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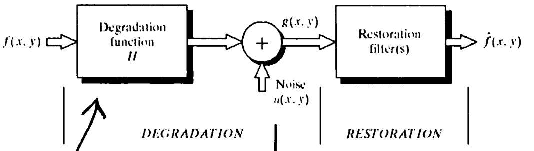

FIGURE 5.1 A model of the image degradation/restoration process

limits us to certain types of degradation

additive noise

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

or $$G(u,v) = H(u,v)\,F(u,v) + N(u,v)$$

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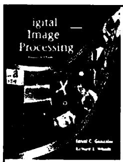


Chapter 5
Image Restoration

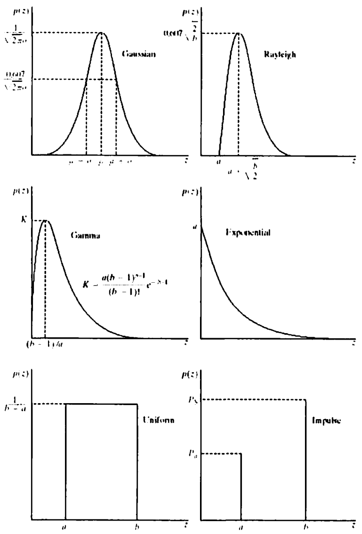


FIGURE 5.2 Some important probability density functions.

Gaussian

$$p(z) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$$

Rayleigh

$$p(z) = \begin{cases} \frac{2}{b}(z-a)\, e^{-\frac{(z-a)^2}{b}} & z \geq a \\ 0 & z < a \end{cases}$$

used for skewed histograms

Gamma

$$p(z) = \begin{cases} \frac{a^b z^{b+1}}{(b-1)!} e^{-az} & z \geqslant 0 \\ 0 & z < 0 \end{cases}$$

Exponential

$$p(z) = \begin{cases} a e^{-az} & z \geqslant 0 \\ 0 & z < 0 \end{cases}$$

Uniform (white)

$$p(z) = \begin{cases} \frac{1}{b-a} & a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

Impulse

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & z = b \\ 0 & \text{otherwise} \end{cases}$$

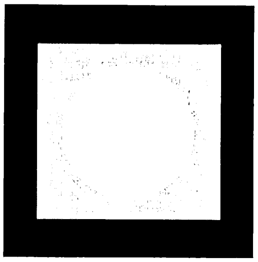
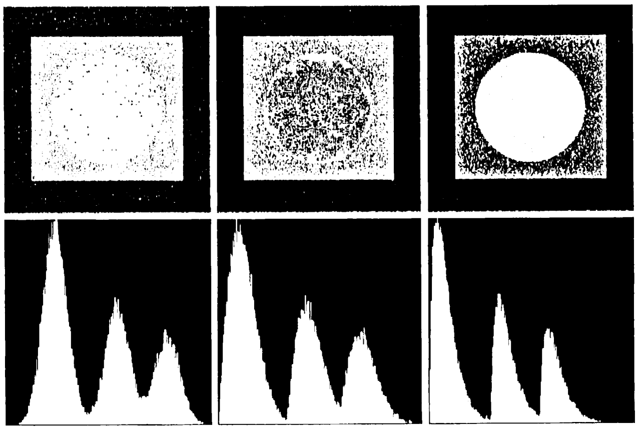


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

Chapter 5
Image Restoration



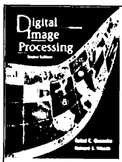
Gaussian Rayleigh Gamma

distributions in each area of image.

a b c
d e f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Hard to visually tell (spatially) the effects of different noise sources apart.



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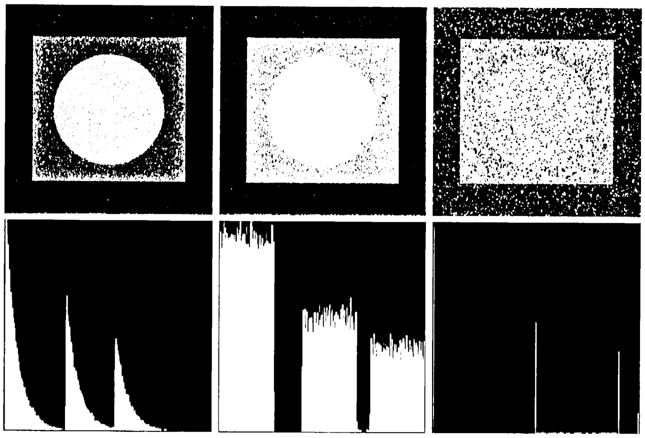
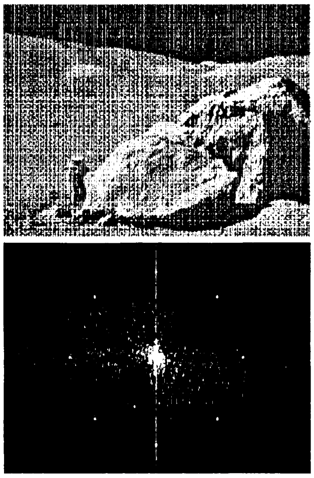
Chapter 5
Image Restoration



g h i
j k l

FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.

Chapter 5
Image Restoration



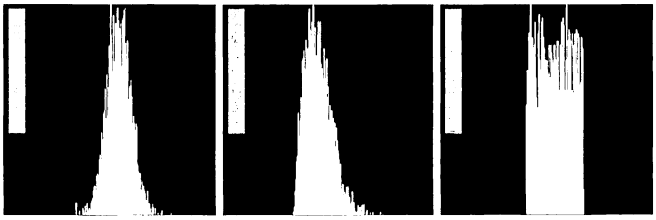
a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

note pattern of noise sources in image.

This is an example of periodic noise.

Chapter 5
Image Restoration



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Gaussian Rayleigh uniform

Compute mean and variance and relate them to the distribution parameters.

$$\mu_z = \sum_{z_i} z_i \, p(z_i)$$

$$\sigma_z^2 = \sum_{z_i} (z_i - \mu_z)^2 \, p(z_i)$$

probability of gray level from histogram

gray level

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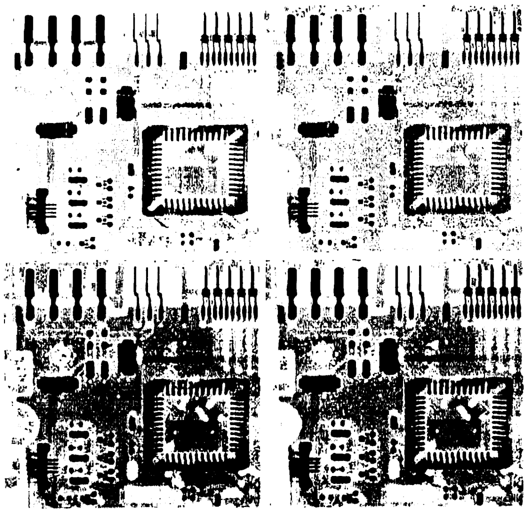
Chapter 5

Image Restoration



a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

arithmetic mean filter $\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$

$$\frac{1}{mn}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

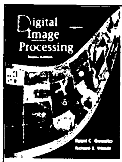
value of restored pixel at (x,y)

geometric mean filter

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t)\right]^{\frac{1}{mn}}$$

just multiply pixels in the subimage window and raise to the power $\frac{1}{mn}$

smoothing comparable to arithmetic mean filter but without losing as much image detail



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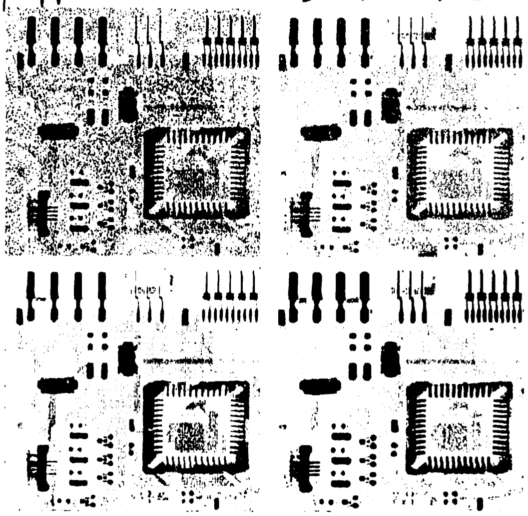
Chapter 5
Image Restoration

pepper noise salt noise



a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3 × 3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

contraharmonic
$Q = 1.5$
eliminates pepper

contraharmonic
$Q = -1.5$
eliminates salt

salt noise random 1's (white)
pepper noise random 0's (black)

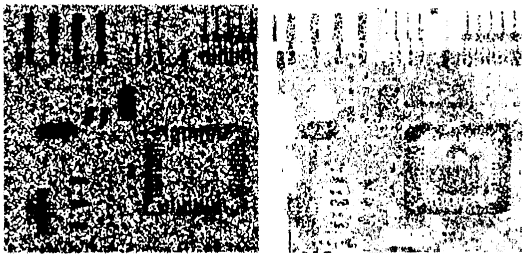
<u>contraharmonic mean filter</u>

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

$\sum g^1 = \sum g$
reduces to arithmetic mean filter if $Q=0$
$\sum g^0 \rightarrow$ # g pixels

use $Q>0$ for eliminating pepper noise
$Q<0$ for eliminating salt noise
but cannot eliminate both simultaneously

Chapter 5
Image Restoration

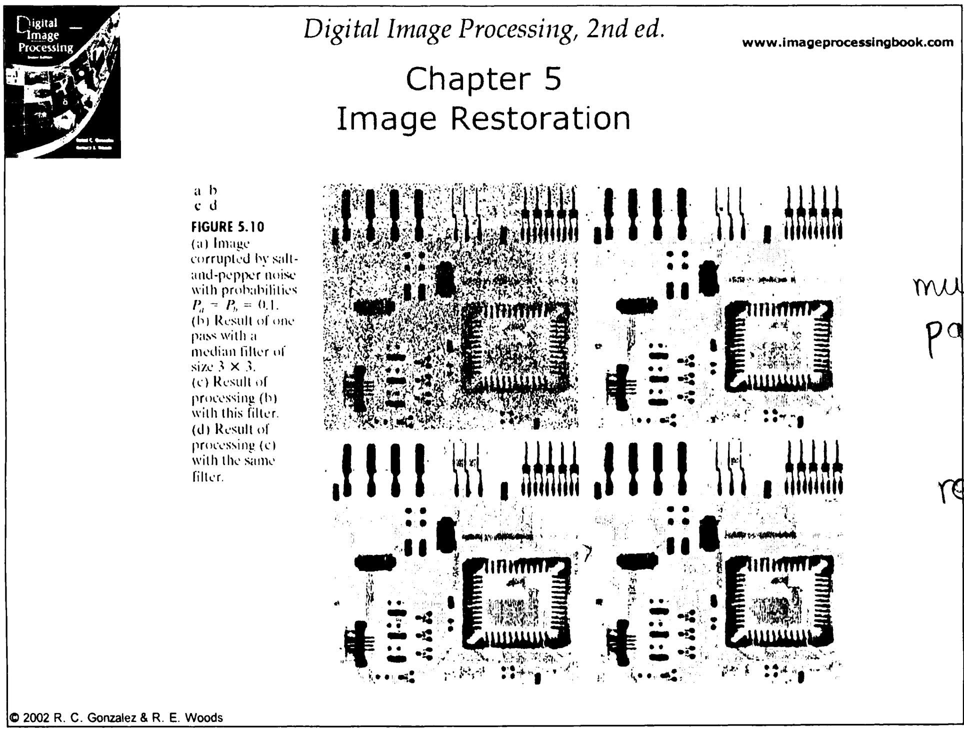


a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3 × 3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

trying to filter pepper noise with wrong sign of Q

trying to filter salt noise with wrong sign of Q



multiple passes of median filter

repeated passes tend to blur

<u>Order statistics filters</u> – based on ordering (ranking) pixels in a neighborhood

median — $$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}} \{g(s,t)\}$$

remember median is the middle element in the list

other order statistics filters

max $$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

best for finding bright points

min $$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

best for finding dark points

midpoint $$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}} \{g(s,t)\} + \min_{(s,t)\in S_{xy}} \{g(s,t)\}\right]$$

works best for Gaussian or uniform noise

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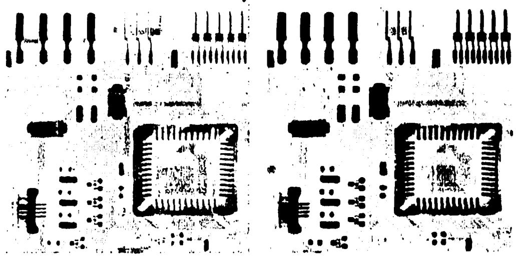
Chapter 5
Image Restoration

Notice difference in finger sizes

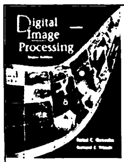


max filter min filter

a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.

The min and max filters do a reasonable job on removing impulsive noise but also remove dark pixels from the borders of dark objects light pixels from the borders of light objects



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Chapter 5
Image Restoration

additive uniform noise →
$\mu=0$, $\sigma=800$

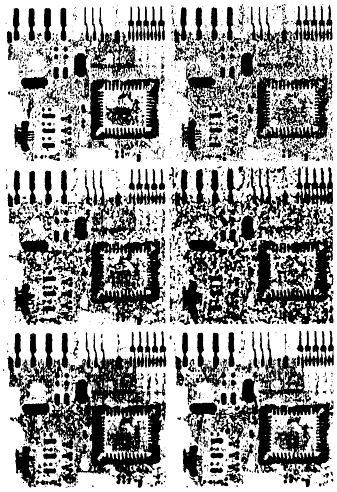
additive uniform PLUS salt-and-pepper noise

5x5 arithmetic mean filter

geometric mean filter

median

alpha-trimmed mean with d = 5
approaches median filter as d increases but also smooths.



a b
c d
e f
FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image in (b) filtered with a 5 × 5: (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

Alpha trimmed mean filter

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

delete $\frac{d}{2}$ lowest and $\frac{d}{2}$ highest values of $g(s,t)$ giving remainder $g_r(s,t)$

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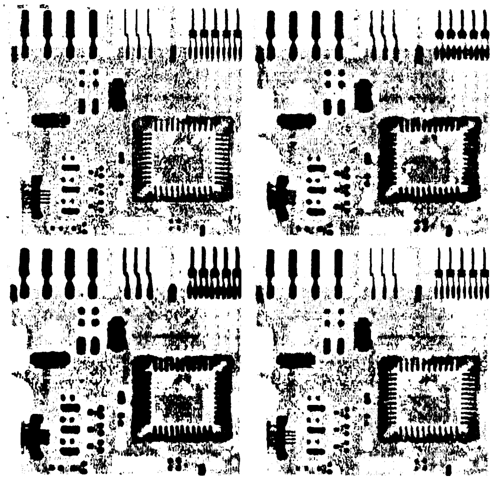
Chapter 5
Image Restoration

additive Gaussian $\sigma_\eta^2 = 1000,\ \mu = 0$ →



a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7 × 7.

7×7 arithmetic mean (visible blurring)

geometric mean

adaptive noise reduction

The results are worse when you estimate σ_η^2

Adaptive noise reduction filter:

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

σ_η^2 = noise variance over the entire image (estimate)

m_L = local mean
σ_L^2 = local variance
} computed locally

The idea is that

when $\sigma_\eta^2 = \sigma_L^2$ it returns the local mean, i.e., averaging out the noise

$\sigma_\eta^2 \ll \sigma_L^2$ this is probably the location of an edge and we should return the edge value, i.e., $g(x,y)$

$\sigma_\eta^2 = 0$ no noise we return $g(x,y)$

if $\sigma_\eta^2 < \sigma_L^2$ we get problems; i.e., producing negative gray levels

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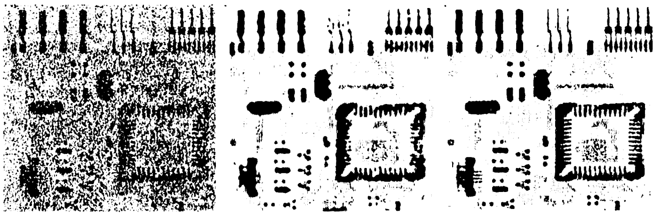
Chapter 5
Image Restoration



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

Very high impulse noise. | 7x7 median filtering lots of loss of detail | adaptive median filtering with $S_{max} = 7$ much better detail

Adaptive median filter: (varies S_{xy})

z_{min} = min. gray value in S_{xy}
z_{max} = max. gray value in S_{xy}
z_{med} = median gray value in S_{xy}
z_{xy} = gray level value at (x,y)
S_{max} = max allowed size of S_{xy}

level A: $A1 = z_{med} - z_{min}$
$A2 = z_{med} - z_{max}$
IF $A1 > 0$ AND $A2 < 0$ THEN go to level B
ELSE increase the window size S_{xy}.
IF window size $\leq S_{max}$ repeat level A
ELSE output z_{xy}.

If $z_{max} > z_{med} > z_{min}$ (in (x,y)) then z_{med} is NOT impulsive. Go to B. Loop continues to increase S_{xy} untill z_{med} is not impulsive.

level B: $B1 = z_{xy} - z_{min}$
$B2 = z_{xy} - z_{max}$
IF $B1 > 0$ AND $B2 < 0$, output z_{xy}
ELSE output z_{med}.

If $z_{max} > z_{xy} > z_{min}$ (at x,y) then z_{xy} is not impulsive and we output z_{xy} otherwise output the median

The idea is we want to avoid outputting impulsive outputs unless they are real. As S_{xy} increases, z_{min} and z_{max} should increase to include anything but "real" noise impulses.

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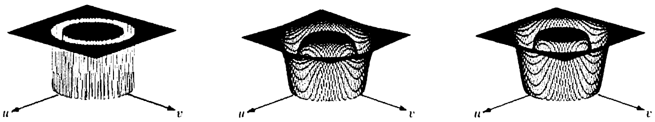
Image Restoration



a b c

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

D_0 is center of stop band
W is full width of stop band

Ideal:

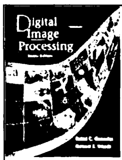
$$H(u,v) = \begin{cases} 1 & D(u,v) < D_0 - \frac{W}{2} \\ 0 & D_0 - \frac{W}{2} \le D(u,v) \le D_0 + \frac{W}{2} \\ 1 & D(u,v) > D_0 + \frac{W}{2} \end{cases}$$

Butterworth

$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)\,W}{D^2(u,v) - D_0^2} \right]^{2n}}$$

Gaussian

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D^2(u,v) - D_0^2}{D(u,v)\,W}\right]^2}$$



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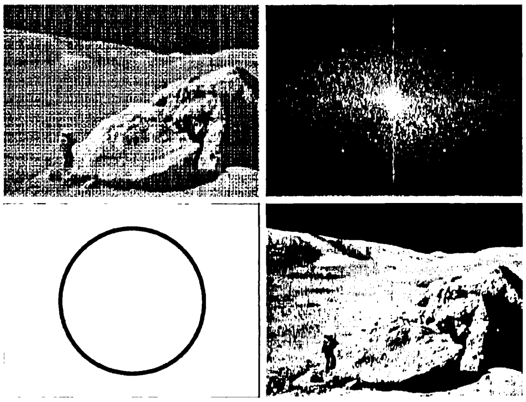
Chapter 5
Image Restoration

notice strong frequency components in a ring



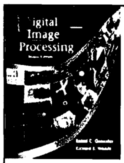
a b
c d

FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

very impressive improvement.

You can not get such results using a spatial domain approach with small filter masks

Usually don't do band reject because it can remove too much image detail.



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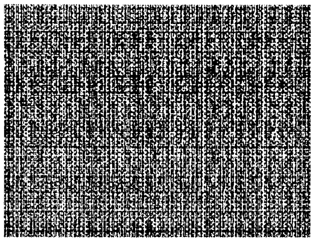
Chapter 5
Image Restoration

FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



Bandpass is the opposite of band reject. This is the image of the noise found in 5.16(a).

$$H_{bp}(u,v) = 1 - H_{br}(u,v)$$

Bandpass filtering is often used to identify noise patterns.