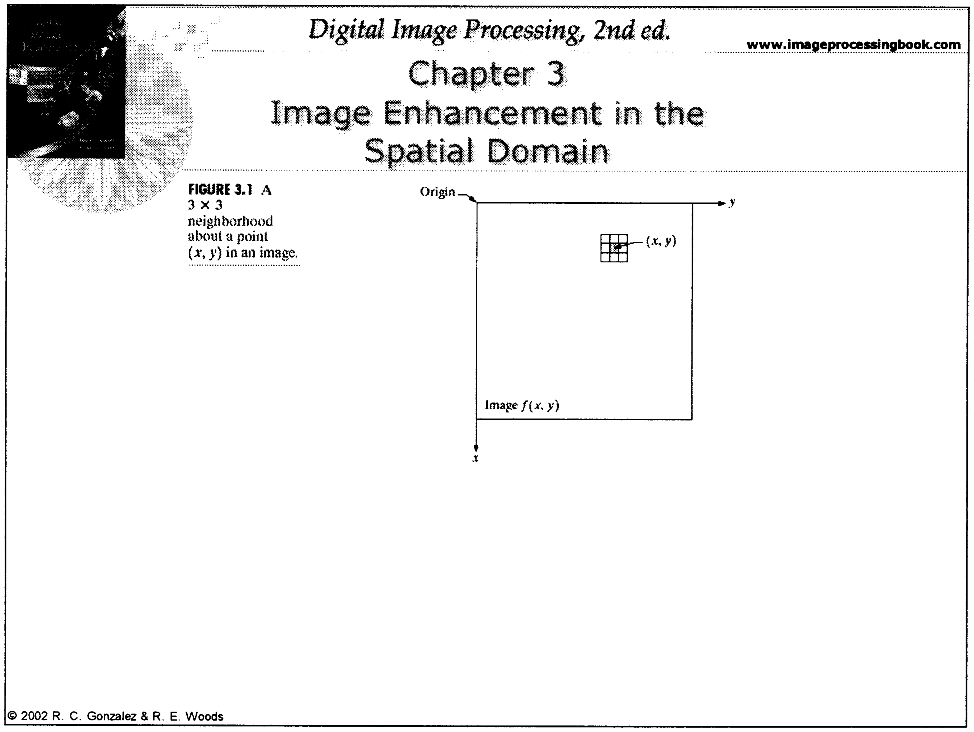

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FIGURE 3.1 A 3 × 3 neighborhood about a point (x, y) in an image.

Neighborhoods usually 3x3 but often larger
5x5
7x7
odd so centered on (x,y)

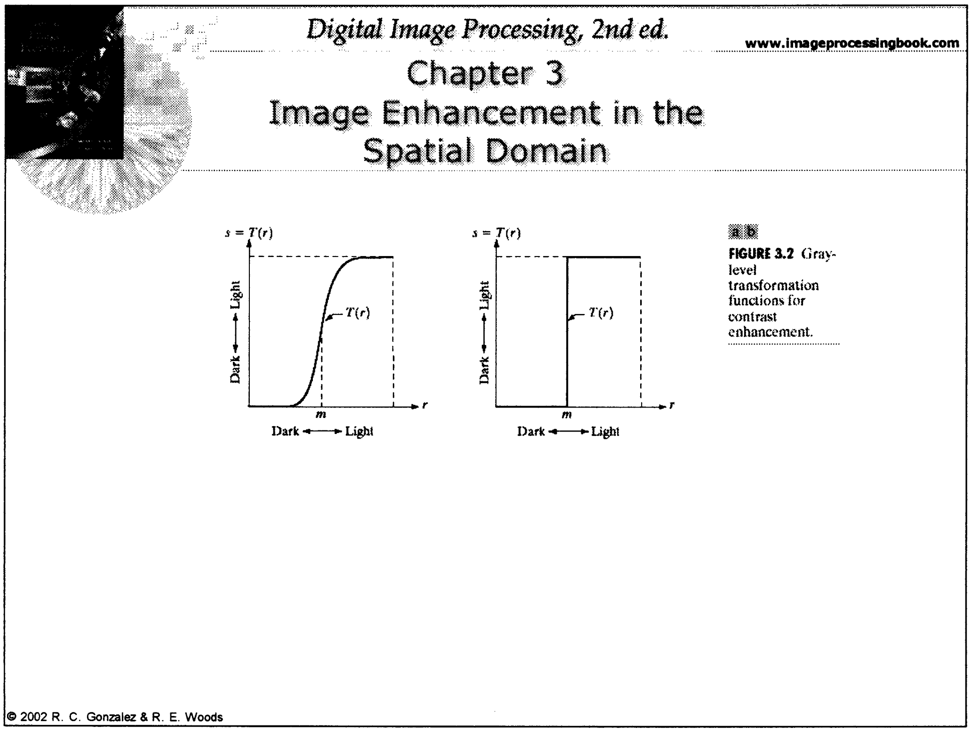

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FIGURE 3.2 Gray-level transformation functions for contrast enhancement.

These are point to point intensity transformations.

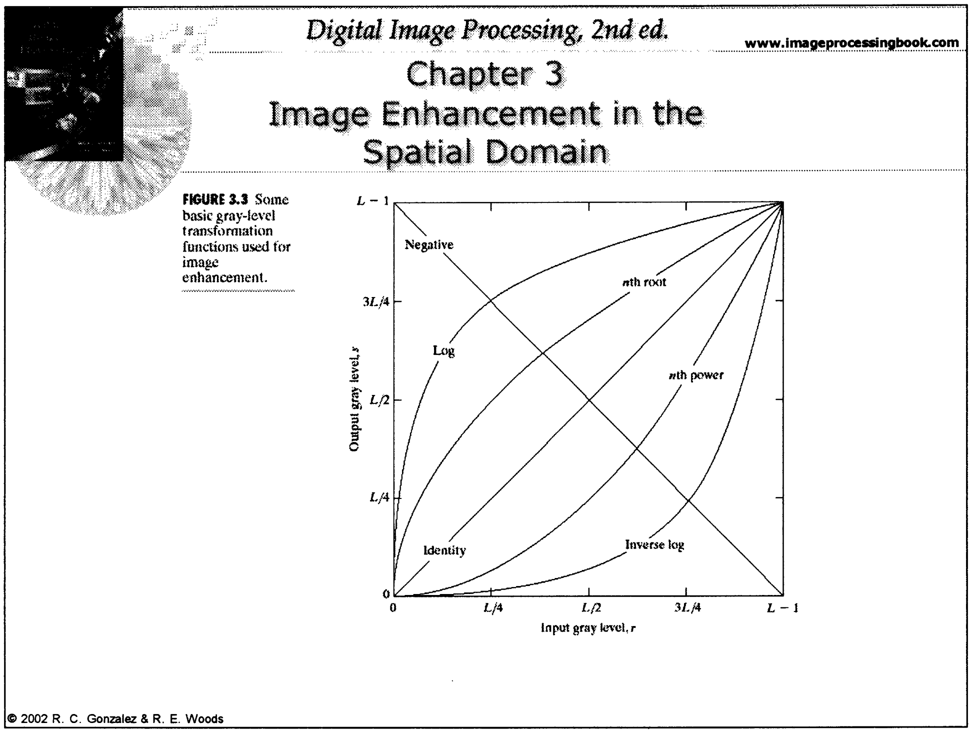

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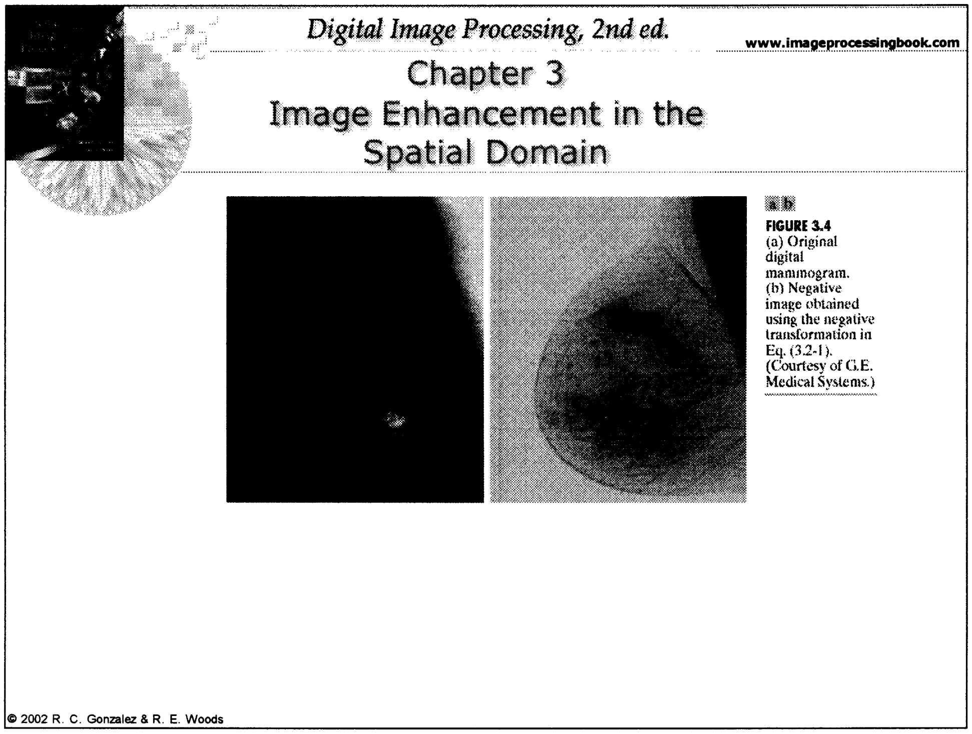
FIGURE 3.3 Some basic gray-level transformation functions used for image enhancement.

This is a logarithmic gray level transformation.

$$\underset{\text{output gray level}}{s} = \underset{\text{constant}}{c} \log(1 + \underset{\text{input gray level}}{r})$$

This type of transform expands/compresses the gray levels of the input.

It can expand dark pixel values.



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a b

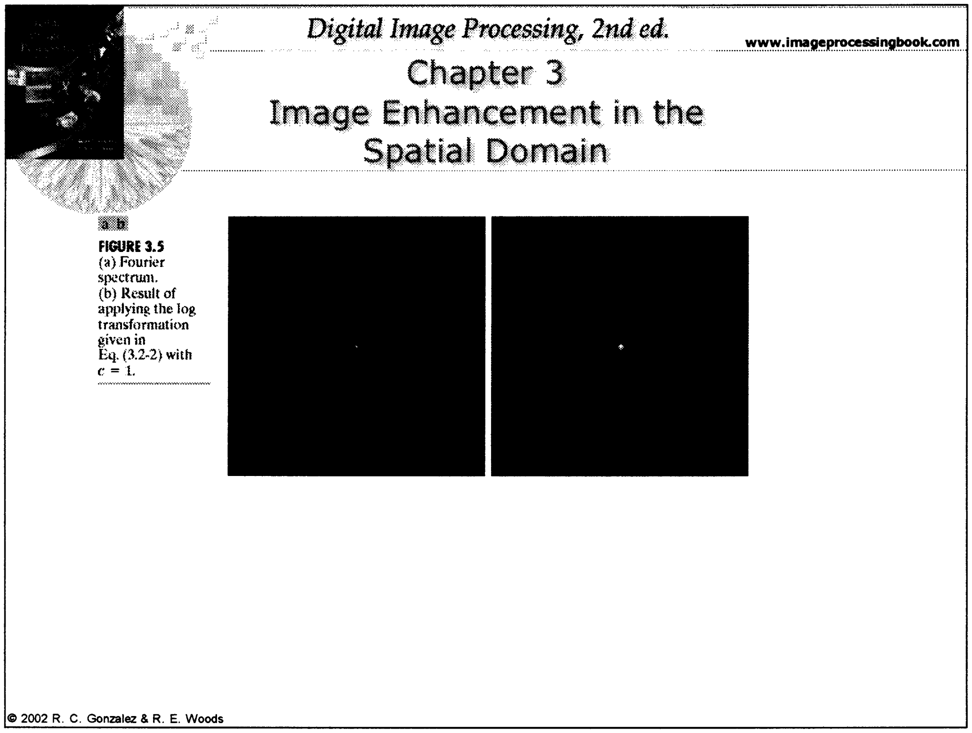
FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

(b) is simply the negative of (a), i.e.

$s = L - 1 - r$ where $L = 2^k$

\# of bits

\# of gray levels



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a b

FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.

This is an example of $s = \log(1+r)$. used to make dark information on monitor more visible.

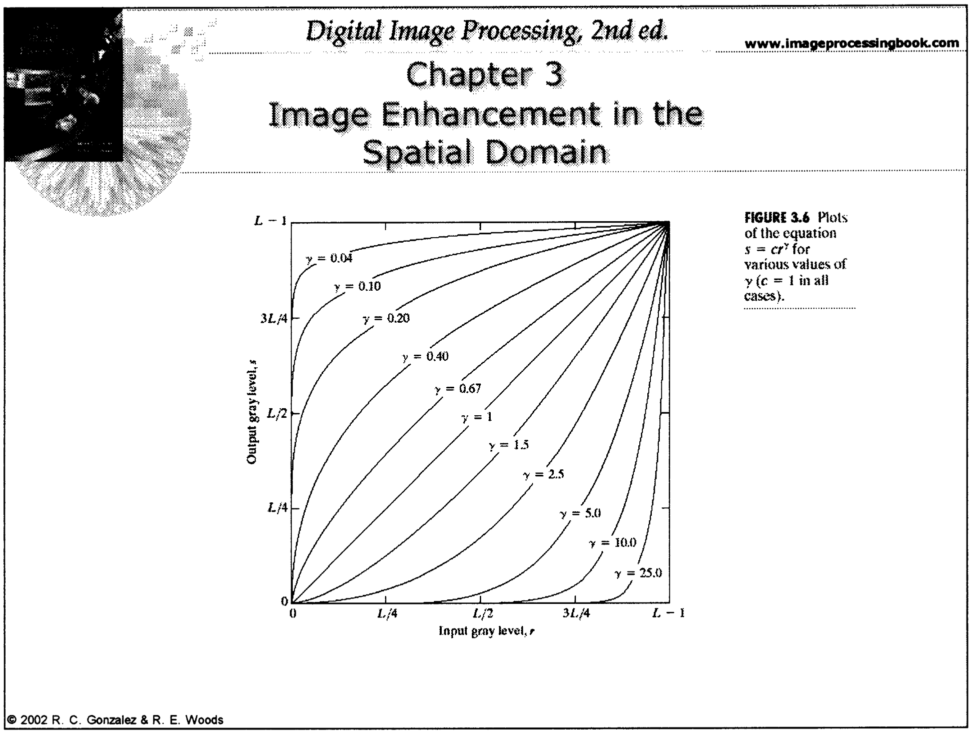

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FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

Power law transform $s = c\,r\underset{\text{gamma}}{\overset{\gamma}{\uparrow}}$

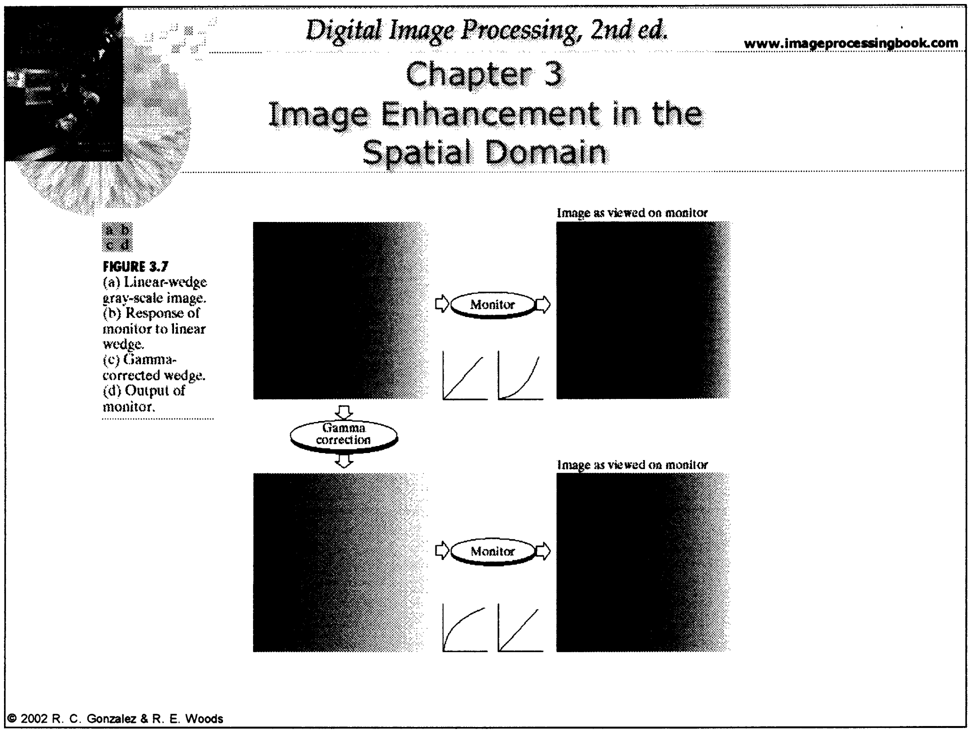

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FIGURE 3.7 (a) Linear-wedge gray-scale image. (b) Response of monitor to linear wedge. (c) Gamma-corrected wedge. (d) Output of monitor.

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This is an example of using a γ transformation to correct for a monitor produced $s = r^{2.5}$

First transform the computer data to get $r' = r^{\frac{1}{2.5}}$

Then, $s = (r')^{2.5}$ by the monitor

$$\underbrace{s = \left(r^{\frac{1}{2.5}}\right)^{2.5} = r}_{\text{This is called } \gamma \text{ correction.}}$$

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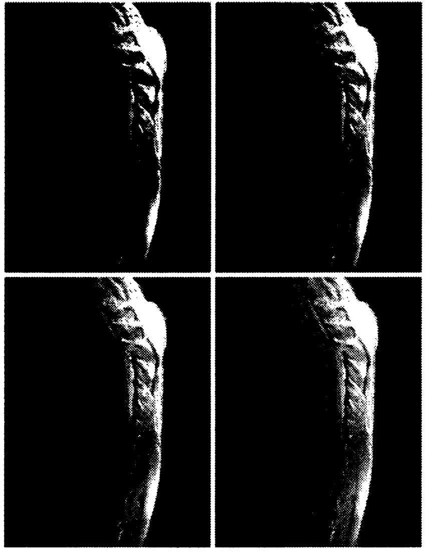

a b
c d

FIGURE 3.8 (a) Magnetic resonance (MR) image of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and 0.3, respectively. (Original image for this example courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

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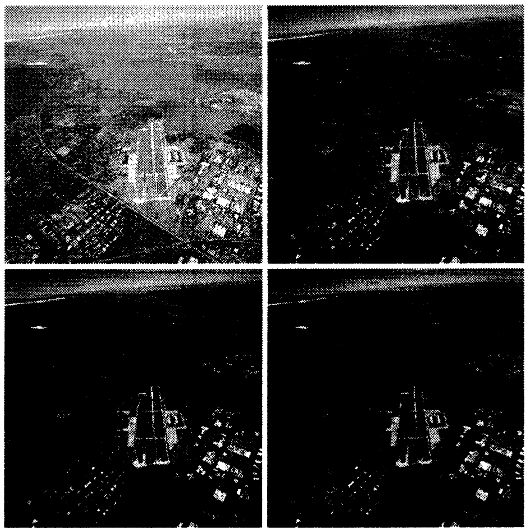

a b
c d

FIGURE 3.9
(a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

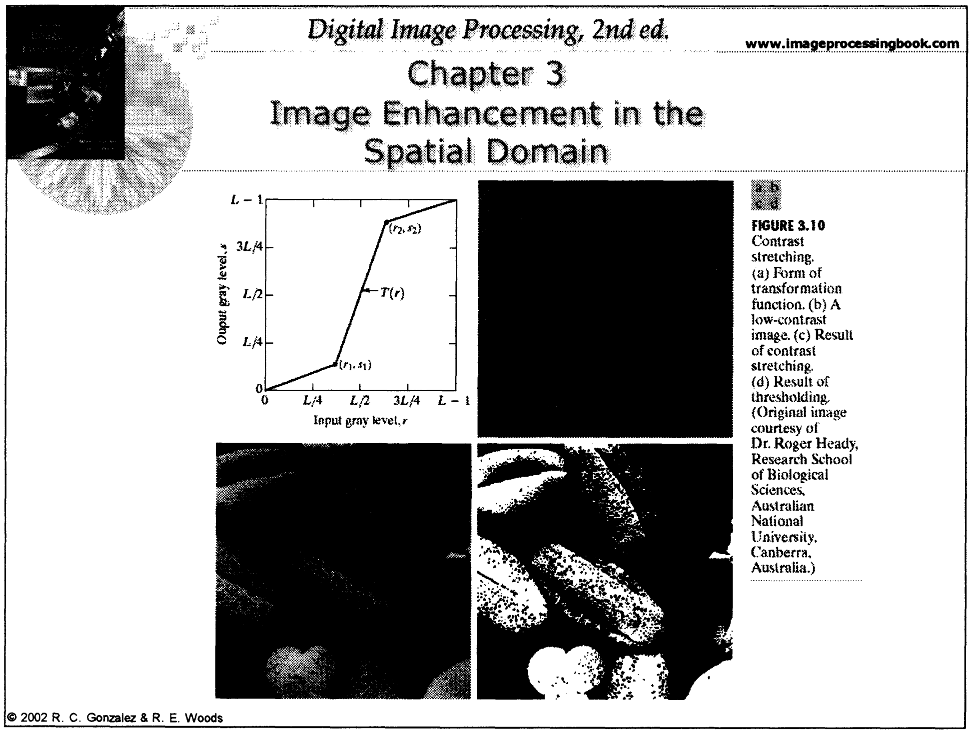

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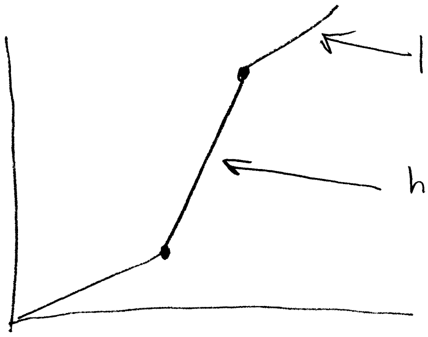
FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)



lower slope decreased dynamic range.

higher slope increases dynamic range

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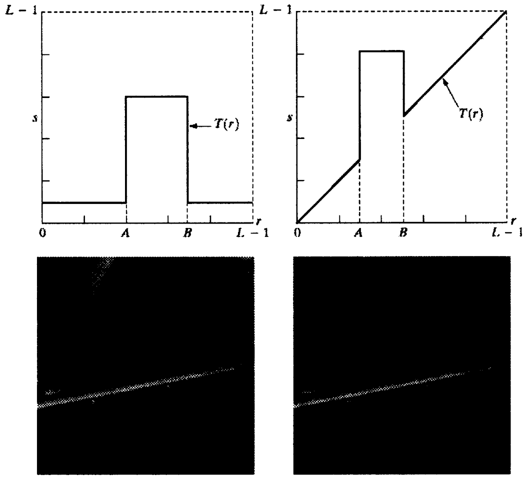

FIGURE 3.11 (a) This transformation highlights range [A, B] of gray levels and reduces all others to a constant level. (b) This transformation highlights range [A, B] but preserves all other levels. (c) An image. (d) Result of using the transformation in (a).

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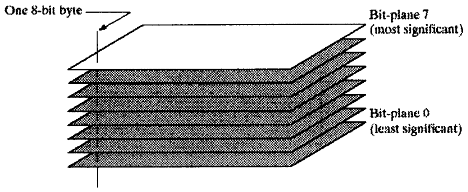

FIGURE 3.12 Bit-plane representation of an 8-bit image.

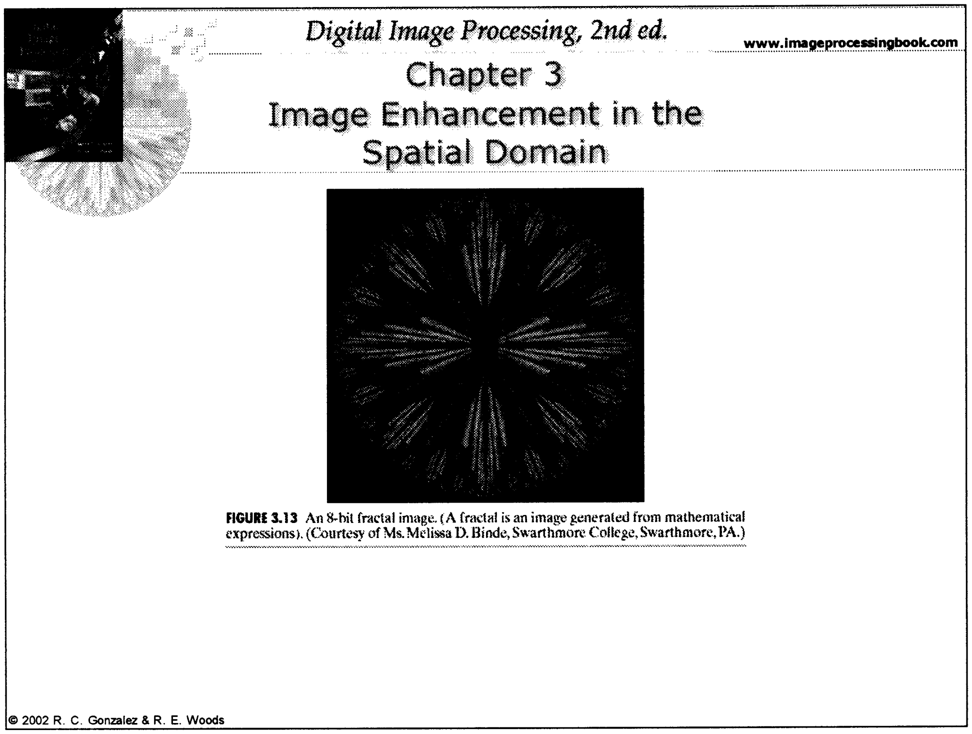

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FIGURE 3.13 An 8-bit fractal image. (A fractal is an image generated from mathematical expressions). (Courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA.)

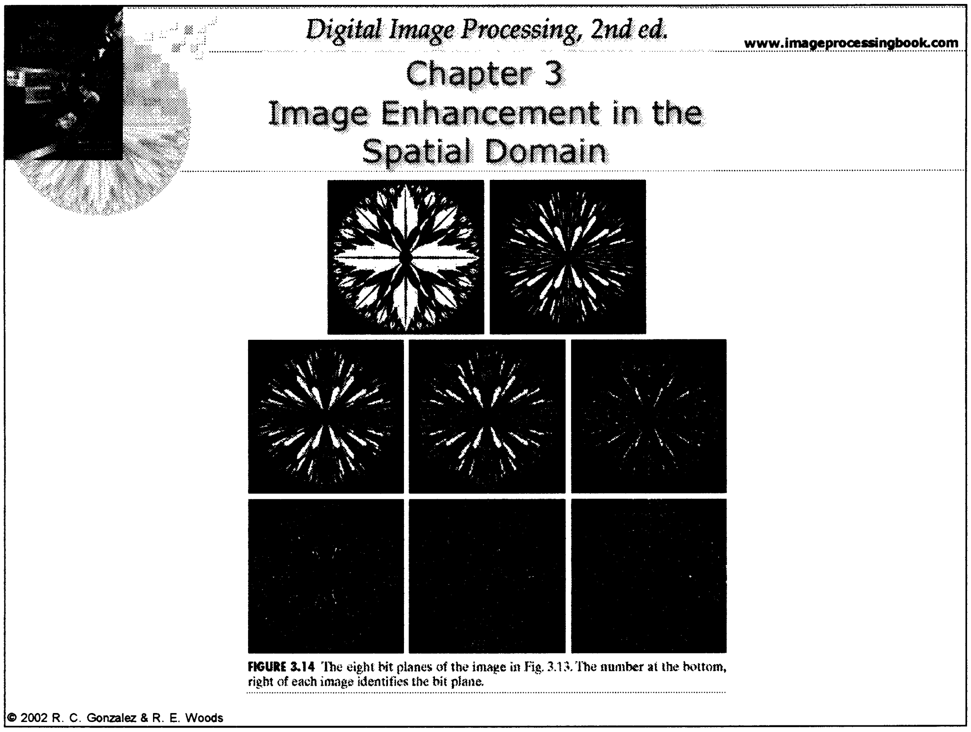

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FIGURE 3.14 The eight bit planes of the image in Fig. 3.13. The number at the bottom, right of each image identifies the bit plane.

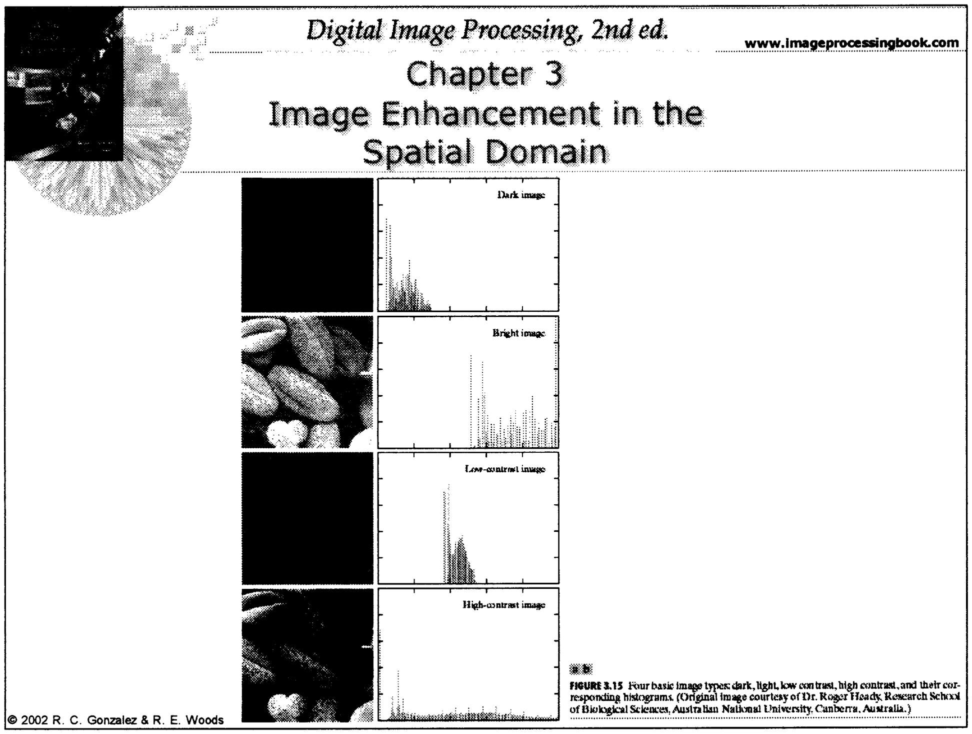

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FIGURE 3.15 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

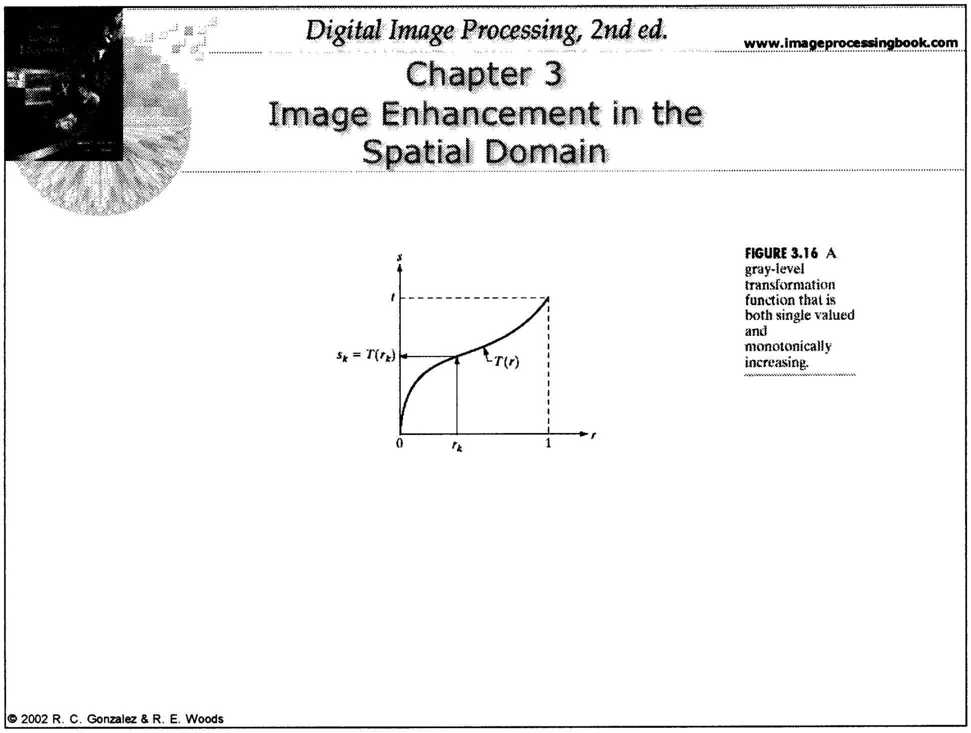

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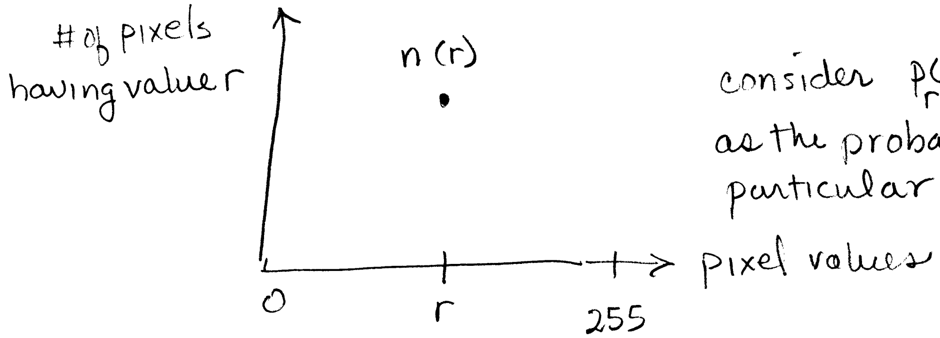
FIGURE 3.16 A gray-level transformation function that is both single valued and monotonically increasing.



consider $p_r(r) = \frac{n(r)}{n}$ as the probability of a particular gray level r.

Then $$s = T(r) = \underbrace{\int_0^r p_r(w)\,dw}$$

This is a cumulative distribution function if we consider $p_r(r)$ to be a probability density function.

T(r) is single valued and monotonically increasing. for $0 \le r \le 1$

$0 \le T(r) \le 1$ for $0 \le r \le 1$

From probability theory if $s = T(r)$.

$$P_s(s) = P_r(r)\left|\frac{dr}{ds}\right|$$

Returning to our transform $s = T(r)$

$$\frac{ds}{dr} = \frac{d(T(r))}{dr} = \frac{d}{dr}\left[\int_0^r P_r(w)\,dw\right] = P_r(r)$$

Then

$$P_s(s) = P_r(r)\left|\frac{1}{p_r(r)}\right| = 1.$$

This is a uniform probability function, i.e. all pixels are equally distributed.

for discrete values. this is the histogram.

$$P_r(r_k) = \frac{n_k}{n}$$

$n_k \leftarrow$ # of pixels of value r_k

$n \leftarrow$ # of pixels in image.

for $k = 0, 1, 2, \ldots, L-1$

histogram equalization since $P_S(s_k) = 1$

$$S_K = T(r_k) = \sum_{j=0}^{k} P_r(r_j)$$

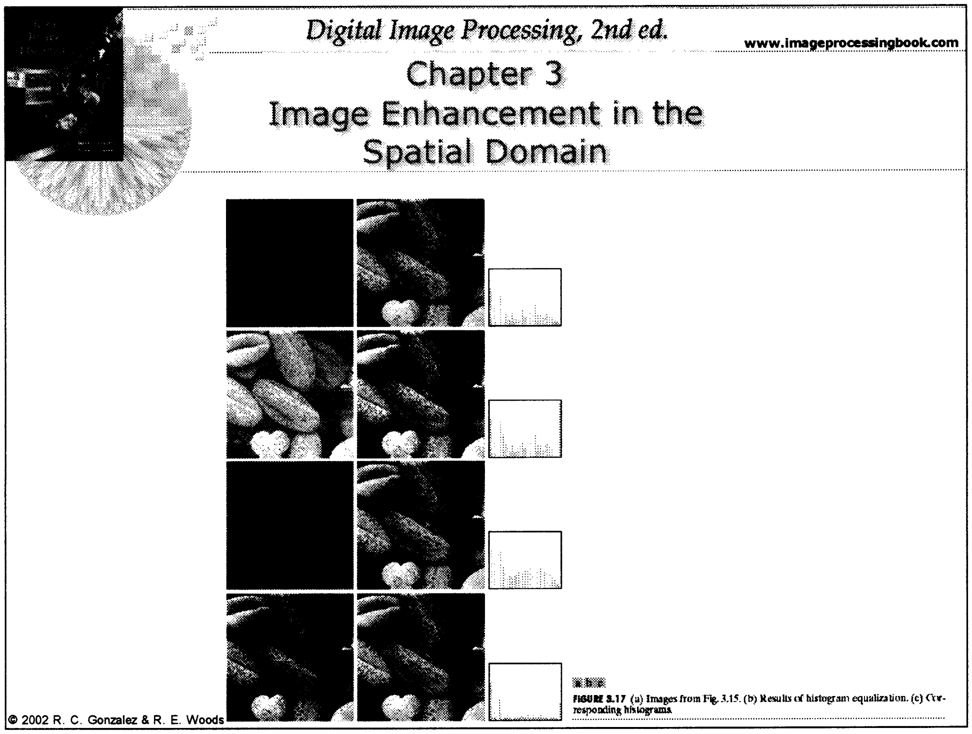
---

summing to k-th value

$$s_K = \sum_{j=0}^{k} \frac{n_j}{n} \qquad k = 0, \ldots, L-1$$

So simply transform r_k to s_K for each gray level k

Author normally uses k differently. # of graylevels $L = 2^{K = \text{\# of bits}}$



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a b c

FIGURE 3.17 (a) Images from Fig. 3.15. (b) Results of histogram equalization. (c) Corresponding histograms.

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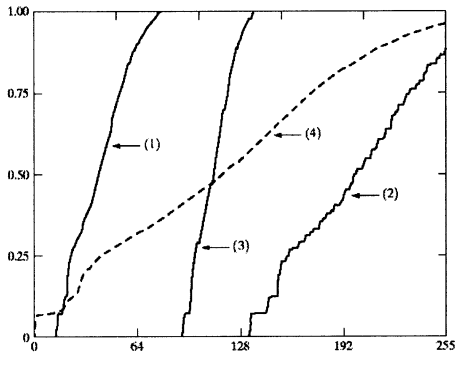
FIGURE 3.18 Transformation functions (1) through (4) were obtained from the histograms of the images in Fig.3.17(a), using Eq. (3.3-8).



3.3.2 Histogram Matching (specification)

We may want to transform a image's gray scale histogram to a particular shape, i.e., specify it.

This is a generalization of histogram equalization

output image input image

these are the probability distribution functions, i.e., histograms.

$P_z(z)$ $P_r(r)$

↑ ↑

This is the desired distribution

This is the original distribution

Consider a new random variable s given by

$$s = T(r) = \int_0^r P_r(w)\,dw \qquad (1)$$

just like we did for histogram equalization.

Now define another random variable z

$$G(z) = \int_0^z P_z(t)\,dt = s \qquad (2)$$

$$\therefore \quad G(z) = T(r)$$

$$\text{or} \quad z = G^{-1}(s) = G^{-1}[\underbrace{T(r)}]$$

This can be computed using (1) from the input image

G can be similarily computed from (2) for the desired image distribution.

The real issue is how to invert G to get G^{-1}

This can be done readily for digital (discrete functions) k-th value of transformation.

k-th value of r_k (gray level)

↓

$$s_k = T(r_k) = \sum_{j=0}^{k} P_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n}$$

↑

This is the cumulative distributive function for each gray level k

↑

probability of each gray level k

$$v_k = G(z_k) = \sum_{i=0}^{k} P_z(z_i) = s_k$$

↑

This is the cumulative distribution function for the desired distribution $P_z(z_i)$

Combining these

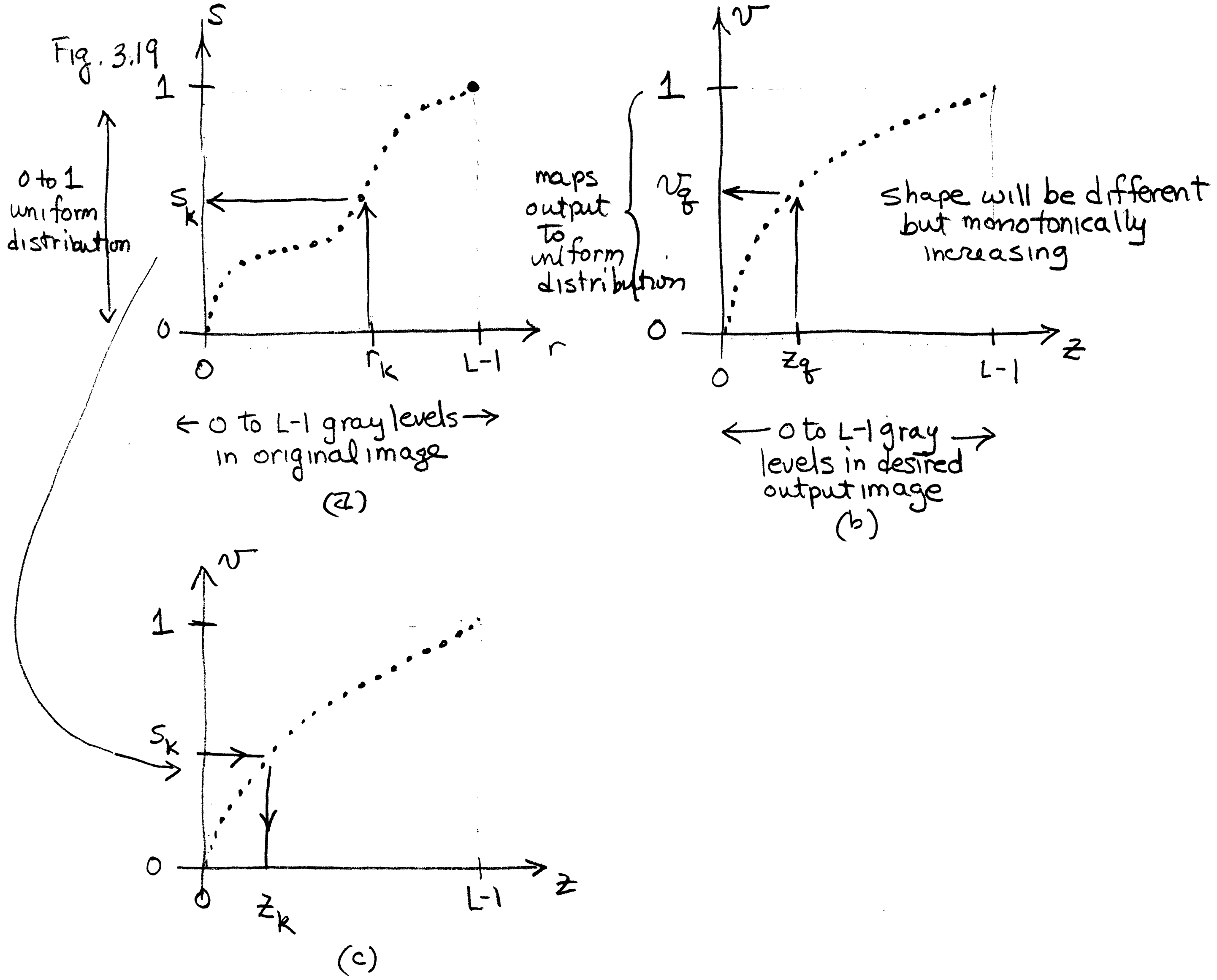
output (desired) gray scales

$$G(z_k) = s_k = T(r_k)$$

Invert to get the transform

$$z_k = G^{-1}\left[T(r_k)\right]$$

Fig. 3.19



Algorithm

① compute histogram of original image

② compute the uniform mapping s_k by computing C.D.F.

③ compute the uniform transformation for the desired output transformation

④ starting at $s_k = 0$ iterate (increase) to find smallest value $\hat{z}_k$ which approximately satisifies (c); repeat for all s_k up to 1

⑤ combine $r_k \rightarrow s_k$ with results of ④ to get $r_k \rightarrow \hat{z}_k$ transformation and transform table.

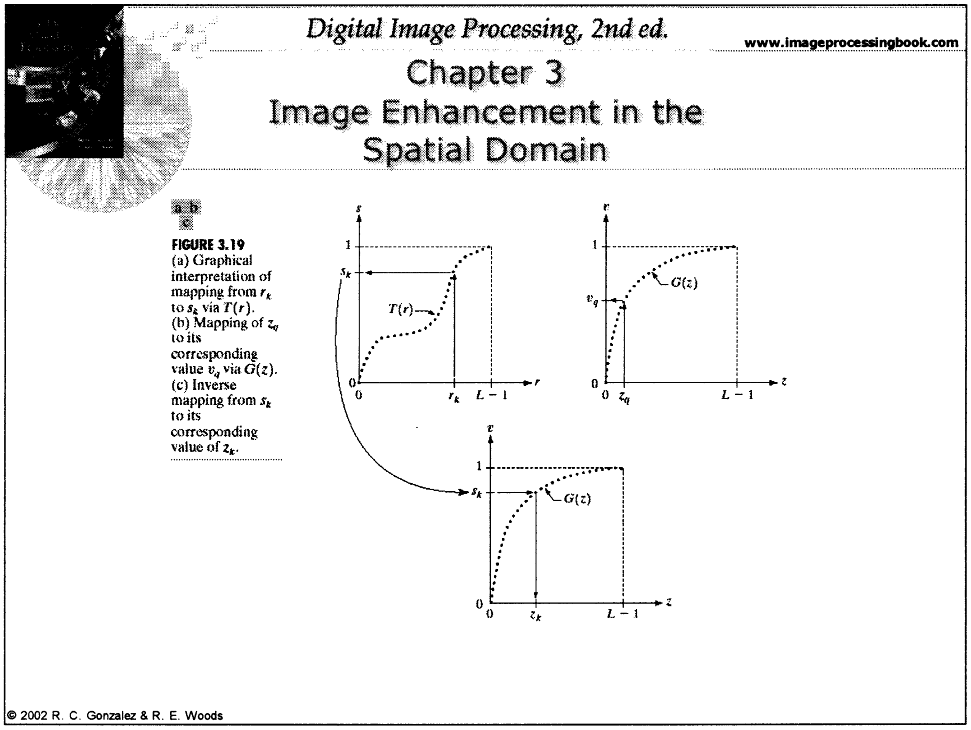


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FIGURE 3.19 (a) Graphical interpretation of mapping from r_k to s_k via $T(r)$. (b) Mapping of z_q to its corresponding value v_q via $G(z)$. (c) Inverse mapping from s_k to its corresponding value of z_k.

Algorithm

① compute histogram of original image

② compute uniform mapping s_k onto [0,1] by computing cumulative distribution function (CDF)

③ similarily compute the CDF for the desired output transformation

④ starting at $s_k = 0$ iterate (increase) to find value $\hat{z}_k$ which approximately satisfies the function in (c). Repeat for all s_k up to 1.

⑤ combine the $r_k \rightarrow s_k$ mapping with the results of ④ to get the functional mapping $r_k \rightarrow \hat{z}_k$ and construct a transformation table

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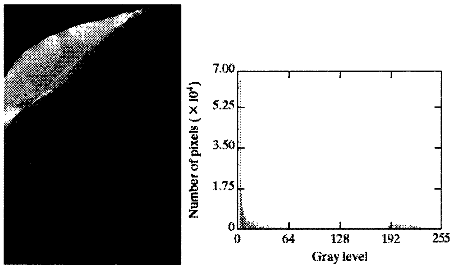

FIGURE 3.20 (a) Image of the Mars moon Photos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

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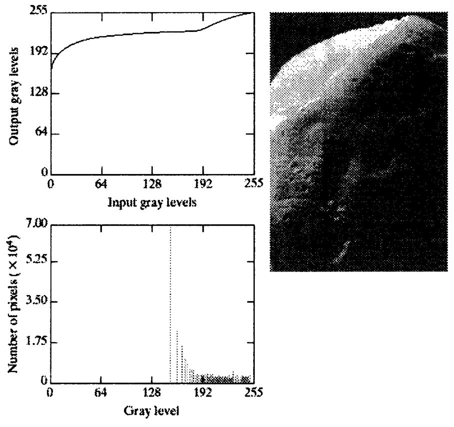

FIGURE 3.21
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

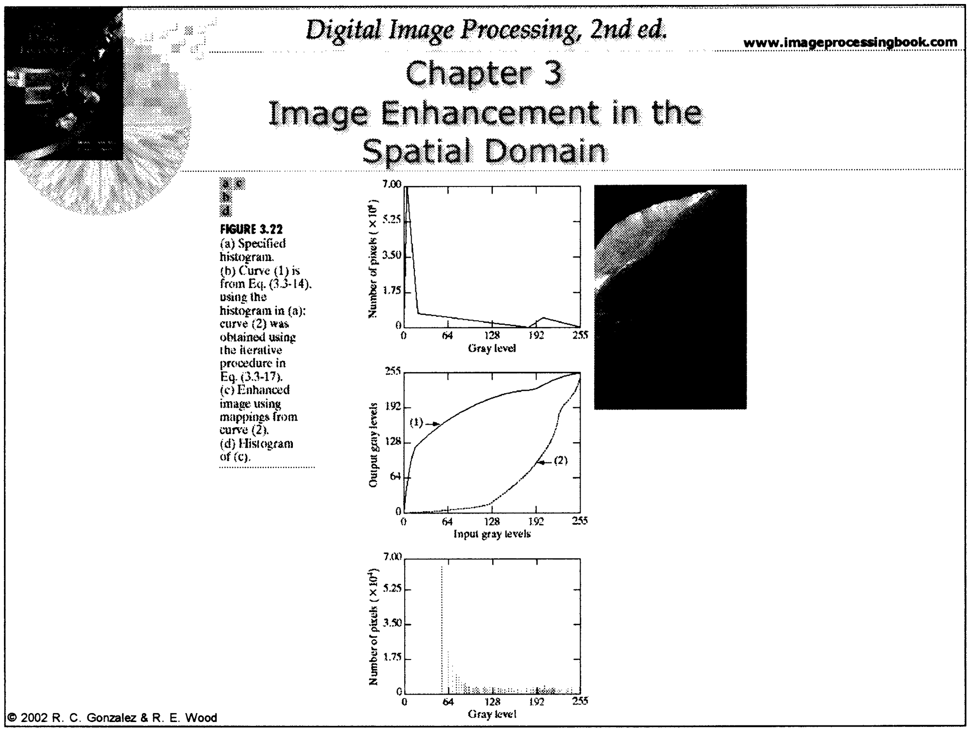

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FIGURE 3.22 (a) Specified histogram. (b) Curve (1) is from Eq. (3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3-17). (c) Enhanced image using mappings from curve (2). (d) Histogram of (c).

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(a) This is the desired manually generated histogram. Note that this is a continuous function.

(b) (1) is the computed C.D.F. of the desired histogram given in (a).
(2) is the transformation resulting from applying the algorithm of Figure 3.19 to this image.

(c) is the result of using (b)(2) to transform the original image.

(d) is the histogram of the image produced in (c)

Note that the histogram does not appear uniform because our derivation was for continuous variables whereas our images and solution are discrete.

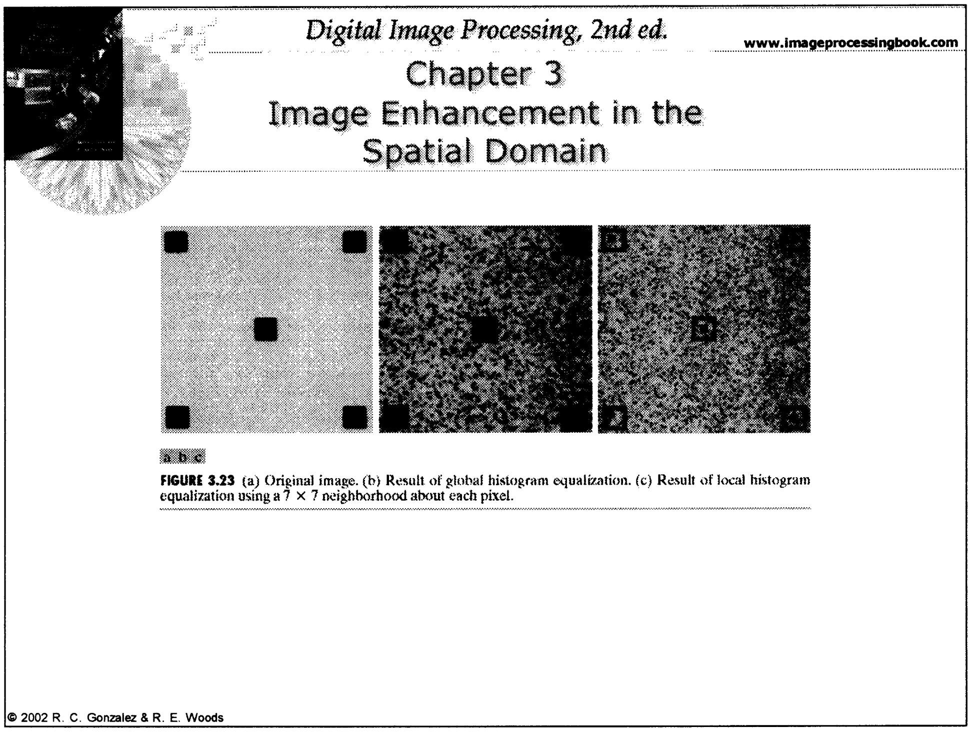

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FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7 × 7 neighborhood about each pixel.

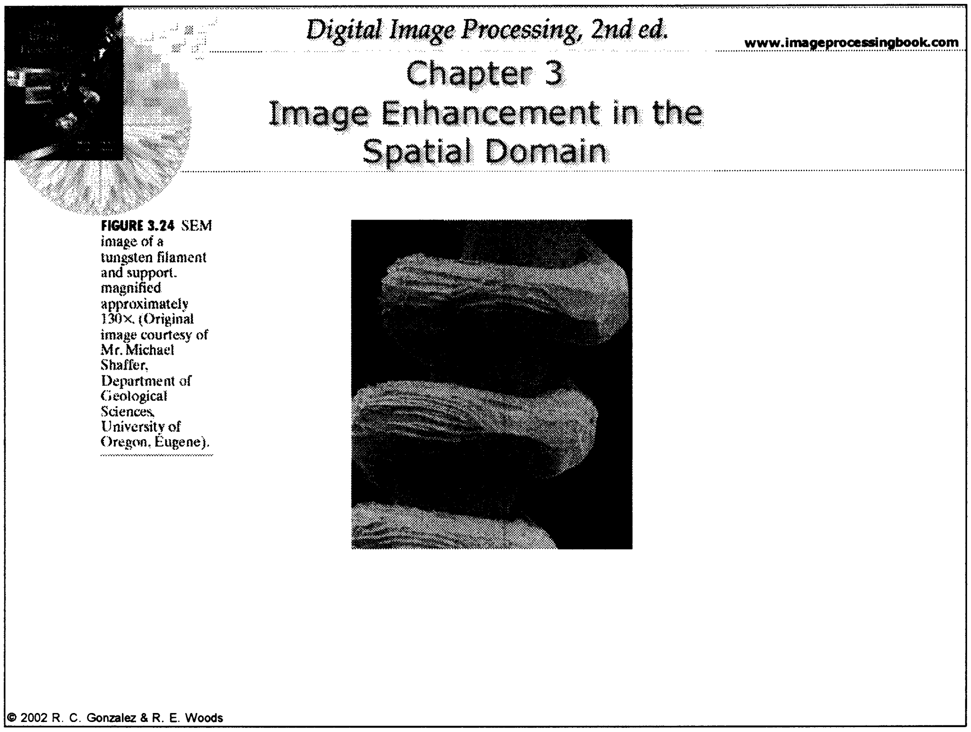

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FIGURE 3.24 SEM image of a tungsten filament and support, magnified approximately 130×. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).

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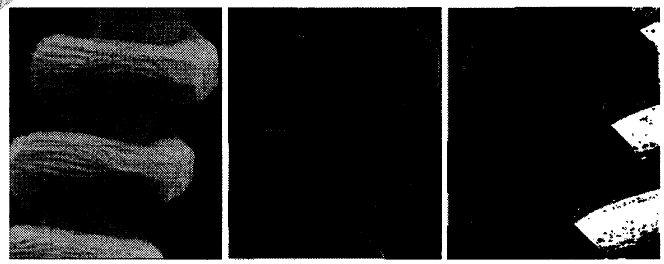

FIGURE 3.25 (a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.

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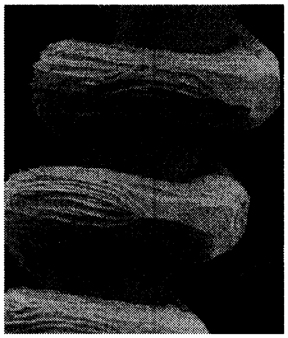

FIGURE 3.26 Enhanced SEM image. Compare with Fig. 3.24. Note in particular the enhanced area on the right side of the image.

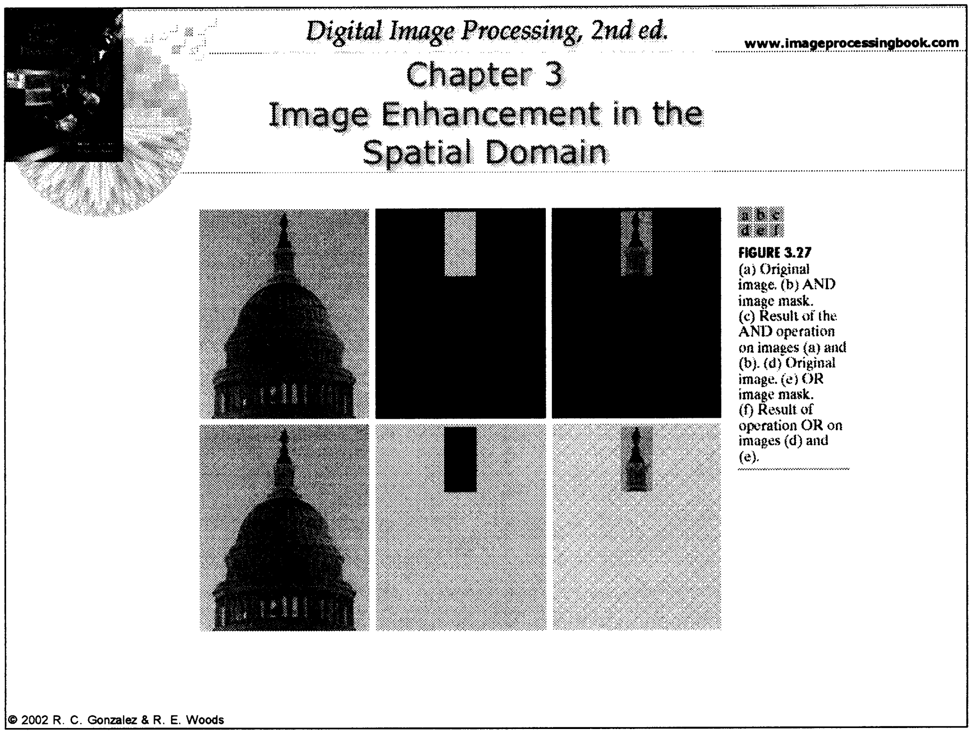

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FIGURE 3.27 (a) Original image. (b) AND image mask. (c) Result of the AND operation on images (a) and (b). (d) Original image. (e) OR image mask. (f) Result of operation OR on images (d) and (e).

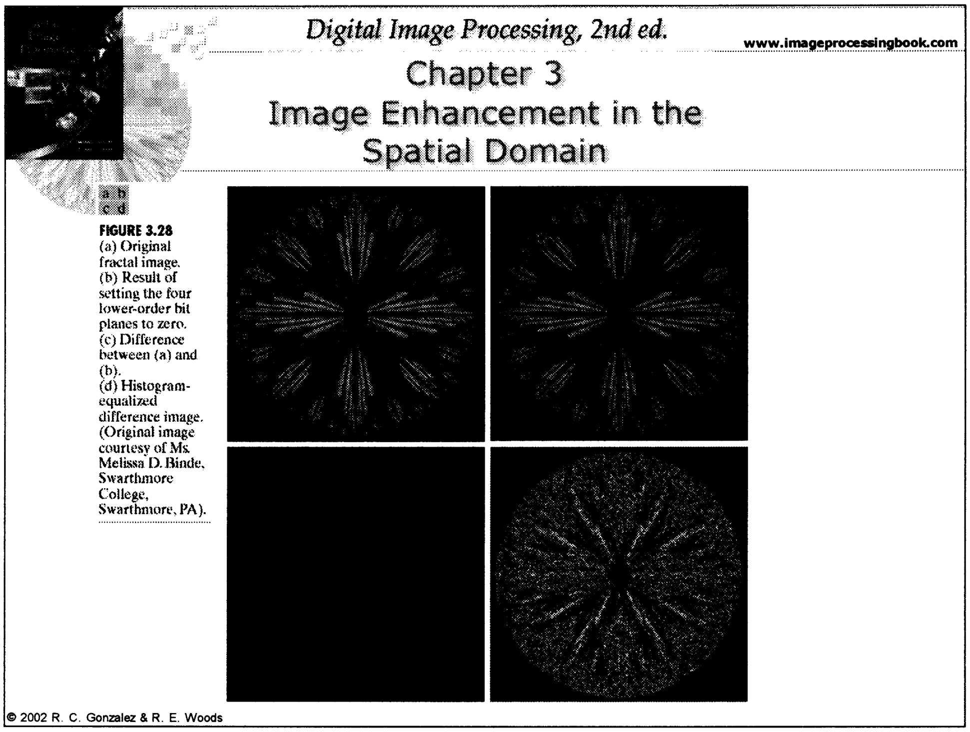

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FIGURE 3.28 (a) Original fractal image. (b) Result of setting the four lower-order bit planes to zero. (c) Difference between (a) and (b). (d) Histogram-equalized difference image. (Original image courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA).

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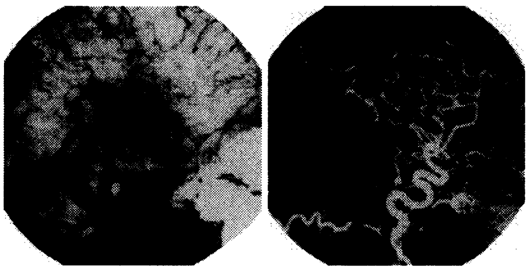

a b

FIGURE 3.29 Enhancement by image subtraction. (a) Mask image. (b) An image (taken after injection of a contrast medium into the bloodstream) with mask subtracted out.

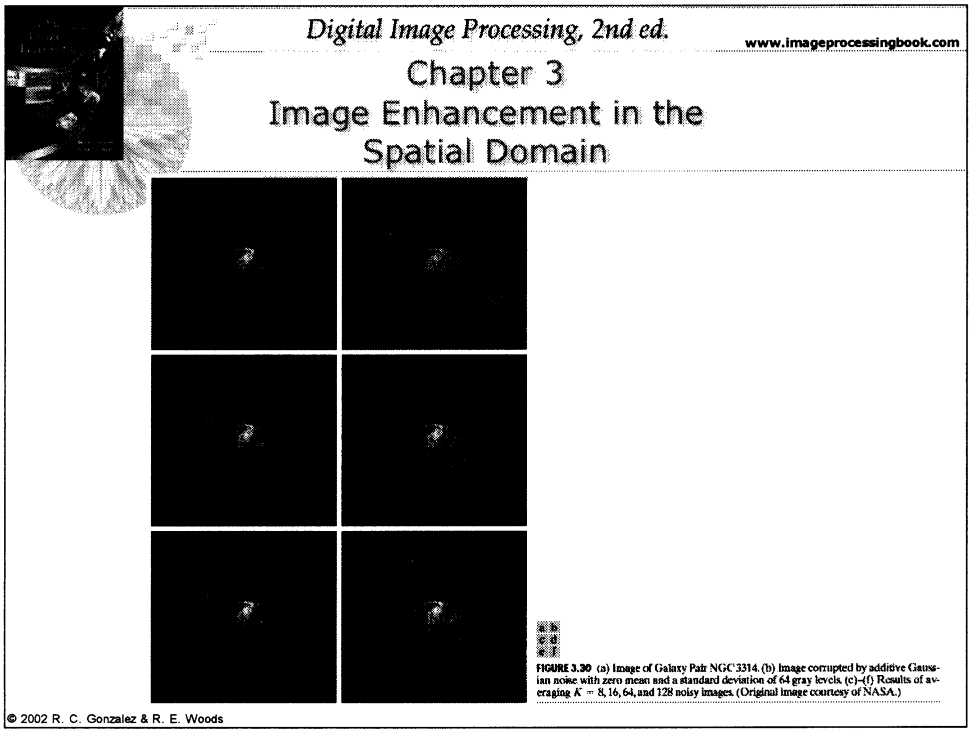

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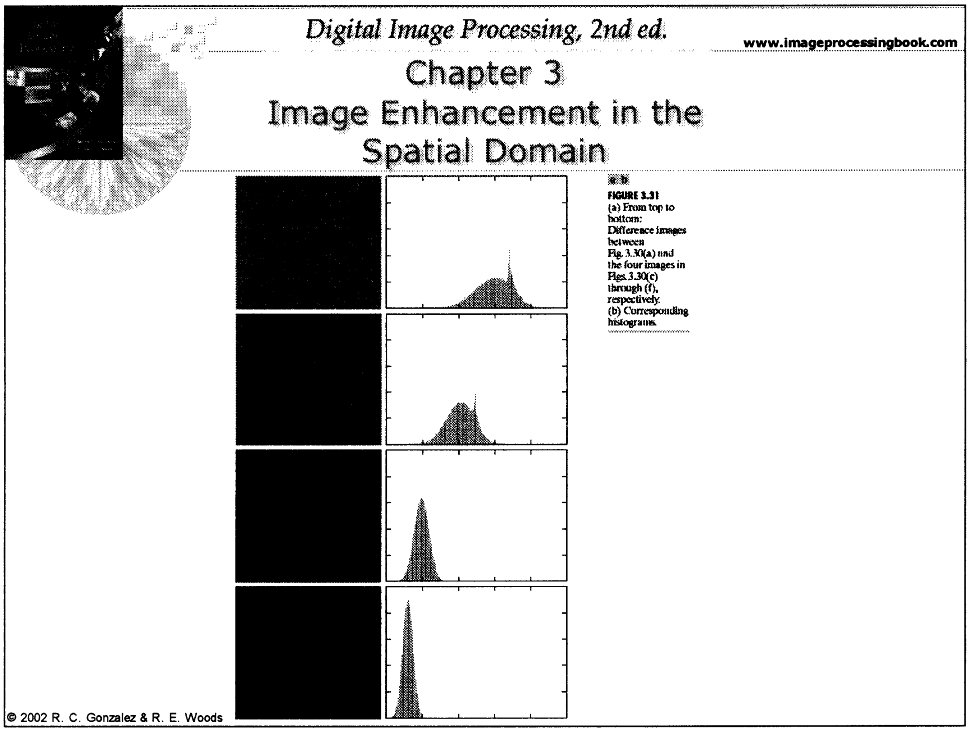
FIGURE 3.30 (a) Image of Galaxy Pair NGC 3314. (b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels. (c)–(f) Results of averaging $K = 8, 16, 64,$ and 128 noisy images. (Original image courtesy of NASA.)



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a b

FIGURE 3.31
(a) From top to bottom: Difference images between Fig. 3.30(a) and the four images in Figs. 3.30(c) through (f), respectively.
(b) Corresponding histograms.

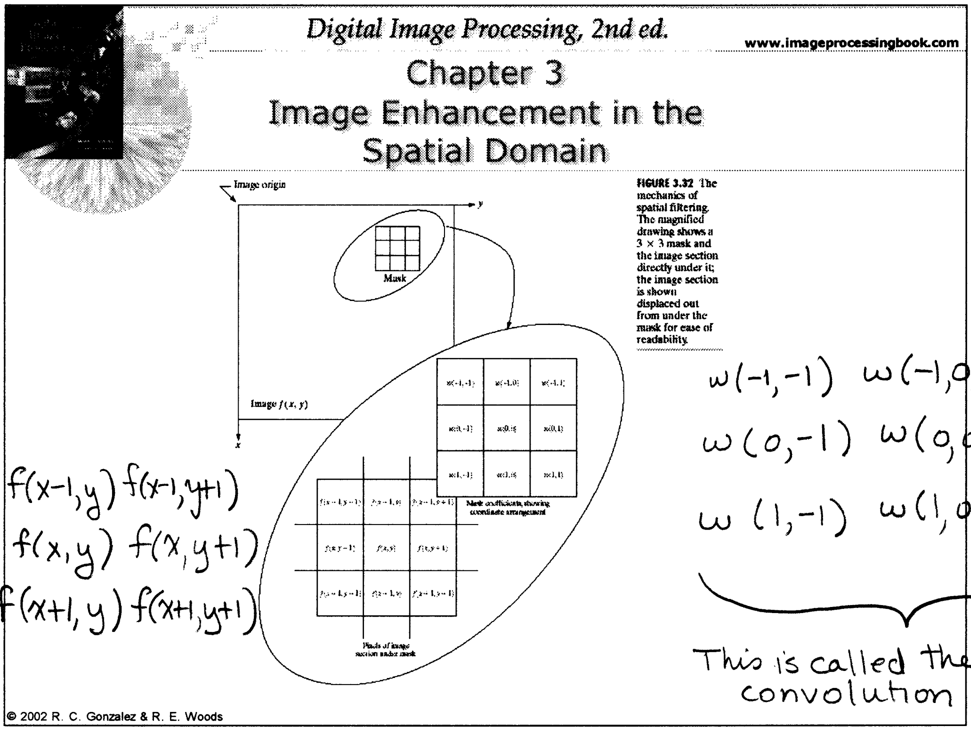

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FIGURE 3.32 The mechanics of spatial filtering. The magnified drawing shows a 3 × 3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

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$$\begin{matrix} f(x-1,y-1) & f(x-1,y) & f(x-1,y+1) \\ f(x,y-1) & f(x,y) & f(x,y+1) \\ f(x+1,y-1) & f(x+1,y) & f(x+1,y+1) \end{matrix}$$

$$\underbrace{\begin{matrix} w(-1,-1) & w(-1,0) & w(-1,1) \\ w(0,-1) & w(0,0) & w(0,1) \\ w(1,-1) & w(1,0) & w(1,1) \end{matrix}}$$

This is called the convolution mask.

Linear filtering of an image f of size M×N with a filter mask w of size m×n is given by

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t)$$

This is essentially a digital convolution.

In practice, we multiply each element of the mask by the corresponding image element (underneath it) We then slide the mask one to the right and repeat At the end of each row we move the template to the beginning of the next row and repeat.

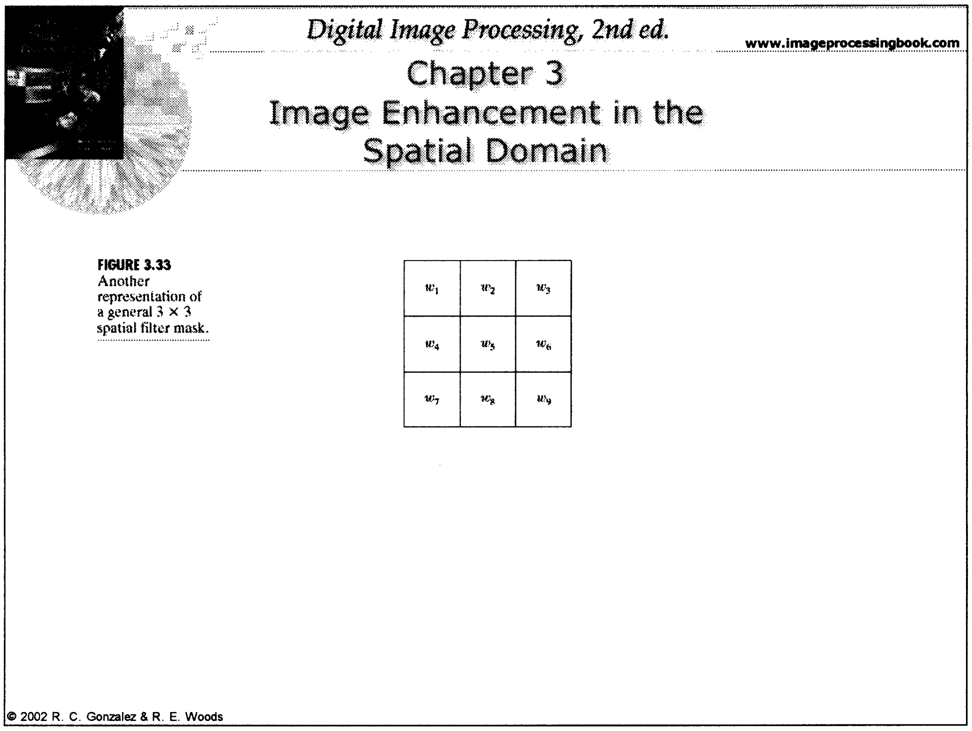

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FIGURE 3.33 Another representation of a general 3 × 3 spatial filter mask.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

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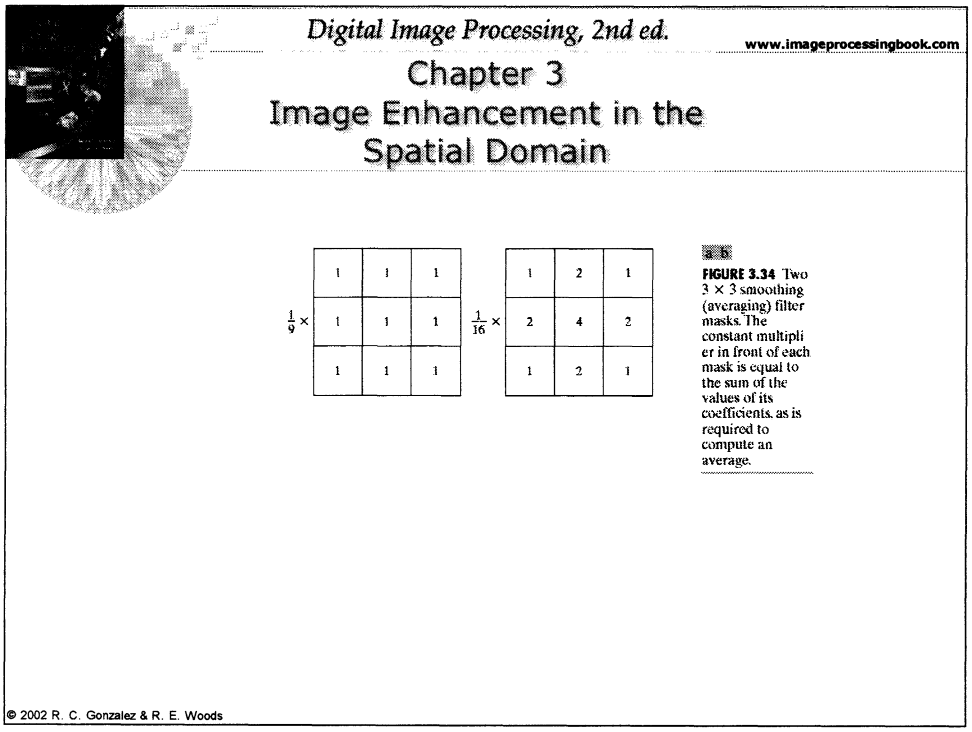
In essence, the response is a sum of products given by

$$R = \sum_{i=1}^{9} w_i z_i \qquad \text{for a } 3\text{x}3 \text{ mask}$$

But what happens when the filter runs out of pixels at the edge of the image.

- simply limit the mask so it can't go outside the image (this reduces the image size!)
- filter all pixels only with the part of the mask fully contained in the image, i.e., a partial mask
- padding – either replicate rows & columns at the edges, or extend the image by adding rows & columns of some constant gray level such as "1"



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$$\frac{1}{9}\times\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\qquad\frac{1}{16}\times\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix}$$

a b

FIGURE 3.34 Two 3 × 3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to the sum of the values of its coefficients, as is required to compute an average.

One of the simplest things you can do is average the pixels contained in the mask neighborhood. This has the effect of smoothing and will be shown to be a low-pass filter.

From Fig 3.33 $\sum_{i=1}^{9} w_i z_i$

a real average would be $\frac{1}{9}\sum_{i=1}^{9} z_i$

$\uparrow$

Scaling constant to make sure result is in range [0,1] or [0,255] as appropriate.

Other constants can be used. Or you can use other weights as shown in (b). This is a center weighted average.

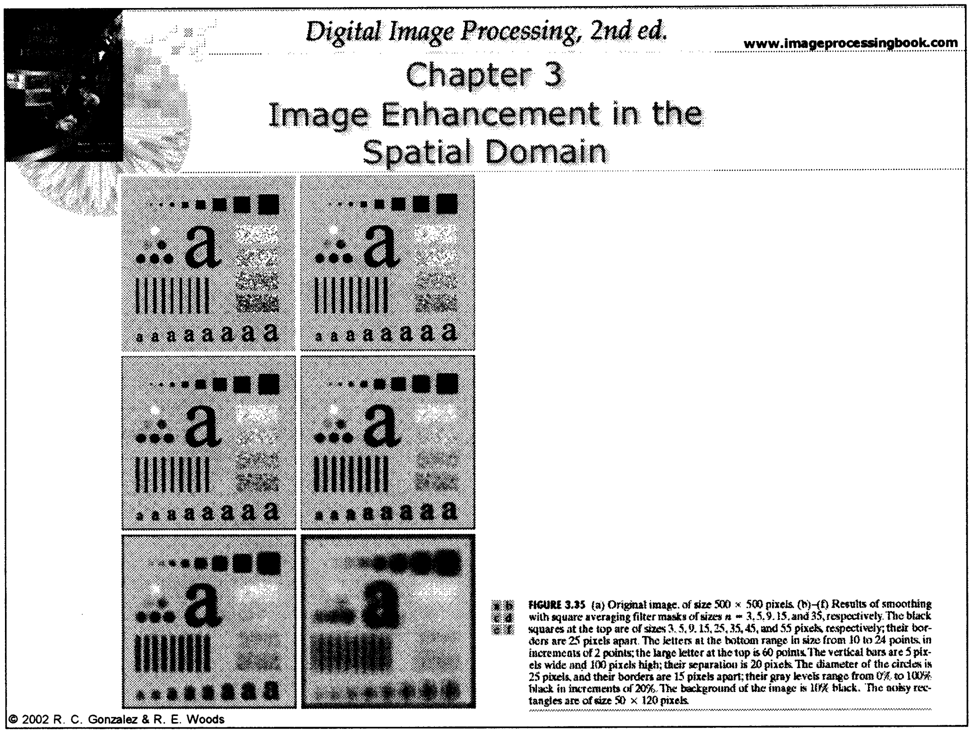

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FIGURE 3.35 (a) Original image. of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15,$ and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

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The size of the mask determines the amount of smoothing.

(b) 3x3 The smallest amount of smoothing

(c) 5x5

(d) 9x9

(e) 15x15

(f) 35x35 most smoothing

vertical bars are 5x 100 pixels with 20 pixel separation

Squares are 3x3 to 55x55

noisy rectangles are 50 x 120 pixels

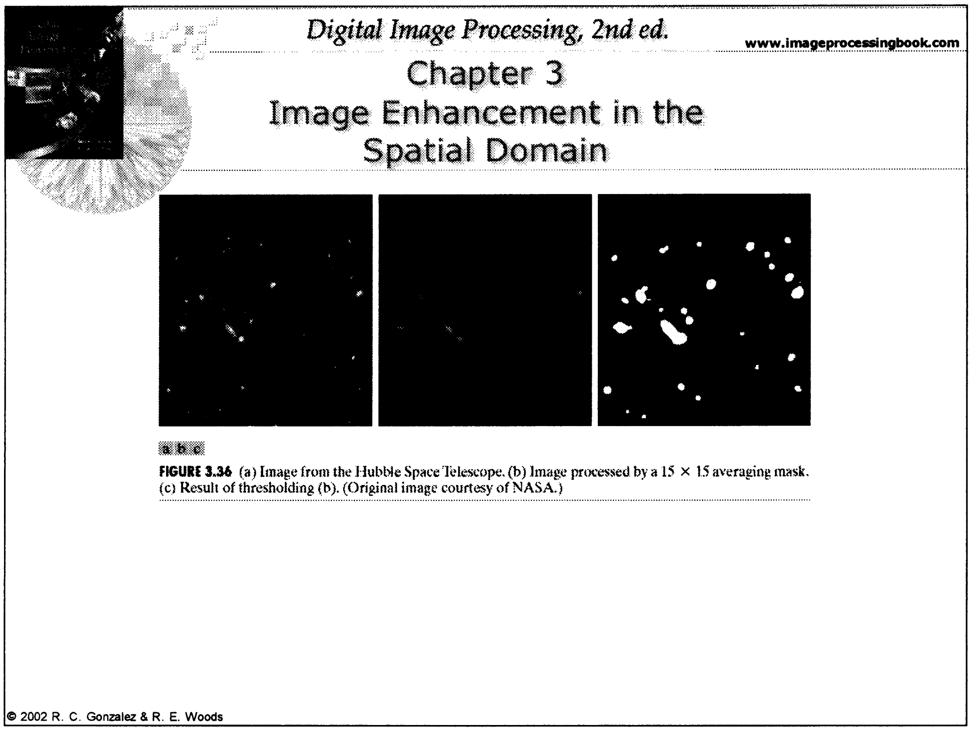

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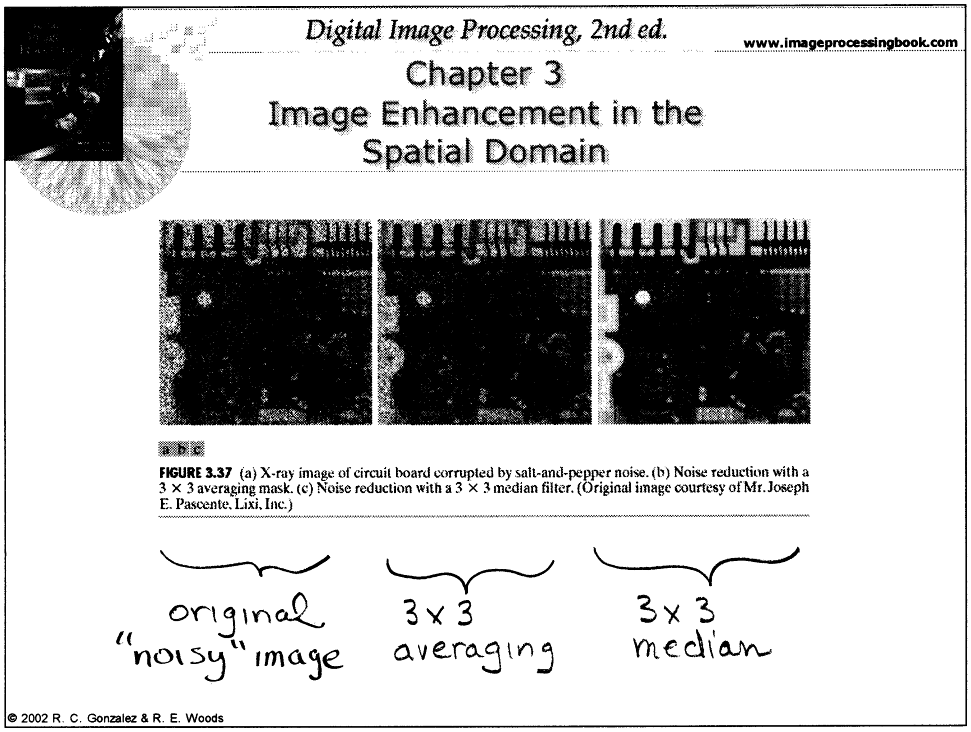
FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

original

15x15 averaging

thresholding



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a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

original "noisy" image

3 x 3 averaging

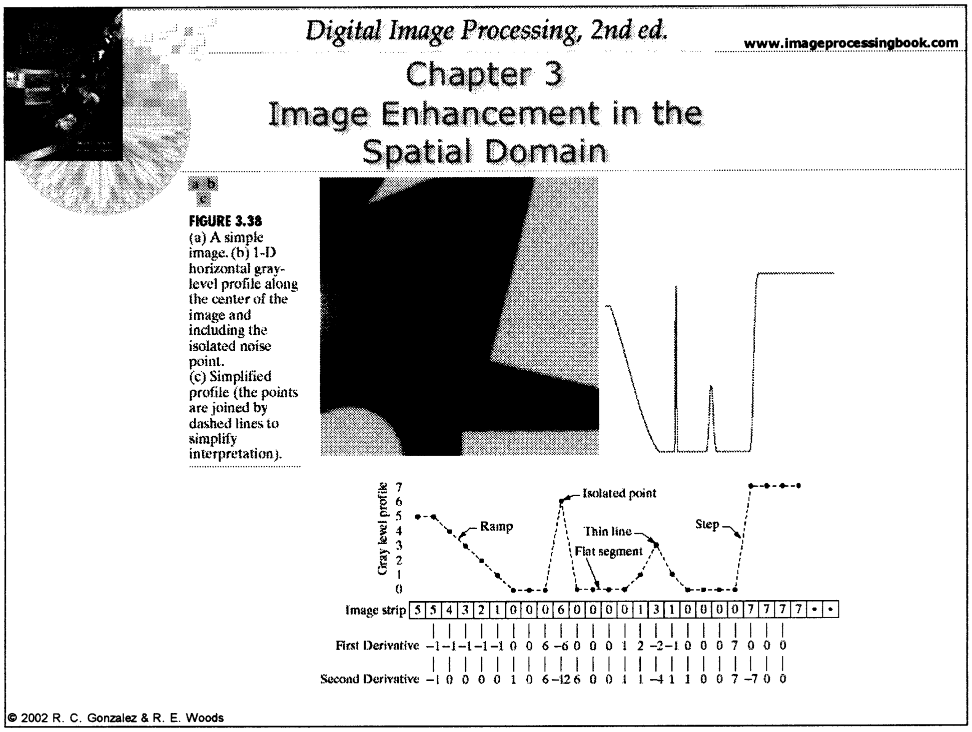
3 x 3 median

This is an example of an order-statistics filter. Produce a sorted list of the 9 pixel values in the mask and replace the input pixel by some function of the ranking.

median filter — replace the input pixel by the median (middle) of the 9 pixel values (for a 3x3 mask)

good match to eliminating certain types of noise
salt & pepper (impulse noise)

x-ray detectors have a lot of impulse noise



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a b
c

FIGURE 3.38 (a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point. (c) Simplified profile (the points are joined by dashed lines to simplify interpretation).

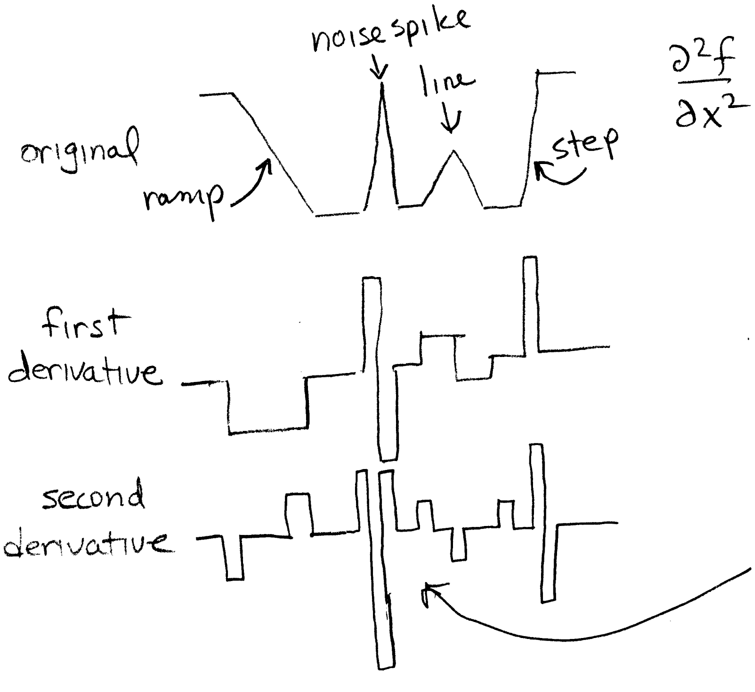
Image strip	5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7	•	•
First Derivative		−1	−1	−1	−1	−1	0	0	6	−6	0	0	0	1	2	−2	−1	0	0	0	7	0	0	0			
Second Derivative		−1	0	0	0	0	1	0	6	−12	6	0	0	1	1	−4	1	1	0	0	7	−7	0	0			

first derivative $\frac{\partial f}{\partial x} = f(x+1) - f(x)$

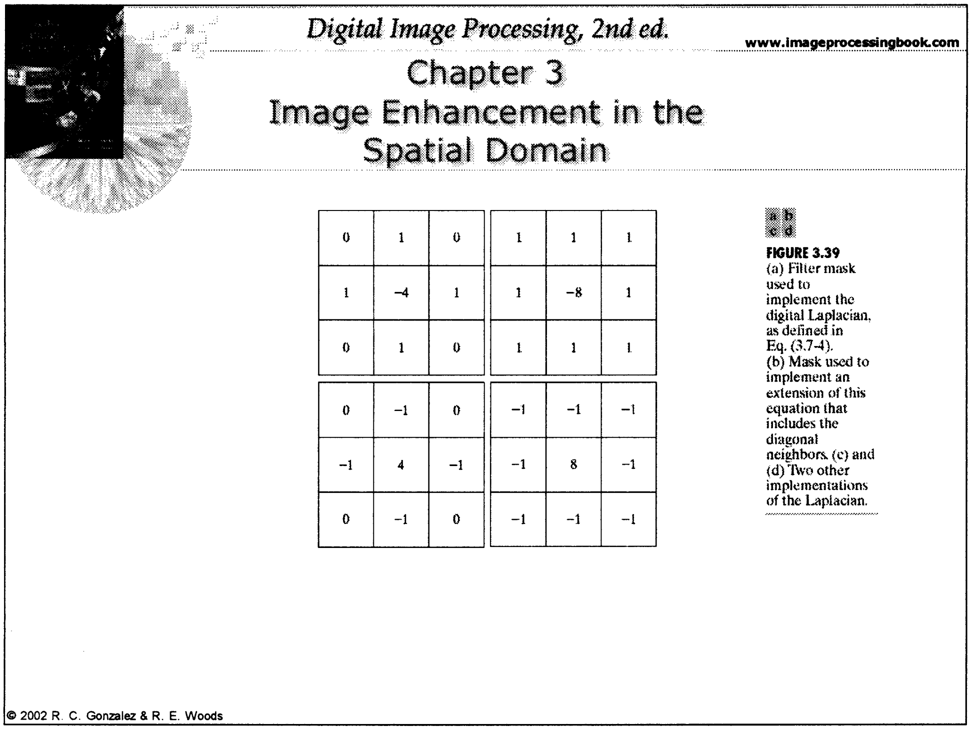
second derivative $\frac{\partial^2 f}{\partial x^2} = \frac{\partial f}{\partial x}(x) - \frac{\partial f}{\partial x}(x-1)$

$$= \left[f(x+1) - f(x)\right] - \left[f(x) - f(x-1)\right]$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$



note classic response of 2nd derivative to a spike.



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0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

a b
c d

FIGURE 3.39 (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

Laplacian $\nabla^2 f = \dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2}$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

$$\Rightarrow \nabla^2 f = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

	1	
1	-4	1
	1	

} mask implementation

(b) This can be extended to include diagonals (smoother)

(c),(d) Negative Laplacians are useful in more complex operations.

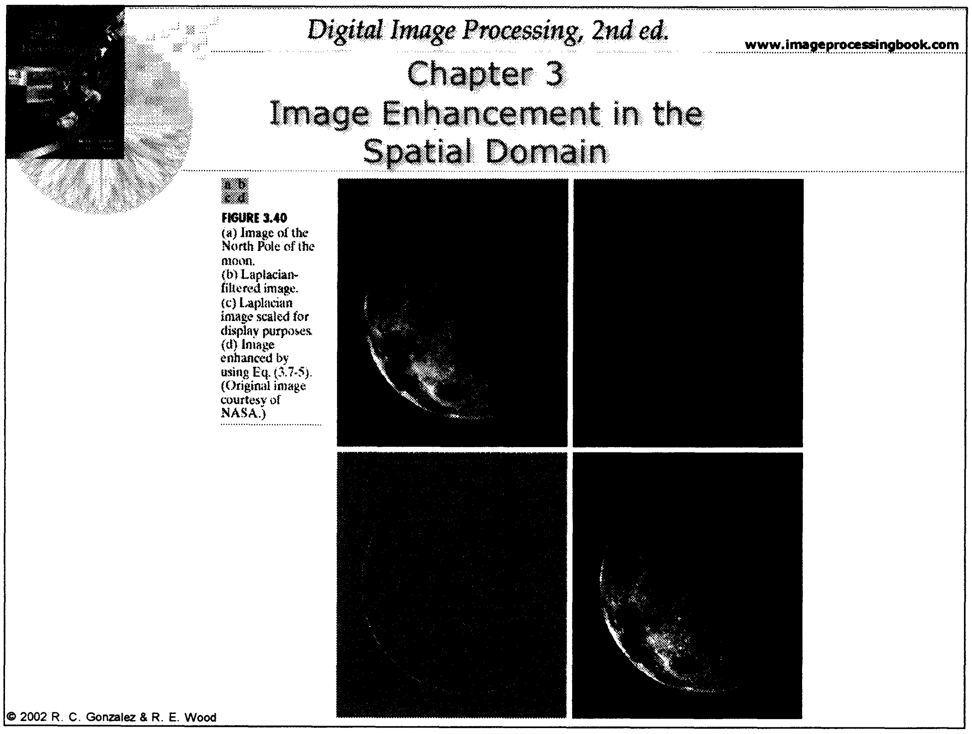

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FIGURE 3.40 (a) Image of the North Pole of the moon. (b) Laplacian-filtered image. (c) Laplacian image scaled for display purposes. (d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)

(a) original image

(b) Laplacian filtered image

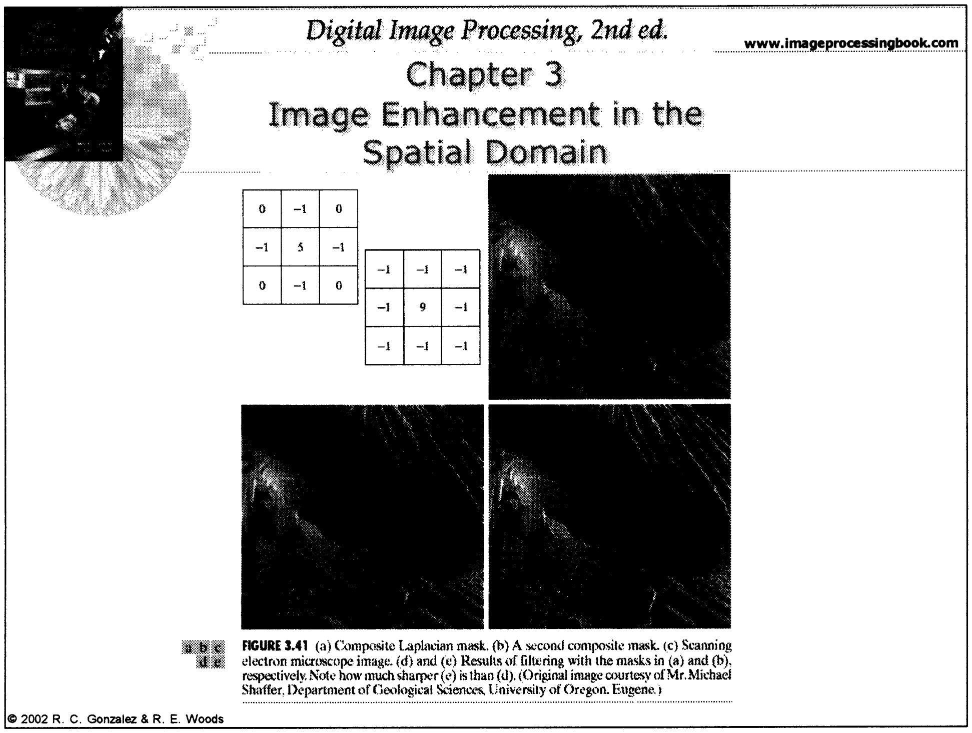
(c) Laplacian filtered image scaled to show detain on monitor

(d) sharpening filter

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \text{Laplacian postive center coeff.} \\ f(x,y) + \nabla^2 f(x,y) & \text{negative center coeff,} \end{cases}$$

See sharpening of crater detail.

Note: $g(x,y)$ can be implemented as a single mask.



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0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

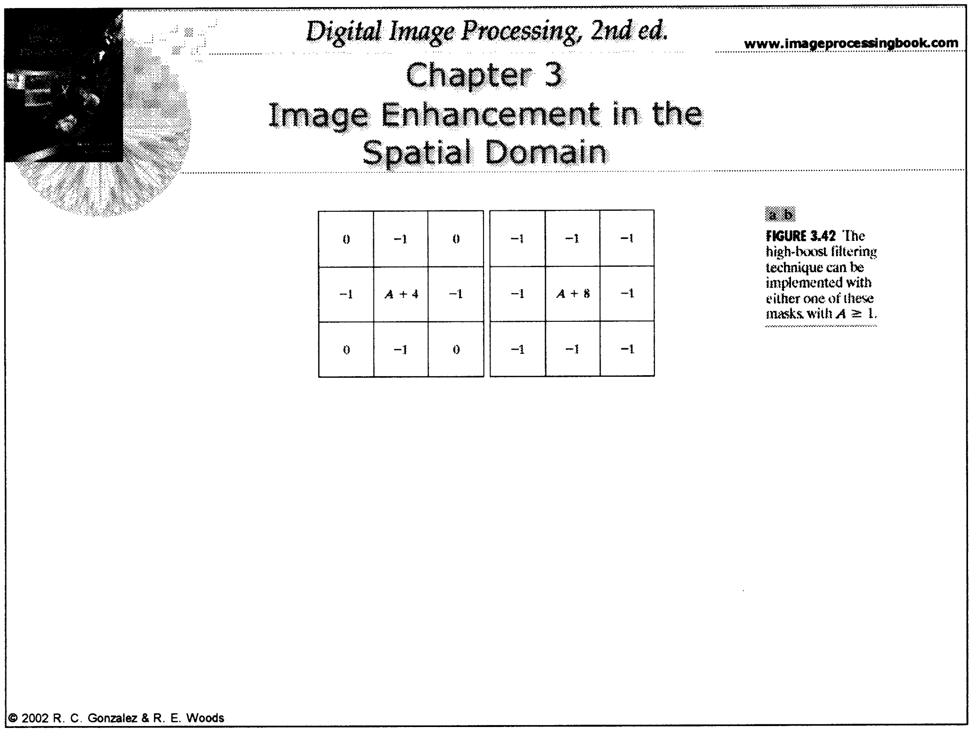
Laplacian with diagonal terms (b) usually yields sharper detail.

(c) original SEM image

(d) image filtered with (a), Laplacian w/o diagonals

(e) image filtered with (e), Laplacian w/ diagonals

As expected (e) is much sharper than (d)



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0	−1	0
−1	A + 4	−1
0	−1	0

−1	−1	−1
−1	A + 8	−1
−1	−1	−1

a b

FIGURE 3.42 The high-boost filtering technique can be implemented with either one of these masks, with $A \geq 1$.

unsharp masking $f_s(x,y) = f(x,y) - \underbrace{\bar{f}(x,y)}_{\text{blurred (averaged) version of } f(x,y)}$

high boost filtering

$$f_{hb}(x,y) = Af(x,y) - \bar{f}(x,y)$$

$$f_{hb}(x,y) = (A-1)f(x,y) + \underbrace{f(x,y) - \bar{f}(x,y)}_{f_s(x,y)}$$

which can come from Laplacian

$f(x,y) \mp \nabla^2 f(x,y)$

$$f_{hb} = \begin{cases} Af(x,y) - \nabla^2 f(x,y) \\ Af(x,y) + \nabla^2 f(x,y) \end{cases}$$

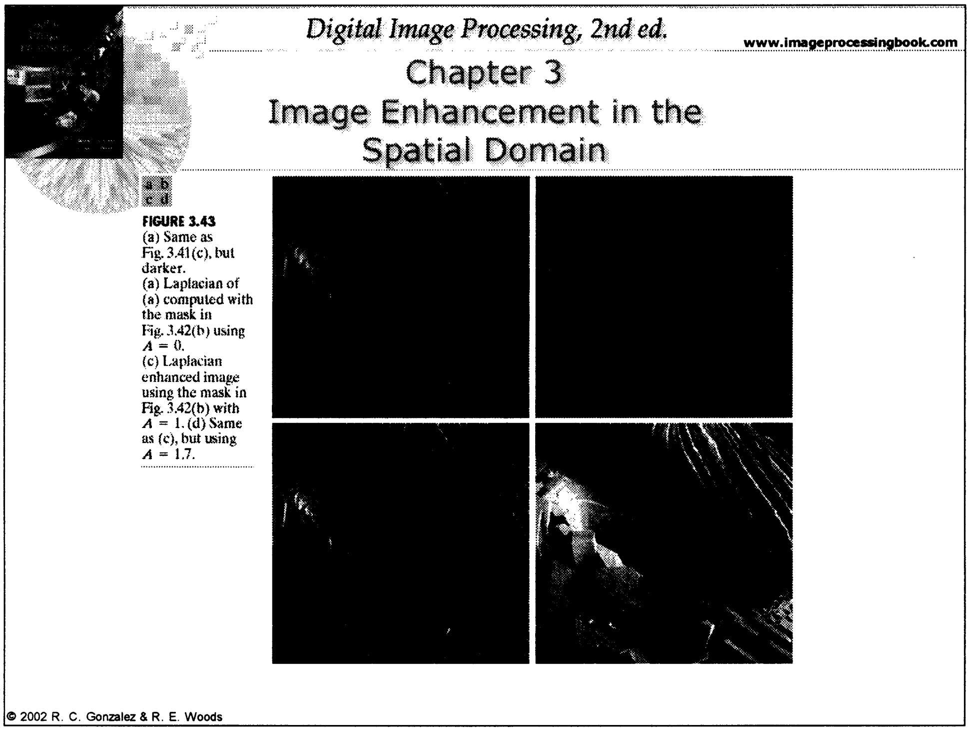

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FIGURE 3.43 (a) Same as Fig. 3.41(c), but darker. (a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$. (c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.

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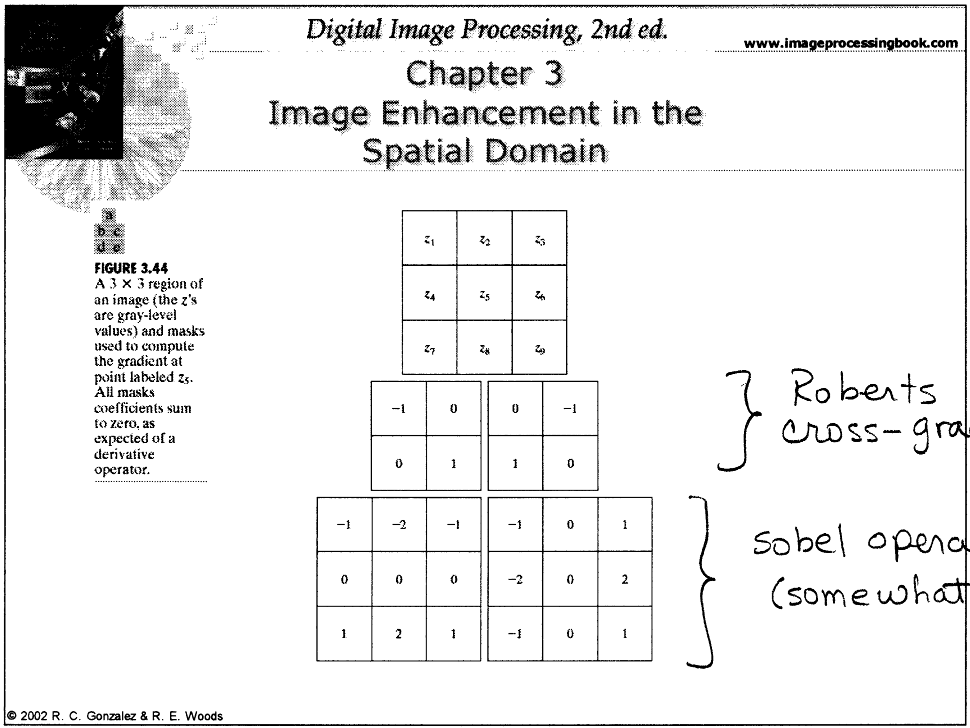
(a) original image (dark in appearance)

(b) Laplacian computed with using A = 0

−1	−1	−1
−1	A+8	−1
−1	−1	−1

(c) boost using above mask with A=1 (brighter)

(d) boost using above mask with A=1.7 (even brighter)



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a
b c
d e

FIGURE 3.44 A 3 × 3 region of an image (the z's are gray-level values) and masks used to compute the gradient at point labeled z_5. All masks coefficients sum to zero, as expected of a derivative operator.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

} Roberts cross-gradient operators

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

} Sobel operators (somewhat center weighted)

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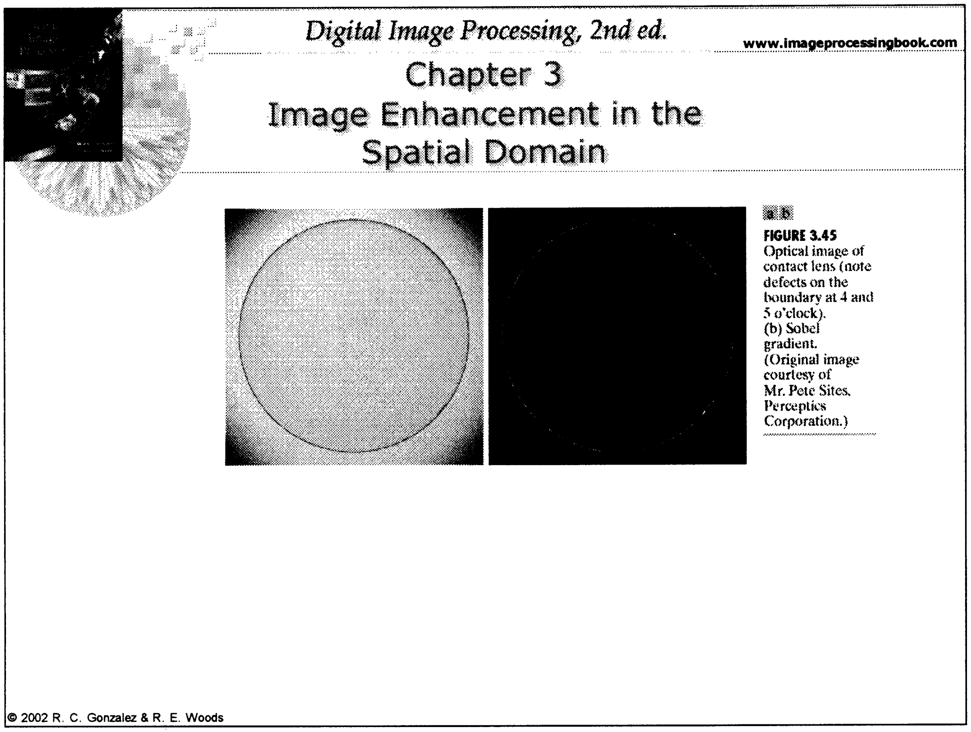
Gradient $$\nabla \underline{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\nabla f = |\nabla \underline{f}| = \sqrt{G_x^2 + G_y^2} = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

This is computationally a lot of work so approximate

$$\nabla f \approx |G_x| + |G_y|$$

This loses isotropy but is faster.



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a b

FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

(a) Original image with side lighting.

(b) Sobel gradient
  - excellent boundary for automated inspection analysis
  - gradient also brings out small specks in uniform background area

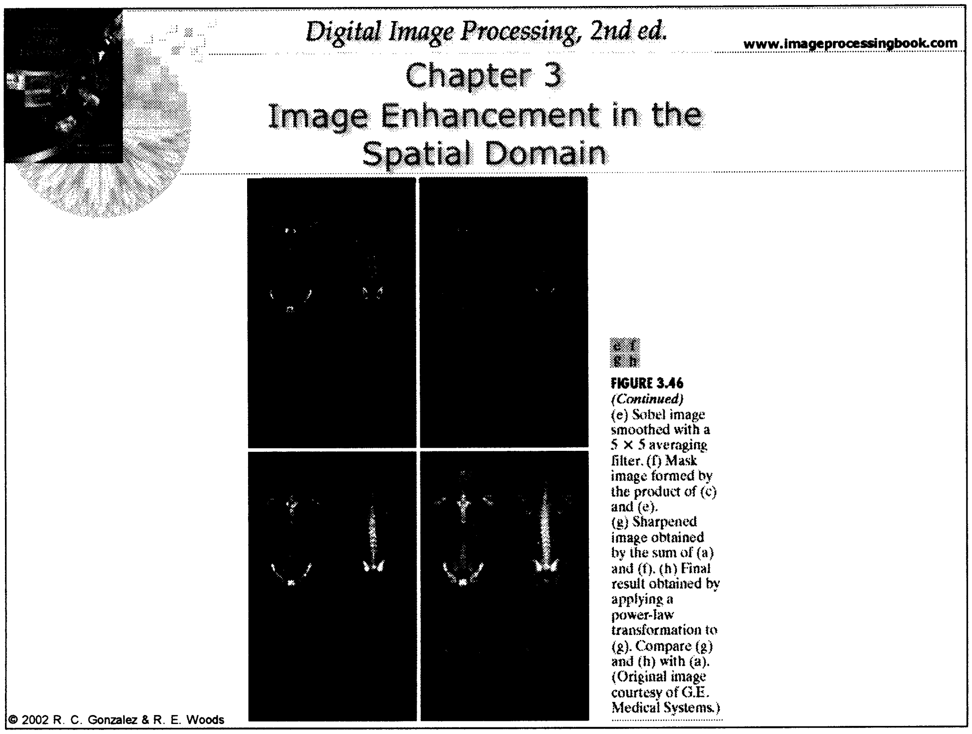

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FIGURE 3.46 *(Continued)* (e) Sobel image smoothed with a 5 × 5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

(e) Sobel smoothed by 5x5 averaging filter.

(d) & (e) are brighter than Laplacian images because there is a lot of edges in image.

(f) multiplying sobel smoothed image by Laplacian gives a masked sharpened image.

(g) Adding the sharpened image to the original image gives sharpened image.
Notice more detail in ribs, spinal cord, etc.

(h) use power law to increase dynamic range of displayed image.

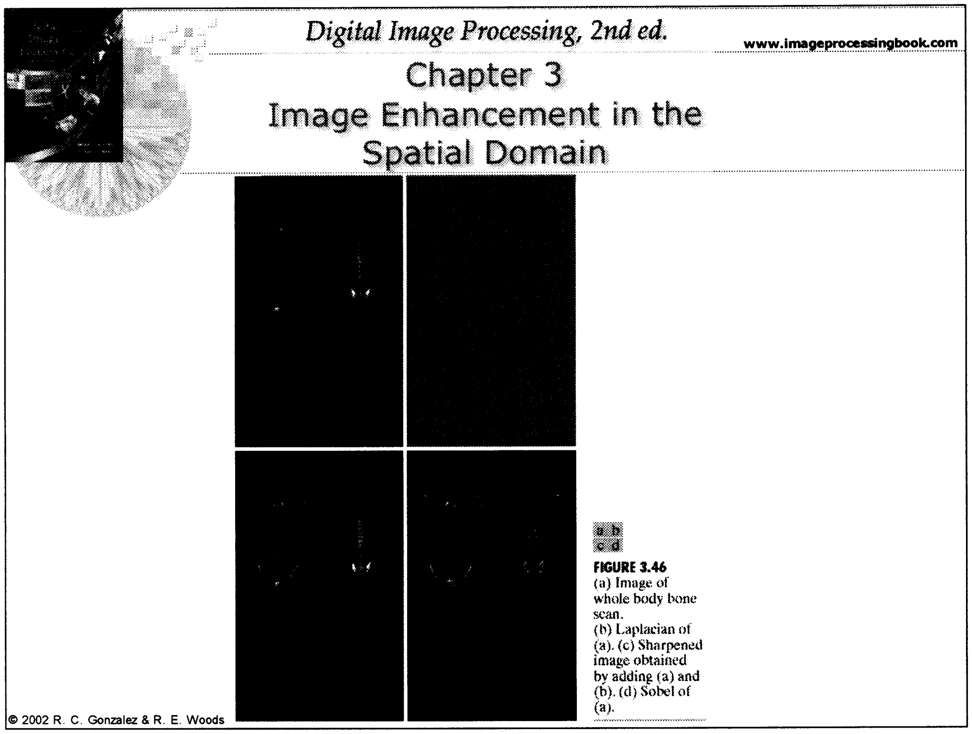

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FIGURE 3.46 (a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a).

(a) original image (used to detect bone disease)

(b) Laplacian of (a), Negative center.

(c) Sharpened image (a)+(b)
Shows noise but we can't use median filter for medical images.

⇒ use a smoother version of the gradient

(d) sobel of (a), Edges more dominant than in Laplacian.

Laplacian is superior in enhancing fine detail.