Spatial Domain Filtering

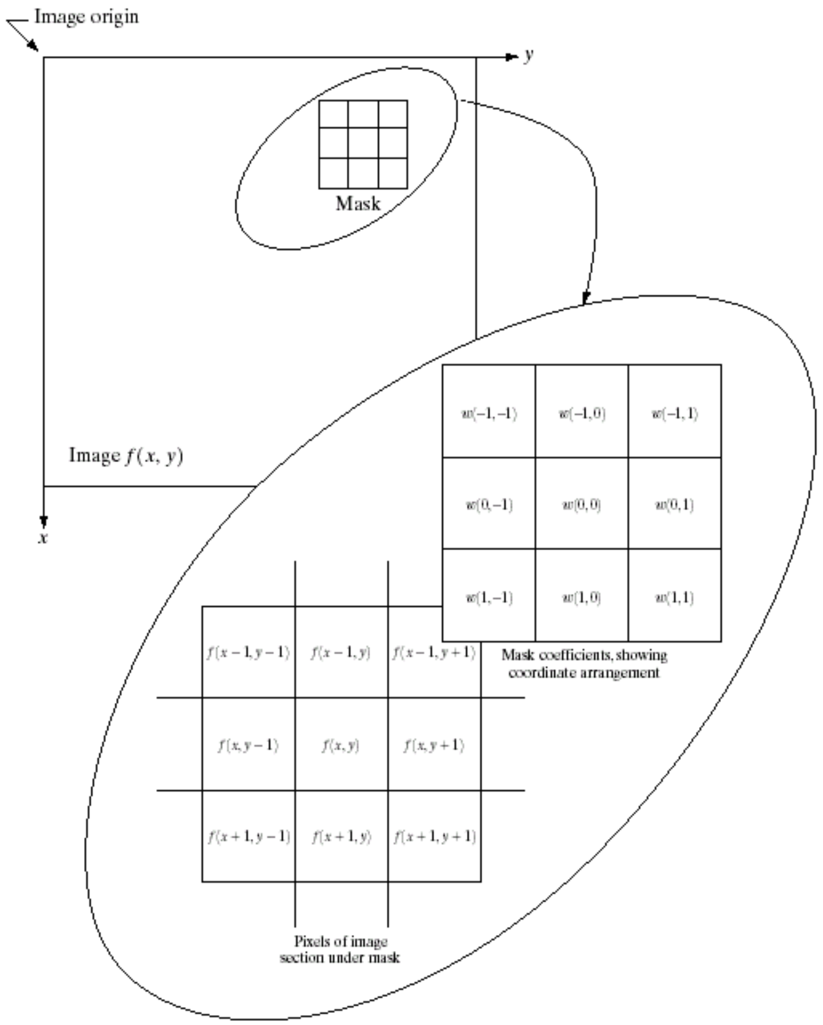


FIGURE 3.32 The mechanics of spatial filtering. The magnified drawing shows a 3×3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

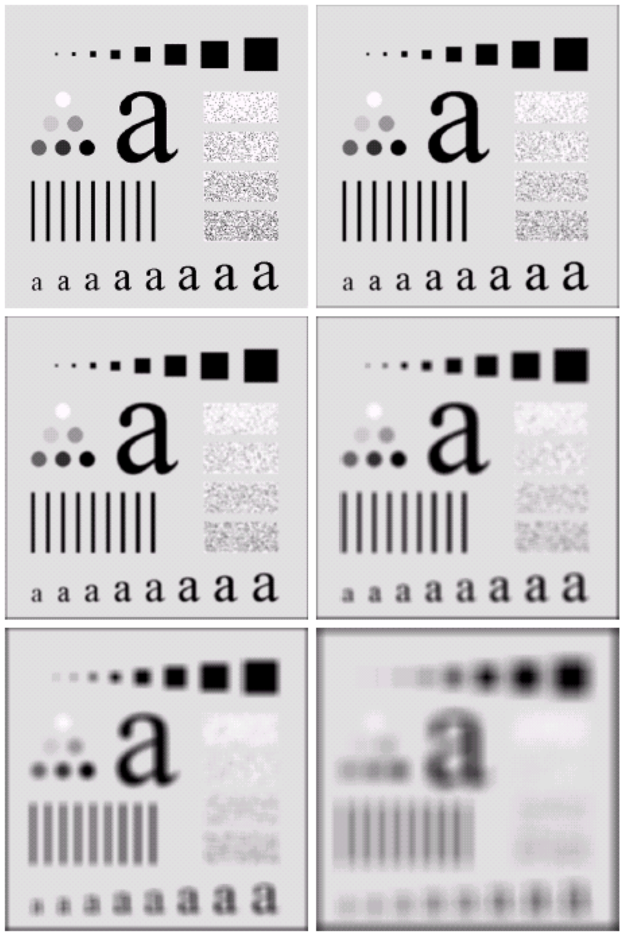
Spatial Domain Filtering

$$\frac{1}{9} \times \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

a b

FIGURE 3.34 Two 3 × 3 smoothing (averaging) filter masks. The constant multipli er in front of each mask is equal to the sum of the values of its coefficients, as is required to compute an average.

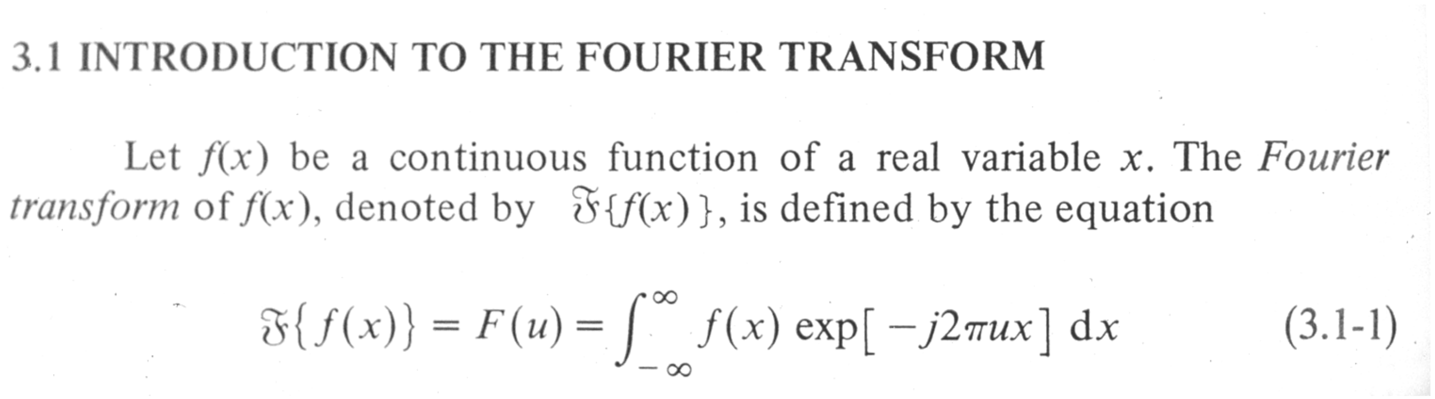
Spatial Domain Filtering



a b
c d
e f

FIGURE 3.35 (a) Original image, of size 500×500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50×120 pixels.

Fourier Transform



3.1 INTRODUCTION TO THE FOURIER TRANSFORM

Let $f(x)$ be a continuous function of a real variable x. The *Fourier transform* of $f(x)$, denoted by $\mathfrak{F}\{f(x)\}$, is defined by the equation

$$\mathfrak{F}\{f(x)\} = F(u) = \int_{-\infty}^{\infty} f(x) \exp[-j2\pi ux]\, dx \qquad (3.1\text{-}1)$$

Example: Consider the simple function shown in Fig. 3.1(a). Its Fo
transform is obtained from Eq. (3.1-1) as follows:

$$F(u) = \int_{-\infty}^{\infty} f(x) \exp[-j2\pi ux]\, dx$$

$$= \int_{0}^{X} A \exp[-j2\pi ux]\, dx$$

$$= \frac{-A}{j2\pi u}\left[e^{-j2\pi ux}\right]_0^X = \frac{-A}{j2\pi u}\left[e^{-j2\pi uX} - 1\right]$$

$$= \frac{A}{j2\pi u}\left[e^{j\pi uX} - e^{-j\pi uX}\right]e^{-j\pi uX}$$

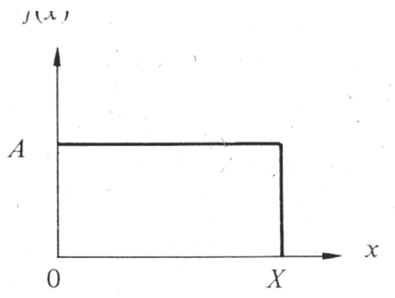
$$= \frac{A}{\pi u} \sin(\pi uX)\, e^{-j\pi uX}$$

which is a complex function. The Fourier spectrum is given by

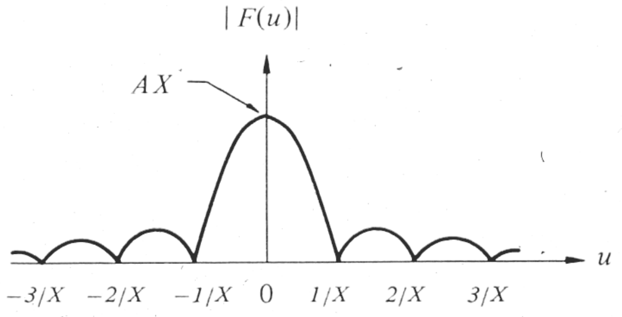
$$|F(u)| = \frac{A}{\pi u} |\sin(\pi uX)|\, |e^{-j\pi uX}|$$

$$= AX \left| \frac{\sin(\pi uX)}{(\pi uX)} \right|$$

A plot of $|F(u)|$ is shown in Fig. 3.1(b).



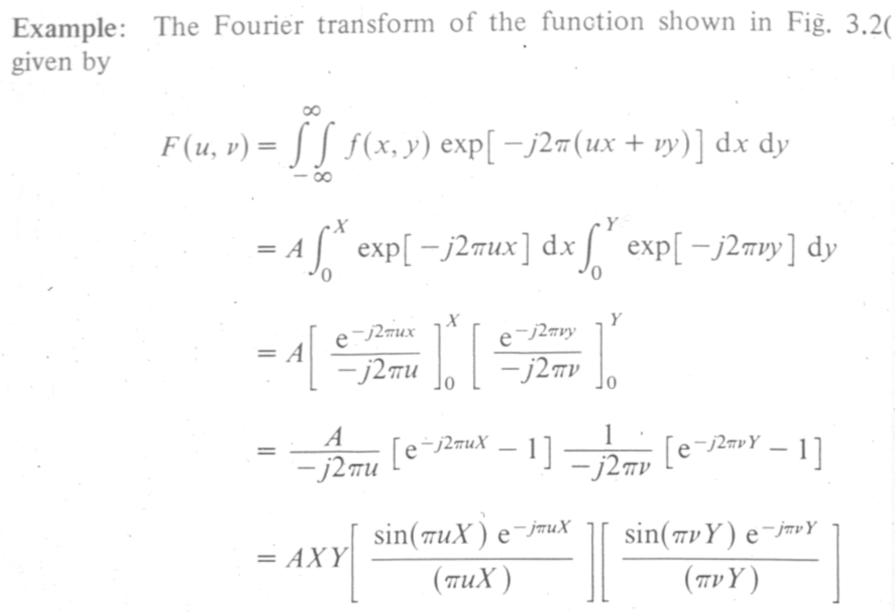
(a)



(b)

Figure 3.1. A simple function and its Fourier spectrum.

2-D Fourier Transform



Example: The Fourier transform of the function shown in Fig. 3.2(
given by

$$F(u, v) = \iint_{-\infty}^{\infty} f(x, y) \exp[-j2\pi(ux + vy)]\, dx\, dy$$

$$= A \int_0^X \exp[-j2\pi ux]\, dx \int_0^Y \exp[-j2\pi vy]\, dy$$

$$= A \left[\frac{e^{-j2\pi ux}}{-j2\pi u} \right]_0^X \left[\frac{e^{-j2\pi vy}}{-j2\pi v} \right]_0^Y$$

$$= \frac{A}{-j2\pi u} [e^{-j2\pi uX} - 1] \frac{1}{-j2\pi v} [e^{-j2\pi vY} - 1]$$

$$= AXY \left[\frac{\sin(\pi uX)\, e^{-j\pi uX}}{(\pi uX)} \right] \left[\frac{\sin(\pi vY)\, e^{-j\pi vY}}{(\pi vY)} \right]$$

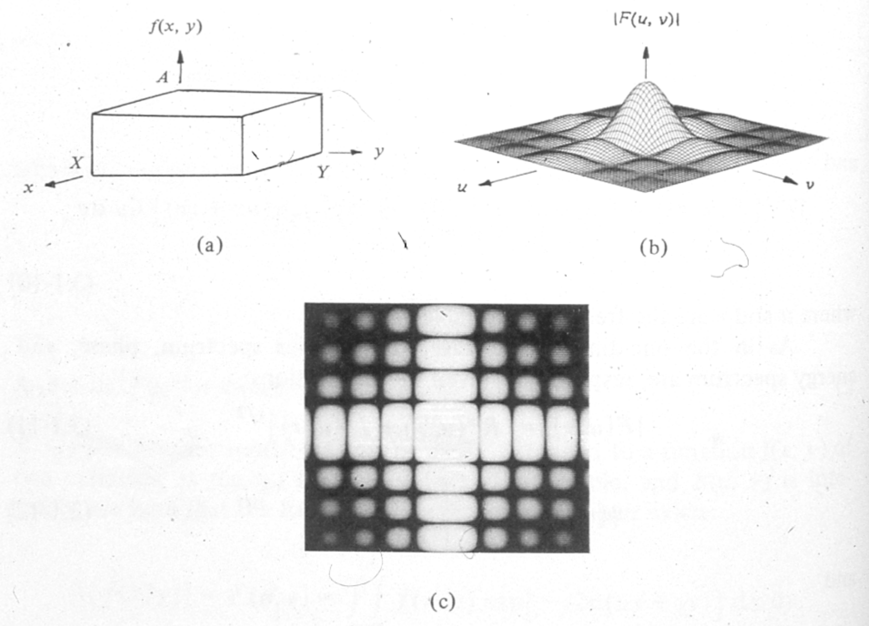
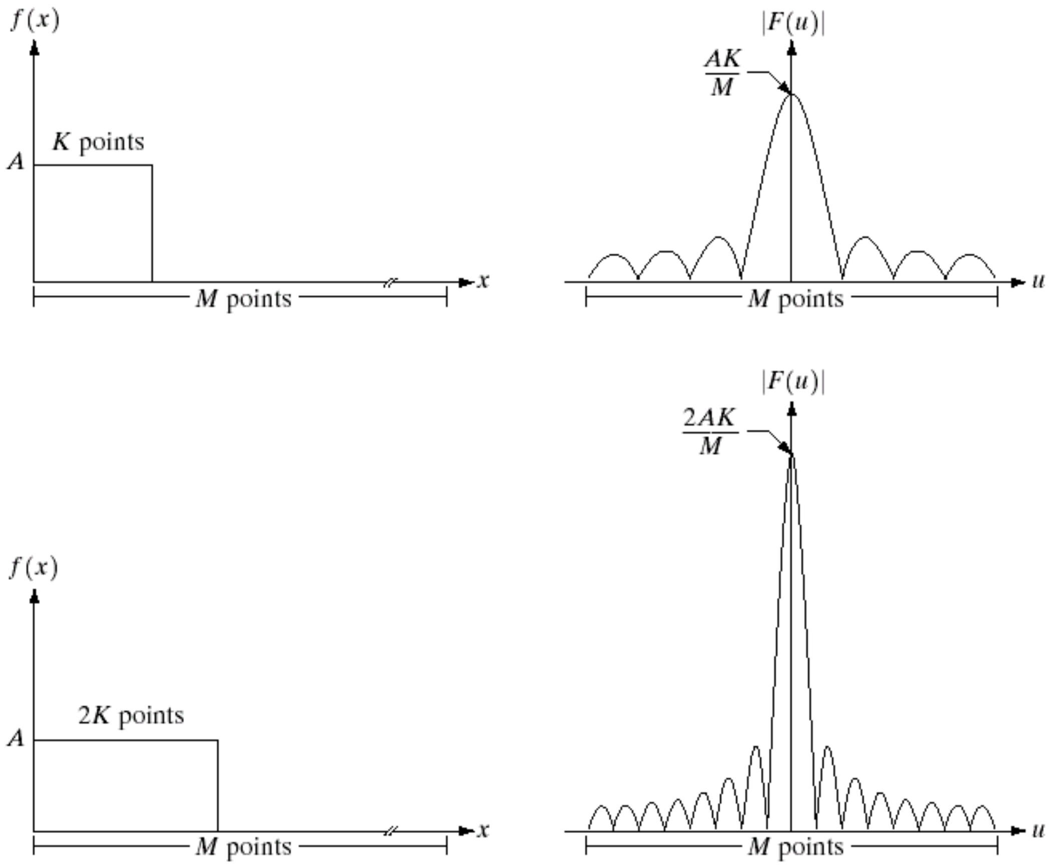


Figure 3.2. (a) A two-dimensional function, (b) its Fourier spectrum, and (c) the spectrum displayed as an intensity function.

Spatial Frequency



a b
c d

FIGURE 4.2 (a) A discrete function of *M* points, and (b) its Fourier spectrum. (c) A discrete function with twice the number of nonzero points, and (d) its Fourier spectrum.

$$\Delta u = \frac{1}{M\,\Delta x}$$

Fourier Transform Shift

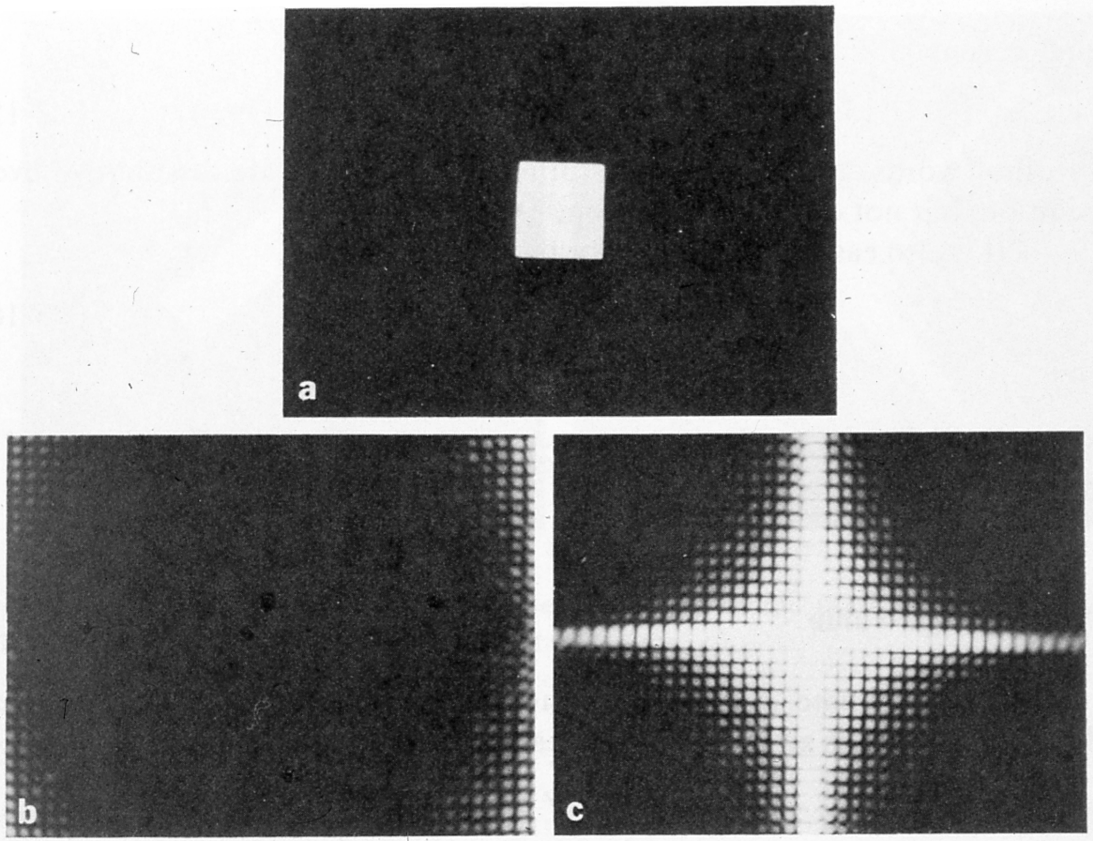
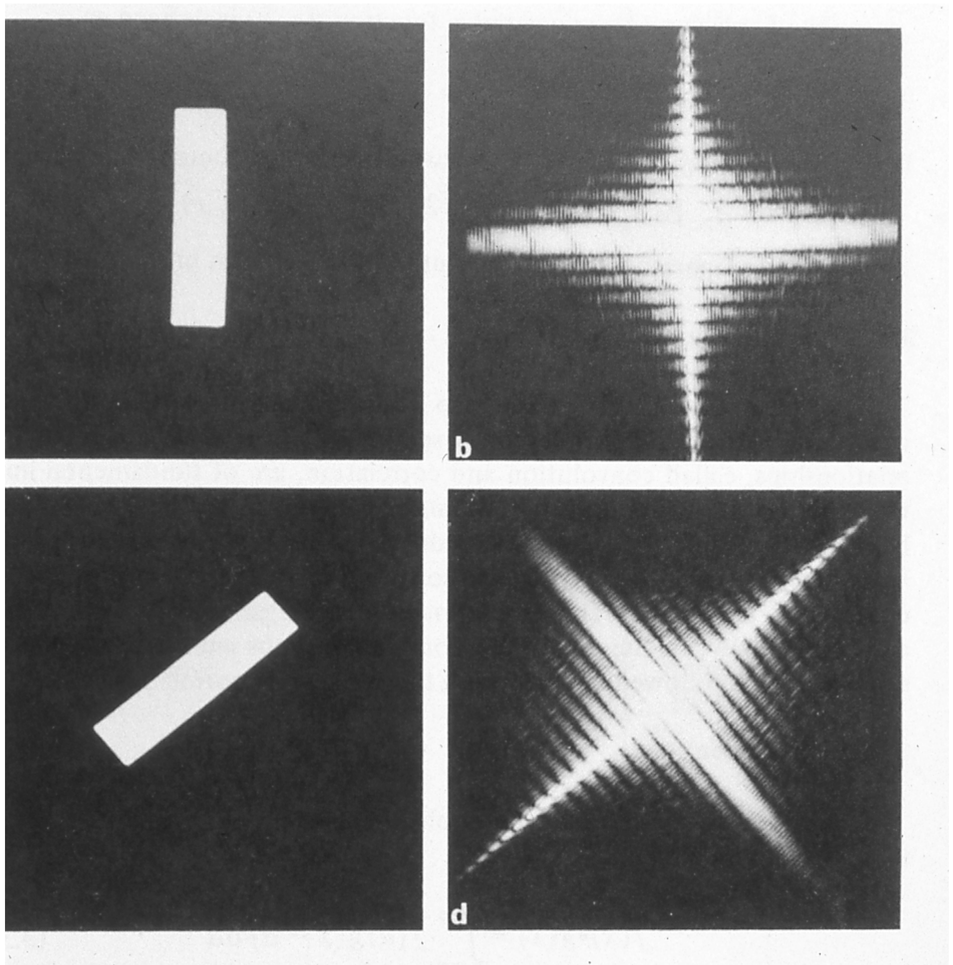


Figure 3.ll. (a) A simple image. (b) Fourier spectrum without shifting. (c) Fourier spectrum shifted to the center of the frequency square.

Fourier Transform Rotation



re 3.12. Rotational properties of the Fourier transform. (a) A simple image. (b) :trum. (c) Rotated image. (d) Resulting spectrum.

Sample 2-D Fourier Transforms

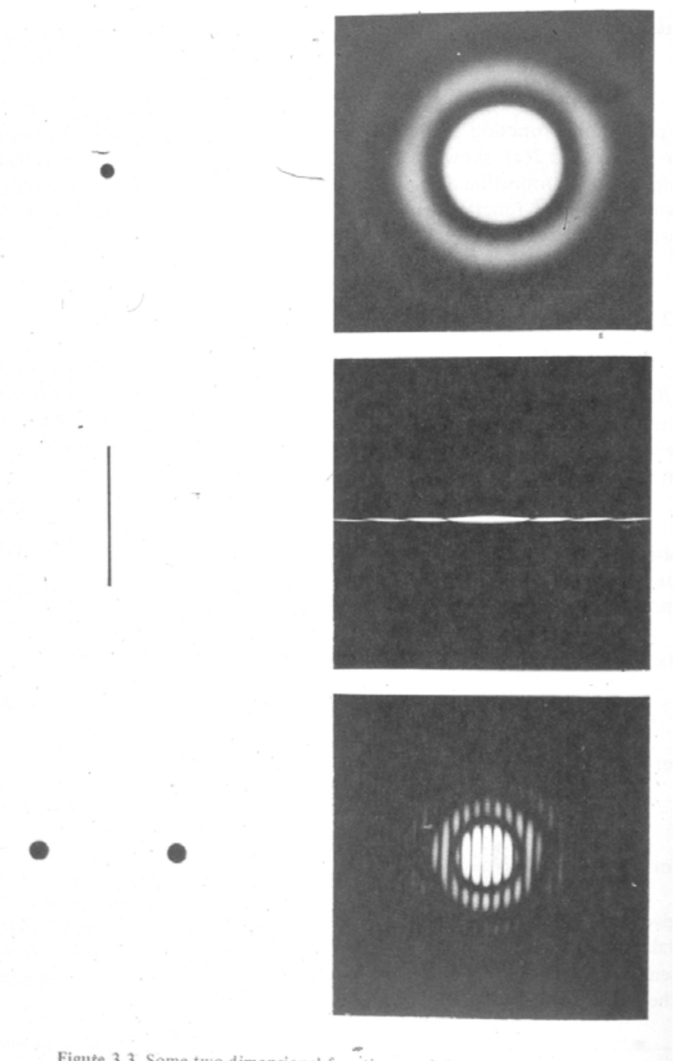


Figure 3.3. Some two-dimensional functions and their Fourier spectra.

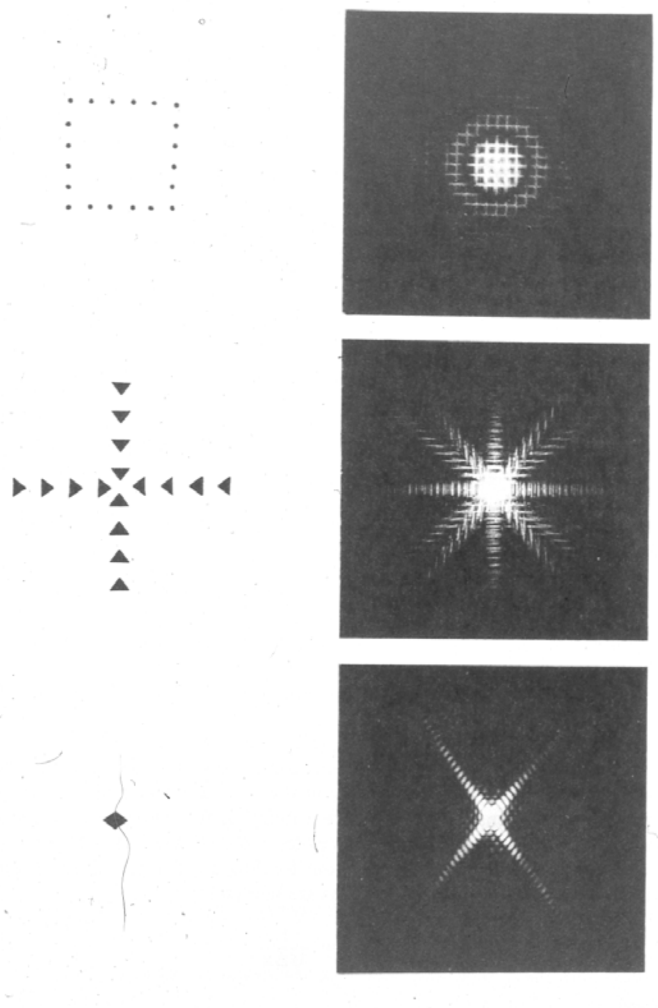


Figure 3.3. (Continued.)

Power Spectra

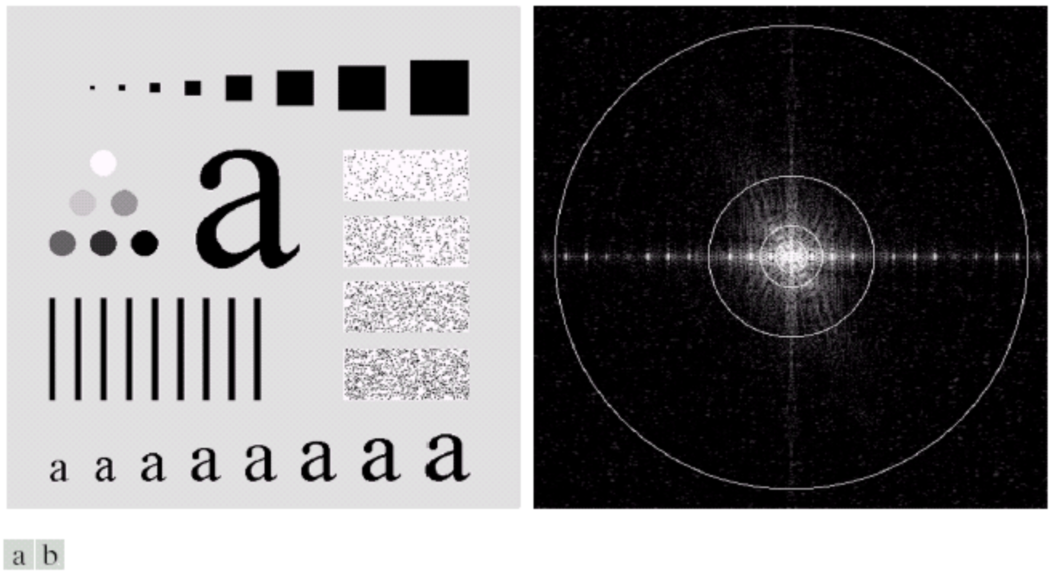


FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

Image Enhancement in the Frequency Domain

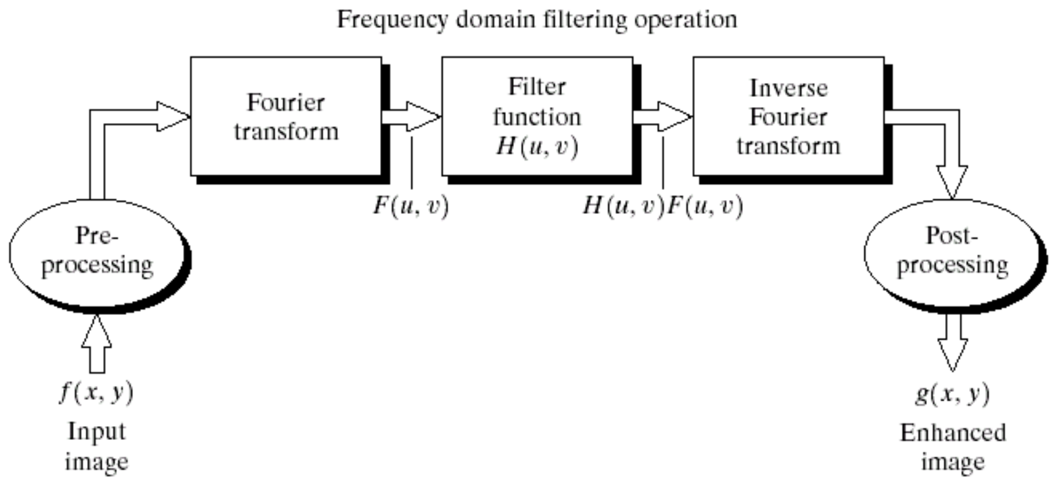
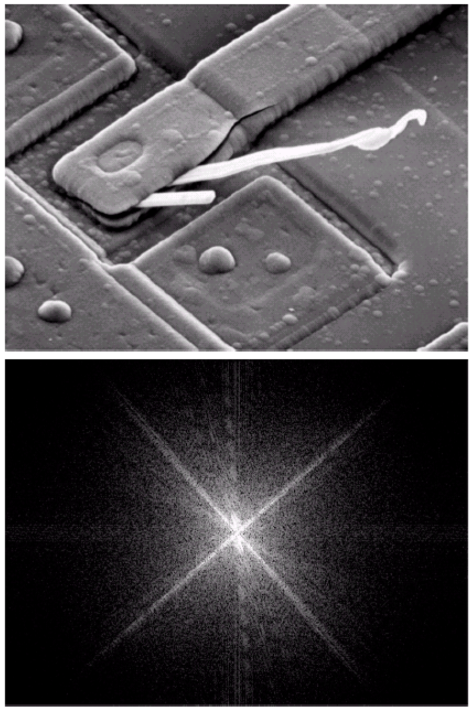


FIGURE 4.5 Basic steps for filtering in the frequency domain.

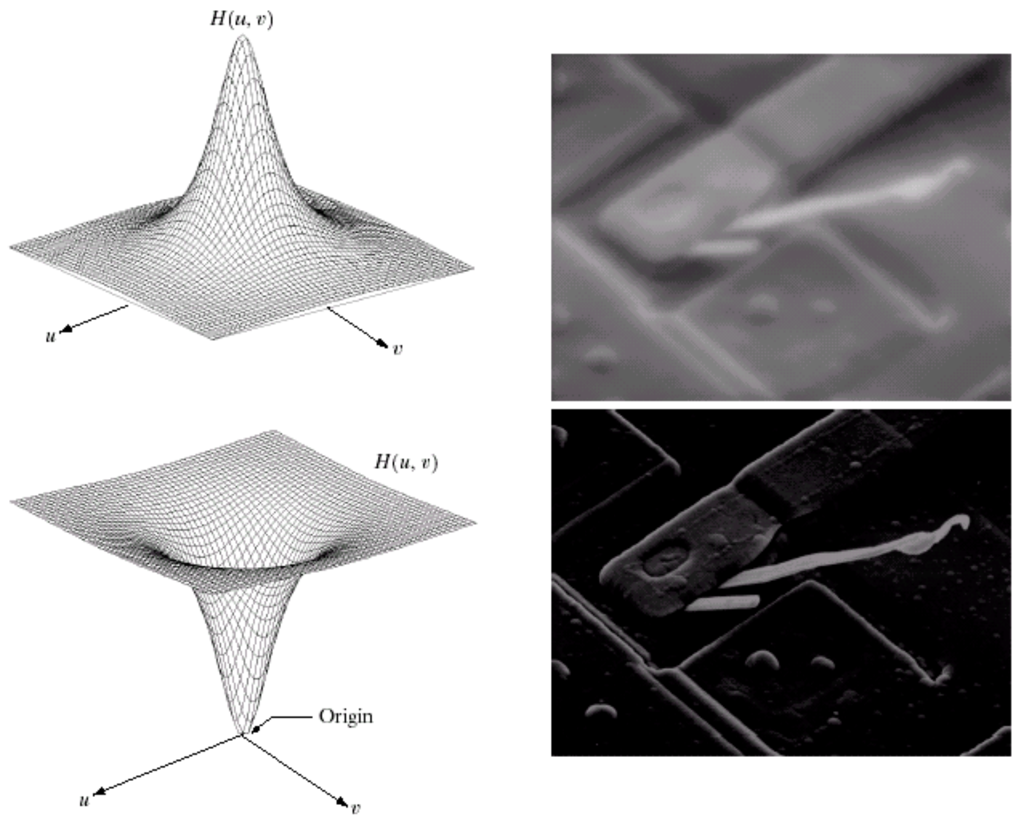
2-D Fourier Transform



a
b

FIGURE 4.4 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

2-D High- & Low-Pass Filters



a b
c d

FIGURE 4.7 (a) A two-dimensional lowpass filter function. (b) Result of lowpass filtering the image in Fig. 4.4(a). (c) A two-dimensional highpass filter function. (d) Result of highpass filtering the image in Fig. 4.4(a).

2-D Notch Filter

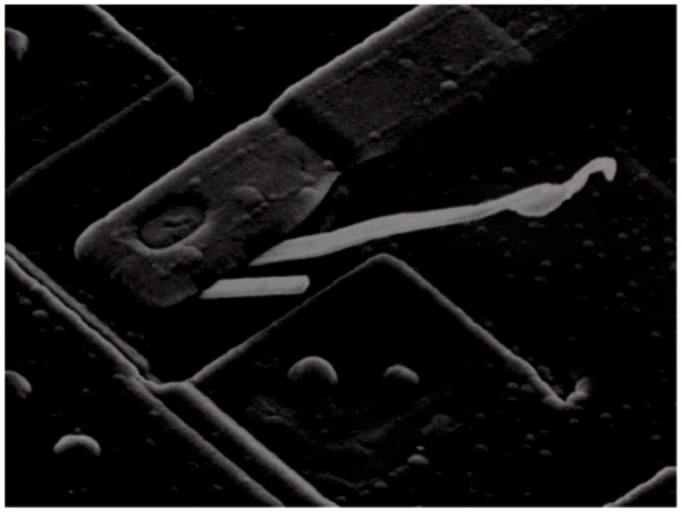
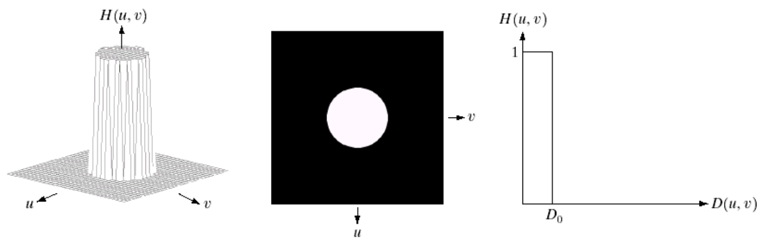


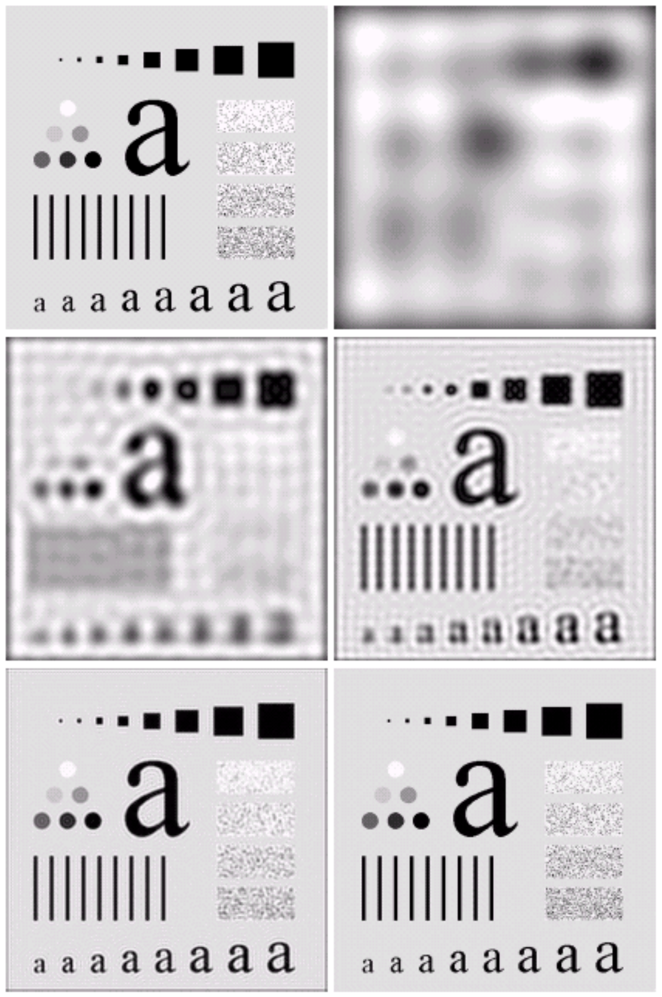
FIGURE 4.6 Result of filtering the image in Fig. 4.4(a) with a notch filter that set to 0 the $F(0, 0)$ term in the Fourier transform.

Low-Pass Filtering in Frequency Domain



a b c

FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.



a b
c d
e f

FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

Non-linear Filtering

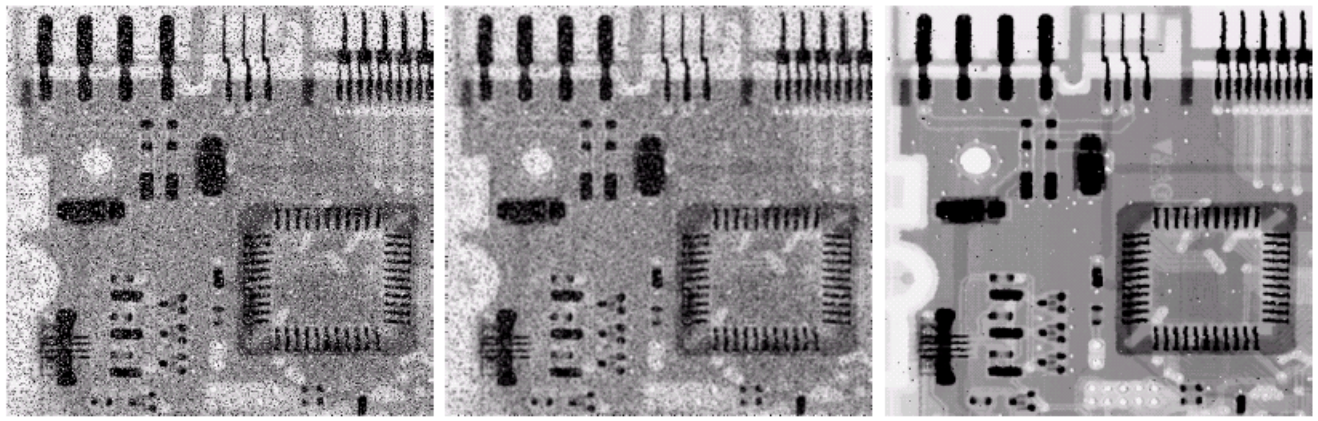

a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

1D Convolution

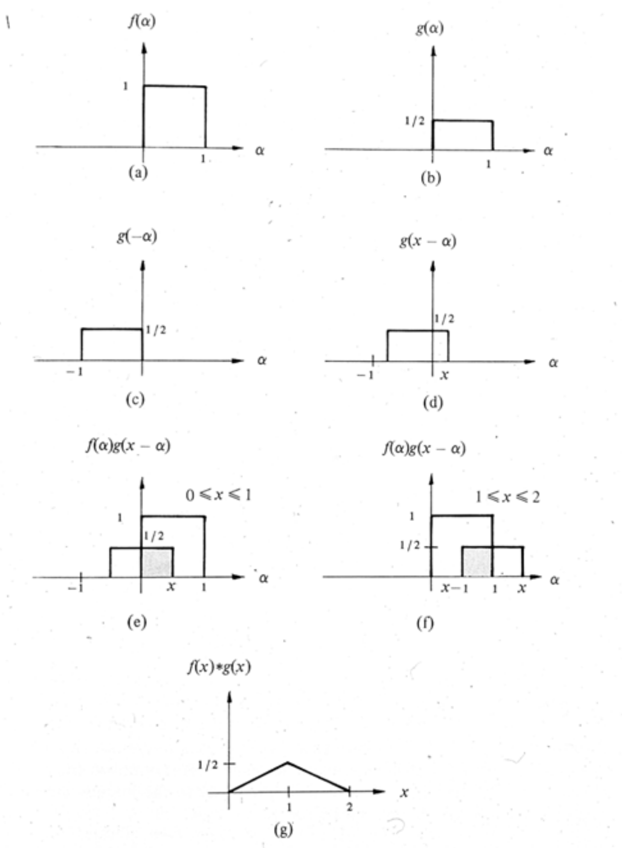
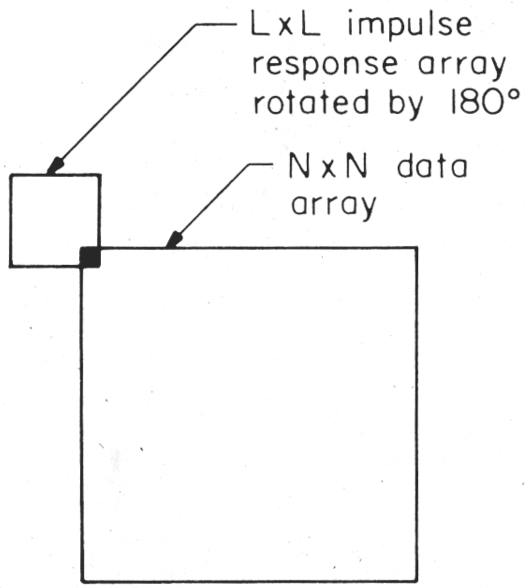


Figure 3.13. Graphical illustration of convolution. The shaded areas indicate regions where the product is not zero.

2D Convolution

Finite Area Superposition Operator



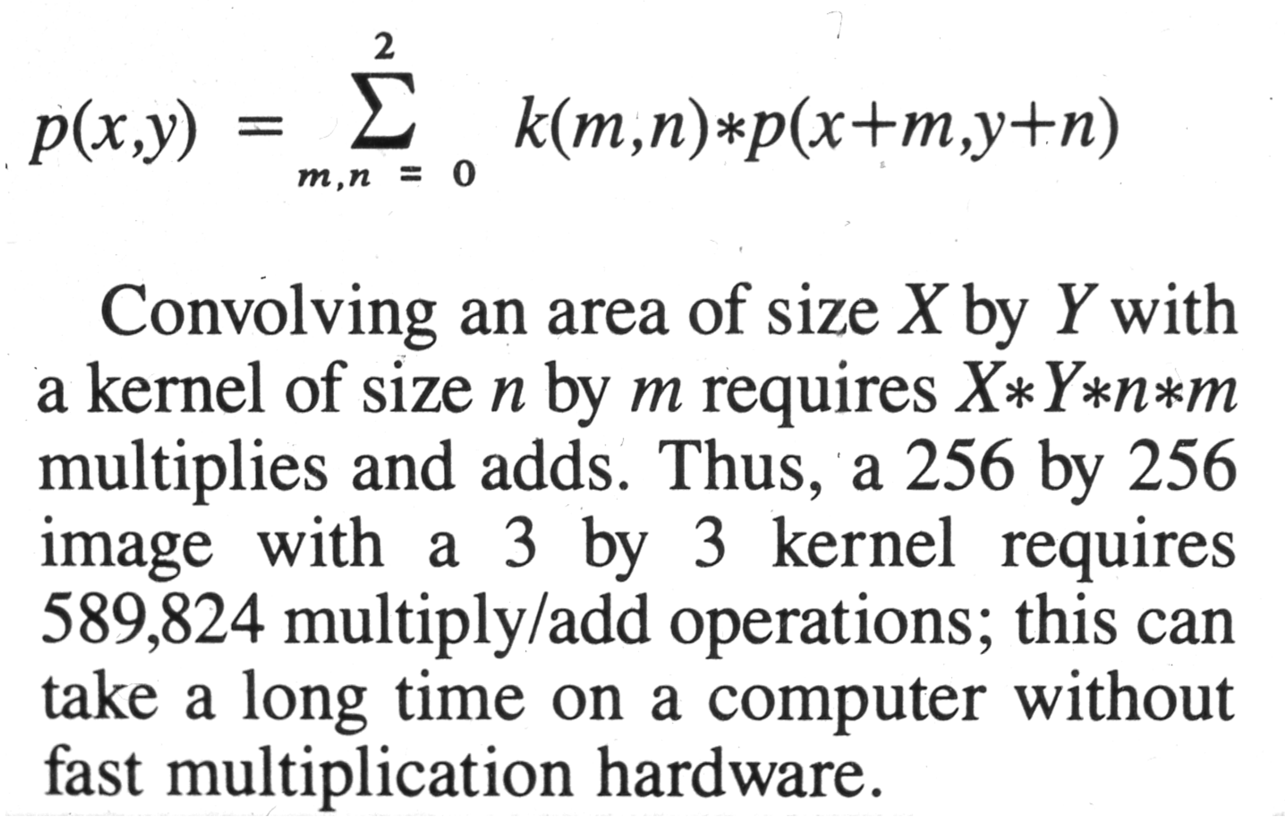
$$\begin{aligned} p(1,1) = {} & p(0,0)*k(0,0) + p(1,0)*k(1,0) \\ & + p(2,0)*k(2,0) + p(0,1)*k(0,1) \\ & + p(1,1)*k(1,1) + p(2,1)*k(2,1) \\ & + p(0,2)*k(0,2) + p(1,2)*k(1,2) \\ & + p(2,2)*k(2,2) \end{aligned}$$

or

$$p(1,1) = \sum_{m,n=0}^{2} k(m,n)*p(m,n)$$

FIGURE 9.1-1. Relationships between input data array and impulse response array for finite area superposition.

Computation Requirements



$$p(x,y) = \sum_{m,n=0}^{2} k(m,n)*p(x+m,y+n)$$

Convolving an area of size X by Y with a kernel of size n by m requires $X*Y*n*m$ multiplies and adds. Thus, a 256 by 256 image with a 3 by 3 kernel requires 589,824 multiply/add operations; this can take a long time on a computer without fast multiplication hardware.

2D Transforms as 1D Computations

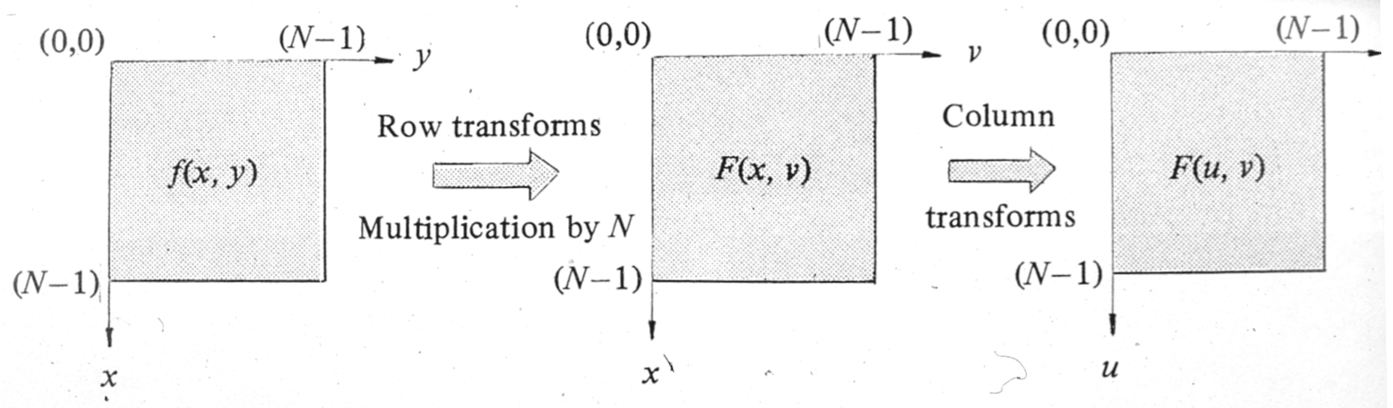
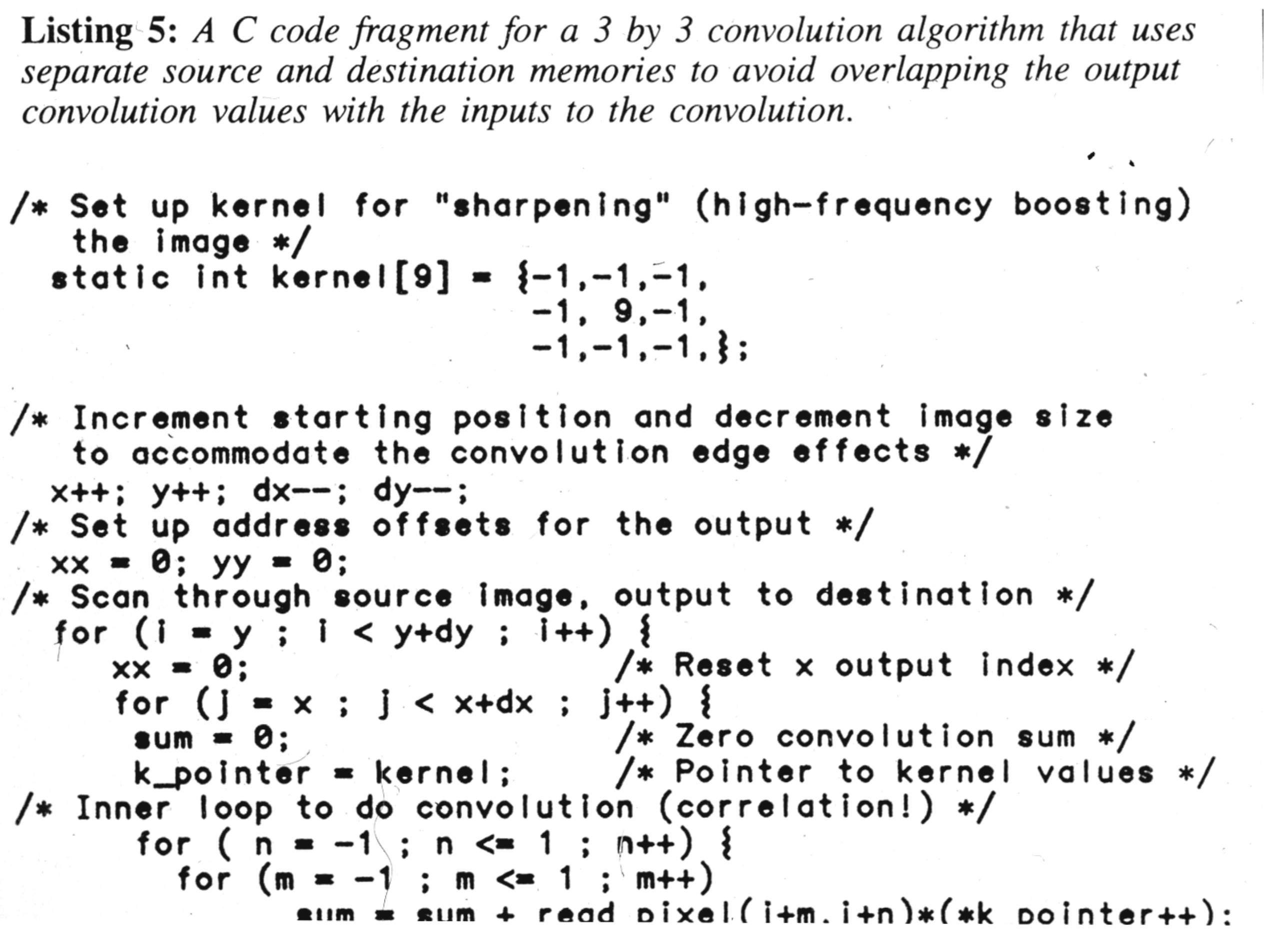


Figure 3.9. Computation of the two-dimensional Fourier transform as a series of o dimensional transforms.

Sample C Code for Spatial Processing

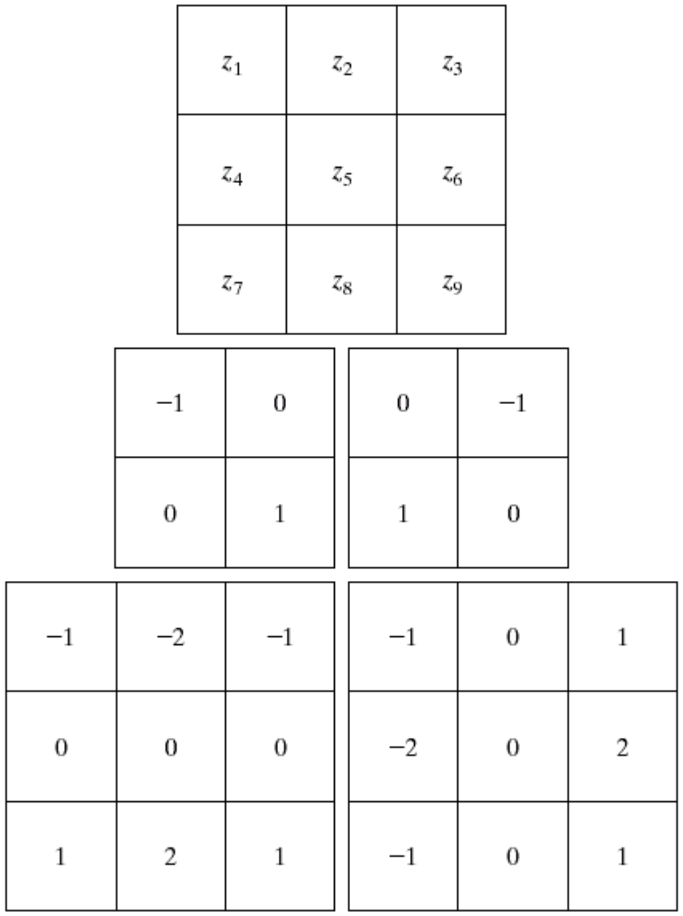


Listing 5: *A C code fragment for a 3 by 3 convolution algorithm that uses separate source and destination memories to avoid overlapping the output convolution values with the inputs to the convolution.*

```
/* Set up kernel for "sharpening" (high-frequency boosting)
   the image */
  static int kernel[9] = {-1,-1,-1,
                          -1, 9,-1,
                          -1,-1,-1,};

/* Increment starting position and decrement image size
   to accommodate the convolution edge effects */
  x++; y++; dx--; dy--;
/* Set up address offsets for the output */
  xx = 0; yy = 0;
/* Scan through source image, output to destination */
  for (i = y ; i < y+dy ; i++) {
     xx = 0;                  /* Reset x output index */
     for (j = x ; j < x+dx ; j++) {
      sum = 0;                /* Zero convolution sum */
      k_pointer = kernel;     /* Pointer to kernel values */
/* Inner loop to do convolution (correlation!) */
      for ( n = -1 ; n <= 1 ; n++) {
        for (m = -1 ; m <= 1 ; m++)
             sum = sum + read pixel(i+m,j+n)*(*k pointer++);
```

Spatial Derivatives



z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

a
b c
d e

FIGURE 3.44 A 3 × 3 region of an image (the z's are gray-level values) and masks used to compute the gradient at point labeled z_5. All masks coefficients sum to zero, as expected of a derivative operator.

Edge Processing

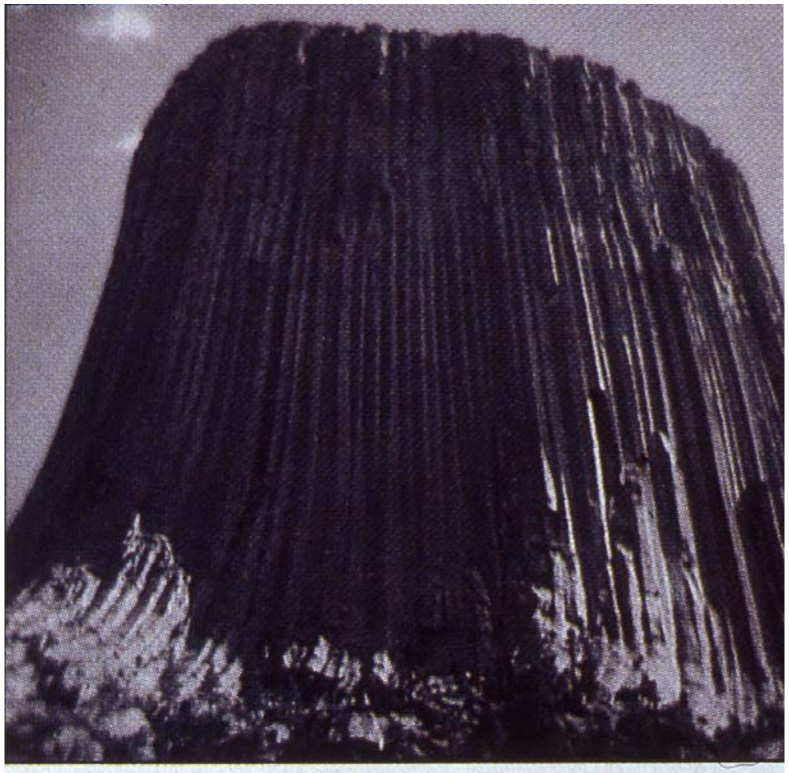


Photo 7: *An image of Devil's Tower National Monument in Wyoming, before image processing.*

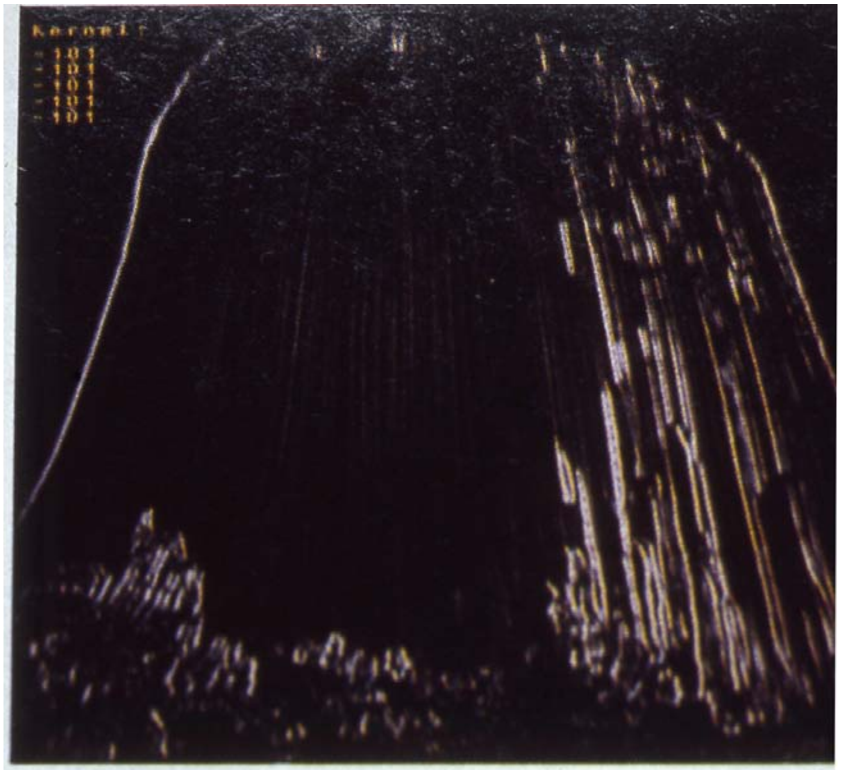
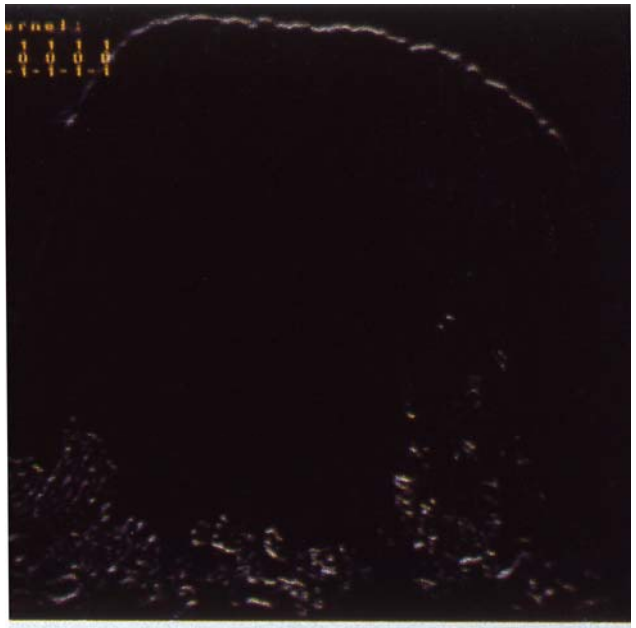


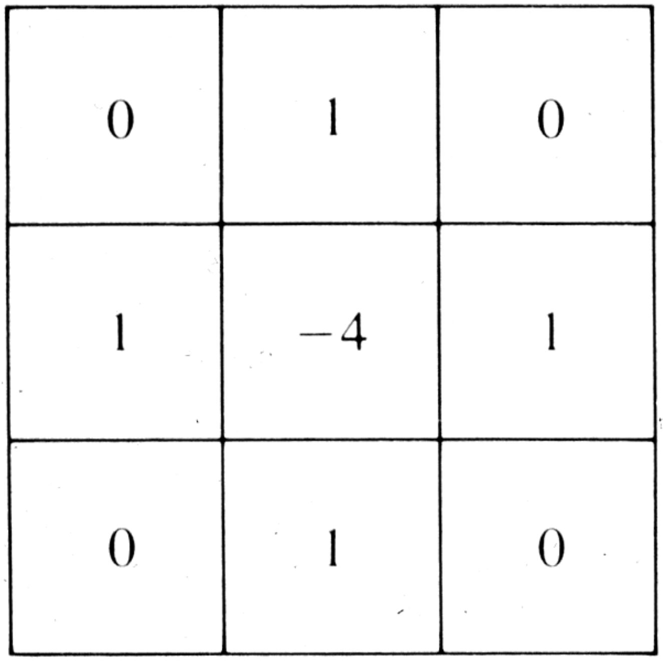
Photo 8: *Convolution of photo 7 with a kernel (shown in the upper left corner) that amplifies vertical edges.*

More Edge Processing



oto 9: *Convolution of photo 7 with*
ernel (shown in the upper left
rner) that amplifies horizontal edges
you can see, this image doesn't
ve many horizontal edges.

Second Order Derivatives



0	1	0
1	−4	1
0	1	0

Figure 7.37 Mask used to compute the Laplacian.

2D Edge Finding

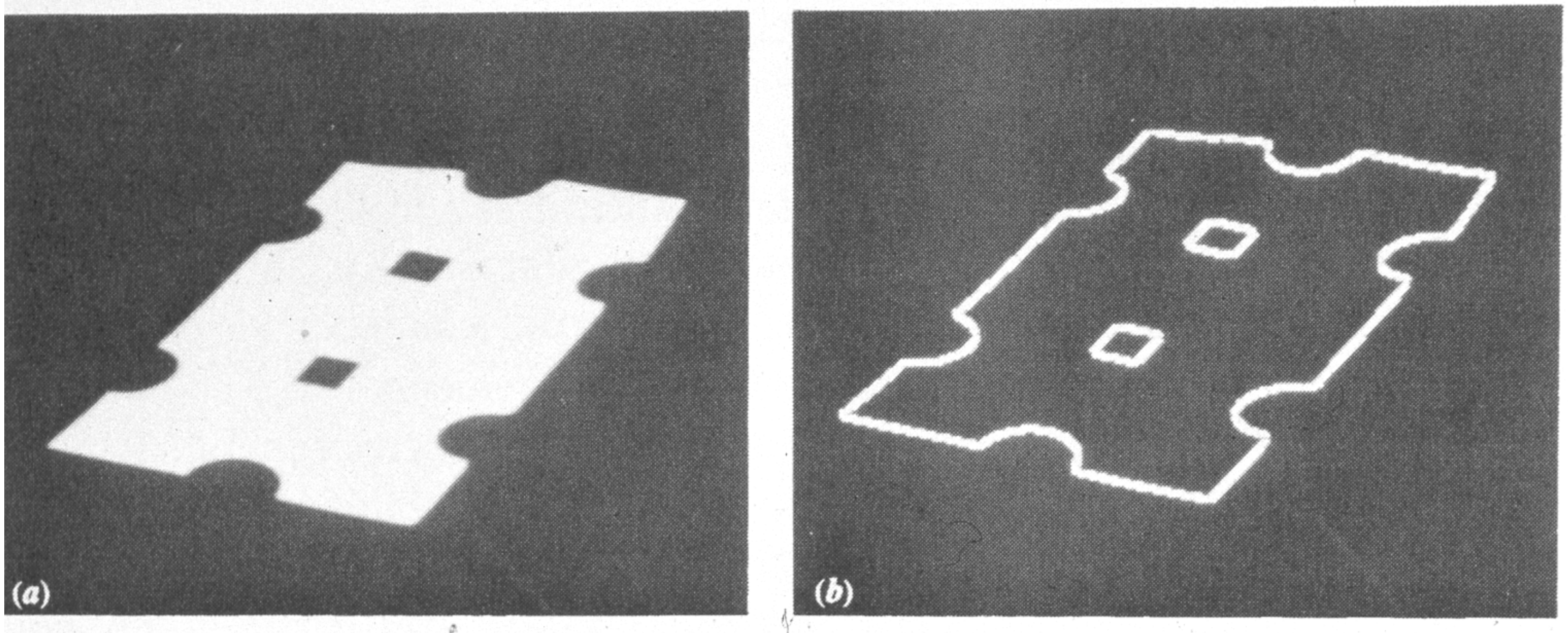


Figure 7.36 (*a*) Input image. (*b*) Result of using Eq. (7.6-44).

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.39 (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

Edge Location

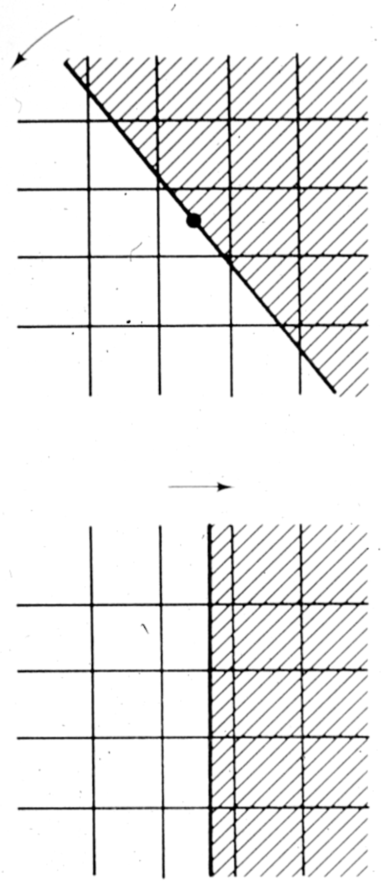
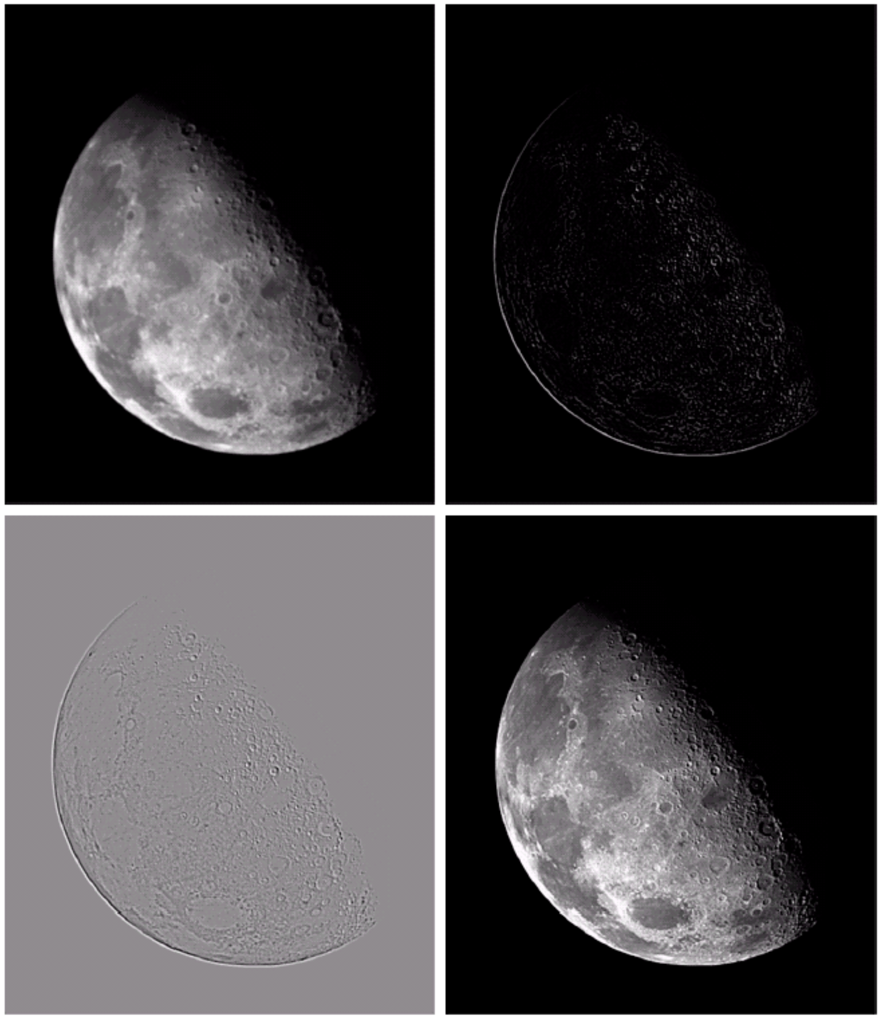


Fig. 3.11 Edge models for orientation and displacement sensitivity analyses.

Sharpening



a b
c d

FIGURE 3.40 (a) Image of the North Pole of the moon. (b) Laplacian-filtered image. (c) Laplacian image scaled for display purposes. (d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)