Maxwell's Equations (general differential)

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$ [1.11a]

$$\nabla \cdot \bar{D} = \tilde{\rho}$$ [1.11b]

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$ [1.11c]

$$\nabla \cdot \bar{B} = 0$$ [1.11d]

Maxwell's Equations (time harmonic)

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B}$$ [1.11a]

$$\nabla \cdot \mathbf{D} = \rho$$ [1.11b]

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$ [1.11c]

$$\nabla \cdot \mathbf{B} = 0$$ [1.11d]

Maxwell's Equations (integral)

$$\oint_C \bar{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot d\mathbf{s}$$ [1.1]

$$\oint_S \bar{D} \cdot d\mathbf{s} = \int_V \rho dv$$ [1.2]

$$\oint_C \bar{H} \cdot d\mathbf{l} = \int_S \bar{J} \cdot d\mathbf{s} + \int_S \frac{\partial \bar{D}}{\partial t} \cdot d\mathbf{s}$$ [1.3]

$$\oint_S \bar{B} \cdot d\mathbf{s} = 0$$ [1.4]

Electromagnetic Boundary Conditions

$$\hat{\mathbf{n}} \times \left[\bar{E}_1 - \bar{E}_2\right] = 0$$ [1.12]

$$\hat{\mathbf{n}} \times \left[\bar{H}_1 - \bar{H}_2\right] = 0$$ [1.13]

$$\hat{\mathbf{n}} \times \bar{H}_1 = \bar{J}$$ [1.14]

$$\hat{\mathbf{n}} \cdot \left[\bar{D}_1 - \bar{D}_2\right] = \tilde{\rho}_s$$ [1.15]

$$\hat{\mathbf{n}} \cdot \left[\bar{B}_1 - \bar{B}_2\right] = 0$$ [1.16]

$$\hat{\mathbf{n}} \cdot \left[\bar{J}_1 - \bar{J}_2\right] = -\frac{\partial \tilde{\rho}_s}{\partial t}$$ [1.17]

Reflection & transmission (simple dielectric)

$$\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{n_1 - n_2}{n_1 + n_2}$$ [3.2]

$$T = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2n_1}{n_1 + n_2}$$ [3.3]

Basic waves

$$n \equiv \frac{c}{v_p} = \beta \frac{c}{\omega} = \sqrt{\mu_r \varepsilon_r} = \sqrt{\frac{\varepsilon}{\varepsilon_o}}$$ p.135

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_p} = \omega\sqrt{\mu\varepsilon}$$

$$f\lambda = v_p = \frac{1}{\sqrt{\mu\varepsilon}}; \quad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

Uniform plane waves in arbitrary direction

$$\underline{H}(\underline{r}) = \frac{1}{\eta}\hat{k} \times \underline{E}(\underline{r})$$

$$\underline{E}(\underline{r}) = -\eta\hat{k} \times \underline{H}(\underline{r})$$ [2.61]

Reflection & transmission (multiple dielectrics)

$$\Gamma_{eff} = \frac{(\eta_2 - \eta_1)(\eta_3 + \eta_2) + (\eta_2 + \eta_1)(\eta_3 - \eta_2)e^{-2j\beta_2 d}}{(\eta_2 + \eta_1)(\eta_3 + \eta_2) + (\eta_2 - \eta_1)(\eta_3 - \eta_2)e^{-2j\beta_2 d}}$$ [3.7]

$$T_{eff} = \frac{4\eta_2\eta_3 e^{-j\beta_2 d}}{(\eta_2 + \eta_1)(\eta_3 + \eta_2) + (\eta_2 - \eta_1)(\eta_3 - \eta_2)e^{-2j\beta_2 d}}$$ [3.8]

$$Z_2(-d) = \eta_2 \frac{\eta_3 + j\eta_2 \tan(\beta_2 d)}{\eta_2 + j\eta_3 \tan(\beta_2 d)} = \eta_{23}$$ [3.9]

$$\Gamma_{eff} = \rho_{eff} e^{j\phi_r} = \frac{Z_2(-d) - \eta_1}{Z_2(-d) + \eta_1} = \frac{\eta_{23} - \eta_1}{\eta_{23} + \eta_1}$$ [3.10]

Plane waves in lossy materials

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} \qquad [2.19]$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]^{1/2} \qquad [2.20]$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]^{1/2} \qquad [2.21]$$

Lossy materials

Polarization currents:

$$\varepsilon_c = \varepsilon' - j\varepsilon'' \qquad [p.46]$$

$$\tan\delta_c = \frac{\sigma_{eff}}{\omega\varepsilon'} = \frac{\varepsilon''}{\varepsilon'} \qquad [p.48]$$

Conduction Currents:

$$\sigma_{eff} = \sigma + \omega\varepsilon'' \qquad [p.48]$$

$$\tan\delta_c = \frac{\sigma + \omega\varepsilon''}{\omega\varepsilon'} \qquad [p.48]$$

Use $\tan\delta_c$ instead of $\frac{\sigma}{\omega\varepsilon}$ in expression for complex impedance

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_{eff}}} = \sqrt{\frac{\mu}{\varepsilon - j\frac{\sigma}{\omega}}} = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\left(1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)^{1/4}} e^{j(1/2)Tan^{-1}(\sigma/\omega\varepsilon)} \qquad [2.22]$$

Good conductor approximations

$$\alpha = \beta \approx \sqrt{\frac{\omega\mu\sigma}{2}} \qquad [p.54]$$

$$\delta \equiv \frac{1}{\alpha} \approx \sqrt{\frac{2}{\omega\mu\sigma}} = \frac{1}{\sqrt{\pi f\mu\sigma}} \qquad [2.26]$$

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\left(1-j\frac{\sigma}{\omega\varepsilon}\right)^{1/2}} = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}e^{j45^\circ} \qquad [p.54]$$

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Poynting Theorem

$$\int_V \mathcal{E}\cdot\mathcal{J}\,dv = -\frac{\partial}{\partial t}\int_V\left(\frac{1}{2}\mu|\mathcal{H}|^2 + \frac{1}{2}\varepsilon|\mathcal{E}|^2\right)dv - \oint_S(\mathcal{E}\times\mathcal{H})\cdot d\mathbf{s} \qquad [2.31]$$

$$\mathbf{S}_{av} = \hat{z}\frac{E_0^2}{2\eta} \qquad [2.32]$$

$$\mathbf{S}_{av} = \frac{1}{2}\mathcal{R}e\{\mathbf{E}\times\mathbf{H}^*\} \qquad [2.43]$$

Arbitrarily directed uniform plane waves

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 e^{-j\beta\hat{\mathbf{k}}\cdot\mathbf{r}} \qquad [2.58]$$

$$\hat{\mathbf{k}} = \frac{\hat{\mathbf{x}}\beta_x + \hat{\mathbf{y}}\beta_y + \hat{\mathbf{z}}\beta_z}{\omega\sqrt{\mu\varepsilon}} \qquad [p.97]$$

$$\mathbf{H}(\mathbf{r}) = \frac{1}{\eta}\hat{\mathbf{k}}\times\mathbf{E}(\mathbf{r}) \qquad [2.61]$$

Reflection and refraction of oblique waves at planar dielectric interfaces

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{v_{p1}}{v_{p2}} = \sqrt{\frac{\varepsilon_2\mu_2}{\varepsilon_1\mu_1}} = \frac{n_2}{n_1} \quad \text{(known as Snell's Law)} \qquad [3.19]$$

$$\Gamma_\perp = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{\cos\theta_i - \sqrt{\varepsilon_2/\varepsilon_1 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\varepsilon_2/\varepsilon_1 - \sin^2\theta_i}} \qquad [3.24]$$

$$T_\perp = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{2\cos\theta_i}{\cos\theta_i + \sqrt{\varepsilon_2/\varepsilon_1 - \sin^2\theta_i}} \qquad [3.25]$$

$$\Gamma_{\parallel} = \frac{E_{r0}}{E_{i0}} = \frac{-\eta_1\cos\theta_i + \eta_2\cos\theta_t}{\eta_1\cos\theta_i + \eta_2\cos\theta_t} = \frac{-\cos\theta_i + \varepsilon_1/\varepsilon_2\sqrt{\varepsilon_2/\varepsilon_1 - \sin^2\theta_i}}{\cos\theta_i + \varepsilon_1/\varepsilon_2\sqrt{\varepsilon_2/\varepsilon_1 - \sin^2\theta_i}} \qquad [3.26]$$

$$T_{\parallel} = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2\cos\theta_i}{\eta_1\cos\theta_i + \eta_2\cos\theta_t} = \frac{2\sqrt{\varepsilon_1/\varepsilon_2}\cos\theta_i}{\cos\theta_i + \sqrt{\varepsilon_1/\varepsilon_2}\sqrt{1-\varepsilon_1/\varepsilon_2\sin^2\theta_i}} \qquad [3.27]$$

Total internal reflection

$$\sin\theta_{ic} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \frac{n_2}{n_1} \qquad [3.29]$$

$$\text{for } \theta_i > \theta_{ic} \quad \cos\theta_t = \pm j\sqrt{\frac{\varepsilon_1}{\varepsilon_2}\sin^2\theta_i - 1} \qquad [3.31]$$

$$\Gamma_\perp = \frac{\cos\theta_i + j\sqrt{\sin^2\theta_i - \varepsilon_2/\varepsilon_1}}{\cos\theta_i - j\sqrt{\sin^2\theta_i - \varepsilon_2/\varepsilon_1}} = 1e^{j\phi_\perp} \qquad [3.32]$$

$$\tan\frac{\phi_\perp}{2} = \frac{\sqrt{\sin^2\theta_i - \varepsilon_2/\varepsilon_1}}{\cos\theta_i} \qquad [p.192]$$

$$\Gamma_\parallel = -\frac{\varepsilon_2/\varepsilon_1 \cos\theta_i + j\sqrt{\sin^2\theta_i - \varepsilon_2/\varepsilon_1}}{\varepsilon_2/\varepsilon_1 \cos\theta_i - j\sqrt{\sin^2\theta_i - \varepsilon_2/\varepsilon_1}} = 1e^{j\phi_\parallel} \qquad [3.33]$$

$$\tan\frac{\phi_\parallel}{2} = \frac{\sqrt{\sin^2\theta_i - \varepsilon_2/\varepsilon_1}}{\varepsilon_2/\varepsilon_1 \cos\theta_i} \qquad [p.192]$$

Normal incidence on a lossy medium

$$\gamma_2 = j\omega\sqrt{\mu\varepsilon_{eff}} \approx j\omega\left(\frac{\mu_2\sigma_2}{j\omega}\right)^{1/2} = \left(j\omega\mu_2\sigma_2\right)^{1/2} = \frac{1+j}{\delta} \qquad [3.39]$$

$$\eta_c = Z_s = R_s + jX_s = \sqrt{\frac{\mu_2}{\varepsilon_{eff}}} \approx \left(\frac{j\omega\mu_2}{\sigma_2}\right)^{1/2} = \frac{\gamma_2}{\sigma_2} = \frac{1+j}{\sigma_2\delta} \qquad [3.40]$$

$$\Gamma_\parallel = -\frac{\varepsilon_2/\varepsilon_1 \cos\theta_i + j\sqrt{\sin^2\theta_i - \varepsilon_2/\varepsilon_1}}{\varepsilon_2/\varepsilon_1 \cos\theta_i - j\sqrt{\sin^2\theta_i - \varepsilon_2/\varepsilon_1}} = 1e^{j\phi_\parallel} \qquad [3.33]$$

$$\tan\frac{\phi_\parallel}{2} = \frac{\sqrt{\sin^2\theta_i - \varepsilon_2/\varepsilon_1}}{\varepsilon_2/\varepsilon_1 \cos\theta_i} \qquad [p.192]$$

Parallel plate waveguide

Parallel-plate TE_m modes: m=0,±1,±2,…

$$E_y = C_1 \sin\left(\frac{m\pi}{a}x\right)e^{-\bar{\gamma}z} \qquad [4.12a]$$

$$H_z = -\frac{1}{j\omega\mu}\frac{\partial E_y}{\partial x} = -\frac{m\pi}{j\omega\mu a}C_1\cos\left(\frac{m\pi}{a}x\right)e^{-\bar{\gamma}z} \qquad [4.12b]$$

$$H_x = -\frac{\bar{\gamma}}{j\omega\mu}E_y = -\frac{\bar{\gamma}}{j\omega\mu}C_1\sin\left(\frac{m\pi}{a}x\right)e^{-\bar{\gamma}z} \qquad [4.12c]$$

Parallel-plate TM_m modes: m=0,±1,±2,…

$$H_y = C_4\cos\left(\frac{m\pi}{a}x\right)e^{-\bar{\gamma}z} \qquad [4.13a]$$

$$E_x = \frac{\bar{\gamma}}{j\omega\varepsilon}H_y = \frac{\bar{\gamma}}{j\omega\varepsilon}\cos\left(\frac{m\pi}{a}x\right)e^{-\bar{\gamma}z} \qquad [4.13b]$$

$$E_z = \frac{jm\pi}{\omega\varepsilon a}C_4\sin\left(\frac{m\pi}{a}x\right)e^{-\bar{\gamma}z} \qquad [4.13c]$$

Parallel-plate TEM mode

$$H_y = C_4 e^{-\bar{\gamma}z} \qquad [4.14a]$$

$$E_x = \frac{\bar{\gamma}}{j\omega\varepsilon}C_4 e^{-\bar{\gamma}z} \qquad [4.14b]$$

$$E_z = 0 \qquad [4.14c]$$

Propagation constants

$$f_{c_m} = \frac{m\upsilon_p}{2a} = \frac{m}{2a\sqrt{\mu\varepsilon}} \qquad [4.15]$$

$$\bar{\gamma} = j\bar{\beta}_m = j\sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2} = j\beta\sqrt{1-\left(\frac{f_{c_m}}{f}\right)^2},\ f > f_{c_m} \qquad [4.16]$$

$$\bar{\gamma} = \bar{\alpha}_m = j\sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon} = \beta\sqrt{\left(\frac{f_{c_m}}{f}\right)^2 - 1},\ f < f_{c_m} \qquad [4.14c]$$

$$\bar{\lambda}_m = \frac{2\pi}{\bar{\beta}_m} = \frac{\lambda}{\sqrt{1-\left(f_{c_m}/f\right)^2}} \qquad [4.18]$$

$$\bar{\upsilon}_{p_m} = \frac{\omega}{\bar{\beta}_m} = \frac{\bar{\upsilon}_p}{\sqrt{1-\left(f_{c_m}/f\right)^2}} \qquad [4.18]$$

Conduction losses

$$\alpha_{c_{TEM}} = \frac{R_s}{\eta a} = \frac{1}{\eta a}\sqrt{\frac{\omega\mu_o}{2\sigma}} \qquad [4.22]$$

$$\alpha_{c_{TE_m}} = \frac{2R_s\left(\frac{f_{c_m}}{f}\right)^2}{\eta a\sqrt{1-\left(\frac{f_{c_m}}{f}\right)^2}} \qquad [4.23]$$

$$\alpha_{c_{TM_m}} = \frac{2R_s}{\eta a\sqrt{1-\left(\frac{f_{c_m}}{f}\right)^2}} \qquad [4.24]$$

Dielectric losses

$$\alpha_d = \frac{\omega\sqrt{\mu\varepsilon'}\,\varepsilon''/\varepsilon'}{2\sqrt{1-\left(\frac{\omega_{c_m}}{\omega}\right)^2}} \qquad [4.27]$$

For parallel plate TE modes the total power through the guide is

$$P_{av} = \int_0^b\int_0^a \frac{\overline{\beta}C_1^2}{2\omega\mu}\sin^2\left(\frac{m\pi}{a}x\right)dxdy = \frac{\overline{\beta}C_1^2ab}{4\omega\mu} \qquad [p.277]$$

For the parallel plate TEM (TM_0) mode the total power through the guide is

$$P_{av} = \frac{1}{2}\eta C_4^2 ba \qquad [p.277]$$

Dielectric slab waveguide

TM Modes The non-zero field components are E_z, E_x, and H_y

For x≤-d/2

$$E_z^0(x) = C_o\sin(\beta_x x) + C_e\cos(\beta_x x) \qquad [4.34]$$

where the transverse propagation constant is given by

$$\beta_x^2 = \omega^2\mu_d\varepsilon_d - \overline{\beta}^2 = h_d^2 \qquad [4.35]$$

For x≥-d/2

$$E_z^0(x) = \begin{cases} C_a e^{-\alpha_x(x-d/2)}, x \ge d/2 \\ C_a e^{\alpha_x(x+d/2)}, x \le -d/2 \end{cases} \qquad [4.36]$$

where the transverse attenuation constant is given by

$$\alpha_x^2 = \overline{\beta}^2 - \omega^2\mu_0\varepsilon_0 = -h_0^2 \text{ or } \alpha_x = \sqrt{\overline{\beta}_x^2 - \omega^2\mu_0\varepsilon_0} \qquad [4.37]$$

The cutoff frequencies are given by

$$f_{C_{TMm}} = \frac{(m-1)}{2d\sqrt{\mu_d\varepsilon_d - \mu_0\varepsilon_0}} \quad \begin{matrix} m = 1,3,5,\ldots odd \\ m = 2,4,6\ldots even \end{matrix} \qquad [4.45]$$

TE Modes The non-zero field components are H_z, H_x, and E_y

For x≤-d/2

$$H_z^0(x) = C_o\sin(\beta_x x) + C_e\cos(\beta_x x) \qquad [4.46]$$

where the transverse propagation constant

$$\beta_x^2 = \omega^2\mu_d\varepsilon_d - \overline{\beta}^2 = h_d^2$$

For x≥-d/2

$$H_z^0(x) = \begin{cases} C_a e^{-\alpha_x(x-d/2)}, x \ge d/2 \\ C_a e^{\alpha_x(x+d/2)}, x \le -d/2 \end{cases}$$

where the transverse propagation constant

$$\alpha_x^2 = \overline{\beta}^2 - \omega^2\mu_0\varepsilon_0 = -h_0^2 \text{ or } \alpha_x = \sqrt{\overline{\beta}_x^2 - \omega^2\mu_0\varepsilon_0} \qquad [4.37]$$

For Odd TM Modes:

$$\frac{\alpha_x}{\beta_x} = \frac{\varepsilon_0}{\varepsilon_d}\tan\left(\frac{\beta_x d}{2}\right) \qquad [4.40]$$

For Even TM Modes:

$$\frac{\alpha_x}{\beta_x} = -\frac{\varepsilon_0}{\varepsilon_d}\cot\left(\frac{\beta_x d}{2}\right) \qquad [4.44]$$

For ALL TM Modes:

$$\alpha_x = \sqrt{\omega^2(\mu_d\varepsilon_d - \mu_0\varepsilon_0) - \beta_x^2} \qquad [4.42]$$

The cutoff frequencies are given by

$$f_{C_{TEm}} = \frac{(m-1)}{2d\sqrt{\mu_d\varepsilon_d - \mu_0\varepsilon_0}} \quad \begin{matrix} m = 1,3,5,\ldots odd \\ m = 2,4,6\ldots even \end{matrix} \qquad [4.49]$$

Dielectric covered ground plane

$$f_{C_{TEm}} = \frac{(m-1)}{2d\sqrt{\mu_d \varepsilon_d - \mu_0 \varepsilon_0}} \begin{matrix} m = 1,3,5,...odd\_TM_m \\ m = 2,4,6...even\_TE_m \end{matrix} \qquad [4.50]$$

Dielectric slab waveguide ray theory

$$\tan\theta_i = \frac{\overline{\beta}}{\beta_x} \qquad [p.306]$$

Detailed example of odd TM Modes for slab dielectric waveguide: free space above the guide

For x≥d/2

$$E_z^0(x) = C_0 \sin\left(\frac{\beta_x d}{2}\right) e^{-\alpha_x (x - d/2)} \qquad [4.39a]$$

$$E_x^0(x) = -\frac{j\overline{\beta}}{\alpha_x} C_0 \sin\left(\frac{\beta_x d}{2}\right) e^{-\alpha_x (x - d/2)} \qquad [4.39b]$$

$$H_y^0(x) = -\frac{j\omega\varepsilon_0}{\alpha_x} C_0 \sin\left(\frac{\beta_x d}{2}\right) e^{-\alpha_x (x - d/2)} \qquad [4.39c]$$

For |x|≤d/2

$$E_z^0(x) = C_0 \sin(\beta_x x) \qquad [4.39d]$$

$$E_x^0(x) = -\frac{j\overline{\beta}}{\beta_x} C_0 \cos(\beta_x x) \qquad [4.39e]$$

$$H_y^0(x) = \frac{j\omega\varepsilon_d}{\beta_x} C_0 \cos(\beta_x x) \qquad [4.39f]$$

For x≤-d/2

$$E_z^0(x) = C_0 \sin\left(\frac{\beta_x d}{2}\right) e^{-\alpha_x (x + d/2)} \qquad [4.39g]$$

$$E_x^0(x) = -\frac{j\overline{\beta}}{\alpha_x} C_0 \sin\left(\frac{\beta_x d}{2}\right) e^{-\alpha_x (x + d/2)} \qquad [4.39h]$$

$$H_y^0(x) = -\frac{j\omega\varepsilon_0}{\alpha_x} C_0 \sin\left(\frac{\beta_x d}{2}\right) e^{-\alpha_x (x + d/2)} \qquad [4.39i]$$

Rectangular waveguides: TM modes

$$E_z = C \sin\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-j\overline{\beta}_{mn} z} \qquad [5.13a]$$

$$E_x = -\frac{j\overline{\beta}_{mn} C}{h^2} \frac{m\pi}{a} \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-j\overline{\beta}_{mn} z} \qquad [5.13b]$$

$$E_y = -\frac{j\overline{\beta}_{mn} C}{h^2} \frac{n\pi}{b} \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\overline{\beta}_{mn} z} \qquad [5.13c]$$

$$H_x = \frac{j\omega\varepsilon C}{h^2} \frac{n\pi}{b} \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\overline{\beta}_{mn} z} \qquad [5.13d]$$

$$H_y = -\frac{j\omega\varepsilon C}{h^2} \frac{m\pi}{a} \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-j\overline{\beta}_{mn} z} \qquad [5.13e]$$

$$Z_{TM_{mn}} = \frac{E_x^0}{H_y^0} = \eta\sqrt{1 - \left(\frac{f_{c_{mn}}}{f}\right)^2} \qquad [5.16]$$

Rectangular waveguides: TE modes

$$H_z = C \cos\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\overline{\beta}_{mn} z} \qquad [5.21a]$$

$$H_x = \frac{j\overline{\beta}_{mn}}{h^2} C \frac{m\pi}{a} \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\overline{\beta}_{mn} z} \qquad [5.21b]$$

$$H_y = \frac{j\overline{\beta}_{mn}}{h^2} C \frac{n\pi}{b} \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-j\overline{\beta}_{mn} z} \qquad [5.21c]$$

$$E_x = \frac{j\omega\mu}{h^2} C \frac{n\pi}{b} \cos\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) e^{-j\overline{\beta}_{mn} z} \qquad [5.21d]$$

$$E_y = -\frac{j\omega\mu}{h^2} C \frac{m\pi}{a} \sin\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-j\overline{\beta}_{mn} z} \qquad [5.21e]$$

$$Z_{TE_{mn}} = \frac{\eta}{\sqrt{1 - \left(\frac{f_{c_{mn}}}{f}\right)^2}} \qquad [5.22]$$

For both TM and TE modes:

$$\overline{\gamma} = \sqrt{h^2 - \omega^2\mu\varepsilon} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon} \qquad [5.14]$$

where $h_{mn}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

$$\omega_{c_{mn}} = \frac{1}{\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \qquad [5.15]$$

$$\overline{\beta}_{mn} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} = \beta\sqrt{1 - \left(\frac{f_{c_{mn}}}{f}\right)^2} \qquad [5.16]$$

Dominant TE_{10} mode:

$$H_z = C\cos\left(\frac{\pi x}{a}\right)e^{-j\bar{\beta}_{10}z} \qquad [5.23a]$$

$$H_x = \frac{j\bar{\beta}_{10}aC}{\pi}\sin\left(\frac{\pi x}{a}\right)e^{-j\bar{\beta}_{10}z} \qquad [5.23b]$$

$$E_y = -\frac{j\omega\mu aC}{\pi}\sin\left(\frac{\pi x}{a}\right)e^{-j\bar{\beta}_{10}z} \qquad [5.23c]$$

$$E_x = H_y = 0 \qquad [5.23d]$$

$$\bar{\beta}_{10} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{\pi}{a}\right)^2} = \sqrt{\left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{\pi}{a}\right)^2} \qquad [5.23e]$$

$$\alpha_{c_{TE_{10}}} = \frac{R_s}{\eta b}\frac{\left[1+\frac{2b}{a}\left(\frac{f_{c_{10}}}{f}\right)^2\right]}{\sqrt{1-\left(\frac{f_{c_{10}}}{f}\right)^2}} \qquad [5.29]$$

$$\bar{\lambda}_{10} = \frac{2\pi}{\beta_{10}} = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}} \qquad [5.23f]$$

$$f_{c_{10}} = \frac{1}{2a\sqrt{\mu\varepsilon}} \qquad [5.24]$$

$$Z_{TE_{10}} = \frac{\eta}{\sqrt{1-\left(\frac{f_{c_{10}}}{f}\right)^2}} = \frac{2a\mu f}{\sqrt{4a^2\mu\varepsilon f^2 - 1}} \qquad [5.24]$$

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Circular waveguides: TM modes where $H_z = 0$

$$E_z = C_n J_n\left(\frac{t_{nl}}{a}r\right)\cos(n\phi)e^{-j\bar{\beta}_{nl}z} \qquad [5.46a]$$

$$E_r = -\frac{ja\bar{\beta}_{TM_{nl}}}{t_{nl}}C_n J_n'\left(\frac{t_{nl}}{a}r\right)\cos(n\phi)e^{-j\bar{\beta}_{nl}z} \qquad [5.46b]$$

$$E_\phi = -\frac{ja^2 n\bar{\beta}_{TM_{nl}}}{t_{nl}^2}C_n J_n\left(\frac{t_{nl}}{a}r\right)\sin(n\phi)e^{-j\bar{\beta}_{nl}z} \qquad [5.46c]$$

$$H_r = -\frac{j\omega\varepsilon na^2}{t_{nl}^2 r}C_n J_n\left(\frac{t_{nl}}{a}r\right)\sin(n\phi)e^{-j\bar{\beta}_{nl}z} \qquad [5.46d]$$

$$H_\phi = -\frac{j\omega\varepsilon a}{t_{nl}}C_n J_n'\left(\frac{t_{nl}}{a}r\right)\cos(n\phi)e^{-j\bar{\beta}_{nl}z} \qquad [5.46e]$$

where

$$\bar{\beta}_{TM_{nl}} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{t_{nl}}{a}\right)^2} \qquad [5.44]$$

$$f_{c_{TM_{nl}}} = \frac{t_{nl}}{2\pi a\sqrt{\mu\varepsilon}} \qquad [5.45]$$

Circular waveguides: TE modes where $E_z = 0$

$$H_z = C_n J_n\left(\frac{s_{nl}}{a}r\right)\cos(n\phi)e^{-j\bar{\beta}_{nl}z} \qquad [5.48a]$$

$$H_r = -\frac{ja\bar{\beta}_{TE_{nl}}}{s_{nl}}C_n J_n'\left(\frac{s_{nl}}{a}r\right)\cos(n\phi)e^{-j\bar{\beta}_{nl}z} \qquad [5.48b]$$

$$H_\phi = \frac{jna^2\bar{\beta}_{TE_{nl}}}{s_{nl}^2 r}C_n J_n\left(\frac{s_{nl}}{a}r\right)\sin(n\phi)e^{-j\bar{\beta}_{nl}z} \qquad [5.48c]$$

$$E_r = -\frac{ja^2\omega\mu n}{s_{nl}^2 r}C_n J_n\left(\frac{s_{nl}}{a}r\right)\sin(n\phi)e^{-j\bar{\beta}_{nl}z} \qquad [5.48d]$$

$$E_\phi = -\frac{ja\omega\mu}{s_{nl}}C_n J_n'\left(\frac{s_{nl}}{a}r\right)\cos(n\phi)e^{-j\bar{\beta}_{nl}z} \qquad [5.48e]$$

where

$$\bar{\beta}_{TE_{nl}} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{s_{nl}}{a}\right)^2} \qquad [p.359]$$

$$f_{c_{TE_{nl}}} = \frac{s_{nl}}{2\pi a\sqrt{\mu\varepsilon}} \qquad [p.359]$$

Table 5.2 *l*th roots (t_{nl}) of $J_n(.)=0$

	n=							
l=	0	1	2	3	4	5	6	7
1	2.405	3.832	5.136	6.380	7.588	8.771	9.936	11.086
2	5.520	7.016	8.417	9.761	11.065	12.339	13.589	14.821
3	8.654	10.173	11.620	13.015	14.372	15.700	17.004	18.288
4	11.792	13.323	14.796	16.223	17.616	18.980	20.321	21.642

Table 5.3 *l*th roots (s_{nl}) of $J_n'(.)=0$

	n=							
l=	0	1	2	3	4	5	6	7
1	3.832	1.841	3.054	4.201	5.317	6.416	7.501	8.578
2	7.016	5.331	6.706	8.015	9.282	10.520	11.735	12.932
3	10.173	8.536	9.969	11.346	12.682	13.987	15.268	16.529
4	13.324	11.706	13.170	14.586	15.964	17.313	18.637	19.942