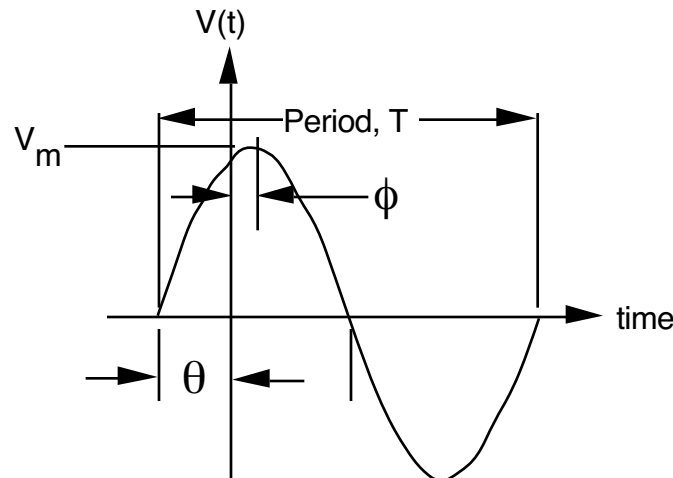


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AC voltage



The maximum value of an ac waveform is its amplitude V_m . This is also called its peak value. The offset of the zero crossing from $t=0$ is called the phase angle θ . The mathematical expression of the voltage waveform shown above is $y(t)=V_m\sin(\omega t+\theta)$. In general, electrical engineers will use the cosine rather than the sine function to express ac functions. The reason for this has to do with working with complex numbers. Expressing as a cosine, the above waveform could be written as $y(t)=V_m\sin(\omega t+\theta) = V_m\cos(\omega t+\theta -90^\circ) = V_m\cos(\omega t-\phi)$.

RMS Voltages and Currents

The effective, or RMS, value of a waveform is that voltage or current which has the same heating effect in a resistor as a dc voltage or current of the same value.

The effective values of voltage and current are given by

$$P = \frac{1}{T} \int_0^T i^2 R \, dt = R \left(\frac{1}{T} \int_0^T i^2 \, dt \right) = R I_{\text{RMS}}^2$$

where $i^2 R$ is the instantaneous power and the term

$$\frac{1}{T} \int_0^T i^2 \, dt$$

is the definition of a RMS current. The appropriate definitions of RMS voltage and current are then

$$I_{\text{RMS}} = I_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T i^2 \, dt}$$

and

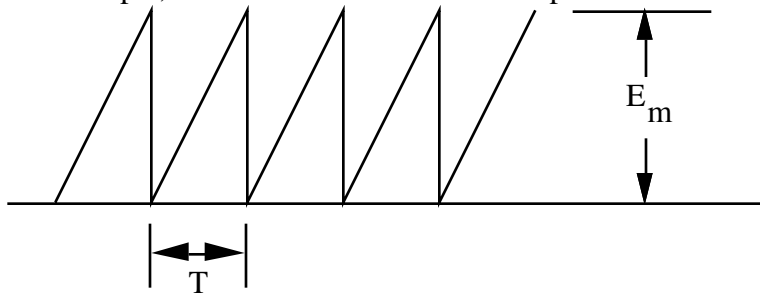
$$E_{\text{RMS}} = E_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T v^2 \, dt}$$

For sinusoidal signals these expressions are simply evaluated to give

$$E_{\text{RMS}} = \frac{E_m}{\sqrt{2}}, \quad I_{\text{RMS}} = \frac{I_m}{\sqrt{2}}$$

where I_m and E_m are the maximum, or peak, values of the voltage and current respectively. For any other waveform the relationship between the peak and RMS quantities is different and must be evaluated using the definitions.

For example, for the sawtooth waveform of period T shown in the diagram below



$$E_{RMS} = \sqrt{\frac{1}{T} \int_0^T \left(\frac{E_m t}{T}\right)^2 dt} = \sqrt{\frac{1}{T} \left(\frac{E_m}{T}\right)^2 \left[\frac{T^3}{3} - \frac{0^3}{3}\right]} = \sqrt{\frac{1}{T} \frac{E_m^2}{T^2} \frac{T^3}{3}} = \frac{E_m}{\sqrt{3}}$$

The effective value of a current which is composed of any number of sinusoidal components of different frequencies can be determined from the following expression

$$I_{RMS} = \sqrt{I_{1,RMS}^2 + I_{2,RMS}^2 + \dots + I_{N,RMS}^2}$$

A similar expression can be written for voltage waveforms. It is very important that because of the way RMS voltages and currents are defined a DC voltage or current is EXACTLY its RMS value.

Example:

If $v(t)$ is the time varying sinusoidal waveform given below

$$v(t) = 2\cos(1000t + 0.1\pi) - \sin(1000t - 0.15\pi)$$

what is the RMS value of $v(t)$?

The first part in solving this problem is to convert both sinusoids to cosines so that we can combine them.

$$v(t) = 2\cos(1000t + 18^\circ) + \cos(1000t - 90^\circ - 27^\circ)$$

$$v(t) = 2\cos(1000t)\cos(18^\circ) - 2\sin(1000t)\sin(18^\circ) + \cos(1000t)\cos(117^\circ) + \sin(1000t)\sin(117^\circ)$$

$$v(t) = [2\cos(18^\circ) + \cos(117^\circ)]\cos(1000t) + [-2\sin(18^\circ) + \sin(117^\circ)]\sin(1000t)$$

The phase factors were also converted to degrees to make the problem simpler. In the second expression the -90° term accounts for the sine function and the negative sign preceding the term in the original equation. Expanding the terms

$$v(t) = 2\cos(1000t)\cos(18^\circ) - 2\sin(1000t)\sin(18^\circ) + \cos(1000t)\cos(117^\circ) - \sin(1000t)\sin(117^\circ)$$

$$v(t) = [2\cos(18^\circ) + \cos(117^\circ)]\cos(1000t) + [-2\sin(18^\circ) + \sin(117^\circ)]\sin(1000t)$$

$$v(t) = 1.4481\cos(1000t) + 0.2730\sin(1000t)$$

This is a tricky problem in that the rms value of each term cannot be simply added together or even combined using the rule for combining the rms values of sinusoids at different frequencies. The general form of the combined sinusoid is

$$A\cos(\omega t + \phi)$$

which, when expanded, becomes

$$A\cos(\omega t + \phi) = A\cos(\omega t)\cos(\phi) - A\sin(\omega t)\sin(\phi)$$

Equating this expression with that for $v(t)$ given above we get

$$A\cos(\phi) = 1.4481$$

and

$$-A\sin(\phi) = 0.2730$$

which can be solved simultaneously to give

$A = 1.4736$ and $\phi = -10.6752^\circ$. The rms value of $v(t)$ is then

$$V_{\text{RMS}} = \frac{1.4736}{\sqrt{2}} = 1.0420$$

NOTE: This problem can be more conveniently solved using phasors:

$$\hat{v} = 2\angle 18^\circ + 1\angle -117^\circ = 1.9021 + j0.6180 - 0.4540 - j0.8910$$

$$\hat{v} = 1.4481 - j0.2730 = 1.4736\angle -10.6763^\circ$$

The RMS voltage is the magnitude of this vector divided by the square root of 2, i.e.

$$V_{\text{RMS}} = \frac{1.4736}{\sqrt{2}} = 1.0420$$

Example:

$$v(t) = 2\cos(1000t) + \sin(500t + 0.2\pi)$$

This example is distinctly different from the previous example in that the terms are at different frequencies. There is no longer any need to combine the sinusoids to add the voltages. Since both waveforms are sinusoids the RMS voltages of the first and second terms are, respectively,

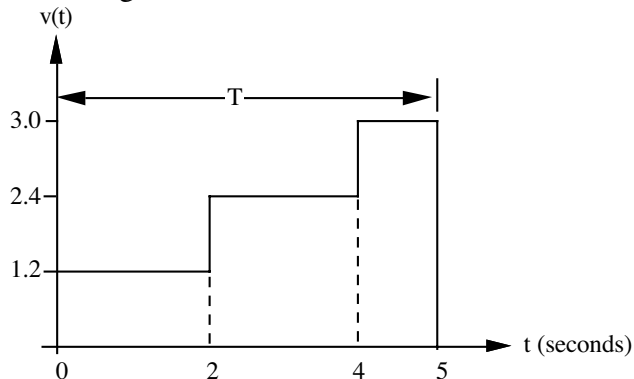
$$V_{1,\text{RMS}} = \frac{2}{\sqrt{2}}, \quad V_{2,\text{RMS}} = \frac{1}{\sqrt{2}}$$

and the total RMS voltage is

$$V_{\text{RMS}} = \sqrt{\left(\frac{2}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1.58 \text{ volts}$$

Example:

Occasionally we are given an unusual voltage waveform which requires one to use the definition of RMS. Digital waveforms such as that shown below are one such case.



Although looking very complicated the RMS value of $v(t)$ is easily computed using the definition

$$V_{\text{RMS}} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$$

$$V_{\text{RMS}} = \sqrt{\frac{1}{5} \{ (1.2)^2 \times 2 + (2.4)^2 \times 2 + (3.0)^2 \times 1 \}} = 2.16 \text{ volts RMS}$$

If this signal is applied across a 10Ω resistor, the average power is

$$P = \frac{V_{\text{RMS}}^2}{R} = \frac{(2.16)^2}{10\Omega} = 0.466 \text{ watts}$$

How much energy is dissipated in the 10Ω resistor in 25 seconds?

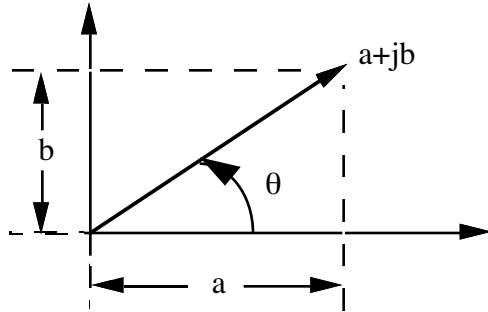
$$\text{Energy} = 0.466 \frac{\text{joules}}{\text{sec}} \times 25 \text{ seconds} = 11.65 \text{ joules}$$

Complex numbers

The ability to convert complex numbers from a rectangular format ($a+jb$) to a polar form $Ae^{j\theta}$ is essential to describing the behavior of ac electrical networks. This conversion is provided by Euler's identity which states that

$$a + jb = \sqrt{a^2 + b^2} \angle \tan^{-1}\left(\frac{b}{a}\right)$$

and can be readily understood by the diagram shown below.



where

$$a + jb = \sqrt{a^2 + b^2} \angle \tan^{-1}\left(\frac{b}{a}\right)$$

and

$$\tan\theta = \frac{b}{a}$$

The rectangular form is $a+jb$; the polar (or phasor) form is $c\angle\theta$ where

$$c = \sqrt{a^2 + b^2}$$

Electrical engineers use the Euler identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$

for complex numbers to simplify the expression and manipulation of time dependent voltages and currents. Specifically, a time-dependent waveform

$$v(t) = A\cos\omega t$$

can be written using the Euler identity as

$$v(t) = \text{Re}\{Ae^{j\omega t}\} = \text{Re}\{A\cos\omega t + jA\sin\omega t\}$$

where Re denotes "the real part of". This converts the "real" magnitude into a "real" and an "imaginary" complex quantity. Since the original $v(t)$ was real, the operator Re is used to select only the "real" part of the complex quantity. This can be extended to cases where A is a complex number, i.e. $A = Be^{j\phi}$, to give

$$v(t) = \text{Re}\{Ae^{j\omega t}\} = \text{Re}\{Be^{j\phi}e^{j\omega t}\} = \text{Re}\{Be^{j(\phi+\omega t)}\} = B\cos(\omega t + \phi)$$

This relationship is usually abbreviated as

$$v(t) \leftrightarrow Be^{j\phi}$$

where the $e^{j\omega t}$ is assumed.

The real (as opposed to complex) time-dependent waveform can be written as $v \cos(\omega t + \theta) = \text{Re}\{ve^{j\omega t + \theta}\}$ where Re denotes the real part of the rectangular form obtained using the Euler identity. A complex voltage $V(t) = V_m \cos(\omega t + \theta)$ can be written in phasor form as

$$\hat{V} = |V|e^{j\theta} = V_me^{j\theta}$$

where the vertical bars denote the magnitude of V and $e^{j\omega t}$ is assumed.

The advantages of phasor notation will be shown in the following section in which ac problems will be easily solved using Ohm's Law and complex currents, voltages and impedances expressed in phasor form.

AC circuits, phasors, impedance

The real voltage (or any other type of waveform) is the real part of the phasor voltage

$$v = \operatorname{Re}\{\hat{v}e^{j\omega t}\}, \text{ etc.}$$

	instantaneous	phasor
R:	$v = iR$	$\hat{v} = R\hat{i}$
L:	$v = L \frac{di}{dt}$	$\hat{v} = j\omega L \hat{i}$
C:	$i = C \frac{dv}{dt}$	$\hat{i} = j\omega C \hat{v}$

where $\omega = 2\pi f$ radians/second and f is in Hertz.

$$\hat{v} = R\hat{i} + j\omega L\hat{i} + \frac{\hat{i}}{j\omega C} = \left(R + j\omega L + \frac{1}{j\omega C}\right)\hat{i}$$

The input impedance is then given by

$$\hat{Z} = \frac{\hat{v}}{\hat{i}} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

The circuit is in series resonance when

$$\omega L - \frac{1}{\omega C} = 0$$

The frequency for which this occurs is the resonant frequency ω_0 and is given by

$$\omega_0 L - \frac{1}{\omega_0 C} = 0$$

Therefore,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

and

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

The input admittance is simply the reciprocal of the impedance

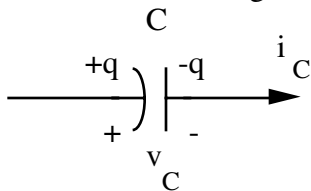
$$\hat{Y} = \frac{\hat{i}}{\hat{v}} = \frac{1}{\hat{Z}}$$

Capacitors

A capacitor can be simply defined by the relationship

$$q = Cv$$

where C is the capacitance in farads, q is the charge on the plates and v is the voltage between the plates; however, just like an inductor the voltage current relationship for a capacitor cannot be written without using integrals or derivatives.



Using the above definitions we can write the "Ohm's Law" for a capacitor as

$$i_C = \frac{dq}{dt} = C \frac{dv}{dt}$$

or, in integral form, as

$$v(t) = v(t_0) + \frac{1}{C} \int_{t_0}^t i_C(t) dt$$

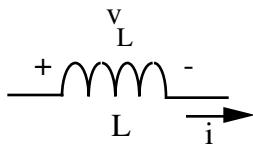
The energy stored in the capacitor can be computed as

$$W_C = \int_{t_0}^t v(t)i(t) dt = \int_{t_0}^t v \left(C \frac{dv}{dt} \right) dt = \int_{v(t_0)}^{v(t)} C v dv$$

which is usually written in the more useful form

$$W_C = \frac{1}{2} C [v^2(t) - v^2(t_0)] \text{ joules}$$

Inductance



An inductor, unlike a simple resistor, requires a differential or integration relation to describe the relationship between voltage and current. For the inductor, this form of "Ohm's Law" is

$$v_L = L \frac{di}{dt}$$

or, in integral form,

$$i(t) = i(t_0) + \frac{1}{L} \int_{t_0}^t v_L(t) dt$$

The energy stored in the inductor is

$$W_L = \int_{t_0}^t p(t) dt = \int_{t_0}^t \left(L \frac{di}{dt} \right) (i dt) = \int_{i(t_0)}^{i(t)} L i di$$

which is more commonly written in the more useful form

$$W_L = \frac{1}{2} L [i^2(t) - i^2(t_0)] \text{ joules}$$

Combining inductances and capacitances

Inductors and capacitors obey rules very similar to resistors for combining. Remember that you CANNOT combine inductors AND capacitors. You can only combine like components.

For inductors in series:

$$L_{\text{equivalent}} = \sum_{n=1}^N L_n$$

For inductors in parallel:

$$L_{\text{equivalent}} = \frac{1}{\sum_{n=1}^N \frac{1}{L_n}}$$

For capacitors in series:

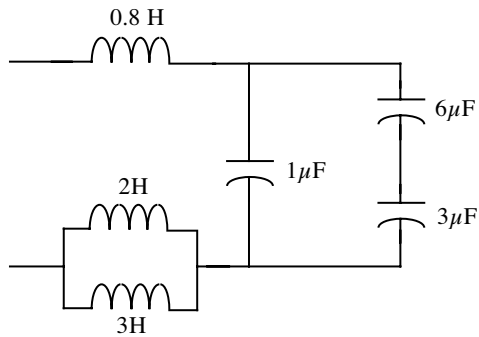
$$C_{\text{equivalent}} = \frac{1}{\sum_{n=1}^N \frac{1}{C_n}}$$

For capacitors in parallel:

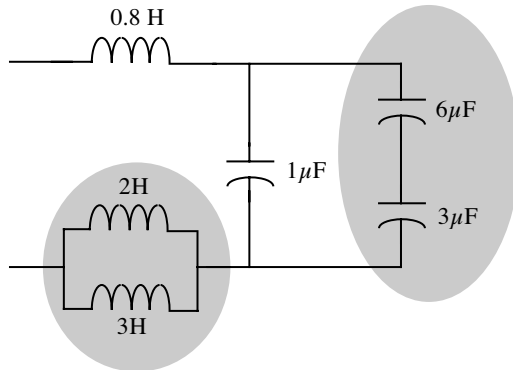
$$C_{\text{equivalent}} = \sum_{n=1}^N C_n$$

Example:

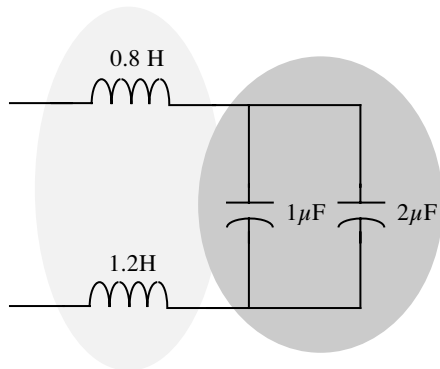
Simplify the complex network of capacitors and inductors shown below.



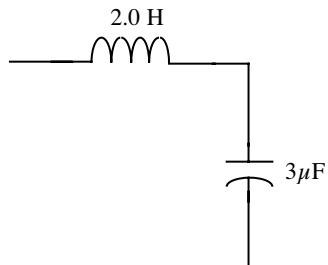
which can be reduced using the above expressions



Combining the two series capacitors and the two parallel inductances:



Where the parallel capacitances and series inductors can again be combined to give:

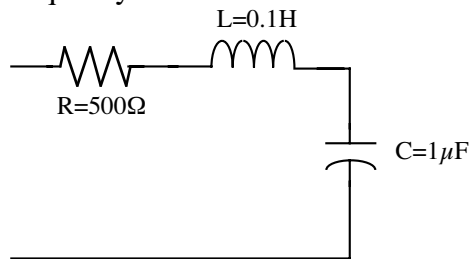


The series combination of the inductances in this last circuit may not be possible for certain applications.

Series Circuits

Example:

If the frequency is $\omega=1000$ radians/second, determine the input impedance and the resonant frequency



$$\hat{Z} = 500 + j(10^3)(0.1) - j \frac{1}{(10^3)(10^{-6})} = 500 + j100 - j1000$$

$$\hat{Z} = 500 - j900 = 1029 \angle -60.9^\circ$$

$$|\hat{Z}| = 1029 \Omega$$

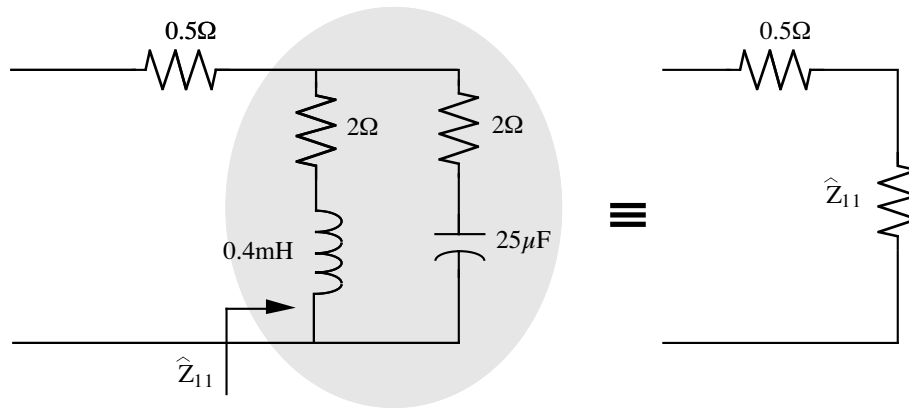
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(10^{-1})(10^{-6})}} = 3162 \frac{\text{radians}}{\text{second}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(10^{-1})(10^{-6})}} = 503 \text{ Hz}$$

Parallel Circuits

Example:

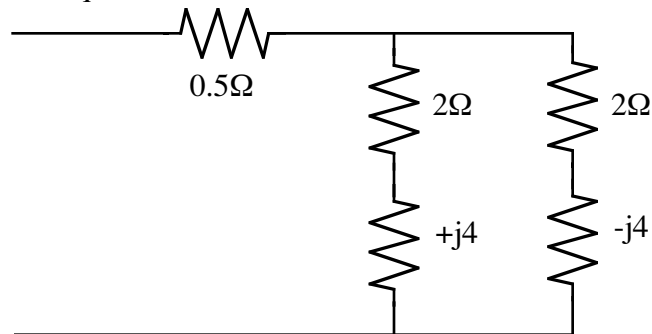
At a frequency of $\omega=10^4$ radians/second, determine the input impedance of the circuit shown below.



$$j\omega L = j(10^4)(0.4 \times 10^{-3}) = j4 \Omega$$

$$\frac{1}{j\omega C} = \frac{1}{j(10^4)(25 \times 10^{-6})} = \frac{1}{j0.25} = -j4 \Omega$$

The equivalent circuit is then



This is essentially two complex impedances in parallel with an equivalent parallel impedance given by

$$\hat{Z}_{11} = \frac{\hat{Z}_1 \times \hat{Z}_2}{\hat{Z}_1 + \hat{Z}_2} = \frac{(2+j4)(2-j4)}{(2+j4) + (2-j4)} = \frac{4+16}{4} = 5 \Omega$$

The input impedance of this circuit is the parallel impedance added to the 0.5Ω series resistance, i.e.

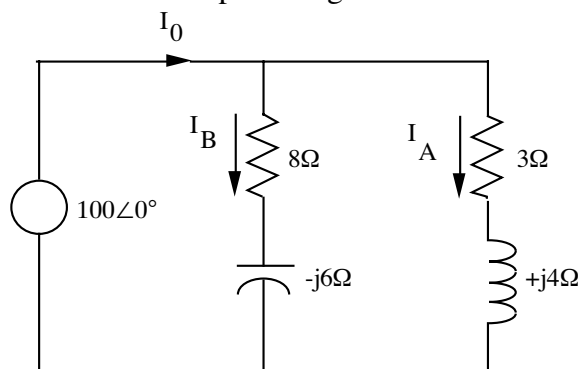
$$\hat{Z}_{IN} = 0.5 + \hat{Z}_{11} = 0.5 + 5 = 5.5 \Omega$$

Since the input impedance is real, this circuit is at resonance. The input admittance is

$$\hat{Y}_{IN} = \frac{1}{\hat{Z}_{IN}} = \frac{1}{5.5} = 0.182 \text{ mhos}$$

Example:

Find I_A and I_B as real currents in the circuit shown below where the source voltage is specified to be 100 volts at a phase angle of 0°



Using the Euler identity $Ve^{j(\omega t + \theta)} = V\cos(\omega t + \theta) + jV\sin(\omega t + \theta)$

$$\hat{I}_A = \frac{\hat{V}}{\hat{Z}} = \frac{100\angle 0^\circ}{3+j4} = \frac{100\angle 0^\circ}{5\angle 53.2^\circ} = 20\angle -53.2^\circ$$

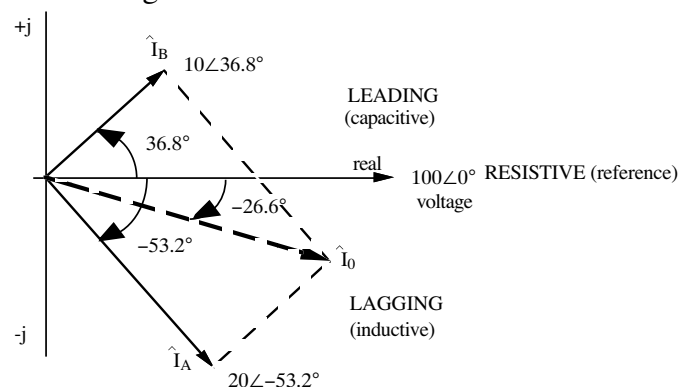
$$\hat{I}_B = \frac{\hat{V}}{\hat{Z}} = \frac{100\angle 0^\circ}{8-j6} = \frac{100\angle 0^\circ}{10\angle -36.8^\circ} = 10\angle +36.8^\circ$$

$$\hat{I}_0 = \hat{I}_A + \hat{I}_B = 20\angle -53.2^\circ + 10\angle +36.8^\circ$$

These currents must be combined vectorially as shown in the diagram below:

$$\begin{aligned} &= 20(e^{-j53.2^\circ}) + 10(e^{+j36.8^\circ}) \\ &= 20(\cos 53.2^\circ - j\sin 53.2^\circ) + 10(\cos 36.8^\circ + j\sin 36.8^\circ) \\ &= 20\cos 53.2^\circ - j20\sin 53.2^\circ + 10\cos 36.8^\circ + j10\sin 36.8^\circ \\ &= 11.98 - j16.01 + 8 + j5.99 \\ &= 19.98 - j10.02 \\ &= 22.35\angle -26.6^\circ \end{aligned}$$

Phasor diagram:



Recalling that $Re\{e^{j\omega t}\} = Re\{\cos\omega t + j\sin\omega t\} = \cos\omega t$, we can write the time dependent voltages in the circuit as

$$v(t) = 100 \cos(\omega t + 0^\circ)$$

$$i_A(t) = 20 \cos(\omega t - 53.2^\circ)$$

$$i_B(t) = 10 \cos(\omega t + 36.8^\circ)$$

$$i_0(t) = 22.35 \cos(\omega t - 26.6^\circ)$$

where the voltages and currents are now instantaneous.

CIRCUITS 14

Given: $3\cos(10t) - 4\sin(10t-45^\circ) + X(t) = 0$ $X(t)$ is

- (a) $6.4\cos(10t)$
- (b) $1.0\cos(10t-135^\circ)$
- (c) $6.4\cos(10t+154^\circ)$
- (d) $1.0\cos(10t+135^\circ)$
- (e) $6.4\cos(10t-154^\circ)$

Solve the problem using phasors. Before using the Euler identity to convert the expression into phasor form, we must use the identity $\sin\theta = \cos(\theta-90^\circ)$ to convert the expression $4\sin(10t-45^\circ) = 4\cos(10t-45^\circ-90^\circ) = 4\cos(10t-135^\circ)$. Substituting this into the given equation:

$$3\cos(10t) - 4\cos(10t-135^\circ) + X(t) = 0$$

Converting to phasor form:

$$3\angle 0^\circ - 4\angle -135^\circ + X = 0$$

Solving for X and converting back to real:

$$X = -3\angle 0^\circ + 4\angle -135^\circ = -3 + (-2.828 - j2.828) = -5.828 - j2.828 = 6.478\angle -154^\circ$$

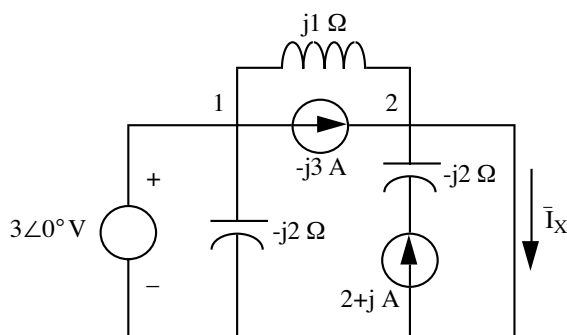
$$X = \text{Re}\{X\} = \text{Re}\{6.478\angle -154^\circ\} = 6.478\cos(10t-154^\circ)$$

The correct answer is (e).

CIRCUITS 21

\bar{I}_X is

- (a) $2.7 - j6.2$ amps
- (b) $2.7 + j6.2$ amps
- (c) $2 + j3$ amps
- (d) $2 - j3$ amps
- (e) $5.4\angle -68^\circ$



Solution:

The trick of this problem is to notice the short from node 2 to the circuit common (the voltage reference). Because of this short the voltage at node 2 is zero and the voltage at node 1 (and across the $j1 \Omega$ inductor) is 3 volts.

Using KCL at node 2

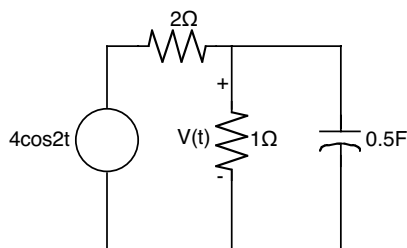
$$\bar{I}_X = (2+j) - j3 + \frac{3}{j} = 2 - j5 = 5.4\angle -68^\circ$$

The correct answer is (e).

CIRCUITS 19

$V(t)$ is

- (a) $1.13\cos(2t+33.6^\circ)$
- (b) $2.24\cos(2t-78^\circ)$
- (c) $1.13\cos(2t-33.6^\circ)$
- (d) $2.24\cos(2t+78^\circ)$
- (e) $2.60\cos(2t)$



Solution:

Replacing all circuit elements the impedances of the 1Ω resistor and the $0.5F$ capacitor can be combined using the rule for impedances in parallel as

$$\hat{Z} = \frac{(1)(-j)}{(1) + (-j)} = \frac{-j}{1-j} = 0.5 - j0.5 = 0.707\angle -45^\circ$$

This impedance Z is in series with the 2Ω resistor and forms a complex voltage divider. The voltage V across Z is then given by

$$\hat{V} = 4 \left(\frac{0.5 - j0.5}{2 + 0.5 - j0.5} \right) = 4 \left(\frac{0.5 - j0.5}{2.5 - j0.5} \right) = 4(0.231 - j0.154) = 4(0.277\angle -33.6^\circ) = 1.108\angle -33.6^\circ$$

and using the Euler identity to convert this answer to a waveform

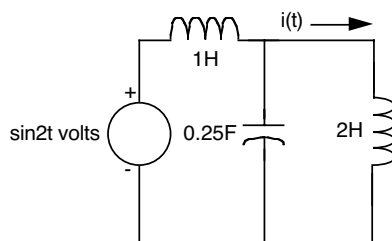
$$V = \text{Re}\{V\} = \text{Re}\{1.108\angle -33.6^\circ\} = 1.108\cos(2t-33.6^\circ)$$

The correct answer is (c).

CIRCUITS 20

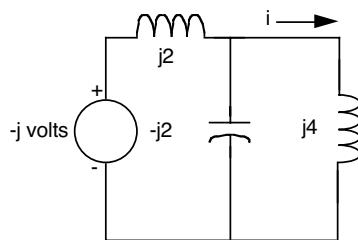
$i(t)$ is

- (a) $-0.5\cos(2t)$ amps
- (b) $0.5\cos(2t)$ amps
- (c) $\sin(2t)$ amps
- (d) $0.5\cos(2t) + 0.5\sin(2t)$
- (e) $\cos(2t)$



Solution:

Re-draw the circuit using complex impedances and phasors and solve for i using Ohm's Law. Note that we used the fact that $\omega=2$ to compute the impedances.



The impedance of the overall circuit as seen by the voltage source is

$$\hat{Z} = j2 + \frac{(-j2)(j4)}{-j2 + j4} = j2 + \frac{(-j2)(j4)}{-j2 + j4} = j2 + (-j4) = -j2$$

The total current i_{TOT} is given by Ohm's Law

$$\hat{i}_{TOT} = (-j)/(-j2) = 0.5 \text{ amp}$$

The current through the inductor is found using a current divider

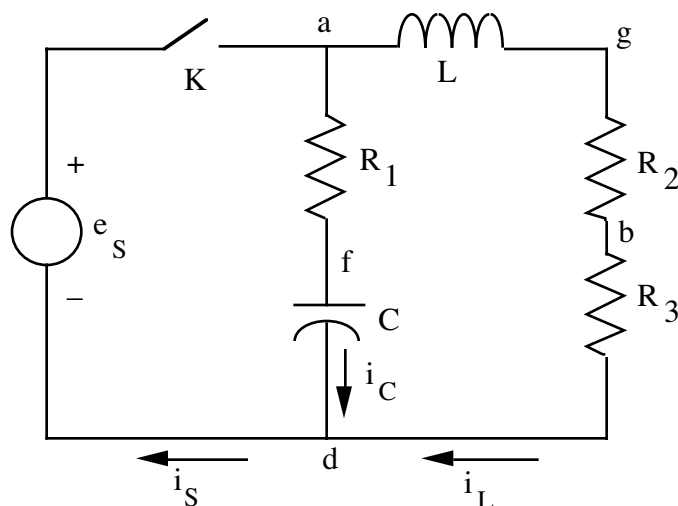
$$\hat{i}_{IND} = \frac{-j2}{-j2 + j4} \hat{i}_{TOT} = \frac{-j2}{-j2 + j4} (0.5 \text{ amp}) = -0.5 \text{ amp}$$

Using the Euler identity to convert this answer to a waveform

$$I = \text{Re}\{I\} = \text{Re}\{-0.5\angle 0^\circ\} = -0.5\cos(2t)$$

The correct answer is (a).

EXAMPLE PROBLEM:



You are given that $e_s(t) = 30 + 40\sin(500t) + 40\sin(1000t)$, $L = 10$ millihenries, $C = 200$ microfarads, $R_1 = 10$ ohms, $R_2 = 5.0$ ohms and $R_3 = 5.0$ ohms

For questions 15-20 assume that switch K has been closed for a sufficient time for steady-state conditions to be established in the circuit and answer questions 15-20 based on that assumption.

15. A high impedance instrument with a D'Arsonval movement and calibrated to read volts is connected between points f and b. The indication on the instrument is most nearly:

- (A) 3.2 volts
- (B) 13 volts
- (C) 15 volts
- (D) 18 volts
- (E) 30 volts

A D'Arsonval meter movement reads the RMS value of the signal so the voltmeter reading will be the RMS voltage. The high impedance of the meter means that the meter will not affect the circuit (i.e. change the currents or voltages) in any way.

Since there are three frequency components to e_s we must compute the voltage component from each frequency separately and use the mean square formula to get the final result. The dc component of e_s is 30 volts; the signal component at $\omega = 500$ radians/second has a peak value of 40 volts and an rms value of $40/\sqrt{2} = 28.28$ volts. The signal component at $\omega = 1000$ radians/second also has a peak value of 40 volts and a RMS value of 28.28 volts. Since the meter and all our calculations are in RMS volts, we will use these RMS values throughout the problem.

For dc, the capacitor charges to the dc potential giving 30 volts on the capacitor after the switch has been closed for a long time.

The dc voltage at point b is given by the voltage divider formula. The inductor acts as a short for dc.

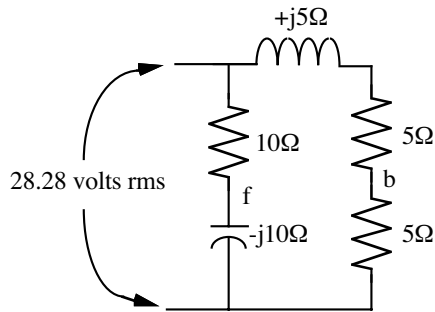
$$V_b = 30 \left(\frac{5}{5+5} \right) = 15 \text{ volts}$$

Therefore, $V_{fb} = V_f - V_b = 30 - 15 = 15$ volts.

For $\omega = 500$ ($f = \omega/2\pi = 500/2\pi = 79.6\text{Hz}$) we must use a complex voltage divider where the impedances of the capacitor and the inductor are calculated (for $\omega = 500$) as shown below

$$\frac{1}{j\omega C} = \frac{1}{j(500)(200 \times 10^{-6})} = -j10$$

$$j\omega L = j(500)(0.01) = +j5$$



Using two voltage divider relationships:

$$V_f = \left(\frac{-j10}{10 - j10} \right) 28.2 = \frac{-j28.2}{1 - j} = 14.1 - j14.1$$

Remember the inductor is NOT a short to ac.

$$V_b = \left(\frac{5}{5 + 5 + j5} \right) 28.2 = \left(\frac{5}{10 + j5} \right) 28.2 = \frac{28.2}{2 + j} = 11.28 - j5.64$$

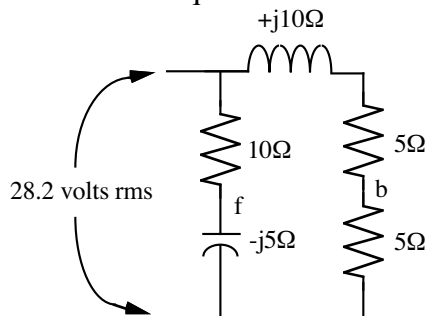
$$V_{fb} = V_f - V_b = 2.82 - j8.46 = 8.92 \angle -71.6^\circ \text{ volts rms}$$

Now, we must repeat this procedure for $\omega=1000$ radians/second realizing that the impedances have changed:

$$\frac{1}{j\omega C} = \frac{1}{j(1000)(200 \times 10^{-6})} = -j5$$

$$j\omega L = j(1000)(0.01) = +j10$$

with the new equivalent circuit becoming:



Note that although the input voltage did not change, the input frequency has changed for this calculation.

$$V_f = \left(\frac{-j5}{10 - j5} \right) 28.2 = \left(\frac{-j}{2 - j} \right) 28.2 \quad V_f = \left(\frac{-j5}{10 - j5} \right) 28.2 = \left(\frac{-j}{2 - j} \right) 28.2 = 5.64 - j11.28$$

$$V_b = \left(\frac{5}{5 + 5 + j10} \right) 28.2 = \left(\frac{5}{10 + j10} \right) 28.2 = \left(\frac{1}{2 + j2} \right) 28.2 = 7.05 - j7.05$$

$$V_{fb} = V_f - V_b = (5.64 - j11.28) - (7.05 - j7.05) = -1.41 - j4.23 = 4.45 \angle -108^\circ \text{ volts rms}$$

Now we must combine the three rms frequency components to get what the meter will read. Finding their effective RMS value

$$V_{fb, \text{ total rms}} = \sqrt{(15)^2 + (8.92)^2 + (4.45)^2} = 18.0 \text{ volts rms}$$

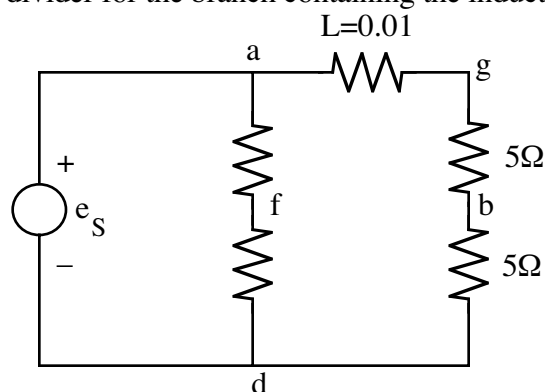
The answer is (D).

If all questions were like this one there would probably not be enough time to do the exam.

16. A high impedance moving-iron-vane instrument with adequate frequency range to accurately determine applied voltage (and calibrated in rms volts) is connected between points b and d. The indication on the instrument is most nearly

- (A) 20 volts
- (B) 28 volts
- (C) 30 volts
- (D) 33 volts
- (E) 53 volts

This problem is easier than the previous one since V_{bd} can be calculated using a single voltage divider for the branch containing the inductor as shown in the diagram below



Just as in question #15, we must evaluate the voltage divider for dc, $\omega=500$ rad/sec and $\omega=1000$ rad/sec and then combine our results.

At dc the inductor is a short, i.e. its impedance is zero, so that

$$V_{bd, dc} = \frac{5}{5+5} 30 = 15 \text{ volts}$$

At $\omega=500$ rad/sec the impedance of the inductor is $+j5 \Omega$ and, using the voltage divider formula,

$$V_{bd, \omega=500} = \frac{5}{5+5+j5} 28.2 = \frac{5}{10+j5} 28.2 = \frac{1}{2+j} 28.2$$

$$V_{bd, \omega=500} = 11.28 - j5.64 = 12.6 \angle -26.5^\circ \text{ volts rms}$$

At $\omega=1000$ rad/sec the impedance of the inductor is $+j10 \Omega$ and again using the voltage divider formula.

$$V_{bd, \omega=1000} = \frac{5}{5+5+j10} 28.2 = \frac{1}{2+j2} 28.2 = 7.05 - j7.05 = 9.97 \angle -45^\circ \text{ volts rms}$$

To get the meter reading we must combine the different frequency rms voltages to get

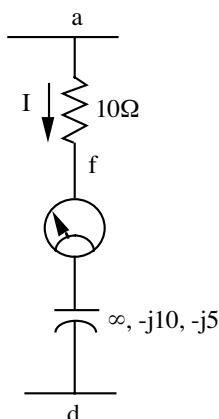
$$E_{\text{eff}} = \sqrt{(15)^2 + (12.6)^2 + (9.97)^2} = 21.98 \text{ volts rms}$$

The closest correct answer is (A)

17. An instrument with negligible impedance and calibrated to indicate rms values of current is inserted in the circuit at point f. The reading of the instrument is most nearly

- (A) 1.3 amperes
- (B) 2.4 amperes
- (C) 2.8 amperes
- (D) 3.3 amperes
- (E) 4.2 amperes

The solution to this problem is gotten by solving Ohm's Law at three different frequencies and computing the mean square sum to get the total rms current.



At dc $I=0$ since no dc can get through the capacitor.

$$I_{\omega=500} = \frac{V}{Z} = \frac{28.2}{10 - j10} = 1.41 + j1.41 = 1.99 \angle 45^\circ$$

$$I_{\omega=1000} = \frac{V}{Z} = \frac{28.2}{10 - j5} = 2.256 + j1.128 = 2.52 \angle 26.6^\circ$$

The total rms current is then:

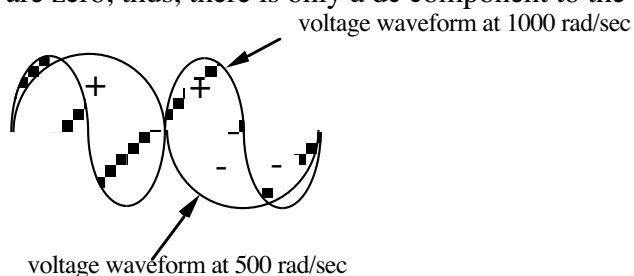
$$I_{\text{eff}} = \sqrt{(0)^2 + (1.99)^2 + (2.52)^2} = 3.2 \text{ amps}$$

The closest answer is (D).

18. Consider an instant when the fundamental component of source voltage e_s is zero and changing from negative to positive in value. The energy stored in the 200-microfarad capacitor is most nearly

- (A) 4.0×10^{-4} joules
- (B) 5.7×10^{-4} joules
- (C) 6.4×10^{-3} joules
- (D) 9.0×10^{-2} joules
- (E) 4.0×10^2 joules

This is a very unusual question with a simple answer and a lot of thought. Consider the capacitor-resistor circuit between points a and d. There is a dc voltage across the capacitor as well as ac voltages at $\omega=500$ rad/sec and $\omega=1000$ rad/sec. The two ac signals have phase constants of zero degrees relative to each other. When the voltage at $\omega=500$ is going from - to +, the voltage at $\omega=1000$ is going from - to + also. Looking at the diagram below we see that the waveform must be at the end of a complete cycle and the net energy from the ac voltages is zero since the ac voltages are zero; thus, there is only a dc component to the energy.



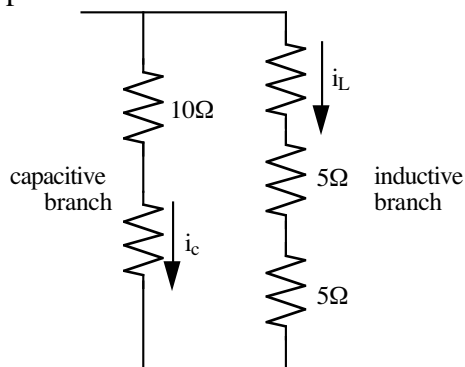
$$W = \frac{1}{2} C v^2 = \frac{1}{2} (200 \times 10^{-6}) (30)^2 = 0.09 = 9 \times 10^{-2} \text{ joules}$$

The answer is (D).

20. If the magnitudes of E , E_1 and E_2 are such that $i_C = 0.71\sin(500t+45^\circ) + 0.45\sin(1000t+26.6^\circ)$ amperes and $i_L = 2.0+0.9\sin(500t-26.6^\circ) + 0.354\sin(1000t-45^\circ)$ amperes, the power output of the source e_S is most nearly

- (A) 48 watts
- (B) 56 watts
- (C) 106 watts
- (D) 160 watts
- (E) 220 watts

In short, we are given the current through each branch of the circuit and we must find the REAL power.



In the capacitive branch, since there is no dc current through the capacitor, the power is simply the power dissipated in the resistor due to the two frequency components

$$P = \frac{1}{2}i^2R = \frac{1}{2}(0.71)^2(10) + \frac{1}{2}(0.45)^2(10) = 2.52 + 1.0125 \text{ watts}$$

In the inductive branch there is a dc component of the power. Note that the dc power is, by definition, a rms quantity and does not need a factor of 1/2.

$$P = \frac{1}{2}i^2R = (2)^2(10) + \frac{1}{2}(0.9)^2(10) + \frac{1}{2}(0.354)^2(10) = 40 + 4.05 + 0.63 \text{ watts}$$

The total power is then $P_T = 2.52 + 1.01 + 40 + 4.05 + 0.63 = 48.21$ watts. The answer is (A).