

Segmentation (Chap. 10 topic)

If we want to segment an image based on color the hue (H) image is the most natural to use.

1st thresholding - identify saturated colors: ^{1 - white saturated} ^{0 - black - unsaturated}
 products - regions of significant color
 color distribution
 histogram of regions of significant color

2nd thresholding will identify ~~all~~ colors.
 Colors > 0.9 are colors of interest.
 colors near red



Chapter 6 Color Image Processing

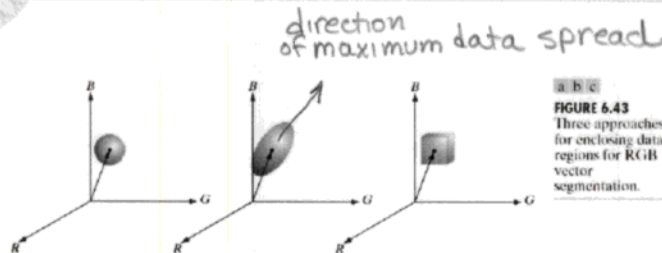


FIGURE 6.43
Three approaches for enclosing data regions for RGB vector segmentation.

spherical

ellipsoidal

box

happens when you use C^{-1}
 C = covariance matrix

computationally much more effective since no squares or square roots

choose sides proportional to standard deviations.

Segmentation divides an image into constituent regions or objects.

Segmentation usually works better in RGB space than in HSI space.

Objective - classify each color pixel as having a color in the specified range or not

Use Euclidean distance

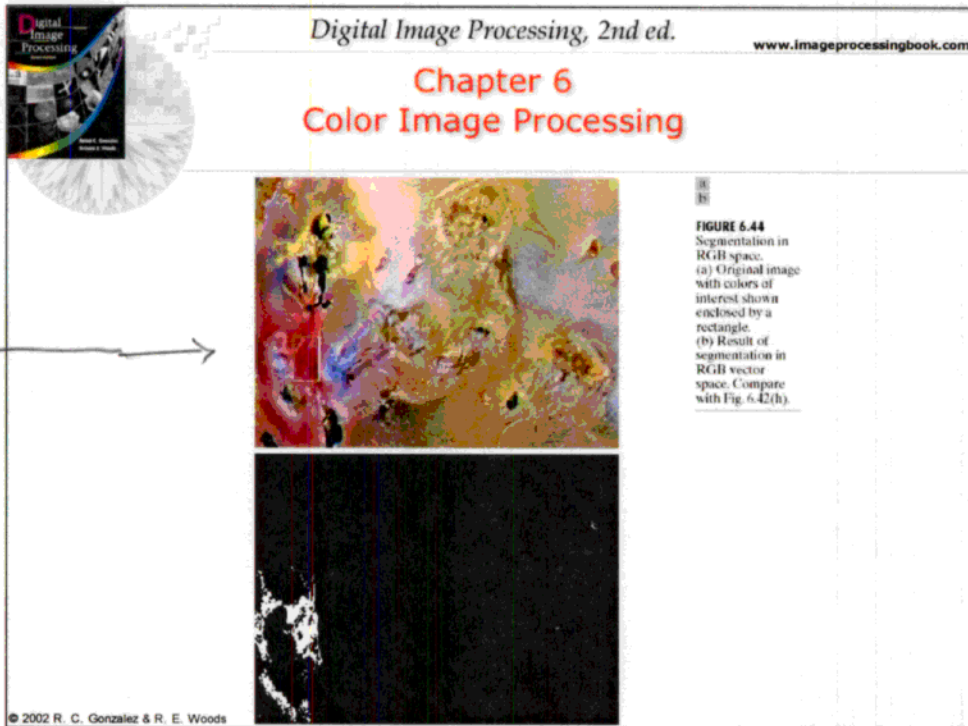
$$D(\underline{z}, \underline{a}) = \|\underline{z} - \underline{a}\| = \sqrt{(\underline{z} - \underline{a})^T (\underline{z} - \underline{a})}$$

$$D(\underline{z}, \underline{a}) = \sqrt{(z_R - a_R)^2 + (z_G - a_G)^2 + (z_B - a_B)^2}$$

A generalized distance measure is

$$D(\underline{z}, \underline{a}) = \sqrt{(\underline{z} - \underline{a})^T \underset{\substack{\uparrow \\ \text{covariance matrix}}}{C^{-1}} (\underline{z} - \underline{a})}$$

a = average color vector for what
we are interested in
i.e. cluster cen



Same image and goal as figure 6.42 but done in RGB space.

Procedure

1. Compute mean and standard deviations of the color contained in the sample rectangle.

mean \underline{a}
std deviations $\sigma_R, \sigma_G, \sigma_B$

2. using box in color space

$$a_R \pm 1.25\sigma_R$$

$$a_G \pm 1.25\sigma_G$$

$$a_B \pm 1.25\sigma_B$$

classify pixel as white if inside this box,
or white outside

Compare results with 6.42(h)

Digital Image Processing, 2nd ed. www.imageprocessingbook.com

Chapter 6 Color Image Processing

RGB edges aligned

RGB edges not aligned

FIGURE 6.45 (a)–(c) *R*, *G*, and *B* component images and (d) resulting RGB color image. (f)–(g) *R*, *G*, and *B* component images and (h) resulting RGB color image.

If we simply add the gradients they would be the same at the center point. Intuitively they cannot be the same.

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Computing edges in individual color planes vs. computing directly in color space.

Alternative vector definition

S. Di Zenzo (1986)

A Note on the Gradient of a Multi-Image
Computer Vision, Graphics and Image Processing
Vol. 33, pp. 116-125

We want to define the gradient (magnitude and direction) of the vector $\underline{c}(x, y)$

For a scalar function $f(x, y)$ the gradient is a vector pointing in the direction of maximum rate of change of f at (x, y)

$\hat{r}, \hat{g}, \hat{b}$ be unit vectors along the R, G, B axis of a RGB color space

define

$$\underline{u} = \frac{\partial R}{\partial x} \hat{r} + \frac{\partial G}{\partial x} \hat{g} + \frac{\partial B}{\partial x} \hat{b}$$
$$\underline{v} = \frac{\partial R}{\partial y} \hat{r} + \frac{\partial G}{\partial y} \hat{g} + \frac{\partial B}{\partial y} \hat{b}$$

} These can be computed using a Sobel operator.

further define

$$g_{xx} = \underline{u} \cdot \underline{u} = \underline{u}^T \underline{u} = \left| \frac{\partial R}{\partial x} \right|^2 + \left| \frac{\partial G}{\partial x} \right|^2 + \left| \frac{\partial B}{\partial x} \right|^2$$

$$g_{yy} = \underline{v} \cdot \underline{v} = \underline{v}^T \underline{v} = \left| \frac{\partial R}{\partial y} \right|^2 + \left| \frac{\partial G}{\partial y} \right|^2 + \left| \frac{\partial B}{\partial y} \right|^2$$

$$g_{xy} = \underline{u} \cdot \underline{v} = \underline{u}^T \underline{v} = \frac{\partial R}{\partial x} \frac{\partial R}{\partial y} + \frac{\partial G}{\partial x} \frac{\partial G}{\partial y} + \frac{\partial B}{\partial x} \frac{\partial B}{\partial y}$$

Implemented by
GWE as
color gradient.

The maximum rate of change of $\underline{c}(x, y)$ at (x, y) is in the direction given by

$$\Theta(x, y) = \frac{1}{2} \tan^{-1} \left[\frac{2g_{xy}}{g_{xx} - g_{yy}} \right]$$

and the rate of change in the direction $\Theta(x, y)$ is given by

$$F(\theta) = \sqrt{\frac{1}{2}(g_{xx} + g_{yy}) + (g_{xx} - g_{yy})\cos 2\theta + 2g_{xy}\sin 2\theta}$$

θ is given in two orthogonal directions. One is a maximum for F and the other is a minimum.



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original image

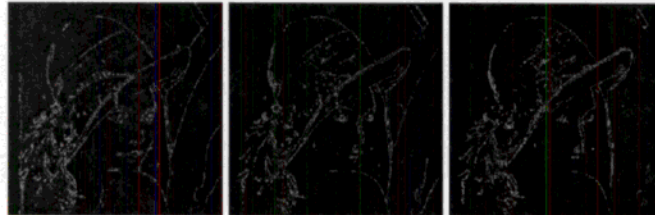
colored edges computed by adding derivatives in each color plane together

color edges computed used vector method in RGB color space

difference between (b) and (c)



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a b c

FIGURE 6.47 Component gradient images of the color image in Fig. 6.46. (a) Red component, (b) green component, and (c) blue component. These three images were added and scaled to produce the image in Fig. 6.46(c).

Individual RGB gradient images
Added and scaled to produce RGB gradient image.



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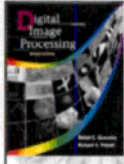
(a) (b)
(c) (d)

FIGURE 6.48
(a)–(c) Red, green, and blue component images corrupted by additive Gaussian noise of mean 0 and variance 800. (d) Resulting RGB image. [Compare (d) with Fig. 6.46(a).]



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Added Gaussian noise to the R, G, and B color planes.
RGB image with noise shown in (d).



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a b c

FIGURE 6.49 HSI components of the noisy color image in Fig. 6.48(d). (a) Hue. (b) Saturation. (c) Intensity.

significantly degraded
due to non-linearity of
cosine and min in
transformations

intensity is average
which tends to
reduce noise.

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HSI components of the noisy RGB image.



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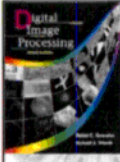
RGB image
Salt & pepper noise
in Green channel



FIGURE 6.50
(a) RGB image with green plane corrupted by salt-and-pepper noise. (b) Hue component of HSI image. (c) Saturation component. (d) Intensity component.

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noise spreads to all HSI component images



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original
image



FIGURE 6.51
Color image
compression.
(a) Original RGB
image. (b) Result
of compressing
and
decompressing
the image in (a).

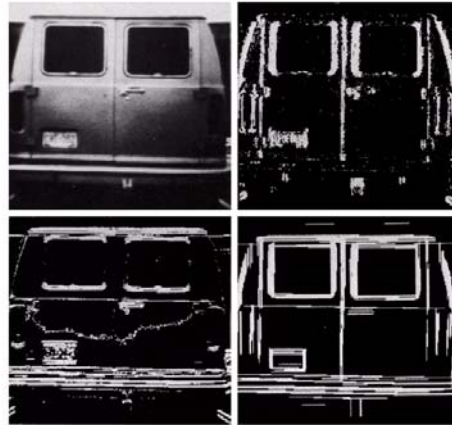
Compressed and
decompressed
using JPEG-2000
slight blurring
due to lossy technique

From Pixels to Lines

Chapter 10 Image Segmentation

a b
c d

FIGURE 10.16
(a) Input image.
(b) G_x component
of the gradient.
(c) G_y component
of the gradient.
(d) Result of edge
linking. (Courtesy
of Perceptics
Corporation.)



The final image is the result of linking all points that had a gradient value >25 and whose direction did not differ by more than 15°

Additional processing to link line segments separated by short breaks and deleting short isolated segments.

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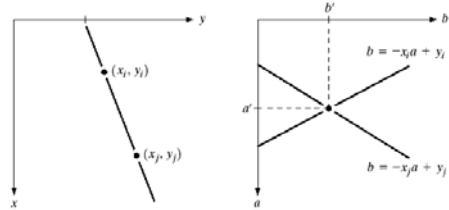


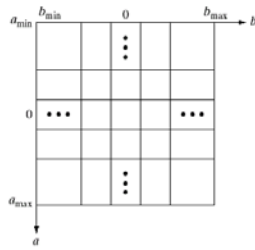
FIGURE 10.17
(a) xy -plane.
(b) Parameter space.

Hough Transform

1. Quantize parameter space between appropriate maxima and minima for c and m
2. Form an accumulator array $A[c,m]:=0$
3. For each **point** (x,y) in an edge-enhanced image such that $E(x,y)>T$, increment all points in $A[c,m]$ along the appropriate **line** in m - c space, i.e.,
 $A[c,m]:=A[c,m]+1$ for $c=-mx+y$
4. **Local maxima** in $A[c,m]$ space correspond to collinear points (i.e., lines) in the image array. Values in $A[c,m]$ correspond to how many points exist on that line.

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FIGURE 10.18
Subdivision of the
parameter plane
for use in the
Hough transform.



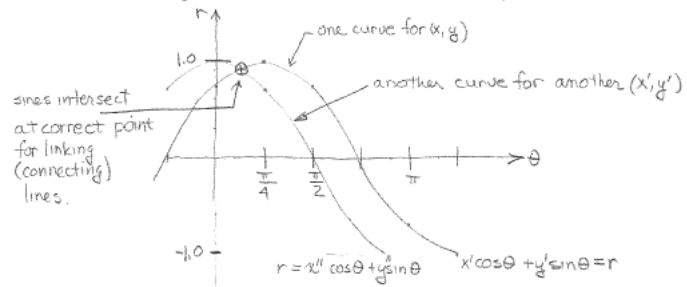
Hough Transforms

Problem: m is unbounded $-\infty < m < +\infty$

Solution: convert to polar coordinates BEFORE transforming.

$$\text{i.e. } r = x \cos \theta + y \sin \theta$$

This gives sine waves instead of straight lines in Hough space.



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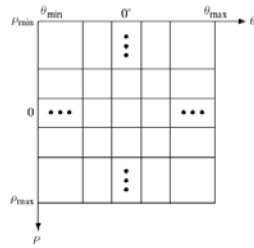
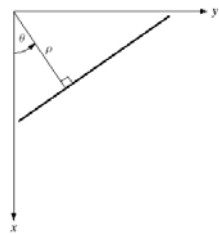


FIGURE 10.19
(a) Normal representation of a line.
(b) Subdivision of the ρ - θ -plane into cells.

Hough Transforms

the Hough transform can be applied to any curve of the form

$$f(\underline{x}, \underline{a}) = 0$$

position vector \uparrow \uparrow parameter vector

For example, $(x-a)^2 + (y-b)^2 = r^2$
is a three parameter space (a, b, r)

This approach is impractical for too many parameters

Effectively it is a matched filtering process

Consider looking for a circle of 1's (edge only)

$A(a, b, r)$ is the correlation with that circle template

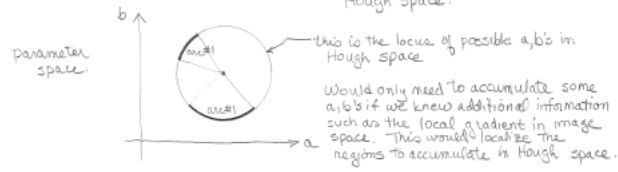
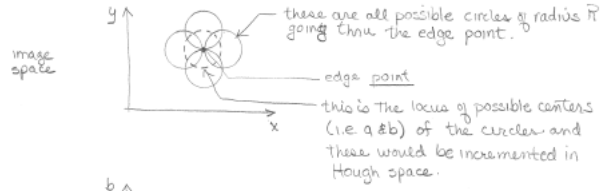
\uparrow possible radii
possible centers

SPECIAL techniques needed to find end points of a line segment.

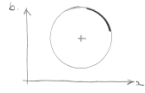
Hough Transform

4.3.1. The Hough transform is traditionally slow.
Use gradient direction to reduce computations.

Example: find a circle of fixed radius R



Hough Transform



Consider equation of circle
 $(x-a)^2 + (y-b)^2 = r^2$
 differentiating $2(x-a) dx + 2(y-b) dy = 0$
 $\frac{dy}{dx} = \frac{x-a}{y-b}$ (1)

define $\frac{dy}{dx} = \tan \phi$ (the gradient of the edge in image space)

Solve for a & b in terms of x, y, R and ϕ

$$(x-a)^2 + (y-b)^2 = r^2$$

substituting (1) $\left[\frac{dy}{dx}(y-b)\right]^2 + (y-b)^2 = r^2$

$$(y-b)^2 \left[\left(\frac{dy}{dx}\right)^2 + 1\right] = r^2$$

$$(y-b)^2 [\tan^2 \phi + 1] = r^2 = (y-b)^2 \frac{1}{\cos^2 \phi} = r^2$$

$\therefore (y-b)^2 = r^2 \cos^2 \phi$
 $y-b = \pm r \cos \phi$
 $b = y \mp r \cos \phi$

similarly for a using the equation of the circle and (1)

$$(x-a)^2 + (y-b)^2 = r^2$$

$$(x-a)^2 + \left[\frac{x-a}{\tan \phi}\right]^2 = r^2$$

$$(x-a)^2 \left[1 + \frac{1}{\tan^2 \phi}\right] = r^2$$

$$(x-a)^2 [1 + \cot^2 \phi] = r^2 = (x-a)^2 \frac{1}{\sin^2 \phi}$$

$$(x-a)^2 = r^2 \sin^2 \phi$$

$$a = x \mp r \sin \phi$$

\Rightarrow Use $a = x \mp r \sin \phi$ R is known in this example
 $b = y \mp r \cos \phi$ ϕ is known from gradient
 (x, y) is location of edge candidate.
 Although this gives a single point in (a, b) space. Do an arc
 trace (a, b) to cover errors in ϕ due to quantization,
 noise, etc

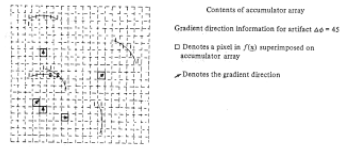


Fig 4.6 Reduction in comparison with gradient information

Generalized Hough Transform

basic premise of Hough
 edge point data in image space \rightarrow loci of possible points
 in parameter space.



R-table: gradient angle θ
 θ_1
 θ_2
 \vdots
 θ_m

locations reference points
 from arbitrary points $L = (L_1, L_2, L_3, \dots, L_n)$

Algorithm 4.3 Generalized Hough

- a. Construct an R-table for the shape to be located
1. Initialize Accumulate array $A(x_{\min}, x_{\max}, y_{\min}, y_{\max})$ for all possible reference points
2. For each edge point:
 - 2.1 Compute $\theta(x)$ direction of gradient
 - 2.2a. calculate all possible centers, i.e. $x_c = x + \rho \cos(\alpha(\theta))$ } look up in table and compute
 - 2.2b. $y_c = y + \rho \sin(\alpha(\theta))$
 - 2.2b. increment the accumulate array $A(x_c, y_c) := A(x_c, y_c) + 1$
3. Possible locations of shape are local maxima of A .

can handle scale ρ and rotation θ by increasing dimensionality of accumulate arrays.



Hough Transform

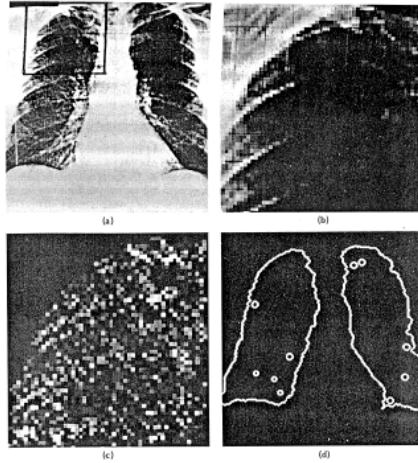
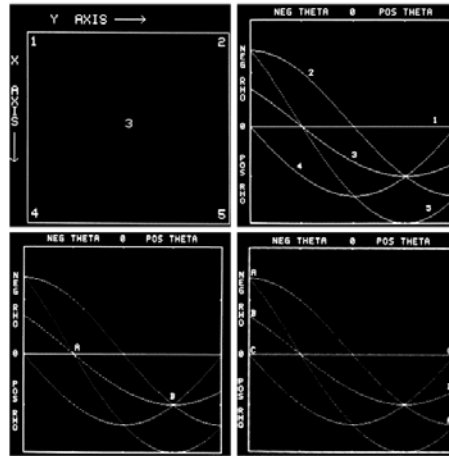


Fig. 4.7 Using the Hough technique for circular shapes. (a) Radiograph. (b) Window. (c) Accumulator array for $r = 3$. (d) Results of maxima detection.

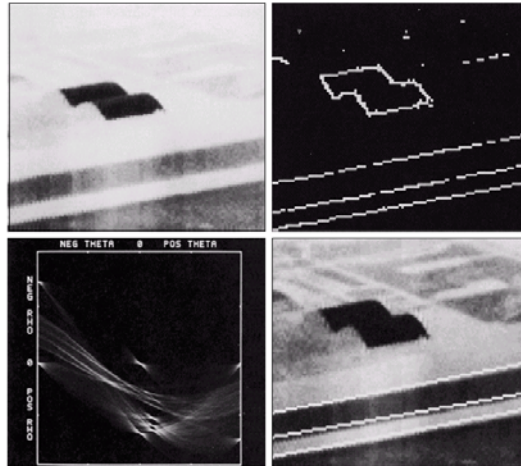
Chapter 10 Image Segmentation

a b
c d

FIGURE 10.20
Illustration of the
Hough transform.
(Courtesy of Mr.
D. R. Cate, Texas
Instruments, Inc.)



Chapter 10 Image Segmentation



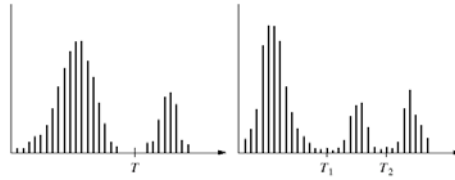
a b
c d
FIGURE 10.21
(a) Infrared image.
(b) Thresholded gradient image.
(c) Hough transform.
(d) Linked pixels.
(Courtesy of Mr. D. R. Cate, Texas Instruments, Inc.)

Chapter 10 Image Segmentation



FIGURE 10.25
Image of noisy
chromosome
silhouette and
edge boundary
(in white)
determined by
graph search.

Chapter 10 Image Segmentation



a b

FIGURE 10.26 (a) Gray-level histograms that can be partitioned by (a) a single threshold, and (b) multiple thresholds.