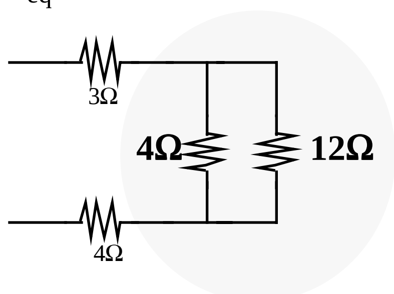


Looks like two resistors in parallel:

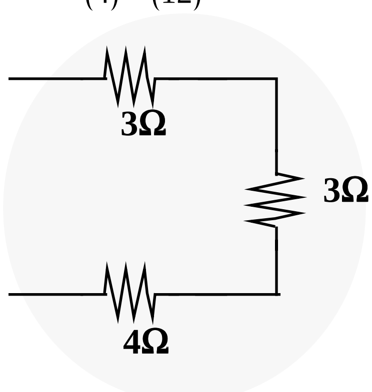
$$R_{eq} = \frac{(10)(15)}{(10) + (15)} = 6\ \Omega$$



Looks like three resistors in series
$R_{eq} = 4 + 6 + 2 = 12\Omega$



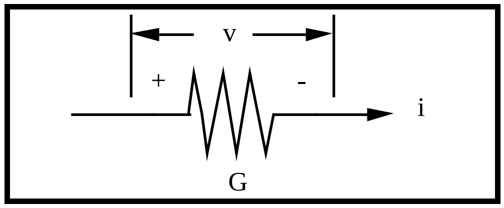
Looks like two resistors in parallel:
$$R_{eq} = \frac{(4)(12)}{(4)+(12)} = 3\ \Omega$$



Looks like three resistors in series:
$R_{eq} = 3 + 3 + 4 = 10\Omega$

Conductivity and conductance

Ohm's Law can also be written in terms of conductance which is simply 1/R, i.e. v=i/G where G is in mhos, the unit of conductance.

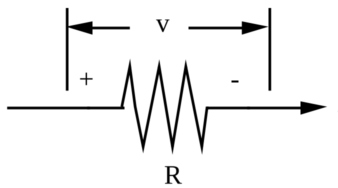


Ohm's Law

Electrical resistance

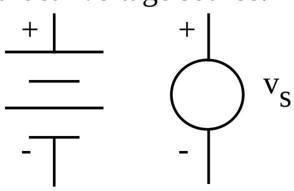
$$\boxed{v=iR}$$

where R is in units of ohms.

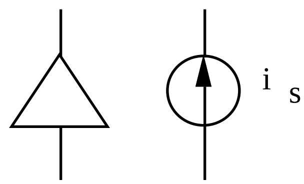


Energy sources

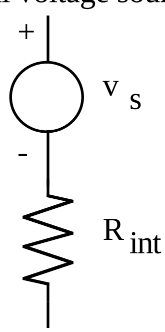
independent ideal voltage source:



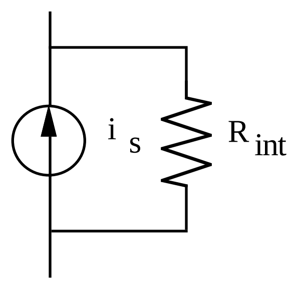
independent ideal current source:



independent real voltage source:



independent real current source:



Perfect voltage and current sources have the following characteristics:

A perfect (ideal) voltage source has $R_{int}=0$.

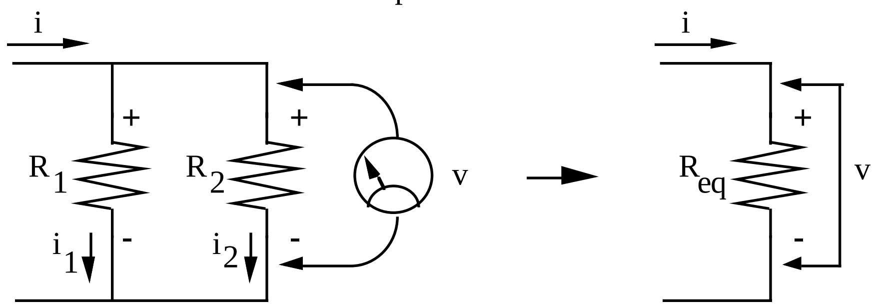
A perfect (ideal) current source has $R_{int}=\infty$.

Voltage Sources in Series and Parallel

Voltage sources that are in series (even if there are intervening resistances) can be algebraically combined into a single equivalent resistance.

Voltage and Current dividers

Current division between two resistors in parallel:



Since the resistors are in parallel they MUST have the same voltage across them

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$v = iR_{eq} = \frac{R_1 R_2}{R_1 + R_2} i$$

$$\boxed{\begin{aligned} i_1 &= \frac{v}{R_1} = \frac{R_2}{R_1 + R_2} i \\ i_2 &= \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} i \end{aligned}}$$

This is known as a current divider.

Voltage division between two resistors in series (see the figure below).
As the resistors are in series they MUST have the same current thru them.

$$R_{eq} = R_1 + R_2$$

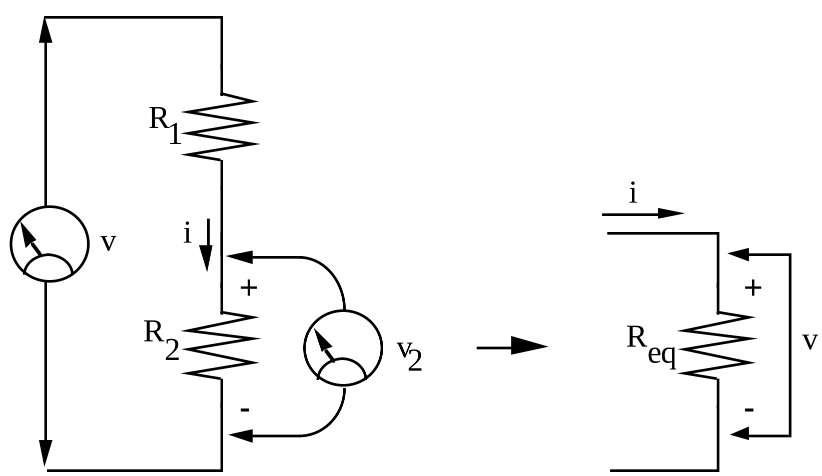
Using Ohm's Law

$$i = \frac{v}{R_{eq}} = \frac{v}{R_1 + R_2}$$

Knowing the current through each resistor we can apply Ohm's Law again to get

$$\boxed{v_2 = iR_2 = \frac{v}{R_1 + R_2} R_2 = \frac{R_2}{R_1 + R_2} v}$$

This result is known as a voltage divider.



Power
Power may also be defined, using our previous relationships, as

$$p=vi.$$

The unit of power is joules/second, or watts. 746 watts = 1 horsepower is a very common conversion.

Decibels
Decibels are units used to express power ratios

$$db = 10 \log_{10}\left(\frac{P_2}{P_1}\right)$$

or voltage and current ratios

$$db = 20 \log_{10}\left(\frac{V_2}{V_1}\right)$$

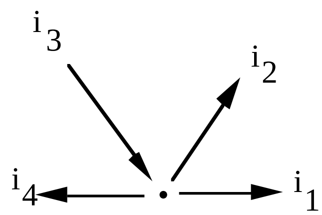
$$db = 20 \log_{10}\left(\frac{I_2}{I_1}\right)$$

Kirchoff’s Laws

Kirchoff's current Law: the algebraic sum of all currents entering or leaving a node is zero. Mathematically,

$$\boxed{\sum_{j=1}^{N} i_j = 0}$$

For a simple example: $i_3-i_1-i_2-i_4=0$ where we used the negative sign to indicate current leaving the node. IMPORTANT: It does not matter whether you use the positive or negative sign to indicate current leaving the node AS LONG AS YOU ARE CONSISTENT.

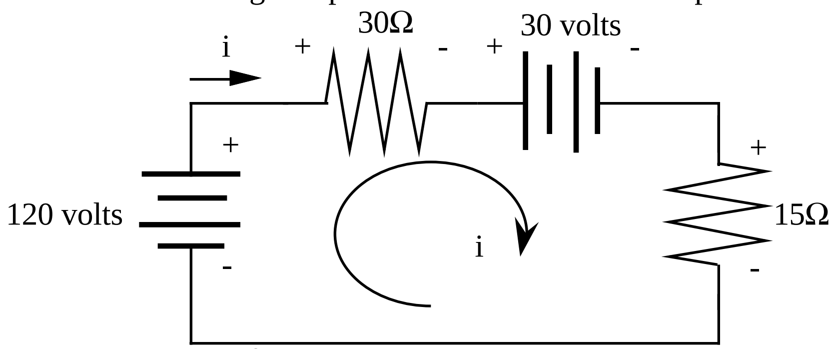


Kirchoff's voltage law states that the algebraic sum of the voltages around any closed path in a circuit is zero, i.e.

$$\boxed{\sum_{j=1}^{N} v_j = 0}$$

Simple Series Circuit

Determine the voltage drop across the resistors and the power delivered by the batteries.



Note that the sign of the voltages drops agrees with our previously defined convention. Summing the voltages around the circuit in a clockwise direction:

$$-120 + 30i + 30 + 15i = 0$$
$$45i = 90$$
$$i = 2 \text{ amps}$$

The voltage drop across the 30Ω resistor is:

$$v_{30\Omega} = iR = (2)(30) = 60 \text{ volts}$$
$$v_{15\Omega} = iR = (2)(15) = 30 \text{ volts}$$

The power supplied by the batteries is:

$$P_{120V} = vi = (120)(2) = 240 \text{ watts}$$
$$P_{30V} = vi = (30)(2) = 60 \text{ watts}$$

The power dissipated by the resistors is:

$$P_{30\Omega} = i^2R = (2)^2(30) = 120 \text{ watts}$$
$$P_{15\Omega} = i^2R = (2)^2(15) = 60 \text{ watts}$$

where we used the following formula to calculate the power:

$$\boxed{P = vi = i^2R = \frac{v^2}{R}}$$

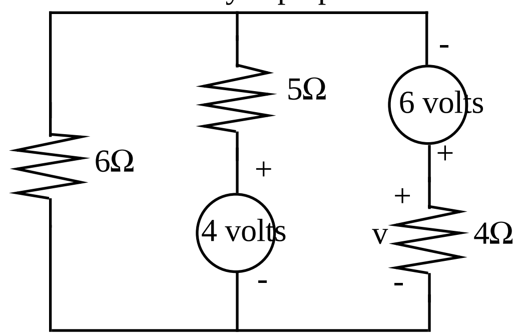
Note that the batteries use a lot of power just canceling each other out.

Superposition

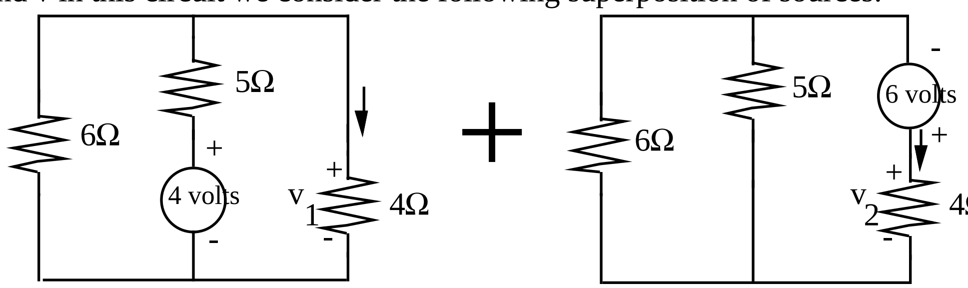
Superposition can be used to determine node voltages but is usually more complex than the loop current method discussed in the next section. To solve a problem by superposition we consider only one voltage or current source at a time (all the other voltage sources are replaced by shorts, all the other current sources are replaced by open circuits) and sum up the resulting voltages or currents. Superposition works because electrical sources are linear.

Example:

Find the voltage v across the 4Ω resistor by superposition.



To find v in this circuit we consider the following superposition of sources:



(i) (ii)

In circuit (i), the total resistance seen by the source is 5 + 4||6 = 5 + 2.4 = 7.4Ω. The total current is then i=4volts/7.4Ω=0.54 ampere. This current goes through the 5Ω resistor and, then, splits with 0.54(6/(4+6)) = 0.324 amperes going through the 4Ω resistor. The voltage drop v_1 is then v_1= iR = (0.324 amperes)(4Ω)= 1.3 volts.

In circuit (ii), the total resistance seen by the source is 4 + 5||6 = 4 + 2.73 = 6.73Ω. The total current is i=6volts/6.73Ω=0.892 ampere. This current goes through the 4Ω resistor directly below the source. The voltage drop v_2 across the 4Ω resistor is then v_2= iR = (0.892 amperes)(4Ω)= 3.57 volts.
Since the currents going through the 4Ω resistor are in the same direction they <u>add</u> giving, for the original circuit, $v = v_1 + v_2$ = 1.3 + 3.57 = 4.87 volts.

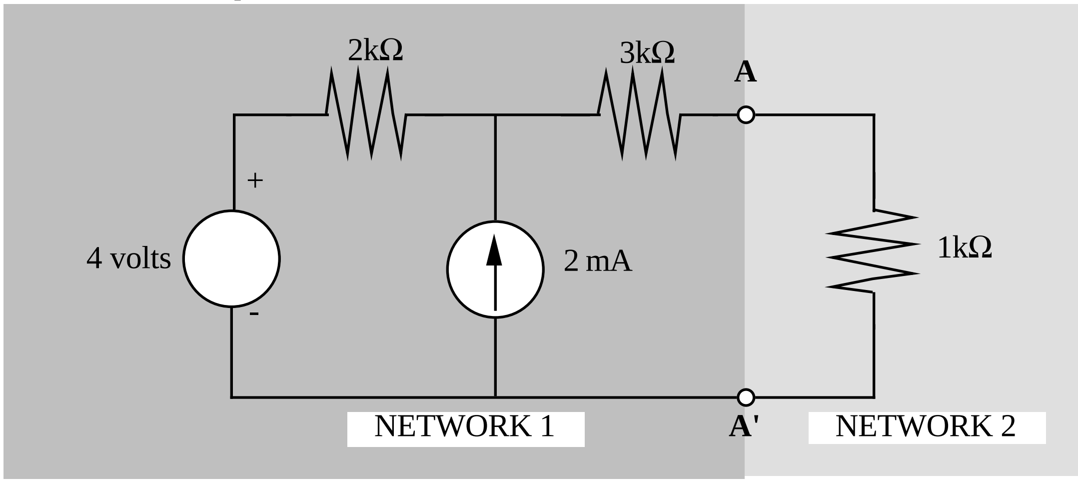
Norton's Theorem

Norton's Theorem (formal definition)

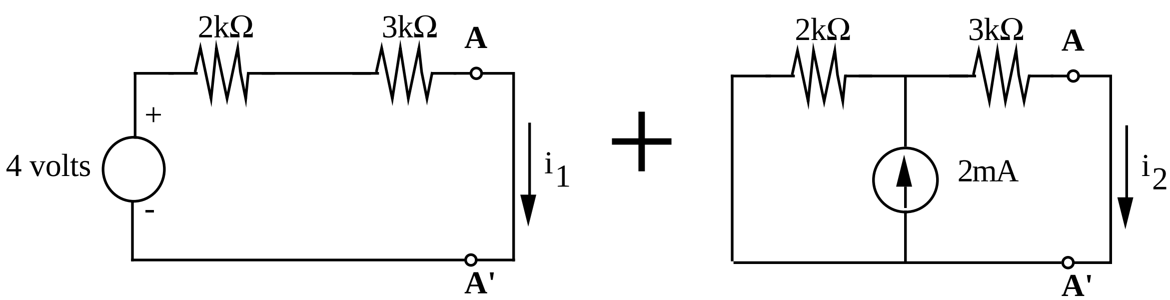
Given any linear circuit, rearrange it in the form of two networks 1 and 2 that are connected together by two zero resistance conductors. Define a current i_{sc} as the short-circuit current which would appear at the terminals A and A' of network 1 if network 2 were replaced by a short circuit. Then, all the currents and voltages in network 2 will remain unchanged if network 1 is killed (all independent voltage sources and current sources in network 1 are replaced by short circuits and open circuits, respectively) and an independent current source i_{sc} is connected, with proper polarity, in parallel with the equivalent resistance of the dead (inactive) network 1.

Example:

Find the Norton equivalent circuit of Network 1 shown below.



Replace network 2 by a short-circuit and superimpose the 4 volt and the 2 ma sources to find i_{sc}.



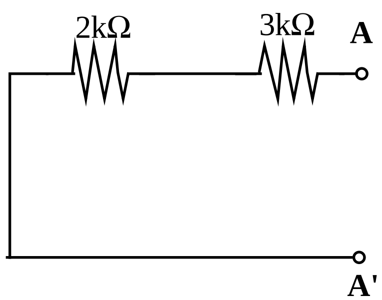
The current i_1 = 4 volts/5kΩ = 0.8mA. The current i_2 is found using the current divider relationship.

$$i_2 = 2\text{mA}\left(\frac{2\text{k}\Omega}{2\text{k}\Omega + 3\text{k}\Omega}\right) = 2\text{mA}\left(\frac{2}{5}\right) = 0.8\text{ mA}$$

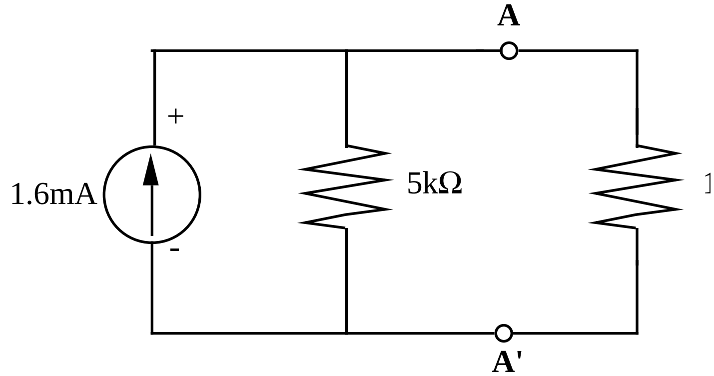
Both currents are in the same direction so $i_{sc} = i_1 + i_2$ = 0.8mA + 0.8mA = 1.6mA.

Shorting out the voltage sources and opening the current sources yields:

Shorting out the voltage sources and opening the current sources yields:



The terminal resistance $R_T = 2k\Omega + 3k\Omega = 5k\Omega$
The final Norton equivalent circuit is then:

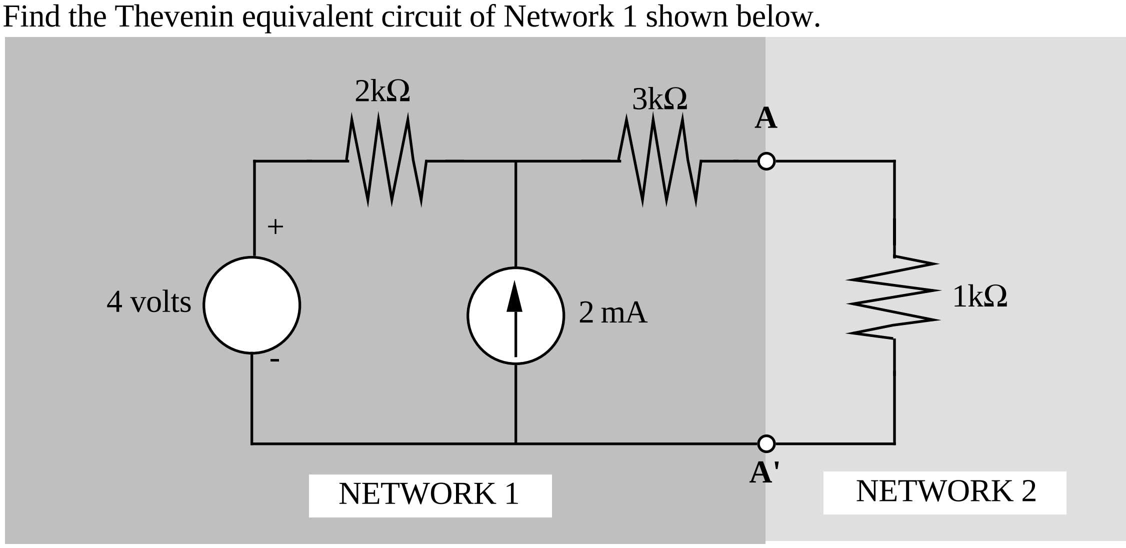


which is certainly easier to analyze than the original circuit with multiple sources.

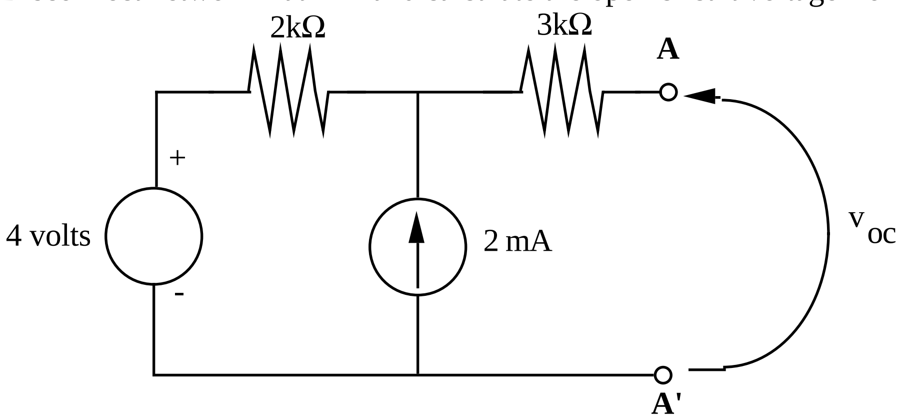
Thevenin's Theorem

Thevenin's Theorem (formal definition):
Given any linear circuit, rearrange it in the form of two networks 1 and 2 that are connected together by two zero resistance conductors at A and A'. Define a voltage v_{oc} as the open-circuit voltage which would appear between the terminals A and A' if network 2 were disconnected so that no current is drawn from network 1. Then, all the currents and voltages in network 2 will remain unchanged if network 1 is killed (i.e., all independent voltage sources and current sources in network 1 are replaced by short circuits and open circuits, respectively) and an independent voltage source v_{oc} is connected, with proper polarity, in series with the equivalent resistance of the dead (inactive) network 1.

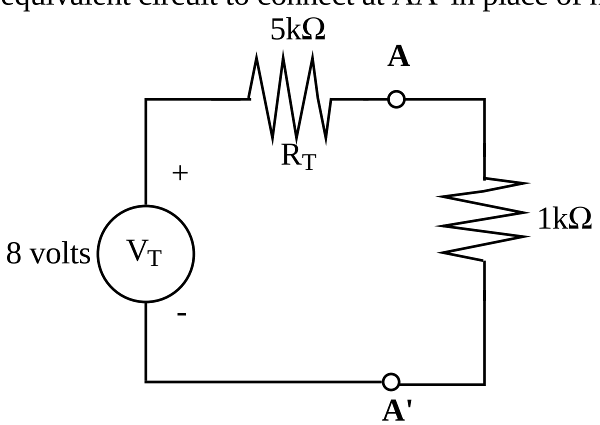
Example:
Find the Thevenin equivalent circuit of Network 1 shown below.



Disconnect network 2 at AA' and calculate the open circuit voltage from network 1.

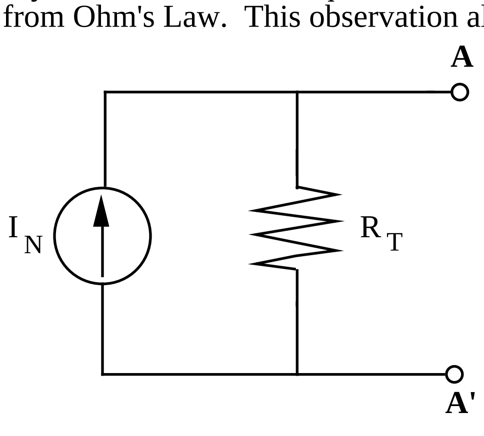


No current flows through the 3kΩ resistor. Use superposition to find v_{oc}. From the voltage source only, $v_1 = 4$ volts. From the current source only $v_2 = (2k\Omega)(2mA) = 4$ volts. The voltages are of the same polarity so $v_{oc} = 4 + 4 = 8$ volts. The equivalent resistance is found by replacing the 4 volt source by a short and the 2mA current source by an open and computing the resultant Thevenin resistance $R_T = 2k\Omega + 3\ k\Omega = 5k\Omega$. The Thevenin equivalent circuit to connect at AA' in place of network 1 is then:

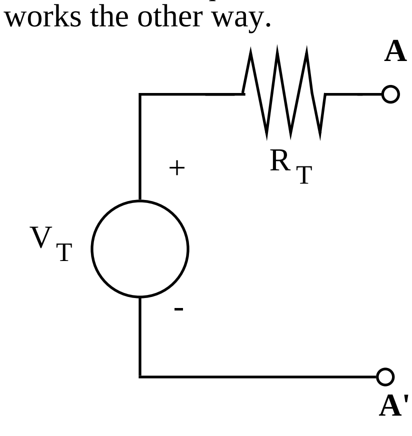


<u>Equivalence of Norton and Thevenin Equivalent Circuits</u>

If you know the Norton equivalent circuit, the Thevenin equivalent circuit is directly computable from Ohm's Law. This observation also works the other way.



Norton equivalent circuit



Thevenin equivalent circuit

Note that R_T is the same in both circuits and $V_T = I_N R_T$.

Perfect voltage and current sources have the following characteristics:

A perfect (ideal) voltage source has $R_T=0$.

A perfect (ideal) current source has $R_T=\infty$.

For a perfect (ideal) source,

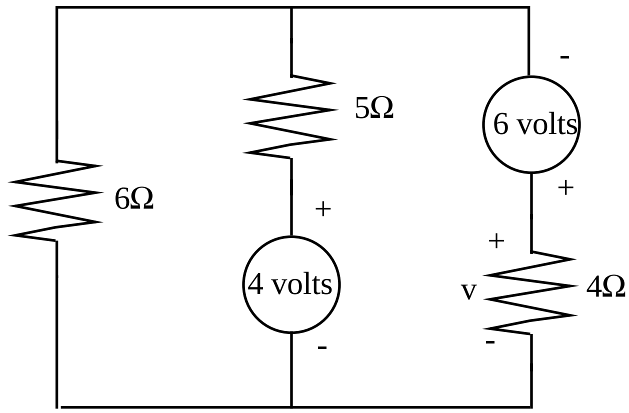
if $V_T=0$ then the source is replaced by a short, $R_T=0$.

if $I_N=0$ then the source is replaced by an open, $R_T=\infty$.

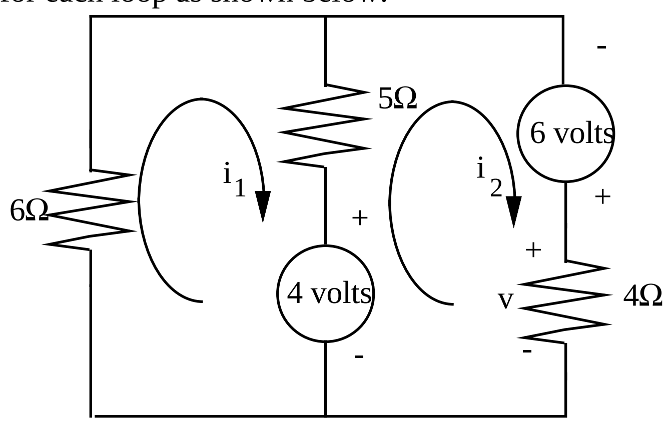
Loop Current Method

Example:

Find the voltage v across the 4Ω resistor by the loop current method.



To find v in this circuit we assume current directions for the chosen loops and write Kirchoff's voltage law for each loop as shown below:



Writing the loop equations:

loop1: $6i_1 + 5i_1 - 5i_2 + 4\text{volts} = 0$

loop2: $-4\text{volts} + 5i_2 - 5i_1 - 6\text{volts} + 4i_2 = 0$

Simplifying,

loop1: $11i_1 - 5i_2 = -4\text{volts}$

loop2: $-5i_1 + 9i_2 = +10\text{volts}$

which can be solved to give

$i_2 = +1.216$ amperes and $i_1 = 0.19$ amperes.

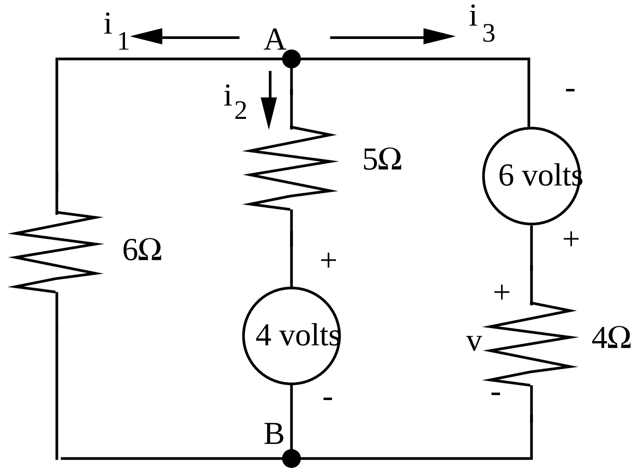
The voltage v across the 4Ω resistor is then

$v = i_2(4\Omega) = (1.216)(4) = 4.864$ volts

Node Voltage Method

Example:

Find the voltage v across the 4Ω resistor by the node voltage method.



To find v in this circuit we assume that the node B is ground (0 volts). Typically, the ground is associated with the negative side of a voltage source or other voltage reference. In this case, ground is the negative side of the voltage drop v. The only other node in the circuit is A and we will define the voltage at A to be V_A. Using the node voltage method we apply Kirchoff's current law to node A.

At A: $i_1 + i_2 + i_3 = 0$

where

$V_A = i_1(6\Omega)$, or $\quad i_1 = \frac{V_A}{6}$

$V_A = i_2(5\Omega) + 4$ volts, or $\quad i_2 = \frac{V_A - 4}{5}$

$V_A = i_3(4\Omega) - 6$ volts, or $\quad i_3 = \frac{V_A + 6}{4}$

Substituting these results into Kirchoff's current law and solving for V_A:

$$\frac{V_A}{6} + \frac{V_A - 4}{5} + \frac{V_A + 6}{4} = 0$$

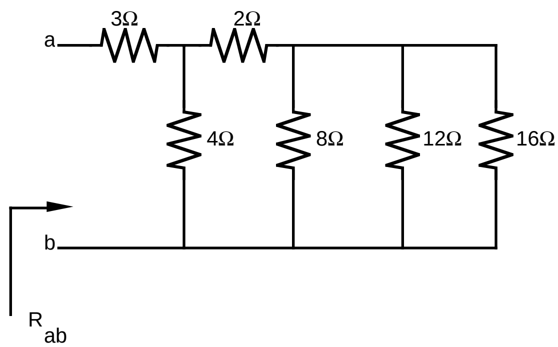
$V_A = -1.135$ volts.

Then, $i_3 = (-1.135+6)/4 = 1.216$ amperes and $v = i_3(4\Omega) = (1.216 \text{ amperes})(4\ \Omega) = 4.864$ volts.

Notice that all three examples give the same answer. You may judge whether one is particularly easier or faster than the others.

CIRCUITS 1

The equivalent resistance R_{ab} is closest to
(a) 2 ohms
(b) 4 ohms
(c) 6 ohms
(d) 8 ohms
(e) 10 ohms



Solution:
R_{eq} for the 8Ω, 12Ω and 16Ω resistors in parallel is

$$R_{eq} = \frac{1}{\frac{1}{8} + \frac{1}{12} + \frac{1}{16}} = 3.68\Omega$$

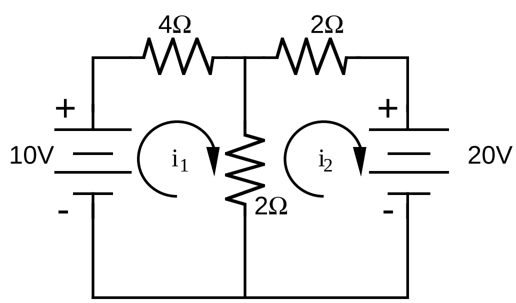
and

$$R_{ab} = 3 + 4||(2+3.68) = 3 + \frac{4\times(2+3.68)}{4+(2+3.68)} = 3 + 2.35 = 5.35\Omega$$

The correct answer is (c).

CIRCUITS 4

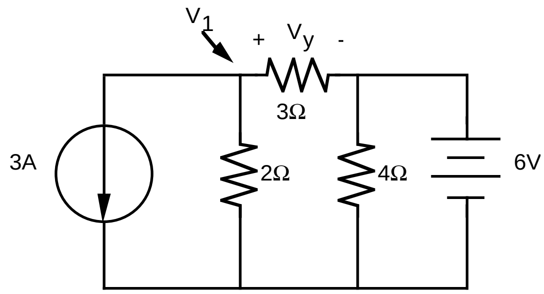
The power supplied by the 10 volt source is
(a) 12 watts
(b) 0 watts
(c) -12 watts
(d) 16 watts
(e) -16 watts



Solution:
Call the clockwise loop currents i_1 and i_2 as shown in the drawing above. Use KCL to obtain two equations in two unknowns
$6i_1 - 2i_2 = 10$
$-2i_1 + 4i_2 = -20$
Multiplying the first equation by two gives
$12i_1 - 4i_2 = 20$
and adding the last two equations we get the solution that $i_1=0$.
$P_{10\ volt\ source} = i_1(10\ volts) = 0$
The correct answer is (b).

CIRCUITS 5

The voltage V_y is closest to
(a) 0 volts
(b) 3.6 volts
(c) -1.2 volts
(d) 7.2 volts
(e) -7.2 volts



Solution:
Call the voltage drop (from top to bottom) across the 2Ω resistor V_1. Using KCL to sum the currents at the node pointed to by V_1 in the above drawing gives the following expression

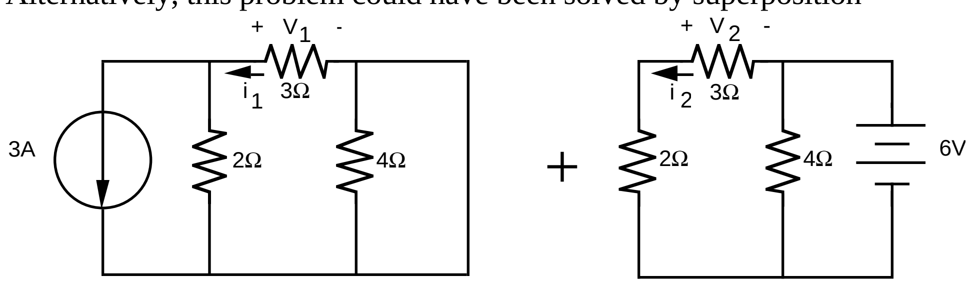
$$\frac{V_1}{2} + \frac{V_1 - 6}{3} = -3$$

Note that we used the + sign for currents coming out of the node. Solving for V_y gives V:

$$V_y = V_1 - 6 = -7.2 \text{ volts}$$

or V_1= -1.2 volts. The correct answer is (e).

Alternatively, this problem could have been solved by superposition



Since the 4Ω resistor is shorted out by the 6 volt source in the circuit on the left we can solve for the loop current as

$$i_1 = \frac{2}{2+3}(3A) = 1.2 \text{ A}$$

The voltage across the 3Ω resistor is then

$$V_1 = -\left(\frac{6}{5}A\right)(3\Omega) = -\frac{18}{5} \text{ A}$$

Examing the right hand circuit we can solve for the current i_2 as

$$i_2 = \frac{6 \text{ V}}{5\Omega} = \frac{6}{5} \text{ A}$$

The voltage across the 3Ω resistor is then given by

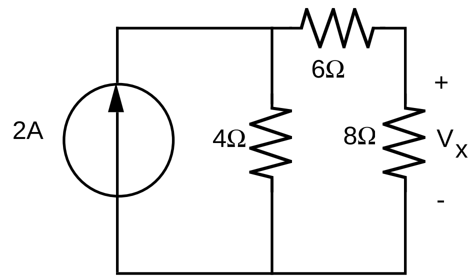
$$V_2 = -\left(\frac{6}{5}A\right) 3\Omega = -\frac{18}{5} \text{ volts}$$

Vy is then the sum of the voltages across the 3Ω resistor, i.e.

$$V_y = V_1 + V_2 = -\frac{18}{5} - \frac{18}{5} = -\frac{36}{5} = -7.2 \text{ volts}$$

CIRCUITS 2

The voltage V_x is closest to
(a) 16 volts
(b) 8 volts
(c) 3.55 volts
(d) 6.42 volts
(e) 4.65 volts



Solution:
Using current division the current through the 8Ω resistor is

$$i_{8\Omega} = 2 \times \frac{4}{4 + (6+8)} = \frac{8}{18} \text{ Amps}$$

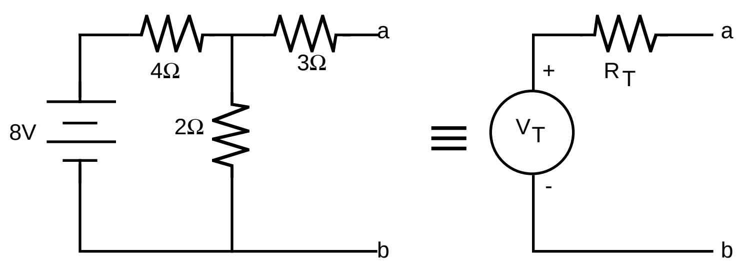
The voltage across the 8Ω resistor is then given by Ohm's Law as

$$V_x = i_{8\Omega} \times 8\Omega = \frac{8}{18} \times 8 = \frac{64}{18} = 3.55 \text{ Volts}$$

The correct answer is (c).

CIRCUITS 10

The Thevenin equivalent at terminals a-b is closest to
(a) V_T=5.33 volts, R_T=5Ω
(b) V_T=1.84 volts, R_T=4.33Ω
(c) V_T=1.84 volts, R_T=5Ω
(d) V_T=5.33 volts, R_T=4.33Ω
(e) V_T=2.67 volts, R_T=4.33Ω



Solution:
V_T is the open-circuit voltage from terminals a-b. The open circuit voltage at a-b is given by the voltage divider relationship

$$V_T = 8 \times \frac{2}{2+4} = 2.67 \text{ Volts}$$

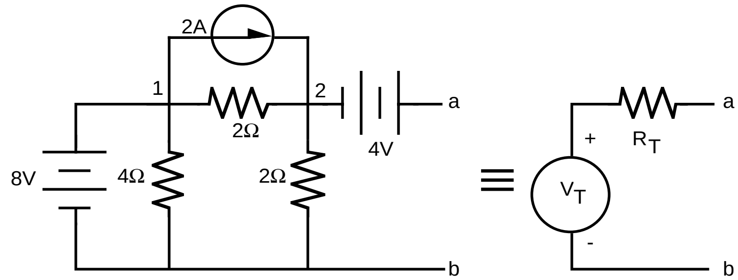
Note that the 3Ω resistor does enter this relationship since no current flows through it if terminals a-b are open. R_T is the equivalent resistance with all the sources replaced by their equivalent impedances. For a voltage source this is zero, a short, placing the 4Ω and 2Ω resistors in parallel and their resultant in series with the 3Ω resistor to give

$$R_T = 3 + \frac{2 \times 4}{2 + 4} = 3 + \frac{8}{6} = 4.33\ \Omega$$

The correct answer is (e).

CIRCUITS 11

The Thevenin equivalent at terminals a-b is closest to
(a) V_T=10 volts, R_T=1.5 Ω
(b) V_T=12 volts, R_T=1.0 Ω
(c) V_T=12 volts, R_T=1.5 Ω
(d) V_T=10 volts, R_T=1.0 Ω
(e) V_T=0 volts, R_T=1.0 Ω



Solution:
Since this is a more complex circuit than the previous example we must find V_T indirectly by first finding the voltage V_2 at node 2. Using KCL at node 2 gives the relationship for V_2 as

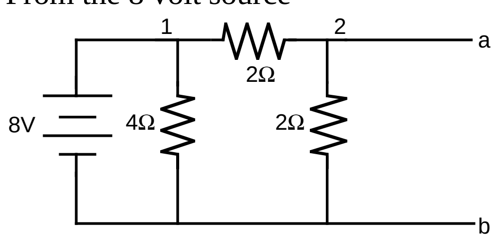
$$\frac{V_2}{2} + \frac{V_2 - 8}{2} - 2 = 0$$

where currents leaving the node are positive. Note that V_2-8 is the voltage across the resistor between nodes 1 and 2 since the voltage at node 1 must be 8 volts. Solving for V_2, V_2=6 volts. Using KVL across the 2Ω output resistor and the 4V voltage source we can get the voltage V_{ab} as $V_{ab}=V_2 + 4 = 6 + 4 = 10$ Volts.
R_T is relatively easy to determine. Replacing all of the sources by their equivalent impedances, the voltage sources are replaced by shorts and the current source is replaced by an open. The 4Ω resistor is shorted out by the 8V voltage source leaving only two 2Ω resistors in parallel. R_T is

$$R_T = 2\|2 = \frac{2 \times 2}{2 + 2} = 1\Omega$$

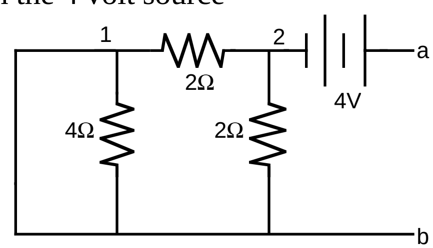
The correct answer is (d).
V_T could also be determined by superposition; however, there will be three components to V_T due to the fact that three sources are present. These contributions are shown below:

From the 8 volt source



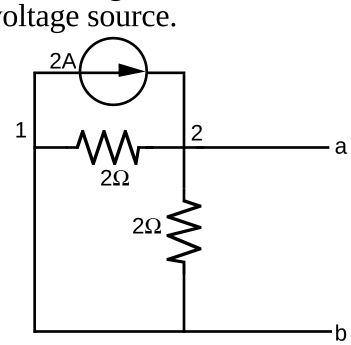
V_{ab}=+4 volts

From the 4 volt source

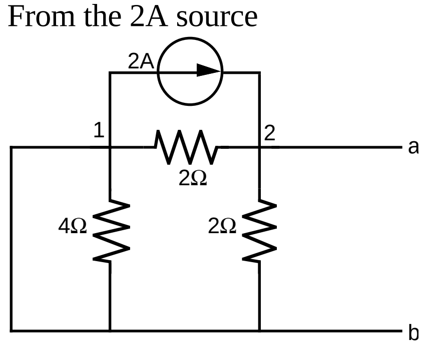


V_{ab}= +4 volts

Redrawing the original circuit to explicitly show the 2Ω resistors in parallel as a result of the shorting out of the 4Ω resistor by the 8V voltage source.



From the 2A source



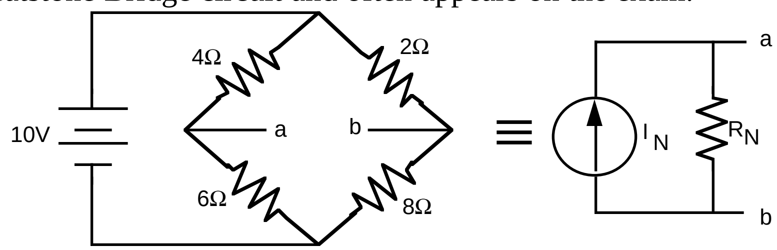
V_{ab}=2A × 1Ω = +2 volts

The resulting V_{ab} is then the sum of all the voltage contributions, i.e.
V_{ab}=4+4+2 = 10 volts as before.

CIRCUITS 12 This is theWheatstone Bridge circuit and often appears on the exam.

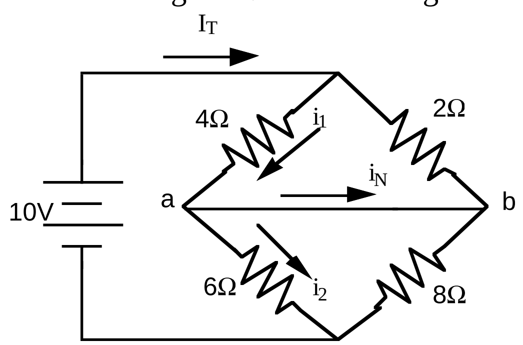
The Norton equivalent at terminals a-b is closest to

(a) I_N=0.2 amps, R_N=4 Ω
(b) I_N=0.2 amps, R_N=5 Ω
(c) I_N=-0.5 amps, R_N=4 Ω
(d) I_N=0.5 amps, R_N=5 Ω
(e) I_N=-0.5 amps, R_N=2 Ω



Solution:

The Norton current I_N is defined as the current between terminals a and b when terminals a and b are shorted together. The resulting circuit looks like a series combination of 4Ω||2Ω and 6Ω||8Ω.



The total current I_T supplied by the 10 volt source is then

$$I_T = \frac{10\text{ volts}}{\frac{4\times2}{4+2}+\frac{8\times6}{8+6}} = \frac{10\text{ volts}}{\frac{8}{6}\Omega+\frac{48}{14}\Omega} = 2.1\text{Amps}$$

Using the current divider relationship the current i_1 through the 4Ω resistor and the current i_2 through the 6Ω resistor can be calculated as

$$i_1 = 2.1\text{ Amps}\times\frac{2}{4+2} = 0.7\text{ Amps}$$

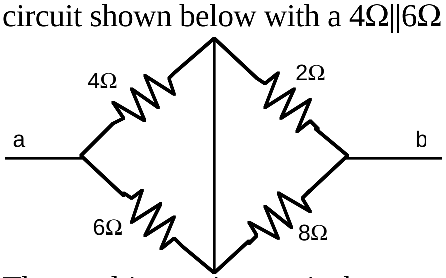
and

$$i_2 = 2.1\text{ Amps}\times\frac{8}{6+8} = 1.2\text{ Amps}$$

The Norton current I_N can then be found by applying KCL to node a

$i_N = i_1 - i_2$= 0.7 Amps - 1.2 Amps = -0.5 Amps

R_N is relatively easy to determine. Replacing the voltage source by a short we are now facing the circuit shown below with a 4Ω||6Ω combination in series with the 2Ω||8Ω combination.



The resulting resistance is then

$$R_N = \frac{4\times6}{4+6}+\frac{8\times2}{8+2} = 4.0\Omega$$

The correct answer is (c).

I_N could alsobe determined by finding the Thevenin equivalent and converting it to a Norton equivalent. The 4Ω and 6Ω resistors form a voltage divider at terminal a. The voltage at terminal a is then

$$V_a = \frac{6}{4+6}(10\text{ volts}) = 6\text{ volts}$$

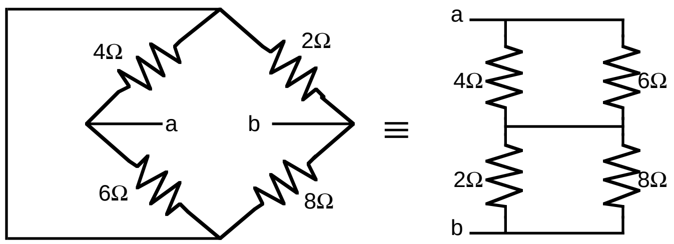
The voltage at terminal b can be found in a similar manner

$$V_b = \frac{8}{2+8}(10\text{ volts}) = 8\text{ volts}$$

The Thevenin voltage is the voltage difference between a and b.

$$V_T = V_a - V_b = 6 - 8 = -2\text{ volts}$$

The Thevenin resistance R_T is found by shorting the voltage source and computing the resistance between terminals a and b as shown below

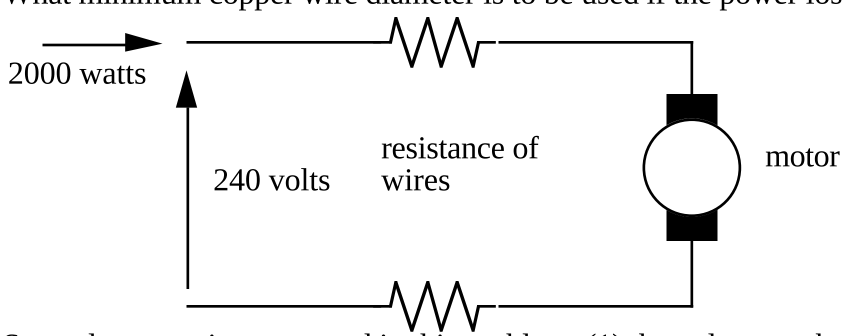


$$R_T = \frac{4\times6}{4+6} + \frac{2\times8}{2+8} = 2.4 + 1.6 = 4.0\Omega$$

Note that R_T=R_N and that V_T and I_N satisify Ohm’s Law

$$I_N = \frac{V_T}{R_T} = \frac{-2\text{ volts}}{4\Omega} = -0.5\text{ Amps}$$

Problem 47-3 A 240 volt motor requiring 2000 watts is located 1 km from a power source. What minimum copper wire diameter is to be used if the power loss is to be kept less than 5%?



Several assumptions are used in this problem: (1) the voltage at the line input is 240 volts, not at the motor; (2) that the 2000 watts are required by the motor, not at the input.

The power loss due to the resistors is 0.05×2000=100 watts. The total power consumed by the circuit is then $P = VI = (240\text{ volts})\, I = 2100\text{ watts}$

Solving for I gives I=8.75 amps. We can now use this current to find the wire resistance.

$P = I^2R = (8.75)^2R = 100\text{ watts}$ or R=1.306Ω. Remember that this is the total resistance of the wire. Calculating the wire resistance using the total wire distance of 2km we can solve for the diameter of the wire.

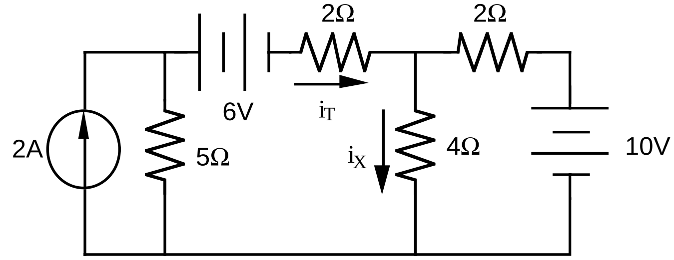
$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi\left(\frac{D}{2}\right)^2} = \frac{\left(0.6788\times10^{-6}\,\frac{\Omega\text{-in}^2}{\text{in}}\right)\left(2\text{km}\times3281\frac{\text{ft}}{\text{km}}\times12\frac{\text{in}}{\text{ft}}\right)}{\pi\left(\frac{D}{2}\right)^2} = 1.306\Omega$$

or D=0.228 inches

CIRCUITS 13

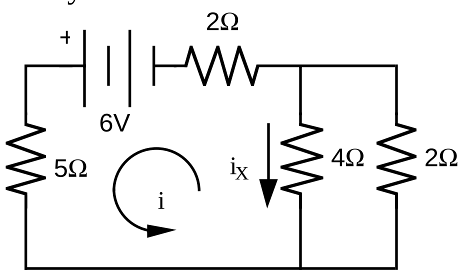
When solving for i_X using superposition, the contribution due to the 6V source is

(a) -0.24 amps
(b) 0.24 amps
(c) 0 amps
(d) 0.72 amps
(e) -0.72 amps



Solution:

To find the contribution from the 6V source, we replace the 2 amp source by an open circuit and the 10 volt source by a short. Redrawing the circuit to show the 6V source circuit a little more clearly



The total current i due to the 6V source is given as

$$i = \frac{6 \text{ volts}}{5 + 4||2 + 2} = \frac{6 \text{ volts}}{5 + \frac{4\times2}{4+2} + 2} = \frac{18}{25} = 0.72 \text{ amps}$$

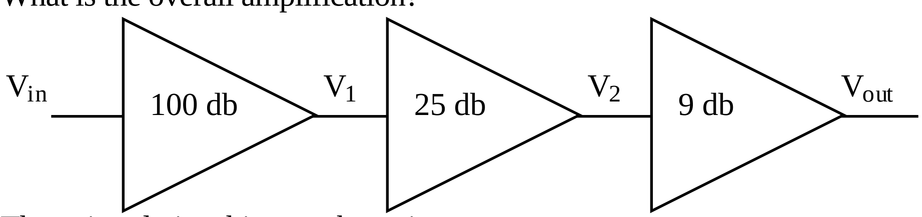
The fraction of this current flowing through the 4Ω resistor is given by a current divider

$$i_X = -\frac{2}{2+4}(0.72 \text{ amps}) = -0.24 \text{ amps}$$

Note the use of the minus sign since I defined i to be in the opposite direction to i_X.

The correct answer is (a).

Problem 47-15 Three cascaded amplifier stages have amplifications of 100 db, 25db and 9db. What is the overall amplification?



The gain relationships can be written as

$$100 \text{ db} = 20 \log\left(\frac{V_1}{V_{in}}\right) = 20 \log V_1 - 20 \log V_{in}$$

$$25 \text{ db} = 20 \log\left(\frac{V_2}{V_1}\right) = 20 \log V_2 - 20 \log V_1$$

$$9 \text{ db} = 20 \log\left(\frac{V_{out}}{V_2}\right) = 20 \log V_{out} - 20 \log V_2$$

and can be combined by adding the right and left hand sides of the above expressions

$$134 \text{ db} = 20 \log\left(\frac{V_{out}}{V_{in}}\right) = 20 \log V_{out} - 20 \log V_{in}$$