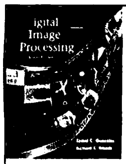

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FIGURE 10.1 A general 3 × 3 mask.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

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We can regard a 3x3 mask as either a filter mask or a correlation mask of sorts.

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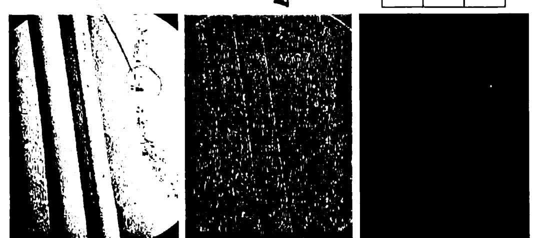
−1	−1	−1
−1	8	−1
−1	−1	−1

single black point we want to find

point is magnified



result after thresholding

a
b c d

FIGURE 10.2 (a) Point detection mask. (b) X-ray image of a turbine blade with a porosity. (c) Result of point detection. (d) Result of using Eq. (10.1-2). (Original image courtesy of X-TEK Systems Ltd.)

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Consider (a) as a mask for a point, i.e., correlator,

$$R = \sum_{i=1}^{9} w_i z_i$$

If $|R| \geq T$ we have found a point

The mask here is also that of a Laplacian operator but we can also consider it as a point correlator.

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FIGURE 10.3 Line masks.

−1	−1	−1	−1	−1	2	−1	2	−1	2	−1	−1
2	2	2	−1	2	−1	−1	2	−1	−1	2	−1
−1	−1	−1	2	−1	−1	−1	2	−1	−1	−1	2
	Horizontal			+45°			Vertical			−45°	

These are examples of correlation (also derivative) masks for single pixel width lines

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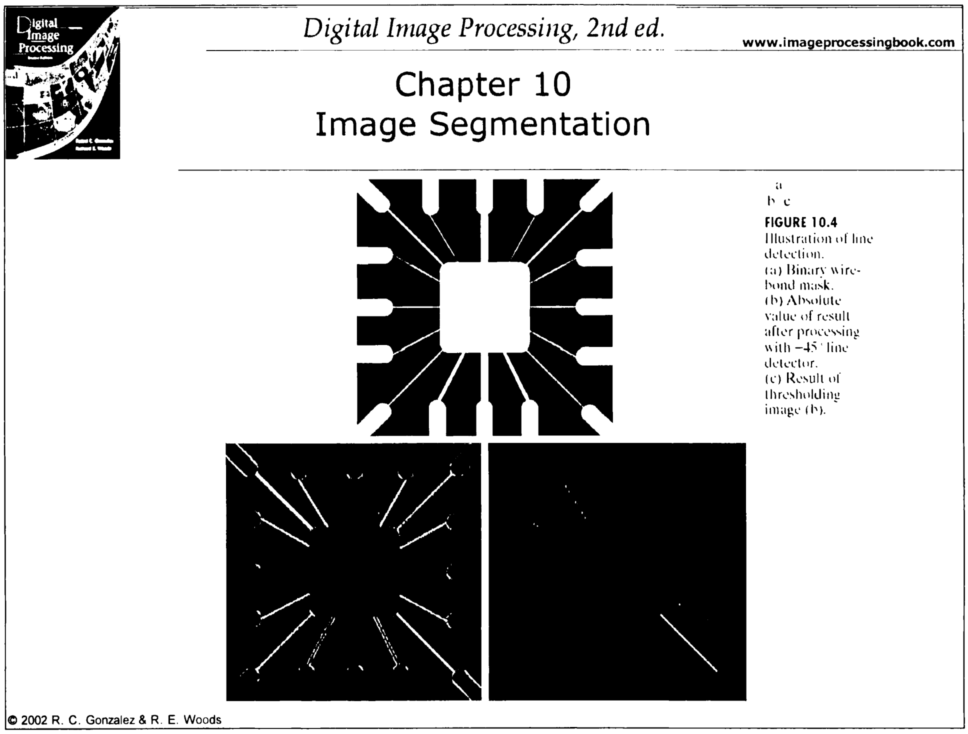


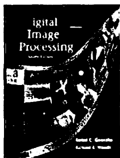
(b) shows the result of applying a −45° line detector mask on the wire bond mask for an integrated circuit.

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

(c) shows the result of thresholding (b).

A good rule of thumb for these masks is to use $T =$ maximum pixel value in original image (a)

The isolated points in (c) could be eliminated using morphological processing.



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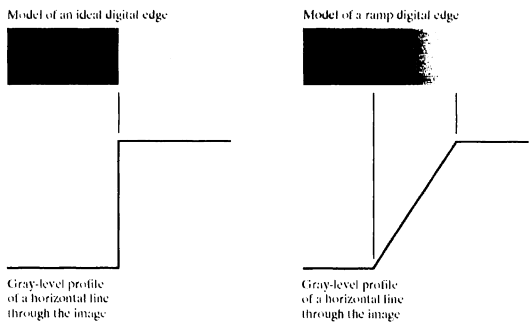

FIGURE 10.5 (a) Model of an ideal digital edge. (b) Model of a ramp edge. The slope of the ramp is proportional to the degree of blurring in the edge.

sharp edge blurred edge

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models of gray scale edges.
The slope is proportional to the amount of blurring.

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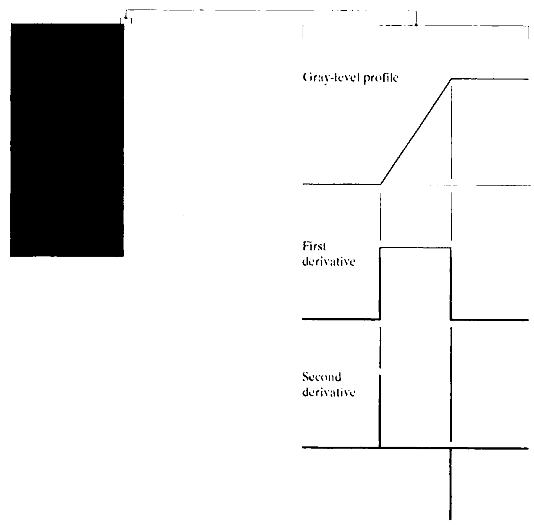

a b
FIGURE 10.6 (a) Two regions separated by a vertical edge. (b) Detail near the edge, showing a gray-level profile, and the first and second derivatives of the profile.

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"blurred" gray scale edge with a gray profile

the magnitude of the first derivative can be used to detect the presence of an edge

the second derivative's zero crossing property is also appropriate for detecting the presence of an edge, i.e., a line drawn between the two extrema must cross through zero near the center of the edge

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$\sigma = 0$

$\sigma = 0.1$

$\sigma = 1$

$\sigma = 10$

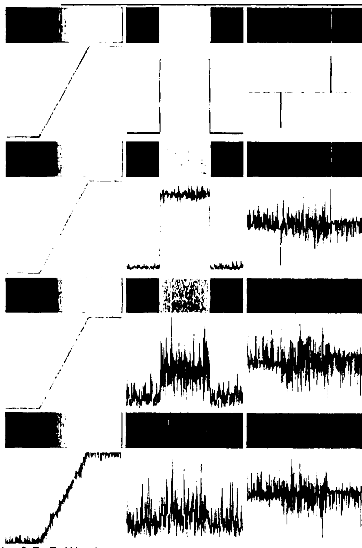

FIGURE 10.7 First column: images and gray-level profiles of a ramp edge corrupted by random Gaussian noise of mean 0 and σ = 0.0, 0.1, 1.0, and 10.0, respectively. Second column: first-derivative images and gray-level profiles. Third column: second-derivative images and gray-level profiles.

a b c d

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gray level edges — first derivative — second derivative

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a
b c
d e
f g

FIGURE 10.8 A 3 × 3 region of an image (the z's are gray-level values) and various masks used to compute the gradient at point labeled z_5.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

-1	0
0	1

0	-1
1	0

Roberts

most commonly used gradient masks

-1	-1	-1
0	0	0
1	1	1

-1	0	1
-1	0	1
-1	0	1

Prewitt

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

Sobel

superior noise characteristics to Prewitt

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Gradient

$$\underline{\nabla f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\nabla f = |\underline{\nabla f}| = \sqrt{G_x^2 + G_y^2}$$

$$\alpha(x,y) = \tan^{-1}\left(\frac{G_y}{G_x}\right)$$

The above operators are attempts to implement first-order partial derivatives

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0	1	1
−1	0	1
−1	−1	0

−1	−1	0
−1	0	1
0	1	1

Prewitt

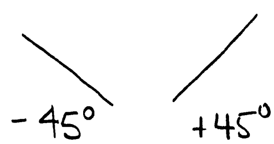
0	1	2
−1	0	1
−2	−1	0

−2	−1	0
−1	0	1
0	1	2

Sobel

a b
c d

FIGURE 10.9 Prewitt and Sobel masks for detecting diagonal edges.



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modified Prewitt & Sobel masks for detecting diagonal edges.

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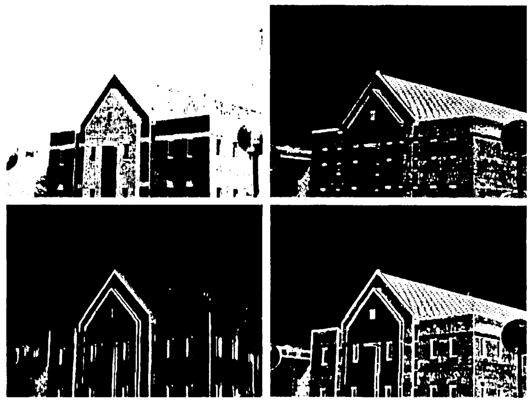
1200×1600 pixel
original image



a b
c d

FIGURE 10.10
(a) Original image. (b) $|G_x|$. component of the gradient in the x-direction. (c) $|G_y|$. component in the y-direction. (d) Gradient image. $|G_x| + |G_y|$.

$|G_x|$

$|G_y|$

gradient approximation
$|G_x| + |G_y|$

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No sure which mask was used – not identified in text

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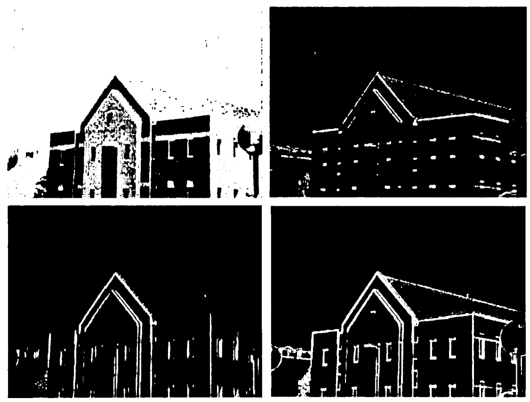
1200x1600

5x5 →
avg. filter



a b
c d

FIGURE 10.11 Same sequence as in Fig. 10.10, but with the original image smoothed with a 5 × 5 averaging filter.

$|G_x|$

$|G_y|$

$|G_x| + |G_y|$ *

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* Averaging causes all the edges to be weaker, but cleaner. Removed lines of bricks.

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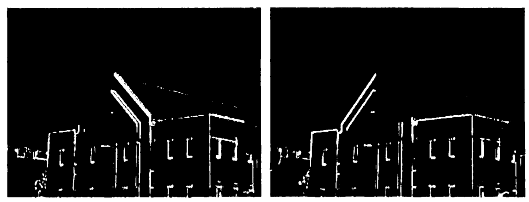

a b

FIGURE 10.12 Diagonal edge detection. (a) Result of using the mask in Fig. 10.9(c). (b) Result of using the mask in Fig. 10.9(d). The input in both cases was Fig. 10.11(a).

-45° sobel $+45^\circ$ sobel

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Both can detect horizontal & vertical edges but with a weaker response than a horizontal or vertical operator.

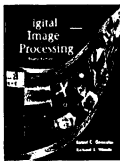

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FIGURE 10.13 Laplacian masks used to implement Eqs. (10.1-14) and (10.1-15), respectively.

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

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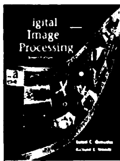
The Laplacian is a second-order derivative which is usually approximated by the above masks.



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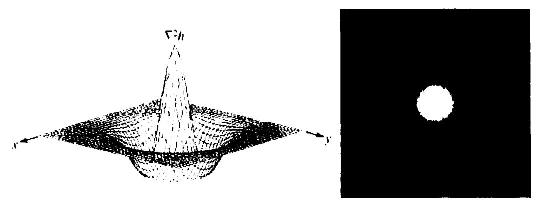
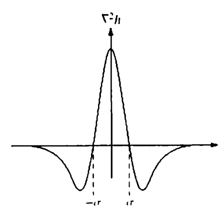
LoG sometimes called the mexican hat function





0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0

a b
c d

FIGURE 10.14 Laplacian of a Gaussian (LoG). (a) 3-D plot. (b) Image (black is negative, gray is the zero plane, and white is positive). (c) Cross section showing zero crossings. (d) 5×5 mask approximation to the shape of (a).

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The Laplacian is never used directly because of its strong noise sensitivity.

We usually use it with a Gaussian smoothing (low-pass) filter to minimize noise. $h(r) = -e^{-\frac{r^2}{2\sigma^2}}$

Combining these operators leads to the Laplacian of a Gaussian (LoG)

$$\nabla^2 h(r) = -\left[\frac{r^2-\sigma^2}{\sigma^4}\right] e^{-\frac{r^2}{2\sigma^2}}$$

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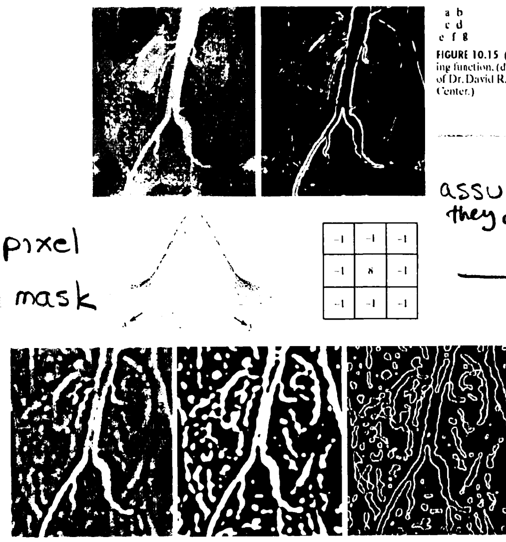

-1	-1	-1
-1	8	-1
-1	-1	-1

FIGURE 10.15 (a) Original image. (b) Sobel gradient (shown for comparison). (c) Spatial Gaussian smoothing function. (d) Laplacian mask. (e) LoG. (f) Thresholded LoG. (g) Zero crossings. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

— Sobel image

assume they did $\sqrt{S_x^2 + S_y^2}$ instead of $|S_x| + |S_y|$

27 x 27 pixel Gaussian smoothing mask

— gradient mask ∇^2

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gray scale LoG result

done as two masks or operations

more control and smaller masks this way

thresholded LoG image

gradient output is + and −

zero-crossings obtained from thresholded image

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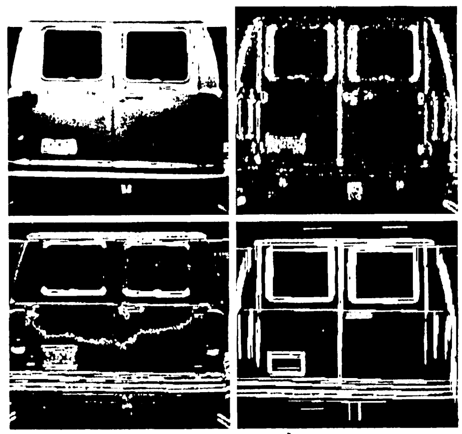
a b
c d

FIGURE 10.16 (a) Input image. (b) G_y component of the gradient. (c) G_x component of the gradient. (d) Result of edge linking. (Courtesy of Perceptics Corporation.)



$|G_x|$

$|G_y|$

simply linked if $\alpha = 15°$ and $|\nabla f| > 25$

detect license plate using 2:1 rectangle ratio

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edge linking – uses similarity of edge pixels to produce meaningful edges

in a neighborhood (usually 3×3 or 5×5)

Edge pixels are similar if neighboring pixels satisfy

magnitude of gradient $$|\nabla f(x,y) - \nabla f(x_0,y_0)| \le E$$

direction $$|\alpha(x,y) - \alpha(x_0,y_0)| < A$$

Remember the edge direction is perpendicular to ∇f.

A list of linked points must be maintained.

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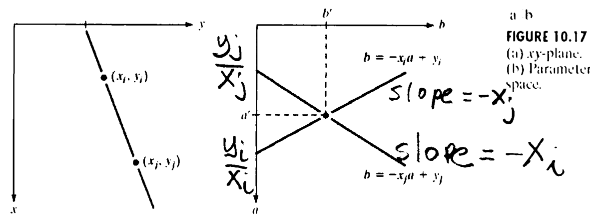

a b
FIGURE 10.17 (a) xy-plane. (b) Parameter space.

equation of line
$y_i = a x_i + b$

rewrite line as $b = -x_i a + y_i$

these come from image edge point

This is a line in a-b parameter space.
Intersections of lines from all image edge points locates lines,

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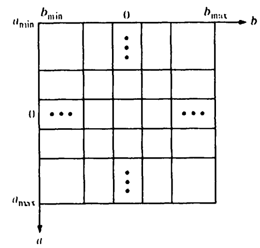

FIGURE 10.18 Subdivision of the parameter plane for use in the Hough transform.

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To compute the hough transform divide the parameter space into accumulator cells which span the expected ranges of a and b. Set all values to zero.

Enter each edge point incrementing all appropriate accumulator cells. Round-off a and b as appropriate.

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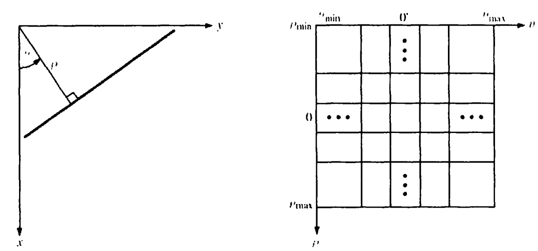

a b
FIGURE 10.19 (a) Normal representation of a line. (b) Subdivision of the $\rho\theta$-plane into cells.

accumulator array for $\rho - \theta$
note zero at center of each axis.

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A major problem with using lines of the form $y = ax + b$ is that the slope $\to \infty$ which exceeds the accumulator array.

Solution is to use the polar form of the line

$$\rho = x\cos\theta + y\sin\theta$$

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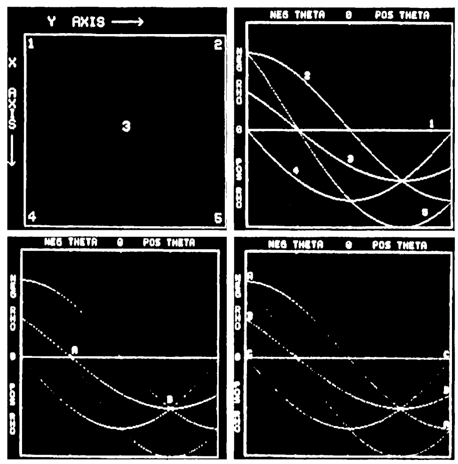
a b
c d

FIGURE 10.20 Illustration of the Hough transform. (Courtesy of Mr. D. R. Cate, Texas Instruments, Inc.)

original image



each edge point gives $x_i \cos\theta + y_i \sin\theta = \rho$ which a sinusoidal curve in $\rho\theta$ space

Note: point 1 is $\rho = 0$

Note how ρ and θ change signs across ρ-θ space

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points 1, 3, 5 intersect at A in ρ-θ space

points 2, 3, 4 intersect at B in ρ-θ space

Note: you can do Hough transforms based upon more complex and generalized functions

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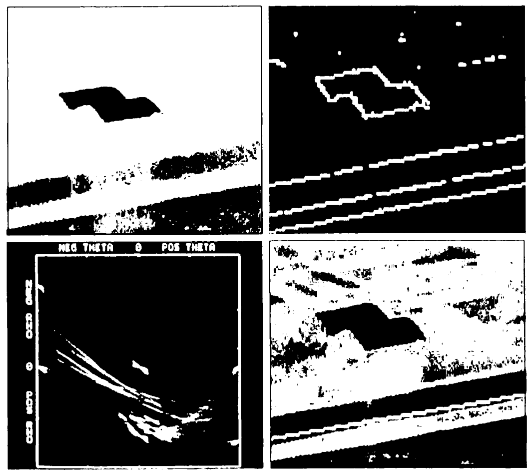

a b
c d

FIGURE 10.21 (a) Infrared image. (b) Thresholded gradient image. (c) Hough transform. (d) Linked pixels. (Courtesy of Mr. D. R. Cate, Texas Instruments, Inc.)

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(a) IR image of a runway and two hangers

(b) thresholded gradient image

(c) Hough transform of (b) using $\rho = x\cos\theta + y\sin\theta$

(d) Linked pixels from strongest points in (c)
No gaps in linked image.

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Graph searching

these are the centers of the pixels

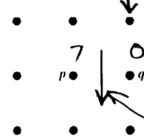

FIGURE 10.22 Edge element between pixels p and q.

goal nodes

each edge element has a cost

This is a cost function for the edge

$$c(p,q) = H - \underbrace{[f(p) - f(q)]}_{\text{gray scale image values}}$$

right − left

highest gray scale value in image

Note cost has a direction.

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let $f(p) = 7$

$f(q) = 0$

$H = 7$

Then $c(p,q) = 7 - [7 - 0] = 0$

cost is low traveling along edge from top to bottom

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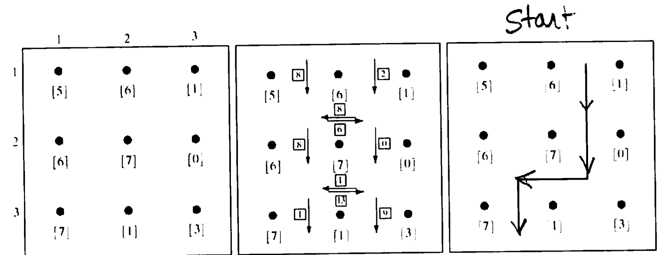
Start



stop

pick lowest starting cost and traverse lowest cost path to bottom

a b c

FIGURE 10.23 (a) A 3 × 3 image region. (b) Edge segments and their costs. (c) Edge corresponding to the lowest-cost path in the graph shown in Fig. 10.24.

edge segments and all computed costs

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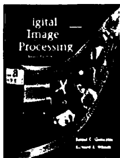

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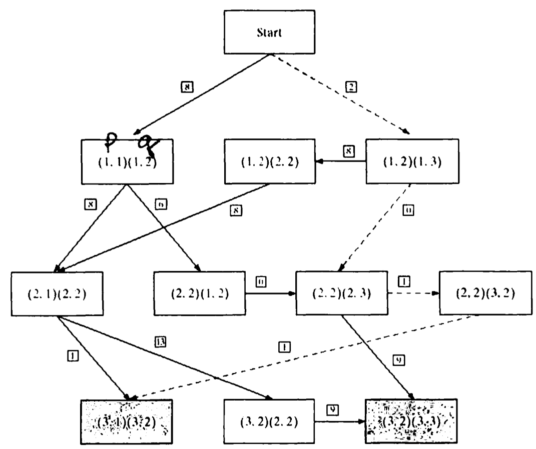

FIGURE 10.24 Graph for the image in Fig. 10.23(a). The lowest-cost path is shown dashed.

shaded blocks (nodes) are end pixels.

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graph of problem in previous figure shows all possible paths.
nodes correspond to edge pixels in figures
arcs are potential linked edges

lowest cost path shown in dashes

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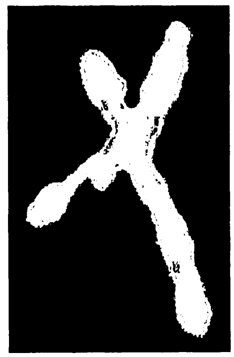

FIGURE 10.25 Image of noisy chromosome silhouette and edge boundary (in white) determined by graph search.

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improved algorithm will estimate the cost to the end as well

$$r(n) = \underbrace{g(n)}_{\text{lowest cost path found to } n} + \underbrace{h(n)}_{\text{estimated cost from } n \text{ to goal node using some heuristic}}$$

estimate of minimum cost path

heuristic used here was to simply use optimum path 5 levels down

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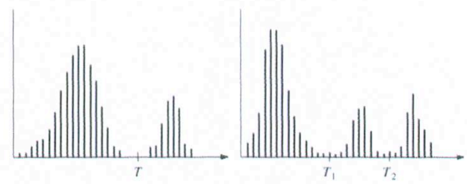

a b

FIGURE 10.26 (a) Gray-level histograms that can be partitioned by (a) a single threshold, and (b) multiple thresholds.

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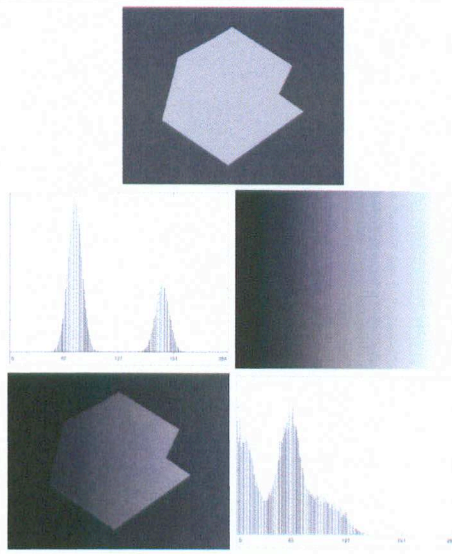

a
b c
d e

FIGURE 10.27 (a) Computer generated reflectance function. (b) Histogram of reflectance function. (c) Computer generated illumination function. (d) Product of (a) and (c). (e) Histogram of product image.

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(a) basic 2 gray level image that can be readily separated by a single threshold – reflectance function of object

(b) histogram of (a)

(c) illumination function $i(x,y)$

(d) actually seen image is product of illumination and reflection

(e) histogram of seen image. f

$$f(x,y) = i(x,y)\, r(x,y)$$

just as for homomorphic filter let

$$z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y) = i'(x,y) + r'(x,y)$$

Papoulis, if i' and x' are independent random variables their probability density functions (pdf's) will convolve (smear) to give z's pdf

If you know $i(x,y)$ such as in an industrial application you can compute $r(x,y) = \dfrac{f(x,y)}{i(x,y)}$ to get back a well behaved function

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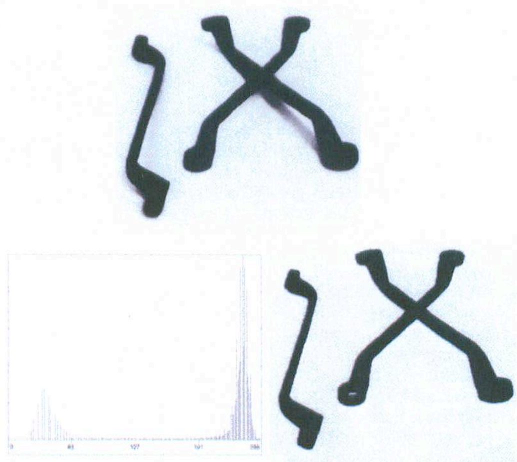

a
b c

FIGURE 10.28 (a) Original image. (b) Image histogram. (c) Result of global thresholding with T midway between the maximum and minimum gray levels.

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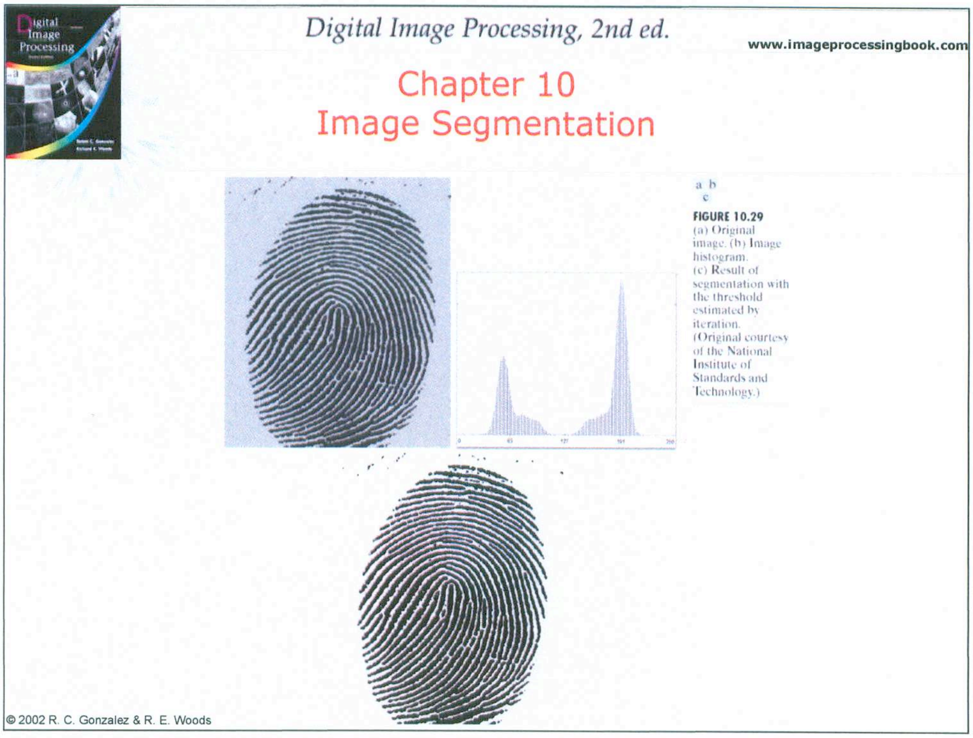
Simple example of global (single common) threshold



Automatic threshold calculation

1. select an initial estimate for T
2. segment image, i.e., threshold
3. compute the average gray scale value for pixels that were converted to 0. Call it μ_1
   compute the average gray scale value for pixels converted to 1. Call it μ_2
4. Compute new threshold $T = \frac{1}{2}(\mu_1 + \mu_2)$
5. Repeat until ΔThreshold less than some tolerance

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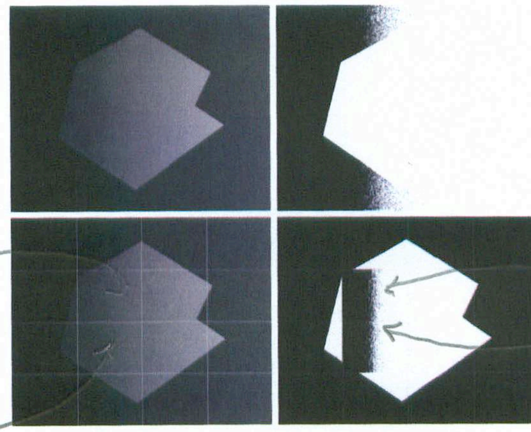
a b
c d

FIGURE 10.30 (a) Original image. (b) Result of global thresholding. (c) Image subdivided into individual subimages. (d) Result of adaptive thresholding.



these subimages did not threshold properly

did not threshold properly

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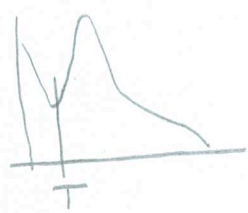
(a) image $f(x,y)$ from Fig 10.27
product of a reflectance and a non-uniform illumination

(b) result of single threshold
placed at valley of histogram
of f



(c) break image down into subimages
compute histogram and threshold for subimages
with $\sigma^2 > 100$, i.e., estimated to contain an edge
use initial T as $\dfrac{\text{gray scale}_{max} - \text{gray scale}_{min}}{2}$

subimages with $\sigma^2 < 100$ were all combined together
and then automatically thresholded.

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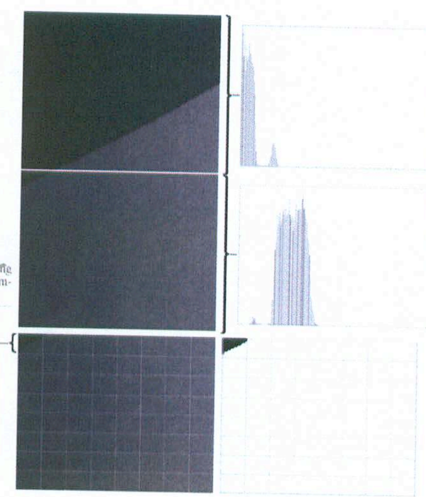
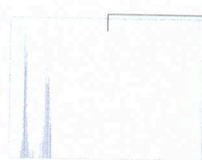
subimages



This one segmented well because it is bimodal

nearly unimodal histogram



a b c
e d f

FIGURE 10.31 (a) Properly and improperly segmented subimages from Fig. 10.30. (b)–(c) Corresponding histograms. (d) Further subdivision of the improperly segmented subimage. (e) Histogram of small subimage at top, left. (f) Result of adaptively segmenting (d).

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further subdivide and process each sub-sub-image in the same manner

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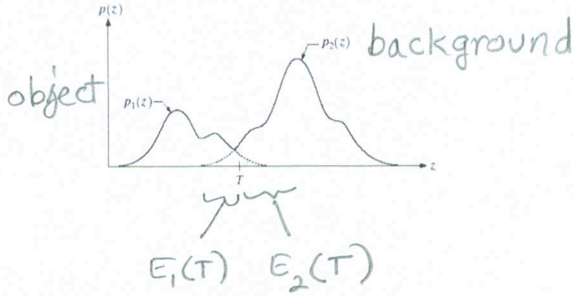

FIGURE 10.32 Gray-level probability density functions of two regions in an image.

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Compute error of wrong classification for given T

$$E(T) = P_2 E_1(T) + P_1 E_2(T)$$

probability of object pixel

probability of background pixel

$$E_1(T) = \int_{-\infty}^{T} p_2(z)\,dz \qquad E_2(T) = \int_{T}^{\infty} p_1(z)\,dz$$

minimize by $\dfrac{\partial E(T)}{\partial T} = 0$

use Gaussian probability density function

If $\sigma_1^2 = \sigma_2^2 = \sigma^2$

$$T = \frac{\mu_1 + \mu_2}{2} + \frac{\sigma^2}{\mu_1 - \mu_2} \ln\left(\frac{P_2}{P_1}\right)$$

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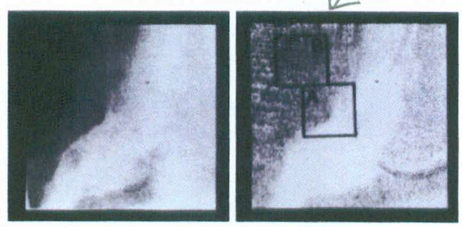
subimage B



subimage A

a b

FIGURE 10.33 A cardioangiogram before and after preprocessing. (Chow and Kaneko.)

X-rays of heart injected with a contrast agent

outline automatically boundary of left ventricle

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Original way of processing image.

1. take log of pixel to counter radioactive absorption which is exponential
2. subtract image of heart before injection to remove spinal column from image
3. average several images to eliminate noise

---

4. divide into 49 64x64 subimages with 50% overlap
5. A & B are ~~two~~ typical subimages

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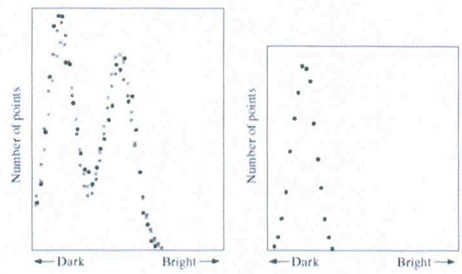

a b

FIGURE 10.34 Histograms (black dots) of (a) region *A* and (b) region *B* in Fig. 10.33(b). (Chow and Kaneko.)

B: unimodal → no boundary

A: bimodal → boundary

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Only process subimages which are bi-modal, i.e. $\sigma^2 >$ some threshold

For bimodal distributions

1. fit Gaussian density functions
2. minimize error function by optimum T

For unimodal distributions estimate thresholds by interpolating from neighboring bimodal subimages

Now interpolate a threshold T_{xy} for every point in image.

Finally, for every point $f(x,y) = \begin{cases} 1 & \text{if } f(x,y) \geq T_{xy} \\ 0 & \text{otherwise} \end{cases}$

Compute gradient of this binary image.

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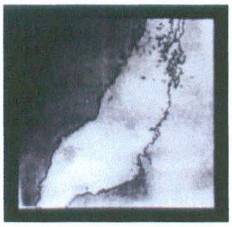

FIGURE 10.35 Cardioangiogram showing superimposed boundaries. (Chow and Kaneko.)

original image with gradient of binary image superimposed

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local thresholding

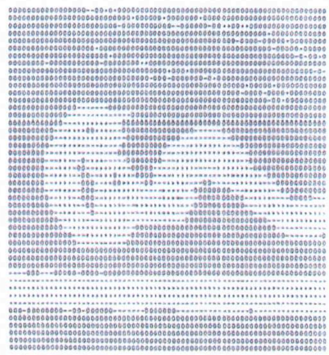

FIGURE 10.36 Image of a handwritten stroke coded by using Eq. (10.3-16). (Courtesy of IBM Corporation.)

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Three level image where

$$s(x,y) = \begin{cases} 0 & \text{if } \nabla f < T \\ + & \text{if } \nabla f \geq T \text{ and } \nabla^2 f \geq 0 \\ - & \text{if } \nabla f \geq T \text{ and } \nabla^2 f < 0 \end{cases}$$

not on an edge

pixels on dark side of edge

pixels on light side of edge

object (1) must be characterized by the following pattern

Assuming light background.

$(\ldots)(-,+)\quad(0 \text{ or } +)\quad(+,-),(\ldots)$

light to dark — dark to light

for a horizontal or vertical scan.

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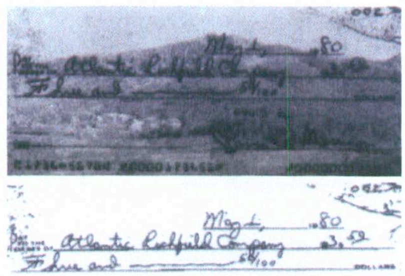
a
b

FIGURE 10.37 (a) Original image. (b) Image segmented by local thresholding. (Courtesy of IBM Corporation.)



using local thresholding

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pick T here

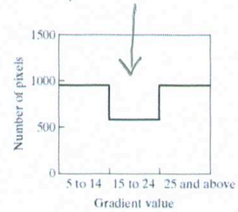

FIGURE 10.38 Histogram of pixels with gradients greater than 5. (Courtesy of IBM Corporation.)

Histogram of gradient values.

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This is <u>local</u> thresholding since gradient & Laplacian are calculated locally.

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a b c

FIGURE 10.39 (a) Original color image shown as a monochrome picture. (b) Segmentation of pixels with colors close to facial tones. (c) Segmentation of red components.

Thresholding for cluster corresponding to skin tones.

Thresholding for cluster corresponding to red.

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Multispectral thresholding

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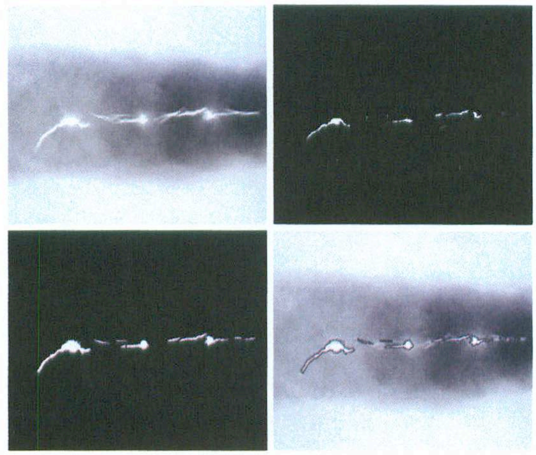
a b
c d

FIGURE 10.40 (a) Image showing defective welds. (b) Seed points. (c) Result of region growing. (d) Boundaries of segmented defective welds (in black). (Original image courtesy of X-TEK Systems, Ltd.).



select as seed points all points with intensity = 255

to be added to region

1. seed(255) − p(x,y) < 64
2. 8-connected to region

boundaries superimposed on original image

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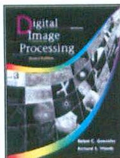

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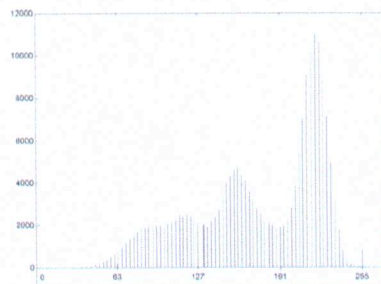

FIGURE 10.41 Histogram of Fig. 10.40(a).

Histogram of weld image
Could not be segmented without using connectivity

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