EECS 412 Electromagnetic Fields III
Fall 2002

Background Exam Part 2:

Due September 30th

Amperes Law	6-9(1)
Biot-Savart Law	6-13
Magnetic loops	6-16(1)
Magnetic fields from current distributions	6-22(2)
Vector potential	6-31
Inductance	6-40(3),6-43

Notes:

1. HINT: Superposition
2. You may leave your answer in the form of an integral.
3. In lieu of Figure 6-38 the wires are of radius a, with their centers at x=±d/2. Assume the wires long dimension is in the z-direction. The wire at x=+d/2 has a current +Iz and the wire at x=–d/2 has a current in the –Iz direction. Hint: Assuming the wires are infinitely long, compute the flux linkage between the wires for a length l.

6.12 PROBLEMS

6-1. Force between two infinitely long wires. A dc transmission line consists of two infinitely long, parallel wires separated by a distance of 50 cm. The two wires carry the same current I in opposite directions. Find I if the force per unit length experienced by each wire is 0.5 N-m^{-1}.

6-2. Forces between two wires. Consider a two-wire transmission line consisting of two parallel conductors of 1 mm diameter separated by 1 cm. If a potential difference of 10 V is applied between the conductors, equal charges of opposite sign would be induced on the two conductors, resulting in an electrostatic force of attraction between them. Is there a value of line current I for which this electric attraction force might be balanced by the magnetic force acting on the wires?

6-3. Bundle clash in a transmission line. Each phase of a three-phase alternating current transmission line consists of a bundle of two parallel wires each 4 cm in diameter and 50 cm apart, carrying current in the same direction. Under normal operation, the currents flowing in the wires are of the order of hundreds of amperes, and the magnetic force of attraction between the wires in the bundle is relatively small. An out-of-control airplane accidentally strikes the power transmission line, causing short-circuit loading, as a result of which the peak current in each wire in the bundle reaches a value of 100 kA. Calculate the peak magnetic force of attraction per unit length on each wire and comment on the possibility of a bundle clash. Note that similar short-circuit loading of power lines can also occur due, for example, to a tree falling on the line or a snake climbing up a power-line pole.

6-4. Magnetic force between current elements. Two elements of current are oriented in air in the same plane and positioned with respect to each other as illustrated in Figure 6.52a. (a) Find the magnitude and direction of the force exerted upon each element by the other. (b) Repeat (a) for the configuration of the current elements shown in Figure 6.52b.

6-5. Square loop. Does a square loop carrying current have a tendency to expand or contract?

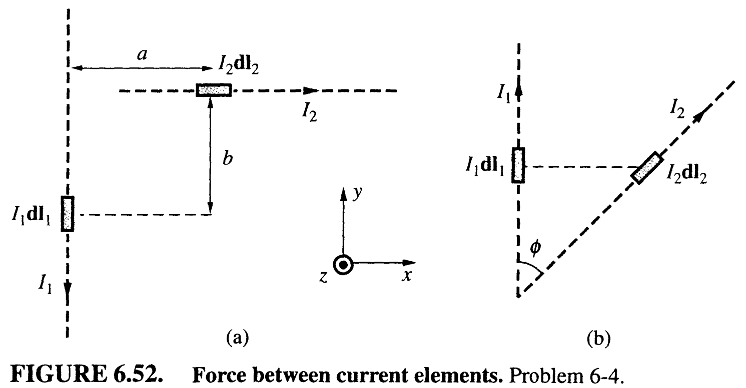

FIGURE 6.52. Force between current elements. Problem 6-4.

6-6. Force on a current-carrying wire. A single conductor of a transmission line extends in the east-west direction and carries a current of 1 kA. The earth's **B** field is directed essentially straight north at the point in question and has a value of ~0.5 T. What is the magnitude of the force per meter on the current-carrying conductor?

6-7. Rigid rectangular loop. A rigid rectangular loop carrying a current of 5 A is located in the xy plane with its four corners at (0,0), (1,0), (1,2), and (0,2). Determine the magnetic force exerted on each side of the loop if the region in which the loop is located is permeated with a **B** field given by (a) $\mathbf{B} = \hat{\mathbf{x}}1.5$ T, (b) $\mathbf{B} = \hat{\mathbf{z}}1.5$ T.

6-8. Two parallel wires. Consider two infinitely long parallel wires, each carrying a steady current of I in the z direction, one passing through the point (2,0,0) and the other through (0,2,0), as shown in Figure 6.53. (a) Find **B** at the origin. (b) Find **B** at point (1,1,0). (c) Find **B** at (2,2,0). (d) Repeat parts (a), (b), and (c) if the direction of the current in the wire on the x axis is switched.

6-9. An infinitely long L-shaped wire. Consider a single wire extending from infinity to the origin along the y axis and back to infinity along the x axis and carrying a current I. Find **B** at the following points: (a) $(-a,0,0)$; (b) $(0,-a,0)$; and (c) $(0,0,a)$.

6-10. Irregular loop. Find **B** at point P due to the wire carrying a current I in free space, as shown in Figure 6.54.

6-11. An N-sided regular loop. A regular polygon–shaped loop, with N sides, carries a current I. (a) Show that the magnetic field at the center of the loop has a magnitude given by $B = [\mu_0 NI/(\pi d)]\tan(\pi/N)$, where d is the diameter of the circle passing through the corners of the polygon. (b) Also show that as $N \to \infty$, B approaches that found in Example 6-6.

6-12. Square loop of current. A current I flows in a square loop of side length a as shown in Figure 6.55. Find the **B** field at the point P(a/4, a/4).

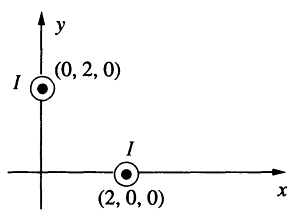

FIGURE 6.53. **Two parallel wires.** Problem 6-8.

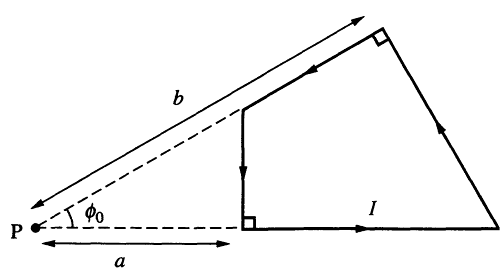

FIGURE 6.54. **Irregular loop.** Problem 6-10.

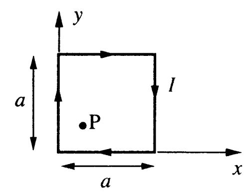

FIGURE 6.55. Square loop of current. Problem 6-12.

6-13. A wire with two circular arcs. Consider a loop of wire consisting of two circular and two straight segments carrying a current I, as shown in Figure 6.56. Find **B** at the center P of the circular arcs.

6-14. Wire with four 90° bends. An infinitely long wire carrying current $I = 1$ A has four sharp 90° bends 1 m apart as shown in Figure 6.57. Find the numerical value (in Wb-m^{-2}) and direction (i.e., the vector expression) of the **B** field at point P(1, 0, 0).

6-15. Infinitely long copper cylindrical wire carrying uniform current. An infinitely long solid copper cylindrical wire 2 cm in diameter carries a total current of 1000 A, distributed uniformly throughout its cross section. Find **B** at points inside and outside the wire.

6-16. Helmholtz coils. Two thin circular coaxial coils each of radius a, having N turns, carrying current I, and separated by a distance d, as shown in Figure 6.58, are referred to as *Helmholtz coils* for the case when $d = a$. This setup is well known for producing an approximately uniform magnetic field in the vicinity of its center of symmetry. (a) Find **B** on the axis of symmetry—the z axis—of the Helmholtz coils.

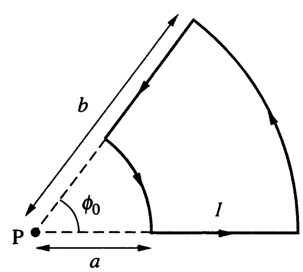

FIGURE 6.56. Wire with two circular arcs. Problem 6-13.

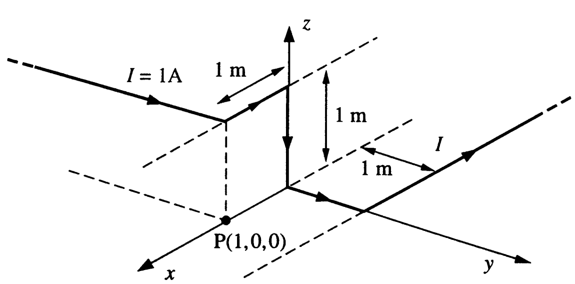

FIGURE 6.57. Wire with four 90° bends. Problem 6-14.

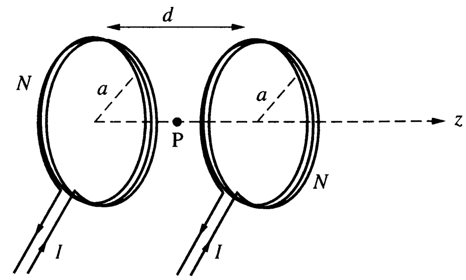

FIGURE 6.58. Helmholtz coils.
Problem 6-16.

(b) Show that $dB_z/dz = 0$ at the point P midway between the two coils. (c) Show that both $d^2B_z/dz^2 = 0$ and $d^3B_z/dz^3 = 0$ at the midway when $d = a$. (Note that d is called the Helmholtz spacing, which corresponds to the coil separation for which the second derivative of B_z vanishes at the center.) (d) Show that B_z at the midpoint P between the Helmholtz coils is

$$B_z \simeq 0.8992 \times 10^{-6} \frac{NI}{a} \quad \text{T}$$

(e) Find B_z at the center of each loop and compare it with the value at the midpoint between the coils.

6-17. Helmholtz coils. Design a pair of Helmholtz coils, separated by 0.5 m, to produce a magnetic field of 100 μT midway between the two coils. Take $I = 1$ A.

6-18. Helmholtz coils. Consider the Helmholtz coils discussed in Problem 6-16. Calculate and sketch B_z along the axis of symmetry between the two coils if one of the coils is connected backward (i.e., its current is switched).

6-19. Square Helmholtz coils. Consider a pair of square coils, similar to Helmholtz coils, each of same side a, having N turns, and carrying current I, separated by a distance d. Show that the Helmholtz spacing d (i.e., the coil separation for which $d^2B_z/dz^2 = 0$ at the center) is equal[80] to $d \simeq 0.5445a$. (Note that the Helmholtz spacing for circular coils is $d = a$, where a is the radius of each coil.) To simplify your solution, take each side to be of unity length and use a simple iterative procedure.

6-20. A circular and a square coil. Consider a pair of coils, similar to Helmholtz coils, separated by a distance d, where one of the coils is circular in shape with radius a and the other square with side $2a$, each having N turns and carrying current I, and $d = a$. For $a = 25$ cm, $I = 1$ A, and $N = 20$, calculate and plot the magnitude of the **B** field along the axis of symmetry between the two coils.

6-21. Magnetic field of a solenoid. For an air-core solenoid of $N = 350$ turns, length $l = 40$ cm, radius $a = 2$ cm, and having a current of 1 A, find the **B** field (a) at the center; and (b) at the ends of the solenoid.

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[80] A. H. Firester, Designs of square Helmholtz coil systems, *Rev. Sci. Instr.*, 37, pp. 1264–1265, 1966.

6-22. Magnetic field of a surface current distribution. A circular disk of radius a centered at the origin with its axis along the z axis carries a surface current flowing in a circular direction around its axis given by

$$\mathbf{J}_s = \hat{\boldsymbol{\phi}} Kr \text{ A-m}^{-2}$$

where K is a constant. Find **B** at a point P on the z axis.

6-23. B inside the solenoid. An air-core solenoid of 750 turns, 20 cm length, and 5 cm^2 cross-sectional area is carrying a current of 1 A. (a) Find **B** along the axis of the solenoid. (b) Find $\mathbf{B}_{\text{ctr}}$ at the center of the solenoid. (c) Sketch $|\mathbf{B}|$ along the axis of the solenoid.

6-24. B inside the solenoid. A long solenoid consists of a tightly wound coil around a magnetic core ($\mu_r = 300$) carrying a current of 10 A. (a) If the **B** field magnitude at the center of the core is $B_{\text{ctr}} \simeq 1.5$ T, find the number of turns of wire wound around the core per centimeter. (b) Repeat part (a) if this was an air-core solenoid.

6-25. Split washer. The thin flat washer shown in Figure 6.59 has an inner radius of $a =$ 10 mm and an outer radius of $b = 50$ mm. A uniform voltage V_0 is applied between the edges of the radial slot, resulting in an angular current of $I = 100$ A. Find the **B** field at the center of the washer. Note that the current density is a function of the radius from the center. Neglect the width of the slot. Note that the application of the voltage between the edges is as shown in Figure 5.6c.

6-26. Long wire encircled by small loop. A long straight wire oriented along the z axis carries a current of 1 A. A loop of 5 cm radius carrying a current of 100 A is located in the xy (i.e., $z = 0$) plane, with its center at the origin, concentric with the wire, so that the axis of the loop coincides with the z axis. Find the **B** field at a point on the xy plane at a distance of 1 m from the origin.

6-27. Flux through a rectangular loop. A long, straight wire carrying a current I and a rectangular loop of wire are separated by a distance a as shown in Figure 6.60. (a) If the sides of the rectangular loop parallel and perpendicular to the straight wire are a and $2a$, respectively, find the magnetic flux Ψ that links the rectangular loop due to the straight current-carrying wire. (b) The rectangular loop is rotated by 90° around its perpendicular symmetry axis. Find the percentage change in Ψ linking the loop.

6-28. Flux through a triangular loop. A long, straight wire carrying a current I and a triangular loop of wire are as shown in Figure 6.61. (a) Find the magnetic flux Ψ that links the triangular loop in terms of a, b, and ϕ_0. (b) Find Ψ if $I = 100$ A, $b = 4a = 40$ cm, and $\phi_0 = 45°$, respectively.

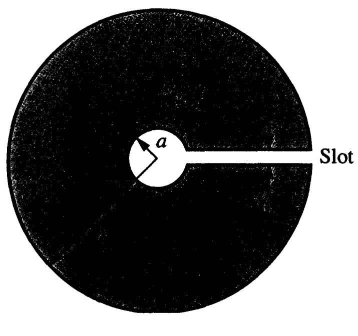

FIGURE 6.59. **Split washer.** Problem 6-25.

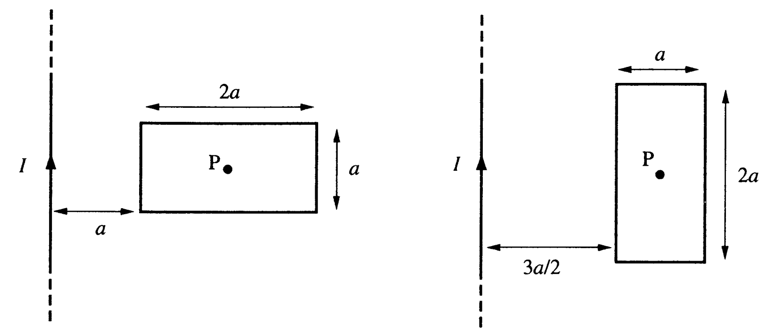

FIGURE 6.60. **Flux through rectangular loop.** Problem 6-27.

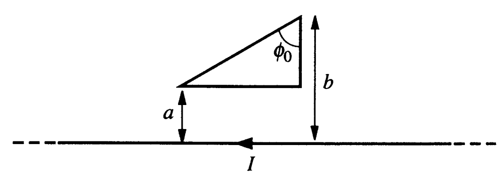

FIGURE 6.61. **Flux through a triangular loop.** Problem 6-28.

6-29. **A toroidal coil around a long, straight wire.** A long, straight wire carrying a current of 100 A coincides with the principal axis of symmetry of a 200-turn rectangular toroid of inner and outer radii $a = 4$ cm and $b = 6$ cm, thickness $t = 3$ cm, core material with $\mu_r = 250$, respectively. No current flows in the toroid. Find the total magnetic flux Ψ linking the toroid due to the current in the long, straight wire.

6-30. **Infinitely long wire with a cylindrical hole.** Consider an infinitely long cylindrical conductor wire of radius b, the cross section of which is as shown in Figure 6.62. The wire contains an infinitely long cylindrical hole of radius a parallel to the axis of the conductor. The axes of the two cylinders are apart by a distance d such that $d + a < b$. If the wire carries a total current I, (a) find the vector magnetic potential **A** and (b) use **A** to find the **B** field in the hole. Compare your result with that found in Example 6-13.

6-31. **Finite-length straight wire: A at an off-axis point.** Consider the straight, current-carrying filamentary conductor of length $2a$, as shown in Figure 6.11. (a) Find the magnetic vector potential **A** at an arbitrary point P(r, ϕ, z). (b) Find the **B** field using $\mathbf{B} = \nabla \times \mathbf{A}$ to verify the result of Example 6-7.

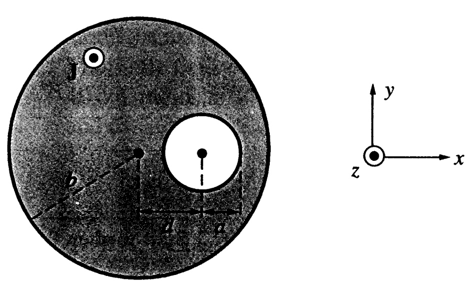

FIGURE 6.62. **Wire with hole.** Problem 6-30.

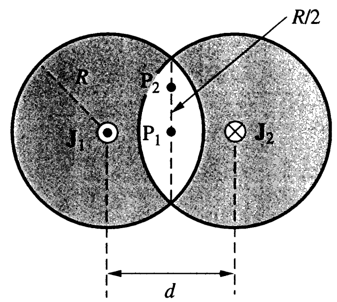

FIGURE 6.63. **Wire with oval-shaped hole.** Problem 6-33.

6-32. Two infinitely long wires. Consider two infinitely long parallel wires oriented in the z direction each carrying a current I in opposite directions respectively. Show that the magnetic vector potential $\mathbf{A}$ is give by $\mathbf{A} = \hat{\mathbf{z}}[\mu_0 I/(2\pi)]\ln(b/a)$, where a and b are the distances from the observation point to the wires.

6-33. Wire with oval-shaped hole. Consider a wire with the cross-sectional shape shown in Figure 6.63, having an oval-shaped axial hole. The wire segments on both sides of the hole carry uniform current densities $\mathbf{J}_1$ and $\mathbf{J}_2$ of equal magnitude J_0 and opposite sign. Find the magnetic field $\mathbf{B}$ at points P_1 and P_2 in the hole.

6-34. Inductance of a short solenoid. Find the inductance of a short air-core solenoid of 500 turns, 10 cm length, and 5.5 cm diameter. Use the Nagaoka formula given in Section 6.7.3.

6-35. Inductance of a finite-length solenoid. The secondary coil of a high-voltage electric generator is an air-core solenoid designed with the following parameters: $l = 1.8$ m, $a = 17.7$ cm, and $N = 780$ turns. Calculate the self-inductance of the coil by treating it (a) as an infinitely long solenoid, (b) as a finite-length solenoid. Compare your results.

6-36. Inductance of a rectangular toroid. Consider the toroidal coil of rectangular cross section shown in Figure 6.19. (a) Show that the inductance of this coil on an air core is in general given by

$$L = 2 \times 10^{-5} t N^2 \ln\left(\frac{b}{a}\right) \quad \text{H}$$

where N is the number of turns and t is the vertical thickness of the core in cm. (Do not assume $r_m \gg b - a$.) (b) If the dimensions of this toroid are $a = 1.2$ cm, $b = 2$ cm, and $t = 1.5$ cm, and $N = 1000$, find the inductance using the expression in part (a). (c) Find the inductance using the approximate expression derived in Example 6-27, assuming $r_m \gg b - a$ and compare it with the result of part (b).

6-37. Inductance of a rectangular toroid. Consider an air-core rectangular toroid as shown in Figure 6.19 with the values of its dimensions given by $a = 3.8$ mm, $b = 6.4$ mm, and $t = 4.8$ mm, respectively. (a) Find the total number of turns to be wound on this core such that its total inductance is around 1 mH. (b) Repeat part (a) for a powdered nickel-iron core (assume $\mu_r = 200$) having the same geometric dimensions.

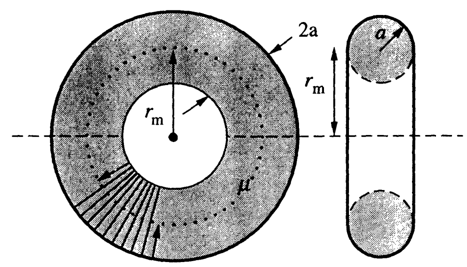

FIGURE 6.64. Inductance of a circular toroid. Problem 6-39.

6-38. Inductance of a rectangular toroid. A 50-mH toroid inductor is to be designed using a molypermalloy powder core with $\mu_r = 125$, $a = 7.37$ mm, and $b = 13.5$ mm, and $t = 11.2$ mm. Find the approximate number of turns N required.

6-39. Inductance of a circular toroid. Consider a toroid of circular cross section of radius a and mean radius r_m as shown in Figure 6.64. Show that the inductance of this coil is given by

$$L = \mu_r \mu_0 N^2 [r_m - (r_m^2 - a^2)^{1/2}]$$

where μ_r is the relative permeability of the core material.

6-40. Inductance of a two-wire line. Determine the inductance per unit length of a two-wire transmission line in air as shown in Figure 6.38, designed for an amateur radio transmitter, with conductor radius $a = 1$ mm and spacing $d = 6$ cm. (b) Repeat part (a) if the conductor spacing is doubled (i.e., $d = 12$ cm).

6-41. Inductance of a thin circular loop of wire. (a) Find the inductance of a single circular loop of wire with $d = 2$ mm and $a = 5$ cm. (b) Repeat part (a) with $d = 0.5$ mm and $a = 10$ cm. Refer to Figure 6.42.

6-42. Mutual inductance between two solenoidal coils. An air-core solenoid 25 cm long and 2.5 cm diameter is wound of 1000 turns of closely spaced, insulated wire. A separate coil of 100 turns, 3 cm long, with about the same diameter is located at the center of the longer coil (where **B** is approximately constant). Find the mutual inductance between the two coils.

6-43. Mutual inductance between a wire and a circular loop. Find the mutual inductance between an infinitely long, straight wire and a circular wire loop, as shown in Figure 6.65.

6-44. Long wire and loop. An infinitely long wire carrying current I passes just under a circular loop of radius a, also carrying the same current I as shown in Figure 6.66. (a) Find

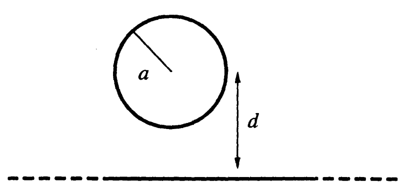

FIGURE 6.65. Wire and circular loop. Problem 6-43.

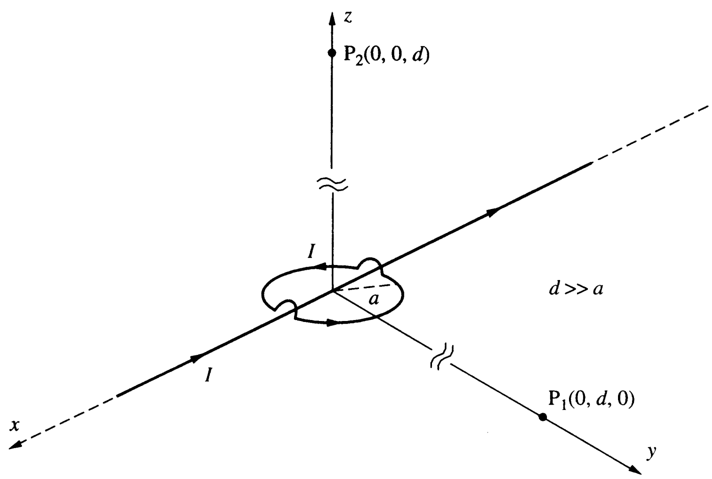

FIGURE 6.66. Long wire and loop. Problem 6-44.

the magnitude and direction of the **B** field at point P_1. Note that $d \gg a$. (b) Repeat (a) for point P_2. (c) What is the mutual inductance between the wire and the loop?

6-45. Inductance and energy of a solenoid. An air-core solenoid 30 cm long and 1 cm diameter is wound of 1000 turns of closely spaced, insulated wire. (a) Find the inductance of the solenoid. (b) Find the energy stored in the solenoid if it is carrying a current of 10 A.

6-46. Two square coils. Two square coils each of side length a are both located on the xy plane such that the distance between their centers is d. (a) Find a simple expression for the mutual inductance between the two loops for the case $d \gg a$. (b) Find the mutual inductance between the two coils for the case $d = 4a$. (c) For $d = 4a$, assume that coil #1 has 10 turns and that coil #2 has 100 turns. If an alternating current of 1 A of frequency 10 kHz is passed through coil #1, find the electromotive force induced across the terminals of coil #2.

6-47. Fixed-length piece of wire. You have a fixed length of copper wire of 1 mm diameter and 20 cm length. You can bend this wire in any shape or form to obtain as large an external self-inductance as you can. (a) What is the value of the maximum inductance, and what arrangement would work best? You cannot use any magnetic materials. (b) If you had the tools necessary to melt the copper wire and form it into a longer wire of diameter 0.1 mm instead, how would your answer to (a) change? Also compare and comment on the resistance of the two different wires.

6-48. Magnetic energy. A conductor consists of a cylinder with radius b with a hole of radius a ($a < b$) drilled coaxially through its axis. The current density is uniform and corresponds to a total current of I. Find the magnetic energy stored inside per unit length of the wire.

6-49. Explosion in a power transformer. A current transformer used for 500 kV transmission lines has a single primary coil which is connected to the high voltage line via two parallel wires that are 20 cm apart and carrying the same current in opposite

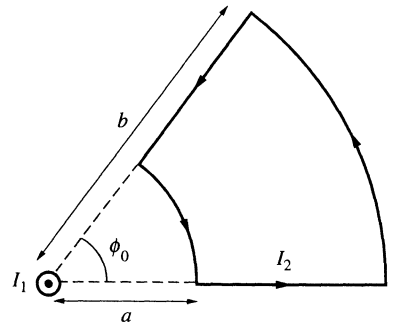

FIGURE 6.67. Force between a long wire and a loop. Problem 6-50.

directions. A fault occurs on the line and causes the current of each wire of the transformer to reach a peak value of 100 kA which results in an explosion in the oil/paper insulation structure of the wires. Calculate the per-unit-force on each wire and explain what happened.

6-50. Force between a long wire and a loop. A long wire extending along the z axis is situated near a rigid loop as shown in Figure 6.67. Find an expression for the force and torque (about the origin) experienced by the loop.