## CIRCUITS 4

The power supplied by the 10 volt source is
(a) 12 watts
(b) 0 watts
(c) -12 watts
(d) 16 watts
(e) -16 watts


Solution:
Call the clockwise loop currents $i_{1}$ and $i_{2}$ as shown in the drawing above. Use KCL to obtain two equations in two unknowns
$6 \mathrm{i}_{1}-2 \mathrm{i}_{2}=10$
$-2 \mathrm{i}+4 \mathrm{i}=-20$
Multiplying the first equation by two gives
$12 i_{1}-4 i_{2}=20$
and adding the last two equations we get the solution that $\mathrm{i}_{1}=0$.
$\mathrm{P}_{10 \text { volt source }}=\mathrm{i}_{1}(10$ volts $)=0$
The correct answer is (b).

## CIRCUITS 5

The voltage $\mathrm{V}_{\mathrm{y}}$ is closest to
(a) 0 volts
(b) 3.6 volts
(c) -1.2 volts
(d) 7.2 volts
(e) -7.2 volts


Solution:
Call the voltage drop (from top to bottom) across the $2 \Omega$ resistor $\mathrm{V}_{1}$. Using KCL to sum the currents at the node pointed to by $\mathrm{V}_{1}$ in the above drawing gives the following expression
$\underline{V_{1}}+\underline{V_{1}-6}=-3$
Note that we used the + sign for currents coming out of the node. Solving for $\mathrm{V}_{\mathrm{y}}$ gives V :
$\mathrm{V}_{\mathrm{y}}=\mathrm{V}_{1}-6=-7.2$ volts
or $\mathrm{V}_{1}=-1.2$ volts. The correct answer is (e).
Alternatively, this problem could have been solved by superposition

## CIRCUITS 11

The Thevenin equivalent at terminals $\mathrm{a}-\mathrm{b}$ is closest to
(a) $\mathrm{V}_{\mathrm{T}}=10$ volts, $\mathrm{R}_{\mathrm{T}}=1.5 \Omega$
(b) $\mathrm{V}_{\mathrm{T}}=12$ volts, $\mathrm{R}_{\mathrm{T}}=1.0 \Omega$
(c) $\mathrm{V}_{\mathrm{T}}=12$ volts, $\mathrm{R}_{\mathrm{T}}=1.5 \Omega$
(d) $\mathrm{V}_{\mathrm{T}}=10$ volts, $\mathrm{R}_{\mathrm{T}}=1.0 \Omega$
(e) $\mathrm{V}_{\mathrm{T}}=0$ volts, $\mathrm{R}_{\mathrm{T}}=1.0 \Omega$


Solution:
Since this is a more complex circuit than the previous example we must find $\mathrm{V}_{\mathrm{T}}$ indirectly by first finding the voltage $\mathrm{V}_{2}$ at node 2 . Using KCL at node 2 gives the relationship for $\mathrm{V}_{2}$ as
$\underline{\mathrm{V}_{2}}+\underline{\mathrm{V}_{2}-8}-2=0$
where currents leaving the node are positive. Note that $\mathrm{V}_{2}-8$ is the voltage across the resistor between nodes 1 and 2 since the voltage at node 1 must be 8 volts. Solving for $\mathrm{V}_{2}, \mathrm{~V}_{2}=6$ volts.
Using KVL across the $2 \Omega$ output resistor and the 4 V voltage source we can get the voltage $\mathrm{V}_{\mathrm{ab}}$ as $\mathrm{V}_{\mathrm{ab}}=\mathrm{V}_{2}+4=6+4=10$ Volts.
$\mathrm{R}_{\mathrm{T}}$ is relatively easy to determine. Replacing all of the sources by their equivalent impedances, the voltage sources are replaced by shorts and the current source is replaced by an open. The $4 \Omega$ resistor is shorted out by the 8 V voltage source leaving only two $2 \Omega$ resistors in parallel. $\mathrm{R}_{\mathrm{T}}$ is
$\mathrm{R}_{\mathrm{T}}=2 \| 2=\underline{2 \times 2}=1 \Omega$
The correct answer is (d).

## CIRCUITS 2

The voltage $\mathrm{V}_{\mathrm{x}}$ is closest to
(a) 16 volts
(b) 8 volts
(c) 3.55 volts
(d) 6.42 volts
(e) 4.65 volts


Solution:
Using current division the current through the $8 \Omega$ resistor is
$\mathrm{i}_{8 \Omega}=2 \times \frac{4}{4+(6+8)}=\frac{8}{18} \mathrm{Amps}$
The voltage across the $8 \Omega$ resistor is then given by Ohm's Law as
$\mathrm{V}_{\mathrm{x}}=\mathrm{i}_{8 \Omega} \times 8 \Omega=\frac{8}{18} \times 8=\frac{64}{18}=3.55$ Volts
The correct answer is (c).

CIRCUITS 12 This is the Wheatstone Bridge circuit and often appears on the exam.
The Norton equivalent at terminals $\mathrm{a}-\mathrm{b}$ is closest to
(a) $\mathrm{I}_{\mathrm{N}}=0.2 \mathrm{amps}, \mathrm{R}_{\mathrm{N}}=4 \Omega$
(b) $\mathrm{I}_{\mathrm{N}}=0.2 \mathrm{amps}, \mathrm{R}_{\mathrm{N}}=5 \Omega$
(c) $\mathrm{I}_{\mathrm{N}}=-0.5 \mathrm{amps}, \mathrm{R}_{\mathrm{N}}=4 \Omega$
(d) $\mathrm{I}_{\mathrm{N}}=0.5 \mathrm{amps}, \mathrm{R}_{\mathrm{N}}=5 \Omega$
(e) $\mathrm{I}_{\mathrm{N}}=-0.5 \mathrm{amps}, \mathrm{R}_{\mathrm{N}}=2 \Omega$


Solution:
The Norton current $\mathrm{I}_{\mathrm{N}}$ is defined as the current between terminals a and b when terminals a and b are shorted together. The resulting circuit looks like a series combination of $4 \Omega|\mid 2 \Omega$ and $6 \Omega| \mid 8 \Omega$.


The total current $\mathrm{I}_{\mathrm{T}}$ supplied by the 10 volt source is then
$\mathrm{I}_{\mathrm{T}}=\frac{10 \text { volts }}{\frac{4 \times 2}{\underline{8 \times 6}}+\frac{10 \text { volts }}{\frac{8}{6} \Omega+\frac{48}{14} \Omega}}=2.1 \mathrm{Amps}$
Using the current divider relationship the current $\mathrm{i}_{1}$ through the $4 \Omega$ resistor and the current $i_{2}$ through the $6 \Omega$ resistor can be calculated as
$\mathrm{i}_{1}=2.1 \mathrm{Amps} \times \frac{2}{4+2}=0.7 \mathrm{Amps}$
and
$\mathrm{i}_{2}=2.1 \mathrm{Amps} \times \frac{8}{6+8}=1.2 \mathrm{Amps}$
The Norton current $\mathrm{I}_{\mathrm{N}}$ can then be found by applying KCL to node a
$\mathrm{i}_{\mathrm{N}}=\mathrm{i}_{1}-\mathrm{i}_{2}=0.7 \mathrm{Amps}-1.2 \mathrm{Amps}=-0.5 \mathrm{Amps}$
$\mathrm{R}_{\mathrm{N}}$ is relatively easy to determine. Replacing the voltage source by a short we are now facing the circuit shown below with a $4 \Omega \| 6 \Omega$ combination in series with the $2 \Omega \| 8 \Omega$ combination.


The resulting resistance is then
$\mathrm{R}_{\mathrm{N}}=\frac{4 \times 6}{}+\frac{8 \times 2}{}=4.0 \Omega$
The correct answer is (c).

## CIRCUITS 31

The switch S closes at $\mathrm{T}=0$. The complete response for $\mathrm{i}(\mathrm{t})$ for $t>0$ is
(a) $2.5+6 \mathrm{e}^{-0.75 \mathrm{t}}$
(b) $2.5-2.5 \mathrm{e}^{-0.75 \mathrm{t}}$
(c) $2.5+2.5 \mathrm{e}^{-1.33 \mathrm{t}}$
(d) 0
(e) $2.5-2.5 \mathrm{e}^{-1.33 \mathrm{t}}$


We use KVL to write the differential equation for the circuit using the correct expression for the impedance of the inductor.
$-10+4 i+3 \frac{\mathrm{di}}{\mathrm{dt}}=0$
Re-writing this in conventional form with the sources on the right hand side of the equation
$3 \frac{\mathrm{di}}{\mathrm{dt}}+4 \mathrm{i}=10$
The dc (or homogeneous) solution is obtained by setting the derivatives equal to zero or, in this case
$4 \mathrm{i}=10$
giving $\mathrm{i}=2.5 \mathrm{amps}$.
The transient solution is always an exponential in form. Substituting $\mathrm{i}(\mathrm{t})=A \mathrm{e}^{\mathrm{kt}}$ into the differential equation and setting the source (the right hand side of the equation) equal to zero we obtain

$$
\begin{aligned}
& 3 \frac{\mathrm{di}}{\mathrm{dt}}+4 \mathrm{i}=0 \\
& \mathrm{kAe}^{\mathrm{kt}}+4 \mathrm{Ae}^{\mathrm{kt}}=0 \\
& 3 \mathrm{k}+4=0 \\
& \mathrm{k}=-4 / 3
\end{aligned}
$$

The total solution is then $\mathrm{i}(\mathrm{t})=\mathrm{i}_{\text {transient }}+\mathrm{i}_{\text {homogeneous }}=\mathrm{Ae}^{-1.33 \mathrm{t}}+2.5$
The coefficient A is solved for by using the boundary condition that $\mathrm{i}\left(0^{+}\right)=\mathrm{i}\left(0^{-}\right)=0$.
This requires that $\mathrm{i}\left(0^{+}\right)=\mathrm{A}+2.5=0$, or that $\mathrm{A}=-2.5$.
Then $\mathrm{i}(\mathrm{t})=2.5-2.5 \mathrm{e}^{-1.33 \mathrm{t}}$ and the correct answer is $(\mathrm{e})$.

## Circuits 36

$i(t)$ is

$$
\begin{array}{ll}
\text { (a) } & \mathrm{e}^{-0.6 \mathrm{t}} \\
\text { (b) } & \mathrm{e}^{-1.67 \mathrm{t}} \\
\text { (c) } & -\mathrm{e}^{-1.67 t} \\
\text { (d) } & -\mathrm{e}^{-0.6 \mathrm{t}} \\
\text { (e) } & 2-\mathrm{e}^{-1.67 t}
\end{array}
$$



Solution:
There are many ways to solve this problem but, perhaps, the easiest way is to Thevenize the left hand side of the circuit (the voltage source and the two $2 \Omega$ resistors) and replace the right hand side of the circuit (the two $8 \Omega$ resistors in parallel) by its equivalent resistance.
Thevenizing the left hand side of the circuit


$$
\mathrm{T}=\frac{2}{2+2} 10=5 \text { volts }
$$

and

$$
\mathrm{R}_{\mathrm{T}}=\frac{2 \times 2}{}=1 \Omega
$$

NOTE: This is a good technique to use to get rid of current sources in problems.
The $8 \Omega$ resistors in parallel can be replaced by a $4 \Omega$ resistor. Redrawing the original circuit and replaceing the left hand side by its Thevenin equivalent and replacing the two $8 \Omega$ resistors by a single $4 \Omega$ resistance, we get the following circuit


Since $\mathrm{V}_{\mathrm{C}}\left(0^{+}\right)=\mathrm{V}_{\mathrm{C}}\left(0^{-}\right)=0$
$\mathrm{i}_{\mathrm{C}}\left(0^{+}\right)=\frac{5 \text { volts }}{1 \Omega+4 \Omega}=1 \mathrm{amp}$
The time constant for the circuit can be directly computed as
$\tau=\mathrm{RC}=(5 \Omega)\left(\frac{1}{3} \mathrm{~F}\right)=1.67$ seconds
The solutions should be of the form
$A e^{-\frac{t}{1.67}}=\mathrm{Ae}^{0.6 t}$
The correct answer is (a).

Example afternoon problem (this is actually tougher than most afternoon problems):


You are given that $\mathrm{e}_{S}(\mathrm{t})=\mathrm{E}+\mathrm{E}_{1} \sin (500 \mathrm{t})+\mathrm{E}_{2} \sin (1000 \mathrm{t}), \mathrm{L}=10$ millihenries, $\mathrm{C}=200$ microfarads, $\mathrm{R}_{1}=10$ ohms, $\mathrm{R}_{2}=5.0$ ohms and $\mathrm{R}_{3}=5.0$ ohms in the above circuit.

For questions 1-5 assume that switch $K$ is closed at $t=0$ and answer the questions for the instant immediately after the switch is closed, i.e. for time $t=0^{+}$.

1. If $E=30 \mathrm{~V}, E_{1}=40 \mathrm{~V}$ and $E_{2}=20 \mathrm{~V}$, the current $i_{C}$ is most nearly
(A) 0.0 amperes
(B) 1.5 amperes
(C) 2.8 amperes
(D) 3.0 amperes
(E) 6.0 amperes

This problem is most easily solved by recalling the initial conditions for capacitors which require that $\mathrm{V}(0-)=\mathrm{V}(0+)$. The initial voltage on the capacitor is 0 volts so the voltage across the capacitor immediately after the switch is closed must also be 0 volts. The applied voltage $e_{S}(t=0+) \approx 30$ since $\sin (0+) \approx 0$. At $t=0+e_{S}$ appears enitrely across $R_{1}$ and the resulting current (which is equal to $i_{C}$ since $R_{1}$ and $C$ are in series) must be given by

$$
\mathrm{i}_{\mathrm{C}}=\frac{\mathrm{e}_{\mathrm{S}}}{\mathrm{R}_{1}}=\frac{30 \text { volts }}{10 \Omega}=3.0 \text { Amperes }
$$

The correct answer is (D).
2. If $E=30 \mathrm{~V}, \mathrm{E}_{1}=40 \mathrm{~V}$ and $\mathrm{E}_{2}=20 \mathrm{~V}$, the magnitude of the voltage between points $a$ and $b$ is most nearly
(A) 5.0 volts
(B) 7.5 volts
(C) 15 volts
(D) 22 volts
(E) 30 volts

The voltage being referred to is across the series combination of the inductor and $\mathrm{R}_{2}$ as shown in the diagram below.


As in question \#1 the voltage es $\left(0^{+}\right) \approx 30$ volts, $\mathrm{i}_{\mathrm{L}}\left(0^{+}\right)=\mathrm{i}_{\mathrm{L}}\left(0^{-}\right)$since the current through inductors is continuous, and $\mathrm{V}_{\mathrm{C}}\left(0^{+}\right)=\mathrm{V}_{\mathrm{C}}\left(0^{-}\right)$since the voltage across capacitors is continuous. Since there is no current flow through L at $\mathrm{t}=0^{+}$the inductor represents an open circuit. The potential at $a$ is +30 volts; the potential at $b$ is zero since it is connected to ground through $\mathrm{R}_{3}$ and no current is flowing through $\mathrm{R}_{3}$. The potential $\mathrm{v}_{\mathrm{ab}}$ is then 30 volts. The correct answer is (E)
3. If $\mathrm{E}, \mathrm{E}_{1}$ and $\mathrm{E}_{2}$ are magnitudes such that $\mathrm{i}_{\mathrm{C}}$ at $\mathrm{t}=0^{+}$is 2.0 amperes, the rate of change of the voltage between points f and d is most nearly
(A) 0.0 volts/second
(B) 20 volts/second
(C) $5 \times 10^{2}$ volts/second
(D) $5 \times 10^{3}$ volts/second
(E) $1.0 \times 10^{4}$ volts/second

This question is a lot simpler than it sounds and is a direct application of of the definition of capcitance. By definition,
$\mathrm{i}_{\mathrm{C}}=\mathrm{C} \frac{\mathrm{dv}}{\mathrm{dt}}$
which includes a direct expression of the rate of change of voltage. Evaluating this expression for $\mathrm{t}=0^{+}$,
$\mathrm{i}_{\mathrm{C}}\left(0^{+}\right)=\left.\mathrm{C} \frac{\mathrm{dv}}{\mathrm{dt}}\right|_{0^{+}}$
which can be solved for the time rate of change of voltage at $t=0^{+}$
$\left.\frac{\mathrm{dv}}{\mathrm{dt}}\right|_{\mathrm{t}=0^{+}}=\frac{\mathrm{i}_{\mathrm{C}}\left(0^{+}\right)}{\mathrm{C}}=\frac{2 \text { Amperes }}{200 \times 10^{-6} \text { Farads }}=10^{4} \frac{\text { volts }}{\text { second }}$
The closest answer is (E).
4. If $E, E_{1}$ and $E_{2}$ are magnitudes such that the voltages between points a and $g$ is 40 volts at $\mathrm{t}=0^{+}$, the rate of change of $\mathrm{i}_{\mathrm{L}}$ is most nearly
(A) $0.0 \mathrm{amps} / \mathrm{second}$
(B) $4 \times 10^{-2} \mathrm{amps} /$ second
(C) $4 \times 10^{3} \mathrm{amps} / \mathrm{second}$
(D) $2 \times 10^{4} \mathrm{amps} /$ second
(E) $4 \times 10^{4} \mathrm{amps} /$ second

The solution of this problem is almost identical to that of problem \#3 with the exception that the voltage expression must be that for an inductor, i.e.
$\mathrm{V}_{\mathrm{L}}=\mathrm{L} \frac{\mathrm{di}}{\mathrm{dt}}$
Evaluating this expression at $\mathrm{t}=0^{+}$
$\left.\frac{\mathrm{di}}{\mathrm{dt}}\right|_{\mathrm{t}=0^{+}}=\frac{\mathrm{V}_{\mathrm{L}}\left(0^{+}\right)}{\mathrm{L}}=\frac{40 \text { volts }}{10 \times 10^{-3} \text { Henrys }}=4000 \frac{\mathrm{amps}}{\text { second }}$
The answer is (C).
5. If $\mathrm{E}=30 \mathrm{~V}, \mathrm{E}_{1}=40 \mathrm{~V}$ and $\mathrm{E}_{2}=20 \mathrm{~V}$, the magnitude of the current $\mathrm{i}_{\mathrm{C}}$ is most nearly
(A) 0.0 amperes
(B) 1.5 amperes
(C) 2.8 amperes
(D) 3.0 amperes
(E) 6.0 amperes
$\mathrm{e}_{\mathrm{S}}(\mathrm{t})=\mathrm{E}+\mathrm{E}_{1} \sin (500 \mathrm{t})+\mathrm{E}_{2} \sin (1000 \mathrm{t})$
$\mathrm{i}_{\mathrm{C}}\left(0^{-}\right)=0$ and $\mathrm{i}_{\mathrm{L}}\left(0^{-}\right)=0$ since there are no voltage sources for $\mathrm{t}<0$. Using our rules for boundary conditions on inductors and capacitors $\mathrm{i}_{\mathrm{L}}\left(0^{+}\right)=\mathrm{i}_{\mathrm{L}}\left(0^{-}\right)$but
$\mathrm{i}_{\mathrm{C}}\left(0^{+}\right) \neq \mathrm{i}_{\mathrm{C}}\left(0^{-}\right)$. At $\mathrm{t}=0+\mathrm{es}\left(0^{+}\right)=30+40 \sin \left(500 \times 0^{+}\right)+20 \sin \left(1000 \times 0^{+}\right) \approx 30$ volts
since the sine of a small number is approximately zero. Since $\mathrm{V}_{\mathrm{C}}\left(0^{+}\right)=\mathrm{V}_{\mathrm{C}}\left(0^{-}\right)$ (remember that the voltage across a capacitor is continuous) the current $\mathrm{i}_{\mathrm{C}}$ through $\mathrm{R}_{1}$ at $\mathrm{t}=0^{+}$must then be given by $\mathrm{i}_{\mathrm{C}}=30$ volts $/ 10 \mathrm{ohms}=3$ amperes. The answer is (D).

## CIRCUITS 14

Given: $3 \cos (10 \mathrm{t})-4 \sin \left(10 \mathrm{t}-45^{\circ}\right)+\mathrm{X}(\mathrm{t})=0 \mathrm{X}(\mathrm{t})$ is
(a) $6.4 \cos (10 t)$
(b) $1.0 \cos \left(10 \mathrm{t}-135^{\circ}\right)$
(c) $6.4 \cos \left(10 t+154^{\circ}\right)$
(d) $1.0 \cos \left(10 t+135^{\circ}\right)$
(e) $6.4 \cos \left(10 \mathrm{t}-154^{\circ}\right)$

Solve the problem using phasors. Before using the Euler identity to convert the expression into phasor form, we must use the identity $\sin \theta=\cos \left(\theta-90^{\circ}\right)$ to convert the expression $4 \sin \left(10 t-45^{\circ}\right)=4 \cos \left(10 t-45^{\circ}-90^{\circ}\right)=+4 \cos \left(10 t-135^{\circ}\right)$. Substituting this into the given equation:
$3 \cos (10 t)-4 \cos \left(10 t-135^{\circ}\right)+X(t)=0$
Converting to phasor form:
$3 \angle 0^{\circ}-4 \angle-135^{\circ}+X=0$
Solving for $X$ and converting back to real:
$X=-3 \angle 0^{\circ}+4 \angle-135^{\circ}=-3+(-2.828-\mathrm{j} 2.828)=-5.828-\mathrm{j} 2.828=6.478 \angle-154^{\circ}$
$\mathrm{X}=\operatorname{Re}\{X\}=\operatorname{Re}\left\{6.478 \angle-154^{\circ}\right\}=6.478 \cos \left(10 \mathrm{t}-154^{\circ}\right)$
The correct answer is (e).

## CIRCUITS 21

$\overline{\mathrm{I}}_{\mathrm{X}}$ is
(a) $2.7-\mathrm{j} 6.2 \mathrm{amps}$
(b) $2.7+\mathrm{j} 6.2 \mathrm{amps}$
(c) $2+\mathrm{j} 3 \mathrm{amps}$
(d) $2-\mathrm{j} 3 \mathrm{amps}$
(e) $5.4 \angle-68^{\circ}$


Solution:
The trick of this problem is to notice the short from node 2 to the circuit common (the voltage reference). Because of this short the voltage at node 2 is zero and the voltage at node 1 (and across the $\mathrm{j} 1 \Omega$ inductor) is 3 volts.

Using KCL at node 2
$\overline{\mathrm{I}}_{\mathrm{X}}=(2+\mathrm{j})-\mathrm{j} 3+\frac{3}{\mathrm{j}}=2-\mathrm{j} 5=5.4 \angle-68^{\circ}$
The correct answer is (e).

## CIRCUITS 20

$i(t)$ is
(a) $-0.5 \cos (2 \mathrm{t}) \mathrm{amps}$
(b) $0.5 \cos (2 \mathrm{t}) \mathrm{amps}$
(c) $\sin (2 \mathrm{t}) \mathrm{amps}$
(d) $0.5 \cos (2 \mathrm{t})+0.5 \sin (2 \mathrm{t})$
(e) $\cos (2 \mathrm{t})$


Solution:
Re-draw the circuit using complex impedances and phasors and solve for i using Ohm's Law. Note that we used the fact that $\omega=2$ to compute the impedances.


The impedance of the overall circuit as seen by the voltage source is
$\widehat{Z}=j 2+\frac{(-j 2)(j 4)}{-j 2+j 4}=j 2+\frac{(-j 2)(j 4)}{-j 2+j 4}=j 2+(-j 4)=-j 2$
The total current $i_{\text {TOT }}$ is given by Ohm's Law
$\hat{\mathrm{i}}_{\text {TOT }}=(-\mathrm{j})(-\mathrm{j} 2)=0.5 \mathrm{amp}$
The current through the inductor is found using a current divider
$\hat{\mathrm{i}}_{\text {IND }}=\frac{-\mathrm{j} 2}{-\mathrm{j} 2+\mathrm{j} 4} \hat{\mathrm{i}}_{\text {TOT }}=\frac{-\mathrm{j} 2}{-\mathrm{j} 2+\mathrm{j} 4}(0.5 \mathrm{amp})=-0.5 \mathrm{amp}$
Using the Euler identity to convert this answer to a waveform
$\mathrm{I}=\operatorname{Re}\{I\}=\operatorname{Re}\left\{-0.5 \angle 0^{\circ}\right\}=-0.5 \cos (2 \mathrm{t})$
The correct answer is (a).

## CIRCUITS 19

$\mathrm{V}(\mathrm{t})$ is
(a) $1.13 \cos \left(2 \mathrm{t}+33.6^{\circ}\right)$
(b) $2.24 \cos \left(2 \mathrm{t}-78^{\circ}\right)$
(c) $1.13 \cos \left(2 t-33.6^{\circ}\right)$
(d) $2.24 \cos \left(2 t+78^{\circ}\right)$

(e) $2.60 \cos (2 \mathrm{t})$

## Solution:

Replacing all circuit elements the impedances of the $1 \Omega$ resistor and the 0.5 F capacitor can be combined using the rule for impedances in parallel as
$\widehat{Z}=\frac{(1)(-\mathrm{j})}{(1)+(-\mathrm{j})}=\frac{-\mathrm{j}}{1-\mathrm{j}}=0.5-\mathrm{j} 0.5=0.707 \angle-45^{\circ}$
This impedance Z is in series with the $2 \Omega$ resistor and forms a complex voltage divider.
The voltage V across Z is then given by
$\widehat{V}=4\left(\frac{0.5-\mathrm{j} 0.5}{2+0.5-\mathrm{j} 0.5}\right)=4\left(\frac{0.5-\mathrm{j} 0.5}{2.5-\mathrm{j} 0.5}\right)=4(0.231-\mathrm{j} 0.154)=4\left(0.277 \angle-33.6^{\circ}\right)=1.108 \angle-33.6^{\circ}$
and using the Euler identity to convert this answer to a waveform

$$
\mathrm{V}=\operatorname{Re}\{V\}=\operatorname{Re}\left\{1.108 \angle-33.6^{\circ}\right\}=1.108 \cos \left(2 \mathrm{t}-33.6^{\circ}\right)
$$

The correct answer is (c).

Problems 2 and 3 refer to the following figure.

2. What is the current, i ?
(A) -0.88 A
(B) -0.25 A
(C) 0 A
(D) 0.25 A

## Solution 2:

The input current in an op amp is so small that it is assumed to be zero.
Answer is C.
3. What is the output voltage, $\mathrm{v}_{\mathrm{o}}$ ?
(A) -7 V
(B) -6 V
(C) -1 V
(D) 6 V

## Solution 3:

This op amp circuit is a summing amplifier. Since $\mathrm{i}=0$,
$i_{f}=\frac{v_{1}}{R_{1}}+\frac{v_{2}}{R_{2}}=\frac{3 \mathrm{~V}}{8 \Omega}+\frac{2 \mathrm{~V}}{4 \Omega}=0.875 \mathrm{~A}$
$\mathrm{V}_{\mathrm{o}}=-\mathrm{i}_{\mathrm{f}} \mathrm{R}_{\mathrm{f}}=-(0.875 \mathrm{~A})(8 \Omega)=-7 \mathrm{~V}$

Answer is A.
5. Evaluate the following amplifier circuit to determine the value of resistor $\mathrm{R}_{4}$ in order to obtain a voltage gain $\left(v_{0} / v_{i}\right)$ of -120 .

(A) $25 \Omega$
(B) $23 \mathrm{k} \Omega$
(C) $24 \mathrm{k} \Omega$
(D) $25 \mathrm{k} \Omega$

## Solution:


$\mathrm{v}_{\mathrm{in}+}$ is grounded, so $\mathrm{v}_{\mathrm{in}-}$ is also a virtual ground.
$\mathrm{v}_{\text {in- }}=0$
Since $v_{\text {in- }}=0, v_{i}=i_{1} R_{1}$ and $i_{1}=v_{i} / R_{1}$.
Since $v_{\text {in }}=0, v_{x}=-i_{2} R_{2}$ and $i_{2}=-v_{x} / R_{2}$.
Similarly,
$v_{x}=-i_{3} R_{3}$
$v_{x}-v_{0}=-i_{4} R_{4}$

From Kirchhoff's current law,

$$
\begin{aligned}
& \mathrm{i}_{4}=\mathrm{i}_{2}+\mathrm{i}_{3} \\
& \frac{v_{x}-v_{o}}{R_{4}}=\frac{-v_{x}}{R_{2}}+\frac{-v_{x}}{R_{3}}
\end{aligned}
$$

Now, $v_{0}=-120 v_{i}$.
Also, $i_{1}=i_{2}$, so
$\frac{v_{i}}{R_{1}}=\frac{-v_{x}}{R_{2}}$
$v_{x}=-\left(\frac{R_{2}}{R_{1}}\right) v_{i}$
$\frac{-\left(\frac{R_{2}}{R_{1}}\right) v_{i}-\left(-120 v_{i}\right)}{R_{4}}=\frac{\left(\frac{R_{2}}{R_{1}}\right) v_{i}}{R_{2}}+\frac{\left(\frac{R_{2}}{R_{1}}\right) v_{i}}{R_{3}}$
$\frac{120\left(\frac{R_{1}}{R_{2}}\right)-1}{R_{4}}=\frac{1}{R_{2}}+\frac{1}{R_{3}}=\frac{R_{2}+R_{3}}{R_{2} R_{3}}$
$R_{4}=\frac{120\left(\frac{R_{1}}{R_{2}}\right)-1}{\frac{R_{2}+R_{3}}{R_{2} R_{3}}}=\frac{120\left(\frac{1 \times 10^{6} \Omega}{5 \times 10^{5} \Omega}\right)-1}{\frac{5 \times 10^{5} \Omega+100 \Omega}{\left(5 \times 10^{5} \Omega\right)(100 \Omega)}}=2.39 \times 10^{4} \Omega(24 \mathrm{k} \Omega)$

## Answer is C.

14. For the circuit shown below, $V_{1}=10 \sin (200 t)$ and $V_{2}=15 \sin (200 t)$. What is $\mathrm{V}_{\text {out }}$ ? The op amp is ideal with infinite gain.


ANSWER:
Any problem with a capacitor (or inductor) in it and sinusoidal voltages immediately indicates that phasors are required. This means that $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ should be represented as phasors, and $\mathrm{C}_{\mathrm{f}}$ should be replaced by an impedance. This problem is not solved very well with the formulas in the Reference Handbook. This circuit is most easily solved using the virtual short assumption ( $\mathrm{V}_{+}=\mathrm{V}_{-}$), and using KCL at the inverting input. Note that the grounding of $\mathrm{V}_{+}$then requires that $\mathrm{V}_{-}=0$. This is also called the virtual short assumption.
$+\frac{V_{2}-0}{R_{2}}+\frac{V_{1}-0}{R_{i}}-\frac{0-V_{\text {out }}}{1 / j \omega C}=0$.
Rationalizing this expression gives $+\frac{V_{2}}{R_{2}}+\frac{V_{1}}{R_{i}}+j \omega C V_{\text {out }}=0$.
Solving for $\mathrm{V}_{\text {out }}$ gives $V_{\text {out }}=-\frac{V_{2}}{j \omega C R_{2}}-\frac{V_{1}}{j \omega C R_{i}}$
It is important to recognize that all sine functions should always be converted to cosines for proper phase in the phasor expressions, i.e.
$\sin (200 t)=\cos \left(200 t-90^{\circ}\right) \leftrightarrow 1 \angle-90^{\circ}=-j$

Using the circuit parameters given,

$$
\begin{aligned}
V_{\text {out }} & =-\frac{-j 15}{j(200)\left(2 \times 10^{-6}\right)\left(0.5 \times 10^{6}\right)}-\frac{-j 10}{j(200)\left(2 \times 10^{-6}\right)\left(0.75 \times 10^{6}\right)} \\
& =\frac{15}{200}+\frac{10}{300}=\frac{3}{40}+\frac{1}{30}
\end{aligned}
$$

The answer is then $V_{\text {out }}(t)=\left(\frac{3}{40}+\frac{1}{30}\right) \cos (200 t)$

## COMPUTERS 2

What is the binary (base-2) representation of (135) $)_{10}$ ?
(A) 101111010
(B) 010111101
(C) 010000111
(D) 101111001

Solution:

$$
\begin{aligned}
(135)_{10} & =1 \times 2^{7} \\
& +0 \times 2^{6} \\
& +0 \times 2^{5} \\
& +0 \times 2^{4} \\
& +0 \times 2^{3} \\
& +1 \times 2^{2} \\
& +1 \times 2^{1} \\
& +1 \times 2^{0} \\
(135)_{10} & =10000111_{2}
\end{aligned}
$$

Answer is C.

## COMPUTERS 2

What is the two's complement of $(-14)_{10}$ ?
(A) 10010
(B) 01101
(C) 11011
(D) 01111

Solution:

$$
\begin{aligned}
& 14_{10}=\left(0 \times 2^{4}\right)+\left(1 \times 2^{3}\right)+\left(1 \times 2^{2}\right)+\left(1 \times 2^{1}\right)+0 \\
& (14)_{10}=(01110)_{2}
\end{aligned}
$$

Standard way to find two's complement is to flip all the bits, then add 1. Flipping all of the digits: 10001

Adding 1: $10001+1=10010$
Answer is A.

## COMPUTERS 4

What is the binary (base-2) representation of the hexadecimal (base-16) number (7704) ${ }_{16}$ ?
(A) 0101101100000100
(B) 0110011100000100
(C) 0111001100000100
(D) 0111011100000100

Solution:
Since $16=(2)^{4}$ each hexadecimal digit will expand into four bits.
$(7)_{8}=(0111)_{2}$
$(7)_{8}=(0111)_{2}$
$(0)_{8}=(0000)_{2}$
$(4)_{8}=(0100)_{2}$
Combine the bits.
0111011100000100
Answer is D.

## COMPUTERS 13

Given $\mathrm{A}=$ true, $\mathrm{B}=$ true, and $\mathrm{C}=$ false, what is the value of the following logical expression?
(A.AND.B).AND.(NOT.(C.OB.A))
(A) true
(B) false
(C) either true or false
(D) neither true nor false

Solution:

Evaluate the terms within the parentheses first.
$(\mathrm{A} . \mathrm{AND} \cdot \mathrm{B})=$ true. AND .true $=$ true
$($ C.OR.A $)=$ false. OR.true $=$ true
$($ NOT.(C.OR.A) $)=$ NOT.true $=$ false
(A.AND.B).AND.(NOT.(C.OR..A)) $=-$ true.AND.false $=$ false

This problem can also be done with zeros and ones.
Answer is B.

## COMPUTERS 14

A and B are inputs to two logic gates, as shown. What is the output?

(A) $\mathrm{AB}+\mathrm{AB}$
(B) AB
(C) $\mathrm{A}+\mathrm{B}$
(D) always 0

Solution:
Two AND gates are shown. The output from each gate is AB. The outputs of the two gates are combined in an OR operation. The output of the OR gate is then

$$
\mathrm{AB}+\mathrm{AB}
$$

This will be equal to 0 if $(\mathrm{AB})$ is 0 ; it will be equal to 1 if $(\mathrm{AB})$ is 1 .
Answer is A.

## COMPUTERS 55

In a spreadsheet, the number in cell A4 is set to 6 . Then A5 is set to A4 + \$A\$4 where $\$$ indicates absolute cell address. This formula is copied into cells A6 and A7. The number shown in cell A7 is most nearly:
(A) 12
(B) 24
(C) 36
(D) 216

|  | A | B |
| :--- | :--- | :--- |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 | 6 |  |
| 5 | A4+\$A\$4 |  |
| 6 | A5+\$A\$4 |  |
| 7 | A6+\$A\$4 |  |

A5 $=\mathrm{A} 4+\$ \mathrm{~A} \$ 4=6+6=12$
A6=A5+\$A\$4=12+6=18
$\mathrm{A} 7=\mathrm{A} 6+\$ \mathrm{~A} \$ 4=18+6=24$
Answer is (B)
16. Which of the following program flowchart symbols is typically used to indicate the end of a process or program?
(A)
(C)

(B)

(D)


Solution:
Termination (end of the program) is commonly represented by the symbol in choice (C).
Answer is C.

## COMPUTERS 56

The program segment

```
INPUT Z,N
S=1
T=1
FOR K=1 TO N
T=T*Z/K
S=S+T
NEXT K
```

calculates the sum:
(A) $\mathrm{S}=1+\mathrm{ZT}+2 \mathrm{ZT}+3 \mathrm{ZT}+\ldots+\mathrm{nZT}$
(B) $\mathrm{S}=1+\mathrm{ZT}+\mathrm{ZT} / 2+\mathrm{ZT} / 3+\ldots+\mathrm{ZT} / \mathrm{n}$
(C) $\mathrm{S}=1+\mathrm{Z} / 1+\mathrm{Z}^{2} / 2+\mathrm{Z}^{3} / 3+\ldots+\mathrm{Z}^{\mathrm{n}} / \mathrm{n}$
(D) $S=1+Z / 1!+Z^{2} / 2!+Z^{3} / 3!+\ldots+Z^{n} / n!$

The key thing here is what the FOR loop does.
For $\mathrm{K}=1, \mathrm{~T}=1 * \mathrm{Z} / 1=\mathrm{Z}$
For $\mathrm{K}=2, \mathrm{~T}=\mathrm{T} * \mathrm{Z} / \mathrm{K}=\mathrm{Z} * \mathrm{Z} / 2=\mathrm{Z}^{2} / 2$
For $\mathrm{K}=3, \mathrm{~T}=\mathrm{T}^{*} \mathrm{Z} / \mathrm{K}=\mathrm{Z}^{2} / 2 * \mathrm{Z} / 3=\mathrm{Z}^{3} / 6$
The $\mathrm{S}=\mathrm{S}+\mathrm{T}$ simply sums the terms. This series is proceeding as (D).

## COMPUTERS 23

How many times will the second line be executed?

|  | $\mathrm{M}=42$ |
| :--- | :--- |
| LOOPSTART | $\mathrm{M}=\mathrm{M}-1$ |
|  | $\mathrm{P}=$ INTEGER PART OF (M/2) |
|  | IF P $>15$, THEN GO TO |
|  | LOOPSTART, OTHERWISE |
|  | GO TO END |
| END | PRINT "DONE" |

(A) 8
(B) 9
(C) 10
(D) 11

Solution:

The values of the variables for each iteration are

| iteration | m | p |
| ---: | ---: | ---: |
| $\mathbf{1}$ | 41 | 20 |
| 2 | 40 | 20 |
| 3 | 39 | 19 |
| 4 | 38 | 19 |
| 5 | 37 | 18 |
| 6 | 36 | 18 |
| 7 | 35 | 17 |
| 8 | 34 | 17 |
| 9 | 33 | 16 |
| 10 | 32 | 16 |
| 11 | 31 | 15 |

When P reaches $15, \mathrm{P}$ is no longer greater than 15. Line 2 is executed 11 times.
Answer is D.

