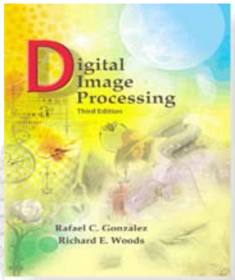
Lecture #28

- Cameras
- Camera calibration

Cameras

- First photographs due to Niepce
- First experiment 1822; first photograph 1826

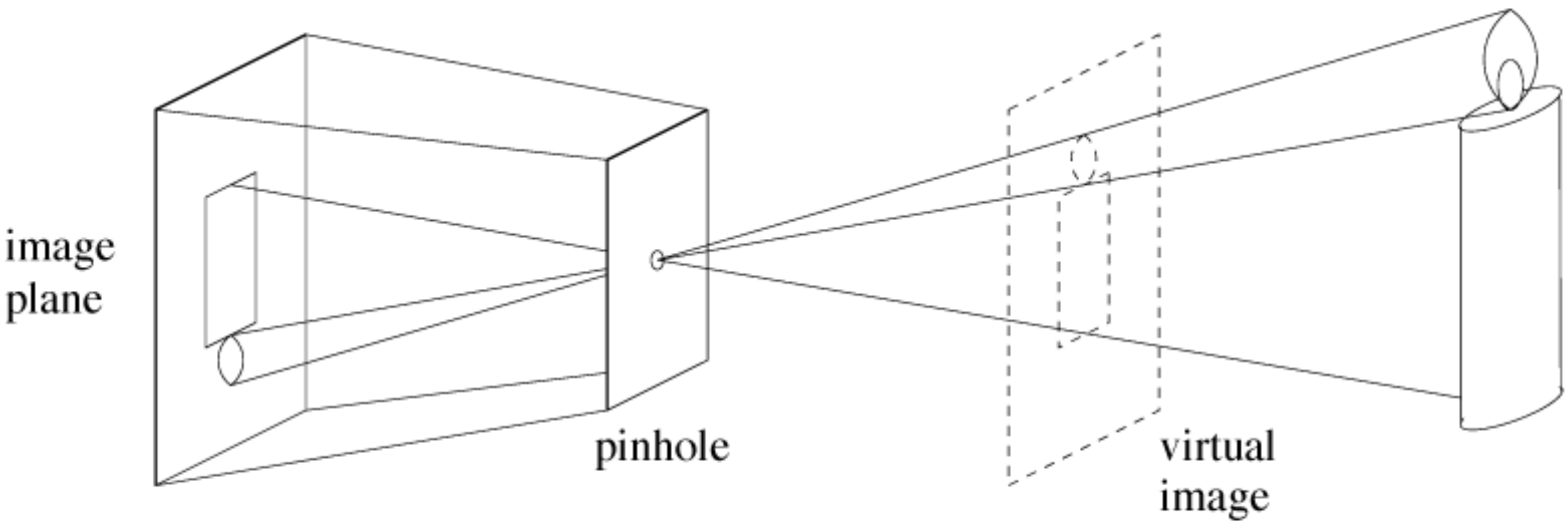
- Basic abstraction is the pinhole camera
  - lenses required to ensure image is not too dark
  - various other abstractions can be applied



Pinhole cameras

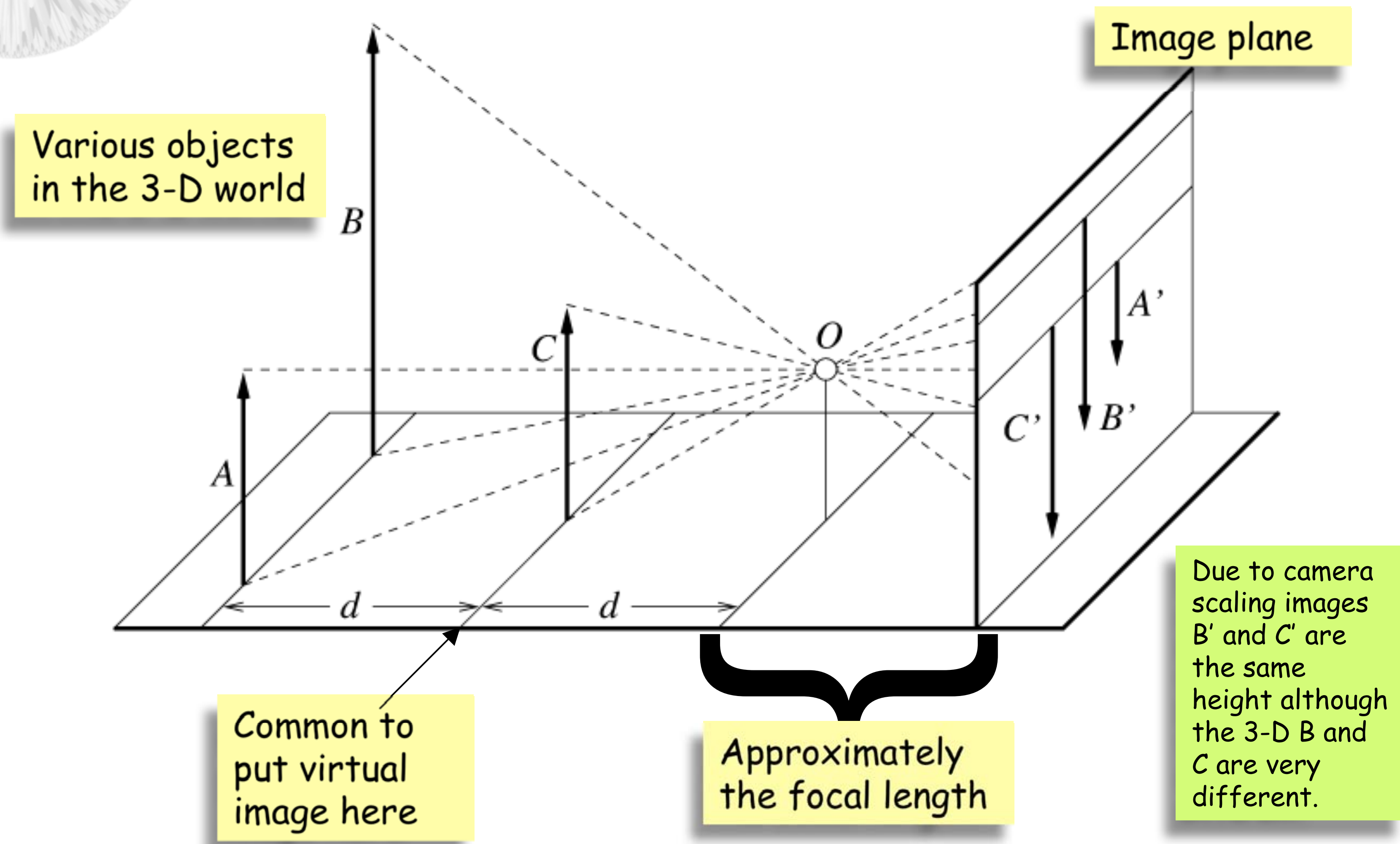
Abstract camera model - box with a small hole in it

Pinhole cameras actually work in practice!

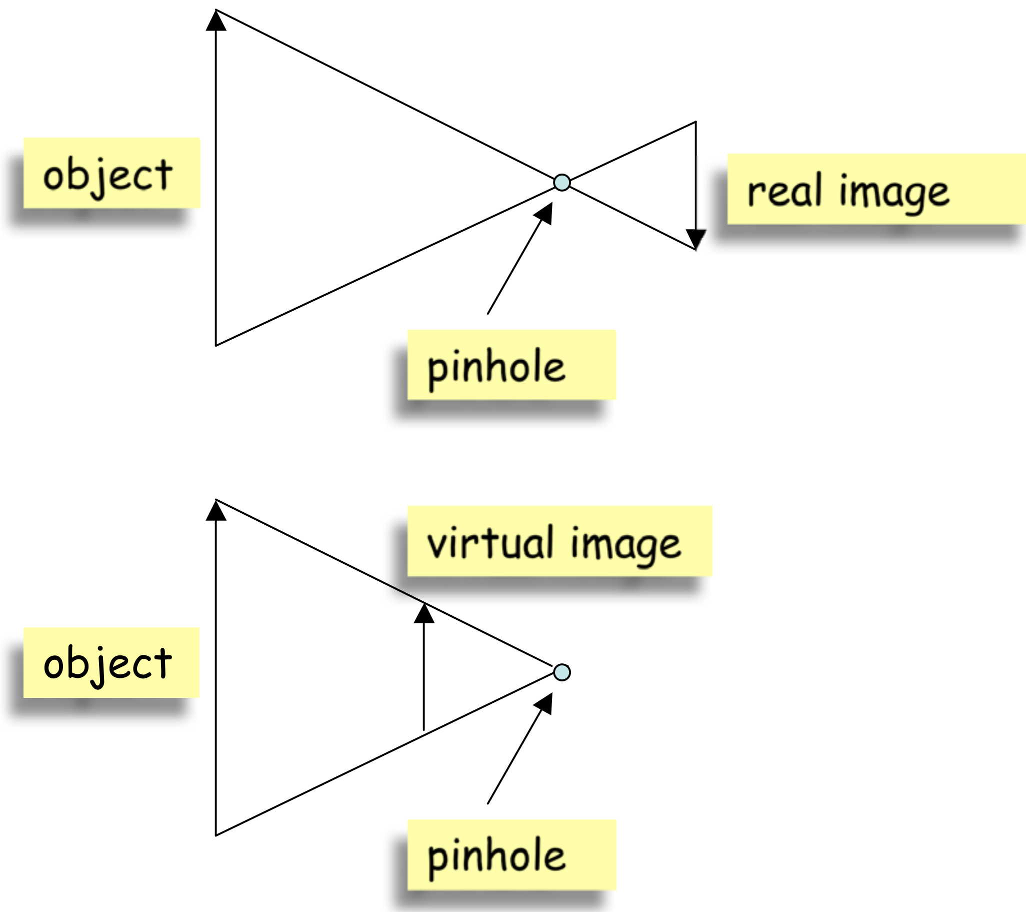


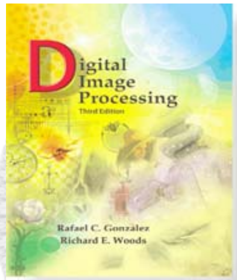
Each point on the image plane sees light from only one direction which passes through the pinhole

Distant objects are smaller



Distant objects are smaller





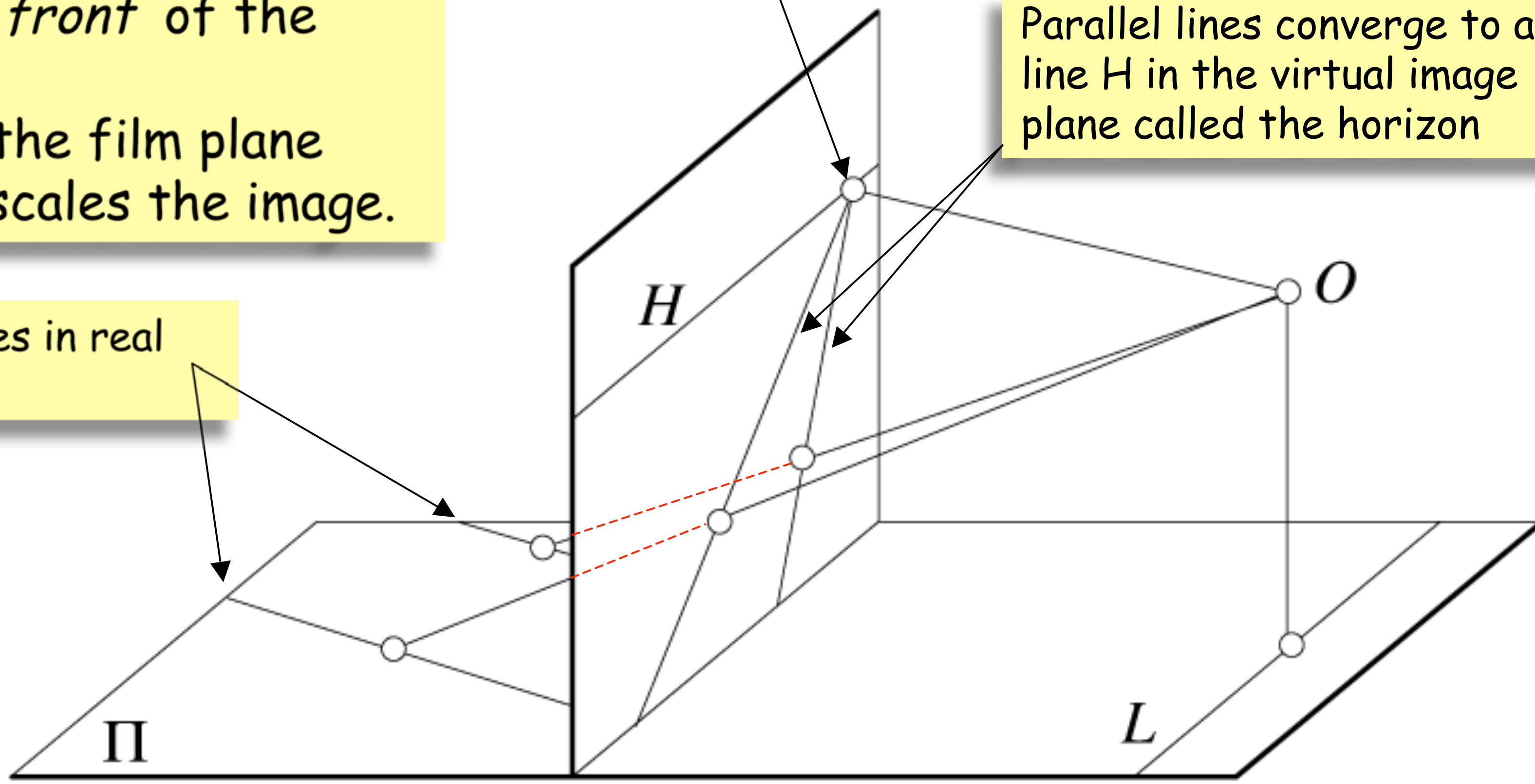
Parallel lines meet

Common to draw film plane *in front* of the pinhole.
Moving the film plane merely scales the image.

Vanishing point

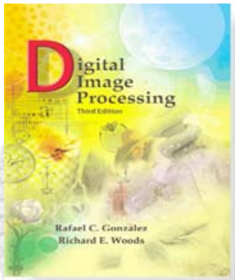
Parallel lines converge to a line H in the virtual image plane called the horizon

Parallel lines in real world



virtual image plane

plane containing pinhole

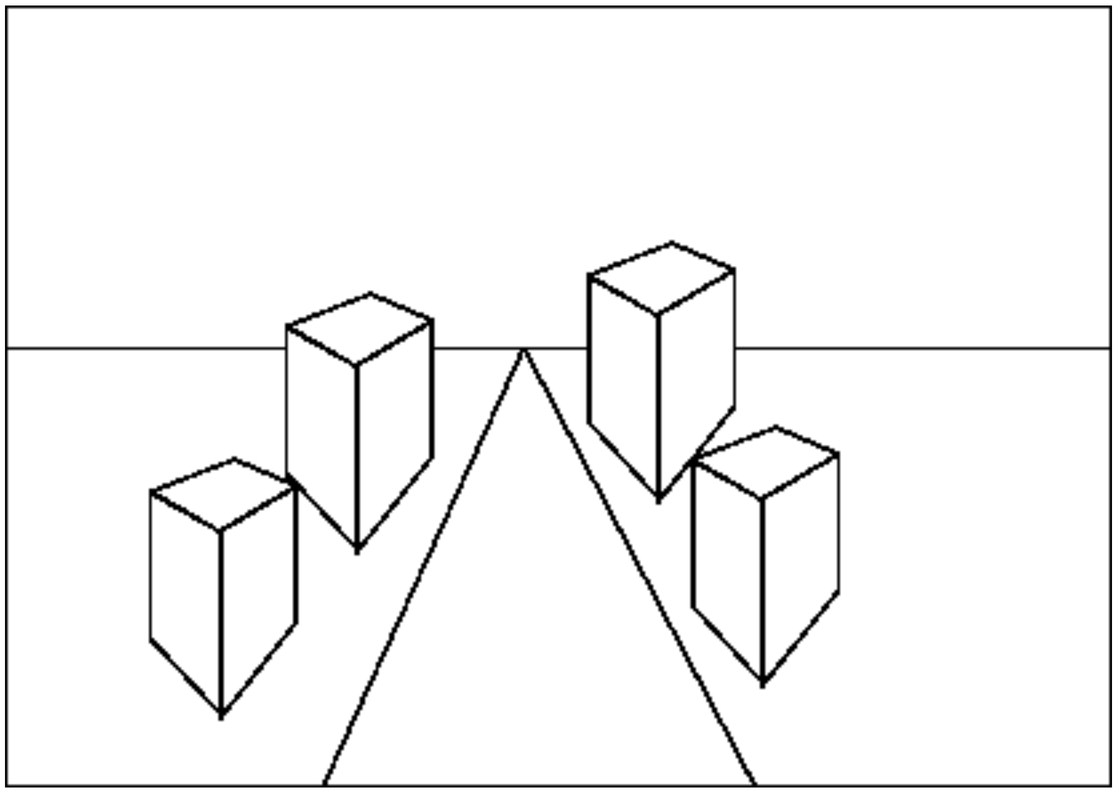


Vanishing points

- Each set of parallel lines (same direction) meets at a different point in the image plane
  - Called the vanishing point for this direction
- Sets of parallel lines on the same plane lead to collinear vanishing points.
  - The line is called the <u>horizon</u> for that plane

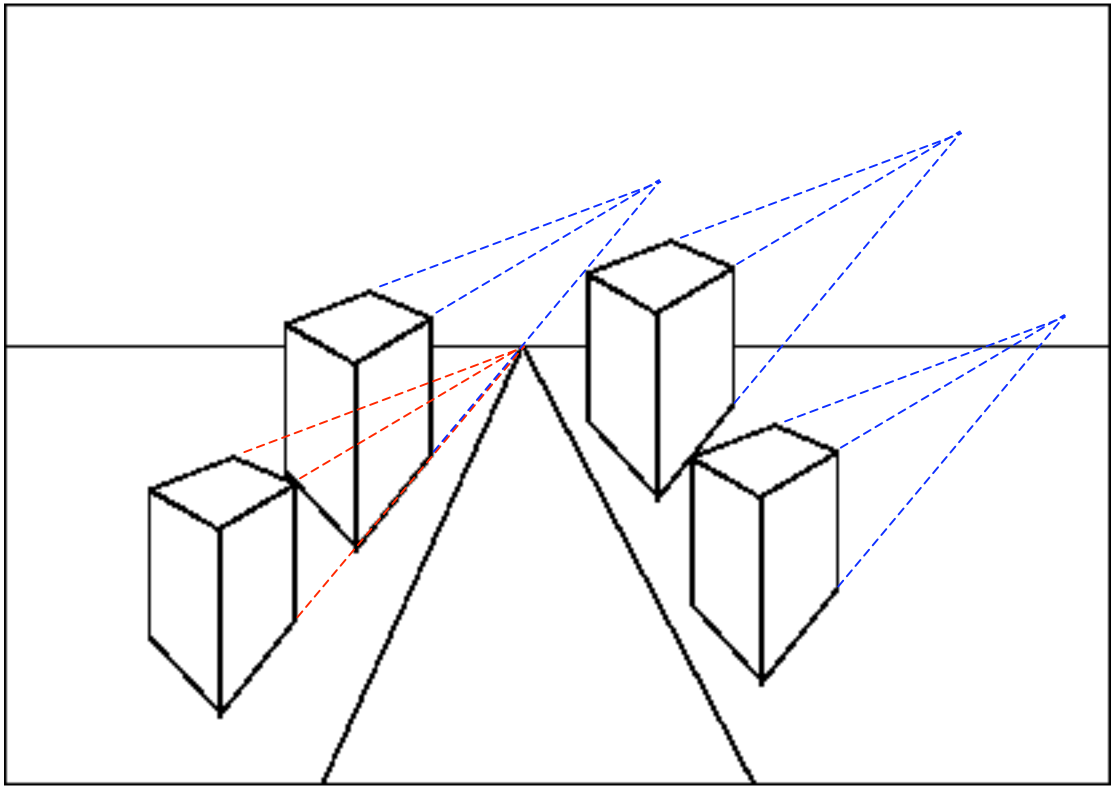
- Good ways to spot faked images
  - scale and perspective don't work
  - vanishing points behave badly
  - supermarket tabloids are a great source of faked images.

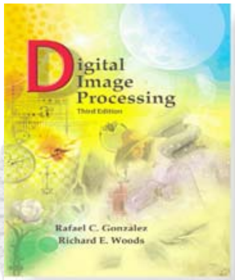
Vanishing points



Vanishing points

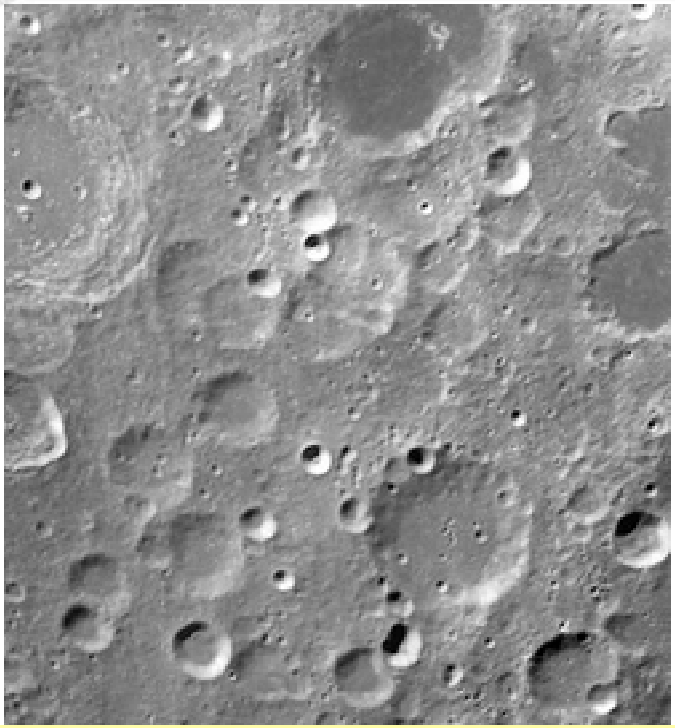
The blocks were obviously cut and pasted. Only one correctly points to the vanishing point on the horizon.





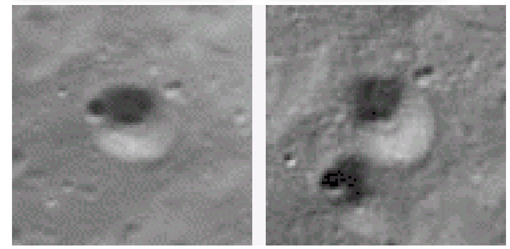
Recent Controversial Image

Chinese Moon Photo Not Fake, But Not Pristine!

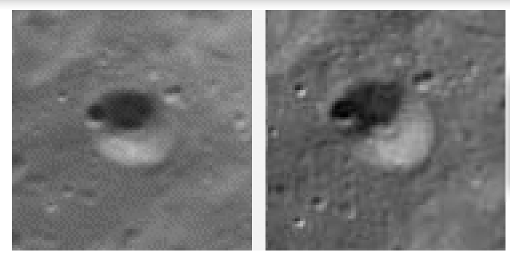


How did that happen? the Chang'e 1 image is actually a mosaic of 19 image strips taken on different orbits. It is incredibly difficult to get every tiny feature in a large mosaic to line up properly; you have to have very, very precise knowledge of the shape of the body being photographed, and we actually don't have very good topographic models of the Moon (Kaguya will change that). So it's no surprise that there are seams in this image. I commented last week when I first posted this image on the beauty of the mosaic, and the fact that there were no obvious seams. It's evident from this little snafu that the seams are there, just blended well to make it difficult to see them.

Clementine | Chang'e

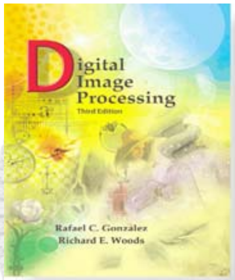


A new feature on the Moon?
A 4-kilometer-diameter crater in a Clementine image of the Moon from 1994 seems to have been struck by another crater in a Chang'e image of the Moon from 2007. Credit: NASA / DOD / CAST

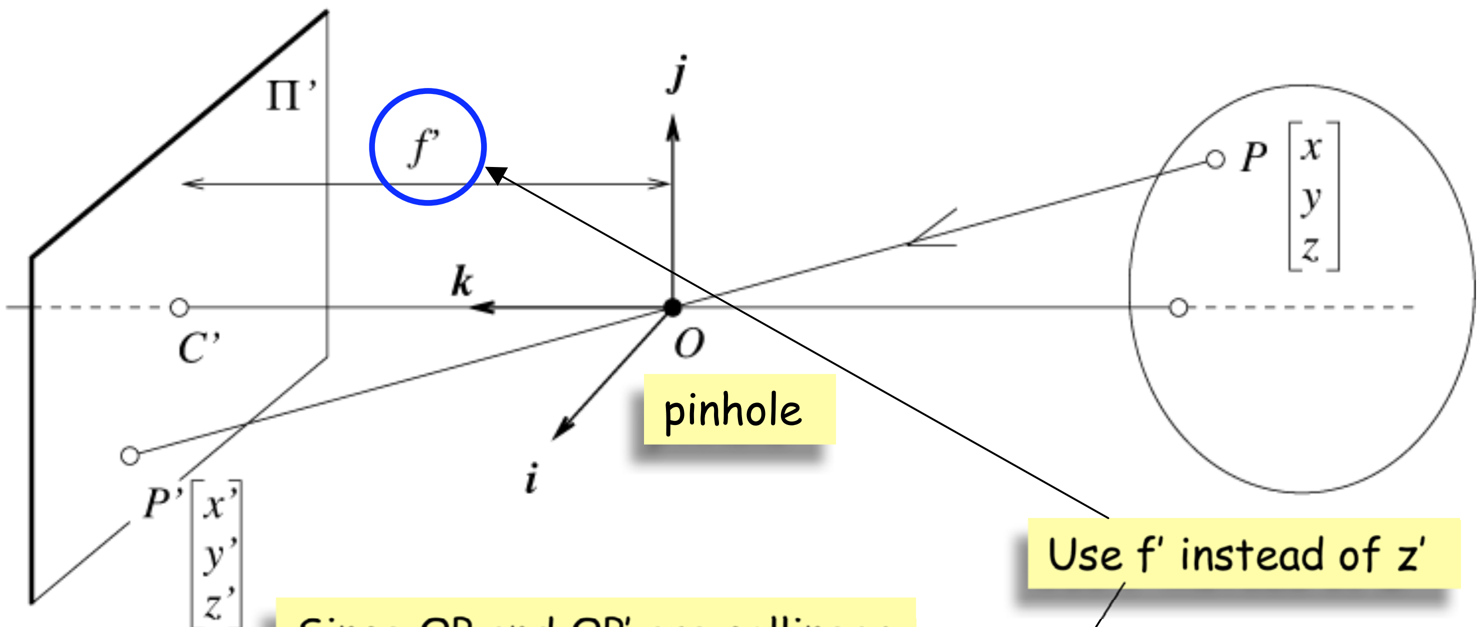


Fixed Chang'e

Not a new feature on the Moon
With a repair to an image seam in a Chang'e 1 image of the Moon, an apparently new feature disappears. Credit: CAST / Emily Lakdawalla



The equation of projection



Since OP and OP' are collinear we can use similar triangles

$x' = \lambda x$

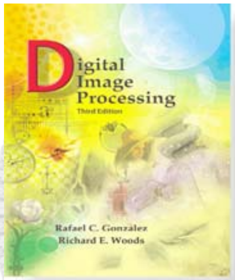
$y' = \lambda x$

$f' = \lambda z$

$$\lambda = \frac{x'}{x} = \frac{y'}{y} = \frac{f'}{z}$$

$$x' = f'\frac{x}{z}$$

$$y' = f'\frac{y}{z}$$



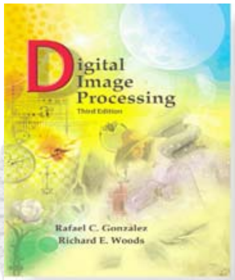
The equation of projection

- Cartesian coordinates:
  - We have, by similar triangles, that

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z}, -f)$$

  - Ignore the third coordinate, and get

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$



Homogenous coordinates

- Add an extra coordinate and use an equivalence relation
- <u>for 2D</u>
  - equivalence relation (X/K,Y/K,1) is the same as (X,Y,K)
- <u>for 3D</u>
  - equivalence relation (X/K,Y/K,Z/K,1) is the same as (X,Y,Z,K)

Basic notion

- Possible to represent points “at infinity”
  - Where parallel lines intersect
  - Where parallel planes intersect
- Possible to write the action of a perspective camera as a matrix

The camera matrix

Re-write previous expression for camera transformation in homogeneous coordinates (HC's)

- HC's for 3D point are (X,Y,Z,T)
- HC's for point in image are (U,V,W)

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

2-D image coordinates

3-D world coordinates

The camera matrix

2-D image coordinates

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

3-D world coordinates

Multiplying

$$U = X$$

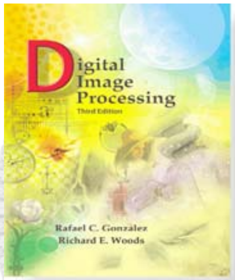
$$V = Y$$

$$W = \frac{Z}{f}$$

Scale factor

$$U = \frac{X}{\frac{Z}{f}} = f\frac{X}{Z}$$

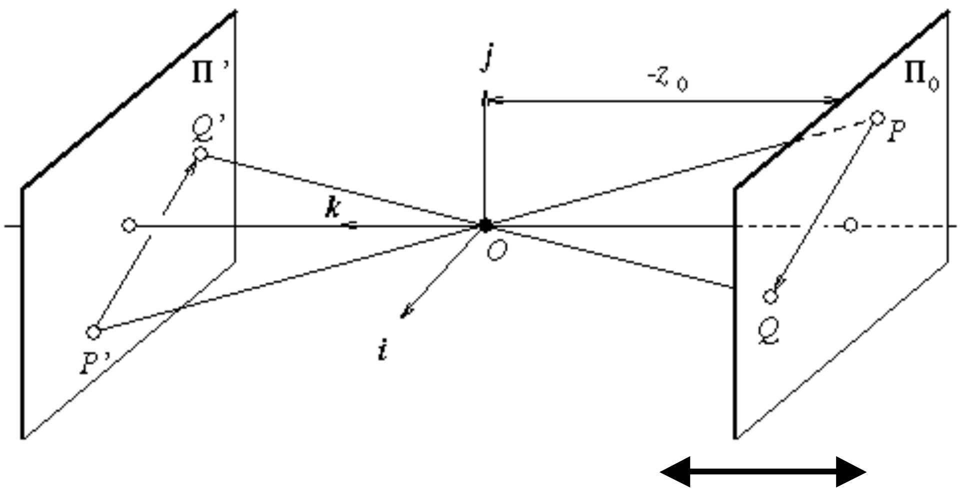
$$V = \frac{Y}{\frac{Z}{f}} = f\frac{Y}{Z}$$



Weak perspective

$$U = f\frac{X}{Z} \approx mX$$

$$V = f\frac{Y}{Z} \approx mY$$

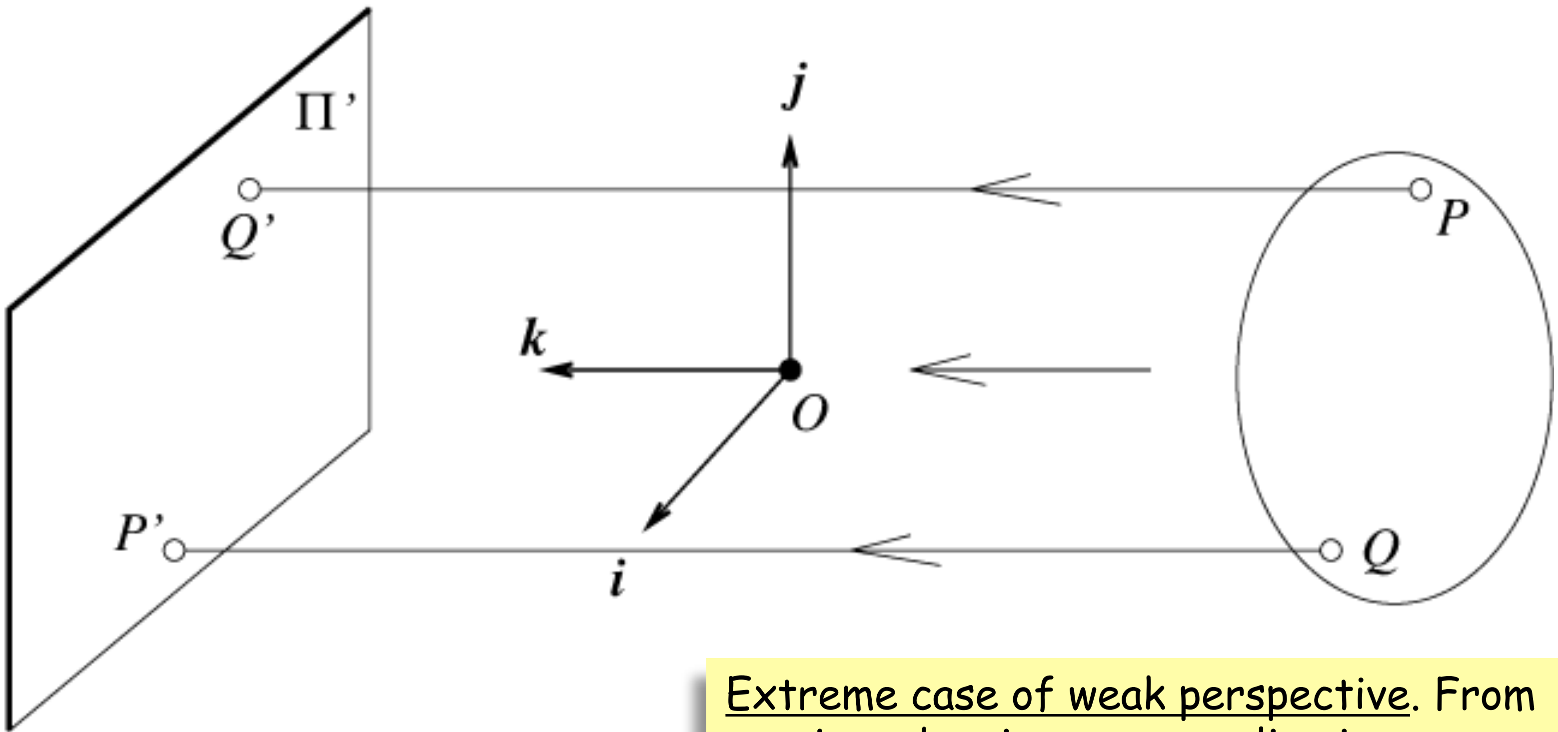


Issue

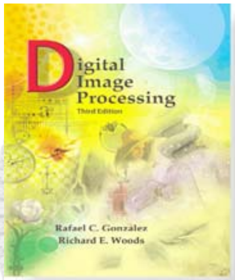
- perspective effects, but not over the scale of individual objects
- collect points into a group at about the same depth, then divide each point by the depth of its group
- Adv: easy
- Disadv: wrong

If depth of field of objects in a group is similar, <u>assume magnification of group is constant.</u>

Orthographic projection



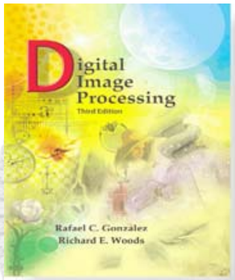
Extreme case of weak perspective. From previous drawing we normalize image coordinates to have magnification m=-1 which looks wrong but is correct.



Projection matrix for orthographic projection

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

This is the mathematical simplification of the previous projection matrix in the limit that f->∞

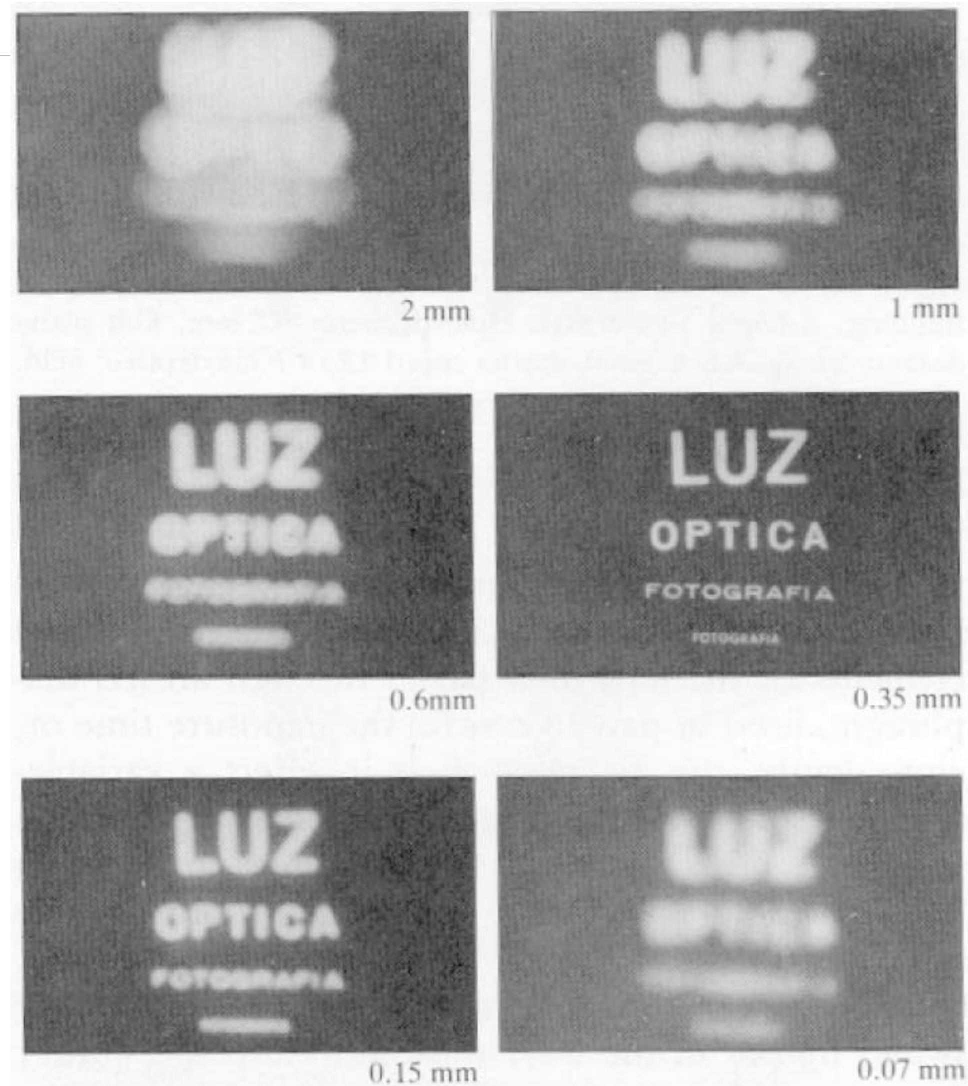


Pinhole imaging

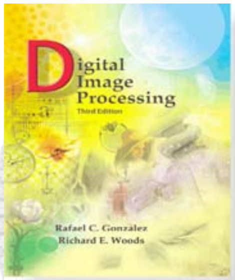
Pinhole too big - many directions are averaged, blurring the image

Pinhole too small- diffraction effects blur the image

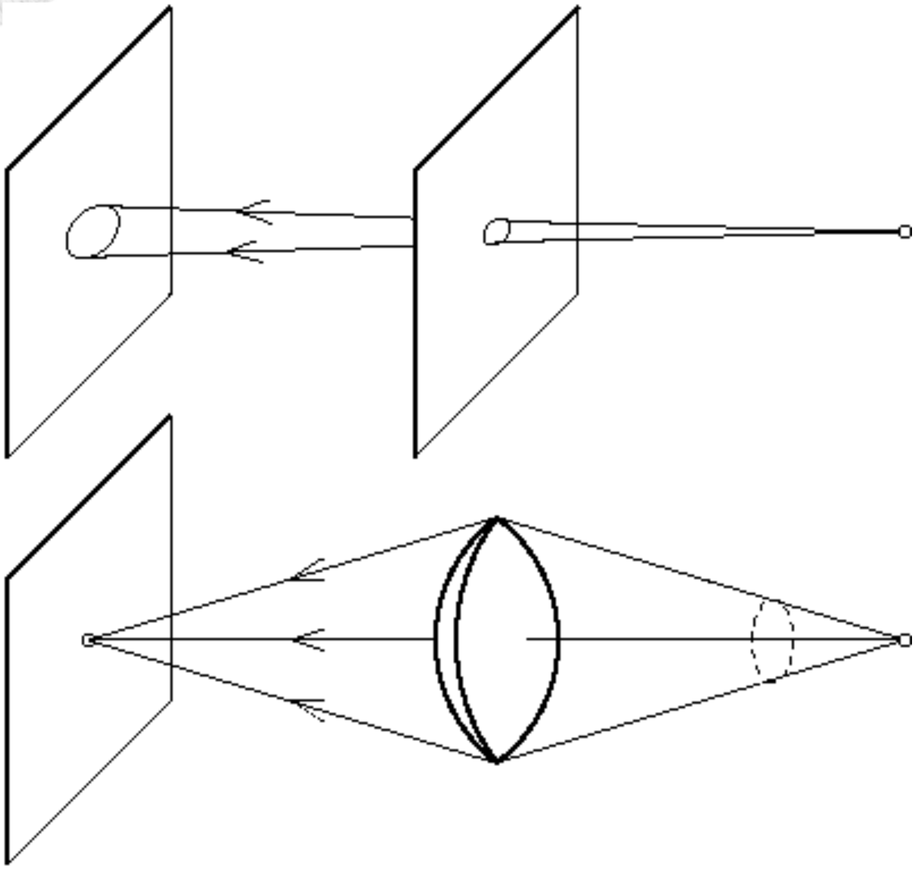
Generally, pinhole cameras are *dark*, because a very small set of rays from a particular point hits the screen.



Computer Vision - A Modern Approach: Cameras
Slides by D.A. Forsyth

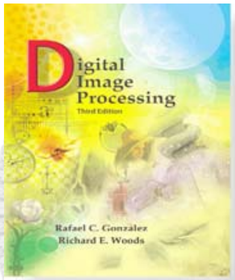


The reason for lenses

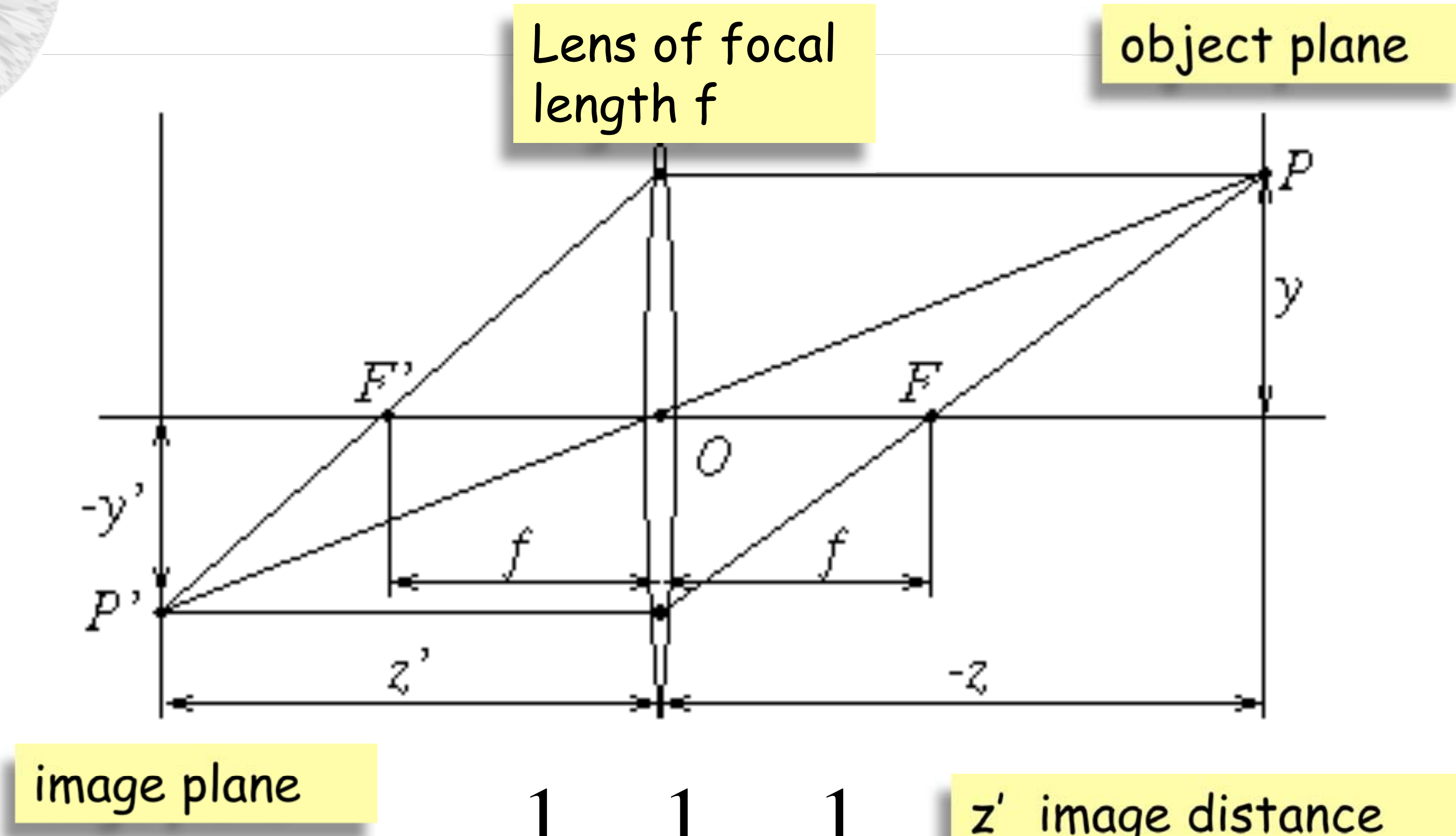


Each point on the image plane sees light from only the one direction which passes through the pinhole

A lens can gather (focus) many light rays to a single image point making a brighter image.



The thin lens



$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

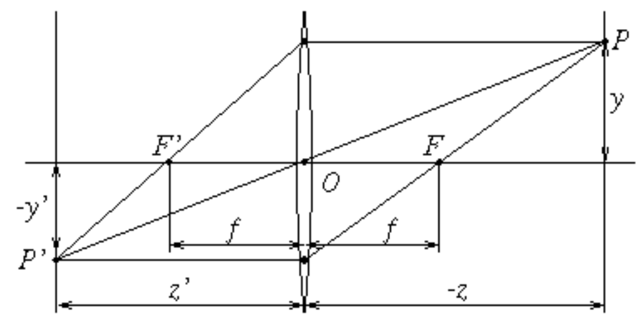
z' image distance
z object distance
f focal length

The lens law

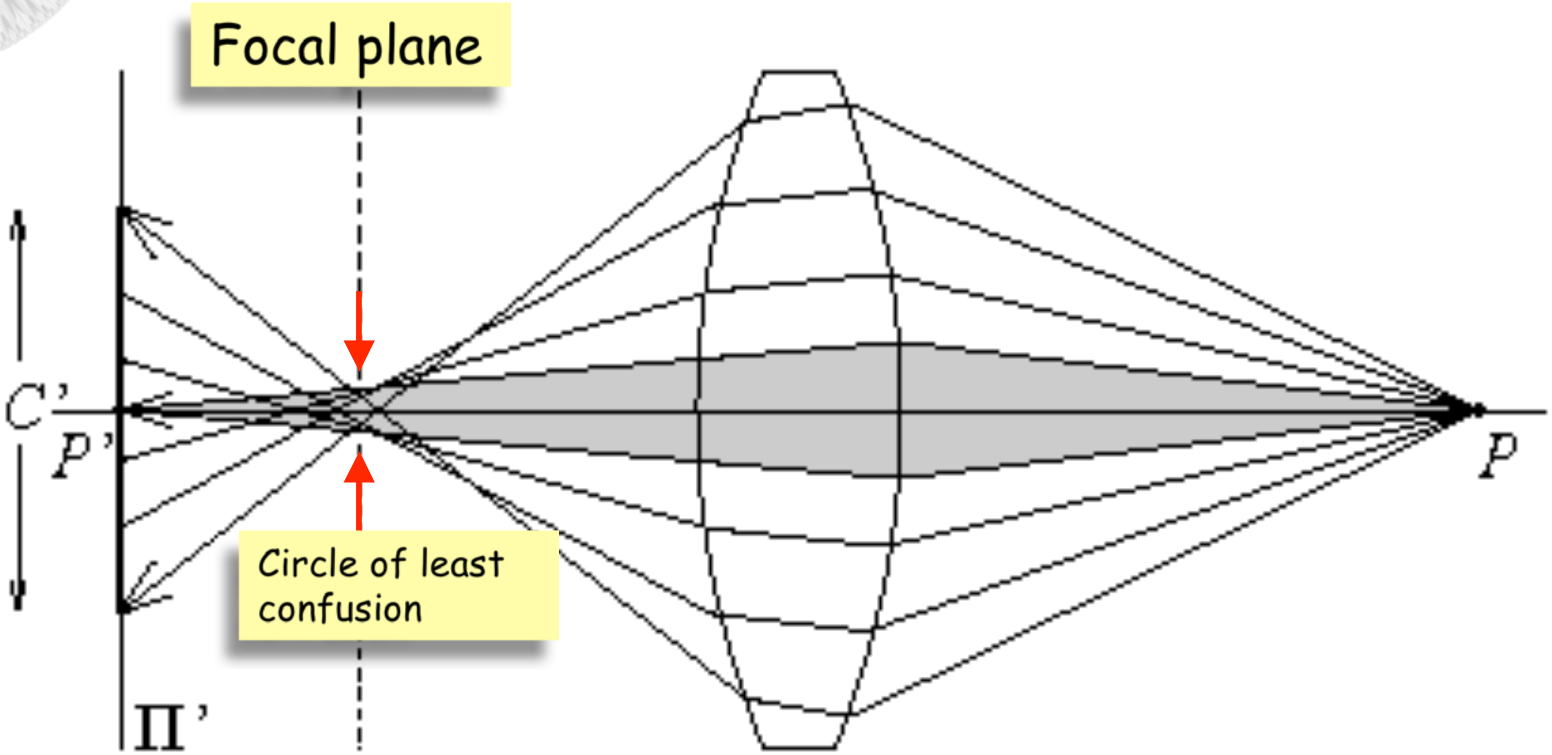
$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

ASSUMPTIONS AND OBSERVATIONS

- From Snell's Law $n_1 \sin\alpha_1 = n_2 \sin\alpha_2$ where n_1 is the index of refraction of air and n_2 is the index of refraction of the lens
- Assumes y and y' are close to the z-axis (paraxial ray assumption) so that $\sin\alpha_1 \approx \alpha_1$
- Ray PP' is not refracted since $\alpha_1 = \alpha_2$ but all other rays are refracted.
- f is the point at which entering rays parallel to the z-axis are focused, i.e., cross the z-axis

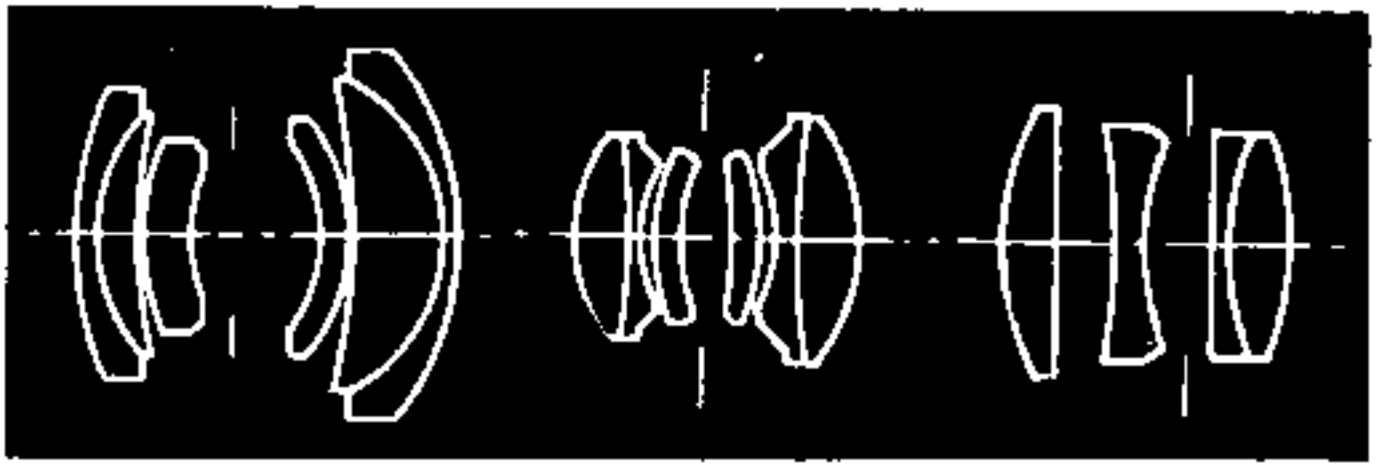


Spherical aberration

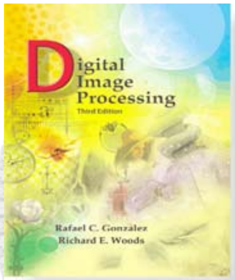


Because the paraxial ray assumptions $\sin\alpha_1 \approx \alpha_1$ and $\sin\alpha_2 \approx \alpha_2$ is only an approximation the focal point for different rays is not at the same z-coordinate for all rays.

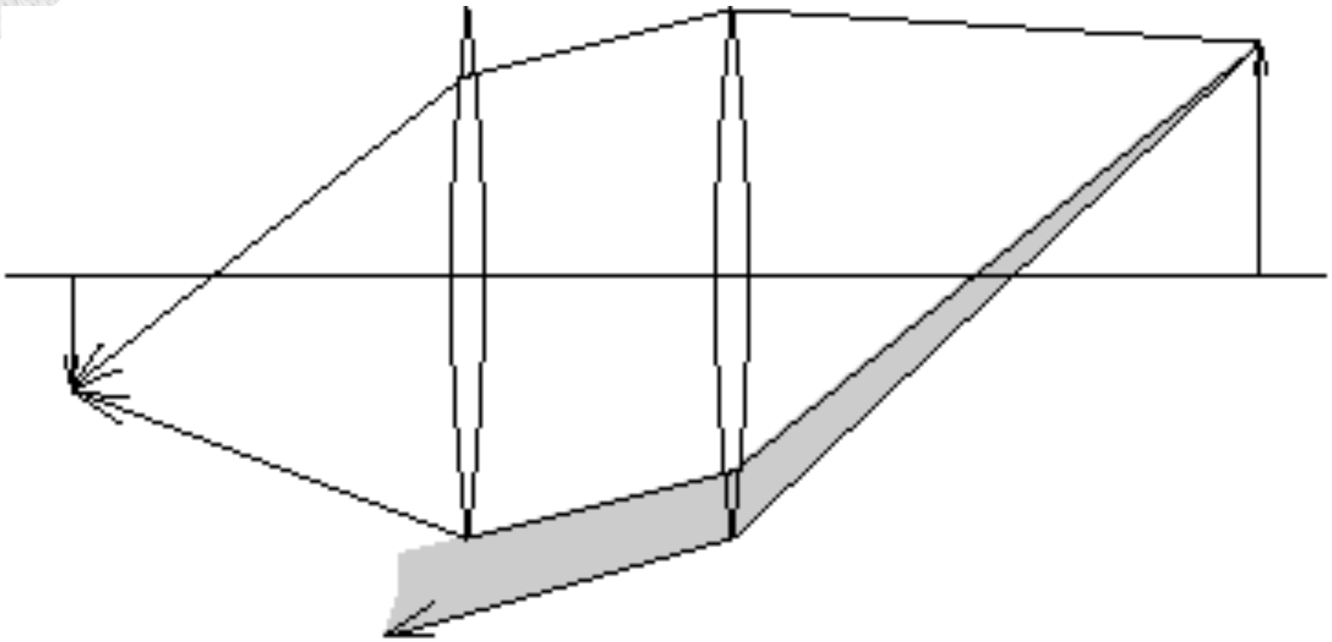
Lens systems



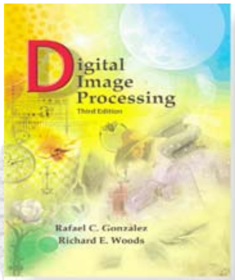
- Complex lens systems are used to minimize aberrations, etc.
- It is common for modern lenses (except the front and rear lenses) to be plastic.
  - Light-weight
  - Easy to manufacture, i.e., low-cost
  - aspheric



Vignetting



- Light follows a path which prevents it from reaching the image plane, i.e., it reaches the limit of the lens or runs into a stop. This can be accidental or deliberate.

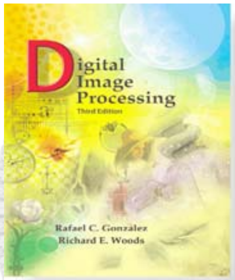


Other (possibly annoying) phenomena

Chromatic aberration
- Light at different wavelengths follows different paths; hence, some wavelengths are defocussed
- Machines: coat the lens
- Humans: live with it

Scattering at the lens surface
- Some light entering the lens system is reflected off each surface it encounters (Fresnel's law gives details)
- Machines: coat the lens, interior
- Humans: live with it (various scattering phenomena are visible in the human eye)

Geometric phenomena (Barrel distortion, etc.)

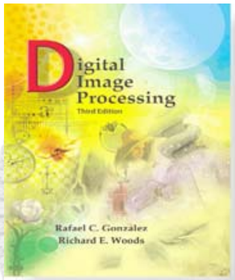


Camera parameters

Issue

- camera may not be at the origin, looking down the z-axis
  - extrinsic parameters
- one unit in camera coordinates may not be the same as one unit in world coordinates
  - intrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{intrinsic parameters} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{extrinsic parameters} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

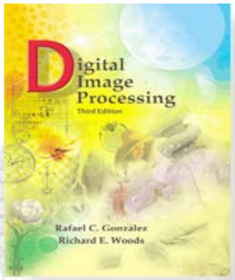


Camera calibration

Issues:
- what are intrinsic parameters of the camera?
- what is the camera matrix? (intrinsic+extrinsic)

General strategy:
- view calibration object
- identify image points
- obtain camera matrix by minimizing error
- obtain intrinsic parameters from camera matrix

Error minimization:
- Linear least squares
  - easy problem numerically
  - solution can be rather bad
- Minimize image distance*
  - more difficult numerical problem
  - solution usually rather good,
  - start with linear least squares
- Numerical scaling is an issue



Camera calibration

Radial alignment constraint (due to Tsai)

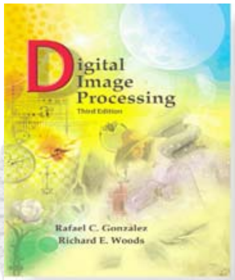
- The idea is that spherical distortion changes the radial scale but not the direction.
- Use least squared error (LSE) to find all camera parameters
  - T_z — z-translation of camera
  - Magnification
  - Distortion parameters (usually we only use the first two terms) where d is the distance between the image point and the image center.

$$\lambda = 1 + \kappa_1 d^2 + \kappa_2 d^4 + \kappa_3 d^6$$

Simple Camera Calibration Example

$$P_i = \begin{bmatrix} w_i x_i \\ w_i y_i \\ w_i \end{bmatrix} = AP_o = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \begin{bmatrix} x_o \\ y_o \\ z_o \\ 1 \end{bmatrix}$$

- Set up basic transformation from 3-D object (homogeneous) coordinates to 2-D image (homogeneous) coordinates.
- Let all matrix parameters be unknown.
- Keep w_i as an unknown in the image plane but explicitly write out scaled coordinates $w_i x_i$ and $w_i y_i$.



Simple Camera Calibration Example

Multiply out

$$w_i x_i = a_{11} x_o + a_{12} y_o + a_{13} z_o + a_{14}$$

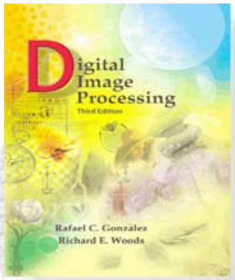
$$w_i y_i = a_{21} x_o + a_{22} y_o + a_{23} z_o + a_{24}$$

$$w_i = a_{31} x_o + a_{32} y_o + a_{33} z_o + a_{34}$$

Eliminate w_i

$$x_i = \frac{a_{11} x_o + a_{12} y_o + a_{13} z_o + a_{14}}{a_{31} x_o + a_{32} y_o + a_{33} z_o + a_{34}}$$

$$y_i = \frac{a_{21} x_o + a_{22} y_o + a_{23} z_o + a_{24}}{a_{31} x_o + a_{32} y_o + a_{33} z_o + a_{34}}$$



Simple Camera Calibration Example

Cross-multiply and rearrange with a_{ij} as unknowns

$$a_{11}x_o + a_{12}y_o + a_{13}z_o + a_{14} = a_{31}x_o x_i + a_{32}y_o x_i + a_{33}z_o x_i + a_{34}x_i$$

$$a_{21}x_o + a_{22}y_o + a_{23}z_o + a_{24} = a_{31}x_o y_i + a_{32}y_o y_i + a_{33}z_o y_i + a_{34}y_i$$

$$x_o a_{11} + y_o a_{12} + z_o a_{13} + a_{14} - x_o x_i a_{31} - x_i y_o a_{32} - x_i z_o a_{33} = a_{34}x_i$$

$$x_o a_{21} + y_o a_{22} + z_o a_{23} + a_{24} - y_i x_o a_{31} - y_i y_o a_{32} - y_i z_o a_{33} = a_{34}y_i$$

We can set $a_{34}=1$ since it simply scales the equations

$$x_o a_{11} + y_o a_{12} + z_o a_{13} + a_{14} - x_o x_i a_{31} - x_i y_o a_{32} - x_i z_o a_{33} = x_i$$

$$x_o a_{21} + y_o a_{22} + z_o a_{23} + a_{24} - y_i x_o a_{31} - y_i y_o a_{32} - y_i z_o a_{33} = y_i$$

Simple Camera Calibration Example

Write in matrix form

$$\begin{bmatrix} x_o & y_o & z_o & 1 & 0 & 0 & 0 & 0 & -x_o x_i & -x_i y_o & -x_i z_o \\ 0 & 0 & 0 & 0 & x_o & y_o & z_o & 1 & -y_i x_o & -y_i y_o & -y_i z_o \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{31} \\ a_{32} \\ a_{33} \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

This is two equations in 11 unknowns. We will need at least six calibration pairs (x_o,y_o,z_o)-(x_i,y_i) to solve this equation. Write the set of equations as UA=X. Since you will have more equations than unknowns use MATLAB pseudo-inverse to calculate A=U\X