utility of edge information

- boundary representation is easy to integrate into object recognition algorithms;

edge operator detects local edges

- most edge operators compute a direction aligned with direction of maximum gray-level change and a magnitude indicating extent of change.

Typical types of edge operators

(1) approximations of gradient operator

(2) template matching at different orientations

(3) curve fitting intensity information to edge models.

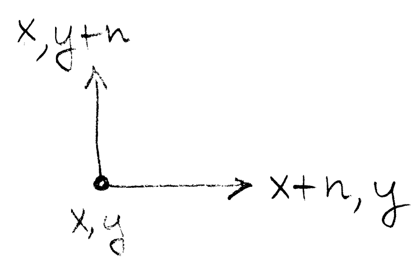
Earliest edge operator — Robert's operator

$$s(\underline{x}) = \sqrt{\Delta_1^2 + \Delta_2^2}$$

$$\phi(\underline{x}) = \tan^{-1}\left(\frac{\Delta_2}{\Delta_1}\right)$$

where $\Delta_1 = f(x+n, y) - f(x, y)$

$\Delta_2 = f(x, y+n) - f(x, y)$



Was very sensitive to noise

Solution — use local averaging to reduce noise effects.

Prewitt operator

-1	0	1
-1	0	1
-1	0	1

1	1	1
0	0	0
-1	-1	-1

Sobel operator

-1	0	1
-2	0	2
-1	0	1

1	2	1
0	0	0
-1	-2	-1

very optimal system

Totally different concept – edge templates.

Kirsch templates (really just correlation)

$f(\underline{x}_k)$

$n = \frac{1}{2}$

$$\begin{array}{|c|c|}\hline -1 & 1 \\ \hline \end{array} \quad \begin{array}{|c|}\hline 1 \\ \hline -1 \\ \hline \end{array} \quad \begin{array}{cc} & \boxed{1} \\ \boxed{-1} & \end{array} \quad \begin{array}{cc} \boxed{1} & \\ & \boxed{-1} \end{array}$$

$n = 1$

$$\begin{array}{|c|c|c|}\hline -1 & & 1 \\ \hline -1 & & 1 \\ \hline -1 & & 1 \\ \hline \end{array} \quad \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline & & \\ \hline -1 & -1 & -1 \\ \hline \end{array} \quad \begin{array}{|c|c|c|}\hline & 1 & 1 \\ \hline -1 & & 1 \\ \hline -1 & -1 & \\ \hline \end{array} \quad \begin{array}{|c|c|c|}\hline 1 & 1 & -1 \\ \hline 1 & & \\ \hline & -1 & -1 \\ \hline \end{array}$$

$n = 2$

$$\begin{array}{|c|c|c|c|c|}\hline -1 & -1 & & 1 & 1 \\ \hline -1 & -1 & & 1 & 1 \\ \hline -1 & -1 & & 1 & 1 \\ \hline -1 & -1 & & 1 & 1 \\ \hline -1 & -1 & & 1 & 1 \\ \hline \end{array} \quad \begin{array}{|c|c|c|c|c|}\hline 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 \\ \hline & & & & \\ \hline -1 & -1 & -1 & -1 & -1 \\ \hline -1 & -1 & -1 & -1 & -1 \\ \hline \end{array} \quad \begin{array}{|c|c|c|c|c|}\hline & 1 & 1 & 1 & 1 \\ \hline -1 & & 1 & 1 & 1 \\ \hline -1 & -1 & & 1 & 1 \\ \hline -1 & -1 & -1 & & 1 \\ \hline -1 & -1 & -1 & -1 & \\ \hline \end{array} \quad \begin{array}{|c|c|c|c|c|}\hline 1 & 1 & 1 & 1 & \\ \hline 1 & 1 & 1 & & -1 \\ \hline 1 & 1 & & -1 & -1 \\ \hline 1 & & -1 & -1 & -1 \\ \hline & -1 & -1 & -1 & -1 \\ \hline \end{array}$$

Kirsch operator

$$S(\underline{x}) = \max\left[1,\ \max \sum_{k=0}^{n-1} \left| f(\underline{x}_k) - f(\underline{x}) \right| \right]$$

This part locates maximum change – the k gives direction

compare it to a threshold.

This is effectively implemented by the above templates.

0	1	2
7	$\underline{x}$	3
6	5	4

← k directions

HUNCH: human vision uses low-level template matching for edge detection

where is an edge? gradient is rarely equal to zero

threshold gradient: edge at $\underline{x}$ iff $\nabla f(\underline{x}) > T$

does this work? Not if edge is weak.

edge activity

Frei-Chen - edge operators are actually orthogonal basis functions of edgeness. Represent local activity as a "Fourier-like" series.

one set of basis functions for 3×3 space

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & -\sqrt{2} & -1 \\ 0 & 0 & 0 \\ 1 & \sqrt{2} & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & -1 & \sqrt{2} \\ 1 & 0 & -1 \\ -\sqrt{2} & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$k=0$ $\quad$ $k=1$ $\quad$ $k=3$ $\quad$ $k=5$

$$\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 1 \\ -\sqrt{2} & 0 & \sqrt{2} \\ -1 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} \sqrt{2} & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & \sqrt{2} \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad \begin{bmatrix} -2 & 1 & -2 \\ 1 & 4 & 1 \\ -2 & 1 & -2 \end{bmatrix}$$

$k=7$ $\quad$ $k=2$ $\quad$ $k=4$ $\quad$ $k=6$ $\quad$ $k=8$

write image in neighborhood of $\underline{x}$ as

$$f(\underline{x}_o) = \sum \underbrace{\frac{\langle f, h_k \rangle}{\langle h_k, h_k \rangle}}_{\text{dot products or correlations}} h_k(\underline{x} - \underline{x}_o)$$

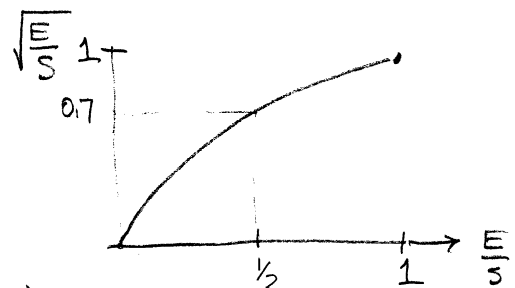
how much edgeness is there

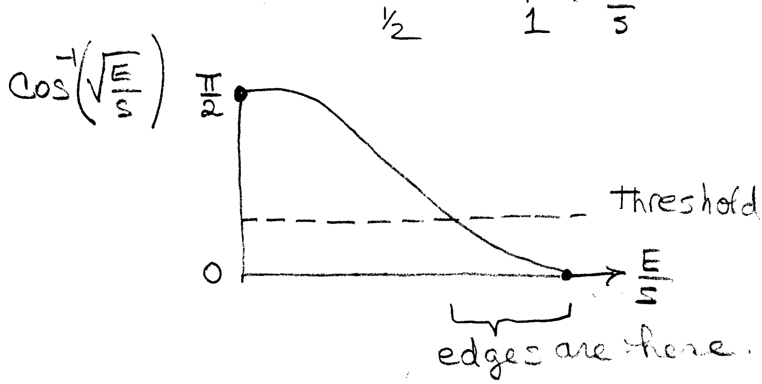
total energy $\quad S = \sum_{k=0}^{8} \langle f, h_k \rangle^2 \quad$ total energy in neighborhood

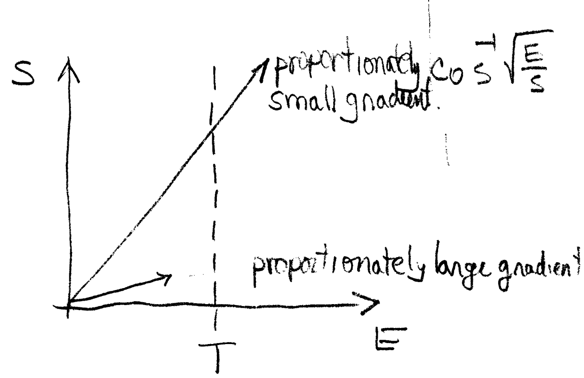
edge energy $\quad E = \sum_{k=1}^{2} \langle f, h_k \rangle^2 \quad$ vertical and horizontal edge energy.

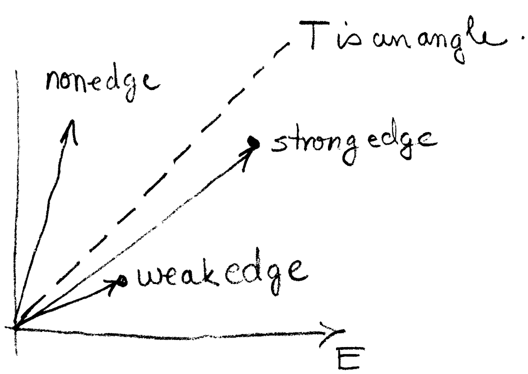
$$\theta = \cos^{-1}\left(\sqrt{\frac{E}{S}}\right)$$

if $\theta < T \Rightarrow$ an edge, otherwise not





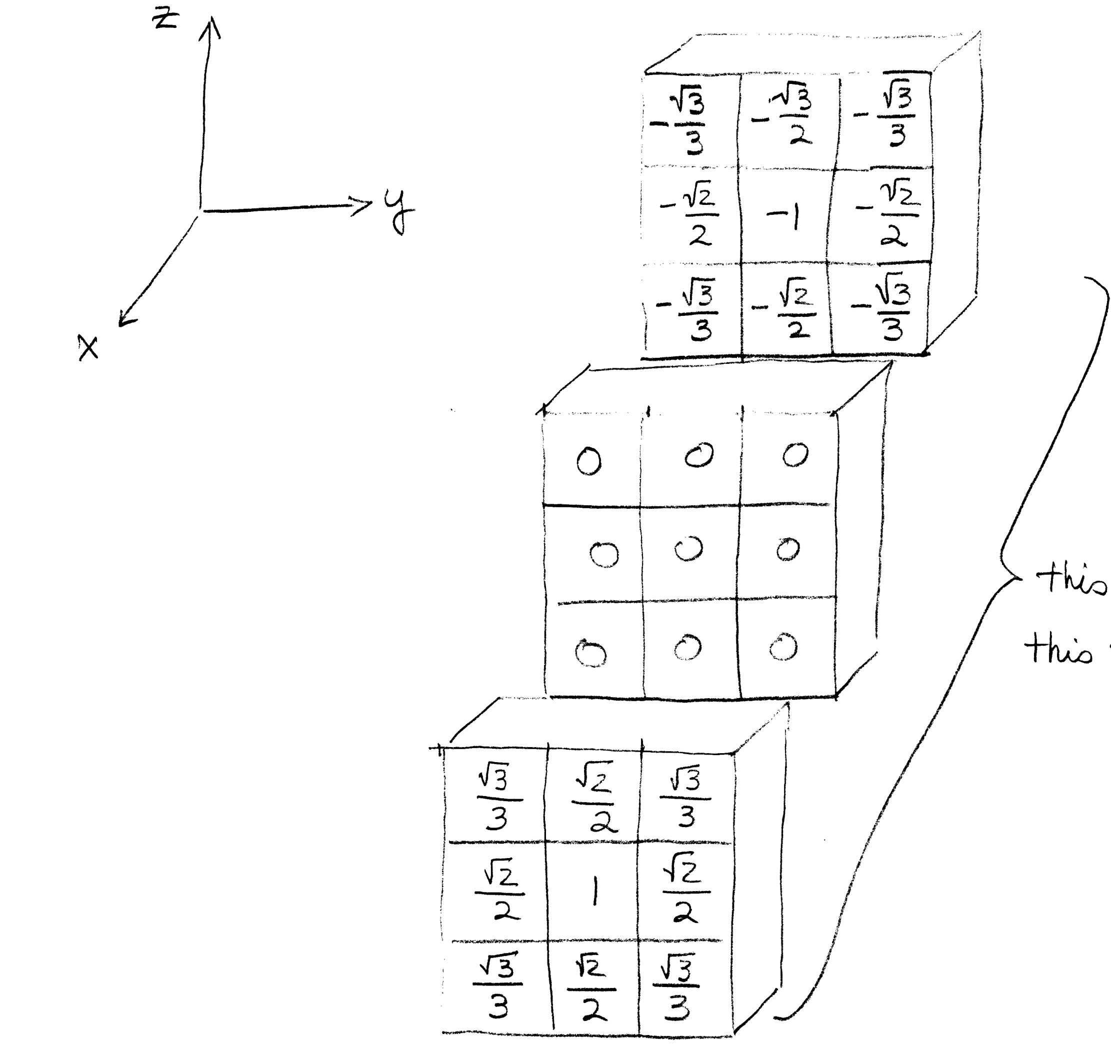




weak edges are lost $\quad$ only strong edges survive.

a large "fraction" of edge energy identifies edges.

three-dimensional edge operators (Zucker-Hummel)



this is $g_1(x,y,z)$
this is x-directed.

$g_1(x,y,z) =$ x-directed basis function
$g_2(x,y,z) =$ y-directed basis function
$g_3(x,y,z) =$ z-directed basis function

surface normal $\underline{n} = (a, b, c)$

$$a = \langle g_1, f(\underline{x}) \rangle$$

$$b = \langle g_2, f(\underline{x}) \rangle$$

$$c = \langle g_3, f(\underline{x}) \rangle$$

surface element detected if $|\underline{n}| > T$, some threshold how good is an edge operator?

Pratt's figure of merit

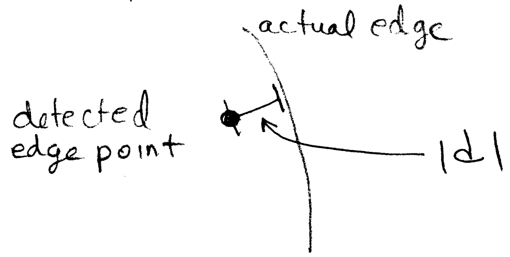
$$F = \frac{1}{\max(N_A, N_I)} \sum_{i=1}^{N_A} \frac{1}{1 + a d_i^2}$$

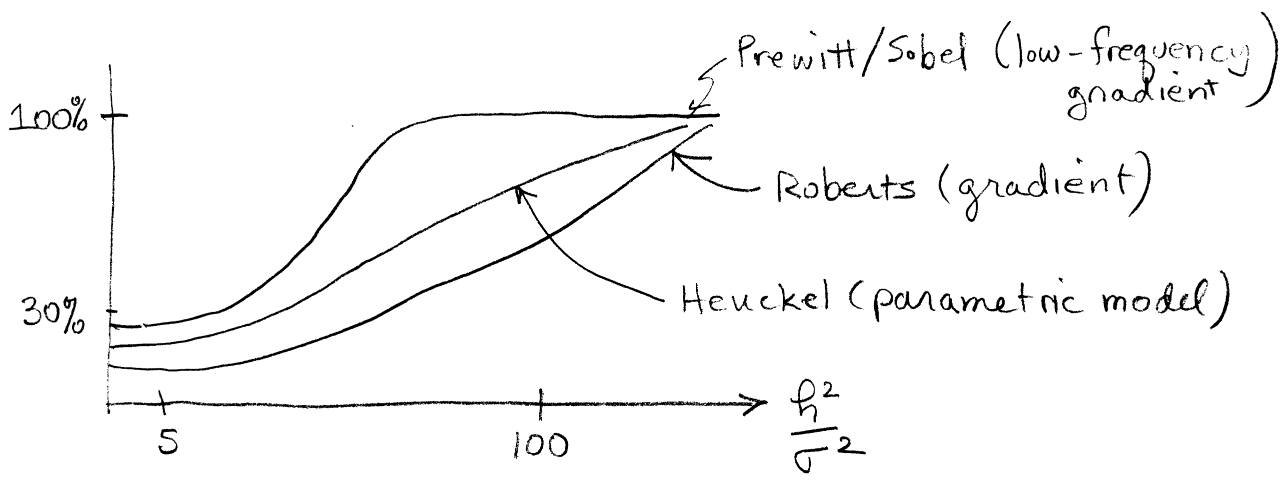
N_A = detected edge points

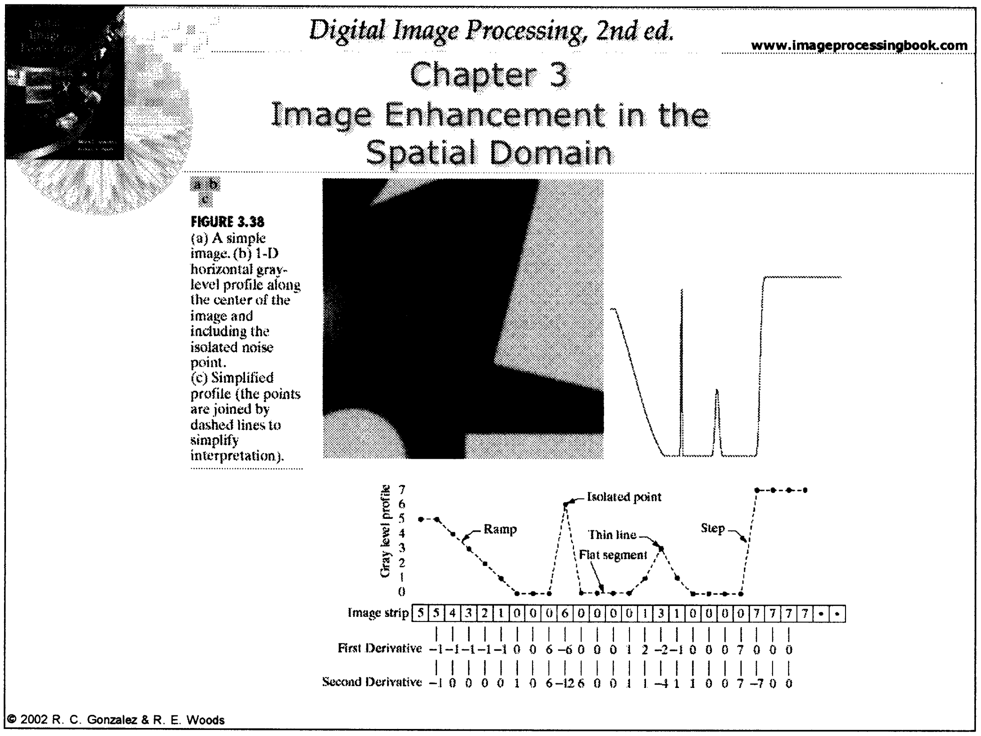
N_I = actual edge points

a = scaling constant, i.e. penalty

d = signed separation distance of an actual edge point normal to a line of ideal edge points





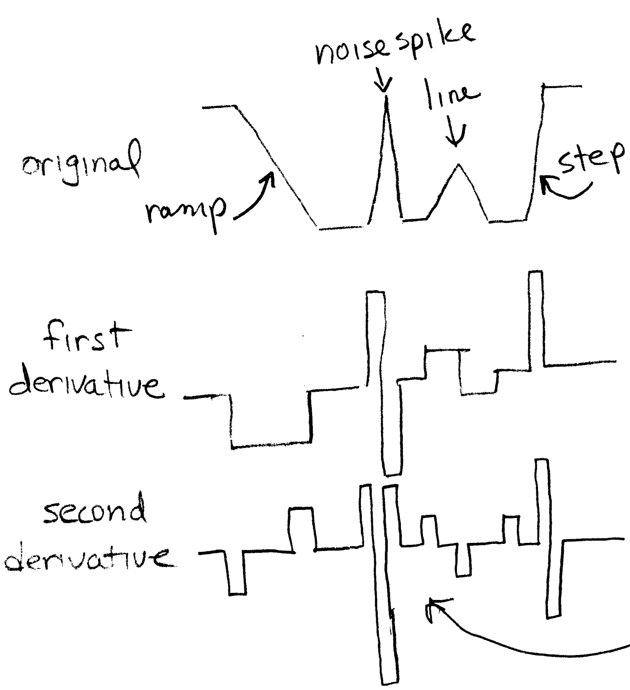


first derivative $\quad \dfrac{\partial f}{\partial x} = f(x+1) - f(x)$

second derivative $\quad \dfrac{\partial^2 f}{\partial x^2} = \dfrac{\partial f}{\partial x}(x) - \dfrac{\partial f}{\partial x}(x-1)$

$$= \left[f(x+1) - f(x)\right] - \left[f(x) - f(x-1)\right]$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$



note classic response of 2nd derivative to a spike.