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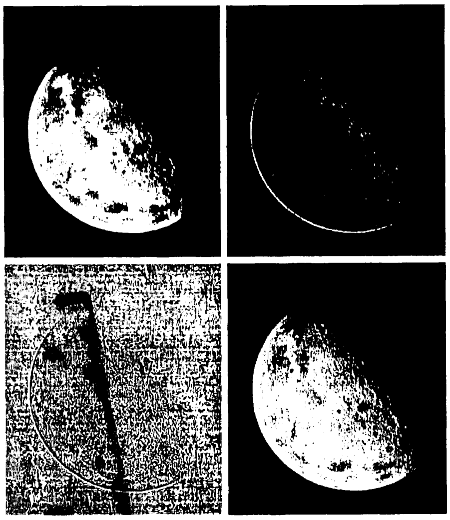
Chapter 4
Image Enhancement in the Frequency Domain



a b
c d

FIGURE 4.28 (a) Image of the North Pole of the moon. (b) Laplacian filtered image. (c) Laplacian image scaled. (d) Image enhanced by using Eq. (4.4-12). (Original image courtesy of NASA.)

scaled image
most positive → 1
most negative → 0

Laplacian
Filtering in the (u,v) domain

$$\nabla^2 f(x,y) = \mathfrak{F}^{-1}\left\{-\left[\left(u-\frac{M}{2}\right)^2+\left(v-\frac{N}{2}\right)^2\right]F(u,v)\right\}$$

(4.4-12)

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$

use one frequency domain mask

$$H(u,v) = 1 - \underbrace{\left[\left(u-\frac{M}{2}\right)^2+\left(v-\frac{N}{2}\right)^2\right]}_{\text{be careful in scaling since these can be} \gg 1}$$

This result is the same one as done before in the spatial domain EXCEPT now done in the frequency domain.

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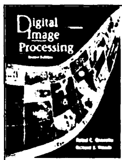

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a b
c d

FIGURE 4.29 Same as Fig. 3.43, but using frequency domain filtering. (a) Input image. (b) Laplacian of (a). (c) Image obtained using Eq. (4.4-17) with $A = 2$. (d) Same as (c), but with $A = 2.7$. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Laplacian high-pass filtered image

high-boost image with A = 2

high-boost image with A increased to 2.7

unshark masking $\quad f_{hp}(x,y) = f(x,y) - f_{lp}(x,y) \quad (1)$

high boost filtering $\quad f_{hb} = Af(x,y) - f_{lp}(x,y)$

re-writing $\quad f_{hb} = (A-1)f(x,y) + \underbrace{f(x,y) - f_{lp}(x,y)}_{f_{hp}(x,y)}$

In the frequency domain we can use a composite filter $\quad H_{hb}(u,v) = (A-1) + H_{hp}(u,v)$ where $A \geq 1$

This is slightly different than previous spatial domain results because this frequency domain representation of the Laplacian does not include diagonal components.

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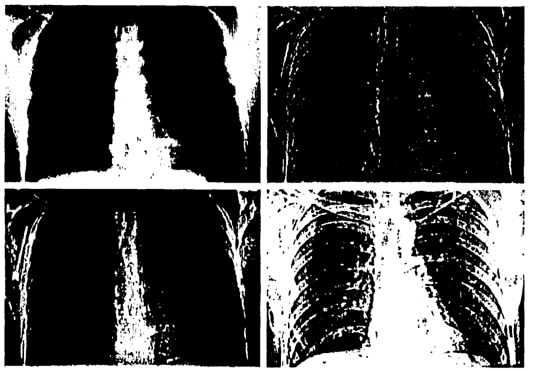

a b
c d

FIGURE 4.30 (a) A chest X-ray image. (b) Result of Butterworth highpass filtering. (c) Result of high-frequency emphasis filtering. (d) Result of performing histogram equalization on (c). (Original image courtesy Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

butterworth high pass
$n=2$

high frequency emphasis
$a=0.5$ $b=2$
keeps low freq. tones

histogram equalization of high frequency emphasis image

high boost $H_{hb}(u,v) = (A-1) + H_{hp}(u,v)$

high-frequency emphasis

$$H_{hfe}(u,v) = a + b H_{hp}(u,v)$$

multiply high frequencies by a constant

add an offset so zero frequency term is not eliminated

typically $.25 < a < .5$ $1.5 < b < 2.0$

when $b=1$ this reduces to high-frequency boost

$b>1$ this emphasizes high frequencies

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a b
FIGURE 4.33 (a) Original image. (b) Image processed by homomorphic filtering (note details inside shelter). (Stockham.)

$\gamma_L = 0.5$

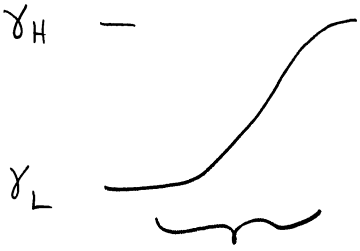
$\gamma_H = 2$

similar to high-frequency emphasis filter

Result of homomorphic processing using a modified Gaussian HP

$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-c\frac{D^2(u,v)}{D_0^2}}\right] + \gamma_L$$

usually $c = \frac{1}{2}$ for a Gaussian filter



D small ←

D → large

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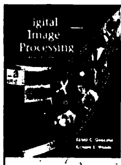

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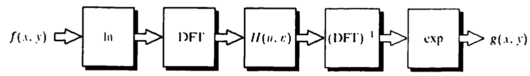


FIGURE 4.31 Homomorphic filtering approach for image enhancement.

simultaneous:

1. dynamic range compression
2. contrast enhancement

$\gamma_H > 1$

$\gamma_L < 1$

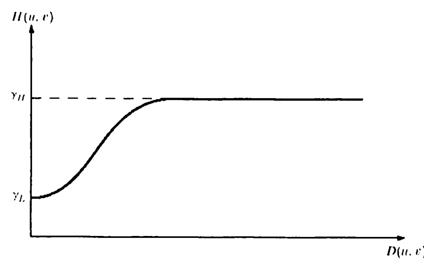
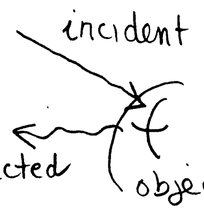


FIGURE 4.32 Cross section of a circularly symmetric filter function. $D(u,v)$ is the distance from the origin of the centered transform.

decrease illumination low frequencies

increase reflectance which varies abruptly for this example



$$f(x,y) = \underbrace{i(x,y)}_{\text{illumination}}\ \underbrace{r(x,y)}_{\text{reflection from object}}$$

define $z(x,y) = \ln(f(x,y)) = \ln i(x,y) + \ln r(x,y)$

(used log to separate i and r)

Fourier transform $\mathfrak{F}\{z(x,y)\} = \mathfrak{F}\{\ln i(x,y)\} + \mathfrak{F}\{\ln r(x,y)\}$

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

Now process by a filter $H(u,v)$

$$S(u,v) = H(u,v)Z(u,v) = \underbrace{H(u,v)F_i(u,v)}_{\text{low frequencies}}^{\text{illumination}} + \underbrace{H(u,v)F_r(u,v)}_{\text{high frequencies}}^{\text{reflection}}$$

inverse

Inverse transforming

$$s(x,y) = \mathfrak{F}^{-1}\{S(u,v)\} = \underbrace{\mathfrak{F}^{-1}\{H(u,v)F_i(u,v)\}}_{i'(x,y)} + \underbrace{\mathfrak{F}^{-1}\{H(u,v)F_r(u,v)\}}_{r'(x,y)}$$

$$s(x,y) = i'(x,y) + r'(x,y)$$

inverse

Use exponential to invert original logarithm

$$g(x,y) = e^{s(x,y)} = e^{i'(x,y)}\, e^{r'(x,y)} = i_0(x,y)\, r_0(x,y)$$

where $i_0(x,y) = e^{i'(x,y)}$ and $r_0(x,y) = e^{r'(x,y)}$

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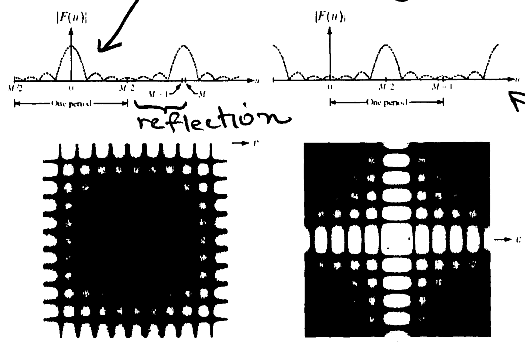
centered on origin due to symmetry



a b
c d

FIGURE 4.34 (a) Fourier spectrum showing back-to-back half periods in the interval $[0, M-1]$. (b) Shifted spectrum showing a full period in the same interval. (c) Fourier spectrum of an image, showing the same back-to-back properties as (a), but in two dimensions. (d) Centered Fourier spectrum.

multiply by $(-1)^x$ before taking transform.

This is the result of fft2

This is what you see after using FFTSHIFT

The DFT is periodic: $F(u,v) = F(u+M, v) = F(u, v+N) = F(u+M, v+N)$

The inverse DFT is periodic

$$f(x,y) = f(x+M, y) = f(x, y+N) = f(x+M, y+N)$$

The DFT is also symmetric

$$F(u,v) = F^*(-u,-v)$$

$$|F(u,v)| = |F(-u,-v)|$$

(a) In (a) we see $F(u)$ is periodic with a period of length M

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)\, e^{-j\frac{2\pi u x}{M}}$$

(b) shows the result of premultiplying by $(-1)^x$ which is simply a shift.

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} (-1)^x f(x)\, e^{-j\frac{2\pi u x}{M}} = \frac{1}{M}\sum_{x=0}^{M-1} f(x)\, e^{-j\frac{\pi}{2}x}\, e^{-j\frac{2\pi u x}{M}}$$

since $-1 = e^{-j\frac{\pi}{2}}$

$$= \frac{1}{M}\sum_{x=0}^{M-1} f(x)\, e^{-j\frac{2\pi u x}{M} - j\frac{M\pi x}{2M}} = \frac{1}{M}\sum_{x=0}^{M-1} f(x)\, e^{-j\left(2\pi u + \frac{M}{2}\right)\frac{x}{M}}$$

shifted by $+\frac{M}{2}$

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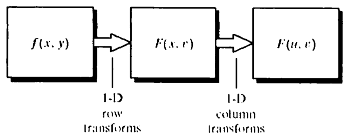


FIGURE 4.35 Computation of the 2-D Fourier transform as a series of 1-D transforms.

can do either first.

This arguement also works for the inverse DFT.

SEPARABILITY

This is one of the most common 2-D DFT implementations

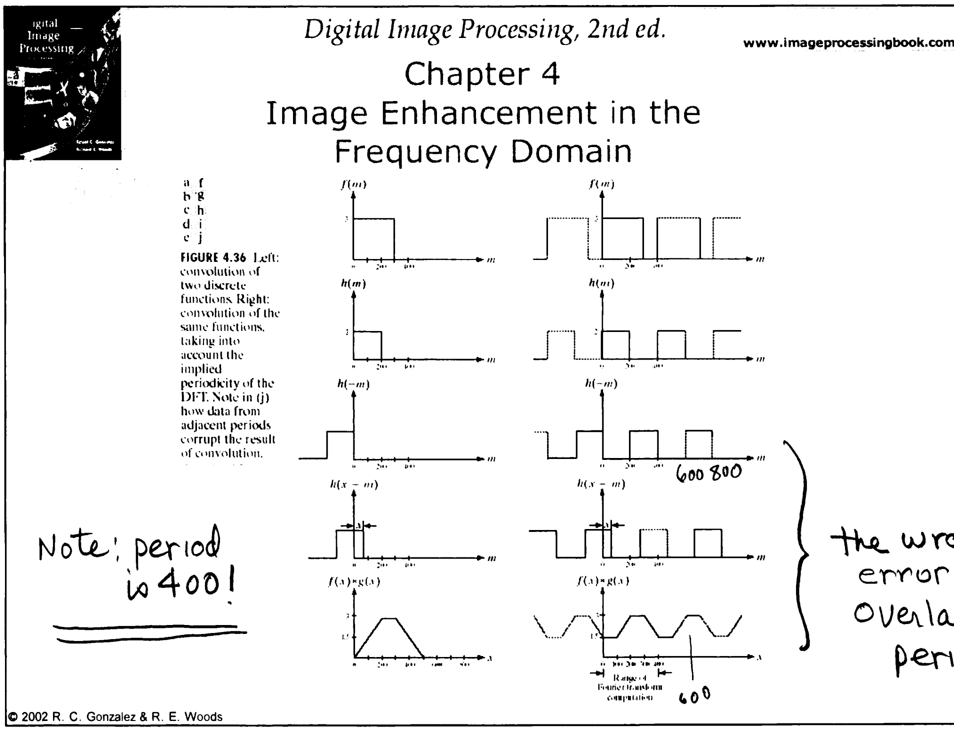
①

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$$= \frac{1}{M} \sum_{x=0}^{M-1} e^{-j2\pi \frac{ux}{M}} \underbrace{\left[\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi \frac{vy}{N}} \right]}_{\text{This is the 1-D DFT of } f(x,y)}$$

$$= \underbrace{\frac{1}{M} \sum_{x=0}^{M-1} F(x,v)\, e^{-j2\pi \frac{ux}{M}}}_{\text{This is the subsequent row DFT. } x \text{ going from 0 to } M} \qquad \underbrace{F(x,v)}_{\text{This is the column DFT of } f(x,y),\ y \text{ going from 0 to } N}$$

② The DFT of the DFT yields $\frac{1}{MN} f^*(x,y)$
Since f is real this is not a problem. Multiply by MN to get $f(x,y)$



Note: period is 400!

the wraparound error comes from overlap of the periodic functions

$\underbrace{\qquad\qquad}_{\text{analytical convolution}}$

Note that if f and g are 400 points f ∗ g goes out to 800 points

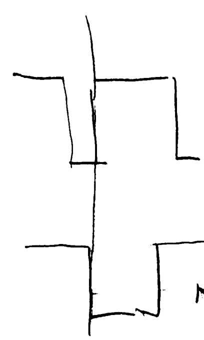
$\underbrace{\qquad\qquad}_{\text{DFT makes functions periodic so}}$

we are really convolving periodic functions

Convolution:

continuous: $f(x) * h(x) = \int_{-\infty}^{+\infty} x(\tau) h(t-\tau)\, d\tau$

discrete: $f(x) * h(x) = \frac{1}{M} \sum_{m=0}^{M-1} f(m)\, h(x-m)$



periodic extension

integral non-zero here

Get rid of this non-zero overlap by simply adding zeros to both functions until the overlap term goes away.

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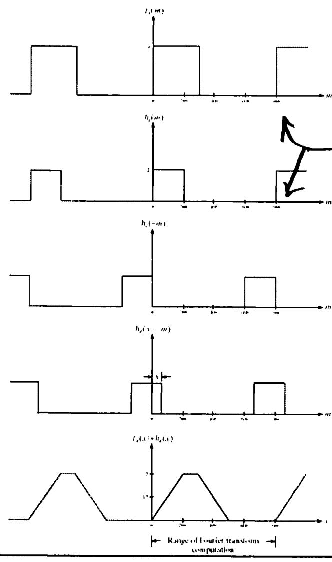


FIGURE 4.37 Result of performing convolution with extended functions. Compare Figs. 4.37(c) and 4.36(e).

extended to 800

convolution is now correct!

Padded functions

$$f_e(x) = \begin{cases} f(x) & 0 \le x \le A-1 \\ 0 & A \le x \le P \end{cases}$$

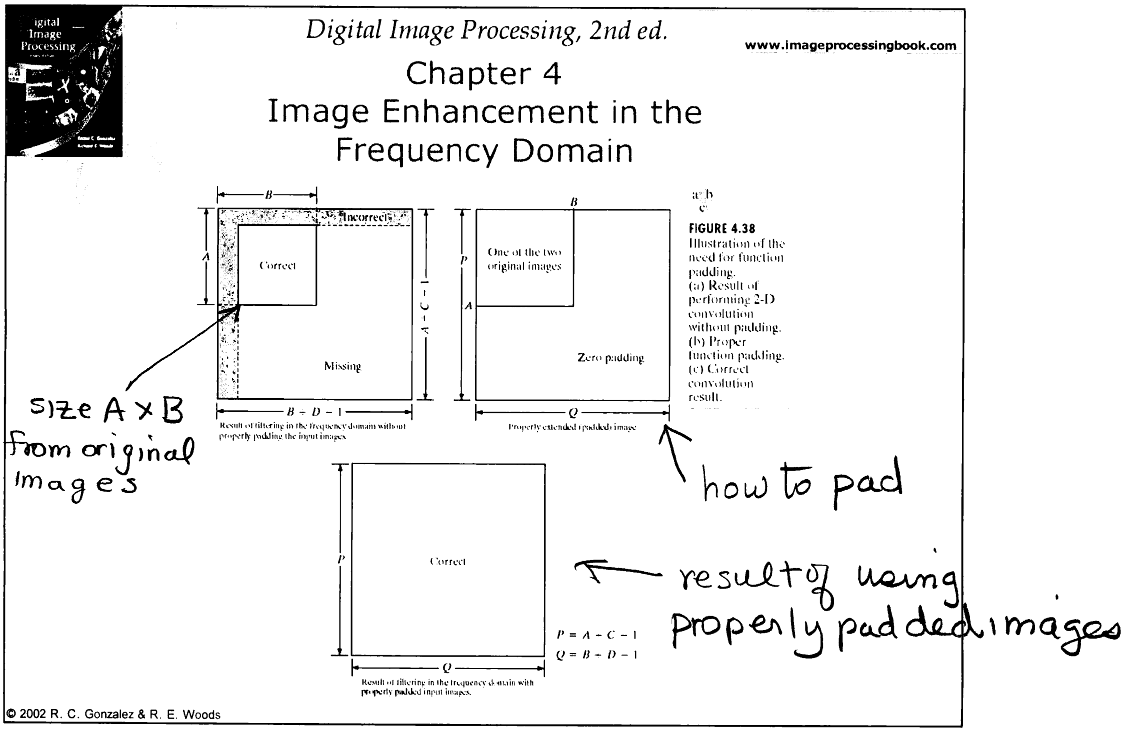
$$g_e(x) = \begin{cases} g(x) & 0 \le x \le B-1 \\ 0 & B \le x \le P \end{cases}$$

both have period P after padding

P is the new <u>extended</u> (or padded) period

If $P = A + B - 1$ there will be no wraparound area

width of one function



(a) f, h are square and the same size

This is the result of $\text{ifft2}\left[\text{fft2}(f) * \text{fft2}(h)\right]$

There is a band of wrap around error.

(b) f, h are properly padded

$P \geq A + C - 1$

$Q \geq B + D - 1$

for images $A \times B$ and $C \times D$

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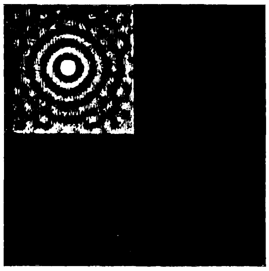


FIGURE 4.39 Padded lowpass filter is the spatial domain (only the real part is shown).

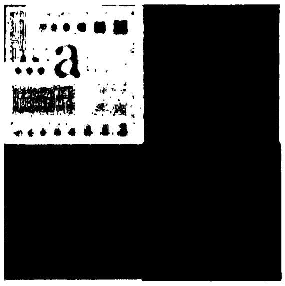
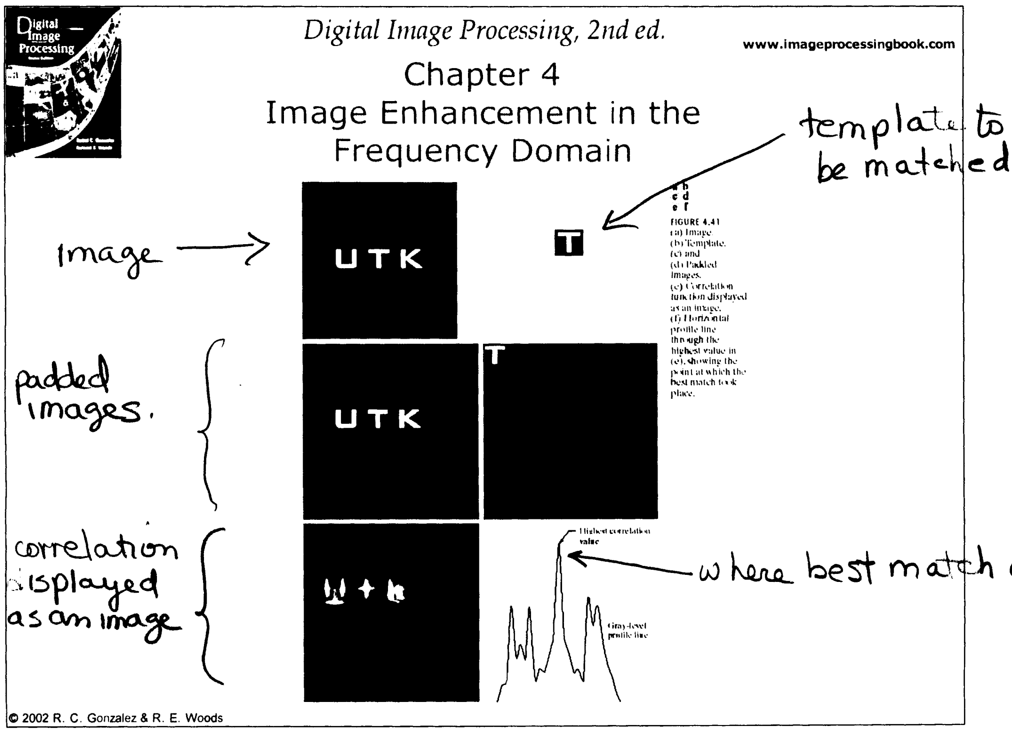


FIGURE 4.40 Result of filtering with padding. The image is usually cropped to its original size since there is little valuable information past the image boundaries.

properly padded low pass filter

result of filtering (using padded filter) a padded test image

Crop back to size of original image.



discrete convolution

$$f(x,y) * h(x,y) \triangleq \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n)\, h(x-m, y-n)$$

$$f(x,y) * h(x,y) \Leftrightarrow F(u,v)\, H(u,v)$$

$$f(x,y)\, h(x,y) \Leftrightarrow F(u,v) * H(u,v)$$

discrete correlation

$$f(x,y) \circ h(x,y) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \underbrace{f^*(m,n)}_{\text{since images are usually real this doesn't affect us}} \underbrace{h(x+m, y+n)}_{\text{not mirrored about origin}}$$

correlation theorem

$$f(x,y) \circ h(x,y) \Leftrightarrow F^*(u,v)\, H(u,v)$$

$$f^*(x,y)\, h(x,y) \Leftrightarrow F(u,v) \circ H(u,v)$$