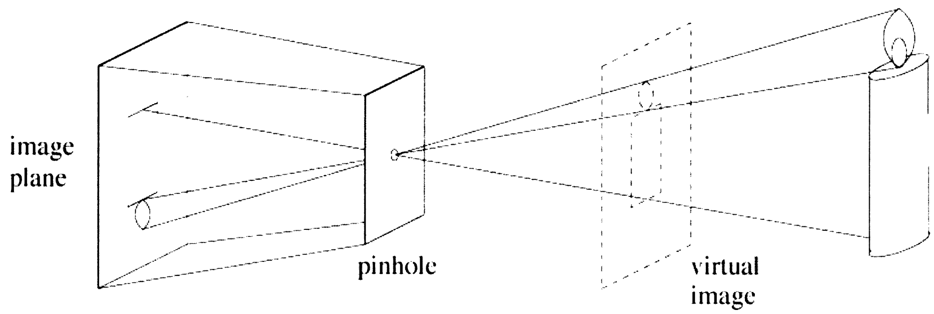
Cameras

- First photograph due to Niepce
- First on record shown in the book - 1822
- Basic abstraction is the pinhole camera
  - lenses required to ensure image is not too dark
  - various other abstractions can be applied

Pinhole cameras

- Abstract camera model - box with a small hole in it
- Pinhole cameras work in practice



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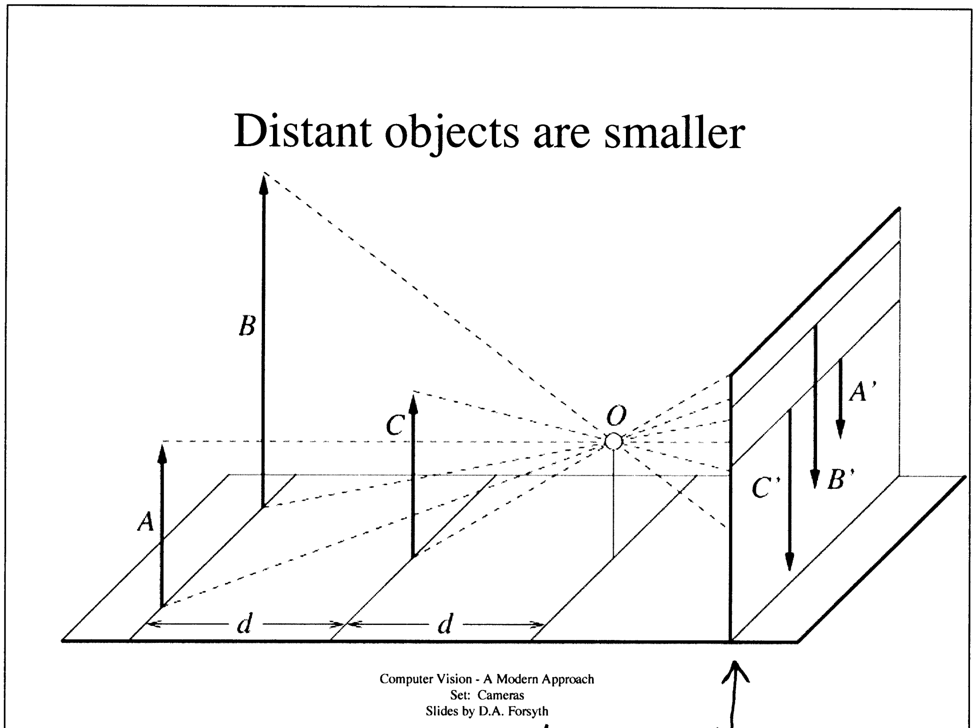
The point to make here is that each point on the image plane sees light from only

one direction, the one that passes through the pinhole.

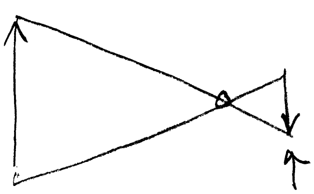


≈ f

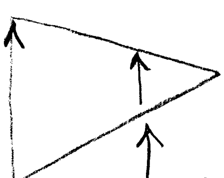
common to put virtual image here

image plane

NOTE: Due to camera scaling B' and C' are shown as the same height although they are different.



real image



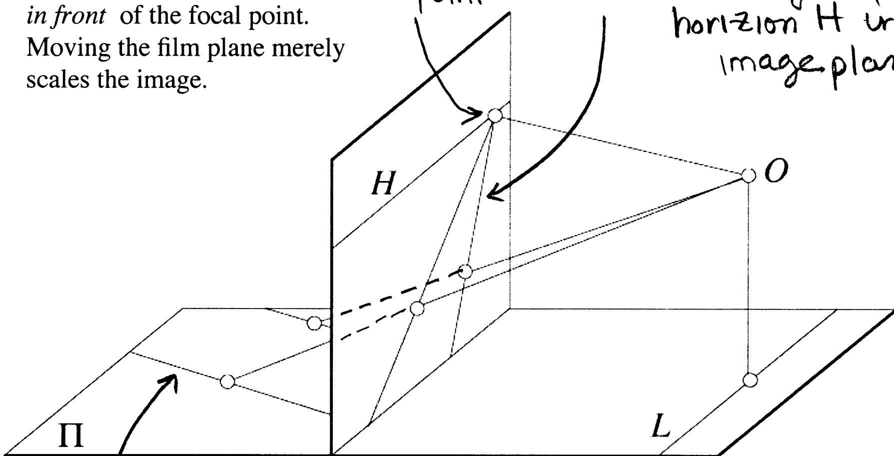
virtual image

Parallel lines meet

Common to draw film plane *in front* of the focal point. Moving the film plane merely scales the image.

Vanishing point

lines converge to a point at horizion H in virtual image plane



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lines parallel in real world

virtual image plane

focal plane

Vanishing points

- each set of parallel lines (=direction) meets at a different point
  - The *vanishing point* for this direction
- Sets of parallel lines on the same plane lead to *collinear* vanishing points.
  - The line is called the *horizon* for that plane

- Good ways to spot faked images
  - scale and perspective don't work
  - vanishing points behave badly
  - supermarket tabloids are a great source.

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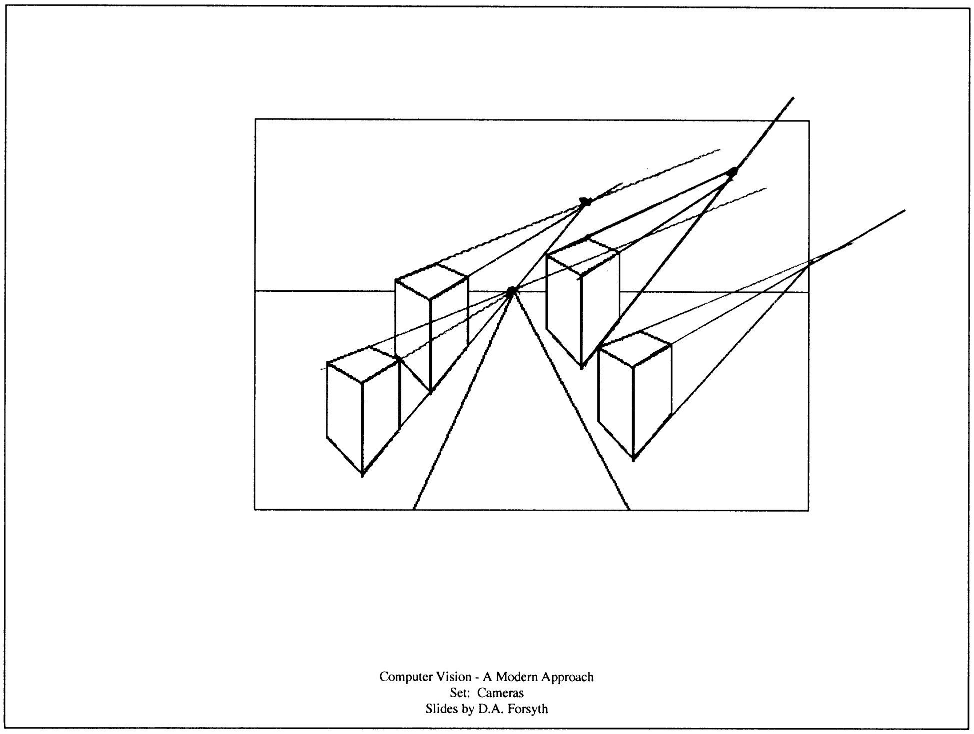
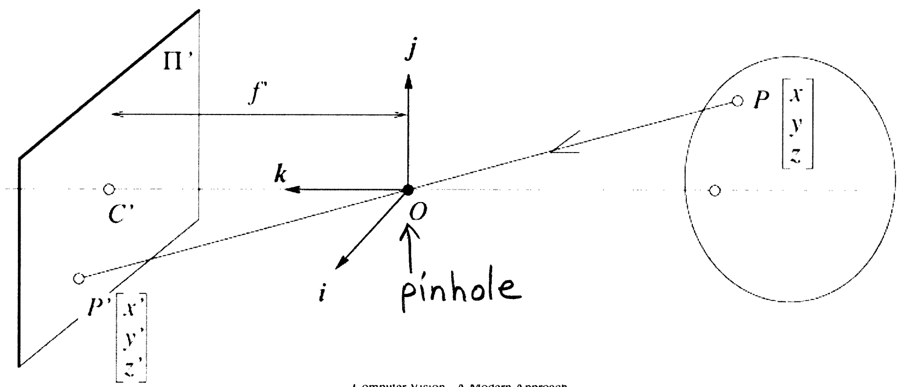
all vanishing points are on same horizion



Is this a perspective image of four identical buildings? why not?

perspective is bad. All vanishing points should be on horizon
same.

The equation of projection



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since $\overrightarrow{OP}$ and $\overrightarrow{OP'}$ are collinear.

$$x' = \lambda x$$
$$y' = \lambda y \quad \Rightarrow \quad \lambda = \frac{x'}{x} = \frac{y'}{y} = \frac{f'}{z} \leftarrow \text{in place of } z'$$
$$f' = \lambda z$$

therefore $x' = f' \dfrac{x}{z}$

$$y' = f' \frac{y}{z}$$

The equation of projection

- Cartesian coordinates:
  - We have, by similar triangles, that (x, y, z) -> (f x/z, f y/z, -f)
  - Ignore the third coordinate, and get

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Homogenous coordinates

- Add an extra coordinate and use an equivalence relation
- for 2D
  - equivalence relation k*(X,Y,Z) is the same as (X,Y,Z)
- for 3D
  - equivalence relation k*(X,Y,Z,T) is the same as (X,Y,Z,T)

- Basic notion
  - Possible to represent points "at infinity"
    - Where parallel lines intersect
    - Where parallel planes intersect
  - Possible to write the action of a perspective camera as a matrix

The camera matrix

- Turn previous expression into HC's
  - HC's for 3D point are (X,Y,Z,T)
  - HC's for point in image are (U,V,W)

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

2-D image

3-D world coordinates

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$U = X$

$V = Y$

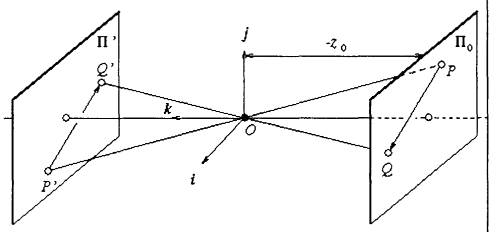
$W = \frac{Z}{f}$ ← scale factor

scaling $U = \frac{X}{\frac{Z}{f}} = f\frac{X}{Z}$

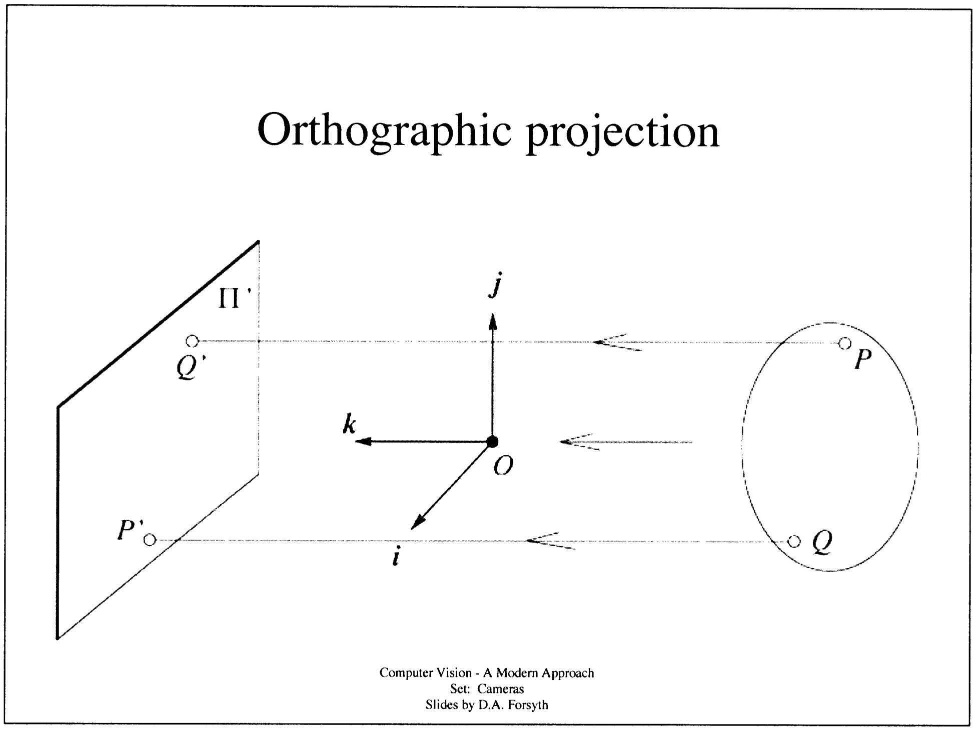
$V = \frac{Y}{\frac{Z}{f}} = f\frac{Y}{Z}$

Weak perspective

- Issue
  - perspective effects, but not over the scale of individual objects
  - collect points into a group at about the same depth, then divide each point by the depth of its group
  - Adv: easy
  - Disadv: wrong



If depth of field of objects in a group is similar, assume magnification for this group is constant.



Extreme case of weak perspective
From previous drawing we normalize image coordinates to have magnification $m = -1$ which looks wrong but is correct.

The projection matrix for orthographic projection

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

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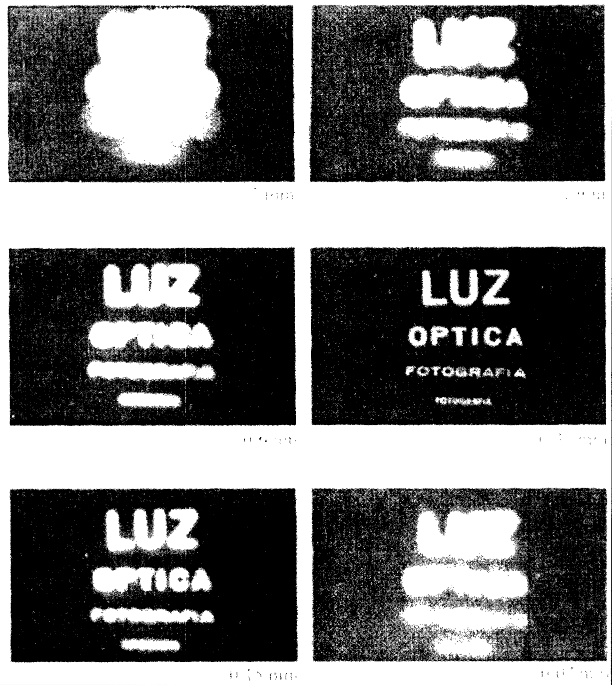
This is a simplification of previous projection matrix in the limit that $f \to \infty$

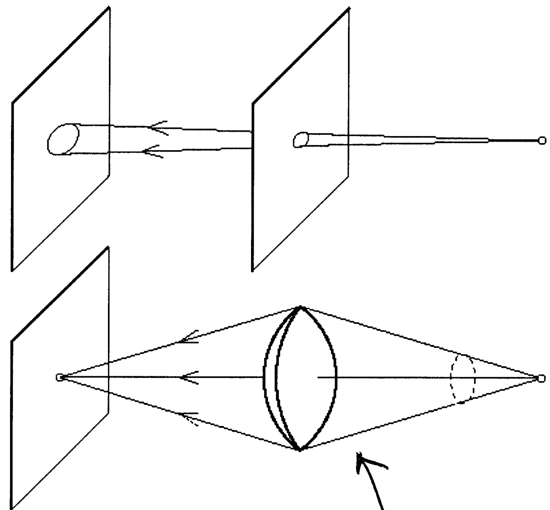
Pinhole too big -
many directions are averaged, blurring the image

Pinhole too small-
diffraction effects blur the image

Generally, pinhole cameras are *dark*, because a very small set of rays from a particular point hits the screen.



The reason for lenses



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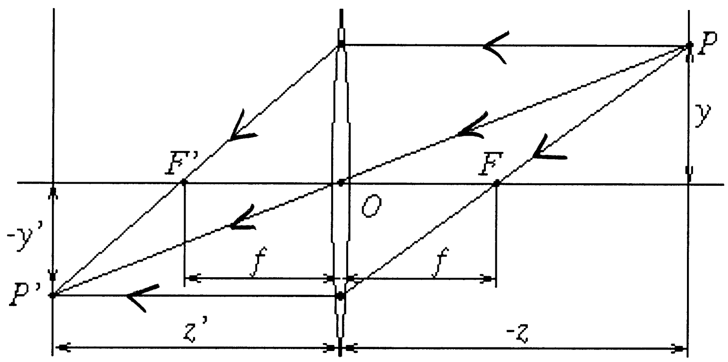
gather many more rays of light

The thin lens



$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$ ← lens law

image distance   object distance

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geometric optics for a thin lens

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

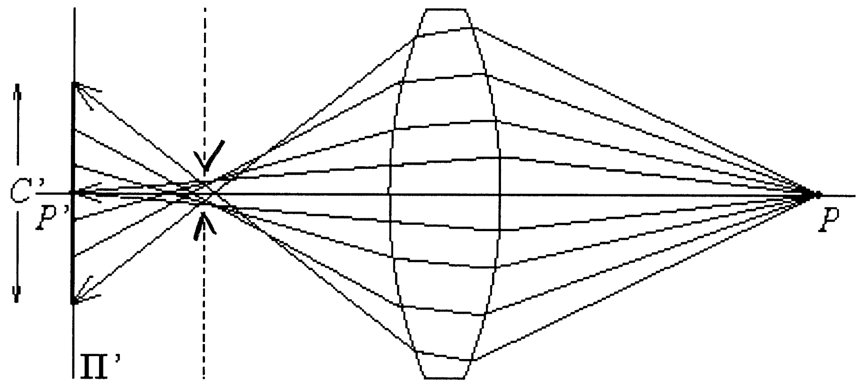
Assumes y and $-y'$ are close to z-axis

ray PP' is not refracted $\alpha_1 = \alpha_2$
but other rays are refracted.

f is the point at which entering rays parallel to the z-axis are focussed, i.e., cross the z-axis

Spherical aberration

circle of least confusion



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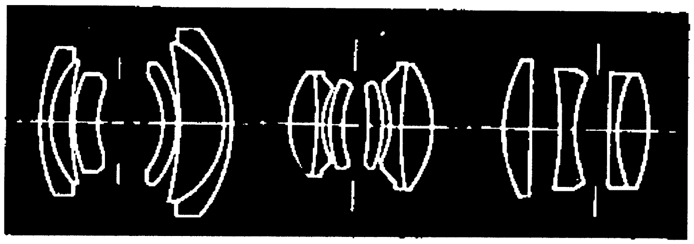
due to assuming $\sin\alpha_1 \approx \alpha_1$
$\sin\alpha_2 \approx \alpha_2$
for paraxial rays

focal point not at same z-coordinate

Lens systems



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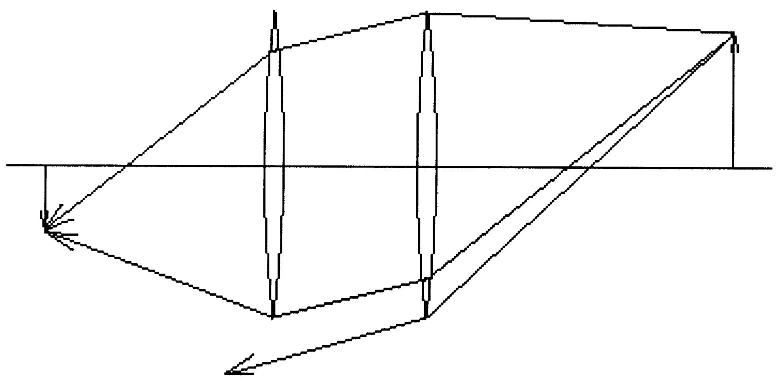
Complicated len systems to minimize aberrations, etc.

Common for modern lenses (except front and rear) to be plastic

- light weight
- aspheric

Vignetting



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light follows a path which prevents it from reaching image plane.
Can be accidental or deliberate

Other (possibly annoying) phenomena

- Chromatic aberration
  - Light at different wavelengths follows different paths; hence, some wavelengths are defocussed
  - Machines: coat the lens
  - Humans: live with it
- Scattering at the lens surface
  - Some light entering the lens system is reflected off each surface it encounters (Fresnel's law gives details)
  - Machines: coat the lens, interior
  - Humans: live with it (various scattering phenomena are visible in the human eye)
- Geometric phenomena (Barrel distortion, etc.)

Camera parameters

- Issue
  - camera may not be at the origin, looking down the z-axis
    - extrinsic parameters
  - one unit in camera coordinates may not be the same as one unit in world coordinates
    - intrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{intrinsic parameters} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{extrinsic parameters} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

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location of camera origin relative to world coordinates

focal length
principal points (thick lens)
etc.

Camera calibration

- Issues:
  - what are intrinsic parameters of the camera?
  - what is the camera matrix? (intrinsic+extrinsic)
- General strategy:
  - view calibration object
  - identify image points
  - obtain camera matrix by minimizing error
  - obtain intrinsic parameters from camera matrix
- Error minimization:
  - Linear least squares
    - easy problem numerically
    - solution can be rather bad
  - Minimize image distance
    - more difficult numerical problem
    - solution usually rather good,
    - start with linear least squares
  - Numerical scaling is an issue

* due to Tsai

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* assume radial alignment constraint

i.e. spherical distortion changes radial <u>scale</u> but not direction

Allows LSE to find all parameters <u>but</u>

- t_z z translation of camera
- magnification
- distortion parameters

$$\lambda = 1 + \underline{\kappa_1 d^2} + K_2 d^4 + K_3 d^6$$

usually this is all we use

d = distance between image point and image center.