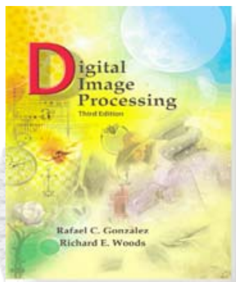
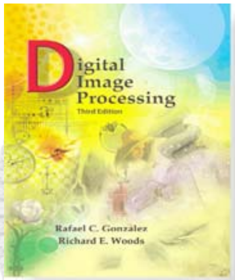
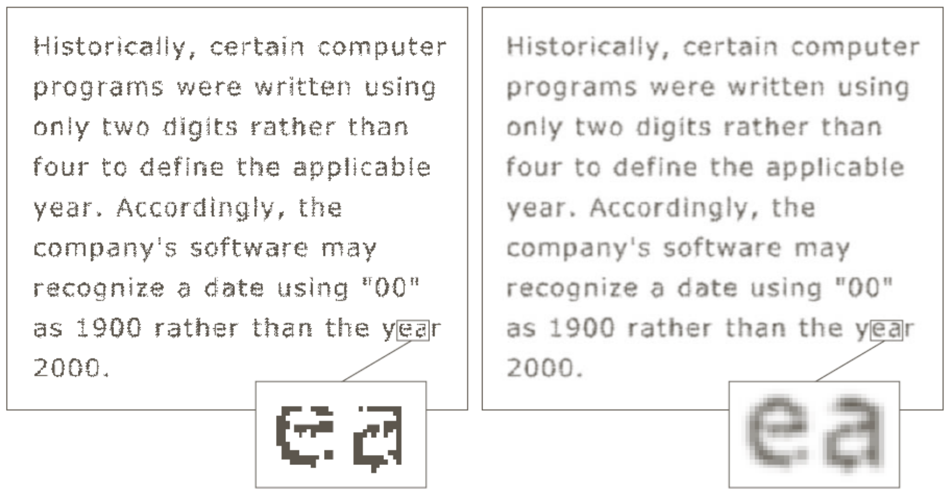


Lecture #11

- Filtering Applications: OCR, scanning
- Highpass filters
- Laplacian in the frequency domain
- Image enhancement using highpass filters
- Homomorphic filters
- Bandreject/bandpass/notch filters
- Correlation (revisited)
- Color basics (Chapter 6)



Filtering for OCR



a b

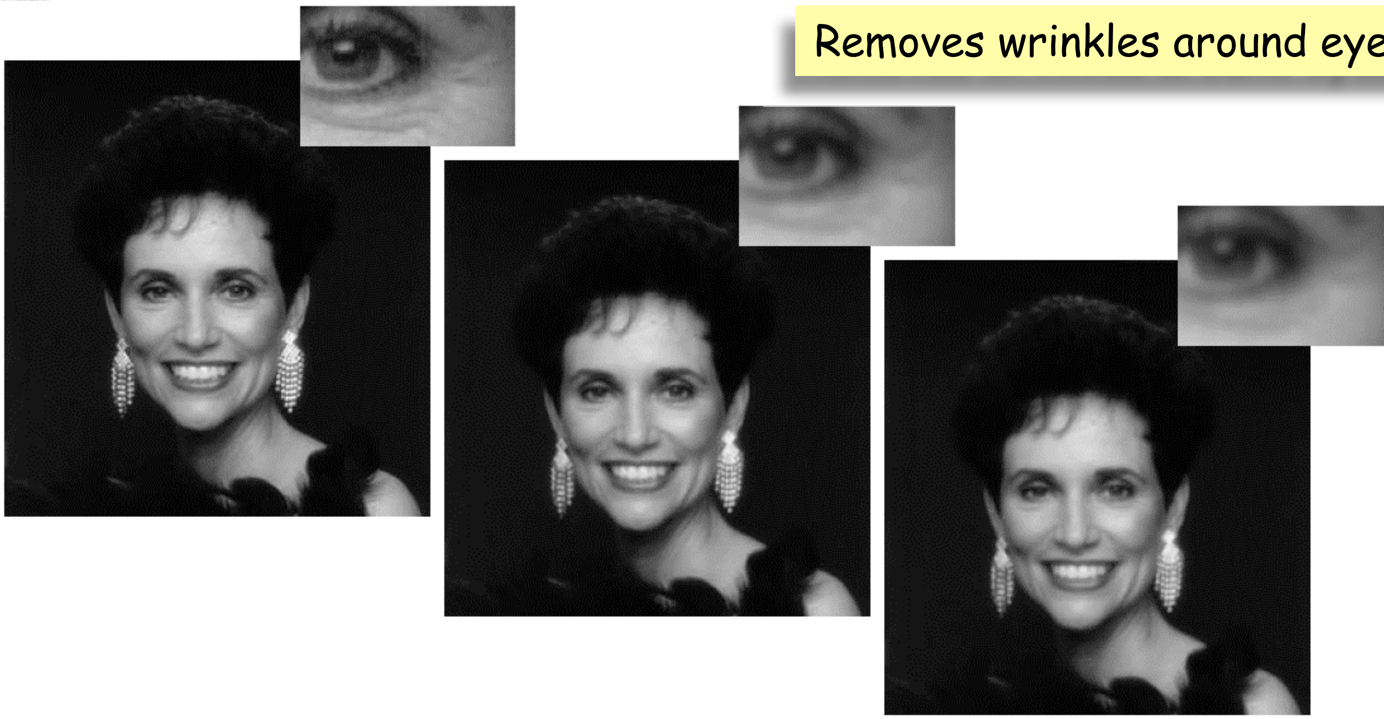
FIGURE 4.49 (a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

No filtering. Broken characters are difficult to recognize.

Filtered with a GLPF with D_0=80. Characters are fuller and filled in.

Filtering for Fun and Profit

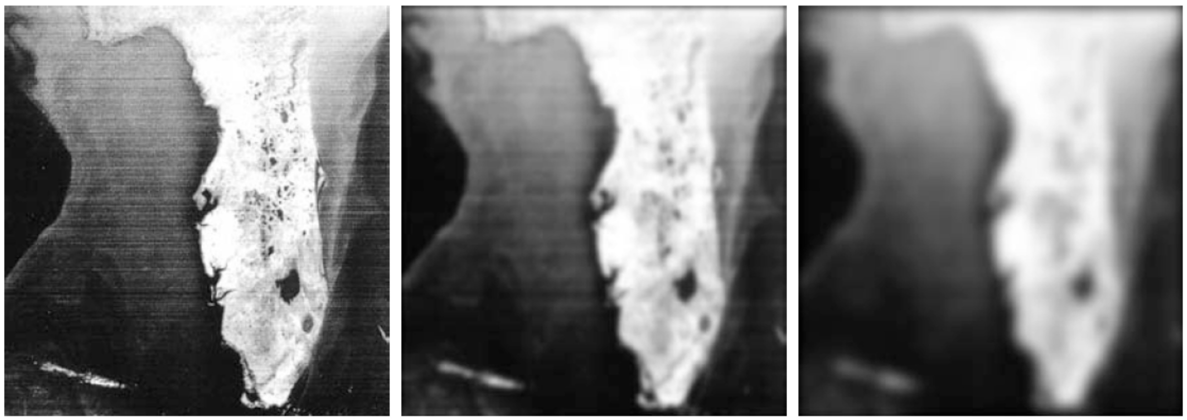
Removes wrinkles around eyes.



a b c

FIGURE 4.50 (a) Original image (784 × 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

Filtering to Remove Scan Lines



a b c

FIGURE 4.51 (a) Image showing prominent horizontal scan lines. (b) Result of filtering using a GLPF with $D_0 = 50$. (c) Result of using a GLPF with $D_0 = 20$. (Original image courtesy of NOAA.)

Highpass Filters (Frequency Domain)

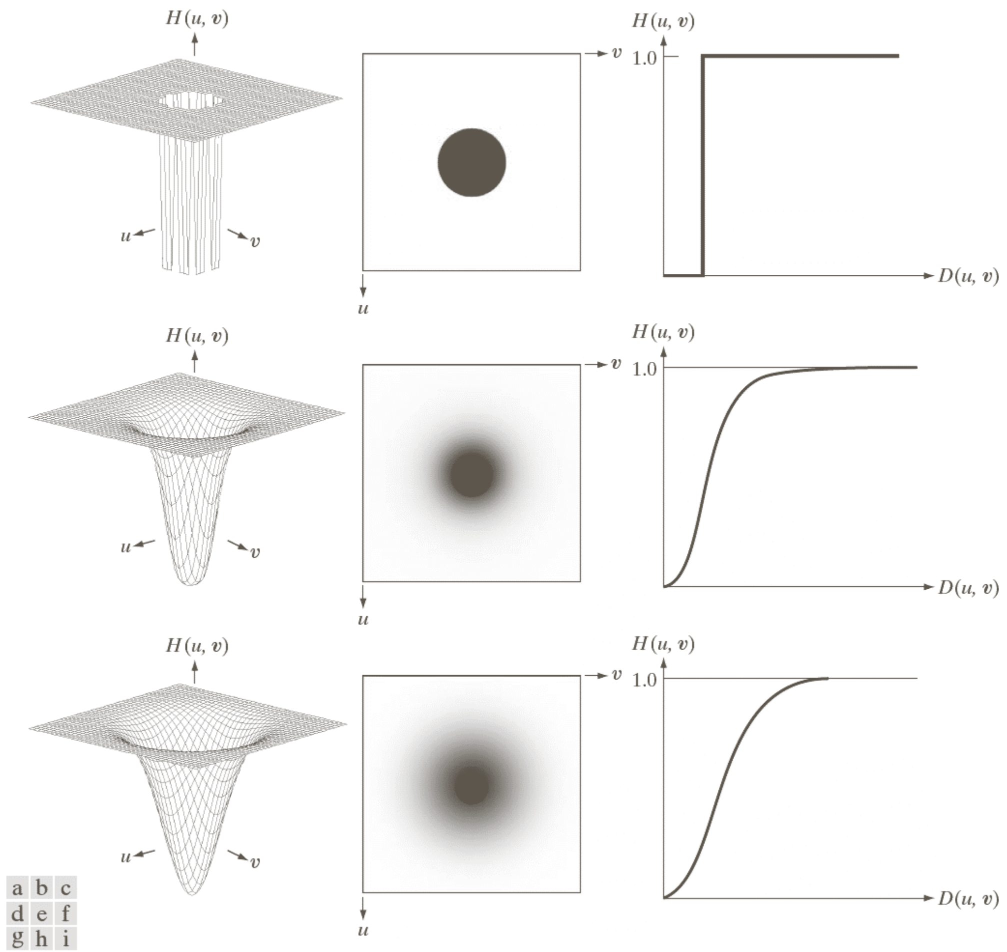
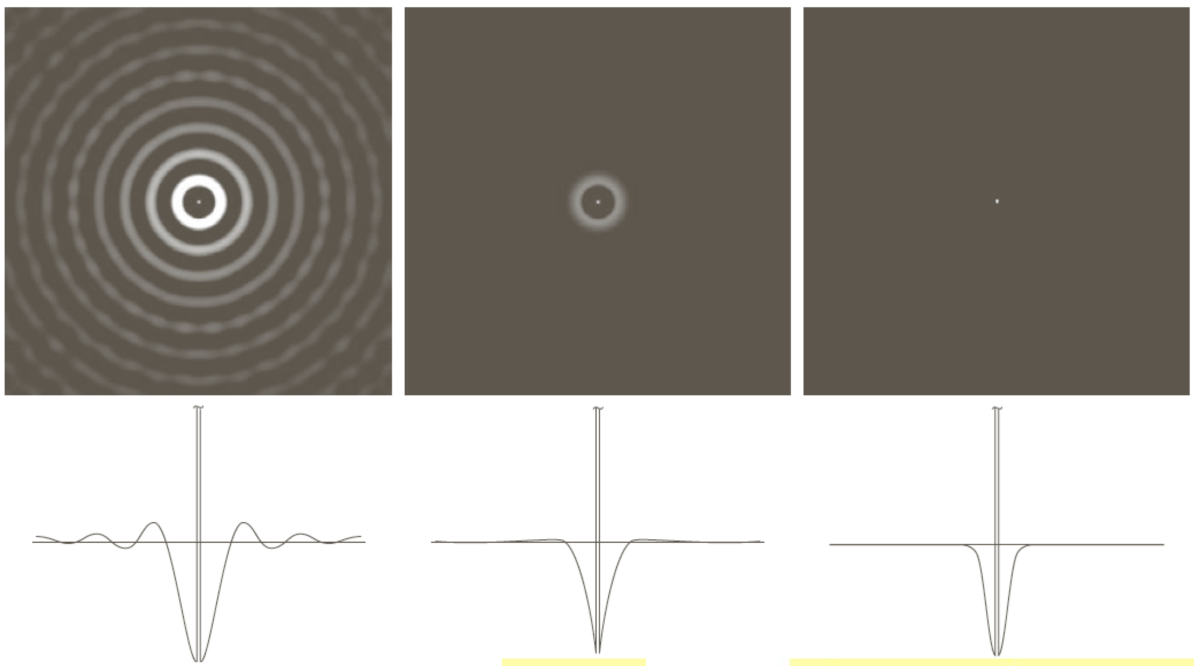


FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

Highpass Filters (Spatial Domain)



a b c

FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

Test Image: Ideal HPF

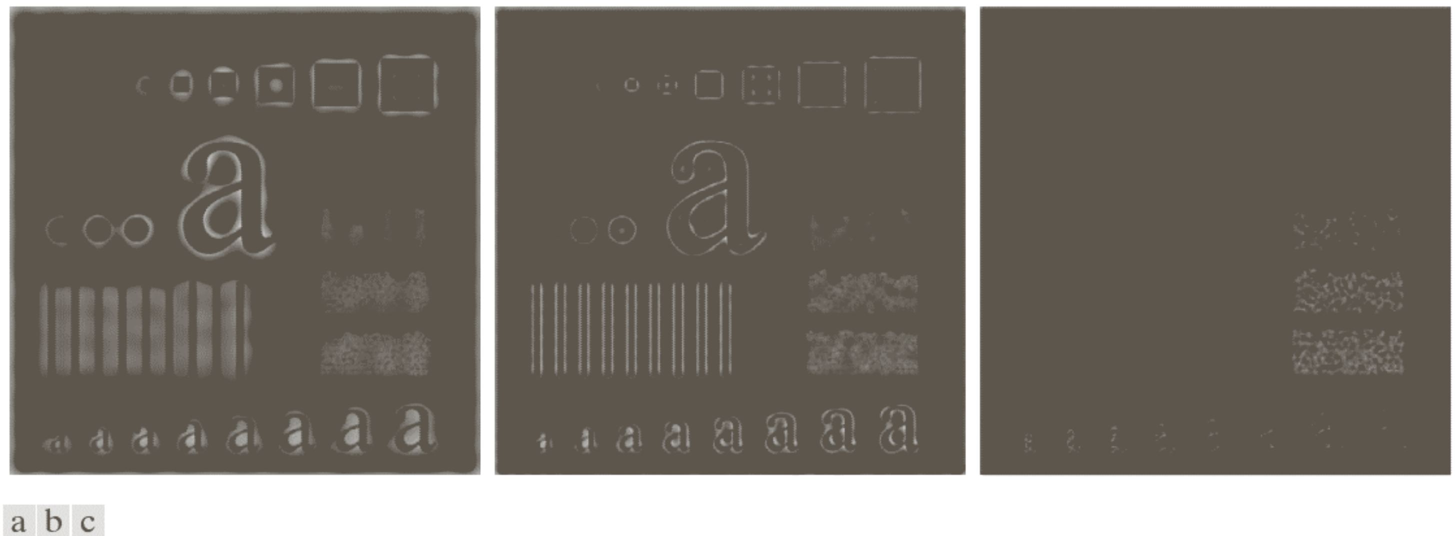
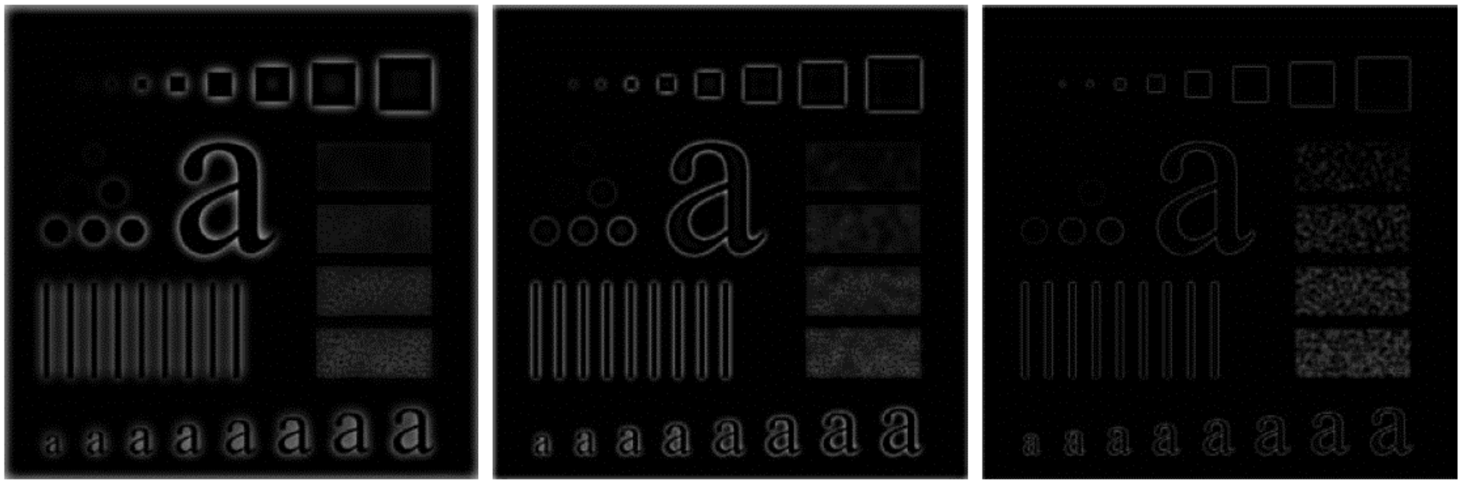


FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30, 60$, and 160.

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

Test Image: Butterworth HPF

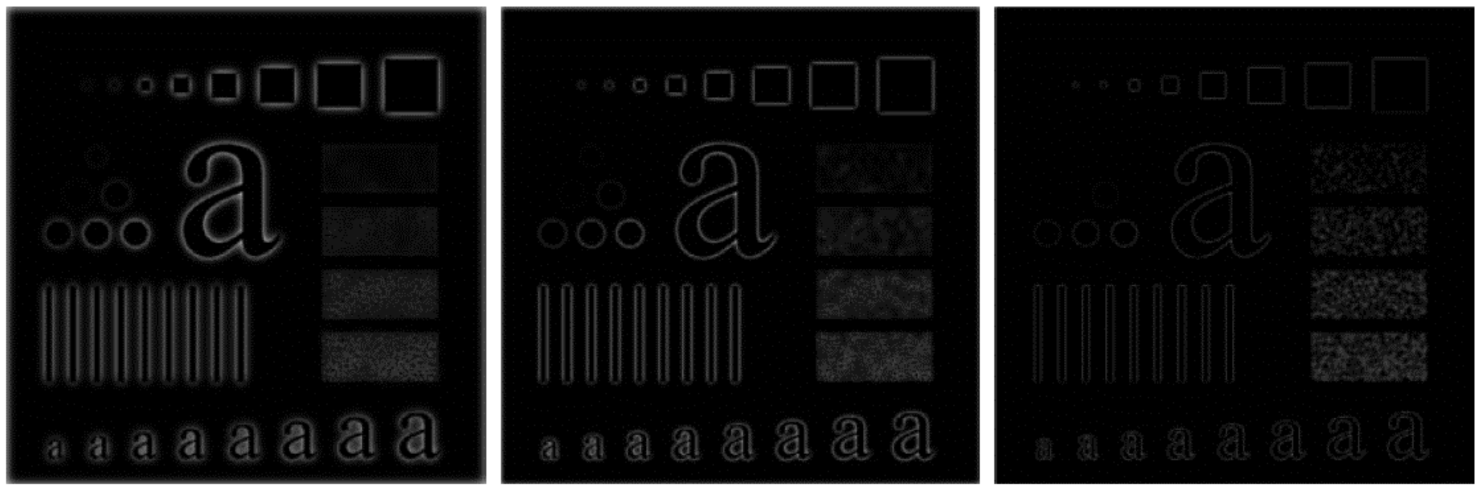


a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60$, and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

$$H(u,v) = \frac{1}{1 + \left[\frac{D_0}{D(u,v)} \right]^{2n}}$$

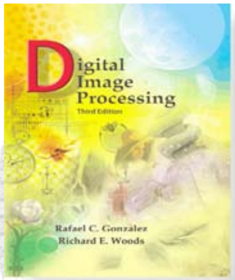
Test Image: Gaussian HPF



a b c

FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.

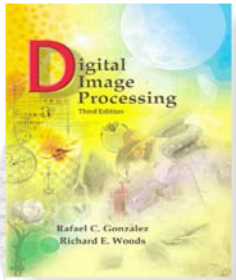
$$H(u,v) = 1 - e^{-\frac{D^2(u,v)}{2D_0^2}}$$



HPF Mathematical Definitions

TABLE 4.5
Highpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D_0/D(u,v)]^{2n}}$	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$



Laplacian Frequency Domain Filter

$$\mathcal{F}\left\{\frac{\partial^2 f(x,y)}{\partial x^2}+\frac{\partial^2 f(x,y)}{\partial y^2}\right\}=(ju)^2F(u,v)+(jv)^2F(u,v)$$

$$\mathcal{F}\left\{\nabla^2 f(x,y)\right\}=-\left(u^2+v^2\right)F(u,v)$$

Note that the inverse Fourier transform of this filer is x-y oriented (no diagonals).

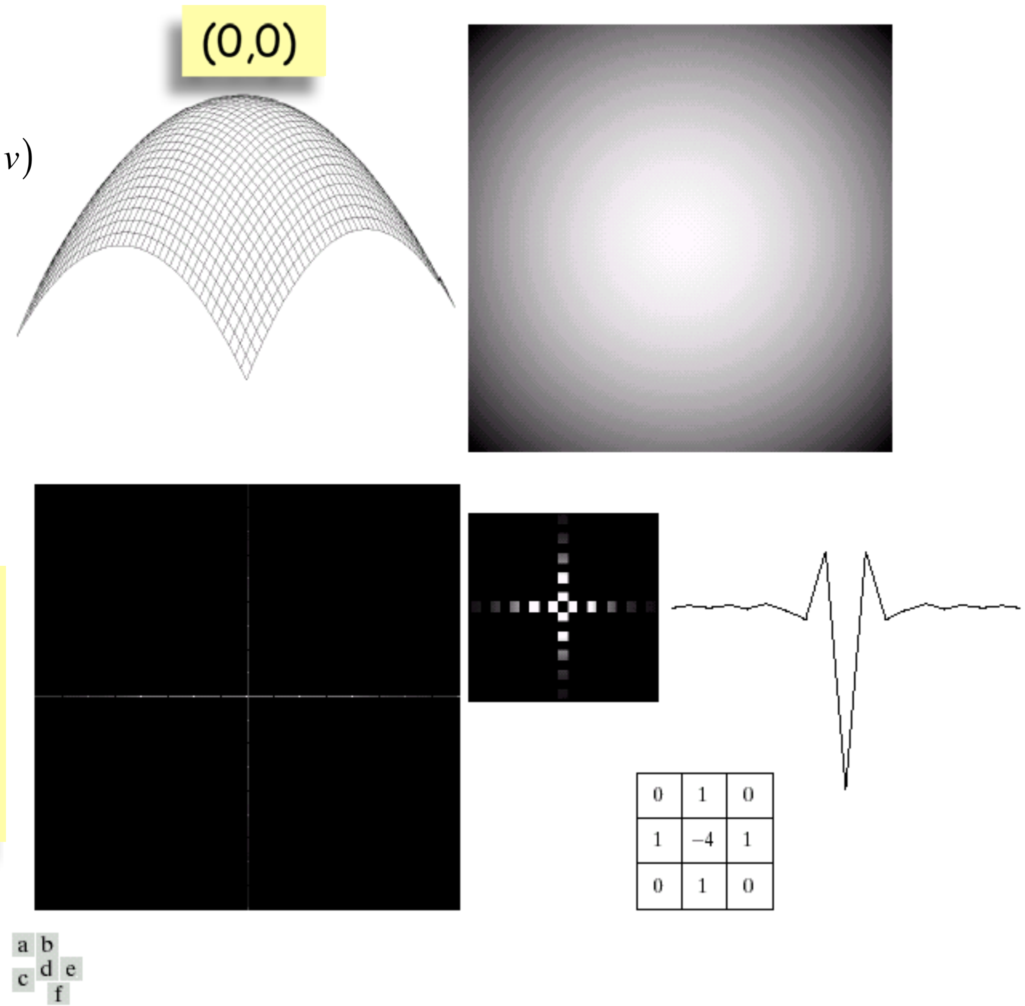


FIGURE 4.27 (a) 3-D plot of Laplacian in the frequency domain. (b) Image representation of (a). (c) Laplacian in the spatial domain obtained from the inverse DFT of (b). (d) Zoomed section of the origin of (c). (e) Gray-level profile through the center of (d). (f) Laplacian mask used in Section 3.7.

HPF for Edge Detection

a b c

FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

Highpass filtering using a Butterworth filter to enhance ridges (high frequencies) and reduce effects of smudging (low frequencies). Since highpass filtering darkens the image thresholding is used to enhance the ridges.

HPF Image Enhancement

Unsharp masking

$$f_s(x,y) = f(x,y) - \bar{f}(x,y)$$

$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

a b

FIGURE 4.58 (a) Original, blurry image. (b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

Sharpening

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$

$$H(u,v) = 1 - \left[\left(u - \frac{M}{2}\right)^2 + \left(v - \frac{N}{2}\right)^2\right]$$

High-Frequency Emphasis

$$H_{hb}(u,v) = (A-1) + H_{hp}(u,v)$$

$$H_{hfe}(u,v) = a + bH_{hp}(u,v)$$

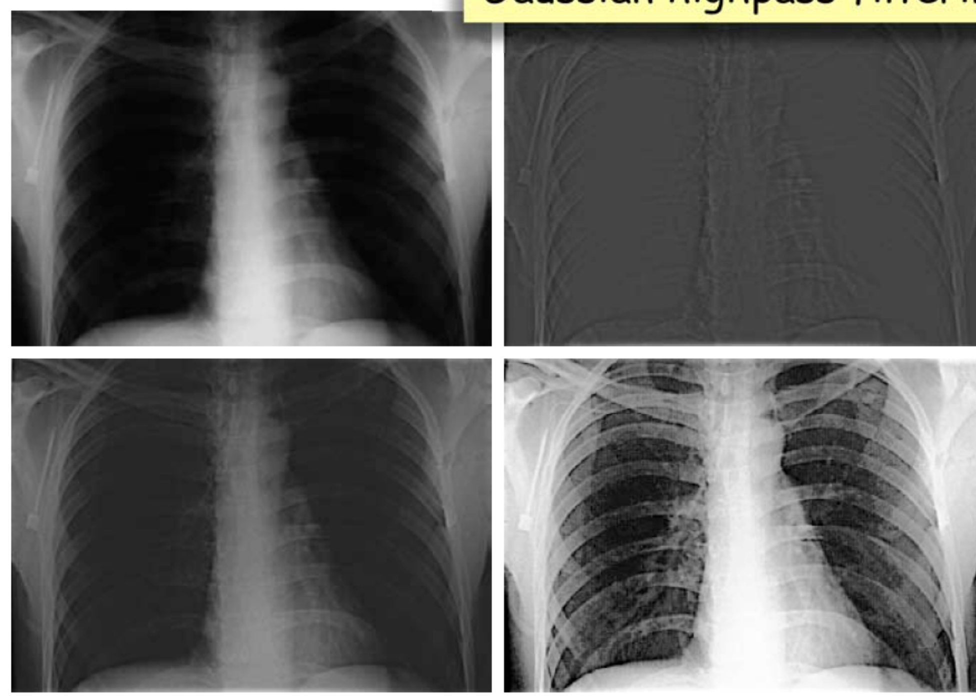
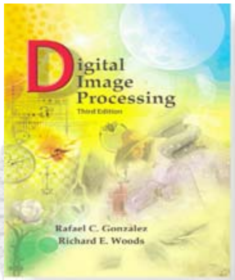


FIGURE 4.59 (a) A chest X-ray image. (b) Result of highpass filtering with a Gaussian filter. (c) Result of high-frequency-emphasis filtering using the same filter. (d) Result of performing histogram equalization on (c). (Original image courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)



Theory of Homomorphic Filtering

$$f(x,y) = i(x,y)r(x,y)$$

Separate image into incident (low frequency) and reflected (high frequency) components

$$z(x,y) = \ln\big(f(x,y)\big) = \ln i(x,y) + \ln r(x,y)$$

Use log to separate frequency components

$$\mathcal{F}\{z(x,y)\} = \mathcal{F}\{\ln i(x,y)\} + \mathcal{F}\{\ln r(x,y)\}$$

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

Fourier transform high and low frequency components

$$S(u,v) = H(u,v)Z(u,v) = H(u,v)F_i(u,v) + H(u,v)F_r(u,v)$$

Filter components

$$s(x,y) = i'(x,y) + r'(x,y)$$

$$g(x,y) = e^{s(x,y)} = e^{i'(x,y)}e^{r'(x,y)} = i_o(x,y)r_o(x,y)$$

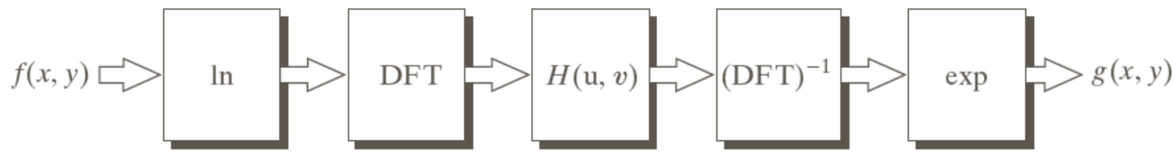
Exponentiate to re-combine

$$s(x,y) = \mathcal{F}^{-1}\{S(u,v)\} = \mathcal{F}^{-1}\{H(u,v)F_i(u,v)\} + \mathcal{F}^{-1}\{H(u,v)F_r(u,v)\}$$

Inverse transform

Homomorphic Filtering

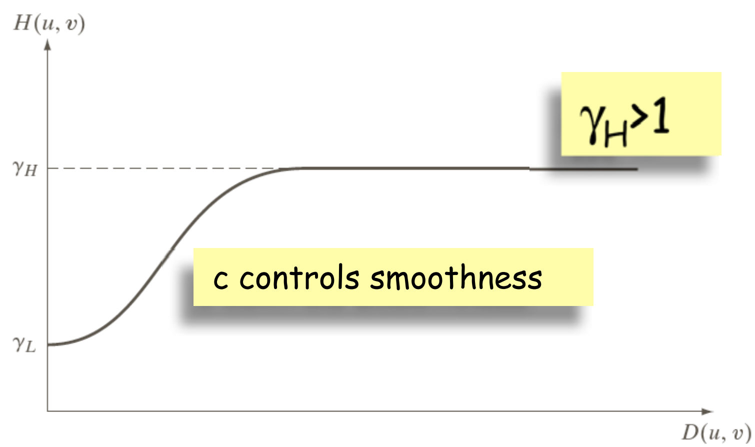
FIGURE 4.60 Summary of steps in homomorphic filtering.



$$H(u,v)=(\gamma_H-\gamma_L)\left[1-e^{-c\frac{D^2(u,v)}{D_0^2}}\right]+\gamma_L$$

Fourier transform and process high and low frequency components differently

$\gamma_L<1$

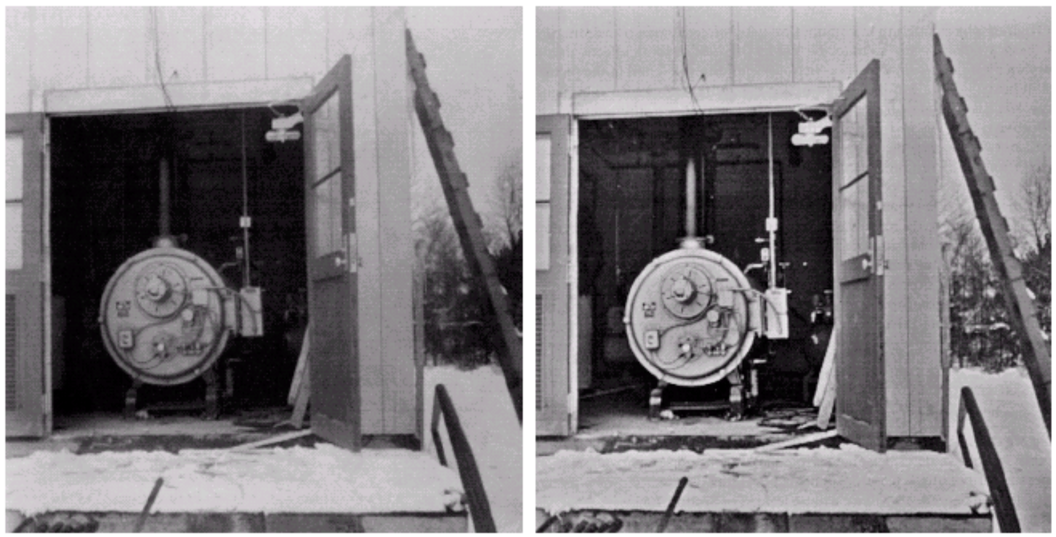


$\gamma_H>1$

c controls smoothness

FIGURE 4.61 Radial cross section of a circularly symmetric homomorphic filter function. The vertical axis is at the center of the frequency rectangle and $D(u, v)$ is the distance from the center.

Homomorphic Filtering

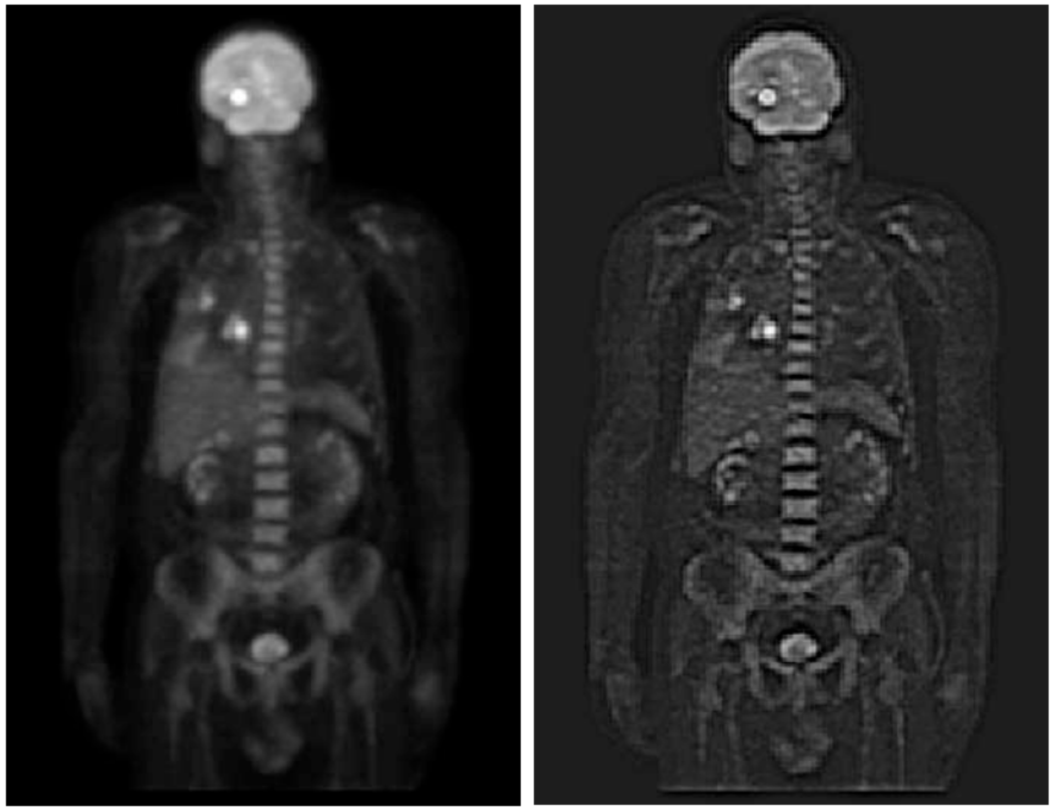


a b

FIGURE 4.33 (a) Original image. (b) Image processed by homomorphic filtering (note details inside shelter). (Stockham.)

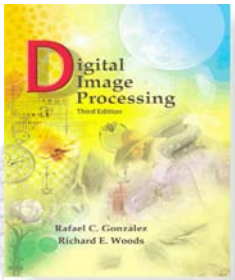
Homomorphic Filtering

Reduce effects of low frequency hot spots



a b

FIGURE 4.62 (a) Full body PET scan. (b) Image enhanced using homomorphic filtering. (Original image courtesy of Dr. Michael E. Casey, CTI PET Systems.)

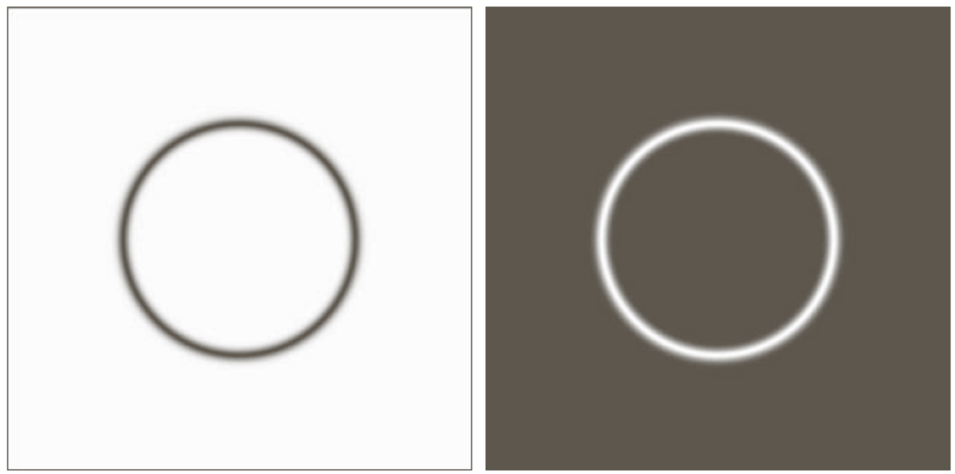


Bandreject Filters

TABLE 4.6
Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

Bandreject/Bandpass Filters



a b

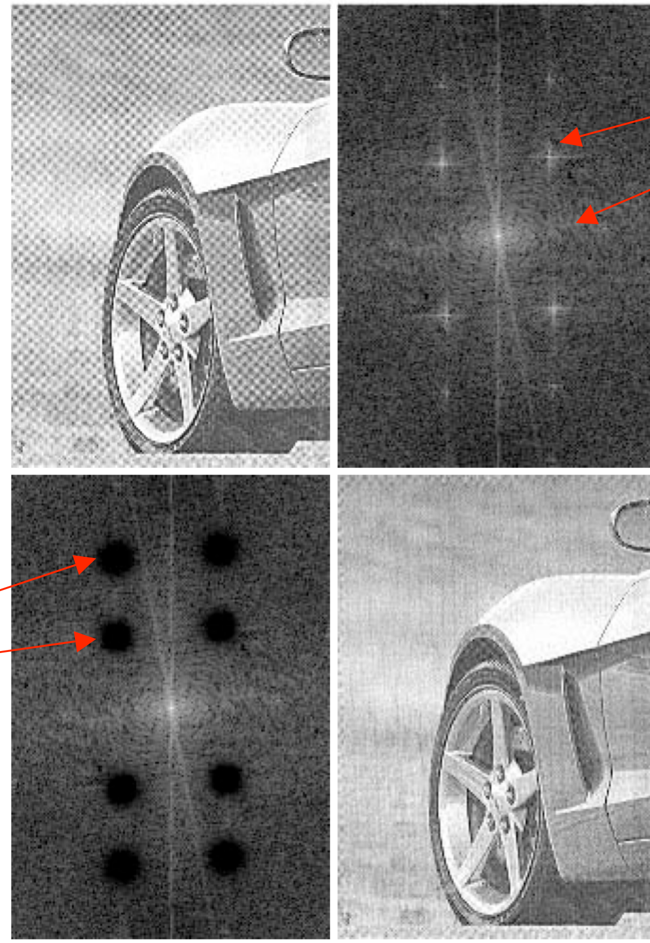
FIGURE 4.63 (a) Bandreject Gaussian filter. (b) Corresponding bandpass filter. The thin black border in (a) was added for clarity; it is not part of the data.

Bandreject

Bandpass

Notch Filtering to Remove Periodic Noise

Frequency components (moire) due to halftone printing grid and scanning grid



a b
c d

FIGURE 4.64 (a) Sampled newspaper image showing a moiré pattern. (b) Spectrum. (c) Butterworth notch reject filter multiplied by the Fourier transform. (d) Filtered image.

Butterworth notches to remove moire effects

Notch Filtering

Strong periodic noise in y-direction

Image corrupted with 60Hz hum

v-axis “notch” filter to remove periodic noise on vertical axis

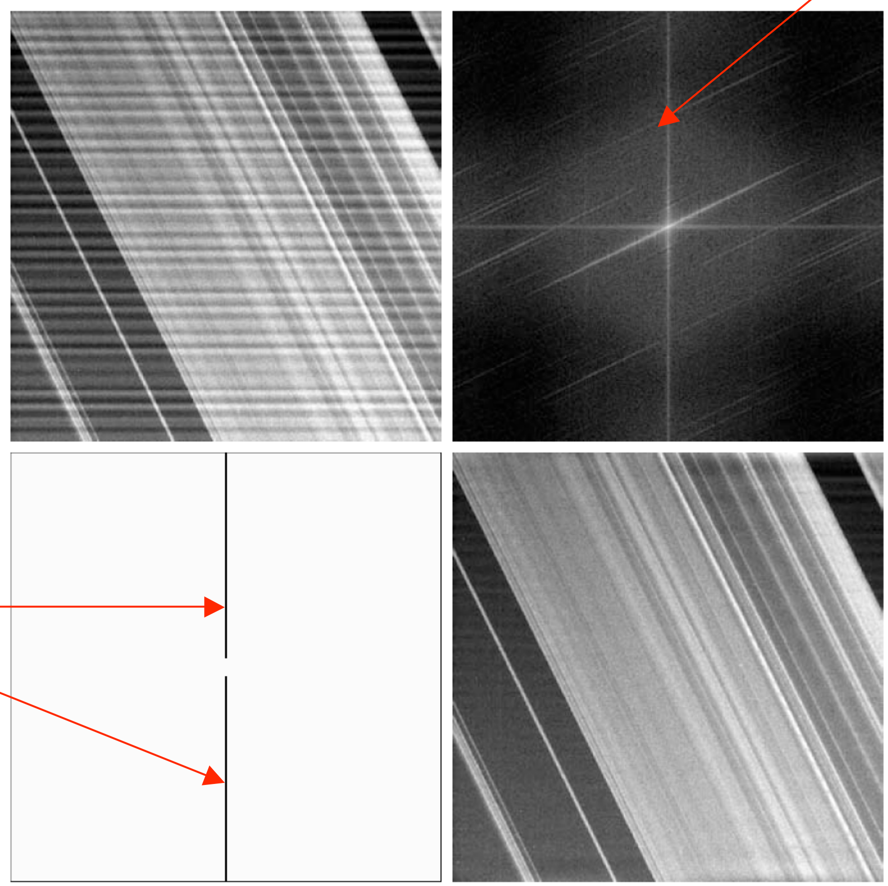
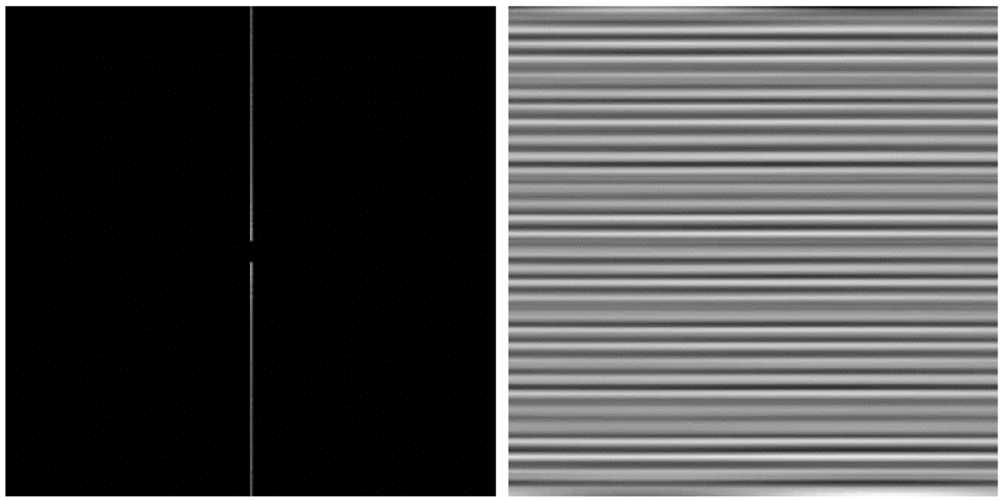


FIGURE 4.65 (a) 674 × 674 image of the Saturn rings showing nearly periodic interference. (b) Spectrum: The bursts of energy in the vertical axis near the origin correspond to the interference pattern. (c) A vertical notch reject filter. (d) Result of filtering. The thin black border in (c) was added for clarity; it is not part of the data. (Original image courtesy of Dr. Robert A. West, NASA/JPL.)

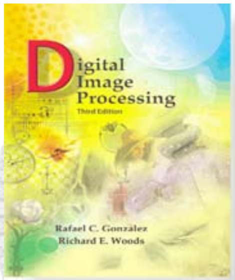
Notch Filtering



a b

FIGURE 4.66 (a) Result (spectrum) of applying a notch pass filter to the DFT of Fig. 4.65(a). (b) Spatial pattern obtained by computing the IDFT of (a).

Convert the notch (reject) filter to a notch pass (invert it), filter the image, and inverse Fourier transform to reveal the nature of the noise.



FFT vs DFT Computational Advantage

For large images you always want to use an FFT due to its large computational advantage.

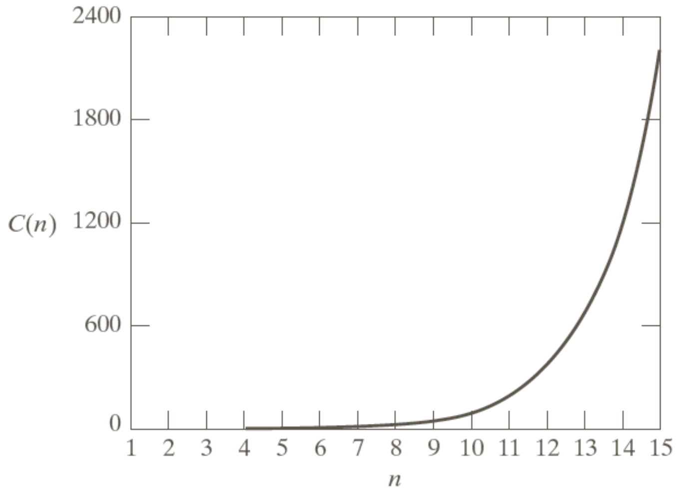
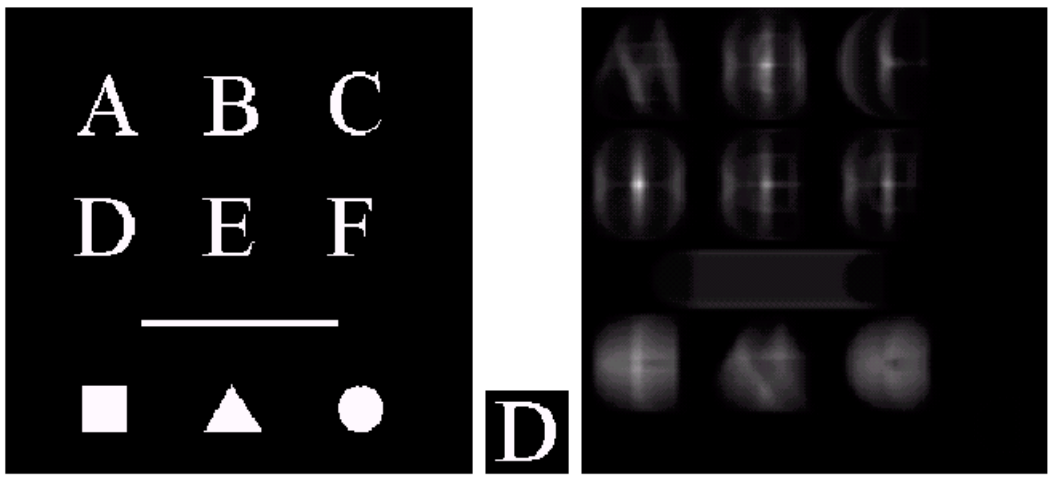


FIGURE 4.67 Computational advantage of the FFT over a direct implementation of the 1-D DFT. Note that the advantage increases rapidly as a function of n.

Ch12 Object Recognition



a b c

FIGURE 12.9 (a) Image. (b) Subimage. (c) Correlation coefficient of (a) and (b). Note that the highest (brighter) point in (c) occurs when subimage (b) is coincident with the letter "D" in (a).

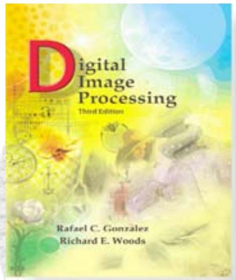
Correlation

- Convolution

$$f_2(m,n)=\sum_{i=0}^{M-1}\sum_{j=0}^{N-1} f_1(i,j)h(m-i,n-j)$$

- Correlation

$$f_2(m,n)=\sum_{i=0}^{M-1}\sum_{j=0}^{N-1} f_1(i,j)h(m+i,n+j)$$

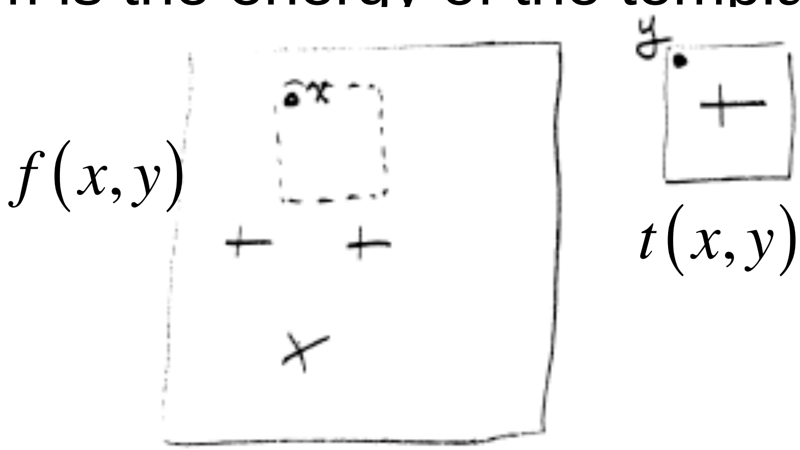


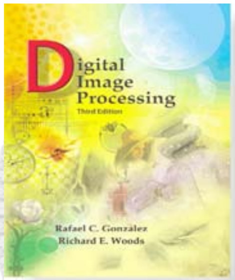
How to find objects of interest in an image

- Define an Euclidian distance $d^2(y)$

$$d^2(\underline{y}) = \sum_{\underline{x}} \left[f(\underline{x}) - t(\underline{x} - \underline{y}) \right]^2 = \sum_{\underline{x}} \left[f^2(\underline{x}) - 2f(\underline{x})t(\underline{x} - \underline{y}) + t^2(\underline{x} - \underline{y}) \right]$$

- The first term can be nearly constant depending upon the spatial uniformity of the image
- The second term is the cross correlation function
- The third term is the energy of the template, a constant.





False errors from correlation

Example:

1	1	1
1	1	1
1	1	1

template

Noisy object

1	1	0	0	0
1	1	1	0	0
1	0	1	0	0
0	0	0	0	0
0	0	0	0	8

Large noise spike

Correlation

7	4	2	x	x
5	3	2	x	x
2	1	9	x	x
x	x	x	x	x
x	x	x	x	x

correct best response (→ 7)

error due to large noise (→ 9)

x = undefined

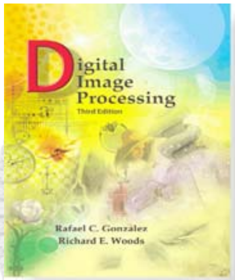
Normalize to prevent false responses

- Images to be matched $f_1(\underline{x})$ $f_2(\underline{x})$
- Patches to be matched q_1 q_2
- Variances $\sigma(q_1) = \sqrt{E(q_1^2) - E^2(q_1)}$ $\sigma(q_2) = \sqrt{E(q_2^2) - E^2(q_2)}$
- Normalized correlation $N(\underline{y}) = \dfrac{E(q_1 q_2) - E(q_1)E(q_2)}{\sigma(q_1)\sigma(q_2)}$

Typically we can think of q_1 as the template, i.e., all of f_1, and q_2 the section of f_2 covered by the displaced q_1

Sequential Similarity Detection Algorithm

- Correlation $$\phi_{ab}\left(\underline{y}\right) = \sum_{i,j} a_{ij}\left(\underline{x}\right) b_{ij}\left(\underline{x} - \underline{y}\right)$$
- If a and b are highly correlated, this summation will be essentially all 1's yielding a large result. Threshold this summation after say 10 terms. If this sum is less than T (the threshold) it is probably uncorrelated and the summation will stop.



Moravec Interest Operator

(produce candidate interest points based upon image activity)

- Define the variance operator

$$Var(\underline{x}) = Var(x,y) = \sqrt{\sum_{k,l\,in\,S} \left[f(x,y) - f(x+k, y+l) \right]^2}$$

The region S is a local region defined to fit the application.

- Algorithm:
- 1. Find the local minimum of the variance

$$IntOpVal(\underline{x}) := \min_{|y| \leq 1} \left[Var\left(\underline{x} + \underline{y} \right) \right]$$

Interest is minimum of variance in some small region

- 2. Find local maxima

$$IntOpVal(\underline{x}) := 0 - unless - IntOpVal(\underline{x}) \geq IntOpVal\left(\underline{x} + \underline{y} \right) for \left| \underline{y} \right| \leq 1$$

Nonzero only if the activity is a local maximum

- 3. Interesting point if

$$IntOpVal(\underline{x}) > T$$

Threshold to get really interesting points

White Light

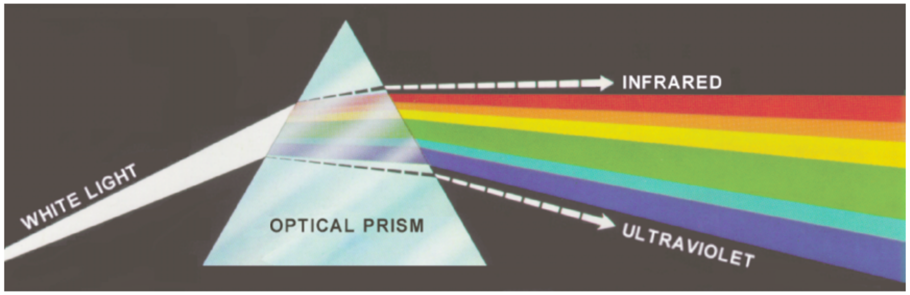
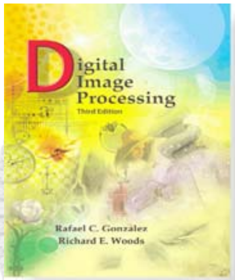


FIGURE 6.1 Color spectrum seen by passing white light through a prism. (Courtesy of the General Electric Co., Lamp Business Division.)

A prism splits white light into its component colors.



Optical Units

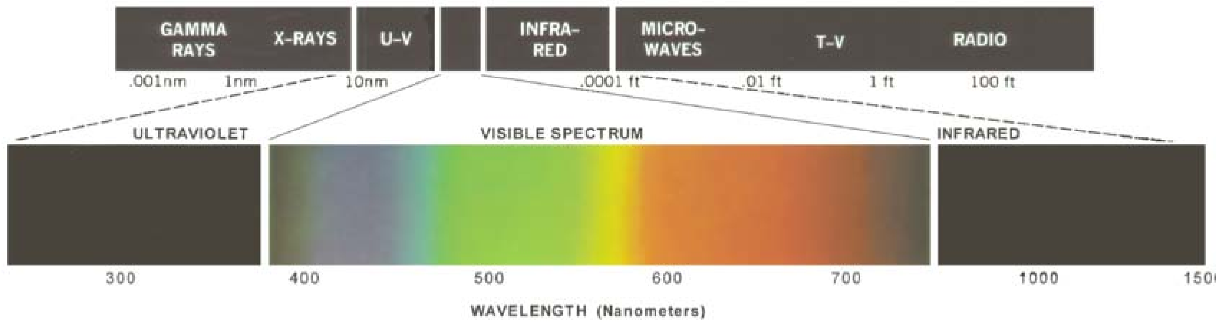


FIGURE 6.2 Wavelengths comprising the visible range of the electromagnetic spectrum. (Courtesy of the General Electric Co., Lamp Business Division.)

<u>Radiometric units:</u>
Radiance (watts/sr/m^2) — total energy emitted by a light source
Irradiance (watts/m^2) — total energy incident on a surface source

<u>Photometric units:</u>
Luminous flux (lumens) — similar to radiance except corrected for wavelength sensitivity of the human eye
Candela — power emitted by a light source in a particular direction corrected for wavelength sensitivity of the human eye
Luminance (candela/m2) — incoming energy as measured by a detector

Human Eye

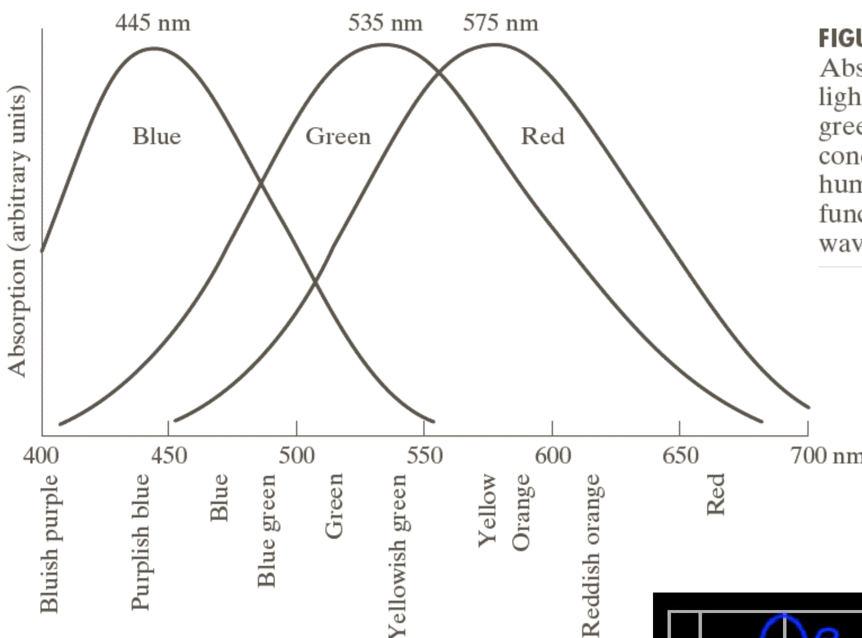
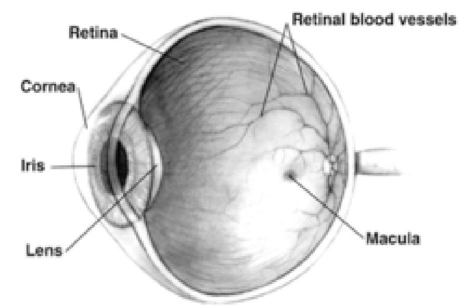
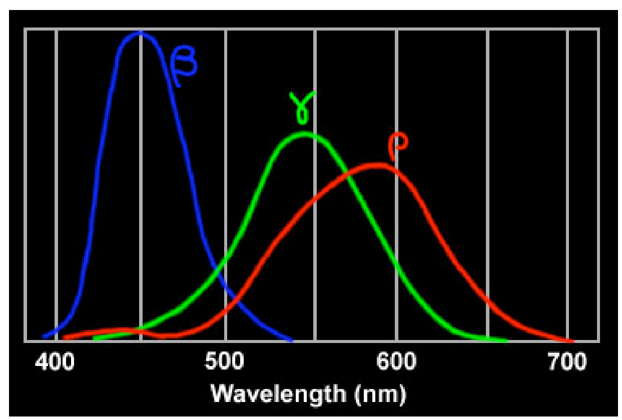


FIGURE 6.3 Absorption of light by the red, green, and blue cones in the human eye as a function of wavelength.

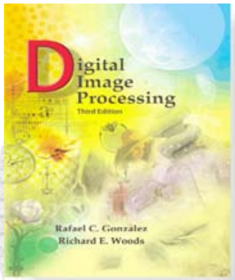
The cones provide all the color sensitivity and are concentrated near the macula.



Relative color sensitivity of the cones.



65% of cones are red sensitive; 33% are green sensitive; 2% are blue sensitive.

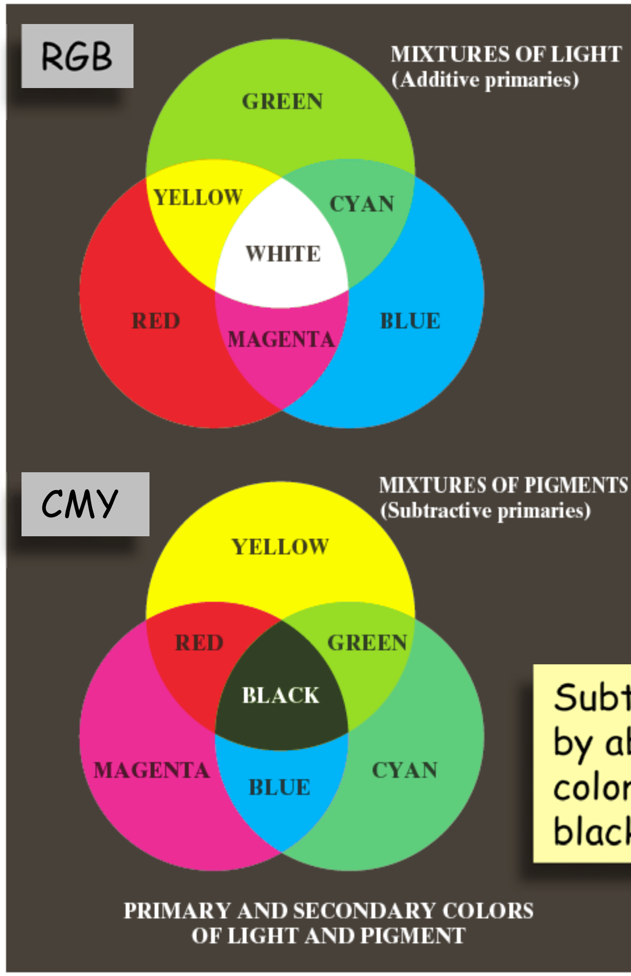


Primary/Secondary Colors

(Primary) additive colors operate in transmission such as found in televisions and computer monitors

(Secondary) subtractive colors operate in reflection such as found in printing

For example, yellow absorbs blue and reflects red+green=yellow



a
b

FIGURE 6.4 Primary and secondary colors of light and pigments. (Courtesy of the General Electric Co., Lamp Business Division.)

Subtractive colors operate by absorbing a primary color. They usually require black.