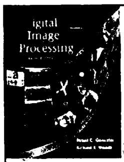

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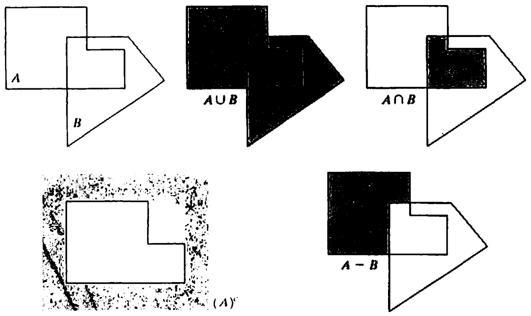

a b c
d e

FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

$$A^c = \{w \mid w \notin A\}$$

$$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$$

morphology — mathematical morphology uses set theory to extract and process image components such as boundaries, skeletons, etc.
The outputs are now attributes rather than a conventional image.

Z^2 set of binary images specified by (x,y) locations

$$C = \{w \mid w = -d, \text{ for } d \in D\}$$

C is the set of elements w, such that w is formed by multiplying each of the two coordinates of all the elements of set D by −1

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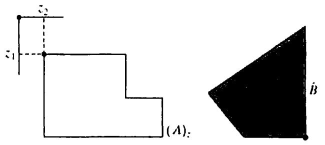
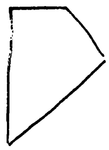


a b

FIGURE 9.2 (a) Translation of A by z. (b) Reflection of B. The sets A and B are from Fig. 9.1.



original B

Image morphology uses two definitions not normally used in set theory.

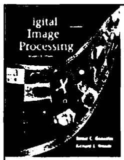
$$\hat{B} = \{ w \mid w = -b, \text{ for } b \in B \}$$

This is the reflection of B about the origin

$$(A)_z = \{ c \mid c = a + z, \text{ for } a \in A \}$$

translate coordinates by $z = (z_1, z_2)$

This is the translation of B.



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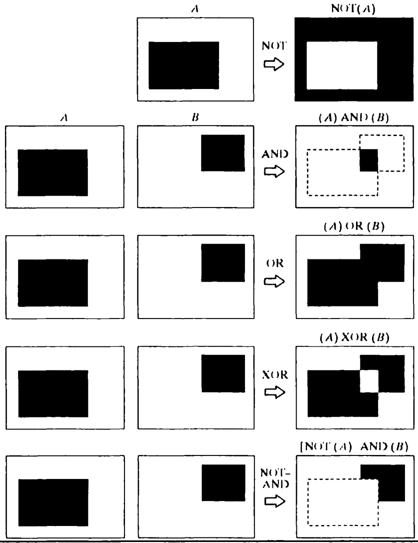

FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

complement of A

AND gives the intersection of the sets

OR gives the union of the sets

This is the exclusive or and basically gives the complement of the OR

more complex

We can perform logic operations between images on a pixel by pixel basis.

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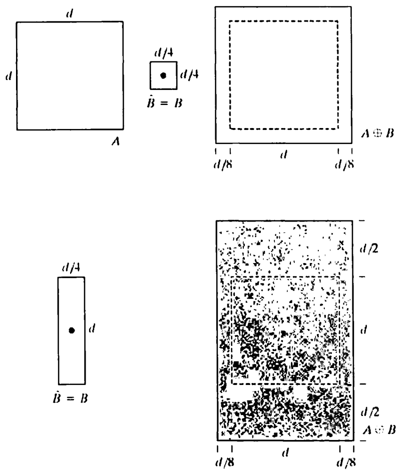

a b c
d e

FIGURE 9.4 (a) Set A. (b) Square structuring element (dot is the center). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element.

dilation of A by B

$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \emptyset \}$$

B is called the structuring element.

Explanation.

- Reflect B about the origin $\hat{B}$
- Shift it by z, $(\hat{B})_z$
- $A \oplus B$ is the set of all displacements z such that $\hat{B}$ and A overlap.

This is very similar to a convolution mask.

Other definitions are possible.

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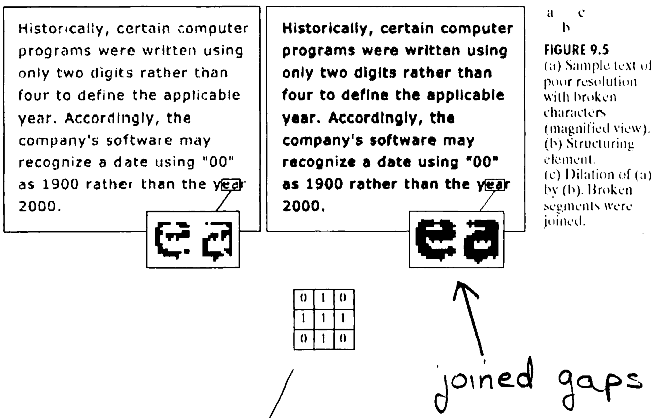

a c
b

FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

structuring element $\hat{B} = B$

Advantage of dilation over low-pass filtering is that result remains binary

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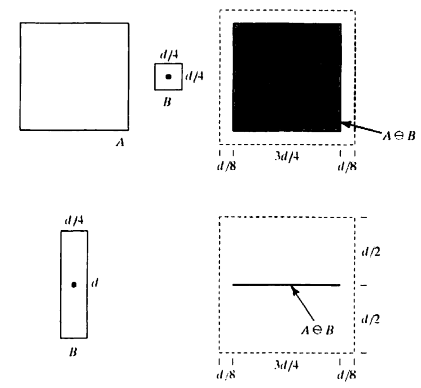
not drawn to scale



no up/down translation is possible so this becomes just a straight line.

FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

Erosion

$$A \ominus B = \{ z \mid (B)_z \subseteq A \}$$

subset of

$A \ominus B =$ set of all points z such that the translation of B by z is contained in A

Note that dilation and erosion are dual processes.

$$(A \ominus B)^c = \{ z \mid (B)_z \subseteq A \}^c$$

$$= \{ z \mid (B)_z \cap A^c = \phi \}^c$$

if $(B)_z$ is in A then $(B)_z \cap A^c = \phi$

$$= \{ z \mid (B)_z \cap A^c \neq \phi \}$$

The complement is just the set for which $(B_z) \cap A^c \neq \phi$

$$= A^c \oplus \hat{B}$$

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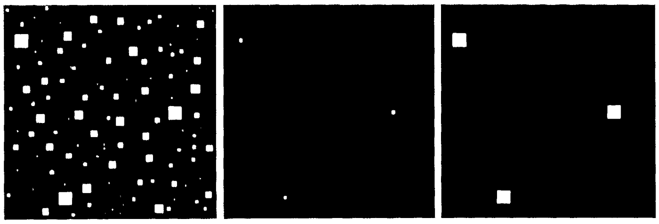

a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

result of erosion by a 13x13 image of 1's

result of dilating the eroded image with a 13x13 all 1's structuring element

Use erosion to eliminate irrelevant (small) detail

Select a structuring element slightly smaller than the details you want to keep.

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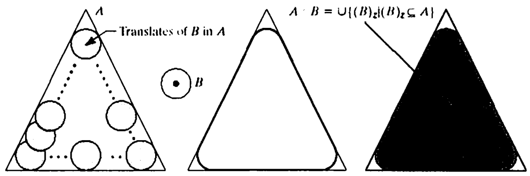

a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

geometric interpretation of opening (rolling ball inside)

Opening

$$A \circ B = (A \ominus B) \oplus B$$

$$= \cup \{ (B)_z \mid (B)_z \subseteq A \}$$

this is the set of all translates of B that fit inside A

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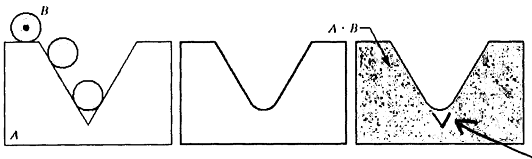

a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

there is no translate of B which can reach inside this region and not intersect A

geometric interpretation of closing
(Roll B on the outside)

$$A \bullet B = (A \oplus B) \ominus B$$

$$A \bullet B = \{ w \mid (B)_z \cap A \neq 0 \}$$

set of all w such that $(B)_z \cap A \neq 0$ for any translate of $(B)_z$ that contains w.

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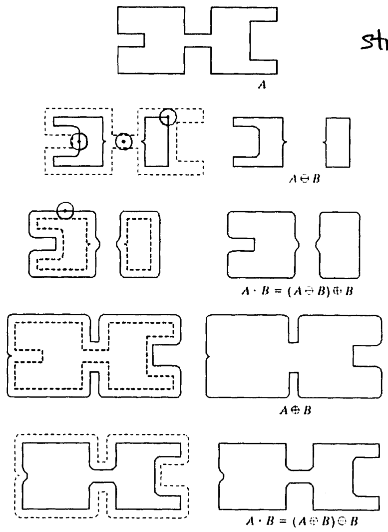


a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

structuring element ⊙ larger than fingers

This is the result of erosion

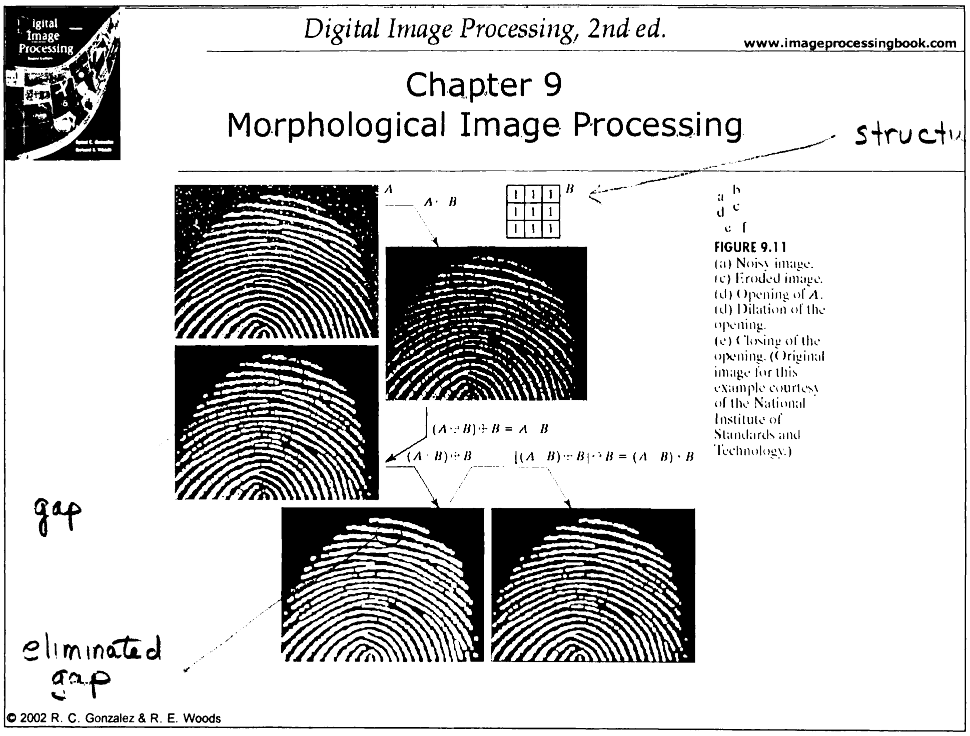
Dilating the above gives the closing.

This is a dilation of the original image.

This shows the erosion of the dilation to complete the opening.

(d), (e) shows the dilation. The topology (connection) was not preserved because B can not fit in A at the connection and the two right fingers.
B must fit inside A to preserve topology.

structuring element (white square)



gap

eliminated gap

Noise is white on black and black on white.

closing

(c) erode A by B
eliminates white on black background noise since the white spots (1's) are smaller than the structuring element but the dark noise in the whorls increased since erosion decreased the size of the white objects

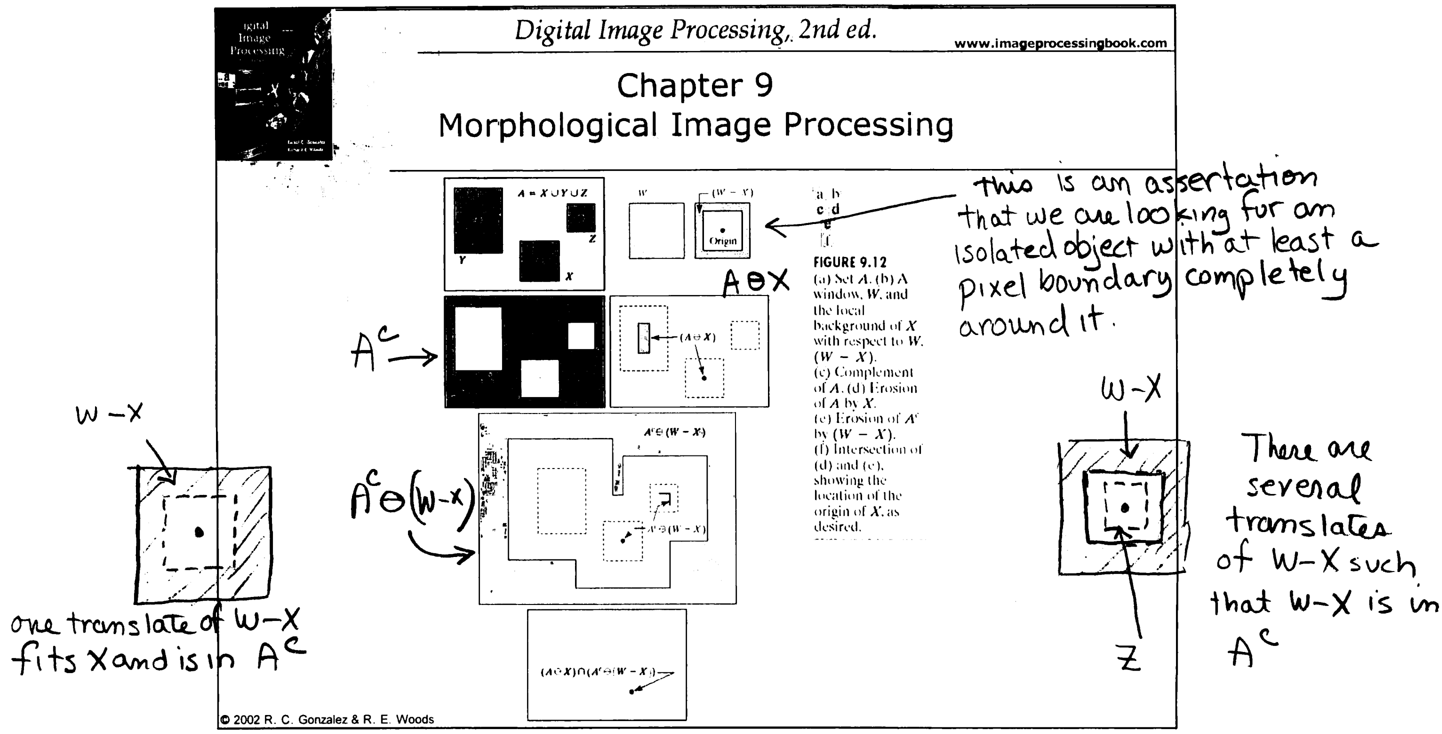
(d) dilate (c) to restore the whorls, i.e. increase the size of the white

opening

(e) dilate (d) to eliminate gaps in the ridges

(f) erode (e) to thin the thickened ridges

Still some gaps because we did not consider how to maintain connectivity.



Use the "hit-or-miss" algorithm to find shapes in images.

(a) We want to find the location of X.

(b) Let X be enclosed by a <u>small</u> background W. Define the local background $W-X$ as shown in (b).

(c) This is simply the complement A^c of the set of all shapes, i.e. $A = X \cup Y \cup Z$

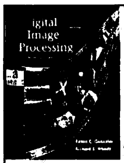
(d) Erode A by X. Since Z is smaller than X it disappears. X eroded by X is a single point, and Y eroded by X is a rectangular region of all locations of X inside Y.

(e) The most complicated thing is the erosion of A^c by $W-X$. It's the set of all translates of $W-X$ such that the center of $W-X$ is in A^c. Note that $W-X$ fits around X giving a center point. Since Z is smaller than X there is also a set of points inside Z corresponding to translates around Z.

(f) Intersection of $A \ominus X$ and $A^c \ominus (W-X)$ is the location of X.

new symbol $\longrightarrow$ $A \circledast B = (A \ominus X) \cap [A^c \ominus (W-X)]$

can be written in other forms as well.



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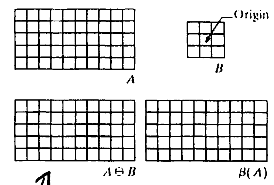
a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

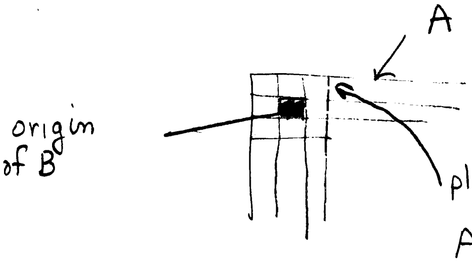


$A \ominus B$ gives the interior of A

$\beta(A) = A - (A \ominus B)$

Boundary extraction:

Erode a set A by an appropriate structuring element B. Subtract this from A to get the boundary $\beta(A)$.



places B 1-pixel inside A giving 1-pixel boundaries. A 5x5 would give 2-pixel boundaries but may eliminate detail.

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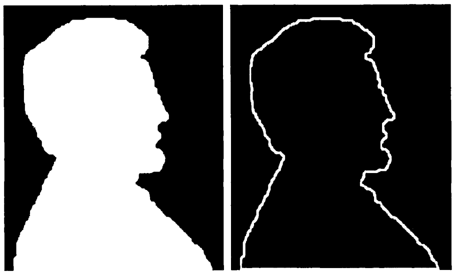

a b
FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Because the structuring element is 3×3 the boundary is 1 pixel thick.

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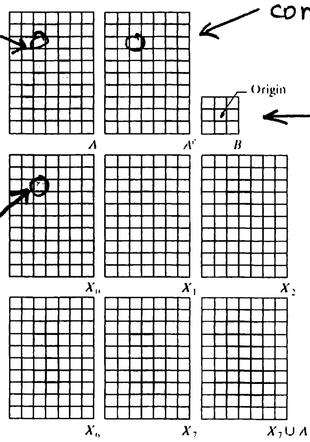
A is a boundary of an object

complement of A limits results.

4 connected filling element.



a b c
d e f
g h i

FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

starting point P inside A.

Region filling

$X_0 = P$. an initial point inside the object

$$X_1 = (\underbrace{P \oplus B}) \cap A^c$$

dilation expands P in four directions

stops all pixels in boundary

this is called conditional dilation

just repeat $X_2 = (\underbrace{X_1 \oplus B}) \cap A^c$

continues to expand (dilate) P

stops all pixels at boundary.

In general $X_k = (X_{k-1} \oplus B) \cap A^c$, $k = 1, 2, 3, \ldots$

stop when $X_k = X_{k-1}$

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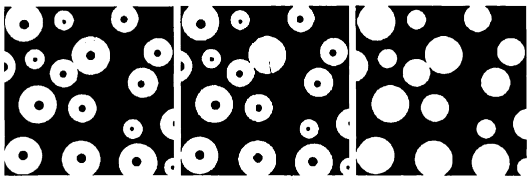
put white dot here



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

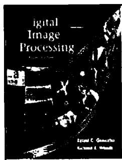
filled

Application of filling

(a) possible thresholded image from ball bearings.
The spots might be due to reflections. One dot inside spot is used to start

(b) Fills up to boundary of circle.

Need additional algorithm to identify circles for this to become automated.



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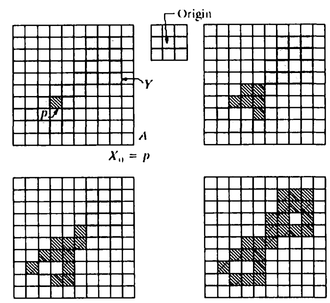

a b c
d e

FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

B as given assumes 8-connectivity of Y.
Previously we used 4-connectivity

Extraction of connected components.

We want to construct a list of connected components in A.

Suppose we are given p which is a part of object Y

Dilate $X_0 = p \oplus B$ to find all points connected in p

now restrict the dilation to points in Y

$X_1 = X_0 \cap A$ gives all the 1's in A connected to P, i.e, elements of Y

$X_2 = (X_1 \oplus B) \cap A$

etc.

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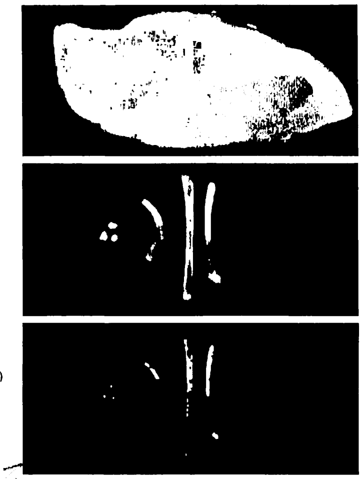
a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5 × 5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)



X-ray of bones in a chicken fillet

Threshholded

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

connected components found by dilating and intersecting

eroded by a 5x5 structuring element of 1's

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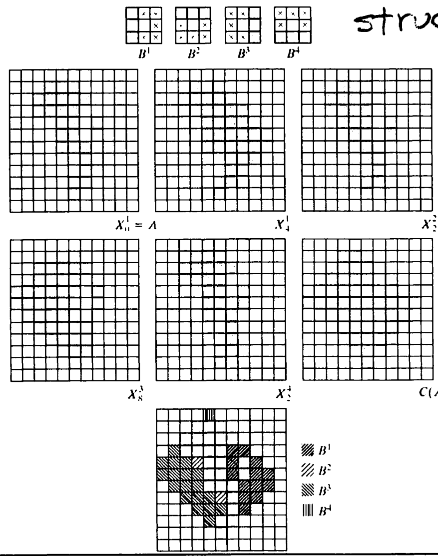
structuring elements.
X = don't care



a
b c d
e f g
h

FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

This is a repetitive "hit or miss" algorithm to construct a convex hull.

You apply B^1 to A, i.e., $X_0^1 \circledast B^1$

which finds the locations of B^1 and you add them to A.

$$X_1^1 = (X_0^1 \circledast B^1) \cup A$$

Continue this process until nothing changes.

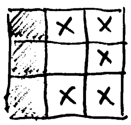
Do this for each shape B^i starting with A

Combine the results to get the convex hull.

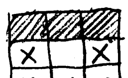
i.e. $X_k^i = (X_{k-1}^i \circledast B^i) \cup A \quad i = 1, 2, 3, \ldots$

$k = 1, 2, 3, 4$

$$C(A) = \bigcup_{i=1}^{4} D^i \quad \text{where} \quad D^i = X^i_{\text{converged}}$$



this basically fills in to the right



this fills in to the bottom

etc.

problem is that this makes objects bigger

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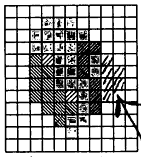

original boundaries

FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

original boundaries.

bounding prevents this growth past the original boundaries.

LIMITING GROWTH

simply restrict maximum growth to initial limits

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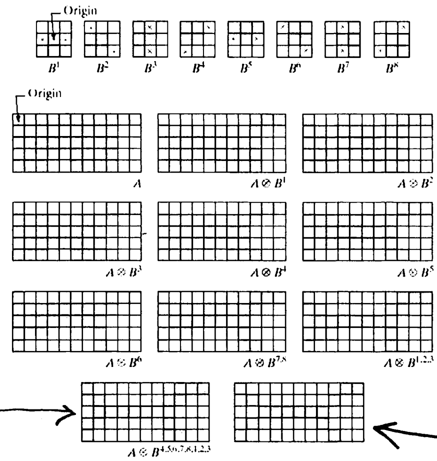
$\otimes$ thin

basically recursively remove



now show result of doing in <u>sequence</u>.

another result of a sequence

converted to m-connectivity

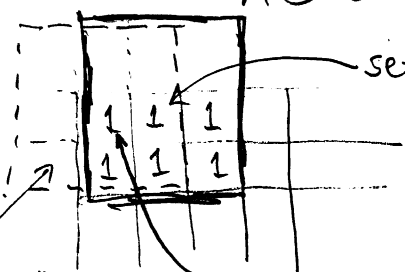
FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to m-connectivity.

We built objects up in last assignment. "Now thin" them down.

Thinning is often used to reduce thick objects to "skeletons"

In this case if the structuring elements match we remove that pixel.

$$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$$



set matches B^1 so remove this pixel.

treat as a "0"

doesn't match B^1 so don't remove

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①

② complement of A

③ now thin the complement of A

④ Now complement the Thinned complement

⑤ remove unconnected points to get thickened object

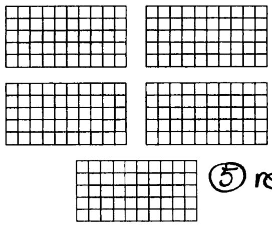
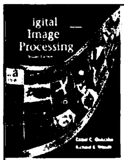


FIGURE 9.22 (a) Set *A*. (b) Complement of *A*. (c) Result of thinning the complement of *A*. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Thickening is the dual of Thinning.

The result of thinning the complement of A is the boundary of the thickened object.

This may result in some disconnected points which need to be removed.



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Skeletons

set of maximum disks which touch the boundary of A in two places or more

a b
c d

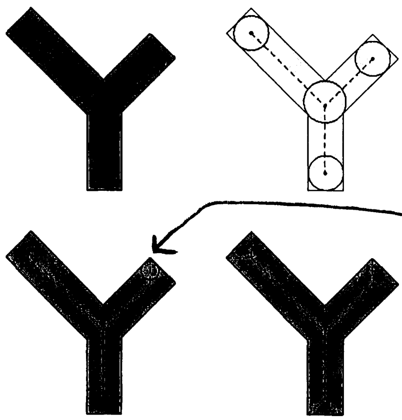


FIGURE 9.23 (a) Set A. (b) Various positions of maximum disks with centers on the skeleton of A. (c) Another maximum disk on a different segment of the skeleton of A. (d) Complete skeleton.

this maximum disk touches the boundary of A at two points

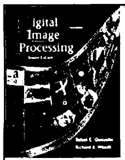
union of all skeleton segments

Mathematically in terms of openings & closings

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

where $S_k(A) = \underbrace{(A \ominus kB)}_{\text{indicates } k \text{ successive erosions of A by B}} - (A \ominus kB) \circ B$

The original object can be recreated by successive dilations from the skeleton.



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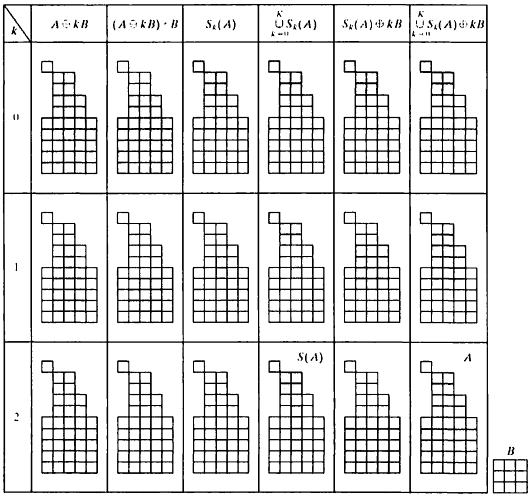

FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

erosions open diff. skeleton dilations union

Using the set definition of skeletons to compute a skeleton.

differences give partial skeletons

this is the result.

1. $A \ominus kB$ means k erosions of A by B and stop just before reaching empty set.

2. $(A \ominus kB) \circ B$ gives the points that fit inside these erosions.

3. Subtract to get the skeleton subset.

4. Combine the skeleton subsets by unioning them.

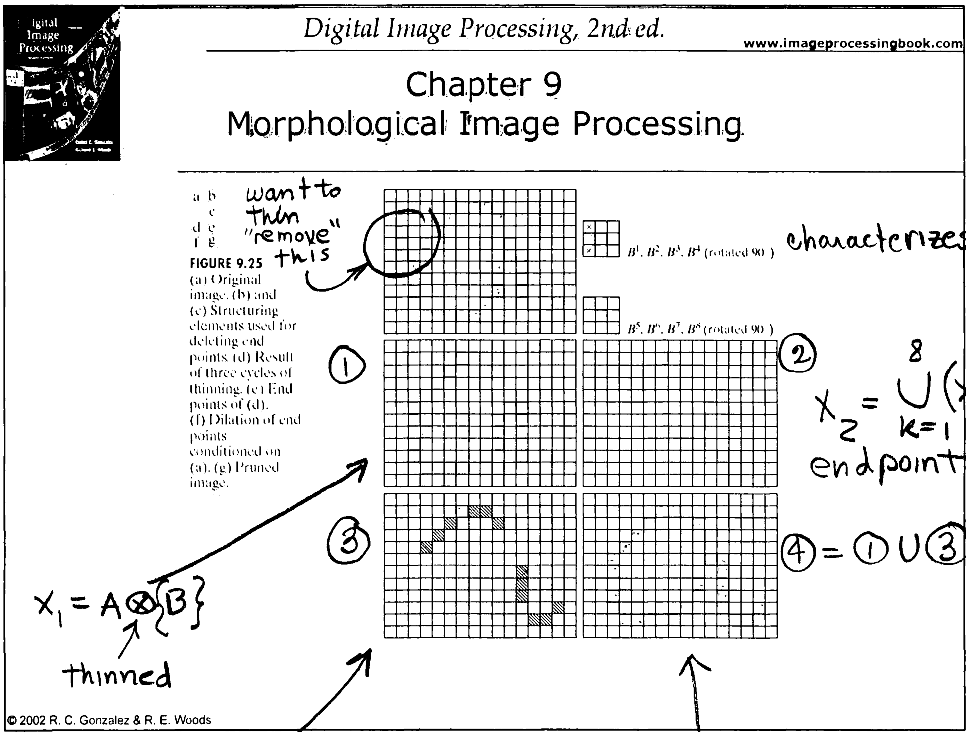

FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

want to thin "remove" this

characterizes projections

① ② ③

$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$

endpoint detection

④ = ① ∪ ③

$X_1 = A \otimes \{B\}$

thinned

$X_3 = (X_2 \oplus H) \cap A$

$X_4 = X_1 \cup X_3$

$H = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Basically detect the end points by
① thinning from end points
② identifying end points by matching
③ dilate from end points (conditional)
④ union of thinned and conditionally dilated images eliminates parasitic branches.

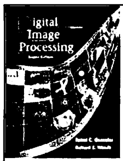

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TABLE 9.2
Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \text{ for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

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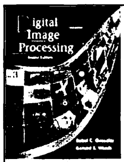
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2) = (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

TABLE 9.2 Summary of morphological results and their properties. *(continued)*



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Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.

TABLE 9.2 Summary of morphological results and their properties. *(continued)*

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Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)	**TABLE 9.2** Summary of morphological results and their properties. *(continued)*
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.	

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commonly used for erosion/dilation

used to exploit connectivity
filling

convex hull

thinning

end points
thinning

thinning

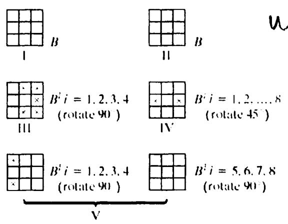

FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

$A \ominus B$ erosion

$A \oplus B$ dilation

$A \circ B$ opening, eliminates small islands and sharp peaks

$A \bullet B$ closing, eliminates holes

$A \circledast B$ "hit or miss" transform

$\beta(A) = A - (A \ominus B)$ boundary detection

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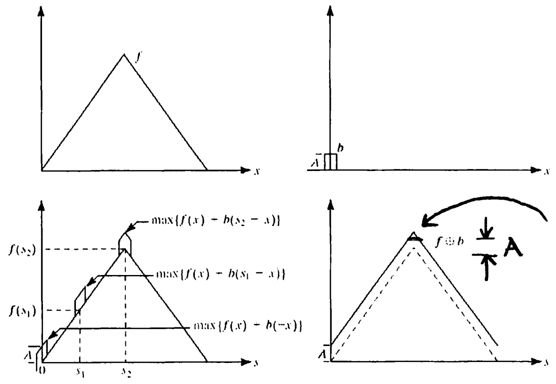


should be flat and width of b

A

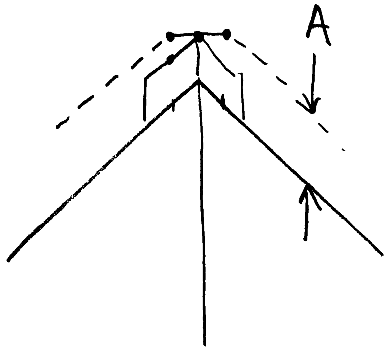
dilation makes objects bigger and brighter

a b
c d

FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).

Note: $$(f \oplus b)(s,t) = \max\{f(s-x, t-y) + b(x,y) \mid (s-x),(t-y) \in D_f;\ (x,y) \in D_b\}$$

This actually defines sliding f by b, but, in practice, we usually slide b by f.



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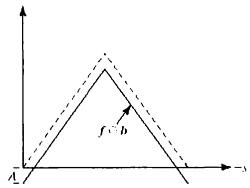

FIGURE 9.28 Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).

erosion makes bright objects dimmer and smaller.

Erosion is defined the same way.

$$(f \ominus b)(s,t) = \min\{f(s+x, t+y) - b(x,y) \mid (s+x),(t+y) \in D_f; (x,y) \in D_b\}$$

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a b
c

FIGURE 9.29 (a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

dilation

1. tends to brighten images
2. dark details (smaller than b) are reduced or eliminated

erosion

1. tends to darken positive images
2. bright details (smaller than b) are reduced or eliminated

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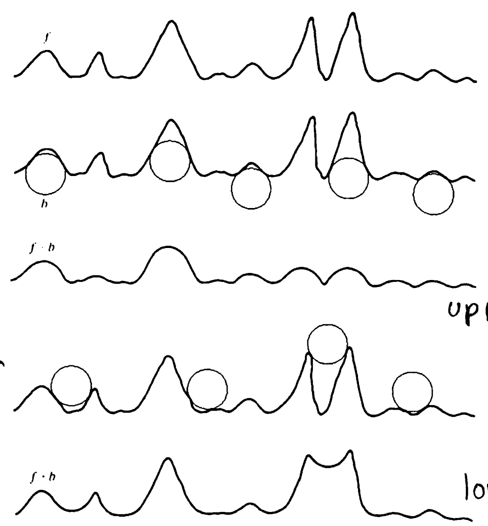

a
b
c
d
e

FIGURE 9.30 (a) A gray-scale scan line. (b) Positions of rolling ball for opening. (c) Result of opening. (d) Positions of rolling ball for closing. (e) Result of closing.

opening
"sharp" details reduced in amplitude and sharpness

upper profile of all balls.

closing
remove "dark" details while leaving bright alone

lower profile of all balls

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a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

opening closing

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FIGURE 9.32 Morphological smoothing of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

smoothing - opening followed by a closing
removes bright and dark details

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FIGURE 9.33 Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

morphological gradient: $g = (f \oplus b) - (f \ominus b)$

dilation − erosion

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note
enhanced
detail

FIGURE 9.34 Result of performing a top-hat transformation on the image of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

top hat $h = f - (f \circ b)$ use a $b =$ flat top

enhances detail in the presence of shading

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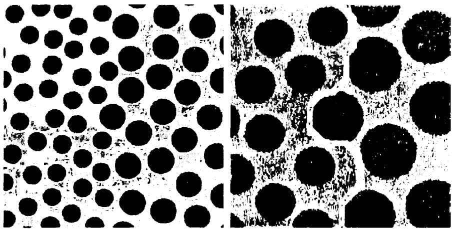
a b

FIGURE 9.35 (a) Original image. (b) Image showing boundary between regions of different texture. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)



Algorithm:

1. repeatedly close the input image to remove dark objects. This will eventually remove all the smaller objects (b is smaller than the circles)
2. Now do a single opening with b larger than the separation between right hand blobs.
3. Now do a thresholding.

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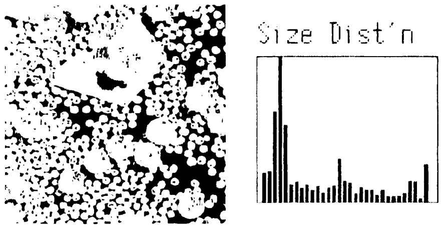

a b
FIGURE 9.36 (a) Original image consisting of overlapping particles; (b) size distribution. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Granulometry - how to determine size distributions

repeatedly open with structuring elements which increase with each iteration
(removes sharp details)

compute differences between original and opened objects

Normalized differences give histogram.