Representing a region:

- external representation – boundary
  - the boundary can be described by features such as length, orientation of the straight line connecting its extreme points, and the number of concavities in the boundary
  - chosen when the focus is on shape
- internal representation – pixels comprising the region
  - color and texture
  - chosen when the primary focus is on regional properties

want features to be insensitive to size, translation and rotation

Representing a region:

- external representation – boundary

  – the boundary can be described by features such as length, orientation of the straight line connecting its extreme points, and the number of concavities in the boundary

  – chosen when the focus is on shape

- internal representation – pixels comprising the region

  – color and texture

  – chosen when the primary focus is on regional properties

want features to be insensitive to size, translation and rotation

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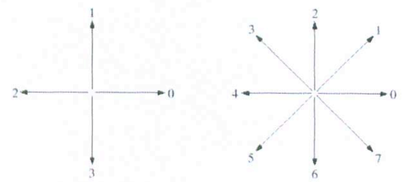

a b

FIGURE 11.1 Direction numbers for (a) 4-directional chain code, and (b) 8-directional chain code.

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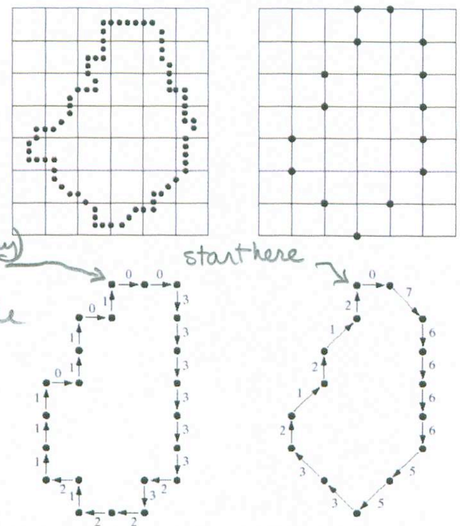
don't want to use original pixels since code would be too long and subject to noise — resample on a larger grid



a b
c d

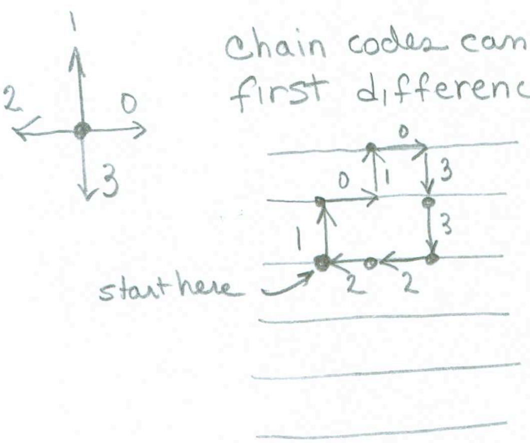
FIGURE 11.2 (a) Digital boundary with resampling grid superimposed. (b) Result of resampling. (c) 4-directional chain code. (d) 8-directional chain code.

4-connected code 00333...

8-connected code - 07666...

chain codes represent a boundary as a connected sequence of straight-line segments of specified length and direction

chain codes can be normalized for rotation by using first difference



1 0 1 0 3 3 2 2 chain code representation

first difference normalizes for rotation

go in counter clockwise direction, count # of $\frac{\pi}{2}$ (or $\frac{\pi}{4}$)

1→0 3 turns

0→1 1

1→0 3

0→3 3

3→3 0

3→2 3

2→2 0

2→1 3 put at beginning

3 3 1 3 3 0 3 0

only rotation invariant if the digitizations are insensitive to rotation

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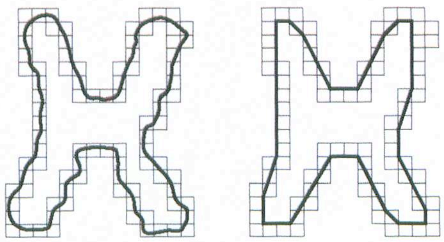


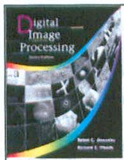
a b

FIGURE 11.3 (a) Object boundary enclosed by cells. (b) Minimum perimeter polygon.

grid of cells which "walls" in boundary

let boundary be a "rubber band"

MINIMUM PERIMETER POLYGONS



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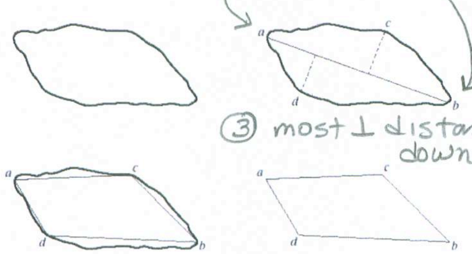
① two most distant points joined by ab

② most ⊥ distance up from ab

③ most ⊥ distance down



a b
c d

FIGURE 11.4 (a) Original boundary. (b) Boundary divided into segments based on extreme points. (c) Joining of vertices. (d) Resulting polygon.

boundary segment splitting – subdivide a segment successively until a specified criterion is satisfied

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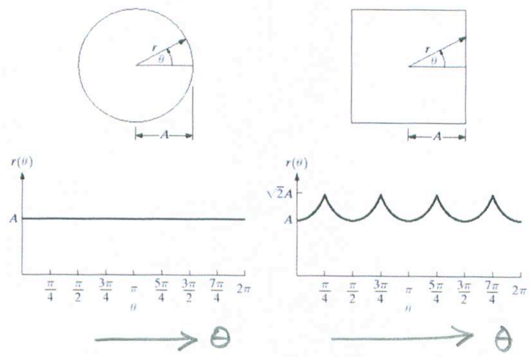

a b

FIGURE 11.5 Distance-versus-angle signatures. In (a) $r(\theta)$ is constant. In (b), the signature consists of repetitions of the pattern $r(\theta) = A \sec\theta$ for $0 \le \theta \le \pi/4$ and $r(\theta) = A \csc\theta$ for $\pi/4 < \theta \le \pi/2$.

signature – 1-D representation of a boundary

These are invariant to translation but
sensitive to rotation and scale (size)

Methods of selecting starting point can make signatures
independent of rotation
- select point farthest from centroid
- select point farthest from centroid along eigenaxis
- use a chain code

Many other types of signatures
ψ–s (i.e. plot the tangent)

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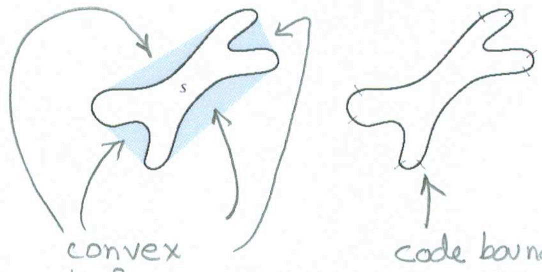
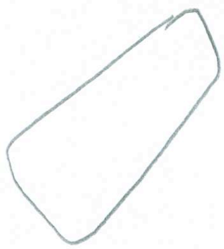


a b

FIGURE 11.6
(a) A region, *S*, and its convex deficiency (shaded).
(b) Partitioned boundary.

convex deficiency

code boundary by points where boundary passes in and out of a convex deficiency



H is the convex hull of S

H - S is the convex defeciency.

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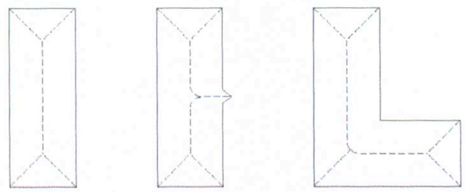


a b c
FIGURE 11.7 Medial axes (dashed) of three simple regions.

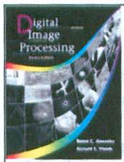
reduce the structure of a shape to a skeleton

morphology will not necessarily keep a skeleton connected

median axis transformation (MAT) algorithm

MAT of a region R with border B
for each point p in R find its closest neighbor on B
if p has more that one "closest" neighbor it belongs
to the <u>medial axis</u> (skeleton) of B

closest is defined using Euclidian distance



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p_9	p_2	p_3
p_8	p_1	p_4
p_7	p_6	p_5

FIGURE 11.8 Neighborhood arrangement used by the thinning algorithm.

0	0	1
1	P_1	0
1	0	1

$N(p_1) = 4$

$T(p_1) = 3$

Algorithm for thinning binary images.

contour point is any pixel with value 1 and at least one 8-neighbor valued 0

Step 1: mark for deletion any contour point p_1 which has

don't delete if end point or inside region — (a) $2 \le N(p_1) \le 6$ — # of non-zero neighbors is between 2 and 6

prevents breaking connected lines — (b) $T(p_1) = 1$ — # of 0-1 transitions in sequence $P_2\, P_3\, P_4\, P_5\, P_6\, P_7\, P_8\, P_9\, P_2$

east or south boundary pt. — (c) $P_2 P_4 P_6 = 0$

northwest corner — (d) $P_4 P_6 P_8 = 0$

(c), (d) says that either P_4 and $P_6 = 0$ or P_2 and $P_8 = 0$

Step 2: mark for deletion if

(a) $2 \le N(p_1) \le 6$

(b) $T(p_1) = 1$

(c') $P_2 P_4 P_8 = 0$

(d') $P_4 P_6 P_8 = 0$

(c'), (d') says that P_2 and $P_4 = 0$ or P_6 and $P_8 = 0$

Iterate by applying step 1 to all border points; deleting marked points; apply step 2 to all remaining border points; and deleting marked points. Repeat until no further points deleted.

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FIGURE 11.9 Illustration of conditions (a) and (b) in Eq. (11.1-1). In this case $N(p_1) = 4$ and $T(p_1) = 3$.

0	0	1
1	p_1	0
1	0	1

$N(p_1) = 4$

$T(p_1) = 3$

$P_2\, P_4\, P_6 = 0$

$P_4\, P_6\, P_8 = 0$

$P_2\, P_4\, P_8 = 0$

$P_2\, P_6\, P_8 = 0$

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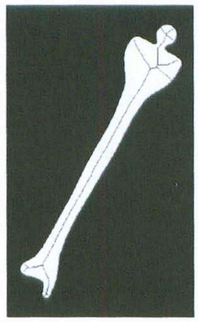
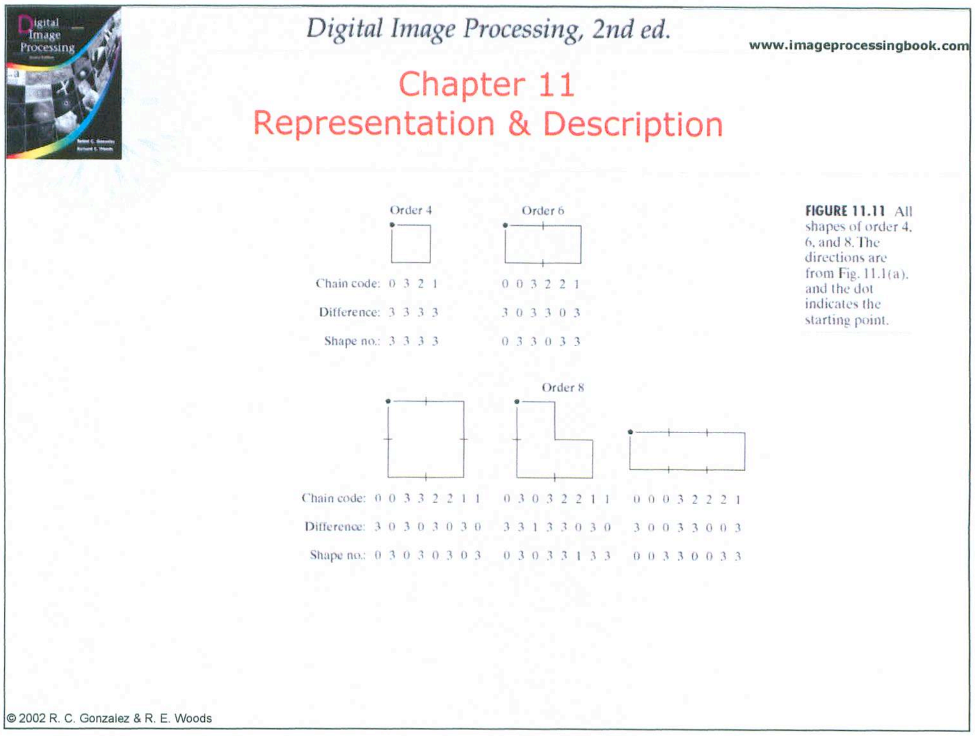


FIGURE 11.10 Human leg bone and skeleton of the region shown superimposed.



n, shape number = first difference of smallest magnitude

n is even for closed boundaries

we can define

diameter $= \max_{i,j} [D(p_i, p_j)]$ — distance

major axis — line segment of length = diameter and connecting two points on boundary

minor axis — line $\perp$ major axis such that a box passing through the four points of intersection with the boundary and the major/minor axes completely encloses the boundary.

basic rectangle — box described above.

eccenctricity — ratio of major to minor axes.

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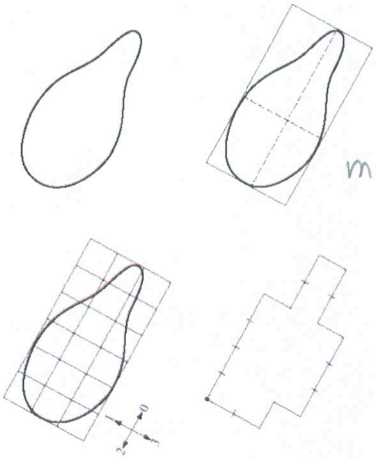


a b
c d

FIGURE 11.12 Steps in the generation of a shape number.

major and minor axes and basic rectangle

rectangle of order 6x3=18

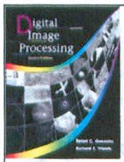
obtain chain code

Chain code: 0 0 0 0 3 0 0 3 2 2 3 2 2 2 1 2 1 1

Difference: 3 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0

Shape no.: 0 0 0 3 1 0 3 3 0 1 3 0 0 3 1 3 0 3

then finally compute difference code and shape number



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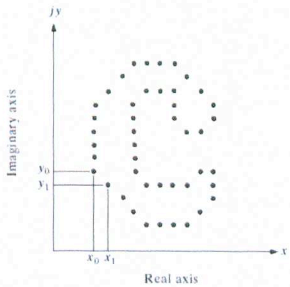


FIGURE 11.13 A digital boundary and its representation as a complex sequence. The points (x_0, y_0) and (x_1, y_1) shown are (arbitrarily) the first two points in the sequence.

represent each point on a digital boundary as

$$s(k) = x(k) + j\,y(k)$$

Compute DFT of the set of boundary points

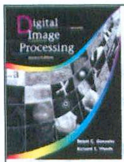
$$a(u) = \frac{1}{K} \sum_{k=0}^{K-1} s(k) e^{-j2\pi uk/K}, \quad u = 0, 1, 2, \ldots, K-1$$

The coefficients $a(u)$ are the Fourier descriptors of the boundary.

$$s(k) = \sum_{u=0}^{K-1} a(u) e^{j2\pi uk/K}$$

We often approximate the boundary by a small set of coefficients

i.e., $$s(k) \cong \hat{s}(k) = \sum_{u=0}^{P-1} a(u) e^{j2\pi uk/K}$$



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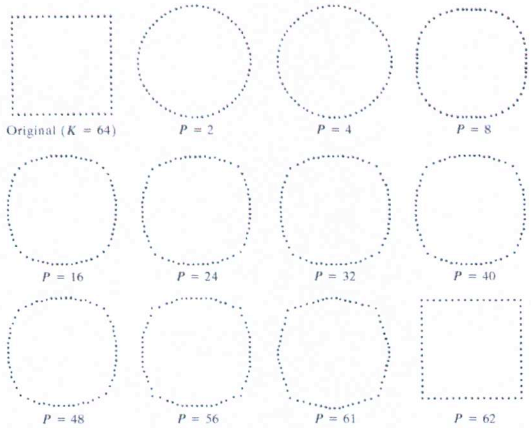
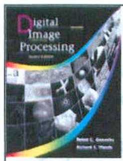


FIGURE 11.14 Examples of reconstruction from Fourier descriptors. P is the number of Fourier coefficients used in the reconstruction of the boundary.

There are the same number of points K in the reconstructed boundary.

However, only P terms we used to reconstruct them.



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Transformation	Boundary	Fourier Descriptor
Identity	$s(k)$	$a(u)$
Rotation	$s_r(k) = s(k)e^{j\theta}$	$a_r(u) = a(u)e^{j\theta}$
Translation	$s_t(k) = s(k) + \Delta_{xy}$	$a_t(u) = a(u) + \Delta_{xy}\delta(u)$
Scaling	$s_s(k) = \alpha s(k)$	$a_s(u) = \alpha a(u)$
Starting point	$s_p(k) = s(k - k_0)$	$a_p(u) = a(u)e^{-j2\pi k_0 u/K}$

TABLE 11.1 Some basic properties of Fourier descriptors.

rotation, scaling and translation of a boundary have simple effects on the Fourier descriptors

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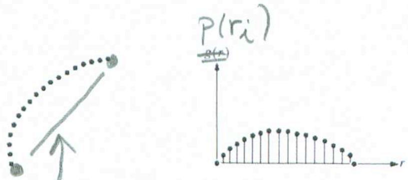


a b
FIGURE 11.15 (a) Boundary segment. (b) Representation as a 1-D function.

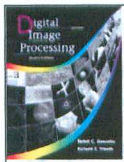
histogram of computed displacements

connect start and stopping points and compute displacement from this line

compute $m = \sum_{i=0}^{A-1} r_i p(r_i)$, mean displacement.

Can compute higher order moments

$$\mu_n(r) = \sum_{i=0}^{K-1} (r_i - m)^n p(r_i)$$



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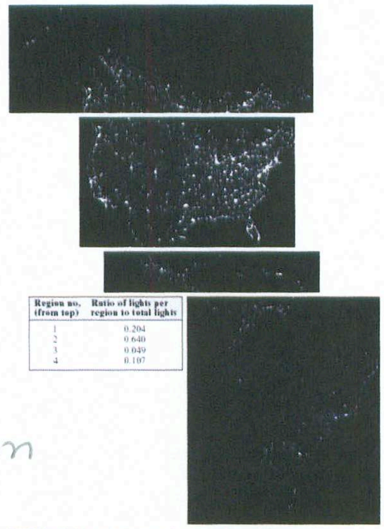


canada.

u s + mexico

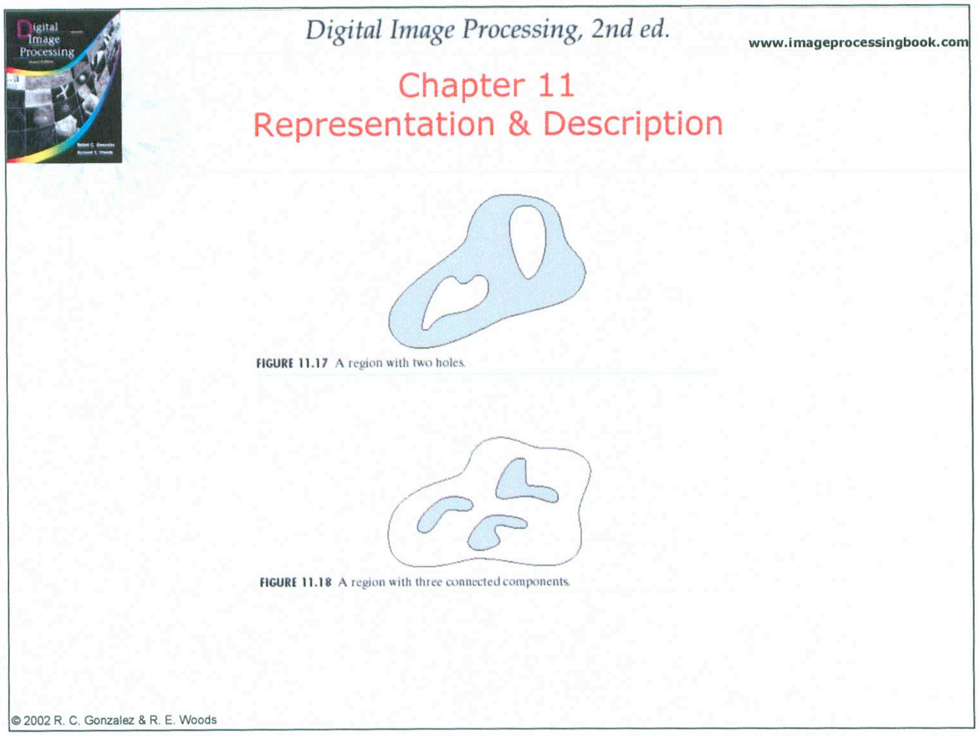
central america

south america

Region no. (from top)	Ratio of lights per region to total lights
1	0.204
2	0.640
3	0.049
4	0.107

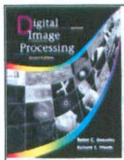
ratio of white pixels
to total area
to estimate electrical
energy consumption

FIGURE 11.16 Infrared images of the Americas at night. (Courtesy of NOAA.)



topology – properties that are unaffected by "rubber sheet" deformation

connected component – for any pixel p in S, the set of pixels that are connected to it in S is called a connected component of S



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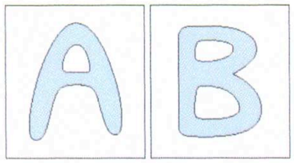

FIGURE 11.19 Regions with Euler number equal to 0 and −1, respectively.

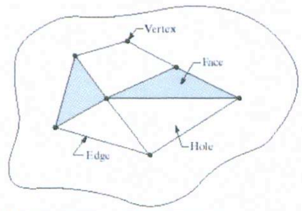

FIGURE 11.20 A region containing a polygonal network.

7 vertices
11 edges
2 faces
3 holes
1 connected region

Euler number $E = C - H$

C: # of connected components

H: # of holes

topological property of a region/object

for polygonal networks

Euler formula $E = V - Q + F = C - H$

V: # of vertices

Q: # of holes.

F: # of faces

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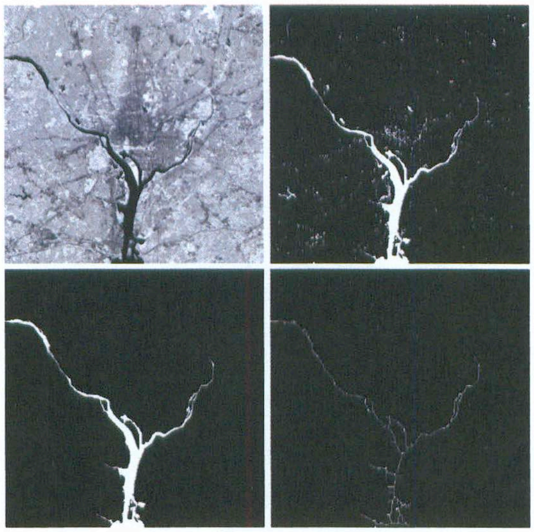
single infrared image



a b
c d

FIGURE 11.21 (a) Infrared image of the Washington, D.C. area. (b) Thresholded image. (c) The largest connected component of (b). Skeleton of (c).

result of Thresholding (largest T before river became disconnected) 1591 connected components

single component with largest number of pixels (8479)

computed skeleton useful for computing length of branches, etc.

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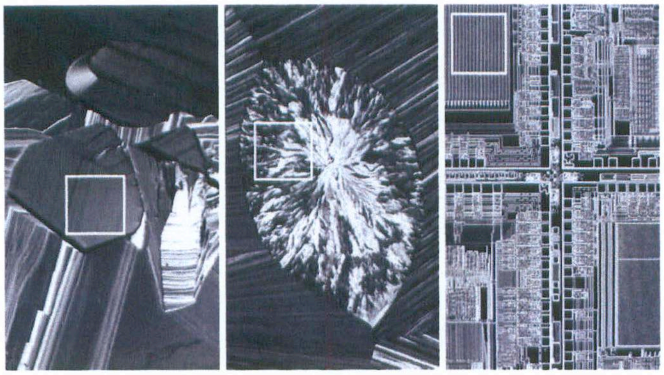

a b c

FIGURE 11.22 The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)

statistical characterizations of texture

structural texture

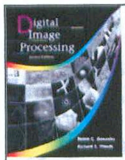

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TABLE 11.2 Texture measures for the subimages shown in Fig. 11.22.

Texture	Mean	Standard deviation	R (normalized)	Third moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	−0.105	0.026	5.434
Coarse	143.56	74.63	0.079	−0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

gives idea of average gray level and not texture

first has less variability, it's smoother, than other two textures

essentially same as standard deviation

let z be a random variable denoting gray levels
let $p(z_i)$, $i = 0, 1, \ldots, L-1$ be the corresponding histogram

mean $\quad m = \sum_{i=0}^{L-1} z_i \, p(z_i)$

measures gray level contrast

Variance $\quad \sigma^2(z) = \mu_2(z) = \sum_{i=0}^{L-1} (z_i - m)^2 p(z_i)$

Smoothness $\quad R = 1 - \dfrac{1}{1+\sigma^2(z)}$

0 for regions of constant intensity where $\sigma^2 \to 0$
1 for regions of large variance

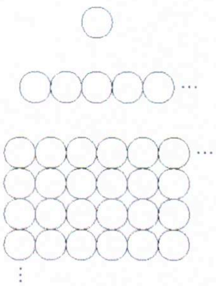
third moment $\quad \mu_3(z) = \sum_{i=0}^{L-1} (z_i - m)^3 p(z_i)$ are grey levels biased toward dark or light

uniformity $\quad U = \sum_{i=0}^{L-1} p^2(z_i)$

average entropy $\quad e = -\sum_{i=0}^{L-1} p(z_i) \ln p(z_i)$ measure of variability

a
b
c

FIGURE 11.23 (a) Texture primitive. (b) Pattern generated by the rule $S \rightarrow aS$. (c) 2-D texture pattern generated by this and other rules.



a

$S \rightarrow aS$ — a = circles to the right

$S \rightarrow bA$
$A \rightarrow cA$
$A \rightarrow c$
$A \rightarrow bS$
$S \rightarrow a$

b = circle down

c = circle to the left

$aaabccbaa \rightarrow$ 

statistical measures of texture carry no information about the relative positions of pixels

structural approaches use a grammar

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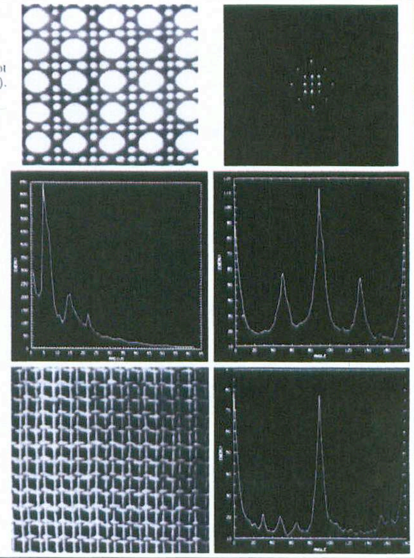
FIGURE 11.24 (a) Image showing periodic texture. (b) Spectrum. (c) Plot of $S(r)$. (d) Plot of $S(\theta)$. (e) Another image with a different type of periodic texture. (f) Plot of $S(\theta)$. (Courtesy of Dr. Dragana Brzakovic, University of Tennessee.)



2 D Fourier spectrum

$S(r)$

$S(\theta)$ peaks at multiples of 45°

another texture

peaks at multiples of 90°

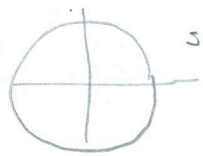
Spectral measures of texture –

Fourier transform good for describing periodic textures

Express spectrum in polar coordinates

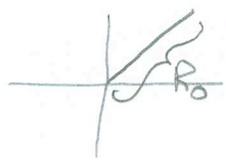
Some more useful measures are

$$S(r) = \sum_{\theta=0}^{\pi} S_\theta(r)$$



sum around a circle

$$S(\theta) = \sum_{r=1}^{R_0} S_r(\theta)$$



sum along a radius

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Invariant (Log)	Original (a)	Half Size (b)	Mirrored (c)	Rotated 2° (d)	Rotated 45° (e)
ϕ_1	6.249	6.226	6.919	6.253	6.318
ϕ_2	17.180	16.954	19.955	17.270	16.803
ϕ_3	22.655	23.531	26.689	22.836	19.724
ϕ_4	22.919	24.236	26.901	23.130	20.437
ϕ_5	45.749	48.349	53.724	46.136	40.525
ϕ_6	31.830	32.916	37.134	32.068	29.315
ϕ_7	45.589	48.343	53.590	46.017	40.470

TABLE 11.3 Moment invariants for the images in Figs. 11.25(a)-(e).

→ stay the same for each transformation

$$\phi_1 = \eta_{20} + \eta_{02}$$

$$\eta_{20} = \frac{\mu_{20}}{\mu_{00}^2} \qquad \eta_{02} = \frac{\mu_{02}}{\mu_{00}^2}$$

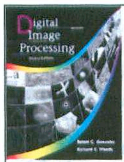
$$\mu_{00} = m_{00} = \sum_x \sum_y f(x,y)$$

$$\bar{x} = \frac{m_{10}}{m_{00}} \qquad \bar{y} = \frac{m_{01}}{m_{00}}$$

$$m_{10} = \sum_x \sum_y x f(x,y) \qquad m_{01} = \sum_x \sum_y y f(x,y)$$

$$\mu_{20} = \sum_x \sum_y (x-\bar{x})^2 f(x,y) = m_{20} - \bar{x} m_{10}$$

$$\mu_{02} = \sum_x \sum_y (y-\bar{y})^2 f(x,y) = m_{02} - \bar{y} m_{01}$$



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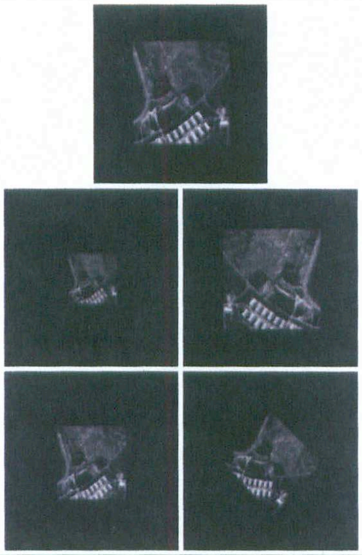

a
b c
d e

FIGURE 11.25 Images used to demonstrate properties of moment invariants (see Table 11.3).

reduced 50%

mirror imaged

rotated by 2°

rotated by 45°

There are seven moments which are invariant to translation, rotation, size change.

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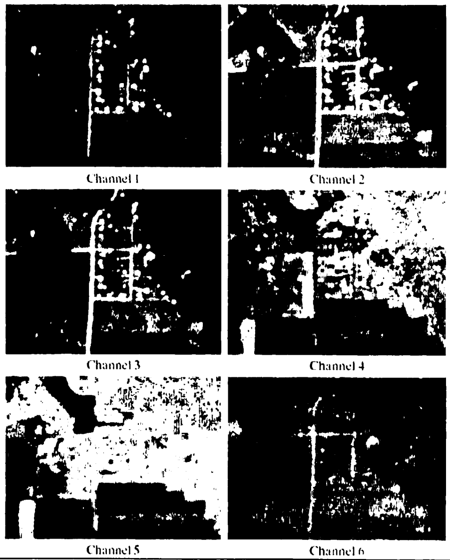
FIGURE 11.26 Six spectral images from an airborne scanner. (Courtesy of the Laboratory for Applications of Remote Sensing, Purdue University.)



384x239 pixel images

i.e. 384x239 $\underline{x}$ vectors
= 91776 vectors

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Images from a six-band multispectral scanner.

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

This is an example of region analysis using eigenvectors.

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TABLE 11.4
Channel numbers and wavelengths

Channel	Wavelength band (microns)
1	0.40–0.44
2	0.62–0.66
3	0.66–0.72
4	0.80–1.00
5	1.00–1.40
6	2.00–2.60

visible

infrared

Six wavebands corresponding to the images in Figure 11.26

11.4 Use of principal components for analysis.

For an RGB image we can write each pixel as

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

For n registered images the corresponding pixel vector will be

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

The mean vector for a population of pixels, i.e., random vectors $\underline{x}$, is

$$\underline{m}_x = E\{\underline{x}\}$$

where we compute the expected value of each element.

The covariance matrix of this vector population is given by

$$\underline{C}_x = E\{(\underline{x}-\underline{m}_x)(\underline{x}-\underline{m}_x)^T\}$$

For k samples from a random population

$$\underline{m}_x = \frac{1}{K}\sum_{k=1}^{K} \underline{x}_k$$

$$\underline{C}_x = \frac{1}{K}\sum_{k=1}^{K} \underline{x}_k \underline{x}_k^T - \underline{m}_x \underline{m}_x^T$$

Example: $\underline{x}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\underline{x}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\underline{x}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\underline{x}_4 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

Using above formulas

$$\underline{m}_x = \frac{1}{4}\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \qquad \underline{C}_x = \frac{1}{16}\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

Transform the data $\underline{x}$ by

$$\underline{y} = \underline{A}(\underline{x} - \underline{m}_x)$$

where $\underline{A} = \begin{bmatrix} \underline{e}_1^T \\ \underline{e}_2^T \\ \vdots \\ \underline{e}_n^T \end{bmatrix}$ where $\underline{e}_1, \ldots \underline{e}_n$ are the eigenvectors of $\underline{C}_x$ and $\lambda_1 > \lambda_2 > \ldots > \lambda_n$

This is called a Hotelling transformation. It is optimum in the sense that it minimizes error between $\underline{x}$ and an eigenvector approximation $\hat{\underline{x}}_k$:

$$\underline{m}_y = E\{\underline{y}\} = 0$$

and $\underline{C}_y = \underline{A}\,\underline{C}_x\,\underline{A}^T$

where $$\underline{C}_y = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

This incidentally indicates that the $\underline{y}$ vectors are uncorrelated.

This can be inverted to give

$$\underline{x} = \underline{A}^T \underline{y} + m_x$$

note: $\underline{A}^{-1} = \underline{A}^T$ because $\underline{A}$ is orthonormal

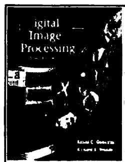
Construct an estimate $\hat{\underline{x}}$ of $\underline{x}$ using only the k largest eigenvalues

$$\hat{\underline{x}} = \underline{A}_k^T \underline{y} + \underline{m}_x$$

$$A = \begin{bmatrix} \underline{e}_1^T \\ \underline{e}_2^T \\ \\ \underline{e}_k^T \end{bmatrix}$$

The rms error between $\hat{x}$ and x is

$$e_{rms} = \sum_{j=k+1}^{n} \lambda_j$$



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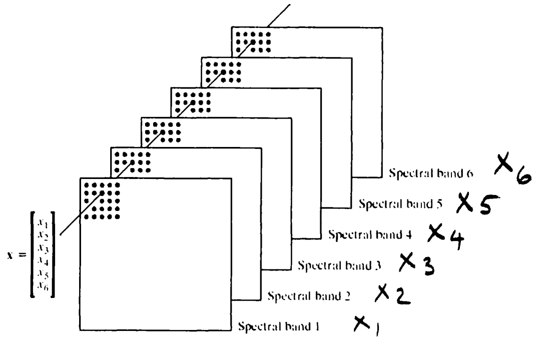

FIGURE 11.27 Formation of a vector from corresponding pixels in six images.

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
3210	931.4	118.5	83.88	64.00	13.40

TABLE 11.5 Eigenvalues of the covariance matrix obtained from the images in Fig. 11.26.

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Compute eigenvalues and eigenvectors of covariance matrix

$$\underline{m}_x = \frac{1}{K}\sum_{k=1}^{K} \underline{x}_k = E\{\underline{x}\} \quad \text{where } \underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

K: 91776

$$\underline{C}_x = E\{(\underline{x}-\underline{m}_x)(\underline{x}-\underline{m}_x)^T\}$$

6x6 matrix

Then compute eigenvectors of $\underline{C}_x$

$$\underline{A} = \begin{bmatrix} \underline{e}_1^T \\ \underline{e}_2^T \\ \underline{e}_3^T \\ \underline{e}_4^T \\ \underline{e}_5^T \\ \underline{e}_6^T \end{bmatrix}$$

each of these is a 1x6 vector.

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$\underline{e}_1$ corresponding to λ_1

$\underline{y} = \underline{e}_1^T(\underline{x} - \underline{m}_x)$

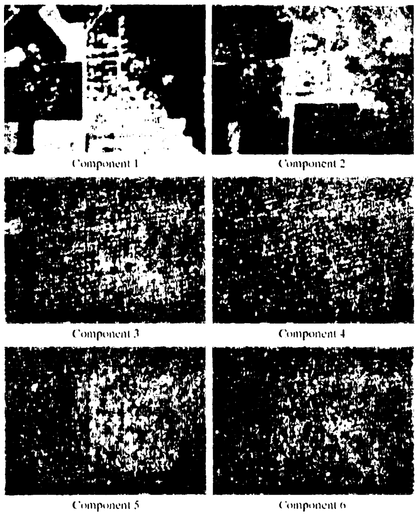

FIGURE 11.28 Six principal-component images computed from the data in Fig. 11.26. (Courtesy of the Laboratory for Applications of Remote Sensing, Purdue University.)

$\underline{e}_2$ corresponding to λ_2

$\underline{e}_6$ corresponding to λ_6

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Principal component images.

What the Purdue researchers did was transform the original image pixels $\underline{x}$ according to

$$\underline{y} = \underline{A}(\underline{x} - \underline{m}_x) \text{ to get a new image.}$$

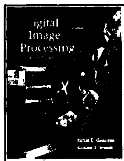
They then approximated the images by $\hat{\underline{y}} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ dropping $y_3 - y_6$.

and inverted to approximate $\hat{x} = \begin{bmatrix} \underline{e}_1^T \\ e_2^T \end{bmatrix}^T \hat{\underline{y}} + \underline{m_x}$

$$\hat{\underline{x}} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = [\underline{e}_1 \ \underline{e}_2] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \underline{m}_x$$

Images in components $\hat{\underline{x}}_1$ $\hat{\underline{x}}_2$ correspond to about 94% of the total variance.

<u>Data compression</u>: store only $\underbrace{\hat{x}_1, \hat{x}_2}_{\text{images}}$, $\underline{m}_x$ and the $\underbrace{\text{first two rows of } \underline{A}}_{\text{the eigenvectors}}$.



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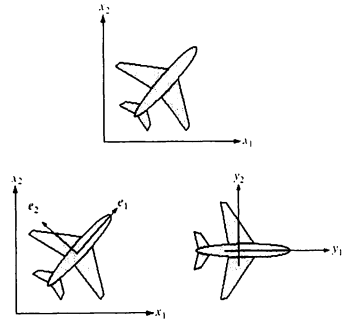

a
b c

FIGURE 11.29 (a) An object. (b) Eigenvectors. (c) Object rotated by using Eq. (11.4-6). The net effect is to align the object along its eigen axes.

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Form 2-D vectors from the coordinates of the boundary or region.

Analyze this set of random vectors $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Compute $\underline{C}_x$ and $\underline{m}_x$

Compute eigenvectors $\underline{e}_1$ and $\underline{e}_2$ of $\underline{C}_x$

Allows us to account for rotation of objects when we do pattern recognition.
Translation handled by $\underline{m}_x$

This is an example of boundary analysis using eigenvectors.

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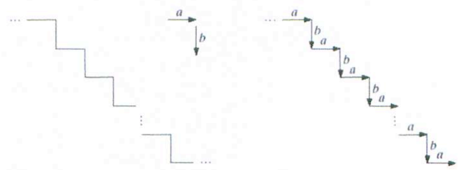


a b

FIGURE 11.30
(a) A simple staircase structure.
(b) Coded structure.

rewriting rules

(a) $S \rightarrow aA$

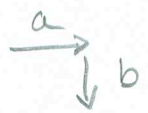
(b) $A \rightarrow bS$

(c) $A \rightarrow b$

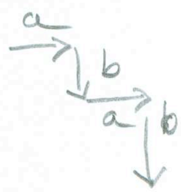
a, b are elements shown above
S is the starting symbol
A is a variable.

for example

S
aA
ab



S
aA
abS
abaA
abab



etc.

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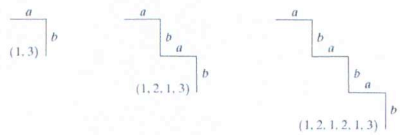

FIGURE 11.31 Sample derivations for the rules $S \rightarrow aA$, $A \rightarrow bS$, and $A \rightarrow b$.

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FIGURE 11.32 Coding a region boundary with directed line segments.

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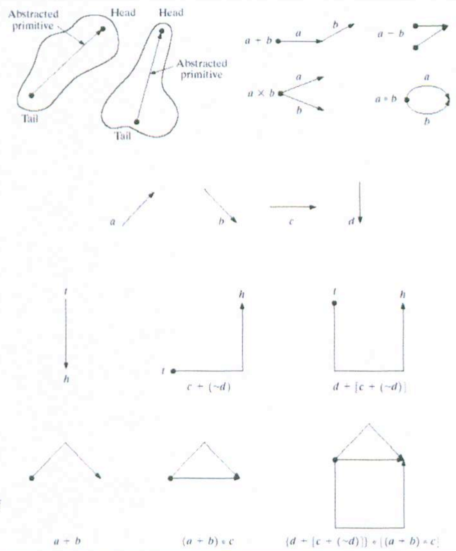

a b
c
d

FIGURE 11.33 (a) Abstracted primitives. (b) Operations among primitives. (c) A set of specific primitives. (d) Steps in building a structure.

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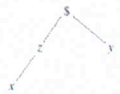
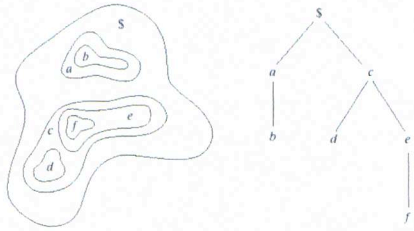


FIGURE 11.34 A simple tree with root $ and frontier *xy*.



a b

FIGURE 11.35 (a) A simple composite region. (b) Tree representation obtained by using the relationship "inside of."

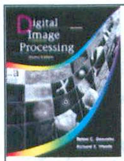


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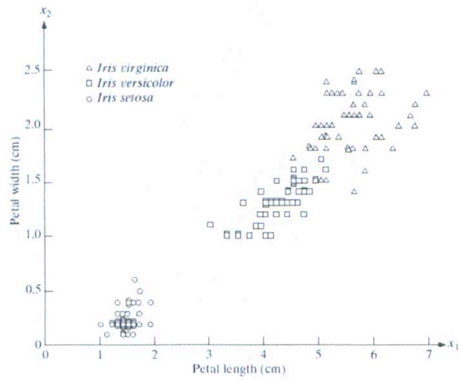
FIGURE 12.1 Three types of iris flowers described by two measurements.



This shows a clearly defined region
for Iris setosa based upon petal length & width

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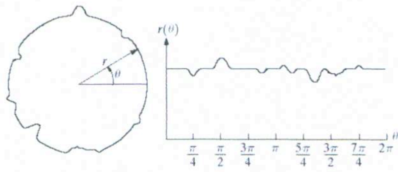

a b
FIGURE 12.2 A noisy object and its corresponding signature.

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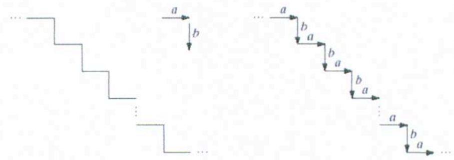

a b

FIGURE 12.3 (a) Staircase structure. (b) Structure coded in terms of the primitives *a* and *b* to yield the string description . . . *ababab* . . . .

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FIGURE 12.4 Satellite image of a heavily built downtown area (Washington, D.C.) and surrounding residential areas. (Courtesy of NASA.)

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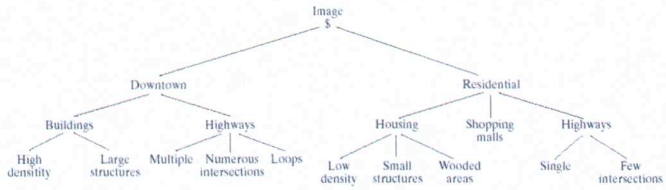
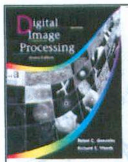


FIGURE 12.5 A tree description of the image in Fig. 12.4.



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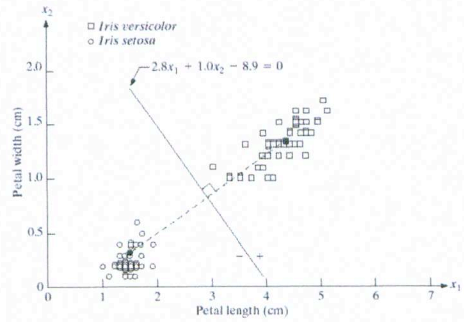


FIGURE 12.6 Decision boundary of minimum distance classifier for the classes of *Iris versicolor* and *Iris setosa*. The dark dot and square are the means.

minimum distance classifiers

prototype of each class is the mean vector of that ~~class~~

$$\underline{m}_j = \frac{1}{N_j} \sum_{x \in \omega_j} x_j \qquad j = 1, 2, \ldots, W$$

compute "closeness" using Euclidian distance

$$D_j(\underline{x}) = \| \underline{x} - \underline{m}_j \| \qquad j = 1, 2, \ldots, W$$

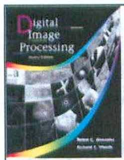
Assign $\underline{x}$ to class j if $D_j(\underline{x})$ is the smallest distance

Analytically,

$$d_{ij}(\underline{x}) = d_i(\underline{x}) - d_j(\underline{x}) = \underline{x}^T(\underline{m}_i - \underline{m}_j) - \frac{1}{2}(\underline{m}_i - \underline{m}_j)^T(\underline{m}_i + \underline{m}_j)$$

$$= 0$$

if $d_{ij}(\underline{x}) > 0$ assign to class j otherwise class i



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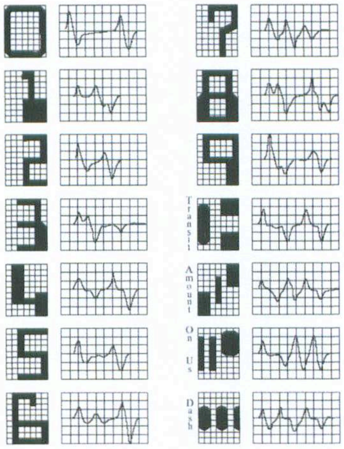
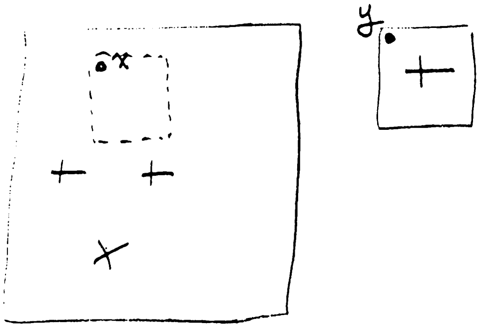


FIGURE 12.7 American Bankers Association E-13B font character set and corresponding waveforms.

Ballard and Brown

some notes on co-relation using a sub-image.



range of $\underline{x}$

how to find objects of interest in image?

$$d^2(\underline{y}) = \sum_{\underline{x}} \left[f(\underline{x}) - t(\underline{x}-\underline{y})\right]^2$$

define an Euclidian distance d

over the image

image function

slide template to location $\underline{y}$

if template matches $d=0$
otherwise $d>0$

$$= \sum_{\underline{x}} \Big[\underbrace{f^2(\underline{x})}_{\text{can be nearly constant depending upon spatial uniformity of image}} - \underbrace{2f(\underline{x})t(\underline{x}-\underline{y})}_{\text{cross-correlation function } \phi_{ft}(\underline{y})} + t^2(\underline{x}-\underline{y})\Big]$$

constant term, the energy of the template

cross-correlation function (looks like a convolution) and can be treated as a filter

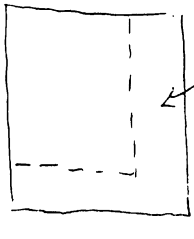
$\phi_{ft}(\underline{y})$

this is a correlation or dot product which is maximized when $t(x-y)$ matches $f(x)$.

two cases of template matching (correlation) – periodic (template wraps around image)

aperiodic (no wraparound can have various amounts of overlap)

to eliminate false responses normalize



can't correlate in this region for a periodic correlation

Normalization of correlation to prevent false errors:

Problems can occur due to intense noise in the image.

Example:

1	1	1
1	1	1
1	1	1

template

1	1	0	0	0
1	1	1	0	0
1	0	1	0	0
0	0	0	0	0
0	0	0	0	8

Correlation

correct best response

error due to large noise

7	4	2	x	x
5	3	2	x	x
2	1	9	x	x
x	x	x	x	x
x	x	x	x	x

x = undefined

To prevent such errors we must normalize according to the statistics of each ~~image~~ sub-image.

$f_1(\underline{x})$ $\quad$ $f_2(\underline{x})$ $\quad$ images to be matched

g_1 $\quad$ g_2 $\quad$ patches to be matched. Typically g_1 is all of f_1; g_1 is the patch of f_1 that is covered by the displaced patch g_2.

$$\sigma(g_1) = \sqrt{E(g_1^2) - E^2(g_1)}$$

$$\sigma(g_2) = \sqrt{E(g_2)^2 - E^2(g_2)}$$

standard deviations. É is the normal expectation operator,

Normalized correlation

$$\underline{N}(\underline{y}) = \frac{E(g_1 g_2) - E(g_1)\,E(g_2)}{\sigma(g_1)\,\sigma(g_2)}$$

← this is some sort of cross correlation

← normalized by the variances

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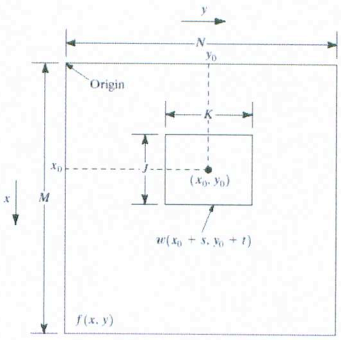

FIGURE 12.8 Arrangement for obtaining the correlation of f and w at point (x_0, y_0).

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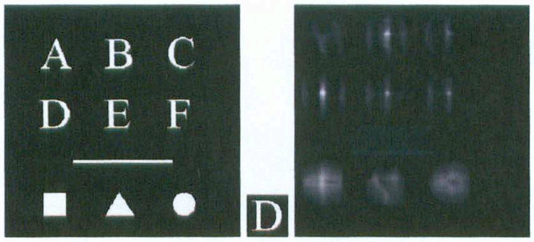

a b c

FIGURE 12.9 (a) Image. (b) Subimage. (c) Correlation coefficient of (a) and (b). Note that the highest (brighter) point in (c) occurs when subimage (b) is coincident with the letter "D" in (a).

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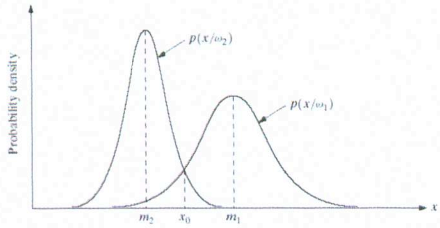
FIGURE 12.10 Probability density functions for two 1-D pattern classes. The point x_0 shown is the decision boundary if the two classes are equally likely to occur.



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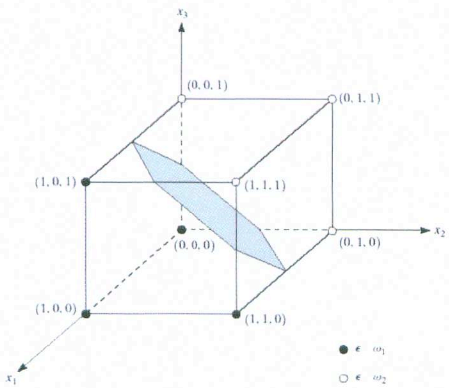

FIGURE 12.11 Two simple pattern classes and their Bayes decision boundary (shown shaded).