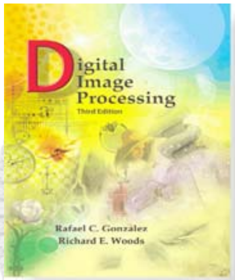
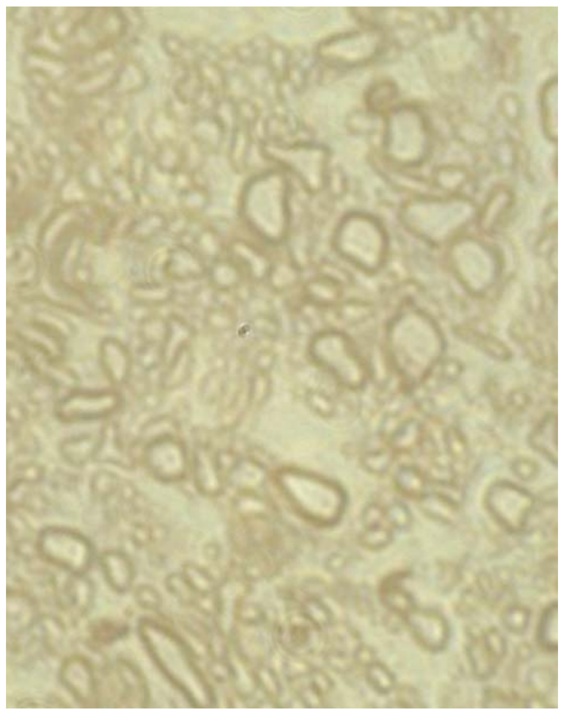


Lecture #15

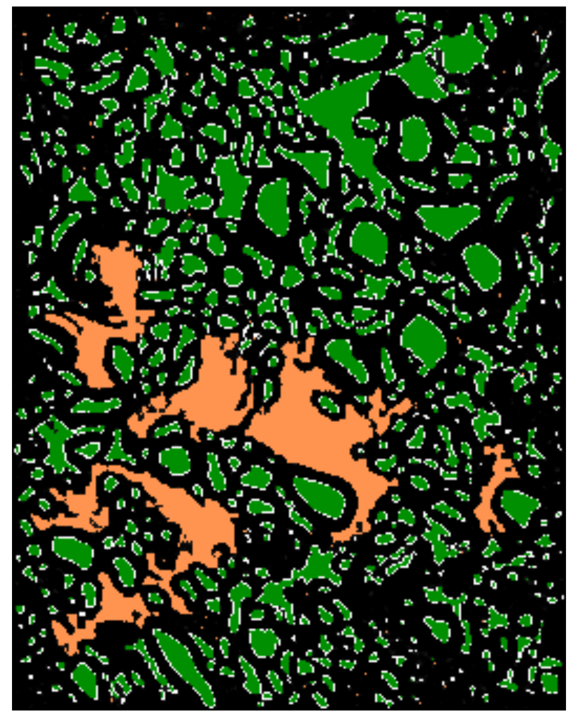
- Adaptive Noise Reduction Filters
- Bandreject and Notch filters
- Optimum Notch Filter
- Modeling Image Degradation
- Inverse filtering
- Wiener Filters

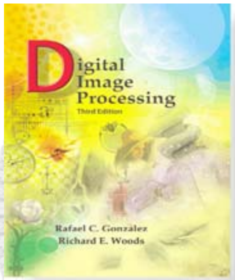
Mid-Term Project

Mouse visual cortex neuron bundle



Example of a segmented image with errors





Adaptive Mean Filter

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

σ_η^2 Noise variance over the entire image (estimated)

m_L Local mean (calculated)

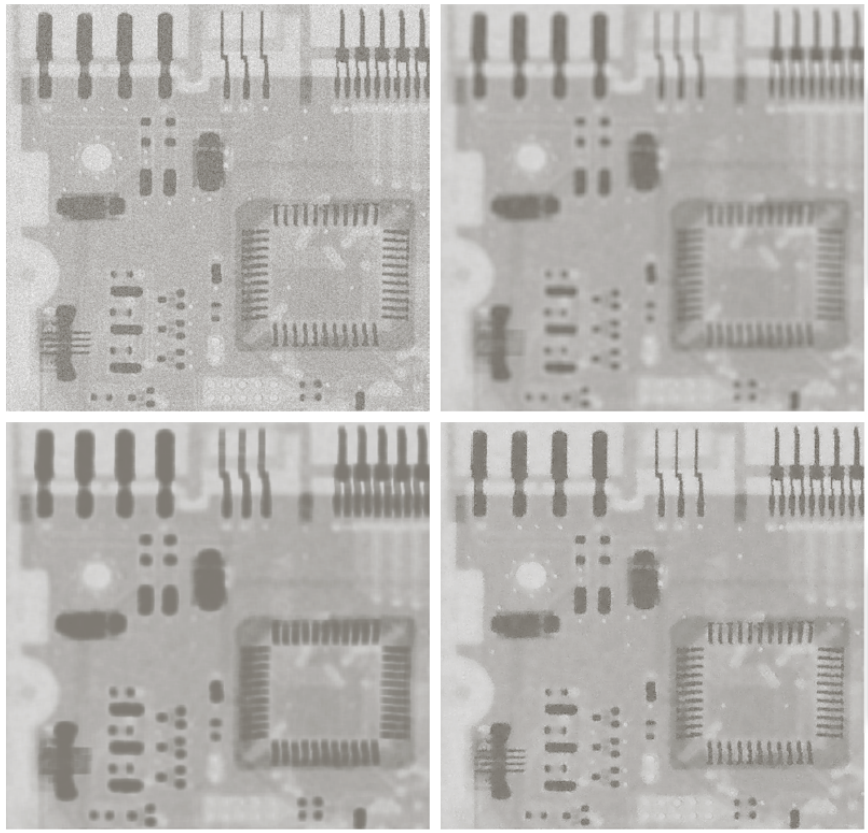
σ_η^2 Local variance (calculated)

DIFFERENT CASES

- If $\sigma_\eta^2 = \sigma_L^2$ the filter returns the local mean thus averaging out the noise
- If $\sigma_\eta^2 << \sigma_L^2$ this is probably the location of an edge and we should return the edge value, i.e., g(x,y)
- If $\sigma_\eta^2 = 0$ there is no noise and we return g(x,y)
- If $\sigma_\eta^2 > \sigma_L^2$ we can get negative gray scale values which is a potential problem

Adaptive Mean Filter

Image +zero-mean Gaussian noise



a b
c d

ian

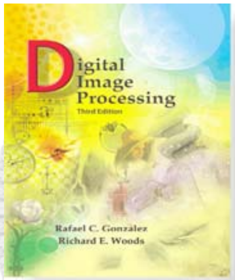
variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7 × 7.

Visible blurring with 7x7 arithmetic mean filter

7x7 geometric mean filter

7x7 adaptive noise reduction filter

Performance can decrease if the estimated overall noise variance σ_η^2 is incorrect



Adaptive Median Filter

varies S_{xy} to reduce impulsive noise

StageA: IF $z_{med} > z_{min}$ and $z_{max} > z_{med}$
    THEN goto StageB
    ELSE increase the window size S_{xy}
IF WindowSize $\leq S_{max}$
    THEN goto LevelA
    ELSE output z_{med}

StageB: IF $z_{xy} > z_{min}$ and $z_{max} > z_{xy}$
    THEN output z_{xy}
    ELSE output z_{med}

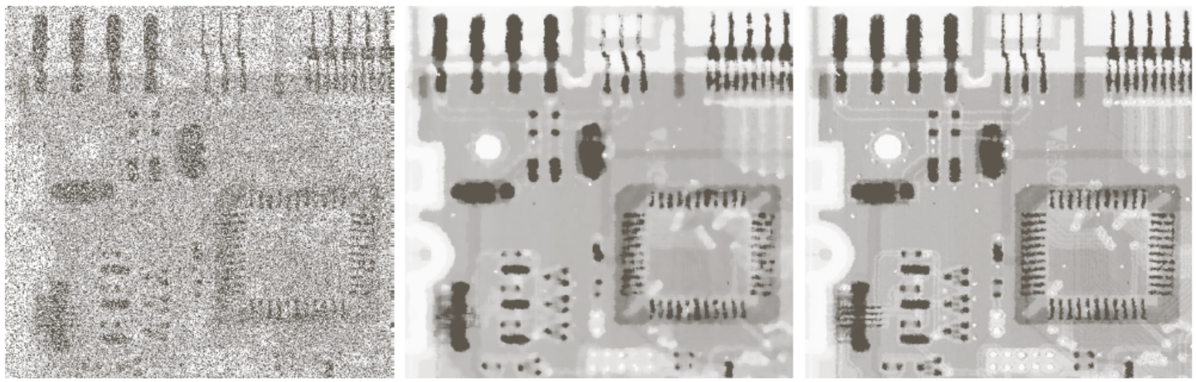
If $z_{max} > z_{med} > z_{min}$ then z_{med} is NOT an impulse and we go to StageB. Otherwise StageA continues to increase the neighborhood S_{xy} until z_{med} is not an impulse.

If $z_{max} > z_{xy} > z_{min}$ then z_{xy} is NOT an impulse and we output z_{xy} otherwise we output the median z_{med}.

z_{min}	min. gray value in S_{xy}
z_{max}	max. gray value in S_{xy}
z_{med}	median gray value in S_{xy}
z_{xy}	gray level value at (x,y)
S_{max}	max. allowed size of S_{xy}

The fundamental idea is to increase the size of the neighborhood S_{xy} until we are sufficiently sure that z_{xy} is impulsive or not. If it is impulsive then output the median otherwise output z_{xy}

Adaptive Median Filter



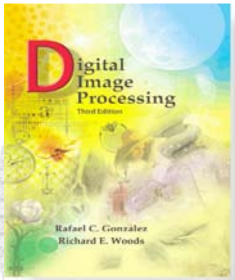
a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

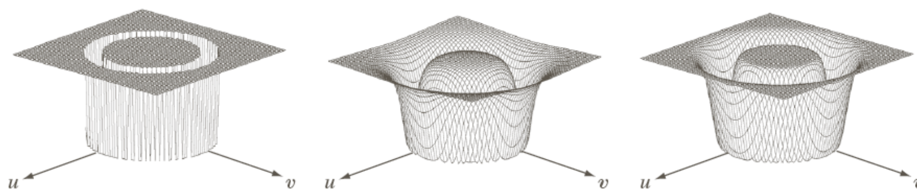
Image + "lots" of salt & pepper noise

7x7 median filtering with loss of detail

Adaptive median filtering (S_{max}=7) with much better detail



Bandreject Filters



a b c

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

Ideal **Butterworth** **Gaussian**

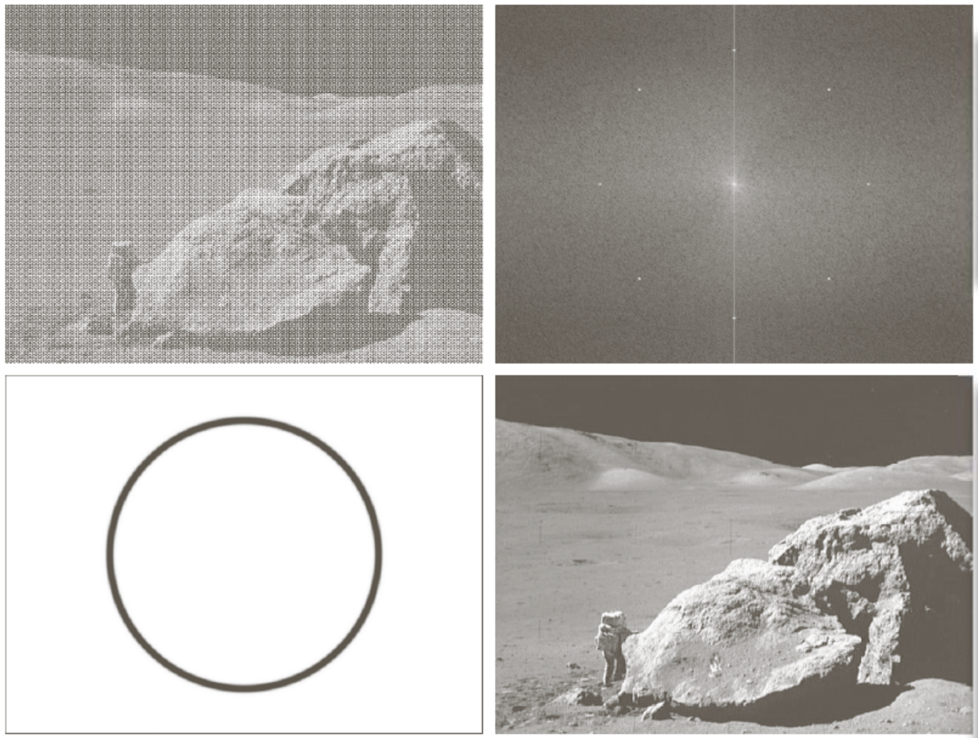
$$H(u,v)=\begin{cases}1 & D(u,v)<\left(D_0-\frac{W}{2}\right)\\ 0 & \left(D_0-\frac{W}{2}\right)<D(u,v)\le\left(D_0+\frac{W}{2}\right)\\ 1 & D(u,v)>\left(D_0+\frac{W}{2}\right)\end{cases}$$

$$H(u,v)=\frac{1}{1+\left[\frac{D(u,v)W}{D^2(u,v)-D_0^2}\right]^{2n}}$$

$$H(u,v)=1-e^{-\frac{1}{2}\left[\frac{D^2(u,v)-D_0^2}{D(u,v)W}\right]^2}$$

D_0 is center of stopband; W is full width of stopband

Bandreject Filtering



a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

Notice strong frequency components in a ring

Butterworth bandreject filter

You cannot get such impressive improvement using a spatial domain approach with small filter masks.

Bandreject filters are not typically used because they can remove too much image detail.

Bandpass Filtering

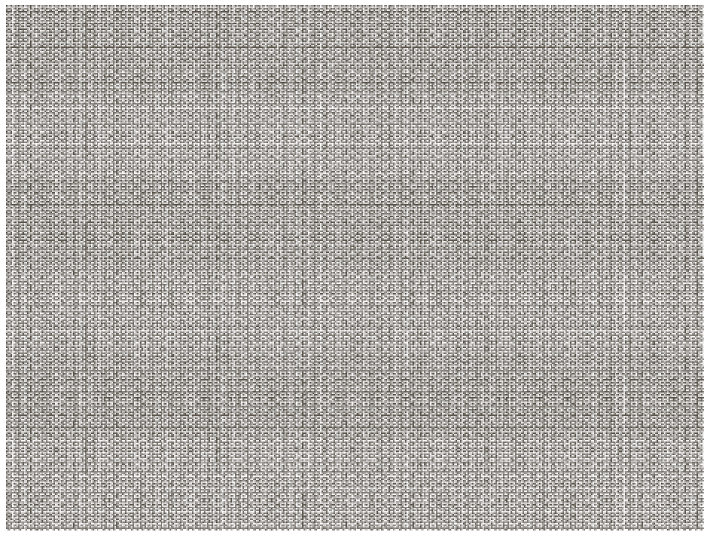
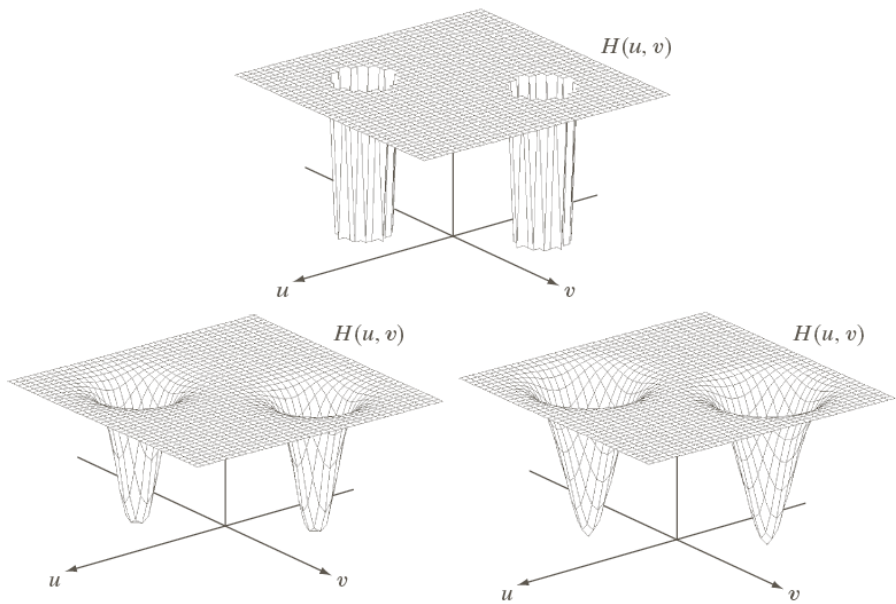


FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.

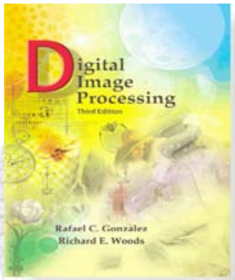
Convert Butterworth bandreject filter on previous page to Butterworth bandpass filter: $H_{bandpass}(u,v)=1-H_{bandreject}(u,v)$. Above image is the result of filtering noisy image with the Butterworth bandpass filter — it is the periodic noise in the spatial domain.

Notch Filters



a
b c

FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.



Notch Filtering

Fourier spectra of original image showing periodic noise on v axis

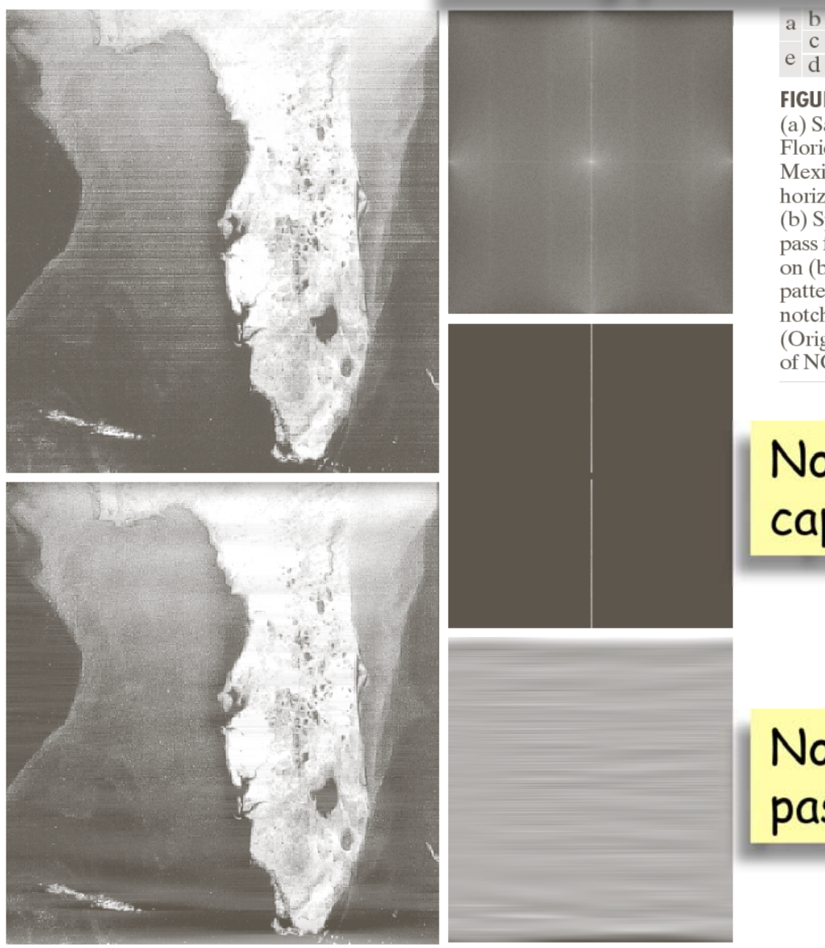
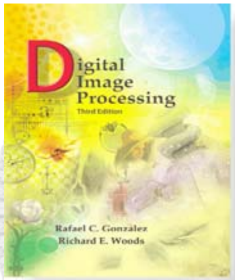


FIGURE 5.19 (a) Satellite image of Florida and the Gulf of Mexico showing horizontal scan lines. (b) Spectrum. (c) Notch pass filter superimposed on (b). (d) Spatial noise pattern. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)

Notch pass filter to capture noise

Noise captured by notch pass filter

Image after notch reject filtering



Optimum Notch Filter

When several interference components are present or if the interference has broad skirts a simply notch filter may remove too much image information.

One solution is to use an optimum filter which minimizes local variances of the restored estimate.

Such "smoothness" constraints are often found in optimum filter design

Optimum Notch Filter

1. Manually place a notch pass filter H_{NP} at each noise spike in the frequency domain. The Fourier transform of the interference noise pattern is

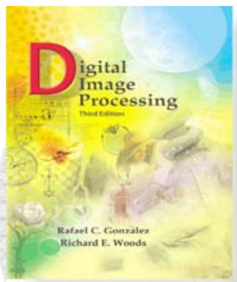
$$N(u,v) = H_{NP}(u,v)G(u,v)$$

2. Determine the noise pattern in the spatial domain

$$\eta(x,y) = \mathcal{F}^{-1}\left\{H_{NP}(u,v)G(u,v)\right\}$$

3. Conventional thinking would be to simply eliminate noise by subtracting the periodic noise from the noisy image

$$\hat{f}(x,y) = g(x,y) - \eta(x,y)$$



Optimum Notch Filter

4. To construct an optimal filter consider

$$\hat{f}(x,y) = g(x,y) - w(x,y)\eta(x,y)$$

where w(x,y) is a weighting function.

5. We use the weighting function w(x,y) to minimize the variance $\sigma^2(x,y)$ of $\hat{f}(x,y)$ with respect to w(x,y)

$$w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \bar{g}(x,y)\bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - \bar{\eta}^2(x,y)}$$

We only need to compute this for one point in each nonoverlapping neighborhood.

Optimum Notch Filter

$$w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \overline{g}(x,y)\overline{\eta}(x,y)}{\overline{\eta^2}(x,y) - \overline{\eta}^2(x,y)}$$

$\overline{\eta(x,y)}$ Mean noise output from the notch filter

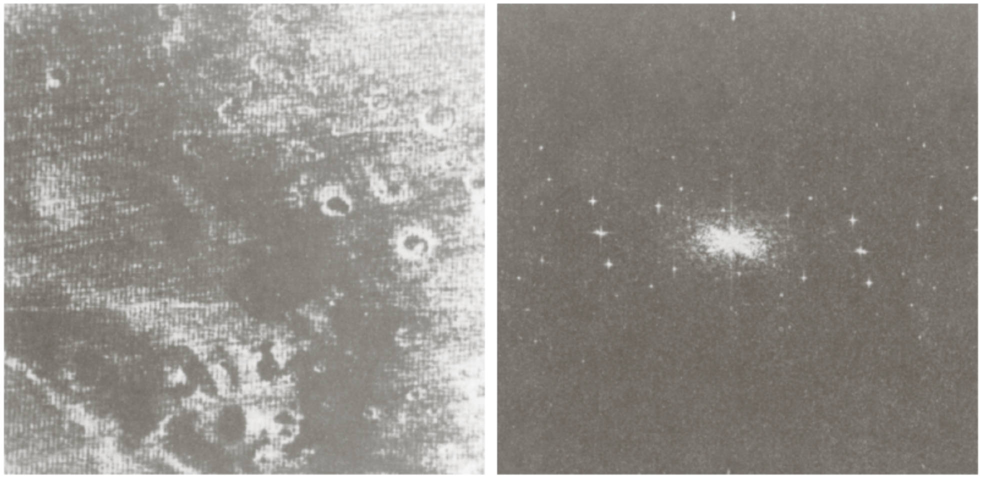
$\overline{\eta^2}(x,y)$ Mean squared noise output from the notch filter

$\overline{\eta}^2(x,y)$ Squared mean noise output from the notch filter

$\overline{g(x,y)}$ Mean noisy image

$\overline{g(x,y)\eta(x,y)}$ Mean product of noisy image and noise

Image Restoration



a b

FIGURE 5.20
(a) Image of the Martian terrain taken by *Mariner 6*.
(b) Fourier spectrum showing periodic interference.
(Courtesy of NASA.)

Centered Fourier transform showing many strong periodic interferers.

Image Restoration

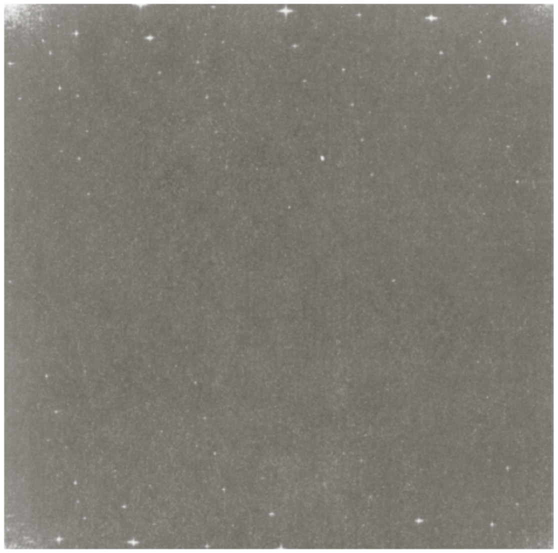
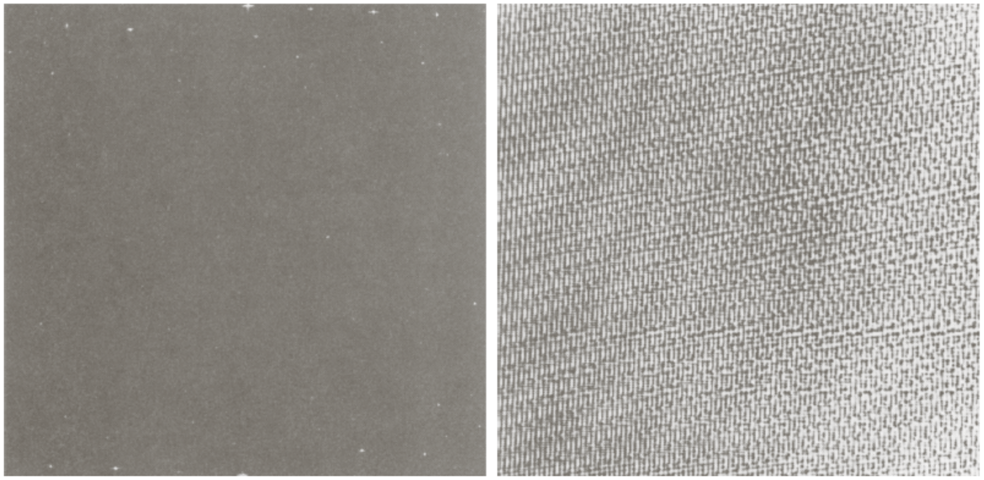


FIGURE 5.21 Fourier spectrum (without shifting) of the image shown in Fig. 5.20(a). (Courtesy of NASA.)

(noncentered) Fourier transform showing the same strong periodic interferers.

Image Restoration



a b

FIGURE 5.22 (a) Fourier spectrum of $N(u, v)$, and (b) corresponding noise interference pattern $\eta(x, y)$. (Courtesy of NASA.)

Fourier spectrum N(u,v) of the noise and its corresponding noise pattern $\eta(x,y)$

Image Restoration

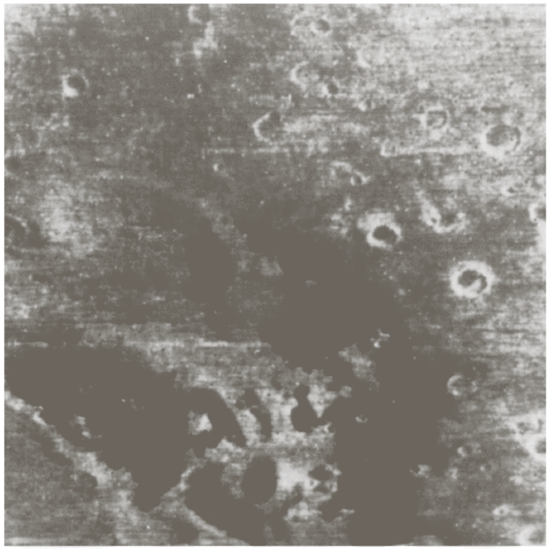


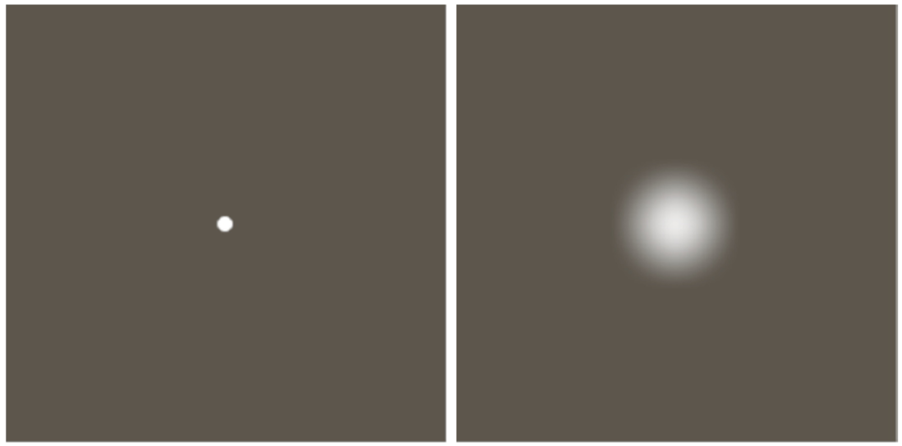
FIGURE 5.23 Processed image. (Courtesy of NASA.)

"Optimum" image constructed by subtracting weighted periodic noise.

Characterization of Degradation

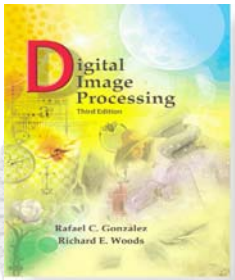
a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.



Original beam of light such as from a laser

Blurring due to passing through an optical system



Modeling of Degradation

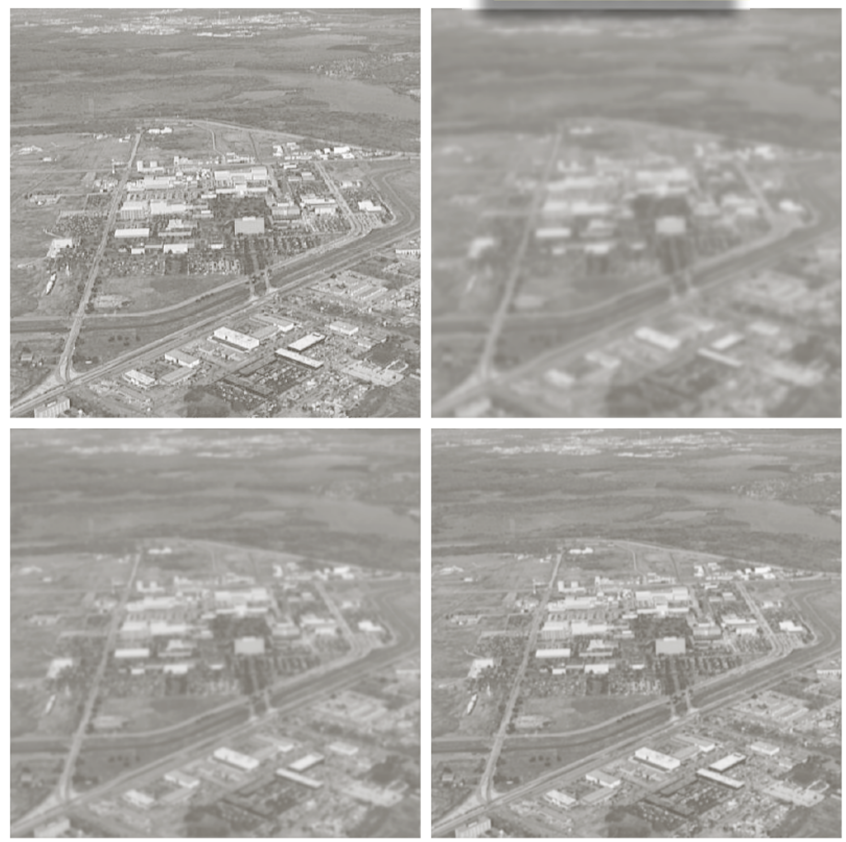
$$H(u,v) = e^{-k\left(u^2+v^2\right)^{\frac{5}{6}}}$$

Atmospheric turbulence degradation model

a b
c d

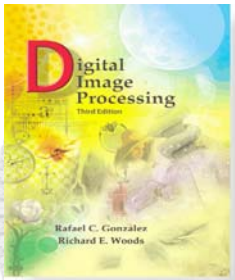
FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

K=0.0025



K=0.001

K=0.00025



Motion Blurring

Model motion in x- and y-directions over a period T for an integrating detector such as a camera.

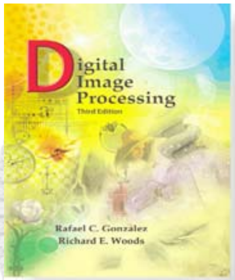
$$g(x,y)=\int_0^T f\left[x-x_0(t),y-y_0(t)\right]dt$$

Fourier transform g(x,y) and reverse order of integration

$$G(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-j2\pi(ux+vy)}dxdy$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_0^T f\left[x-x_0(t),y-y_0(t)\right]dt\right]e^{-j2\pi(ux+vy)}dxdy$$

$$G(u,v)=\int_0^T\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f\left[x-x_0(t),y-y_0(t)\right]e^{-j2\pi(ux+vy)}dxdy\right]dt$$



Motion Blurring

Replace innter term by F(u,v), the Fourier transform of f(x,y)

$$G(u,v)=\int_0^T\left[F(u,v)e^{-j2\pi(ux_0(t)+vy_0(t))}dxdy\right]dt=F(u,v)\int_0^T e^{-j2\pi(ux_0(t)+vy_0(t))}dt$$

Identify the motion blurring transfer function as

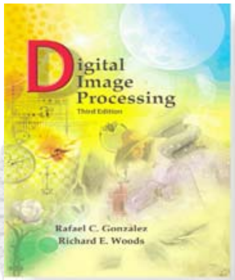
$$H(u,v)=\int_0^T e^{-j2\pi(ux_0(t)+vy_0(t))}dt$$

We can then model motion degradation as

$$G(u,v)=H(u,v)F(u,v)$$

Where, for $x_0(t)=at/T$, $y_0(t)=0$

$$H(u,v)=\int_0^T e^{-j2\pi ux_0(t)}dt=\int_0^T e^{-j2\pi u\frac{at}{T}}dt=\frac{T}{\pi ua}\sin(\pi ua)e^{-j\pi ua}$$



Modeling Image Degradation

Original image (1st edition cover)

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Motion blurring with a=b=0.1 and T=1

Motion blurring transfer function

$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin\left[\pi(ua+vb)\right] e^{-j\pi(ua+vb)}$$

Inverse Filtering

- If degraded image is given by degradation + noise

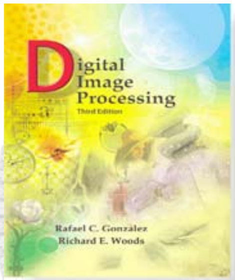
$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

- Estimate the image by dividing by the degradation function H(u,v)

$$\tilde{F}(u,v) = \frac{G(u,v)}{H(u,v)} = \frac{H(u,v)F(u,v) + N(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

We can never recover F(u,v) exactly:

1. N(u,v) is not known since $\eta(x,y)$ is a r.v. — estimated
2. If H(u,v) ->0 then noise term will dominate. Helped by restricting analysis to (u,v) near origin.



Modeling of Degradation

480x480

No radial limiting of H(u,v)

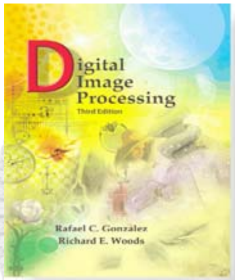
H(u,v) cut off at R=40

(a) Result of using the full filter. (b) Result with *H* cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

H(u,v) cut off at R=70

H(u,v) cut off at R=85

$$\tilde{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)} \quad where \quad H(u,v) = e^{-k\left[\left(u-\frac{M}{2}\right)^2 + \left(v-\frac{N}{2}\right)^2\right]^{\frac{5}{6}}}$$



Wiener Filter

Minimize $e^2 = E\left\{\left(f - \hat{f}\right)^2\right\}$

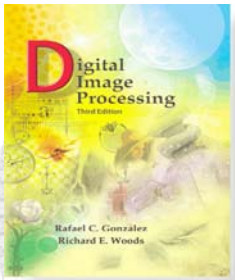
Assuming:
1. f and n are uncorrelated
2. f and/or n is zero mean
3. gray levels in f are a linear function of the gray levels in f

The best estimate $\hat{F}(u,v)$ is then given by

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right]G(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}}\right]G(u,v)$$

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}}\right]G(u,v)$$

H(u,v) = degradation function
H*(u,v) = complex conjugate of H
|H(u,v)|= H*(u,v) H(u,v)
S_η(u,v)=|N(u,v)|^2=power spectrum of noise (estimated)
S_f(u,v)=|F(u,v)|^2=power spectrum of original image (not known)



Modeling of Degradation

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Inverse filtering

$$\frac{N(u,v)}{H(u,v)}$$

Radially limit at D_o=75

Wiener filtering

$$\hat{F}(u,v)=\left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+K}\right]G(u,v)$$

In practice we don't know the power spectrum $S_f(u,v)=|F(u,v)|^2$ of the original image so we replace the S_η/S_f term with a constant K which we vary