PRINCIPLES AND APPLICATIONS OF Electromagnetic Fields

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PREFACE

In view of the number of texts published in the area of electromagnetic fields for undergraduate students in recent years, another book in this area needs some explanation. Generally, we find that the available books suffer from one or more of the following deficiencies:

1. Inadequate coverage of topics
   a. Very little or no material beyond Maxwell's equations
   b. Omission of important derivations
   c. Lack of depth in treatment
2. Lack of separation or distinction between purely mathematical and physical concepts
3. *Ad hoc* treatment of the subject of fields in the presence of material bodies
4. Insufficient number of examples worked out
5. A lack of problems designed to give the student ability and confidence in developing analytical solutions and extensions to the theory in contrast with routine drill problems

The current text represents the authors' efforts to overcome these shortcomings. We have endeavored to produce a book in electromagnetic theory suitable for undergraduate use that is sophisticated enough to establish a firm basis for advanced study in this area. Furthermore, a sufficient number of applications treated in adequate depth are included to illustrate the basic concepts and also provide a nontrivial background in a diverse number of areas of current interest.

We feel that an adequate text should be sufficiently complete and have enough scope to warrant a place on a personal bookshelf of the undergraduate student after he leaves school. In many cases it is desirable to avoid presentation of a lengthy derivation of certain formulas in class. On the other hand, many students wish to know how a given result is obtained, and in this case an outline of the steps to be followed suffices, provided a detailed derivation is included in the text. We have therefore included some material in the text of a more advanced and sophisticated nature for the advanced or curious student and as a supplement to the main course. The chapters have been organized so that this material is reserved to the last sections. These are set in smaller type and may be omitted by the instructor without detriment to the main continuity of the text.

The first nine chapters of the book constitute the basic principles of electromagnetic theory and are more than ample for a junior- or senior-level course of one-semester duration. The level can be varied somewhat by inclusion or elimination of certain of the topics. The organization of these chapters is fairly standard and includes vector analysis, electrostatics, mathematical techniques in the solution of Laplace's equation, current fields, magnetostatics, and time-varying fields (Maxwell's equations), in that order. The material on vector analysis gives greater emphasis to the relationship between fields and their sources. The climax of this development is in the presentation of the Helmholtz theorem. The flux concept is introduced in the same chapter so that the student will understand its general usefulness in representing any type of vector field. Vector-analysis techniques are freely utilized in the main body of the text.

The theory of electrostatics is developed in Chapter 2 for free-space conditions. In this way the electric field and its relationship to charge sources are developed as a basic formulation. The atomic properties of dielectrics are considered in some detail in Chapter 3. The effect of the presence of dielectrics in an electric field is next explained in terms of the induced dipole sources, from which the equivalent charge source is then determined. This procedure is also followed in Chapters 6 and 7, where the subject of magnetostatics is first developed for currents under free-space conditions. The effect of magnetic materials is then introduced in terms of the induced current sources.

Chapter 4 discusses the method of separation of variables and conformal-mapping techniques for obtaining solutions to Laplace's equation with specified boundary conditions. This chapter includes a discussion of cylindrical and spherical functions and sufficient elementary material on functions of a complex variable to be essentially self-contained. Since this chapter presents no new physical concepts, it may be omitted without detriment to the continuity of the text. Simple solutions to Laplace's equation, including image theory, are included in Chapter 3. The uniqueness theorem is developed in Chapter 2.

Chapter 5 discusses currents and Ohm's law. It presents a formal solution to the computation of the resistance of an arbitrary conducting body. The graphical techniques of flux plotting and the use of the electrolytic tank as auxiliary techniques for solution of Laplace's equation are included here.

Energy storage in electric and magnetic fields is discussed in Chapters 3 and 8, respectively. The calculation of the electric or magnetic force by the principle of virtual work is developed. Finally, the Maxwell stress tensor is introduced, and its significance for the field concept discussed.

In Chapter 9 the displacement current is postulated and Maxwell's equations are formulated. The development of vector and scalar potentials, their relationship to the sources, and retardation effects are then discussed. The chapter concludes with a fairly complete description of the relationship between circuit theory and field theory.

Many fields books written for the undergraduate terminate after introducing Maxwell's equations. Since Maxwell's equations form a climax and the bulk of the practical applications involve time-varying fields, it can hardly be considered a wise choice to terminate the book just when the door to a large variety of interesting and important applications has been opened. We feel that the student can proceed to the application of Maxwell's equations most expediently when he does not have to change texts, since this eliminates the necessity of getting acquainted with another author's notation and way of doing things. For this reason we have included three rather complete chapters on applications. These are on wave guiding, radiation, and interaction of fields with charged particles.

The material on wave guiding is included in Chapter 10. This chapter also discusses plane waves, refraction at a plane interface, and cavity resonators. The technique for accounting for losses at good conductors is considered so that attenuation in waveguides and Q of cavities may be computed. The chapter includes a general treatment of the resolution of fields into TEM, TM, and TE modes. The rectangular and circular waveguides are considered.

The subject of radiation is considered in Chapter 11. The simple linear antenna is described, and the fundamental properties of arrays developed. A full description of the receiving antenna and reciprocity is also included.

The final chapter discusses the interaction of fields with charged particles. This includes the subject of electron ballistics and parallel-plane vacuum-tube theory, including the effect of space charge and transit time. Space-charge-wave theory is developed and applied to the klystron and the traveling-wave tube. In the latter case the helix as a slow-wave structure is described. Propagation in gyrotropic media is considered, with specific application to the ionosphere and ferrites. Finally, an introduction to magnetohydrodynamics is given.

The greatest utility of the book is seen as a full-year course which would substantially cover the entire text. This would provide a very firm foundation in electromagnetic theory and also a good insight into a variety of applications. Chapters 9 to 12 may also be used in a one-semester course on applications of electromagnetic fields if the students have a sufficient background in the fundamentals. As with the full course, this would serve as an introduction to advanced graduate study in special topics. The level of this material may in some instances be deemed suitable for an introductory graduate course.

Objections are at times raised as regards presenting field theory in a layered package, that is, electrostatics, stationary currents, magnetostatics, etc. It certainly is feasible to begin with Maxwell's equations and specialize to static fields and then return to time-varying fields. This is essentially a direct-analysis point of view. By beginning with the experimental laws for static fields and building up and generalizing to time-varying fields the whole approach becomes more of a synthesis procedure. This we feel makes the subject material more acceptable from the student's point of view. It also permits the concept of a vector field to be firmly developed in connection with fields that have a relatively simple behavior, for example, electrostatic field vs. time-varying electromagnetic fields.

We do not particularly feel that relativistic electrodynamics should be introduced at the undergraduate level. The practical applications are few in number, and usually insufficient time is available to give anything more than a brief introduction, which probably leaves the student confused rather than informed in the subject. We have adopted the conventional formulation of considering **E** and **B** as the fundamental force vectors and treat magnetization on the Amperian-current basis. This is in contrast to the view adopted by Professor Chu of the Massachusetts Institute of Technology. Although Professor Chu's formulation has certain features to recommend it when considering the four-vector formulation of electrodynamics, we feel it wise to adhere to the conventional formulation. Not only does this keep our presentation similar to that in most other reference books the student will consult; it is also in keeping with the scope of our presentation.

Since this book is based, to a large extent, on the combined work of many earlier contributors, it is impossible to acknowledge this on an individual basis. We should like, however, to express our thanks to those who assisted us during the preparation of the manuscript. We are greatly appreciative of the help received from colleagues at Case Institute of Technology and in particular from Professors Forest E. Brammer and Robert D. Chenoweth. Many fruitful ideas were also received from students who made use of a preliminary version of this book. We are also grateful to Professor John R. Martin, Acting Chairman of the Department of Electrical Engineering, for making available facilities for the preparation of the manuscript. And finally, we are

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It may be worth noting that a toss of a coin determined the order of the authors' names for the book.

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