Appendix E

Smith chart via MATLAB

The development of and the MATLAB program that yields the Smith chart in Figure 6-15 is given in this appendix. We separate the real and the imaginary parts of the normalized impedance as shown in Figure E-1. The circles that represent the real and the imaginary parts of the normalized impedance will be considered separately.

The radius of the real part circle is r and its center is at $(1-r,\ 0)$. We define the coordinates at a point 1 as

$$\mathcal{R}_{r1} = 1 - r + r\cos\theta$$
$$\mathcal{R}_{i1} = r\sin\theta \qquad \text{(E.1)}$$

The radius of the imaginary part circle is x and its center is at $(1,\ x)$. We define the coordinates at a point 2 as

$$\mathcal{R}_{r2} = 1 + x\cos\psi$$
$$\mathcal{R}_{i2} = x + x\sin\psi \qquad \text{(E.2)}$$

The two circles will intersect at a point defined by

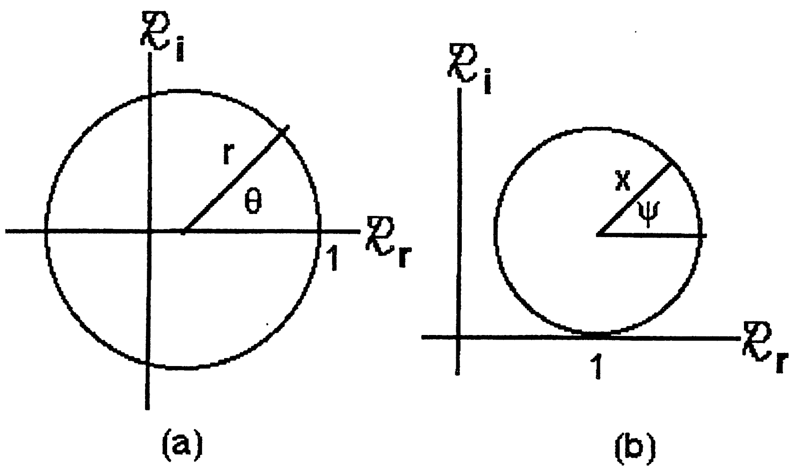

Figure E-1. The circles that represent the real and the imaginary parts of the normalized impedance (a) Real part. (b) Imaginary part.

$$\mathcal{R}_{r2} = \mathcal{R}_{r1} \quad \text{and} \quad \mathcal{R}_{i2} = \mathcal{R}_{i1} \tag{E.3}$$

or

$$\begin{aligned} 1 + x \cos \psi &= 1 - r + r \cos \theta \\ x + x \sin \psi &= r \sin \theta \end{aligned} \tag{E.4}$$

From (E.4), we write

$$\begin{aligned} \cos \psi &= \frac{-r + r \cos \theta}{x} \\ \sin \psi &= \frac{r \sin \theta - x}{x} \end{aligned} \tag{E.5}$$

Using a trig identity, we write

$$1 = \cos^2 \psi + \sin^2 \psi = \left(\frac{-r + r \cos \theta}{x} \right)^2 + \left(\frac{r \sin \theta - x}{x} \right)^2 \tag{E.6}$$

Expanding (E.6), we obtain

$$x^2 = r^2 - 2\, r^2 \cos \theta + r^2 \cos^2 \theta + r^2 \sin^2 \theta - 2rx \sin \theta + x^2 \tag{E.7}$$

or

$$0 = r - r \cos\theta - x \sin \theta \tag{E.8}$$

Expanding (E.8) and using half-angle formulas, we write

$$0 = 2 \sin \frac{\theta}{2} \left(r \sin \frac{\theta}{2} - x \cos \frac{\theta}{2} \right) \tag{E.9}$$

Equation (E.9) will be satisfied if the angle θ is given by

$$\theta = 0 \quad \text{or} \quad \theta = 2 \arctan \left(\frac{\mathrm{x}}{\mathrm{r}} \right) \tag{E.10}$$

Substituting (E.10) into (E.5), we write

$$\psi = \arcsin\left[\frac{r\sin\theta - x}{x}\right] = \arcsin\left[\frac{r\sin\left[2\arctan\left(\frac{x}{r}\right)\right] - x}{x}\right] \tag{E.11}$$

We will employ this relation in writing the MATLAB program.

new program

```
clear
hold on
whitebg
r1=1;r2=1.06;
ang_stp=pi/5;
for ang=0:ang_stp:2*pi
     plot([r1*cos(ang) r2*cos(ang)],[r1*sin(ang) r2*sin(ang)],'k')
end
r1=1 ;r2=1.03;
ang_stp=pi/50;
for ang=0:ang_stp:2*pi
     plot([r1*cos(ang) r2*cos(ang)],[r1*sin(afng) r2*sin(ang)],'k')
end

plot([-1 1],[0 0],'k','linewidth', 1.5)
whitebg
axis equal
axis off
hold on
t=0:.01:2*pi;
plot(cos(t),sin(t),'k','linewidth',2)
text(-1.07,0,'0')
text(-.07,1.07,'+jl')
text(-.07,-1.07,'-jl')
text(-.74,-.85,'-j.5')
text(-.74,.85,'+j.5')
text(.6,-.85,'-j2')
text(.6,.85,'+j2')
text(.96,.39,'+j5')
text(.96,-.39,'-j5')
text(.6,.85,'+j2')
text(.96,.39,'+j5')
```

```
text(.96,-.39,'-j5')
hT=text(.02,.03,'1','fontsize', 10);
hT=text(-.34,.03,'.5','fontsize', 10);
hT=text(.36,.03,'2','fontsize',10);
hT=text(-.65,.03,'.2','fontsize', 10');


for j=(1:11)
      if j==1
            r1=0;  dr=.1; r2=1; x=2; th=1.5;
                  r=sm1 (r1,dr,r2,x,th);
      elseif j==2
            r1=1; dr=.2; r2=2; x=5;  th=.5;
                  r=sm1 (r1,dr,r2,x,th);
      elseif j==3
            r1=.05; dr=.1; r2=.95; x=1; th=.5;
                  r=sm1 (r1,dr,r2,x,th);
      elseif j==4
            r1=1.1; dr=.1; r2=1.9; x=2; th=.5;
                  r=sm1 (r1,dr,r2,x,th);
      elseif j==5
            r1=1;  dr=.5; r2=5; x=2; th=.5;
                  r=sm1 (r1,dr,r2,x,th);
      elseif j==6
            r1=1.1; dr=.2; r2=1.9; x=2; th=.5;
                  r=sm1 (r1,dr,r2,x,th);
      elseif j==7
            r1=5; dr=1; r2=10; x=5; th=.5;
                  r=sm1 (r1,dr,r2,x,th);
      elseif j==8
            r1=1; dr=1; r2=5; x1=-1; x2=1; th=.5;
                  r=sm2(r1,dr,r2,x1,x2,th);
      elseif j==9
            r1=0; dr=.2; r2=1; x1=2; x2=-5; th=.5;
                  r=sm2(r1,dr,r2,x1,x2,th);
      elseif j==10
            r1=0; dr=.5; r2=1; x1=5; x2=-30; th=.5;
                  r=sm2(r1,dr,r2,x1,x2,th);
      else
            r1=5; dr=5; r2=10; x1=-5; x2=-30; th=.5;
                  r=sm2(r1,dr,r2,x1,x2,th);
```

```
        end
end

for j=(1:6)
        if j==1
                r1=0; r2=10; x1=1; dx=1;  x2=5; th=1.5;
                        x=sm4(r1,r2,x1,dx,x2,th);
        elseif j==2
                r1=0; r2=10; x1=5; dx=0; x2=5; th=1.5;
                        x=sm4(r1,r2,x1,dx,x2,th);
        elseif j==3
                r1=0; r2=2; x1=.1; dx=.1; x2=2; th=1.5;
                        x=sm4(r1,r2,x1,dx,x2,th);
        elseif j==4
                r1=0; r2=1; x1=.05; dx=.1; x2=.95; th=.5;
                        x=sm4(r1,r2,x1,dx,x2,th);
        elseif j==5
                r1=0; r2=10; x1=2; dx=.5; x2=5; th=.5;
                        x=sm4(r1,x2,x1,dx,x2,th);
        else
                r1=2; r2=10; x1=5; dx=.5; x2=2; th=.5;
                        x=sm4(r1,r2,x1,dx,x2,th);
        end
end
hold off

% sm1 plots resistance circles.

function r=sm1 (r1,dr,r2,x)
for r=(r1:dr:r2)
        a=1/x;
        b=1/(1+r);
        t1=2*atan (a/b);
        t2=2*pi-t1;
        tr=(t1:.0001:t2);
        plot((1-b)+b*cos(tr),b*sin(tr),'w')
end
```

```
% sm2 plots resistance circles.

function r=sm2(r1,dr,r2,x1,x2)
for r=r1:dr:r2;
      a1=1/x1;
      a2=1/x2;
      b=1/(1+r);
      t1 =2*atan(a1/b);
      t2=2*atan(a2/b);
      tr=(tl:.0001:t2);
      plot((1-b)+b*cos(tr),
      b*sin(tr),'w')
      plot((1-b)+b*cos(tr),-b*sin(tr),'w')
end


% sm3 plots reactance arcs.
function x=sm3(r1,r2,x1,dx,x2)
for x=(x1:dx:x2)
      a=1/x;
      b1=1/(1+r1);
      b2=1/(1+r2);
      tr1=2*atan(a/b1);
      tr2=2*atan(a/b2);
      tx1=asin((b1*sin(tr1)-a)/a);
      tx2=asin((b2*sin(tr2)-a)/a);
      t=pi-tx1:0.0001:pi-tx2;
      plot(1+a*cos(t),a+a*sin(t),'w')
      plot(1+a*cos(t),-a-a*sin (t),'w')
end
```