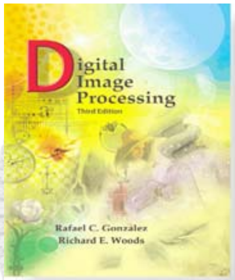
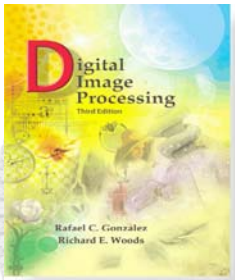


Lecture #18

- Connectivity: convex hull, thinning, thickening, skeletons, end point location
- Geodesic dilation and erosion
- Morphological reconstruction
- Automated hole filling, edge object removal
- Summary of binary morphology
- Morphological operations in MATLAB
- Gray scale morphology: image functions and SE's
- Basic gray scale operations: erosion, dilation, opening, closing
- Advanced operations: smoothing, gradient, top-ha



Connectivity

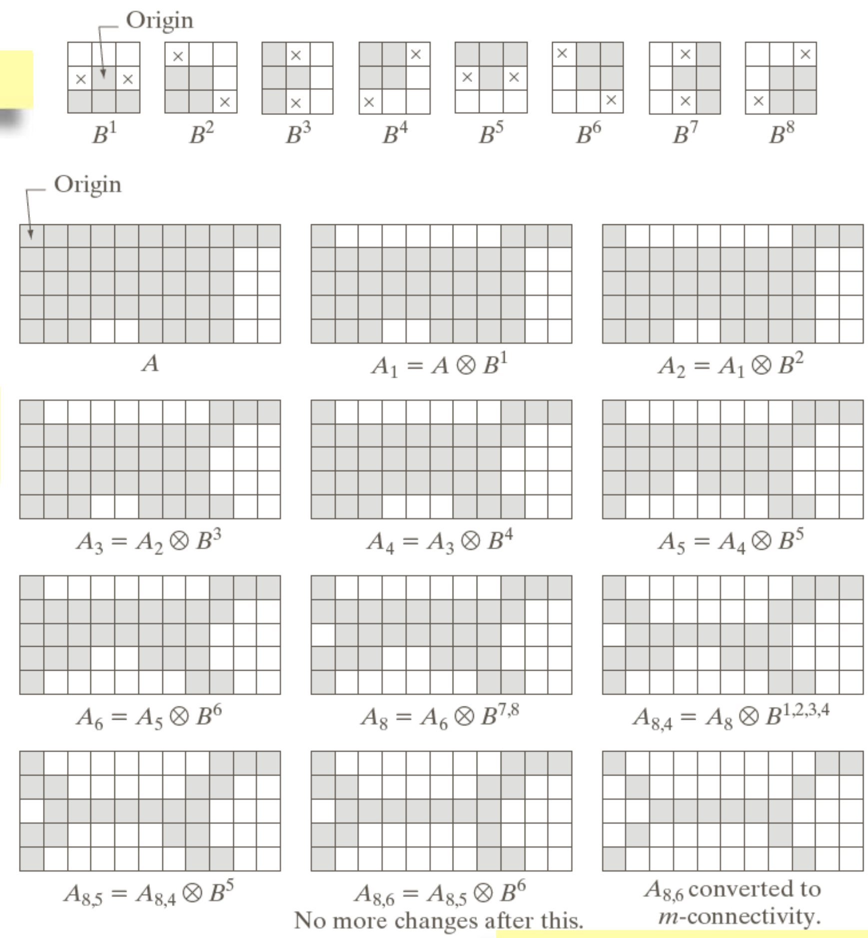
Structuring elements. X's indicate "don't cares".

The convex hull built objects up. We use a thinning operator based upon 8-connectivity to reduce objects to their basic structure.

$A \oplus B = A - (A \circledast B) = A \cap (A \circledast B)^c$

1	1	1
1	1	1

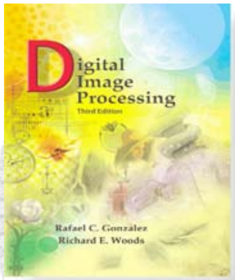
Basically, if the structuring element matches we remove that pixel. In this case of 6 1's B^1 matches so we remove the center pixel.



Converted to m-connectivity

a
b c d
e f g
h i j
k l m

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to m-connectivity.



Connectivity

Thickening is the dual of thinning. The result of thinning the complement of A is the boundary of the thickened object. Thickening can result in some disconnected points which need to be removed.

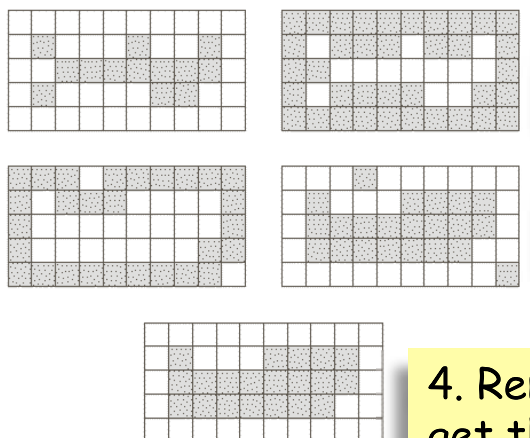


FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Skeletons

A skeleton is the set of maximum disks which fit inside A and touch the boundary of A in two or more places.

Mathematically, a skeleton can be written as a series of openings and closing where k indicates k successive erosions of A by B

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

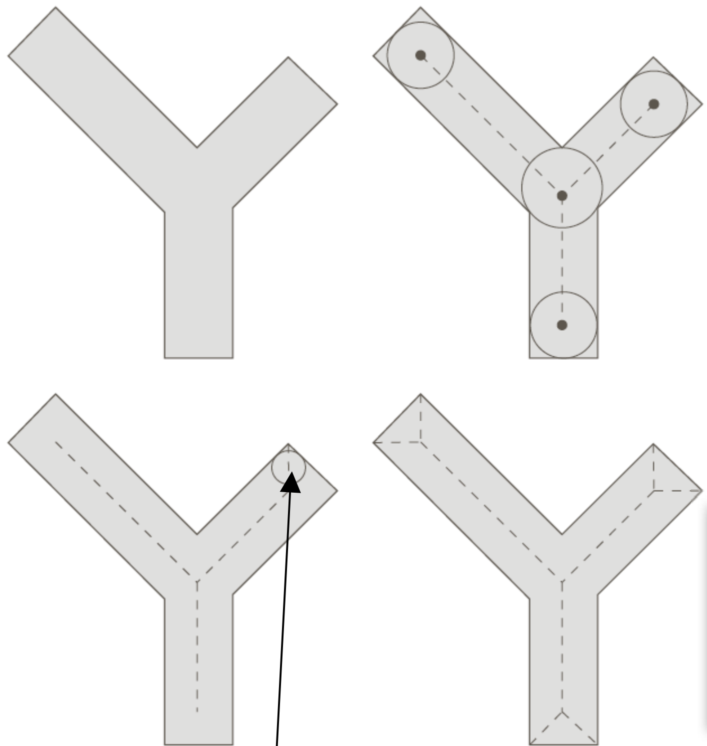
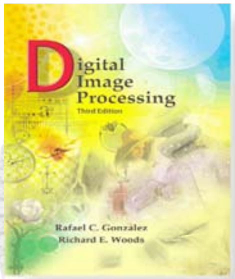


FIGURE 9.23 (a) Set A. (b) Various positions of maximum disks with centers on the skeleton of A. (c) Another maximum disk on a different segment of the skeleton of A. (d) Complete skeleton.

This is the union of all skeleton segments

Note that this maximum disk (which is smaller) also touches the boundary at two points.



Skeletons

Original object

K=0 row: no erosions or dilations since k=0

k=1 row: one erosion or dilation

k=2 row: two successive erosions or dilations

Stops at k=2 since another erosion would simply give the empty set.

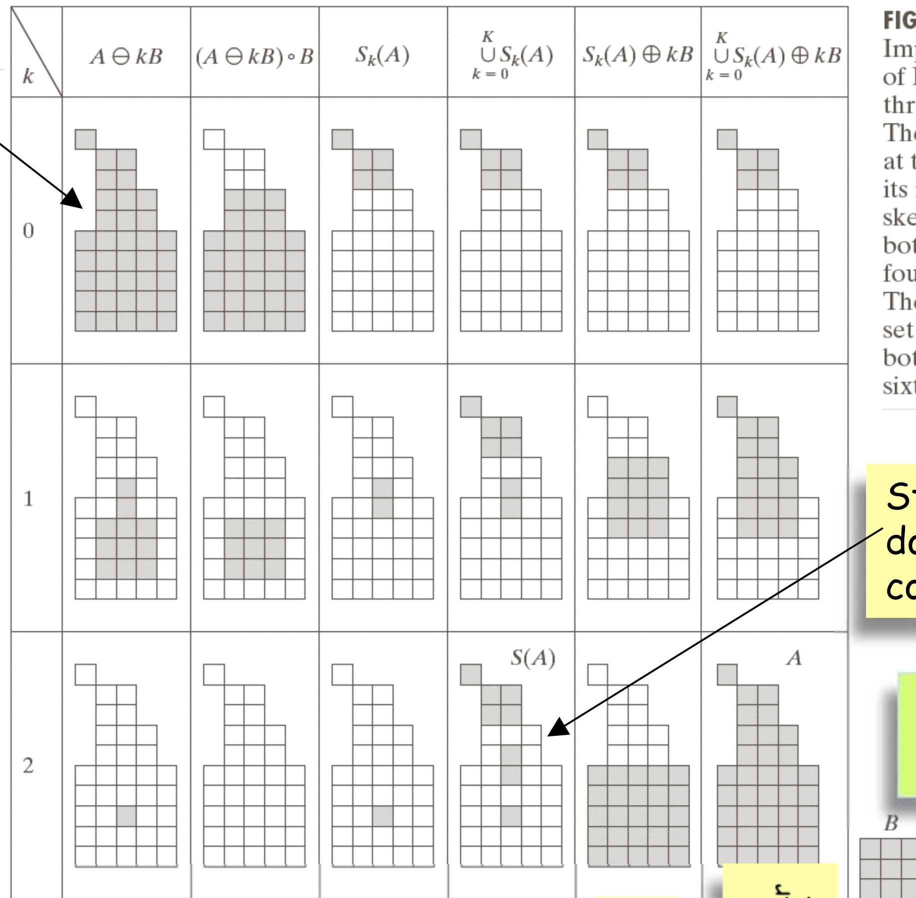


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

Strick skeletoning does not guarantee connectivity

Reconstructed object

1. Successively erode object

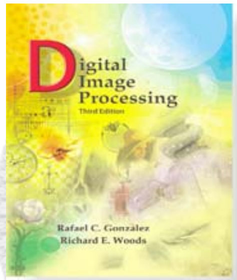
2. Open each erosion with B to compact set

3. Subtract from kB to get skeleton S_k

4. Union of all skeletons

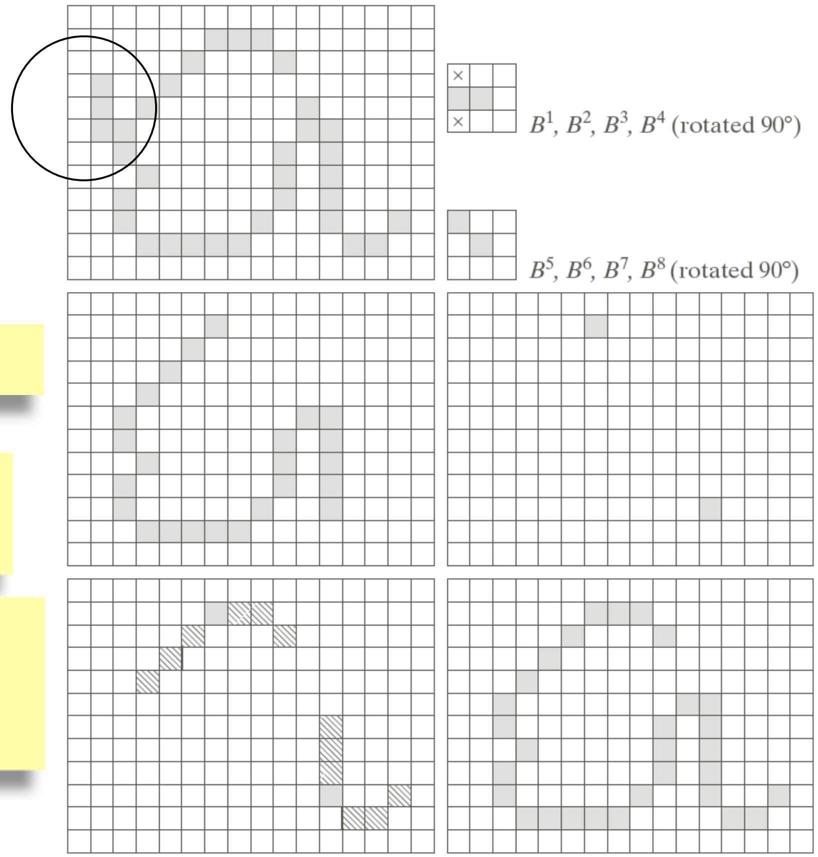
5. Dilate each skeleton S_k by B

6. Union the dilations to recover the original object



Connectivity

Want to remove this



a b
c
d e
f g

FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

1. Thin using B^i

$$X_1 = A \otimes \{B\}$$

{B} specifies using B^1 to B^8 in sequence

2. Identify end points by matching

$$X_2 = \bigcup_{k=1}^{8} \left(X_1 \circledast B^k \right)$$

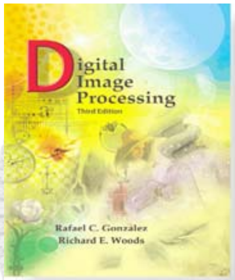
3. Dilation from end points conditioned on A

$$X_3 = \left(X_2 \oplus H \right) \cap A$$

H is a 3x3 of 1's

4. Union of thinned and conditionally dilated images eliminates parasitic branches

$$X_4 = X_1 \cup X_3$$



Geodesic Dilation

The structuring element defines the connectivity

1. Dilate marker image F by structuring element B

The marker image contains the starting point(s)

The mask constrains the transformation

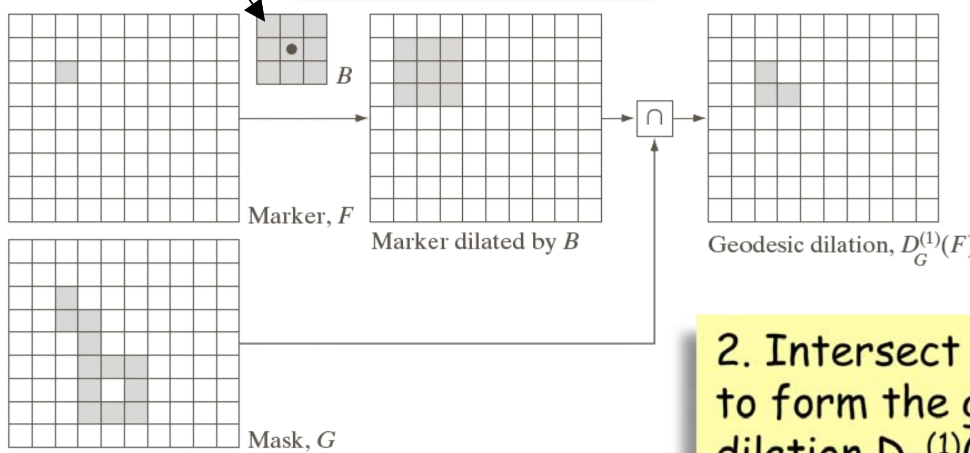
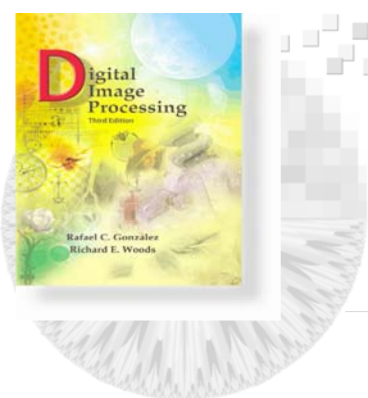


FIGURE 9.26 Illustration of geodesic dilation.

2. Intersect with mask G to form the geodesic dilation $D_G^{(1)}(F)$

This operation can be performed iteratively with the iteration determined by the superscript.



Geodesic Erosion

The geodesic erosion is similar to the geodesic dilation in that it is a erosion of a marker which is constrained by a mask

Erode marker image F by structuring element B

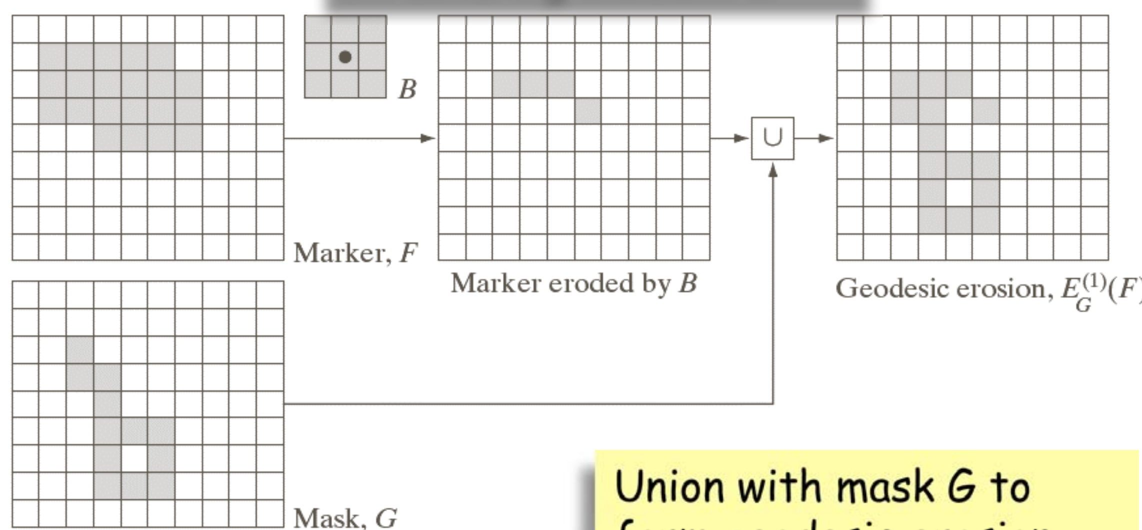
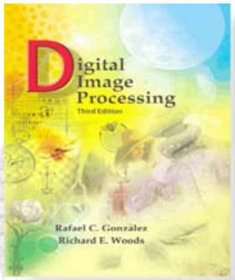


FIGURE 9.27 Illustration of geodesic erosion.

Union with mask G to form geodesic erosion $E_G^{(1)}(F)$

This operation can be performed iteratively with the iteration determined by the superscript.



Morphological Reconstruction

Dilate $D_G^{(1)}(F)$ by B;AND with mask G to get $D_G^{(2)}(F)$

Dilate $D_G^{(2)}(F)$ by B; AND with mask G to get $D_G^{(3)}(F)$

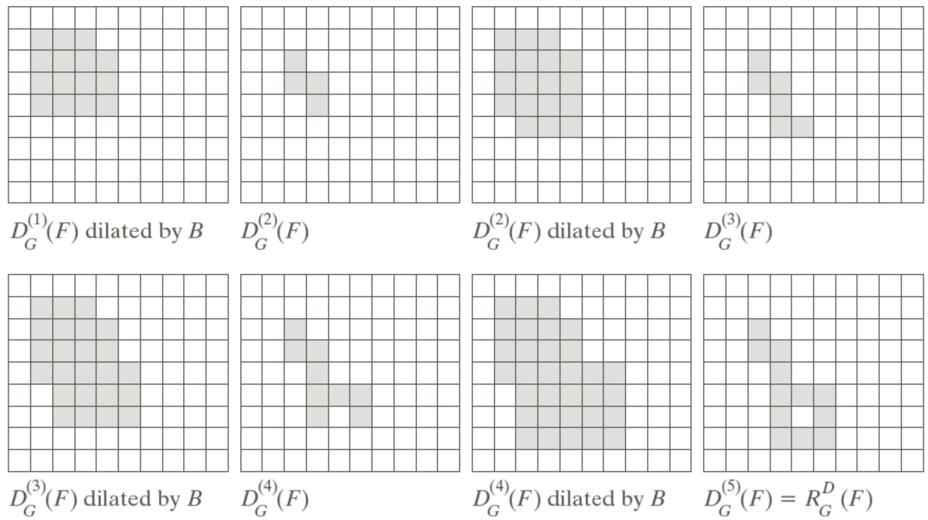
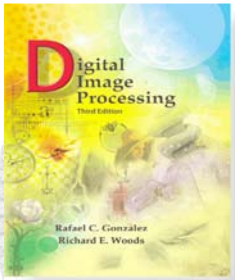


FIGURE 9.28 Illustration of morphological reconstruction by dilation. F, G, B and $D_G^{(1)}(F)$ are from Fig. 9.26.

Dilate $D_G^{(3)}(F)$ by B; AND with mask G to get $D_G^{(4)}(F)$

When $D_G^{(j+1)}(F)= D_G^{(j)}(F)$ we call this $R_G^D(F)$ the reconstruction of the mask by dilation

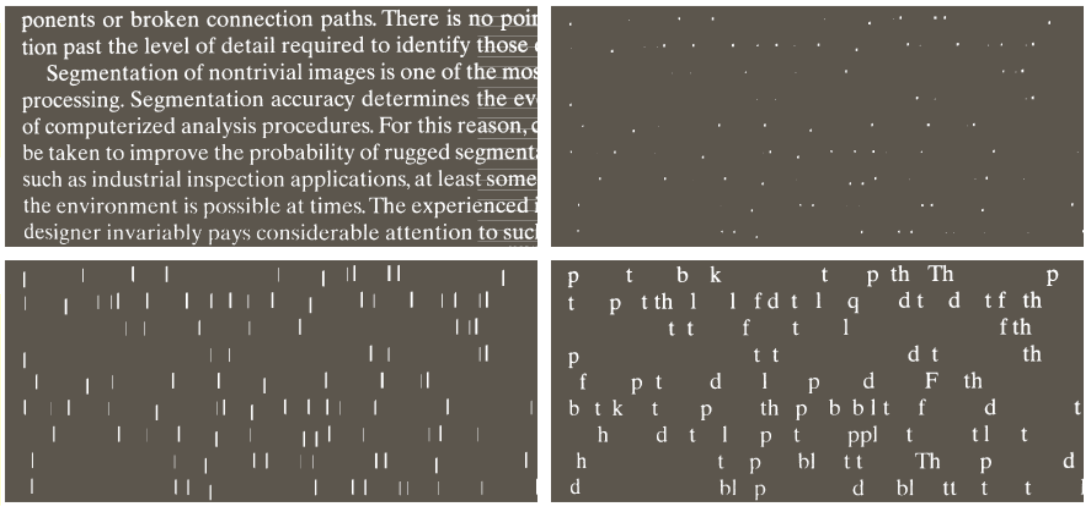
Geodesic dilation can be iterated until the image does not change. The resulting image is called the reconstruction of the mask by dilation.



Locating Tall Characters

Original image A; average character height is 50 pixels

Opening of A with a structuring element of size 51x1 simply gives the vertical stroke of each tall character



a b
c d

FIGURE 9.29 (a) Text image of size 918 × 2018 pixels. The approximate average height of the tall characters is 50 pixels. (b) Erosion of (a) with a structuring element of size 51 × 1 pixels. (c) Opening of (a) with the same structuring element, shown for reference. (d) Result of opening by reconstruction.

1. Erosion of A with a structuring element of size 51x1 to locate tall letters. This is the mask F

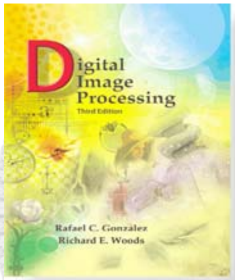
Opening by reconstruction of F with a structuring element of size 51x1

$$O_R^{(n)}(F) = R_F^D[F \ominus nB]$$

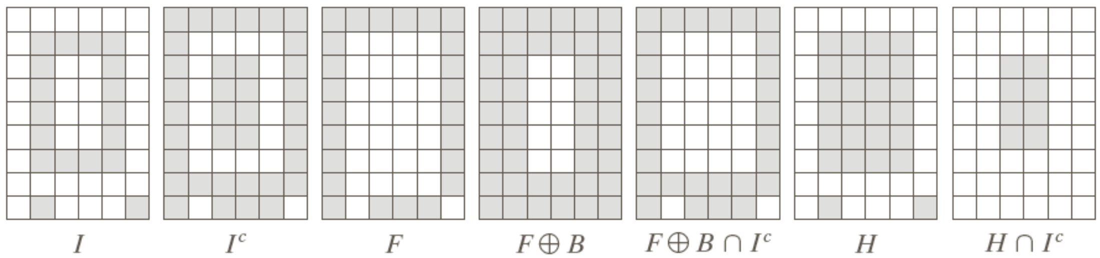
Do n erosions of F by B followed by morphological reconstruction by dilation. This example is for n=1

Goal is to find characters with tall, vertical strokes so we choose a structuring element of 51x1

Morphological reconstruction is a very powerful technique for finding complete objects in an image.



Hole Filling



a b c d e f g

FIGURE 9.30 Illustration of hole filling on a simple image.

Original image

1. Complement original image to build a wall of 1's around hole

2. Construct marker image F(x,y) : 1-I(x,y) on the image border and 0 everywhere else.

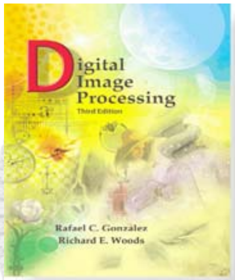
3. Dilate marker F with 3x3 SE of all 1's. Starts outside and works inward.

4. Geodesic dilation of marker image F using complement I^c as mask

5. Repeat dilation until no change in result so stop and complement. Holes are filled.

6. Intersect with complement of image to locate holes that were filled.

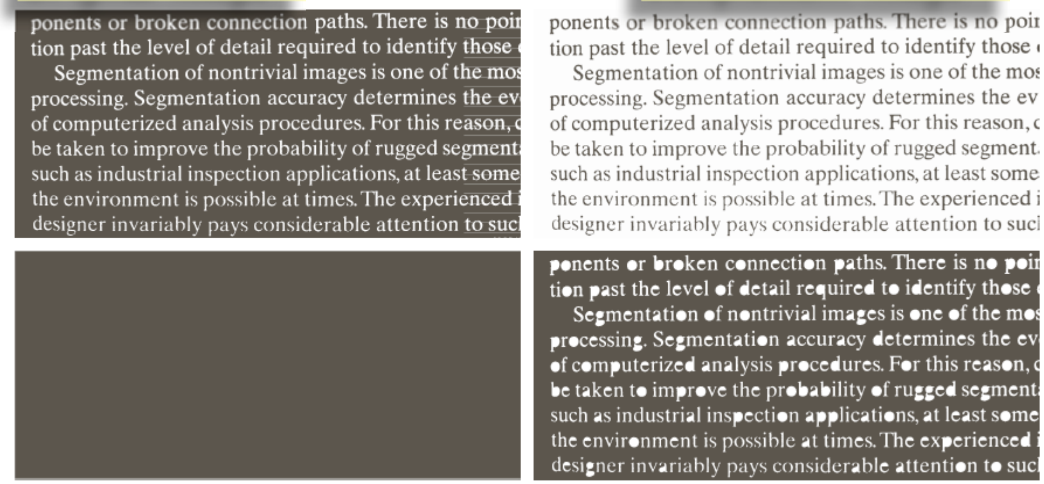
The goal of this example is to fill all holes in a simple image.



Morphological Reconstruction

Original image

Complement image



a b
c d

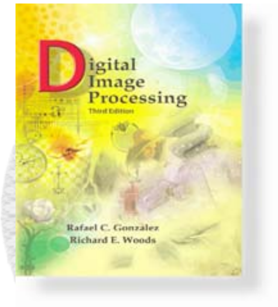
FIGURE 9.31 (a) Text image of size 918 × 2018 pixels. (b) Complement of (a) for use as a mask image. (c) Marker image. (d) Result of hole-filling using Eq. (9.5-29).

Marker image

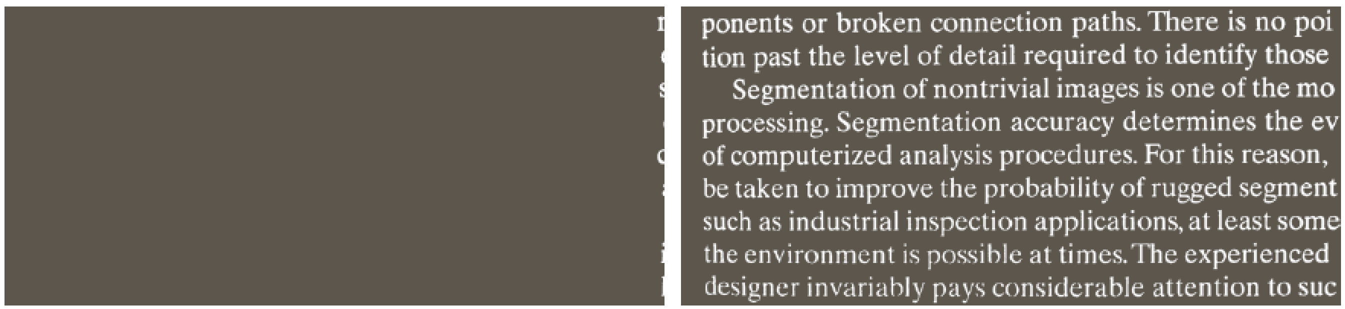
Hole filling using geodesic dilation

$$f(x,y)=\begin{cases}1-I(x,y) & \text{if } (x,y) \text{ is on image border}\\ 0 & \text{elsewhere}\end{cases}$$

This is an example of applying the hole filling algorithm to a more complex image.



Morphological Reconstruction


ponents or broken connection paths. There is no poi
tion past the level of detail required to identify those
Segmentation of nontrivial images is one of the mo
processing. Segmentation accuracy determines the ev
of computerized analysis procedures. For this reason,
be taken to improve the probability of rugged segment
such as industrial inspection applications, at least some
the environment is possible at times. The experienced
designer invariably pays considerable attention to suc

a b

FIGURE 9.32 Border clearing. (a) Marker image. (b) Image with no objects touching the border. The original image is Fig. 9.29(a).

Marker image

$$f(x,y)=\begin{cases} I(x,y) & \text{if } (x,y) \text{ is on image border} \\ 0 & \text{elsewhere} \end{cases}$$

Compute morphological reconstruction $R_I^D(F)$ using marker image to identify objects on edge.

Remove edge objects by simple subtraction $X=I-R_I^D(F)$

This is an example of removing objects on the edge of images.

Basic Binary Structuring Elements

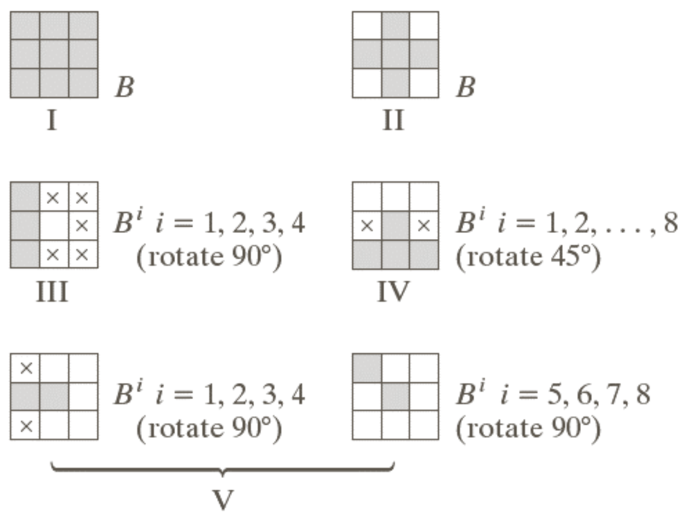
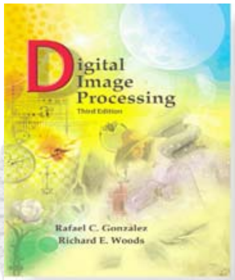


FIGURE 9.33 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.



Basic Morphological Operations

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Translation	$(B)_z = \{w \mid w = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \left\{z \mid (\hat{B}_z) \cap A \neq \varnothing\right\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \left\{z \mid (B)_z \subseteq A\right\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)

(*Continued*)

TABLE 9.1 Summary of morphological operations and their properties.

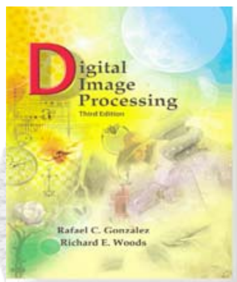


TABLE 9.1
(Continued)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $k = 1, 2, 3, \ldots$	Fills holes in A; X_0 = array of 0s with a 1 in each hole. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $k = 1, 2, 3, \ldots$	Finds connected components in A; X_0 = array of 0s with a 1 in each connected component. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)

(Continued)

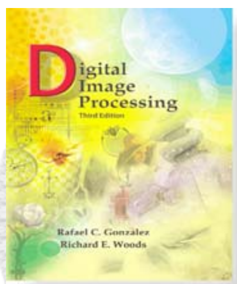
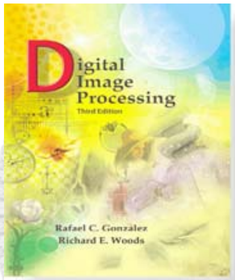


TABLE 9.1
(Continued)

Operation	**Equation**	**Comments** (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.
Geodesic dilation of size 1	$D_G^{(1)}(F) = (F \oplus B) \cap G$	F and G are called the *marker* and *mask* images, respectively.
Geodesic dilation of size n	$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right]$; $D_G^{(0)}(F) = F$	
Geodesic erosion of size 1	$E_G^{(1)}(F) = (F \ominus B) \cup G$	
Geodesic erosion of size n	$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right]$; $E_G^{(0)}(F) = F$	
Morphological reconstruction by dilation	$R_G^D(F) = D_G^{(k)}(F)$	k is such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$
Morphological reconstruction by erosion	$R_G^E(F) = E_G^{(k)}(F)$	k is such that $E_G^{(k)}(F) = E_G^{(k+1)}(F)$
Opening by reconstruction	$O_R^{(n)}(F) = R_F^D[(F \ominus nB)]$	$(F \ominus nB)$ indicates n erosions of F by B.
Closing by reconstruction	$C_R^{(n)}(F) = R_F^E[(F \oplus nB)]$	$(F \oplus nB)$ indicates n dilations of F by B.
Hole filling	$H = \left[R_{I^c}^D(F)\right]^c$	H is equal to the input image I, but with all holes filled. See Eq. (9.5-28) for the definition of the marker image F.
Border clearing	$X = I - R_I^D(F)$	X is equal to the input image I, but with all objects that touch (are connected to) the boundary removed. See Eq. (9.5-30) for the definition of the marker image F.



MATLAB/Image Processing Toolbox

```
MATLAB morphological operations

>> f=imread(‘Fig0911(a)(fingerprint_cleaned).tif’);% load in noisy fingerprint
>> se=strel(‘square’,3);                     % 3x3 structuring element
>> fo=imopen(f,se);                          % open image
>> imshow(f), figure, imshow(fo)             % show both images

>> foc=imclose(fo,se);                       % now close the image
>> figure, imshow(foc)                       % show both images

% strel can be used to make structuring elements of common shapes as
% well as completely arbitrary shapes

% imopen and imclose implement opening and closing
% imerode and imdilate implement erosion and dilation


SEE GWE, Section 9.2 Erosion and Dilation
GWE, Section 9.3 Combining Erosion and Dilation
```

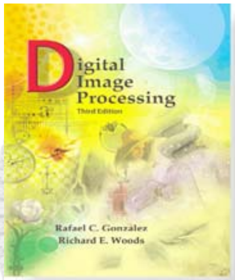
MATLAB/Image Processing Toolbox

MATLAB morphological operations

```
>> h=imread('fingerprint_clean.tif');% load in fingerprint
>> h1=bwmorph(f,'thin',1);            % apply thinning once
>> h2=bwmorph(f,'thin',2);            % apply thinning twice
>> imshow(h1), figure, imshow(h2)     % show both images
% you can actually apply thinning 'inf' times

% the bwmorph function can be used to implement erode, dilate,
% open, close, shrink, skeleton, top-hat and many other
% morphological operations
```

SEE GWE, Section 9.3.4 Function **bwmorph**



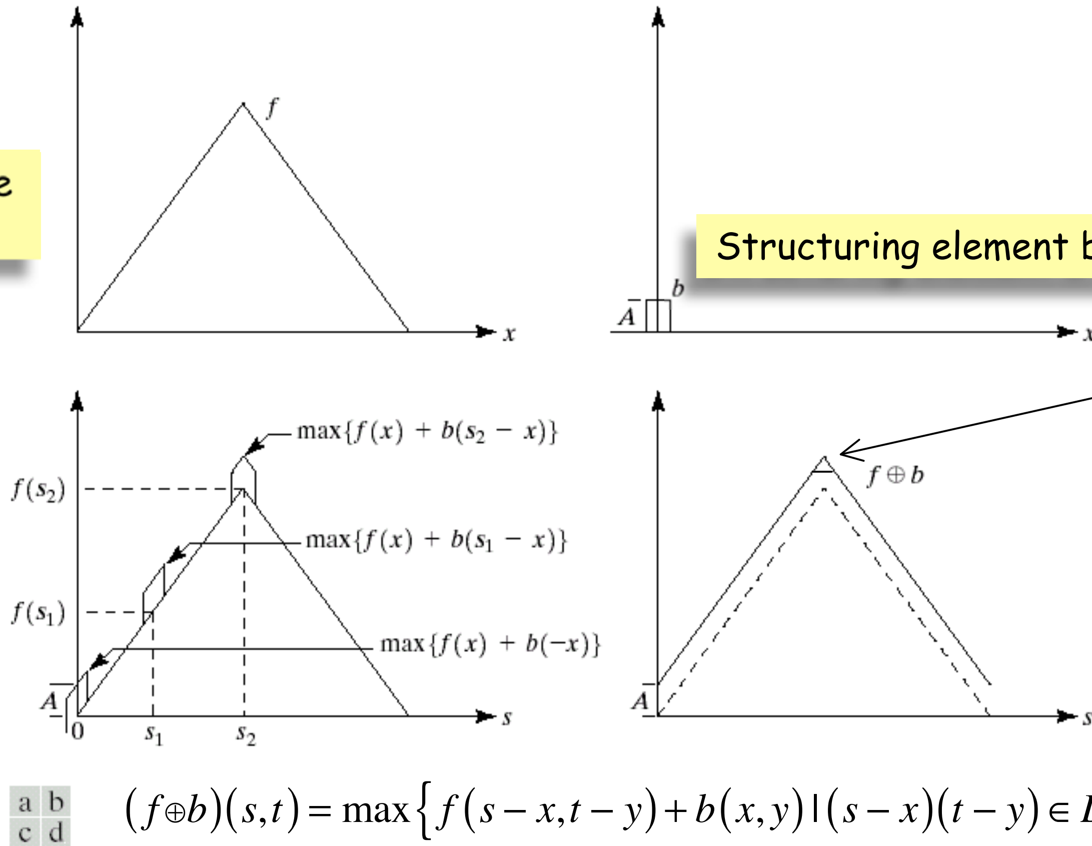
Gray Scale Morphology

gray-scale image function f

Structuring element b

Drawing Error: top should be flat for a distance b

Dilate by sliding the structuring element over the gray-scale function. Taking the max pushes the function up.

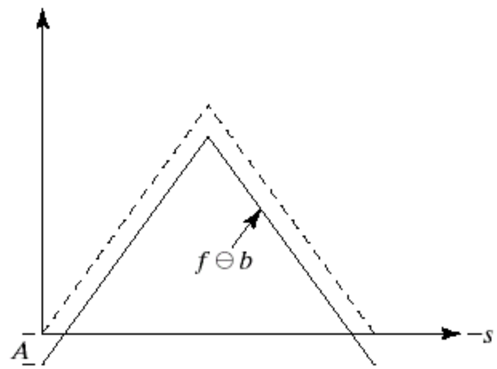


$$(f \oplus b)(s,t) = \max\left\{ f(s-x,t-y) + b(x,y) \mid (s-x)(t-y) \in D_f ; (x,y) \in D_b \right\}$$

FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).

Gray Scale Morphology

FIGURE 9.28 Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).

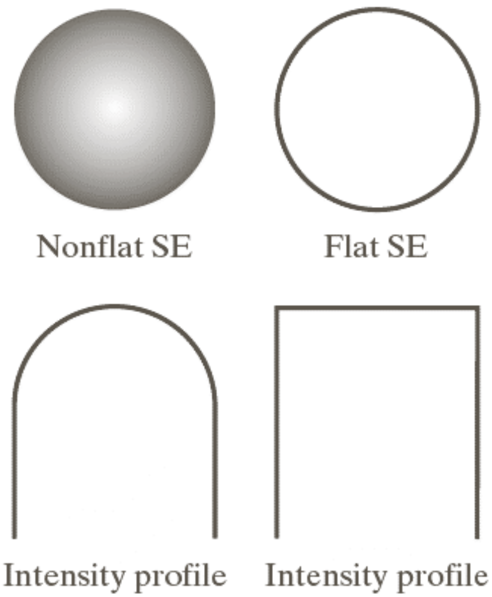


Erode by sliding the structuring element b over the gray-scale function f. Taking the min pushes the function down.

$$(f \ominus b)(s,t) = \min\left\{ f(s+x, t+y) - b(x,y) \mid (s+x)(t+y) \in D_f ; (x,y) \in D_b \right\}$$

Gray-Scale Morphology

Gray-scale structuring elements can have both geometry (connectivity) and intensity.



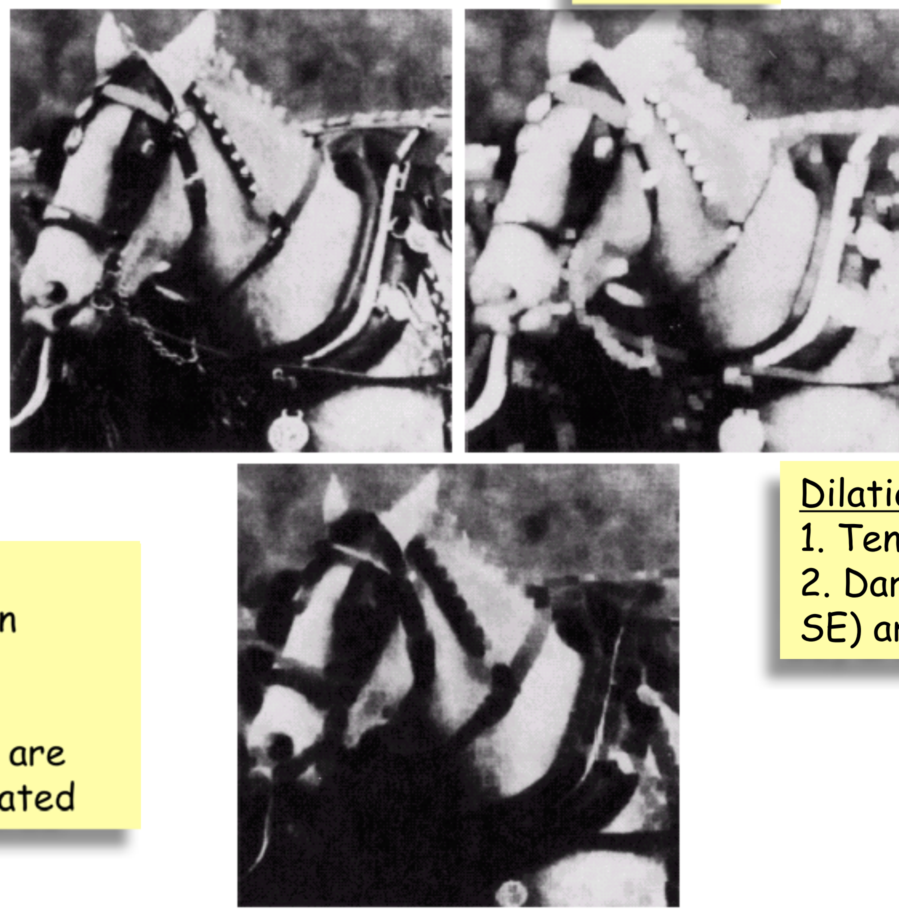
a b
c d

FIGURE 9.34 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their center. All examples in this section are based on flat SEs.

Gray Scale Morphology

dilated

original



a b
c

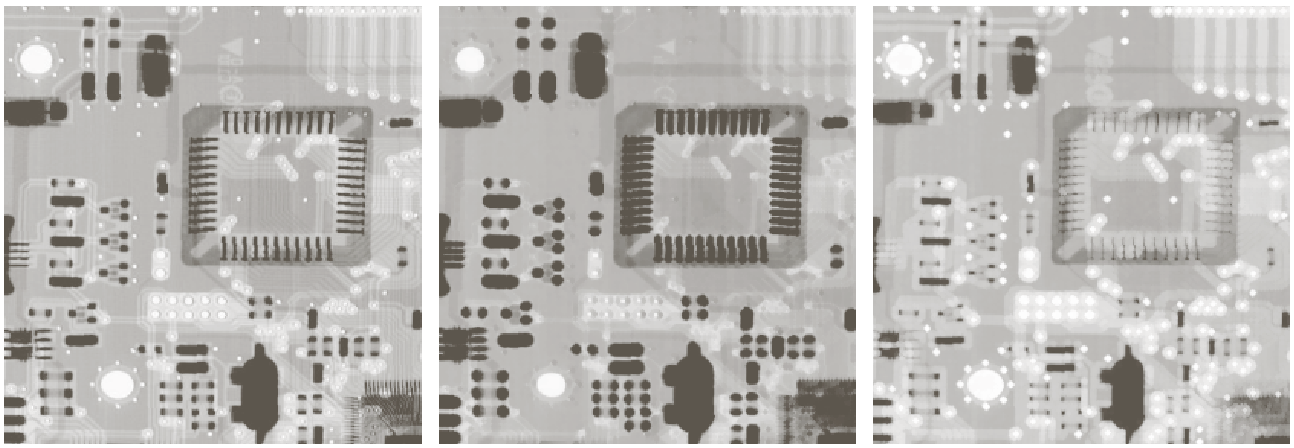
FIGURE 9.29 (a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

eroded

<u>Erosion</u>
1. Tends to darken positive images
2. Bright details (smaller than SE) are reduced or eliminated

<u>Dilation</u>
1. Tends to brighten images
2. Dark details (smaller than SE) are reduced or eliminated

Gray-Scale Morphology



a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

Erosion using a 2-pixel disk SE

Dilation using a 2-pixel disk SE

Gray Scale Morphology

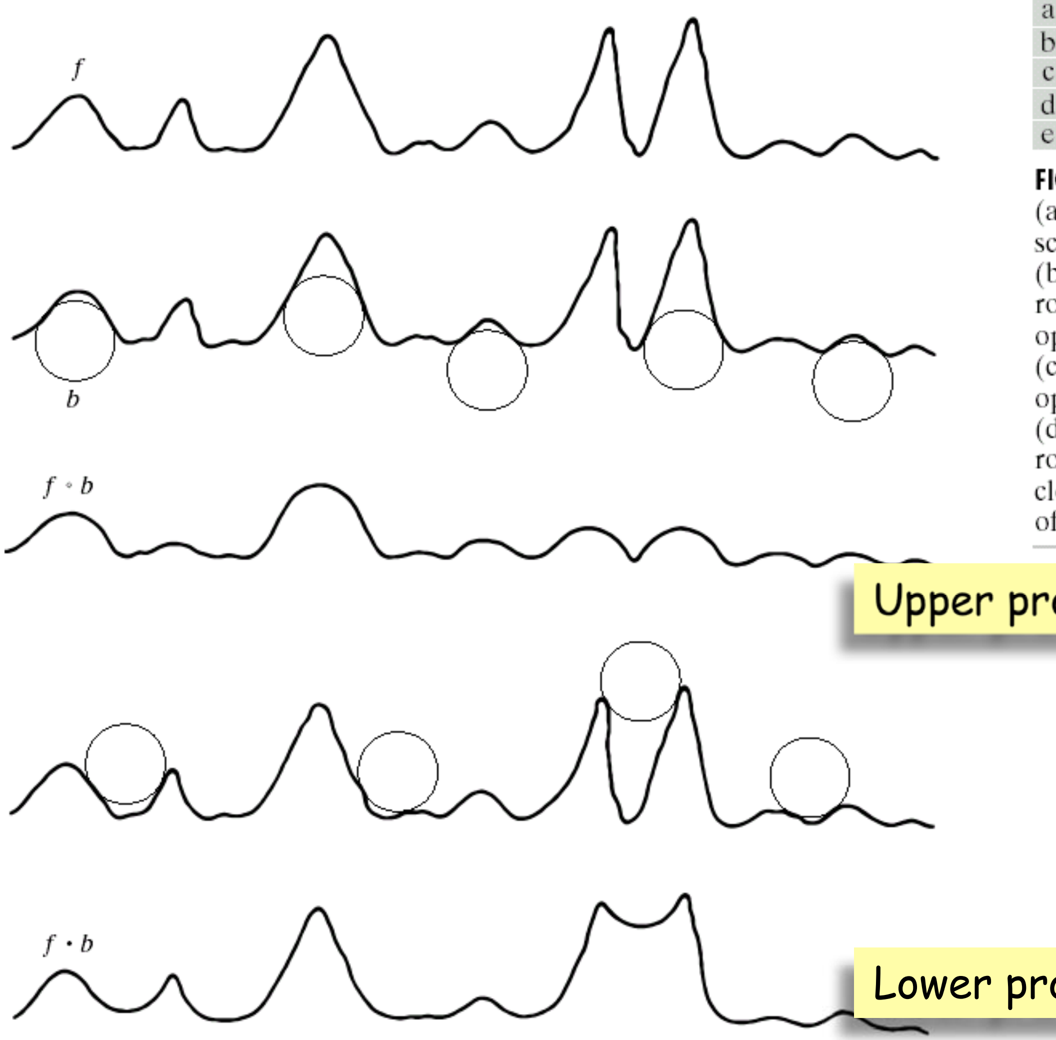


FIGURE 9.30
(a) A gray-scale scan line.
(b) Positions of rolling ball for opening.
(c) Result of opening.
(d) Positions of rolling ball for closing. (e) Result of closing.

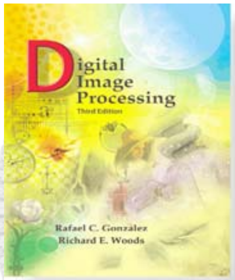
original

Opening
"sharp" details reduced in amplitude and sharpness

Upper profile of all balls

Closing
Removes "dark" details while leaving bright alone

Lower profile of all balls



Gray-Scale Morphology

Opening and closing with flat-top element

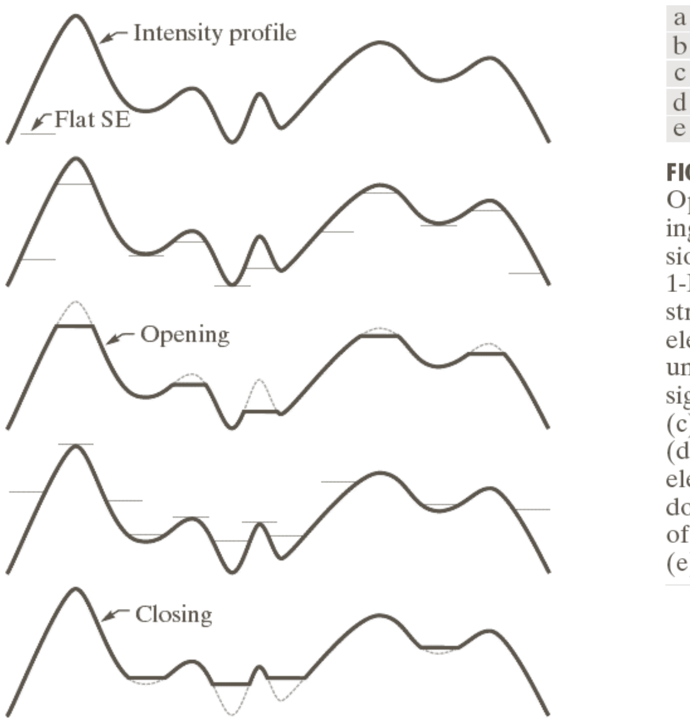
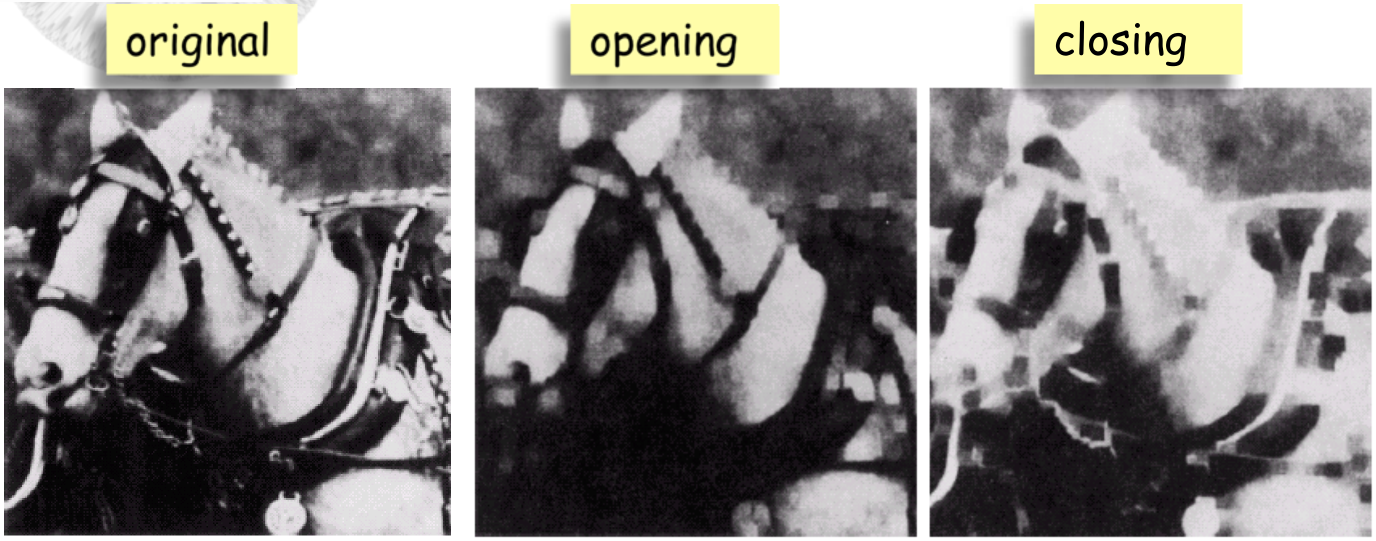


FIGURE 9.36 Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal. (c) Opening. (d) Flat structuring element pushed down along the top of the signal. (e) Closing.

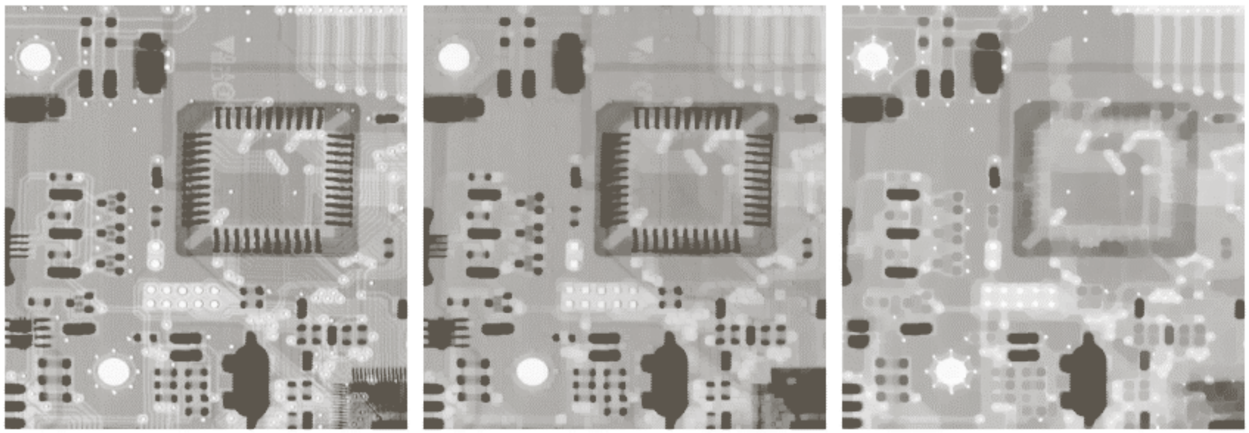
Gray Scale Morphology



a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Gray-Scale Morphology



a b c

FIGURE 9.37 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

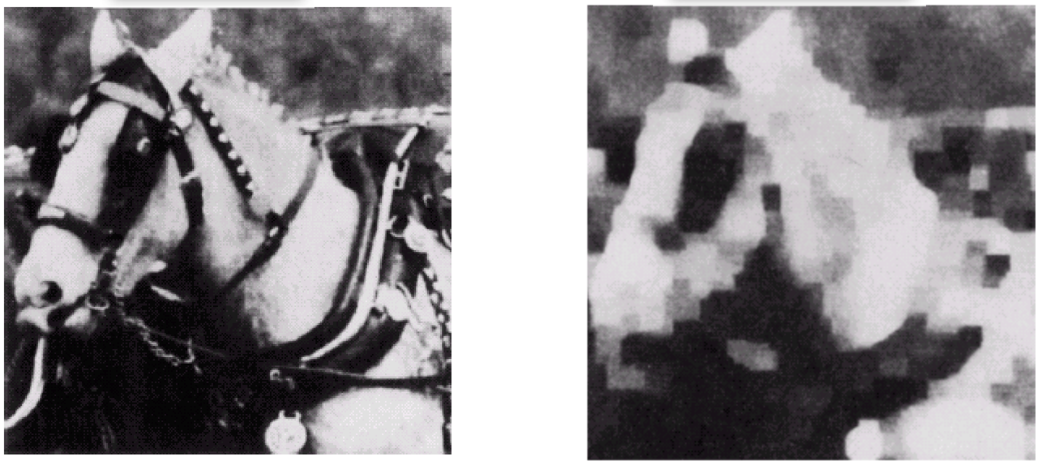
Opening using a 3-pixel disk SE

Closing using a 5-pixel disk SE

Gray Scale Morphology

original

smoothed

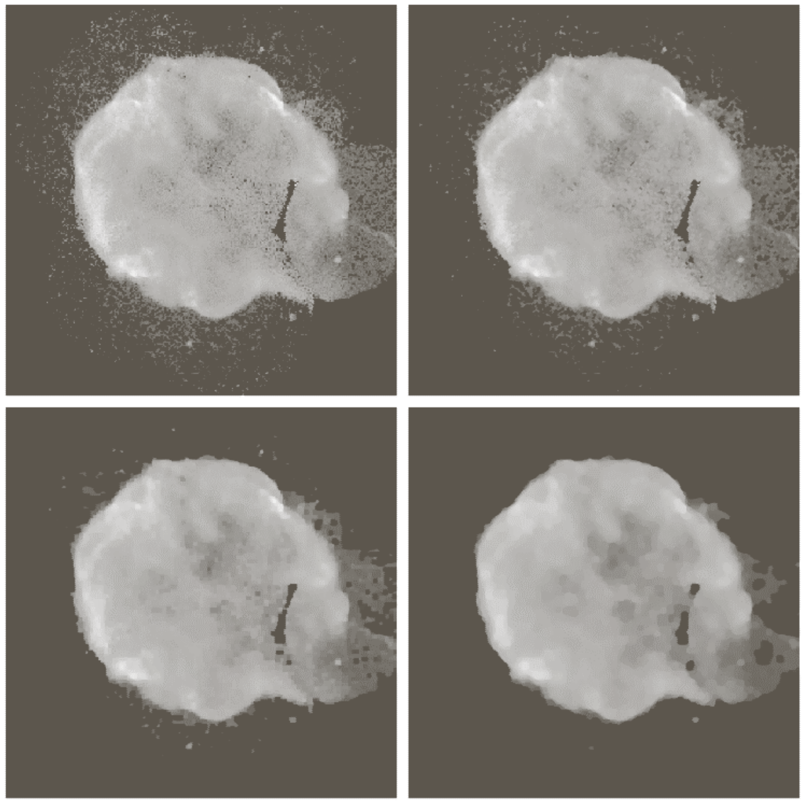


Smoothing
Opening followed by a closing removes "bright" and "dark" details

FIGURE 9.32 Morphological smoothing of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Gray-Scale Morphology

Opening followed by closing ("smoothing") using disk structuring elements of different radii

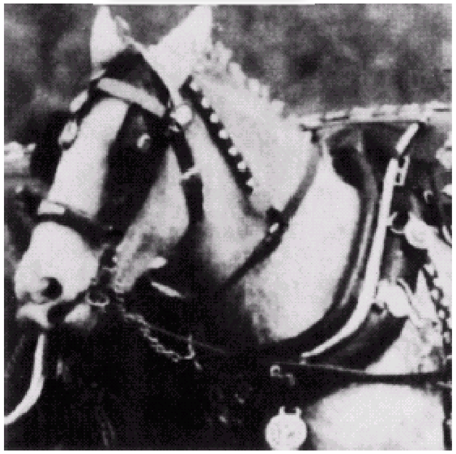


a b
c d

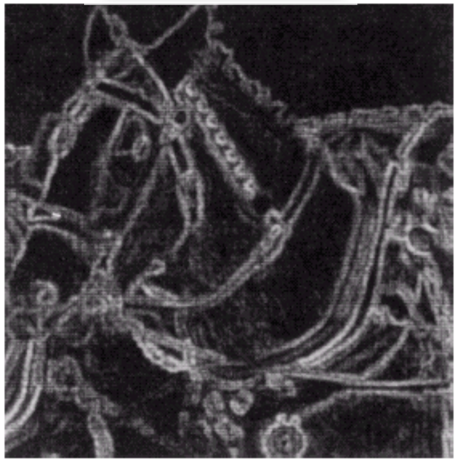
FIGURE 9.38 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

Gray Scale Morphology

original



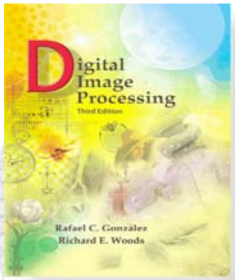
"gradient"



$$g = (f \oplus b) - (f \ominus b)$$

An erosion subtracted from a dilation will give a morphological gradient

FIGURE 9.33 Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)



Gray-Scale Morphology

Dilation

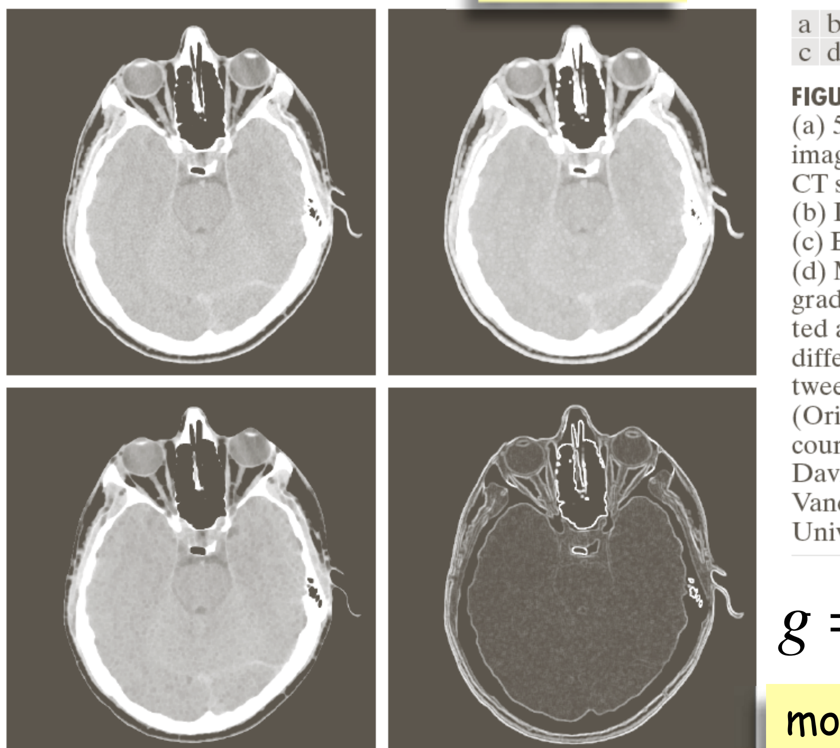


FIGURE 9.39 (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

Erosion

$$g = \left(f \oplus b\right) - \left(f \ominus b\right)$$

morphological gradient

Gray Scale Morphology

original

"top-hat"

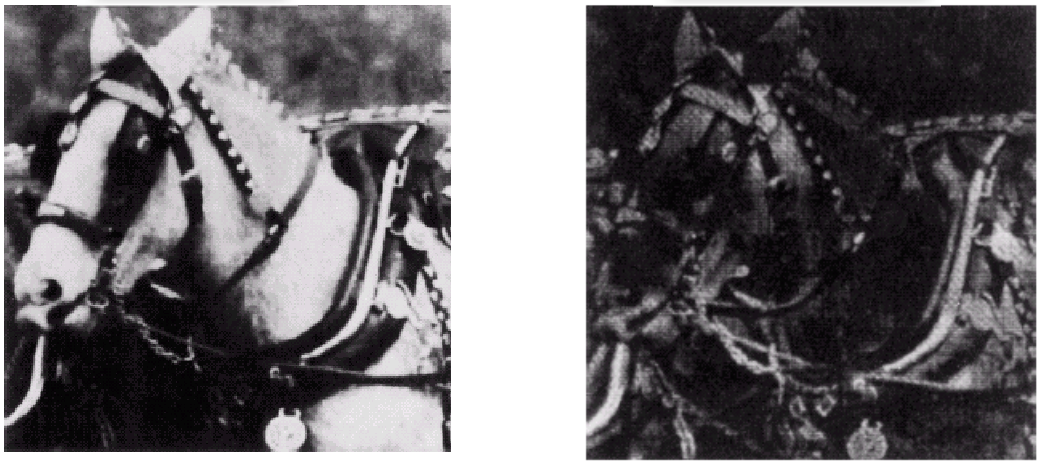


FIGURE 9.34 Result of performing a top-hat transformation on the image of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

$$h = f - (f \circ b)$$

A top-hat transformation subtracts an opening from the image.

Enhances detail in the presence of shading

Gray-Scale Morphology

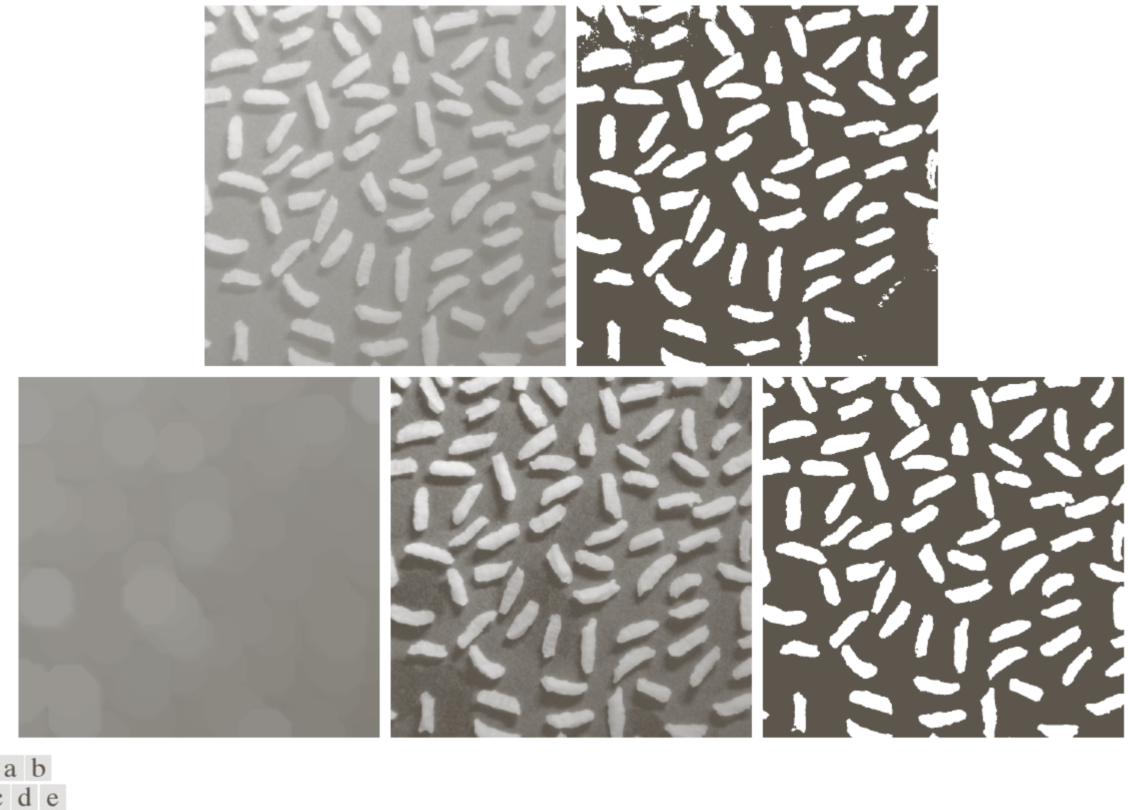


FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600 × 600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.