Appendix D

Simulink simulation of transmission lines

The *Telegraphist's equations* that were discussed in Chapter 6 are two first order partial differential equations that describe the voltage and current wave along the distributed lossless transmission line depicted in Figure D-1. They are written as

$$\left.\begin{aligned}\frac{\partial V}{\partial z} - L\frac{\partial I}{\partial t} = 0\\ \frac{\partial I}{\partial z} - C\frac{\partial V}{\partial t} = 0\end{aligned}\right\} \tag{D.1}$$

where L and C are the inductance and capacitance per unit length respectively. In order to derive (D.1), we have initially used the finite difference scheme of writing the voltage and currents at two adjacent nodes or in two adjacent sections, say $k-1$ and k or alternatively k and $k+1$ that are separated by a finite distance Δz, and then taken the limit as this incremental length $\Delta z \Rightarrow 0$. It is these finite difference representations that will be of interest to us in our intended application rather than the partial differential equations given in (D.1). In addition, the partial derivatives with respect to time $\partial/\partial t$ will be replaced with the Laplace transform variable s. This step is required in order to have the equations conform to the format that is encountered in Simulink.

We first obtain relations for the voltage V_k at the intermediate node k in terms of the voltages at the two adjacent nodes V_{k-1} and V_{k+1}. The current I_k into the node k in Figure D-1 flows through the shunt capacitor or into the next section and it is written as

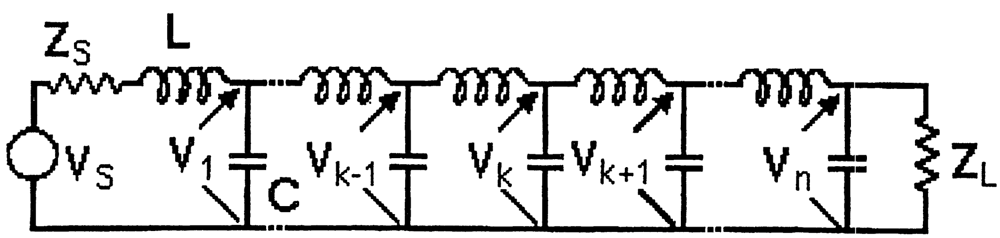

Figure D-1. Distributed transmission lines. The signal generator V_S has an internal impedance Z_S. The load impedance is Z_L. The units of the series inductances L and shunt capacitances C are Henries/unit length and farads/unit length.

$$I_k = C\frac{\partial V_k}{\partial t} + I_{k+1} = sCV_k + I_{k+1} \tag{D.2}$$

The currents I_k and I_{k+1} can be written in terms of the voltages at the appropriate nodes as

$$I_k = \frac{V_{k-1} - V_k}{sL} \quad \text{and} \quad I_{k+1} = \frac{V_k - V_{k+1}}{sL} \tag{D.3}$$

Substituting (D.3) into (D.2) and solving for the voltage V_k, we finally obtain

$$V_k = \frac{1}{2 + s^2 LC}\left[V_{k-1} + V_{k+1}\right] \tag{D.4}$$

Algebraic operations such as addition, subtraction, and multiplication of signals is a feature found in Simulink that we will use in this work.

Equation (D.4) is the basic equation that is required for the simulation of the waves along an infinite homogeneous linear transmission line. The velocity of propagation c of a wave is given by

$$c = \frac{1}{\sqrt{LC}}\left(\frac{\text{sections}}{\text{time}}\right) \tag{D.5}$$

The characteristic impedance Z_c of the transmission line is

$$Z_c = \sqrt{\frac{L}{C}} \ \text{(ohms or } \Omega\text{)} \tag{D.6}$$

As will be shown below, inhomogeneous linear transmission lines can be easily simulated by replacing the terms L and C with L_k and C_k in the derivation of the above equations and then inserting the appropriate numerical values for each section k. If nonlinear waves are to be examined, a more complicated representation for the intermediate section that will be described later is required in the analysis. The representations for the terminating sections, the voltage source with its internal impedance Z_S at one end and the load impedance Z_L at the other end require separate derivations.

The voltage at the first node of the transmission line V_1 will be written in terms of the voltage at the second node and the source voltage V_S. The source voltage in the Simulink library is similar to a function generator found in a laboratory in that the characteristics of the excited signal such as amplitude and frequency can be changed. In addition, the source can be used to excite different signals such as a continuous sine wave, a sine wave burst, or a pulse. The source is also assumed to have a source impedance Z_S. We write the voltage

$$V_S = I_1[Z_S + sL] + V_1 = \left[\frac{V_1 - V_2}{sL} + sCV_1\right][Z_S + sL] + V_1 \tag{D.7}$$

From (D.7), we are able to obtain the voltage V_1 in terms of the voltage V_2 at the next node and the source

$$V_1 = \frac{sL}{s^3 L^2 C + s^2 LC Z_S + s2L + Z_S} V_S + \frac{sL + Z_S}{s^3 L^2 C + s^2 LC Z_S + s2L + Z_S} V_2 \tag{D.8}$$

The voltage V_n across the load impedance Z_L which is in parallel with the final capacitor of a transmission line that contains n sections is determined from

$$V_{n-1} - V_n = sL\, I_n = sL \frac{V_n}{\left(\dfrac{Z_L}{1 + sCZ_L}\right)} \tag{D.9}$$

From (D.9), we write

$$V_n = \frac{Z_L}{s^2 LCZ_L + sL + Z_L} V_{n-1} \tag{D.10}$$

Equations (D.4), (D.8) and (D.10) are in the format that can be analyzed with Simulink™ in that the voltages are related to each other by a term that is a ratio of two polynomials in the Laplace transform variable s. The coefficients of these polynomials can be specified with numerical values in the Simulink menu. As written, these equations represent a distributed linear lossless homogeneous transmission line that is n sections long and terminated at one end with a voltage source with its internal impedance and at the other end with a load impedance. Both impedances could be complex albeit we will assume that they are real in this work.

For the simulation of the current response of the transmission line, the current i_1 in the first loop that includes the voltage source in Figure D-1 can be written in terms of the voltage source V_S and the current in the second loop i_2

$$i_1 = \frac{sC}{s^2 LC + sCZ_S + 1} V_S + \frac{1}{s^2 LC + sCZ_S + 1} i_2 \tag{D.11}$$

The current in an intermediate loop n can written in terms of the current in the previous loop $(n - 1)$ and the following loop $(n + 1)$

$$i_n = \frac{1}{s^2 LC + 2} (i_{n-1} + i_{n+1}) \tag{D.12}$$

The load impedance Z_L is in parallel with the capacitor in the final loop k. The current in this loop is written in terms of the current in the previous loop $(k - 1)$ as

$$i_k = \frac{1 + sCZ_L}{s^3 LC^2 Z_L + s^2 LC + 2sCZ_L + 1} i_k - 1 \tag{D.13}$$

Equations (D.11) to (D.13) determine the elements of a second transmission line.

The complete circuit for the Simulink simulation of a linear homogeneous transmission line that contains $n = 5$ sections is shown in Figure D-2. We have used different values for the total number of sections n in individual examples. The first and the last sections are specified by equations (D-8) and (D-10) respectively and they have their own menu selectability. Each intermediate section is governed by (D-4) and it has its own menu for individual parameter specification. For nonlinear transmission lines, the description of an intermediate section will require more care. Individual sections or a group of sections can be copied, pasted and connected if longer transmission lines are of interest. It is possible to connect an 'oscilloscope' to each node of the transmission line to monitor the wave propagation characteristics. The 'oscilloscope' to which we refer is a standard monitoring tool in Simulink. Rather than use several oscilloscopes, we have found it advantageous to vertically offset signals from juxtaposed nodes with a sequentially increasing stepped 'dc bias voltage' that is added to subsequent signals and observe the propagating wave with one oscilloscope connected to a common bus bar as shown in Figure D-2. This technique will be referred to as a 'bus bar and oscilloscope' in what follows.

In Figure D-3, the simulated wave propagation of a voltage pulse on a linear homogeneous transmission line that consists of $n = 15$ sections is shown. The bus bar and oscilloscope detection scheme was employed. The line was terminated with a source and load impedance $Z_S = Z_L = Z_C$ in order to eliminate any reflection of an incident wave

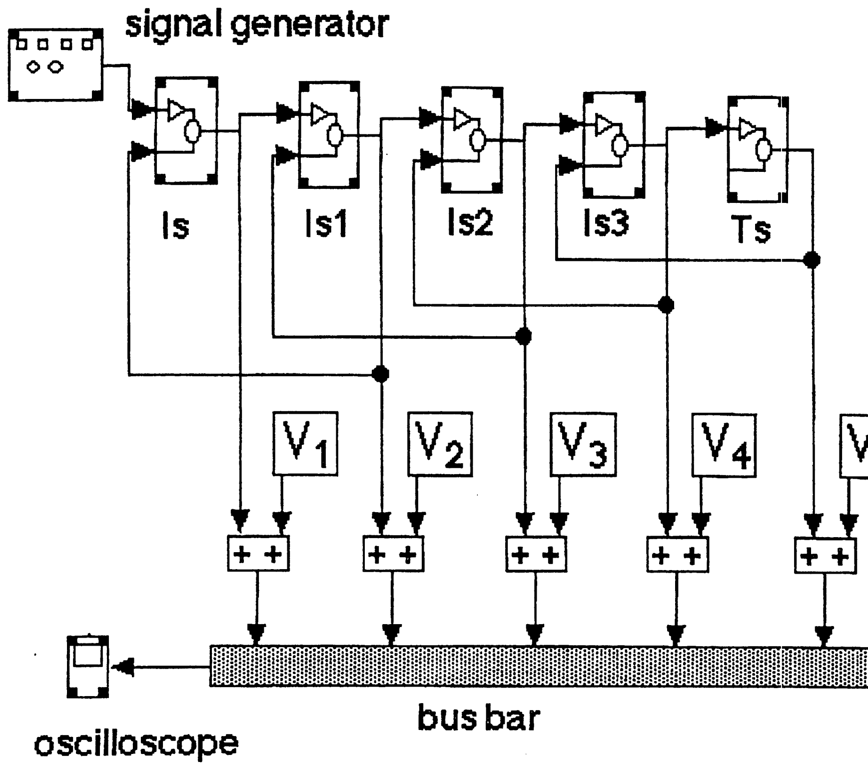

Figure D-2. Simulink simulation of the voltage response of the transmission line shown in Figure D-1. The section Is the initial section [eq D.8]. The sections Is1, Is2, and Is3 are the intermediate sections [eq D.4]. The section Ts is the terminating section. The voltages V_1, V_2, V_3, V_4, and V_5 are used to vertically displace the signals. Their values sequentially increase in equal increments. The number of intermediate sections can be easily changed by copying, pasting and connecting it the desired number of times.

at either terminating end. In Figure D-3, two separate pairs of values for the inductance L and the capacitance C were chosen in order to verify the velocity dependence given in (D.5). Because the source impedance is matched to the characteristic impedance of the transmission line, the amplitude of the propagating wave will only be equal to one-half of the signal generator voltage V_S. Figure D-3a illustrates the propagating wave with the numerical values for the intermediate sections governed by (D.4) of $L = C = 0.2$ and the wave being detected at each section. In this case $Z_C = 1\ \Omega$. The propagation is readily observed. The simulation was repeated with the numerical values $L = C = 0.4$ with all other elements remaining the same. As noted from (D.5) and (D.6), only the velocity of propagation c should decrease by a factor of two and the characteristic impedance should remain at $Z_C = 1\ \Omega$. The results of this simulation are shown in Figure D-3b where the decreased velocity is readily apparent.

The effect of changing the value of the load impedance Z_L was then examined in order to demonstrate that waves can propagate in both directions using this technique.

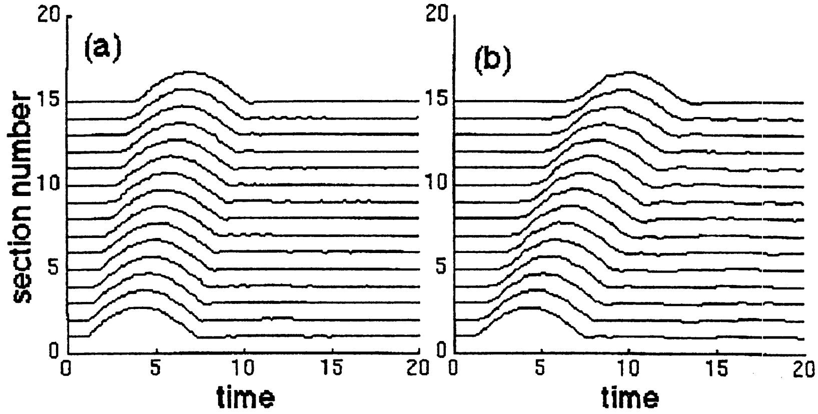

Figure D-3. Simulation of the propagation of a linear voltage pulse. (a) $L = C = 0.2$. (b) $L = C = 0.4$.

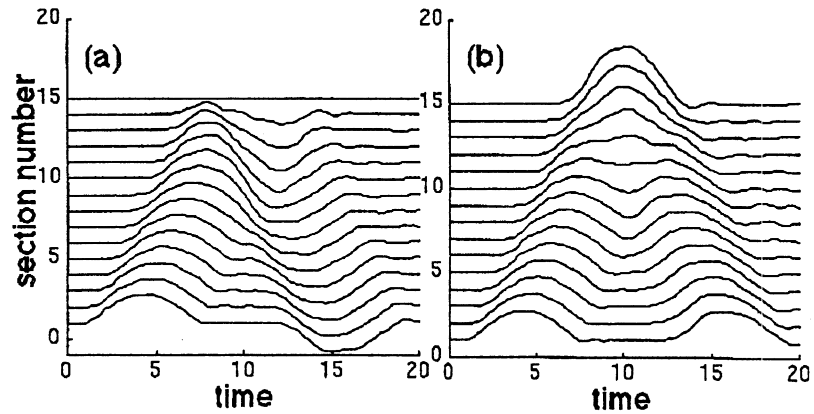

Figure D-4. Simulation of reflection of incident pulses from a load impedance. The transmission line has the values $L = C = 0.4$. (a) Voltage pulse, $Z_L = 0$. (b) Voltage pulse, $Z_L = 100$.

We used the values $L = C = 0.4$ with $Z_L = 0$ [short circuit] and $Z_L = 100$ [open circuit]. The results shown in Figures D-4a and D-4b clearly show the expected behavior for the voltage waves.

Additional examples of wave propagation on linear and nonlinear transmission lines that use this technique are described elsewhere.[1]

[1] B.L. Carter, D.R. Harken, J.H. Parry, S.A. Samson, H.S. Snyder, E.C. Sutton, E.W. Bai, and K.E. Lonngren, "Simulation of linear and nonlinear wave propagation on transmission lines," Electromagnetics, Vol.15, pp.665-677,1995; K.E. Lonngren and E.W. Bai, "Simulink simulation of transmission lines," IEEE Circuits and Devices Journal, Vol.12, No.3, pp.10-15, 1996.