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Chapter 4
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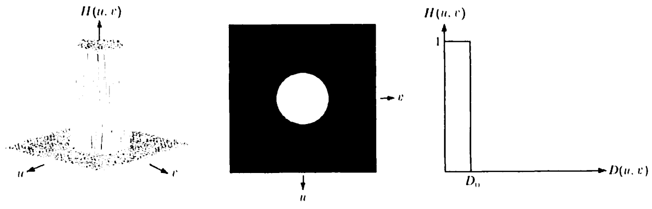


a b c

FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

Ideal Low Pass Filter
(ILPF)

Two-dimensional ideal low-pass filter

$$H(u,v) = \begin{cases} 1 & D(u,v) \le D_0 \\ 0 & D(u,v) \ge D_0 \end{cases}$$

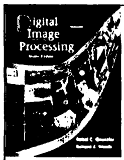
where $D(u,v) = \sqrt{\left(u - \frac{M}{2}\right)^2 + \left(v - \frac{N}{2}\right)^2}$ D_0 is the cutoff frequency and $D(u,v)$ is the radial distance from the center.

Pick cutoff frequencies based upon fraction of image power they remove.

$$P(u,v) = |F(u,v)|^2$$

$$P_T = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} P(u,v)$$

% of image power $\alpha = 100 \left[\sum_u \sum_v \frac{P(u,v)}{P_T} \right]$



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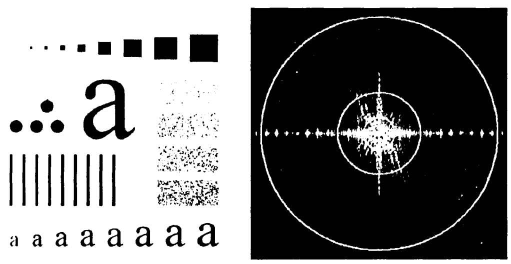


a b

FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

80 230 255

There are five circles shown in the Fourier transform of this test image. These correspond to α = 92.0, 94.6, 96.4, 98 & 99.5%

The corresponding u, v are 5, 15, 30, 80 & 230

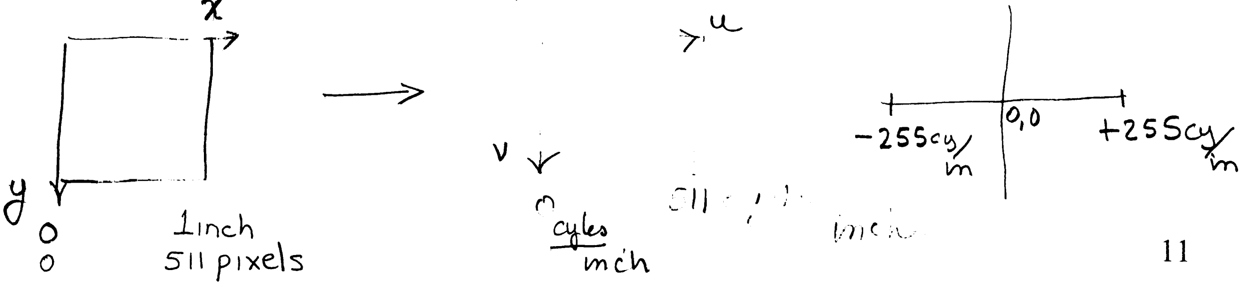
The results of using each filter are shown in the next figure.

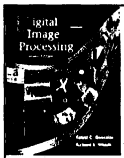
$$\Delta u = \frac{1}{\underline{M \Delta x}}$$

↑ spatial interval. Say $\frac{512 \text{ pixels}}{\text{inch}}$ Then $\frac{1}{512}\frac{\text{inches}}{\text{pixel}}$ = 0.00195 in/pix

$$\Delta u = \frac{}{512 \times .001953} = 1 \text{ cycles/inch}$$

in spatial domain





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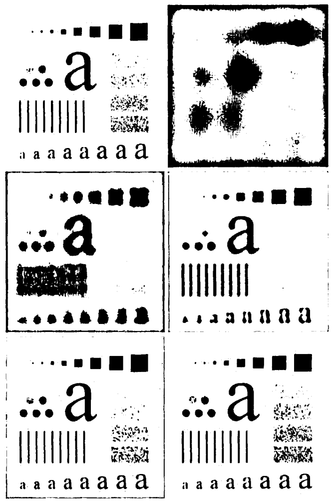

"ringing" shown in all images.

pretty useless except for blurring. all edge information contained in upper 8% of image power.

little edge information in upper 0.5% of power

a b
c d
e f

FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

Lowpass filtering

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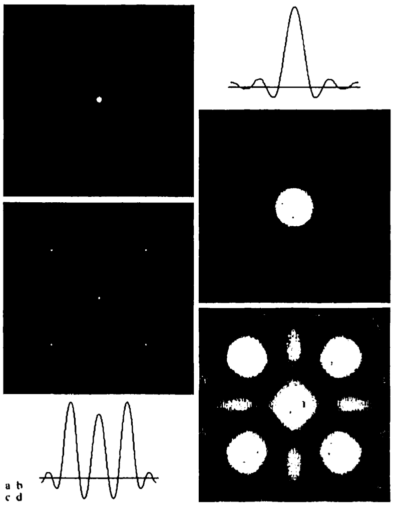


FIGURE 4.13 (a) A frequency-domain ILPF of radius 5. (b) Corresponding spatial filter (note the ringing). (c) Five impulses in the spatial domain, simulating the values of five pixels. (d) Convolution of (b) and (c) in the spatial domain.

illustrates "ringing" due to ILPF (Ideal Low-Pass Filter)

(a) ideal low pass filter of radius 5 pixels

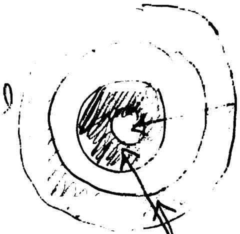
(b) corresponding spatial filter is computed

can do this in freq. domain $(-1)^{u+v} H(u,v)$ for de-centering (and get rid of complex shifts)

inverse DFT to get spatial filter

re-centers $(-1)^{x+y}$ Re [inverse DFT]

take real part since it should be real

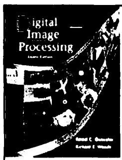


As pixel radius of (a) increases the radius of the spatial rings decreases.

(c) 5 pixels in spatial domain "impulses"

(d) convolution of (b) with (c) showing "sifting" property

i.e., convolution of an impulse with a function "copies" the value of that function to the location of the impulse.



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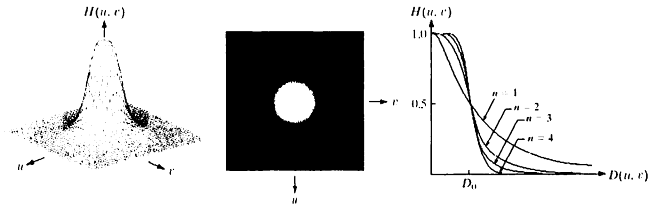


a b c

FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Butterworth LPF
will not exhibit as much ringing as ILPF

Ringing will increase as n increases.

$n=1$ no ringing

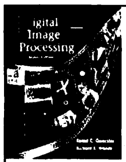
$n=2$ barely perceptible

$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)}{D_0}\right]^{2n}}$$

$D(u,v)$ is the radial distance in the frequency domain

$$D(u,v) = \sqrt{\left(u - \frac{M}{2}\right)^2 + \left(v - \frac{N}{2}\right)^2}$$

D_0 is the cutoff frequency.



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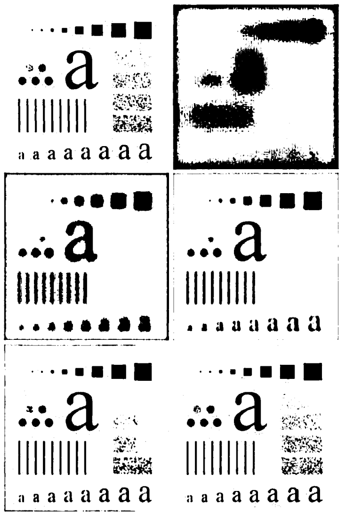

a b
c d
e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

Results of $n=2$ Butterworth low pass filter
Using same radii (D_0) as for ideal low-pass filter in 4.12

Blurring linearly decreases as radius increases,
No ringing visible.

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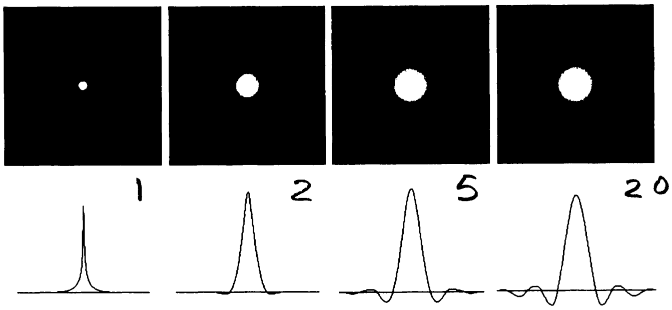
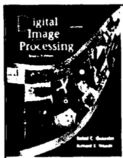


a b c d

FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

Spatial implementation of Butterworth low-pass filters. All filters have $D_0 = 5$

Note that ringing increases with n.



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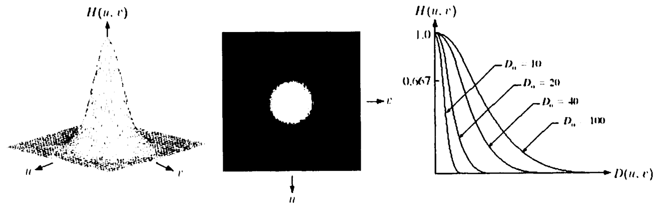

a b c

FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Gaussian Low Pass filter

$$H(u,v) = e^{-\frac{D^2(u,v)}{2\sigma^2}}$$

note constant is 1 for consistency with other filter types

pick $\sigma = D_0$

$$H(u,v) = e^{-\frac{D^2(u,v)}{2D_0^2}}$$

Since its always positive we expect no ringing in the resulting image.

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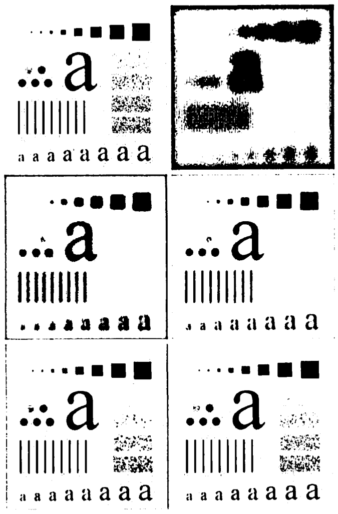

FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

Gaussian
Low Pass Filtering

Blurring linearly decreases as radius (D_0) increases
No ringing visible

Compare 4.18 (c) Gaussian ← picture looks better so not as much smoothing
to 4.15 (c) Butterworth

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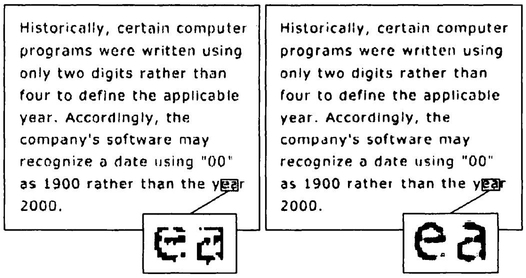
a b

FIGURE 4.19 (a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).



444 x 508 pixels.

broken characters are bad.

fuller and more filled in images using GLPF with $D_0 = 80$

Example of using Gaussian LPF to improve text for OCR.

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a b c

FIGURE 4.20 (a) Original image (1028 × 732 pixels). (b) Result of filtering with a GLPF with $D_0 = 100$. (c) Result of filtering with a GLPF with $D_0 = 80$. Note reduction in skin fine lines in the magnified sections of (b) and (c).

Another commercial application of low pass filtering.
Apply a GLPF to eliminate skin lines and blemishes by selectively smoothing images.

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a b c

FIGURE 4.21 (a) Image showing prominent scan lines. (b) Result of using a GLPF with $D_0 = 30$. (c) Result of using a GLPF with $D_0 = 10$. (Original image courtesy of NOAA.)

588 x 600 pixel original

GLPF $D_0 = 30$

GLPF $D_0 = 10$

might use this image for registration with a map

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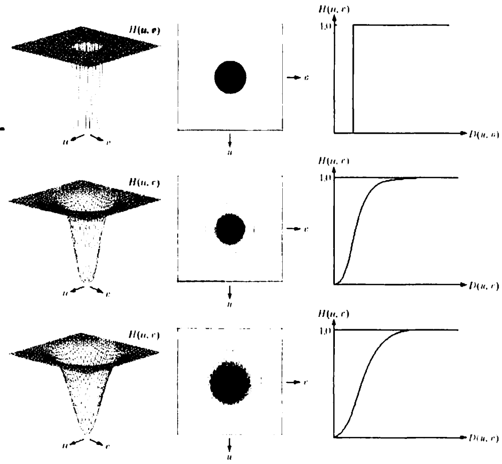
"1" pass —

"0" reject —



ideal high-pass

butterworth high-pass

Gaussian high-pass

a b c
d e f
g h i

FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

In the spatial domain we did unsharp masking as

$$\underset{\text{sharpens detail}}{f_s(x,y)} = f(x,y) - \underbrace{\bar{f}(x,y)}_{\text{by subtracting averaged image}}$$

In the frequency domain we can write something roughly comparable

$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

Generate spatial masks for a HPF just as for a LPF

1. multiply $H(u,v)$ by $(-1)^{u+v}$ to center
2. compute inverse DFT
3. multiply real part of inverse DFT by $(-1)^{x+y}$ to decenter

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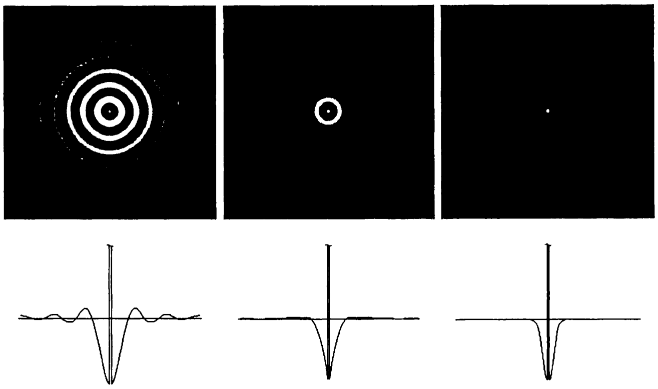

a b c

FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

Spatial filters generated by

1. multiply $H(u,v)$ by $(-1)^{u+v}$ to center
2. inverse DFT
3. multiply inverse DFT by $(-1)^{x+y}$

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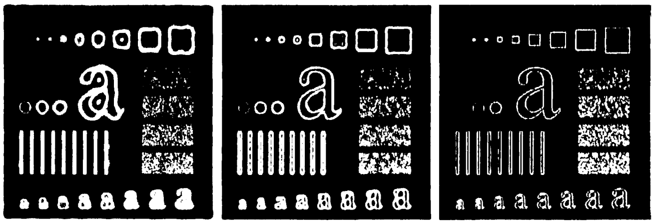
Ideal High Pass Filter

original test image 500 × 500 pixels



a b c

FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).

$D_0 = 15$ $D_0 = 30$ $D_0 = 80$

Nice edges
Good

2D ideal high-pass filter (IHPF)

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \leq D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

high-pass filtering emphasizes edge discontinuity information

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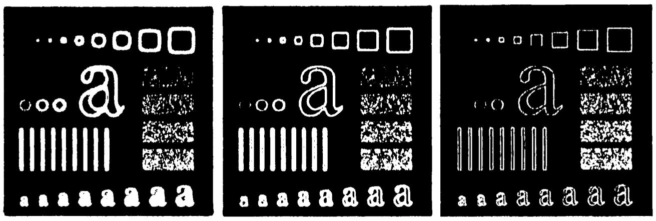
Butterworth HPF



a b c

FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

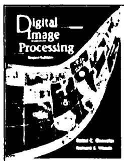
$D_0 = 15$ $D_0 = 30$ $D_0 = 80$

Much less ringing for small D_0 than IHP

Butterworth high pass filter (BHPF) of order n

$$H(u,v) = \frac{1}{1+\left[\frac{D_0}{D(u,v)}\right]^{2n}}$$

inverted compared to BLPF



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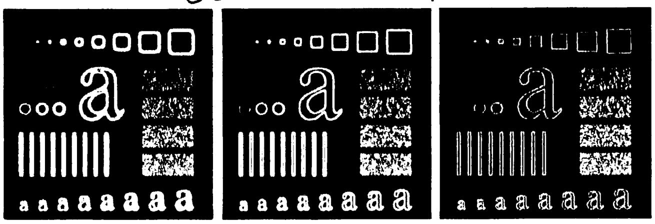
Gaussian HPF



a b c

FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.

$D_0 = 15$ $\quad$ $D_0 = 30$ $\quad$ $D_0 = 80$

Very clean and smooth results.

Gaussian high-pass filter (GHPF)

$$H(u,v) = 1 - e^{-\frac{D^2(u,v)}{2D_0^2}}$$

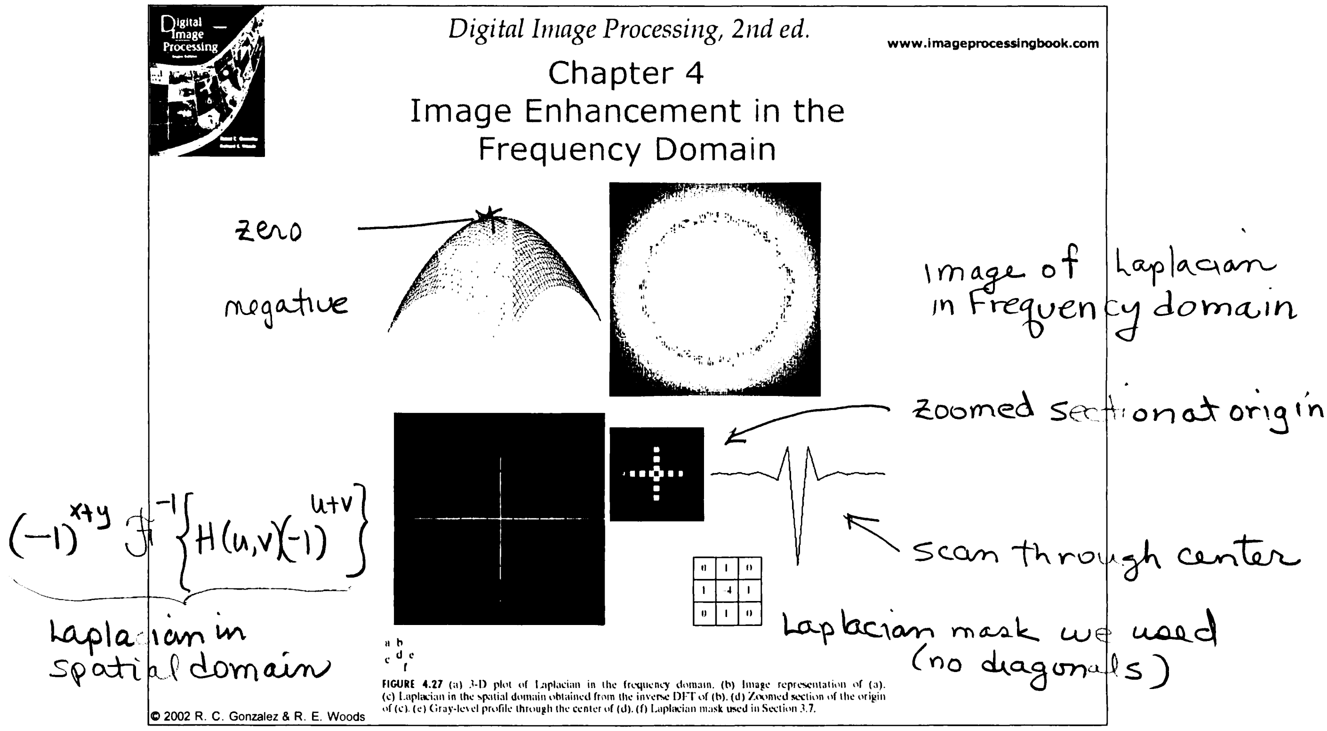


FIGURE 4.27 (a) 3-D plot of Laplacian in the frequency domain. (b) Image representation of (a). (c) Laplacian in the spatial domain obtained from the inverse DFT of (b). (d) Zoomed section of the origin of (c). (e) Gray-level profile through the center of (d). (f) Laplacian mask used in Section 3.7.

What does the Laplacian look like in the frequency domain?

Using $$\mathfrak{F}\left[\frac{d^n f(x)}{dx^n}\right] = (ju)^n F(u)$$

we can write $$\mathfrak{F}\left[\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}\right] = (ju)^2 F(u,v) + (jv)^2 F(u,v)$$

or $$\mathfrak{F}\left[\nabla^2 f(x,y)\right] = -(u^2+v^2)F(u,v)$$

We shift this to the center by multiplying $f(x,y)$ by $(-1)^{x+y}$ before we Fourier transform

$$\Rightarrow \quad \underbrace{\nabla^2 f(x,y)}_{\text{multiply by } (-1)^{x+y}} \iff \underbrace{-\left[\left(u-\frac{M}{2}\right)^2 + \left(v-\frac{N}{2}\right)^2\right]}_{\text{Laplacian plotted in (a)}} F(u,v)$$

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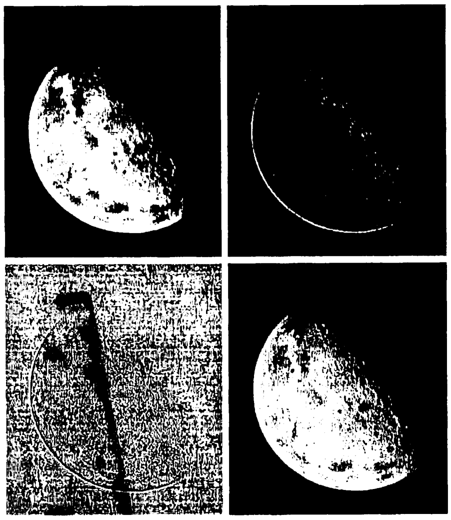

a b
c d

FIGURE 4.28 (a) Image of the North Pole of the moon. (b) Laplacian filtered image. (c) Laplacian image scaled. (d) Image enhanced by using Eq. (4.4-12). (Original image courtesy of NASA.)

scaled image
most positive → 1
most negative → 0

Laplacian
Filtering in the (u,v) domain

$$\nabla^2 f(x,y) = \mathfrak{F}^{-1}\left\{-\left[\left(u-\frac{M}{2}\right)^2+\left(v-\frac{N}{2}\right)^2\right]F(u,v)\right\}$$

(4.4-12)

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$

use one frequency domain mask

$$H(u,v) = 1 - \underbrace{\left[\left(u-\frac{M}{2}\right)^2 + \left(v-\frac{N}{2}\right)^2\right]}_{\text{be careful in scaling since these can be} \gg 1}$$

This result is the same one as done before in the spatial domain EXCEPT now done in the frequency domain.

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a b
c d

FIGURE 4.29 Same as Fig. 3.43, but using frequency domain filtering. (a) Input image. (b) Laplacian of (a). (c) Image obtained using Eq. (4.4-17) with $A = 2$. (d) Same as (c), but with $A = 2.7$. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Laplacian high-pass filtered image

high-boost image with A=2

high-boost image with A increased to 2.7

unsharp masking $f_{hp}(x,y) = f(x,y) - f_{lp}(x,y)$ (1)

high boost filtering $f_{hb} = Af(x,y) - f_{lp}(x,y)$

re-writing $f_{hb} = (A-1)f(x,y) + \underbrace{f(x,y) - f_{lp}(x,y)}_{f_{hp}(x,y)}$

In the frequency domain
we can use a composite filter $H_{hb}(u,v) = (A-1) + H_{hp}(u,v)$ where $A \geq 1$

This is slightly different than previous spatial domain results because this frequency domain representation of the Laplacian does not include diagonal components.