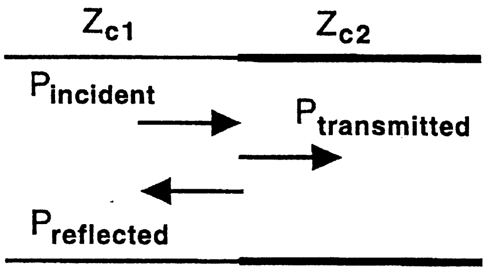
measure the time ΔT that it takes for the reflected pulse to return. From this data, we can compute exactly the unknown distance D from the pulse generator where the reflection took place since $\Delta T = 2D/c$. Imagine that you are trying to locate a fault in an integrated circuit or a short circuit in a cable that is buried underground. Knowing where to probe or dig will save many hours of frustration. This practical technique is called *time domain reflectometry.*

Example 6-14 Using the reflection coefficient $\mathcal{R}$ given in (6.57) and the transmission coefficient $\mathcal{T}$ given in (6.68), show that power is conserved at the junction between two lossless transmission lines.



Answer: Conservation of power implies that

$$P_{\text{inc}} = P_{\text{ref}} + P_{\text{trans}}$$

In terms of the voltages, this becomes

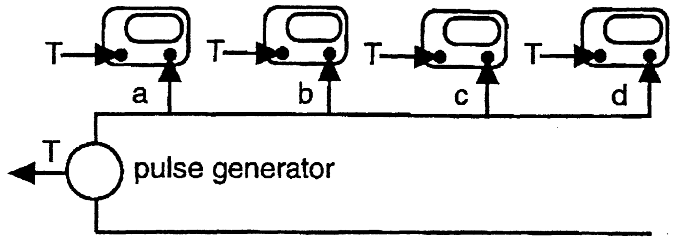
$$\frac{(V_{\text{inc}})^2}{Z_{c_1}} = \frac{(V_{\text{ref}})^2}{Z_{c_1}} + \frac{(V_{\text{trans}})^2}{Z_{c_2}}$$

or

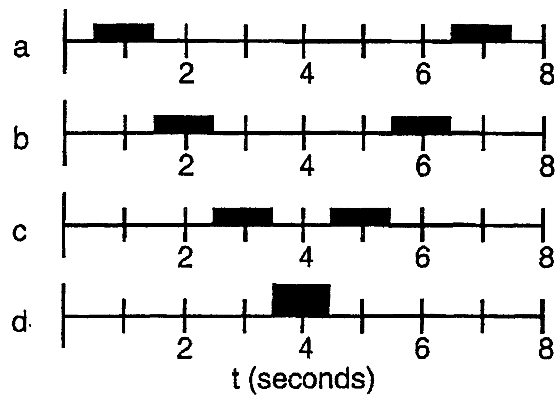
$$1 = \left(\frac{V_{\text{ref}}}{V_{\text{inc}}}\right)^2 + \left(\frac{V_{\text{trans}}}{V_{\text{inc}}}\right)^2 \frac{Z_{c_1}}{Z_{c_2}} =$$

$$= \left(\frac{Z_{c_2} - Z_{c_1}}{Z_{c_2} + Z_{c_1}}\right)^2 + \left(\frac{2Z_{c_2}}{Z_{c_2} + Z_{c_1}}\right)^2 \left(\frac{Z_{c_1}}{Z_{c_2}}\right) = 1$$

Example 6-15. A 1 volt pulse propagates from $z < 0$ on a transmission line. The line is terminated in an open circuit @ $z = 0$. Four oscilloscopes are triggered by the same pulse generator and are located at: $z_a = -6$; $z_b = -4$; $z_c = -2$; and $z_d = 0$ (meters). Find the velocity of propagation and interpret the voltage signals on the oscilloscopes. Sketch the corresponding signals if the transmission line is terminated in a short circuit.



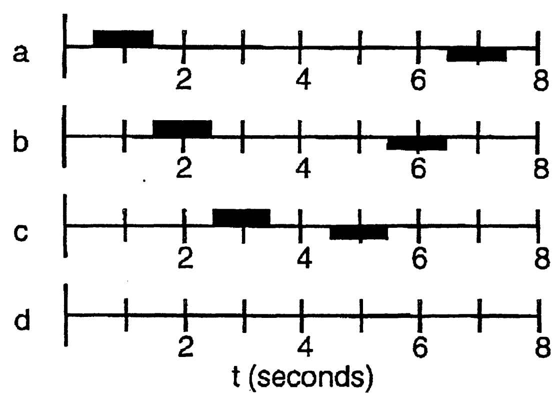
Answer: From the traces on oscilloscopes *a* and *b*, we find the velocity of propagation to be $v = \Delta z / \Delta t$ = (2 meters) / (1 second) = 2 m / s.



Oscilloscope *d* is at the location of the open circuit and the incident and the reflected pulses add together. The signals that are detected after t = 4 seconds are the reflected pulses that propagate toward the pulse generator.

The voltage signals detected by the oscilloscopes if the transmission line is terminated in a short circuit are depicted below. The voltage across the short circuit must be zero, hence the signal at oscilloscope *d* is zero.

This can be also shown using the Simulink application in MATLAB. This is described in Appendix D.



Answer: From (5-79), we write

$$k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2} = \sqrt{\left(\frac{\omega}{\left(\frac{c\pi}{a}\right)}\right)^2 - 1} = \sqrt{\left(\frac{\omega}{\omega_c}\right)^2 - 1}$$

$$k_z = \sqrt{\left(\frac{2\pi \times 1\,\text{GHz}}{2\pi \times 1.2\,\text{GHz}}\right)^2 - 1} = \pm j0.55$$

The additional factor of 'j' implies that the wave will attenuate as $e^{-(0.55)\,z}$.

Example 5-19. Calculate the cutoff frequency for the two lowest-order modes of a parallel plate waveguide where the plates are separated by 3 cm.

Answer: The cutoff frequency is defined from $\omega_c = \frac{n\pi}{a}c$. Hence

$$f_c = \frac{1}{2\pi}\frac{n\pi}{a}c = \frac{1}{2\pi}\frac{\pi}{.03\text{ m}}3\times 10^8\frac{\text{m}}{\text{s}} = 5\times 10^9\,\text{Hz} = 5\text{ GHz}$$

for n = 1. The second mode has n = 2. The cutoff frequency for this mode is

$$f_c = \frac{1}{2\pi}\frac{2\pi}{.03\text{ m}}3\times 10^8\frac{\text{m}}{\text{s}} = 10\times 10^9\,\text{Hz} = 10\text{ GHz}$$

Waves with frequencies between 5 GHz and 10 GHz will propagate only in the lowest n = 1 mode. Waves with frequencies above 10 GHz could propagate in either mode (n = 1 or n = 2).

Wave equation via MATLAB

The numerical solution of the wave equation is a formidable task. One quickly encounters numerical difficulties that are beyond the scope of this text. Fortunately for us, we can be carried on the shoulders of giants in our first encounter with these potential pitfalls. Herein, we will introduce a path through this jungle and develop a numerical program that is written in MATLAB. The resulting figures should aid our understanding of wave phenomena. In Appendix D, we show that waves that propagate on a transmission line can also be investigated with the Simulink feature of MATLAB.

In order to develop a numerical solution for the one dimensional wave equation (5.10),we initially solve a first order partial differential equation. This equation is sometimes called the *advection equation*

$$\frac{\partial \varphi}{\partial t} + c\frac{\partial \varphi}{\partial z} = 0 \qquad (5.83)$$

For the initial condition

$$\varphi\ (z, t = 0) = F(z), \qquad (5.84)$$

the analytical solution of the advection equation is given by

$$\varphi = F(z - ct). \qquad (5.85)$$

We have seen this previously.

Both the wave equation and the advection equation belong to the same family that is called a hyperbolic equation. The diffusion equation is in the parabolic equation family and Laplace's and Poisson's equations are in the elliptic equation family. We will focus our attention here on the advection equation as it is simpler and the procedure and some of the pitfalls along with the bridges that cross these pitfalls will be described.

As shown in Figure 5-24, we consider that the space z and time t can be drawn in a three dimensional figure. The amplitude φ of the wave is specified by the third coordinate.

In Figure 5-24, we set up the numerical grid. First, we have broken the region L in which the wave propagates into N sections. In the figure, we have chosen $N = 4$. Hence we write

$$h \equiv \frac{L}{N} \qquad (5.86)$$

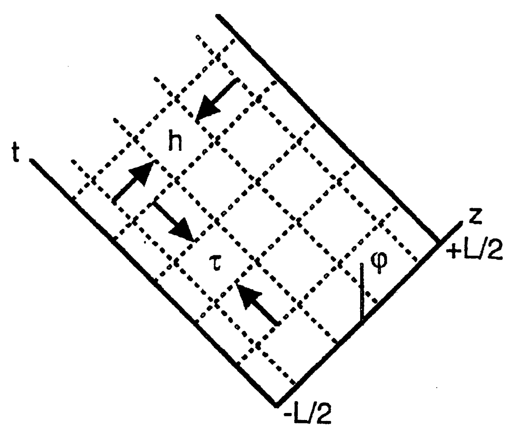


Figure 5-24. Numerical grid that uses periodic boundary conditions.

We assume that the velocity of propagation is c and that it takes a time τ for the wave to propagate a distance h. Therefore

$$h = c\tau \tag{5.87}$$

With these restrictions, we will jump over numerical stability reservations that were noted by Courant–Fredrichs–Lewy (CFL). We will leave it as excercises to examine the cases where $c\tau \neq$ h.

In addition to stability restrictions, we have also invoked *periodic boundary conditions.* This states that once a numerically calculated wave reaches the boundary at $z = +L/2$, it reappears at the same time at $z = -$ L/2 and continues to propagate in the region $-L/2 \leq z \leq +L/2$. As shown in Figure 5-24, we actually do not evaluate the wave at these two edges but at one-half of a spatial increment $h/2$ removed from them at $z = -L/2 + h/2$ and at $z = +L/2 - h/2$.

Let us now convert the advection equation (5.83) to the finite difference form that can be handled by the computer. The time derivative is replaced using the *forward difference method* that was introduced in Chapter 3.

$$\frac{\partial\varphi}{\partial t} \Rightarrow \frac{\varphi(z_i, t_n + \tau) - \varphi(z_i, t_n)}{\tau} \tag{5.88}$$

In this notation with reference to Figure 5-23, we have

$$z_i = (i - 1/2)h - L/2 \text{ and } t_n = (n - 1)\tau \tag{5.89}$$

The space derivative is replaced using the *central difference method.*

$$\frac{\partial\varphi}{\partial z} \Rightarrow \frac{\varphi(z_i + h, t_n) - \varphi(z_i - h, t_n)}{2h} \tag{5.90}$$

Substitute (5.88) and (5.90) into the advection equation (5.83) and obtain

$$\frac{\varphi(z_i, t_n + \tau) - \varphi(z_i, t_n)}{\tau} = -c\frac{\varphi(z_i + h, t_n) - \varphi(z_i - h, t_n)}{2h} \tag{5.91}$$

In (5.91), three of the four terms are evaluated at the same time t_n and one term is evaluated at the next increment in time $t_n + \tau$. From (5.91), we write this term as

$$\varphi(z_i,t_n+\tau)=\varphi(z_i,t_n)-\frac{c\tau}{2h}\left[\varphi(z_i+h,t_n)-\varphi(z_i-h,t_n)\right] \tag{5.92}$$

This is valid in the interior range $2 \le n \le N-1$. In (5.92), we note that all values are known initially at the time $t_0 = 0$. Hence, we use (5.92) to evaluate the values at the next increment in time. With the imposition of *periodic boundary conditions*, we must carefully use (5.92) in order to find the values at the boundaries. This manifests itself with the requirement that

$$\left.\begin{aligned}\varphi(z_1,t_n+\tau)&=\varphi(z_1,t_n)-\frac{c\tau}{2h}\left[\varphi(z_2,t_n)-\varphi(z_N,t_n)\right]\\ \varphi(z_N,t_n+\tau)&=\varphi(z_N,t_n)-\frac{c\tau}{2h}\left[\varphi(z_1,t_n)-\varphi(z_{N-1},t_n)\right]\end{aligned}\right\} \tag{5.93}$$

Example 5-20. Use (5.92) and (5.93) to find the evolution of a square pulse whose initial shape is defined by

$$\left.\begin{aligned}&\varphi\left(-\frac{h}{2}\le z\le+\frac{h}{2},t=0\right)=1\\ &\varphi\left(|z|>\frac{h}{2},t=0\right)=0\end{aligned}\right\}$$

Use the grid depicted in Figure 5-24. The stability requirement $h = c\tau$ is also to be invoked in this calculation.

Answer: We tabulate the computed values to be

z \ t	0	τ	2τ	3τ	4τ	5τ
$-\frac{3L}{8}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{5}{2}$	$+\frac{9}{2}$
$-\frac{L}{8}$	1	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$	$+\frac{1}{2}$
$+\frac{L}{8}$	1	$+\frac{3}{2}$	$+\frac{3}{2}$	$+\frac{1}{2}$	$-\frac{3}{2}$	$+\frac{1}{2}$
$+\frac{3L}{8}$	0	$+\frac{1}{2}$	$+\frac{3}{2}$	$+\frac{5}{2}$	$+\frac{5}{2}$	$+\frac{9}{2}$

Note that the signal becomes distorted and increases in value as it propagates. It is *unstable* for every value of time! Our imposition of the stability requirement that $h = c\tau$ did not insure stability in this case!

Fortunately for us, there is a simple solution to the instability problem that is in Example 5-23. This is the Lax method. It replaces (5.92) with a slightly different iteration equation.

$$\varphi(z_i,t_n+\tau)=\frac{1}{2}\left[\varphi(z_i+h,t_n)+\varphi(z_i-h,t_n)\right]-\frac{c\tau}{2h}\left[\varphi(z_i+h,t_n)-\varphi(z_i-h,t_n)\right] \quad (5.94)$$

The first term on the right side is the *average* of the two neighboring terms. Similarly, the two equations that represent the periodic boundary conditions are modified to

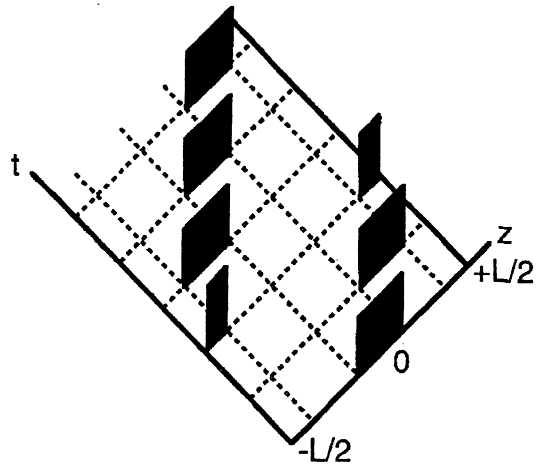
$$\left.\begin{aligned}\varphi(z_i,t_n+\tau)&=\frac{1}{2}\left[\varphi(z_2,t_n)+\varphi(z_N,t_n)\right]-\frac{c\tau}{2h}\left[\varphi(z_2,t_n)-\varphi(z_N,t_n)\right]\\ \varphi(z_N,t_n+\tau)&=\frac{1}{2}\left[\varphi(z_1,t_n)-\varphi(z_{N-1},t_n)\right]-\frac{c\tau}{2h}\left[\varphi(z_1,t_n)-\varphi(z_{N-1},t_n)\right]\end{aligned}\right\} \quad (5.95)$$

Example 5-21. Repeat Example 5-20 using the Lax method and sketch the solution.

Answer: Using (5.94) and (5.95), we compute and tabulate

z \ t	0	τ	2τ	3τ	4τ	5τ
$-\frac{3L}{8}$	0	0	+1	+1	0	0
$-\frac{L}{8}$	+1	0	0	+1	+1	0
$+\frac{L}{8}$	+1	+1	0	0	+1	+1
$+\frac{3L}{8}$	0	+1	+1	0	0	+1

Note that in this case, we have stability. In addition, the pulse is not distorted as it propagates. We plot the solution as



In MATLAB language, we write (5.94) and (5.95) in three steps. The first step finds the new interior values of j in terms of the previous interior values. This iterations are specified to have iterations in the range: $2 \leq i \leq (N-1)$. We write this as

phinew (2:(N–1)) = .5•((phi(3:N)+phi(1:(N–2))+(ct/2h)(phi(3:N)–phi(1:(N–2))

The remaining two steps take care of the periodic boundary conditions at $i = 1$ and at $i = N$. This is written as

phinew(1) = .5•(phi(2)+phi(N))+(ct/2h)(phi(2)–phi(N))
phinew(N) = .5•(phi(1)+phi(N-1))+(ct/2h)(phi(1)–phi(N–1))

Example 5-22. Develop a MATLAB program to illustrate the propagation of a pulse. In addition, the first and the last calulations should be displayed in order to show the stability of the calculation.

Answer. We write

```
%This program illustrates the propagation of a pulse.
clear; clg;
%initial numerical values
      tau=.02;            %size of time step
      N=50;               %number of grid points
      L=1.;               %size of system
      h=L/N;              %spacing of grids = 1/50 =.02
      c=1;                %wave speed

%coefficients required for the Lax method. The time tau equals the time
%that it takes the wave to travel across the grid dimension h = c•tau.
      coef=-ctau/(2h);              %coef=-1/2
```

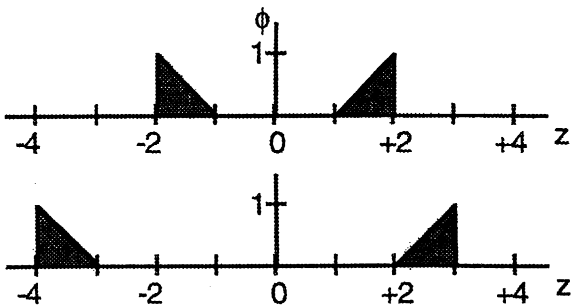
```
%initial pulse shape
    sig=0.1;                        %width of pulse
    z=((1:N)-1/2)•h-L/2;            %value of z @ t = 0
    phi=(sech(z/(2•sig^2))).^2;     %wave shape

%plot variables
    ip=1;                           %plot counter
    phipl(:,1)=phi(:);              %initial state
    tpl(1)=0;                       %initial time

%main loop
    nstep=floor(L/(c•tau));         %number of steps
    plotstep=ceil(nstep/50);        %number of steps between plots

    for ist=1:nstep

        %Lax scheme

        phinew(2:(N-1))=phi(2:(N-1))+coef(phi(3:N)-phi(1:(N-2)))-...
            coef(phi(3:N)+phi(1:(N-2))-2phi(2:(N-1)));

        phinew(1)=phi(1)+coef(phi(2)-phi(N))-...
            coef(phi(2)+phi(N)-2phi(1));

        phinew(N)=phi(N)+coef(phi(1)-phi(N-1))-...
            coef(phi(1)+phi(N-1)-2phi(N));
        phi=phinew;

        if(rem(ist,plotstep))<1
            ip=ip+1;
            phipl(:,ip)=phi(:);  %record for plotting
            tpl(ip)=tauist;
            fprintf('%g out of %g steps completed\n',ist,nstep);
        end
    end
    subplot
    %show first and last calculated value for comparison
        plot(z,phipl(:,1),'-',z,phi,'--');
        xlabel('z','fontsize',18);
        ylabel('amplitude','fontsize',18);
        text(-.4,.75,'(a)','fontsize',18);
        pause;
    subplot(1,2,2)
    %3-d plot of wave
```

Problems

1. In terms of the fundamental units mass M, length L, time T, and change Q, show that $(\mu_0\varepsilon_0)^{-1/2}$ has the units of a velocity (L/T).

2. Let $F(z - ct) = 1$ and $G(z + ct) = -2$ for $|z - ct| \leq 1$ and $|z + ct| \leq 1$ respectively and $F(z - ct) = G(z + ct) = 0$ elsewhere. Accurately sketch the pulse if the velocity $c = 2$ at three times: $t = 0$, $t = 1$, and $t = 3$.

3. Define the functions $F(z-c_1t)$ and $G(z+c_2t)$ from the following sketch which was drawn at the times $t = 0$ and $t = 2$.



4. If the waves in problem 3 were electromagnetic waves, find the ratio of the dielectric constants ε_1 and ε_2 for the two regions $(z < 0)/(z > 0)$ if the relative permeabilities were 1 in the two regions.

5. A displacement wave on a string is described by $0.02 \sin[2\pi(10t - 0.5z)]$ m, where z is in meters and t is in seconds. Find:
   (a) The propagation velocity.
   (b) Wavelength λ and wave number k.
   (c) Frequency f and angular frequency ω.
   (d) The period.
   (e) Direction of propagation.
   (f) Amplitude of the wave.

6. Plot the wave given in Problem 5 as a function of z at $t =$ (a) 0 sec., (b) 0.125 sec., (c) 0.25 sec., and (d) 0.375 sec. Convince yourself that the wave pattern progresses in the positive z direction as time increases.

7. Assume that a wave reflector was installed at $z = 5$ in Figure 5–3. This reflector causes a positive amplitude pulse to be reflected as a positive amplitude pulse. Reflection implies that a wave traveling to increasing values of z would start traveling to decreasing values of z after reflection. Accurately sketch the expected oscilloscope pictures depicted in part a and trajectory

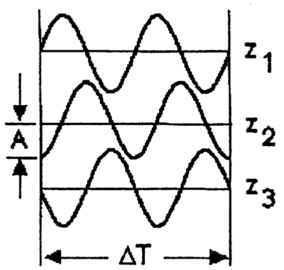
depicted in part b to include the reflected waves. You may assume that the reflected waves are absorbed by the wave maker.

8. Repeat problem 7 with a new wave maker. This wave maker causes an incident positive (or negative) wave to be reflected as a negative (or positive) wave. Examine the waves during the times $0 \leq t \leq 15$.

9. Show that a pulse defined by $f(z, t) = 0.5 \exp[-(z - 5t)^2]$ satisfies a wave equation. Plot this function as a function of z for the three times: $t = 0$, 0.5 sec., and 1.0 sec.

10. Show that (5.20) and (5.21) are related expressions.

11. Snapshots of two cycles of an electromagnetic wave propagating in a vacuum at three locations $z_1 = 0$ cm; $z_2 = 2.25$ cm; and $z_3 = 4.5$ cm are shown. Let $A = 1$ V/m. Find ΔT. Write the equation that describes the electric field.



12. If we know that the magnetic field intensity of an electromagnetic wave is $\mathbf{H} = H_0\, e^{j(\omega t + kz)}\, \mathbf{u}_y$, find the electric field and the direction of power flow.

13. An electromagnetic wave with a frequency $f = 10^6$ Hz propagates in a dielectric material ($\varepsilon_r = 5$, $\mu_r = 1$) and it has an electric field component $E_y = 1.3 \cos(\omega t - kz)$. Find the velocity of the wave, the wave number, **H**, and the characteristic impedance of the material.

14. A helium–neon laser emits light at a wavelength of 6328 Å $= 6.328 \times 10^{-7}$ m in air. Calculate the frequency of oscillation of the laser, the period of the oscillation and the wave number. The symbol Å is called an Ångstrom where 1 Å $= 10^{-10}$ m.

15. Prove that **E** and **H** are orthogonal in a vacuum for an arbitrary function of $(z - ct)$.

16. The electric field of a uniform plane wave propagating in air is given by $\mathbf{E} = E_0(\mathbf{u}_x + j\mathbf{u}_y) \cos(\omega t - kz)$. Using an accurately drawn sketch, show that it is justified to call this wave *circularly polarized.*

17. The electric field of a uniform plane wave propagating in air is given by $\mathbf{E} = \mathbf{E}_0(\mathbf{u}_x + j\, a\, \mathbf{u}_y) \cos(\omega t - kz)$. ($0 < a < \infty$, $a \neq 1$) Using an accurately drawn sketch with $a = 2$, show that it is justified to call this wave *elliptically polarized.*

18. Show that an elliptically polarized wave can be decomposed into two circular polarized waves, one rotating clockwise and the other rotating counterclockwise. This is facilitated if we can find a relation between the complex constants in the two expressions

$$\mathbf{E} = (a\mathbf{u}_x + b\mathbf{u}_y)e^{j(\omega t - kz)}$$

and

$$\mathbf{E} = \left(\hat{a}\mathbf{u}_x + j\hat{a}\mathbf{u}_y\right)e^{j(\omega t - kz)} + \left(\hat{b}\mathbf{u}_x - j\hat{b}\mathbf{u}_y\right)e^{j(\omega t - kz)}$$

19. The magnetic field intensity $\mathbf{H} = -H_0\mathbf{u}_x e^{j(\omega t - kz)}$. Find the electric field. Compute the Poynting vector.

20. In free space, a signal generator launches an electromagnetic wave that has a wavelength of 10 cm. As the same wave propagates in a material, its wavelength is reduced to 8 cm. In the material, the amplitude of the electric field **E** and the magnetic field intensity **H** are measured to be 50 V/m. and 0.1 A/m. respectively. Find the generator frequency and μ_r and ε_r for the material.

21. In free space, a signal generator launches an electromagnetic wave that has a wavelength of 3 cm. As the same wave propagates in a material, its wavelength is reduced to 2 cm. In the material, the amplitude of the electric field **E** and the magnetic field intensity **H** are measured to be 5 V/m. and 0.1 A/m. respectively. Find the generator frequency and μ_r and ε_r for the material.

22. Find explicit expressions for the attenuation constant α and the propagation constant β for an electromagnetic wave propagating in a conducting material.

23. Find the attenuation constant α if the conductivity σ of the material is such that $\sigma = \omega\varepsilon$.

24. Show that Maxwell's equations can be cast in the form of a diffusion equation

$$\frac{\partial^2 \mathrm{E}_y}{\partial z^2} - \mu\sigma\frac{\partial \mathrm{E}_y}{\partial t} = 0$$

Describe when this derivation might be valid.

25. With the substitution $\xi = \frac{z}{\sqrt{At}}$ and using the chain rule, show that the partial differential equation given in problem 24 will transform into an ordinary differential equation. In terms of the fundamental units mass M, length L, time T and charge Q, find the units of the diffusion coefficient $D = 1/\mu\sigma$.

26. An electromagnetic wave with an amplitude of 1 V/m is normally incident from a vacuum into a dielectric having a relative dielectric constant $\varepsilon_r = 4$. Find the amplitude of the reflected and the transmitted electric fields.

27. An electromagnetic wave with an amplitude of 1 V/m is normally incident from a vacuum into a dielectric having a relative dielectric constant $\varepsilon_r = 4$. Find the amplitude of the reflected and the transmitted magnetic field intensities.

28. An electromagnetic wave with an amplitude of 1 V/m is normally incident from a dielectric having a relative dielectric constant $\varepsilon_r = 4$ into a vacuum. Find the amplitude of the reflected and the transmitted electric fields.

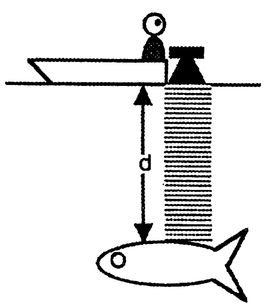
29. An electromagnetic wave with an amplitude of 1 V/m is normally incident from a dielectric having a relative dielectric constant $\varepsilon_r = 4$ into a vacuum. Find the amplitude of the reflected and the transmitted magnetic field intensities.

30. In Example 5–12, a speeder pleads to the judge that because of inclement weather when the radar was tested and calibrated, the calibration was incorrect. If the radar assumed a calibration in a vacuum and said the speeder was traveling at 25% over the speed limit, what would the dielectric constant of ambient space have to be in order that the defendant would go free?

31. A time-harmonic electromagnetic wave in a vacuum is incident upon an ideal conductor located at $z = 0$ and a standing wave is created in the region $z \leq 0$. With a crystal detector connected to a volt meter, we measure a null voltage at equal increments of 10 cm in the region $z \leq 0$. Find the frequency of oscillation of the electromagnetic wave.

32. Compute the skin depth of copper, graphite and germanium at $f = 2.45$ GHz.

33. A fisherman in the sea detects a fish at a depth d with a radar operating at a frequency f. Find d if the detected amplitude just below the air–sea interface is 1% of the incident amplitude at the same point. Assume that fish scales are perfect conductors and the conductivity of the water σ satisfies $\sigma \ll \omega\varepsilon$.



34. Estimate the number of wavelengths of helium–neon laser light ($\lambda = 6328$ Å) where (1 Å $= 10^{-10}$ m) that can be found between the two parallel end plates which are separated by 1 m. You may assume $\varepsilon_r \approx 1$ between the end plates.

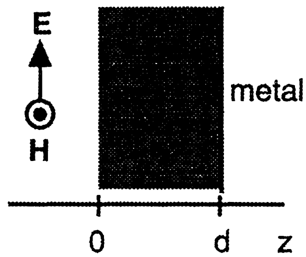
35. The resonant frequency of a Fabry–Perot cavity caused by the introduction of a dielectric is changed from its vacuum value of 10 GHz to 9.9 GHz. Calculate the relative dielectric constant ε_r of the perturbing material.

36. The relative dielectric constant of a slice of lung of thickness ΔL is found to be 1.5. A diseased lung of thickness ΔL as shown in Figure 5–16 is inserted between the plates of a Fabry–Perot cavity. The cavity has a resonant frequency of 9.9 GHz for the undiseased lung and 9.95 GHz for the diseased lung. Find the percentage of the diseased lung that has been eaten away by emphysema..

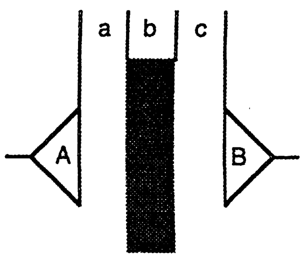
37. A plane wave

$$\mathbf{E}(z,t) = \mathbf{u}_y E_0 \cos(\omega t - kz)$$

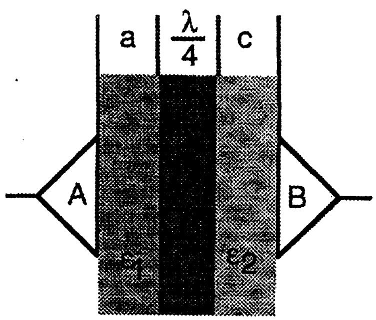
is incident upon a dielectric–metal surface. Determine the thickness d of the dielectric slab (ε_r) that would make the field in the region $z < 0$ the same as the slab were not there.



38 A dielectric slab (ε_r) is inserted between two plane wave launching horns. Waves will be reflected and transmitted at each interface. Determine the ratio of E_B / E_A for a wave that passes through the region b once.

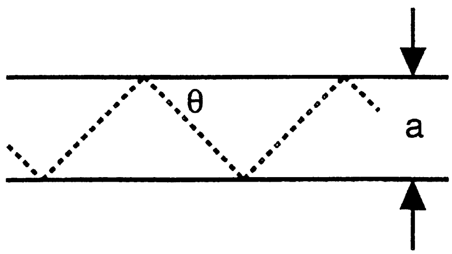


39. A dielectric ε_r that is $\lambda/4$ thick separates two dielectrics. Find the value of ε_r so none of the power launched from A will be reflected back to A.



40. For the infinite parallel plate waveguide depicted in Figure 5–20, determine why an initial assumption for an electric field $E = E_x(x,y)\,\mathbf{u}_x$ will not lead to normal modes.

41. Show that the angle θ between the electric field component wave direction and the conducting sheets in a dielectric filled parallel plate waveguide can be computed from $\theta = \sin^{-1}(\lambda/2a)$ where a is the separation between the two plates.



42. Show that the phase velocity v_ϕ can be written as

$$v_\phi = \frac{c}{\cos\theta} = \frac{1}{\sqrt{\mu_0\varepsilon}\cos\theta}$$

where the angle θ is defined in problem 41.

43. Calculate the cutoff frequency for the two lowest-order modes of a parallel plate waveguide where the plates are separated by 3 cm. The region between the plates is filled with (a) paper or (b) glass.

44. Show that a parallel plate waveguide operating at a frequency equal to the cutoff frequency of a higher order mode can be interpreted in terms of a Fabry-Perot resonator.

45. Repeat Example 5-24 with $(h/c\tau) = 1/2$.

46. Repeat Example 5-24 with $(h/c\tau) = 3/2$.

47. The initial condition at $\tau = 0$ for a wave is

$$\varphi = \varphi_0 \sin(2z)$$

Write a MATLAB program to show the propagation of the wave in the region $0 \le z \le 1$ if $c = 1$.

48. The initial condition at $\tau = 0$ for a wave is

$$\varphi = \varphi_0 \sin(2z)$$

Write a MATLAB program to show the propagation of the wave in the region $0 \le z \le 1$ if $c = 4$.

49. The initial condition at $t = 0$ for a wave is

$$\varphi = \varphi_0 \exp(-2z)$$

Write a MATLAB program to show the propagation of the wave in the region $0 \leq z \leq 1$ if $c = 1$.

50. The initial condition at $\tau = 0$ for a wave is

$$\varphi = \varphi_0 \tanh(2z)$$

Write a MATLAB program to show the propagation of the wave in the region $0 \leq z \leq 1$ if $c = 1$.