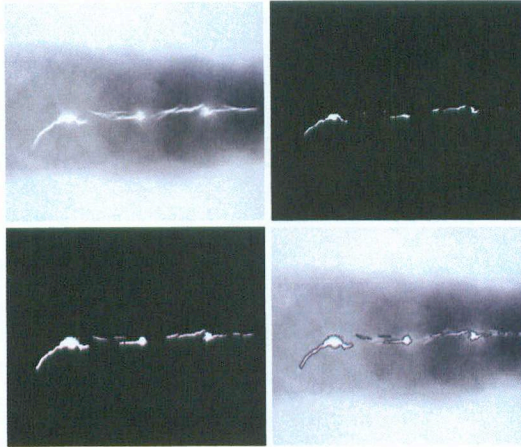


# Chapter 10 Image Segmentation

a b  
c d

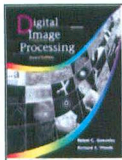
**FIGURE 10.40**  
(a) Image showing defective welds. (b) Seed points. (c) Result of region growing. (d) Boundaries of segmented defective welds (in black). (Original image courtesy of X-TEK Systems Ltd.).



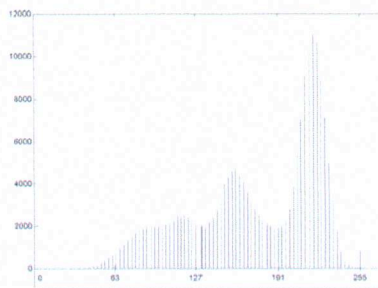
select as seed points  
all points with  
intensity = 255

- to be added to region
1. seed(255) —  $p(x,y) < 64$
  2. 8-connected to region

boundaries  
superimposed on  
original image

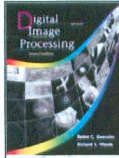


## Chapter 10 Image Segmentation



**FIGURE 10.41**  
Histogram of  
Fig. 10.40(a).

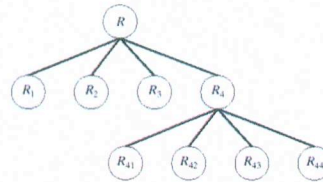
Histogram of weld image  
could not be segmented without using connectivity



## Chapter 10 Image Segmentation

a b  
**FIGURE 10.42**  
(a) Partitioned  
image.  
(b) Corresponding  
quadtrees.

|       |          |          |
|-------|----------|----------|
| $R_1$ | $R_2$    |          |
| $R_3$ | $R_{41}$ | $R_{42}$ |
|       | $R_{43}$ | $R_{44}$ |



split-merge

split image into quadrants

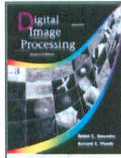
for each quadrant which is not homogeneous

split into sub-quadrants

repeat until every quadrant is homogeneous

Merge any adjacent regions which are similar

repeat split and merge until there are no changes.



## Chapter 10 Image Segmentation

a b c  
**FIGURE 10.43**  
(a) Original image. (b) Result of split and merge procedure. (c) Result of thresholding (a).



original image

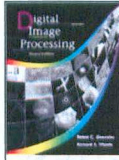
result of  
split-merge

thresholded  
result of split-merge

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Use mean and standard deviation

(Quadrant) Region is uniform if  $|z_j - m_i| \leq 2\sigma_i$   
set all pixels in region  $i$  to  $m_i$



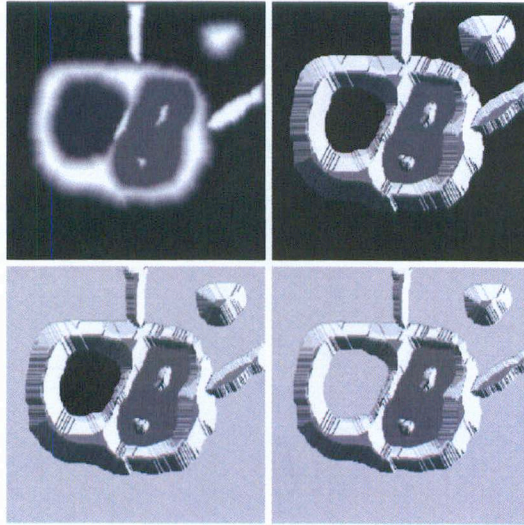
## Chapter 10 Image Segmentation

a b  
c d

**FIGURE 10.44**  
(a) Original image.  
(b) Topographic view.  
(c)-(d) Two stages of flooding.

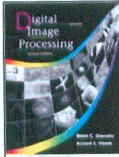
original  
gray scale  
image

punch holes  
in bottom of  
each region  
minimum

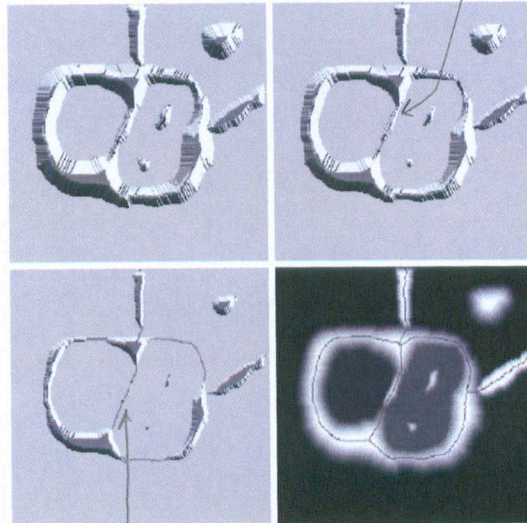


topographic view





# Chapter 10 Image Segmentation

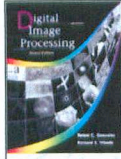


start building dam

e f  
g h  
**FIGURE 10.44**  
(Continued)  
(e) Result of further flooding.  
(f) Beginning of merging of water from two catchment basins (a short dam was built between them). (g) Longer dams (h) Final watershed (segmentation) lines. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

final watershed

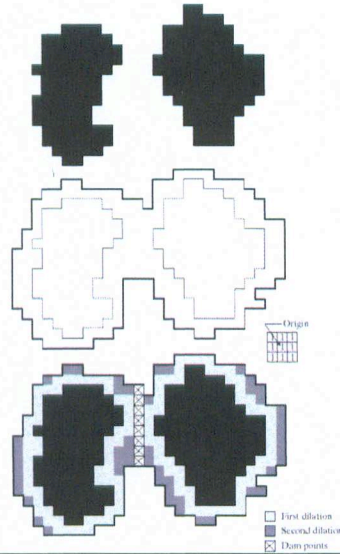
longer dams



## Chapter 10 Image Segmentation

a  
b  
c  
d

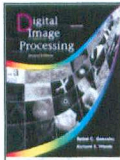
**FIGURE 10.45** (a) Two partially flooded catchment basins at stage  $n - 1$  of flooding. (b) Flooding at stage  $n$ , showing that water has spilled between basins (for clarity, water is shown in white rather than black). (c) Structuring element used for dilation. (d) Result of dilation and dam construction.



start at lowest points

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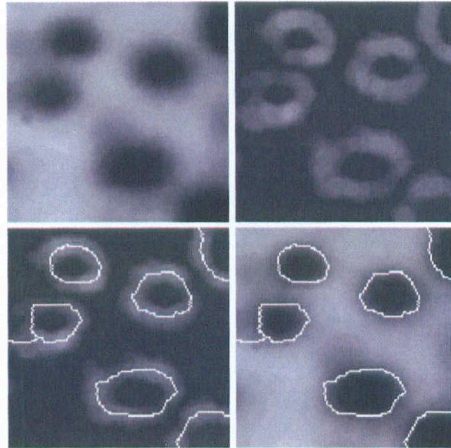
dam construction occurs when dilations overlap.



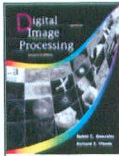
## Chapter 10 Image Segmentation

a b  
c d

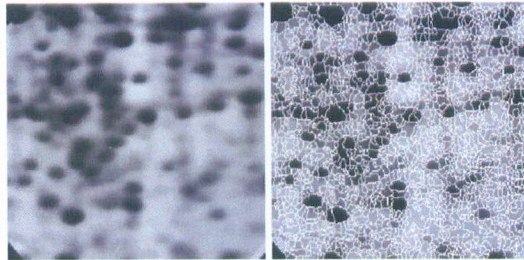
**FIGURE 10.46**  
(a) Image of blobs. (b) Image gradient.  
(c) Watershed lines.  
(d) Watershed lines superimposed on original image.  
(Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)



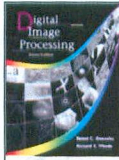




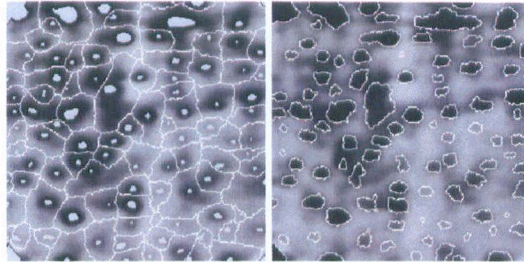
## Chapter 10 Image Segmentation



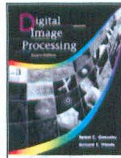
a b  
**FIGURE 10.47**  
(a) Electrophoresis image. (b) Result of applying the watershed segmentation algorithm to the gradient image. Oversegmentation is evident. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)



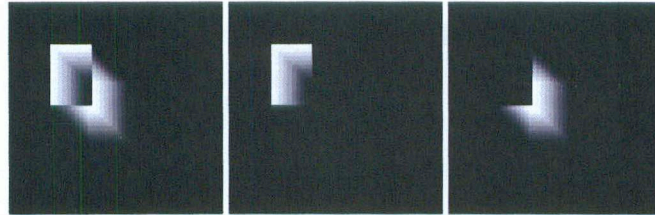
## Chapter 10 Image Segmentation



a b  
**FIGURE 10.48**  
(a) Image showing internal markers (light gray regions) and external markers (watershed lines).  
(b) Result of segmentation. Note the improvement over Fig. 10.47(b).  
(Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)



## Chapter 10 Image Segmentation



a b c  
**FIGURE 10.49** ADIs of a rectangular object moving in a southeasterly direction. (a) Absolute ADI. (b) Positive ADI. (c) Negative ADI.

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ADI - accumulative difference image

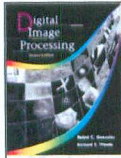
$$A_k(x, y) = \begin{cases} A_{k-1}(x, y) + 1 & \text{if } |R(x, y) - f(x, y, t_k)| > T \\ A_{k-1}(x, y) & \text{otherwise} \end{cases}$$

increase the accumulator if the absolute difference between the reference  $[R(x, y) \text{ is usually } f(x, y, t_0)]$  image and each successive image  $f(x, y, k) = f(x, y, t_k)$

You can also do positive and negative accumulator images

$$P_k(x, y) = \begin{cases} P_{k-1}(x, y) + 1 & \text{if } [R(x, y) - f(x, y, t_k)] > T \\ P_{k-1}(x, y) & \text{otherwise} \end{cases}$$

$$N_k(x, y) = \begin{cases} N_{k-1}(x, y) + 1 & \text{if } [R(x, y) - f(x, y, t_k)] < -T \\ N_{k-1}(x, y) & \text{otherwise} \end{cases}$$

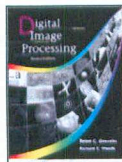


## Chapter 10 Image Segmentation



a b c

**FIGURE 10.50** Building a static reference image. (a) and (b) Two frames in a sequence. (c) Eastbound automobile subtracted from (a) and the background restored from the corresponding area in (b). (Jain and Jain.)



## Chapter 10 Image Segmentation

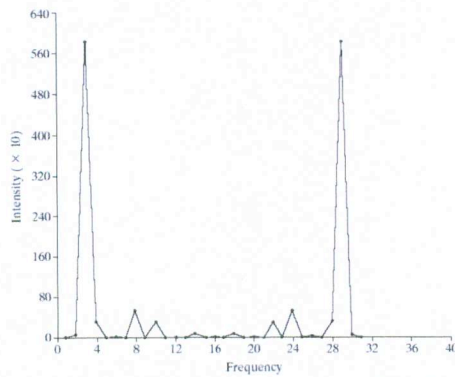


FIGURE 10.53 Spectrum of Eq. (10.6-8) showing a peak at  $u_1 = 3$ . (Rajala, Riddle, and Snyder.)

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↑  
peak at  $u_1 = 3$

$$\therefore v_1 = \frac{u_1}{a_1} = \frac{3}{6} = 0.5 \text{ pixel/frame}$$

Assume images are  $M \times N$

x-axis projection 
$$g_x(t, a_1) = \sum_{x=0}^{m-1} \sum_{y=0}^{N-1} f(x, y, t) e^{j2\pi a_1 x \Delta t} \quad t = 0, 1, \dots, K-1$$

y-axis projection 
$$g_y(t, a_2) = \sum_{x=0}^{N-1} \sum_{y=0}^{m-1} f(x, y, t) e^{j2\pi a_2 y \Delta t} \quad t = 0, 1, \dots, K-1$$

Fourier transform these projections

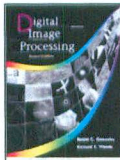
$$G_x(u_1, a_1) = \frac{1}{K} \sum_{t=0}^{K-1} g_x(t, a_1) e^{-j2\pi \frac{u_1 t}{K}} \quad u_1 = 0, 1, \dots, K-1$$

$$G_y(u_2, a_2) = \frac{1}{K} \sum_{t=0}^{K-1} g_y(t, a_2) e^{-j2\pi \frac{u_2 t}{K}} \quad u_2 = 0, 1, \dots, K-1$$

peak at frequency  $u_1 = a_1 v_1 \leftarrow$  x-component velocity

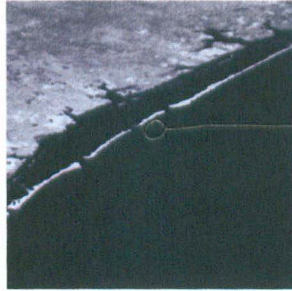
peak at frequency  $u_2 = a_2 v_2 \leftarrow$  y-component velocity





## Chapter 10 Image Segmentation

FIGURE 10.51  
LANDSAT  
frame. (Coward,  
Snyder, and  
Ruedger.)



Somewhere near here.

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There is actually a small moving object with a 9 pixel Gaussian distribution moving with  $v_x = 0.5$  pixel/frame and  $v_y = 1$  pixel/frame.

Basic concept: • single 1-pixel object moving, background = 0 at  $t=0$   
project onto x-axis, i.e. sum columns  
multiply components of array by

$$e^{j2\pi a_1 x \Delta t}$$

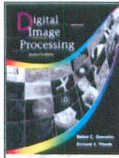
$x = 0, 1, 2, \dots$

↑  
↑  
time interval between planes  
as positive integer

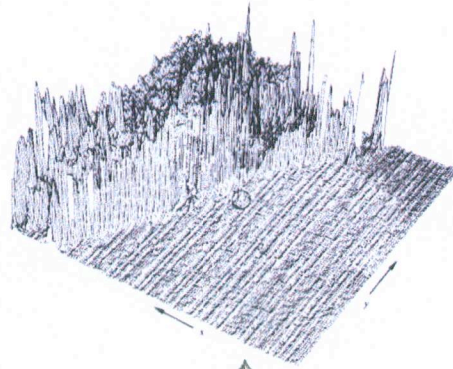
- at  $t=1$  do same thing except object has moved to  $x'+1$
- if constant velocity result is

$$e^{j2\pi a_1 (x'+t)\Delta t} = \cos[2\pi a_1 (x'+t)\Delta t] + j\sin[2\pi a_1 (x'+t)\Delta t]$$

i.e. projections of moving objects give single frequency sinusoids (for constant velocity)



## Chapter 10 Image Segmentation

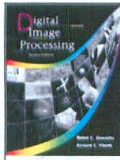


**FIGURE 10.52**  
Intensity plot of  
the image in  
Fig. 10.51, with  
the target circled.  
(Rajala, Riddle,  
and Snyder.)

↑  
moving target is in circle

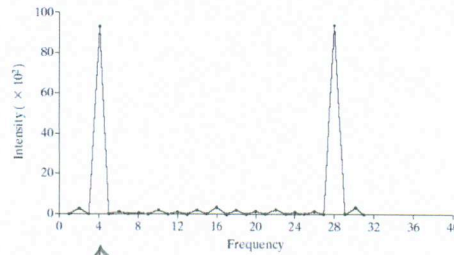
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intensity plot of image



## Chapter 10 Image Segmentation

**FIGURE 10.54**  
Spectrum of  
Eq. (10.6-9)  
showing a peak at  
 $u_2 = 4$ . (Rajala,  
Riddle, and  
Snyder.)



$$a_2 = 4$$

$$u_2 = 4$$

$$\therefore v_2 = \frac{u_2}{a_2} = \frac{4}{4} = 1 \text{ pixel/frame.}$$

## Representing a region:

- external representation - boundary
  - the boundary can be described by features such as length, orientation of the straight line connecting its extreme points, and the number of concavities in the boundary
  - chosen when the focus is on shape
- internal representation - pixels comprising the region
  - color and texture
  - chosen when the primary focus is on regional properties

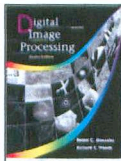
want features to be insensitive to size, translation and rotation

## Representing a region:

- external representation - boundary
  - the boundary can be described by features such as length, orientation of the straight line connecting its extreme points, and the number of concavities in the boundary
  - chosen when the focus is on shape
- internal representation - pixels comprising the region
  - color and texture
  - chosen when the primary focus is on regional properties

want features to be insensitive to size, translation and rotation



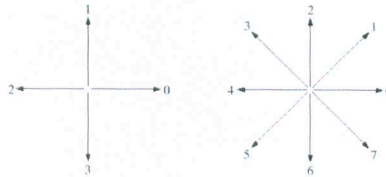


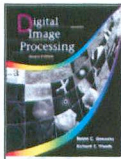
## Chapter 11 Representation & Description

a b

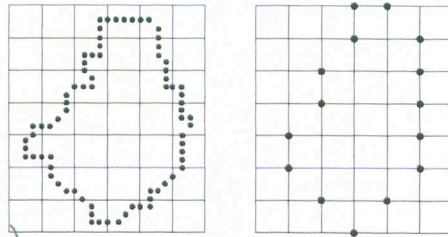
**FIGURE 11.1**

Direction numbers for (a) 4-directional chain code, and (b) 8-directional chain code.





## Chapter 11 Representation & Description

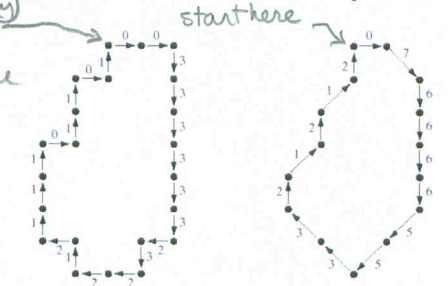


a b  
c d  
**FIGURE 11.2**  
(a) Digital boundary with resampling grid superimposed.  
(b) Result of resampling.  
(c) 4-directional chain code.  
(d) 8-directional chain code.

don't want to use original pixels since code would be too long and subject to noise  
- resample on a larger grid

start here (arbitrary)

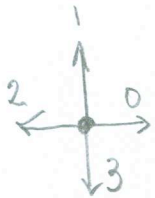
4-connected code  
00333...



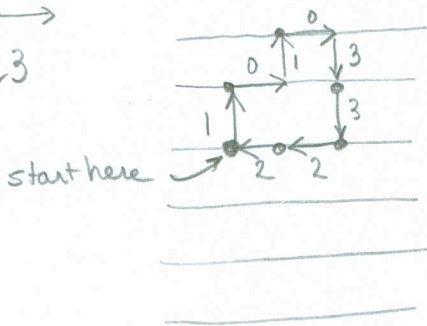
8-connected code -  
07666...

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chain codes represent a boundary as a connected sequence of straight-line segments of specified length and direction



Chain codes can be normalized for rotation by using first difference

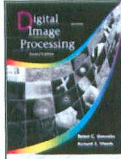


10103322 chain code representation  
first difference normalizes for rotation  
go in counter clockwise direction, count # of  $\frac{\pi}{2}$  (or  $\frac{\pi}{4}$ )

- 1 → 0 3 turns
- 0 → 1 1
- 1 → 0 3
- 0 → 3 3
- 3 → 3 0
- 3 → 2 3
- 2 → 2 0
- 2 → 1 3 put at beginning

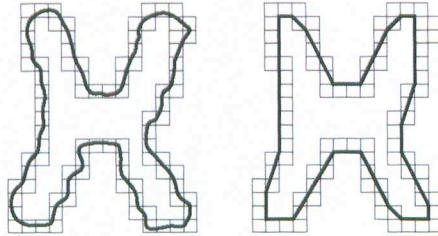
3 3 1 3 3 0 3 0

only rotation invariant if the digitizations are insensitive to rotation



## Chapter 11 Representation & Description

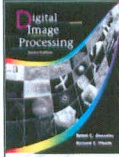
a b  
**FIGURE 11.3**  
(a) Object boundary enclosed by cells.  
(b) Minimum perimeter polygon.



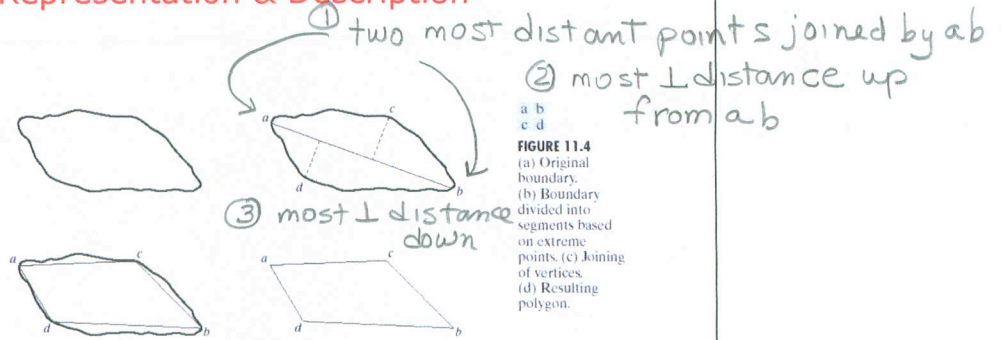
grid of cells which  
"walls" in boundary

let boundary be a  
"rubber band"

MINIMUM PERIMETER POLYGONS

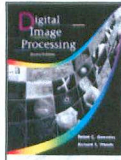


# Chapter 11 Representation & Description



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boundary segment splitting – subdivide a segment successively until a specified criterion is satisfied

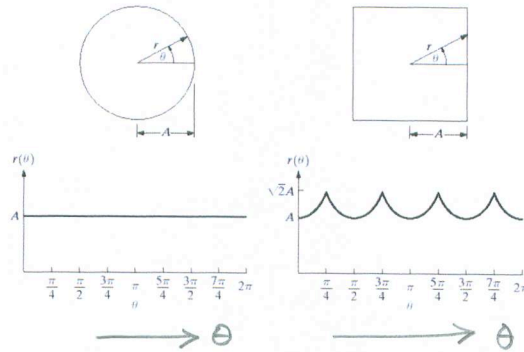


# Chapter 11 Representation & Description

a-b

**FIGURE 11.5**  
Distance-versus-angle signatures. In (a)  $r(\theta)$  is constant. In (b), the signature consists of repetitions of the pattern

$r(\theta) = A \sec \theta$  for  $0 \leq \theta \leq \pi/4$  and  $r(\theta) = A \csc \theta$  for  $\pi/4 < \theta \leq \pi/2$ .



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signature - 1-D representation of a boundary

These are invariant to translation but sensitive to rotation and scale (size)

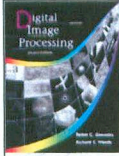
Methods of selecting starting point can make signatures independent of rotation

- select point farthest from centroid
- select point farthest from centroid along eigenaxis
- use a chain code

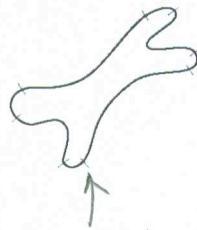
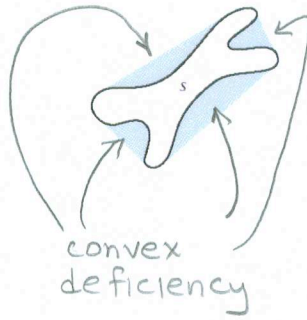
Many other types of signatures

$\psi$ -s (i.e. plot the tangent)





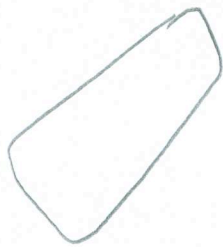
# Chapter 11 Representation & Description



a b  
**FIGURE 11.6**  
(a) A region,  $S$ , and its convex deficiency (shaded).  
(b) Partitioned boundary.

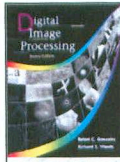
code boundary by points where boundary passes in and out of a convex deficiency

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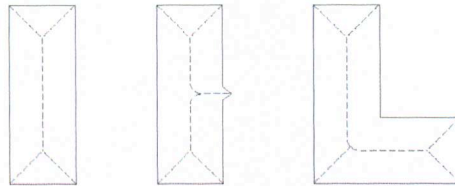


$H$  is the convex hull of  $S$

$H - S$  is the convex deficiency.



## Chapter 11 Representation & Description



a b c  
**FIGURE 11.7**  
Medial axes  
(dashed) of three  
simple regions.

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reduce the structure of a shape to a skeleton

morphology will not necessarily keep a skeleton connected

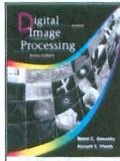
median axis transformation (MAT) algorithm

MAT of a region  $R$  with border  $B$

for each point  $p$  in  $R$  find its closest neighbor on  $B$

if  $p$  has more than one "closest" neighbor it belongs  
to the medial axis (skeleton) of  $B$

closest is defined using Euclidian distance



## Chapter 11 Representation & Description

|       |       |       |
|-------|-------|-------|
| $p_0$ | $p_2$ | $p_3$ |
| $p_6$ | $p_1$ | $p_4$ |
| $p_7$ | $p_5$ | $p_8$ |

**FIGURE 11.8**  
Neighborhood arrangement used by the thinning algorithm.

|   |       |   |
|---|-------|---|
| 0 | 0     | 1 |
| 1 | $p_1$ | 0 |
| 1 | 0     | 1 |

$$N(p_1) = 4$$

$$T(p_1) = 3$$

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Algorithm for thinning binary images.

contour point is any pixel with value 1 and at least one 8-neighbor valued 0

Step 1: mark for deletion any contour point  $p_1$  which has

don't delete if end point or inside region

(a)  $2 \leq N(p_1) \leq 6$

# of non-zero neighbors is between 2 and 6

prevents breaking connected lines

(b)  $T(p_1) = 1$

# of 0-1 transitions in sequence  $p_2 p_3 p_4 p_5 p_6 p_7 p_8 p_9 p_2$

east or south boundary pt.

(c)  $p_2 p_4 p_6 = 0$

northwest corner

(d)  $p_4 p_6 p_8 = 0$

} says that either  $p_4$  and  $p_6 = 0$  or  $p_2$  and  $p_8 = 0$

Step 2: mark for deletion if

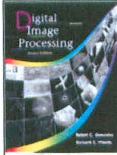
(a)  $2 \leq N(p_1) \leq 6$

(b)  $T(p_1) = 1$

(c')  $p_2 p_4 p_8 = 0$  } says that  $p_2$  and  $p_4 = 0$

(d')  $p_4 p_6 p_8 = 0$  } or  $p_6$  and  $p_8 = 0$

Iterate by applying step 1 to all border points; deleting marked points; apply step 2 to all remaining border points; and deleting marked points. Repeat until no further points deleted. 8



## Chapter 11 Representation & Description

**FIGURE 11.9**  
Illustration of conditions (a) and (b) in Eq. (11.1-1). In this case  $N(p_1) = 4$  and  $T(p_1) = 3$ .

|   |       |   |
|---|-------|---|
| 0 | 0     | 1 |
| 1 | $p_1$ | 0 |
| 1 | 0     | 1 |

$$N(p_1) = 4$$

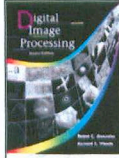
$$T(p_1) = 3$$

$$P_2 P_4 P_6 = 0$$

$$P_4 P_6 P_8 = 0$$

$$P_2 P_4 P_8 = 0$$

$$P_2 P_6 P_8 = 0$$



## Chapter 11 Representation & Description



**FIGURE 11.10**  
Human leg bone  
and skeleton of  
the region shown  
superimposed.