13.3 The camera model used for 3-D measurements

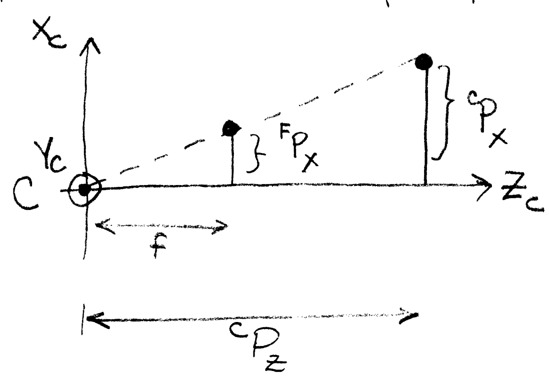
$$^{I}P = {}^{I}_{W}C \; {}^{W}P$$

- ${}^{W}P$: measured world coordinates
- ${}^{I}_{W}C$: the camera matrix from world to image coordinates
- ${}^{I}P$: measured (imaged) image coordinates

$$\begin{bmatrix} s\,{}^{I}P_r \\ s\,{}^{I}P_c \\ s \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & 1 \end{bmatrix} \begin{bmatrix} {}^{W}P_x \\ {}^{W}P_y \\ {}^{W}P_z \\ 1 \end{bmatrix}$$

usually this scale factor (s) is set to 1 as well.

To prove this consider perspective



world & camera coordinates are identical

by similar triangles $\dfrac{^{F}P_x}{f} = \dfrac{^{C}P_x}{^{C}P_z}$ and (not shown) $\dfrac{^{F}P_Y}{f} = \dfrac{^{C}P_Y}{^{C}P_z}$

$$^{F}P_x = \frac{f}{^{C}P_z}\,{}^{C}P_x \qquad {}^{F}P_Y = \frac{f}{^{C}P_z}\,{}^{C}P_Y$$

A camera matrix for perspective

$$\begin{bmatrix} s\,{}^{F}P_x \\ s\,{}^{F}P_Y \\ s\,{}^{F}P_z \\ s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{bmatrix} \begin{bmatrix} {}^{C}P_x \\ {}^{C}P_Y \\ {}^{C}P_z \\ 1 \end{bmatrix}$$

Camera matrix for rotation translation $^{C}P = T(t_x, t_y, t_z)\,R(\alpha, \beta, \gamma)\,{}^{W}P$

The general idea is to model the camera by the geometric transformations followed by a perspective transform.

Three rotations and three translations will look like.

$$\begin{bmatrix} {}^cP_x \\ {}^cP_y \\ {}^cP_z \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^wP_x \\ {}^wP_y \\ {}^wP_z \\ 1 \end{bmatrix}$$

drop this row because there is only a constant value for z.

$$\begin{bmatrix} s\,{}^FP_x \\ s\,{}^FP_y \\ s \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & 1 \end{bmatrix} \begin{bmatrix} {}^wP_x \\ {}^wP_y \\ {}^wP_z \\ 1 \end{bmatrix}$$

does everything but the scaling.

But scaling is a simple matrix and this includes inversion.

$${}^IP = \begin{bmatrix} sr \\ sc \\ s \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -\frac{1}{d_y} & 0 \\ \frac{1}{d_x} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{S} \begin{bmatrix} s\,{}^FP_x \\ s\,{}^FP_y \\ s \end{bmatrix}$$

Final camera matrix

$${}^IP = \underbrace{{}^I_F S}_{\text{scaling}} \; \underbrace{{}^F_C \Pi}_{\text{perspective}} \; \underbrace{{}^C_W TR}_{\text{translation rotation}} \; {}^WP$$

This is an 11-parameter camera model based on affine transforms and perspective

13.4 Best Affine calibration matrix

Use a calibration jig with well known points!

For each calibration point j

$$\begin{bmatrix} su_j \\ sv_j \\ s \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & 1 \end{bmatrix} \begin{bmatrix} x_j \\ y_j \\ z_j \\ 1 \end{bmatrix}$$

or
$$s u_j = c_{11} x_j + c_{12} y_j + c_{13} z_j + c_{14}$$
$$s v_j = c_{21} x_j + c_{22} y_j + c_{23} z_j + c_{24}$$
$$s = c_{31} x_j + c_{32} y_j + c_{33} z_j + 1$$

$$(c_{31} x_j + c_{32} y_j + c_{33} z_j + 1)\, u_j = c_{11} x_j + c_{12} y_j + c_{13} z_j + c_{14}$$
$$(c_{31} x_j + c_{32} y_j + c_{33} z_j + 1)\, v_j = c_{21} x_j + c_{22} y_j + c_{23} z_j + c_{24}$$

Rearranging

$$x_j c_{11} + y_j c_{12} + z_j c_{13} + c_{14} - x_j u_j c_{31} - y_j u_j c_{32} - z_j u_j c_{33} = u_j$$
$$x_j c_{21} + y_j c_{22} + z_j c_{23} + c_{24} - x_j v_j c_{31} - y_j v_j c_{32} - z_j v_j c_{33} = v_j$$

which can be rewritten as

$$\begin{bmatrix} x_j & y_j & z_j & 1 & 0 & 0 & 0 & 0 & -x_j u_j & -y_j u_j & -z_j u_j \\ 0 & 0 & 0 & 0 & x_j & y_j & z_j & 1 & -x_j v_j & -y_j v_j & -z_j v_j \end{bmatrix} \begin{bmatrix} c_{11} \\ c_{12} \\ c_{13} \\ c_{14} \\ c_{21} \\ c_{22} \\ c_{23} \\ c_{24} \\ c_{31} \\ c_{32} \\ c_{33} \end{bmatrix} = \begin{bmatrix} u_j \\ v_j \end{bmatrix}$$

(13.34) of Shapiro

This is an overdetermined set of equations

11 unknowns
≥ 12 equations (i.e., 6 or more points)

Shapiro, p. 433 mentions the pseudoinverse method, but not by name.

Method of evaluating results are residuals

$$\underline{r} = \underline{b} - A\underline{x}$$

$\underline{b}$ ↑ actual image plane coordinates

$A\underline{x}$ ↖ computed image plane coordinates given the world coordinates $\underline{X}$

<u>Methods of calibrating</u>

- calibration jigs – rigid frames w/wires or beads
  rigid boards w/special markings

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Chapter 6
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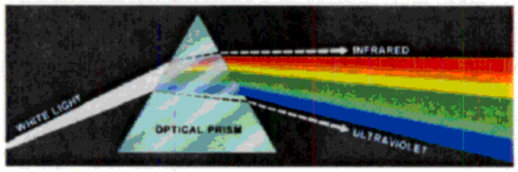

FIGURE 6.1 Color spectrum seen by passing white light through a prism. (Courtesy of the General Electric Co., Lamp Business Division.)

white light split into individual colors using a prism

full-color – image acquired with a full-color sensor

pseudocolor – color assigned to a intensity value or range

Chapter 6
Color Image Processing

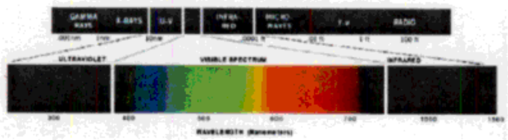

FIGURE 6.2 Wavelengths comprising the visible range of the electromagnetic spectrum. (Courtesy of the General Electric Co., Lamp Business Division.)

blue red

physical units

radiance (watts) – total energy emitted by a light source

luminance (lumens) – incoming energy as measured by the detector

subjective units

brightness

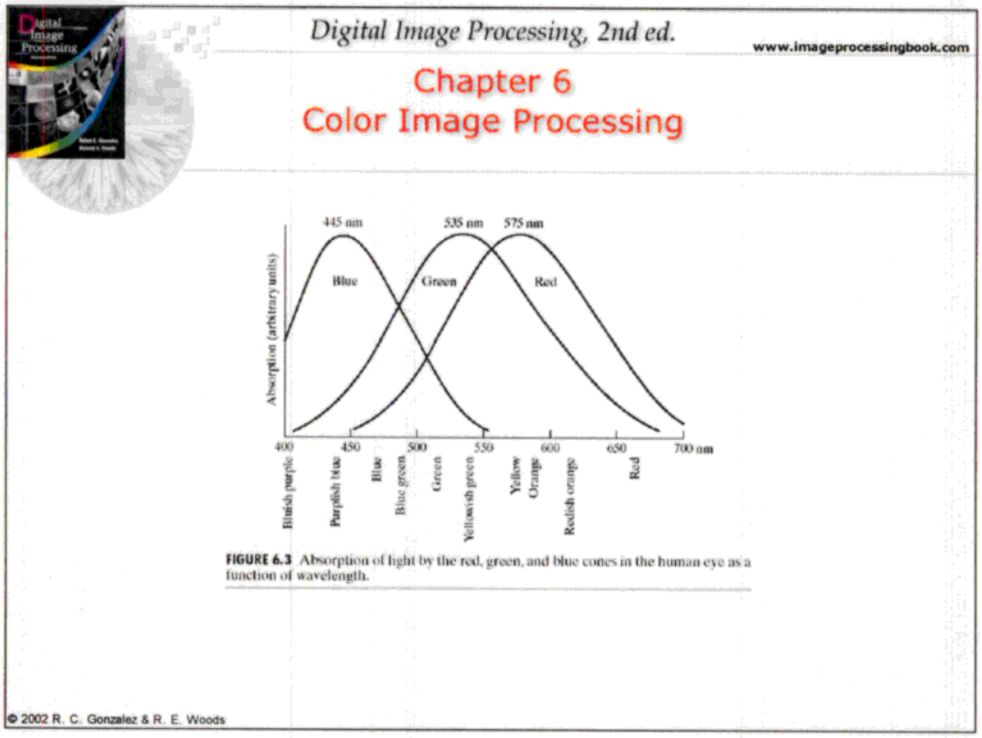

FIGURE 6.3 Absorption of light by the red, green, and blue cones in the human eye as a function of wavelength.

Spectral response of human eye

<u>primary colors</u> do not quite correspond to human eye sensitivity

red (700nm) — 65% of all cones sensitive to red

green (546.1nm) — 33% " green

most sensitive → blue (435.8nm) — 2% " blue

↑ wavelengths were designated in 1931, before good biological data like Fig. 6.3 was available.

Chapter 6
Color Image Processing

(primary)
Additive
colors
transmission

} red-green-blue
television monitors

subtractive
colors
(secondary)
reflection
absorbs a primary color and

} cyan-magenta-yellow
printing
need black in printing

as an example cyan absorbs all red (pigment)

a
b

FIGURE 6.4 Primary and secondary colors of light and pigments. (Courtesy of the General Electric Co., Lamp Business Division.)

yellow – absorbs blue
and transmits
red + green = yellow

<u>characteristics of color</u>

1. brightness

chromaticity {
2. hue — dominant color seen by an observer
3. saturation — amount of white light mixed with the color
}

color $\triangleq$ brightness + chromaticity

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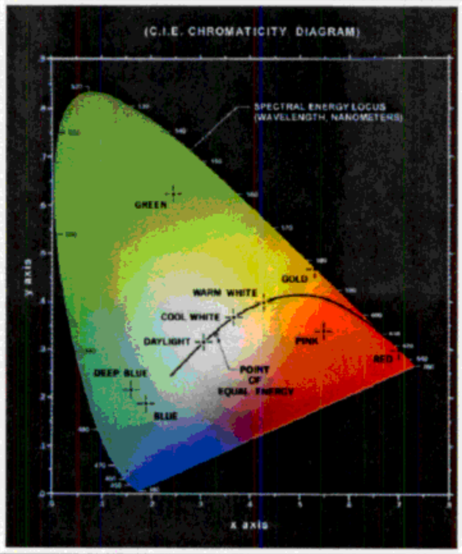
Chapter 6
Color Image Processing

FIGURE 6.5 Chromaticity diagram. (Courtesy of the General Electric Co., Lamp Business Division.)



any point on boundary is fully saturated (saturation = 1)

green ↑

→ red

+green ≈ (.25red, .62green)

point of equal energy – white (.33red, .33green)

Several kinds of white
warm – more red
cool – more blue
natural – (daylight)

A color can be specified by its tristimulus values

This specifies the chromaticity

$$\text{red} \quad x = \frac{X}{X+Y+Z}$$

$$\text{green} \quad y = \frac{Y}{X+Y+Z}$$

$$\text{blue} \quad z = \frac{Z}{X+Y+Z}$$

where $x+y+z=1$

where X, Y, Z are the amounts of red, green, and blue needed to form a color

CIE chromaticity diagram
just specify x, y (red, green) since blue is then determined by $1-x-y$.

point of equal energy = CIE standard for white light (saturation = 0 here)

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Chapter 6
Color Image Processing

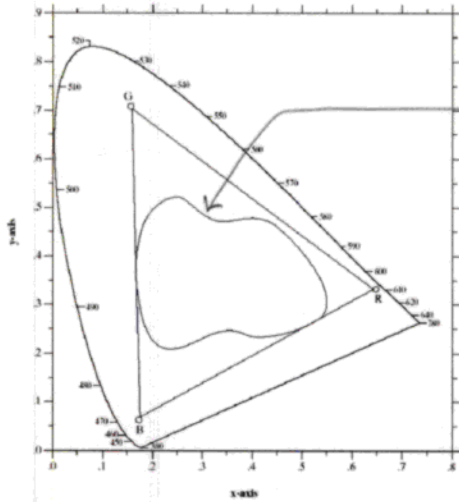


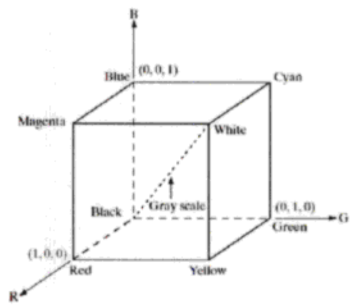
FIGURE 6.6 Typical color gamut of color monitors (triangle) and color printing devices (irregular region).

irregular area is typical color printing area.

The RGB points represent the maximum RGB values of an RGB monitor. Since a monitor is an additive process this RGB monitor can create any color within the triangle.

Chapter 6
Color Image Processing

FIGURE 6.7 Schematic of the RGB color cube. Points along the main diagonal have gray values, from black at the origin to white at point (1, 1, 1).



diagonal has equal amounts of R, G, B so it is gray.

RGB color model

depth is the number of bits used in total

For example, 8-bit RGB is $8 \times 3 = 24$ bit depth.

Three most common color models

- RGB – color monitors
- CMY or CMYK – color printing
- HSI – decouples color and gray scale.