

EECS 412 Electromagnetic Fields III
Fall 2002

Background Exam Part 1:

Due September 20th

Coulomb forces and electric fields	4.3
Potential and electric fields from charge distributions	4.16,4.21,4.24
Capacitance and boundary conditions	4.27, 4.34
Potential	4.39
Energy of electric field distributions	4.42

and is given by $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$, where χ_e is the electric susceptibility, dependent on the particular microscopic atomic, molecular, and orientational properties of the material. The net effect of this induced dipole moment distribution within the dielectric is that the total electric flux density is now given by $\mathbf{D} = \epsilon_0 \mathbf{E} + \epsilon_0 \chi_e \mathbf{E}$, a fact that is typically accounted for by assigning to each material an electric permittivity $\epsilon = \epsilon_0(1 + \chi_e)$ such that $\mathbf{D} = \epsilon \mathbf{E}$.

- **Electrostatic boundary conditions.** Experimentally established laws of electrostatics dictate that the normal component of electric flux density is continuous across the interface between two materials, except when there is free surface charge present, which typically occurs at the surface of metallic conductors. The tangential component of the electric field is always continuous across any interface. In summary, we have

$$D_{1n} - D_{2n} = \rho_s \quad \text{and} \quad E_{1t} = E_{2t}$$

- **Electrostatic energy.** Any configuration of charges stores electrostatic energy of an amount equal to the work that was required to bring the charges together. The total energy stored in a distribution of charge is given by

$$W_e = \frac{1}{2} \int_V \rho \Phi \, dv$$

where V is the volume over which the charge is distributed. Alternatively, the electrostatic energy can be viewed as residing in the fields and can be found from

$$W_e = \frac{1}{2} \int_V \epsilon E^2 \, dv$$

where V is the entire volume over which the electric field is nonzero. The volume energy density of the electrostatic field is $w_e = \frac{1}{2} \epsilon E^2$.

- **Electrostatic forces.** Both conducting and dielectric materials experience forces when in the presence of electric fields. A relatively easy means of calculating the electrostatic force on such objects is to use the principle of virtual work. The physical origin of electrostatic forces on metallic conductors is the attraction or repulsion between induced charges, whereas dielectrics experience a force only when placed in a nonuniform field.

4.15 PROBLEMS

- 4-1. **Two point charges.** Two identical point charges 1 m apart from each other in free space are experiencing a repulsion force of 1 N each. What is the magnitude of each charge?
- 4-2. **Two point charges.** Two small identical spheres have charges of +20 nC and -5 nC, respectively. (a) What is the force between them if they are apart by 10 cm? (b) The two spheres are brought into contact and then separated again by 10 cm. What is the force between them now?

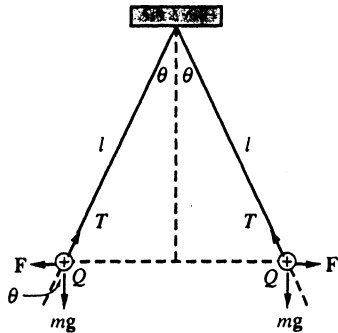


FIGURE 4.63 Two suspended charges. Problem 4-3.

- 4-3. Two suspended charges.** Two small, identical, electrically charged conducting spheres of mass 2.5 g and charge +150 nC each are suspended by weightless strings of length 12 cm each, as shown in Figure 4.63. Calculate the deflection angle θ .
- 4-4. Zero force.** (a) Three point charges of $Q_1 = +40$ nC, $Q_2 = -20$ nC, and $Q_3 = +10$ nC are all situated on the x axis in such a way that the net force on charge Q_2 due to Q_1 and Q_3 is equal to zero. If Q_1 and Q_2 are located at points $(-2,0,0)$ and $(0,0,0)$ respectively, what is the location of charge Q_3 ? (b) Q_3 is moved to a different position on the x axis such that the force on itself due to Q_1 and Q_2 is equal to zero. What is the new position of Q_3 ?
- 4-5. Three charges.** Three identical charges of charge Q are located at the vertices of an equilateral triangle of side length a . Determine the force on one of the charges due to the other two.
- 4-6. Three point charges.** Three point charges of values +150 nC, +100 nC, and +200 nC are located at points $(0,0,0)$, $(1,0,0)$, and $(0,1,0)$, respectively. (a) Find the force on each charge due to the other charges. Which charge experiences the largest force? (b) Repeat part (a) for -100 nC as the second charge.
- 4-7. Two point charges.** Two point charges of $+Q$ and $-Q$ are located at $(0,0,0)$ and at $(0.4a,0)$ respectively. Find the electric field at $P_1(0,0,3a)$ and at $P_2(0,4a,3a)$. Sketch the orientations of the fields.
- 4-8. Two point charges.** Two point charges, $+Q$ and $-3Q$, are located at points $(0,2,0)$ and $(0,-1,0)$, respectively. (a) Find the electric field at the origin. (b) Find the coordinates of the point(s) where $\mathbf{E} = 0$. (c) Repeat parts (a) and (b) for a second charge of $+3Q$.
- 4-9. Zero field from three charges.** Two point charges of $+Q$ each are located at $(10, 0)$ cm, and $(-10,0)$ cm, while a third one of charge $-2Q$ is at $(0, -10)$ cm. Find the coordinates of the point where the electric field is zero.
- 4-10. Three point charges.** Two point charges of +10 nC each are located at points $(0.46,0,0)$ and $(-0.46,0,0)$, respectively. (a) Where should a third point charge of +15 nC be placed such that $\mathbf{E} = 0$ at point $(0,1,0)$? (b) Repeat part (a) for a third charge of -15 nC. (c) With the -15 nC charge located as you determined in part (b), is there another point at which the electric field $\mathbf{E} = 0$? If so, specify this point.
- 4-11. Four point charges on a square.** Four point charges of +50 nC each are located at the corners of a square of side length 10 cm located on the xy plane and centered at the origin. (a) Find and sketch the electric potential on the z axis. (b) Find and sketch the electric field on the z axis.

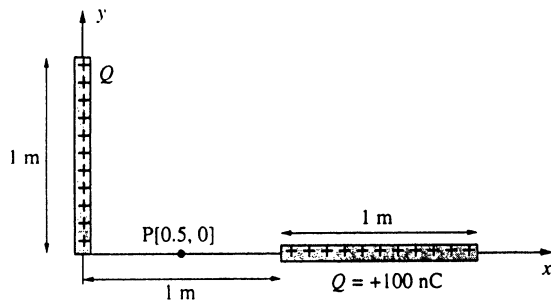


FIGURE 4.64 Two straight-line charges. Problem 4-13.

- 4-12. Six point charges on a hexagon.** Six identical point charges of $+25$ nC each are situated in space at the corners of a regular hexagon whose sides are each of length 6 cm. (a) Find the electric potential at the center of the hexagon. (b) Determine the energy required to move a point charge of -25 nC from infinity to the center of the hexagon.
- 4-13. Two straight-line charges.** Consider two uniformly charged wires, each of length 1 m and a total charge $+100$ nC, with their ends separated by 1 m, as shown in Figure 4.64. (a) Find the electric potential Φ at the point P, midway between the two wires. (b) Find the electric field \mathbf{E} at point P.
- 4-14. Seven point charges on a cube.** Seven identical point charges of $+10$ nC each occupy seven of the eight corners of a cube 3 cm on each side. Find the electric potential at the unoccupied corner.
- 4-15. Two line charges.** A uniform line charge of $\rho_{l1} = -4\pi \times 8.85$ pC \cdot m $^{-1}$ is located between the points $(-5,0)$ and $(-2,0)$ m and another such line of positive charge (i.e., $\rho_{l2} = 4\pi \times 8.85$ pC \cdot m $^{-1}$) between $(5,0)$ and $(2,0)$ m, as shown in Figure 4.65. (a) Find the electric potential Φ at $(1,0)$ m. (b) Find the electric field \mathbf{E} at the same point. At which points, if any, is the electric field \mathbf{E} zero? At which points, if any, is the electric potential Φ zero?
- 4-16. Circular ring of charge.** A total charge of Q_1 is distributed uniformly along a half-circular ring as shown in Figure 4.66. Two point charges, each of magnitude Q_2 , are situated as shown. The surrounding medium is free space. (a) Find Q_2 in terms of Q_1 so that the potential Φ at the center of the ring is zero. (b) Find Q_2 in terms of Q_1 so that the electric field \mathbf{E} at the center of the ring is zero.

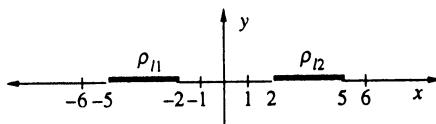


FIGURE 4.65 Two line charges. Problem 4-15.

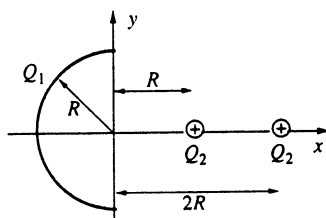


FIGURE 4.66 Circular ring of charge. Problem 4-16.

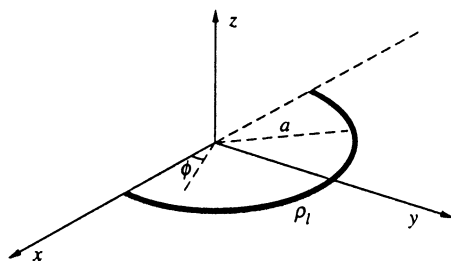


FIGURE 4.67 Semicircular line charge. Problem 4-17.

- 4-17. Semicircular line charge.** A thin line charge of density ρ_l is in the form of a semicircle of radius a lying on the xy plane with its center located at the origin, as shown in Figure 4.67. Find the electric field at the origin for the cases in which (a) the line charge density $\rho_l = \rho_0$ is a constant, and (b) the line charge density varies along the semicircular ring as $\rho_l = \rho_0 \sin \phi$.
- 4-18. Charge on a hemisphere.** The curved surface of a hemisphere of radius a centered at the origin carries a total charge of Q uniformly distributed over its curved surface, as shown in Figure 4.68. (a) Find the electric potential on the z axis. (b) Find the electric field on the z axis. (c) Repeat parts (a) and (b) if the charge Q is uniformly distributed throughout the volume of the hemisphere.
- 4-19. Sheet of charge with hole.** An infinite sheet of uniform charge density ρ_s is situated coincident with the xy plane at $z = 0$. The sheet has a hole of radius a centered at the origin. Find (a) the electric potential Φ and (b) the electric field \mathbf{E} at points along the z axis.
- 4-20. Spherical charge distribution.** A charge density of

$$\rho(r) = Ke^{-br}$$

where K and b are constants, exists in a spherical region of space defined by $0 < r < a$.

- (a) Find the total charge in the spherical region. (b) Find the electric field at all points in space. (c) Find the electric potential at all points in space. (d) Show that the potential found in part (c) satisfies the equation $\nabla^2\Phi = -\rho(r)/\epsilon_0$ for both $r < a$ and $r > a$.
- 4-21. The electron charge density in a hydrogen atom.** According to quantum mechanics, the electron charge of a hydrogen atom in its ground state is distributed like a cloud surrounding its nucleus, extending in all directions with steadily decreasing density such that the total charge in this cloud is equal to q_e (i.e., the charge of an electron). This electron charge distribution is given by

$$\rho(r) = \frac{q_e}{\pi a^3} e^{-2r/a}$$

where a is the *Bohr radius*, $a \approx 0.529 \times 10^{-10}$ m. (a) Find the electric potential and the electric field due to the electron cloud only. (b) Find the total electric potential and

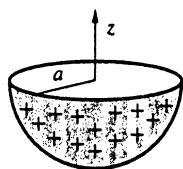


FIGURE 4.68 Charge on a hemisphere. Problem 4-18.

the electric field in the atom, assuming that the nucleus (proton) is localized at the origin.

- 4-22. Spherical shell of charge.** The space between two concentric spheres of radii a and b ($a < b$) in free space is charged to a volume charge density given by

$$\rho(r) = \frac{K}{r^2}; \quad a \leq r \leq b$$

where K is a constant. (a) Find the total charge in the shell. (b) Find the electric field at all points in space. (c) Find the electric potential at all points in space. (d) What happens if $b \rightarrow a$?

- 4-23. Spherical charge distribution.** A spherical charge distribution exists in free space in the region $0 < r < a$ given by

$$\rho(r) = \rho_0 \left(1 - \frac{r^2}{a^2} \right)$$

(a) Find the total charge. (b) Determine \mathbf{E} everywhere. (c) Determine Φ everywhere. (d) Sketch both $|\mathbf{E}|$ and Φ as a function of r .

- 4-24. Spherical charge with a cavity.** A spherical region of radius b in free space is uniformly charged with a charge density of $\rho = K$, where K is a constant. The sphere contains an uncharged spherical cavity of radius a . The centers of the two spheres are separated by a distance d such that $d + a < b$. Find the electric field inside the cavity.
- 4-25. Charge on a hollow metal sphere.** A hollow metal sphere of 20 cm diameter is given a total charge of 1 μC . Find the electric field and the electric potential at the center of the sphere.
- 4-26. A 1-farad capacitor.** To get an idea about the physical size of a 1 F capacitor, consider a parallel-plate capacitor with the two metal plates separated by 1 mm thickness of air. Calculate the area of the metal plates needed so that the capacitance is 1 F.
- 4-27. Gate oxide capacitance of a MOS transistor.** A basic MOS transistor consists of a gate conductor and a semiconductor (which is the other conductor), separated by a gate dielectric. Consider a MOS transistor using silicon dioxide (SiO_2) ($\epsilon_r = 3.9$) as the gate oxide. The gate oxide capacitance can be approximated as a parallel-plate capacitor. The gate oxide capacitance per unit area is given by

$$C_{\text{ox}} = \frac{\epsilon_{\text{ox}}}{t_{\text{ox}}}$$

where ϵ_{ox} and t_{ox} are the permittivity and the thickness of the gate dielectric. (a) If the thickness of the SiO_2 layer is 2×10^{-6} cm, find the gate oxide capacitance per unit area. (b) If the length and the width of the gate region are $L = 5 \times 10^{-4}$ cm and $W = 2 \times 10^{-3}$ cm respectively, find the total gate capacitance.

- 4-28. RG 6 coaxial cable.** A coaxial cable (RG 6) designed for interior use, such as connecting a TV set to a VCR, has a per-unit-length capacitance listed as 17.5 pF/ft. If the relative dielectric constant of the insulator material in the cable is $\epsilon_r \approx 1.64$, find the ratio of the inner and outer radii of the insulator.
- 4-29. Radius of a high-voltage conductor sphere.** Consider an isolated charged metallic conductor sphere in a dielectric medium at an electric potential of 500 kV. Calculate the minimum radius of the sphere such that dielectric breakdown will not occur if the

the electric field in the atom, assuming that the nucleus (proton) is localized at the origin.

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- 4-29. Radius of a high-voltage conductor sphere.** Consider an isolated charged metallic conductor sphere in a dielectric medium at an electric potential of 500 kV. Calculate the minimum radius of the sphere such that dielectric breakdown will not occur if the

surrounding dielectric is (a) air ($E_{BR} = 3 \text{ MV}\cdot\text{m}^{-1}$); (b) a gaseous dielectric such as sulfur hexafluoride (SF_6) ($\epsilon_r \approx 1$ and $E_{BR} = 7.5 \text{ MV}\cdot\text{m}^{-1}$); (c) a liquid dielectric such as oil ($\epsilon_r = 2.3$ and $E_{BR} = 15 \text{ MV}\cdot\text{m}^{-1}$); and (d) a solid dielectric such as mica ($\epsilon_r = 5.4$ and $E_{BR} = 200 \text{ MV}\cdot\text{m}^{-1}$).

- 4-30. Parallel-plate capacitor.** A parallel-plate capacitor is constructed from two aluminum foils of 1 cm^2 area each placed on both sides of rubber ($\epsilon_r = 2.5$ and $E_{BR} = 25 \text{ MV}\cdot\text{m}^{-1}$) of thickness 2.5 mm . Find the voltage rating of the capacitor using a safety factor of 10.
- 4-31. A parallel-plate capacitor with variable ϵ_r .** A parallel-plate capacitor of cross-sectional area A and thickness d is filled with a dielectric material whose relative permittivity varies linearly from $\epsilon_r = 1$ at one plate to $\epsilon_r = 10$ at the other plate. Find the capacitance. (b) Compare the result to the case of the same capacitor filled with air instead of the dielectric.
- 4-32. Coaxial capacitor.** Consider a coaxial capacitor as shown in Figure 4.69. Given $a = 5 \text{ mm}$, $l = 3 \text{ cm}$, and the voltage rating of the capacitor to be 2 kV with a safety factor of 10, what is the maximum capacitance that can be designed using (a) oil ($\epsilon_r = 2.3$ and $E_{BR} = 15 \text{ MV}\cdot\text{m}^{-1}$) and (b) mica ($\epsilon_r = 5.4$ and $E_{BR} = 200 \text{ MV}\cdot\text{m}^{-1}$).
- 4-33. Coaxial capacitor with two dielectrics.** A coaxial capacitor consists of two conducting coaxial surfaces of radii a and b ($a < b$). The space between is filled with two different dielectric materials with relative dielectric constants ϵ_{1r} and ϵ_{2r} , as shown in Figure 4.70. (a) Find the capacitance of this configuration. (b) Assuming that $l = 5 \text{ cm}$, $b = 3a = 1.5 \text{ cm}$, and oil and mica are used, calculate the capacitance. (c) Redo part (b) assuming that only oil is used throughout.

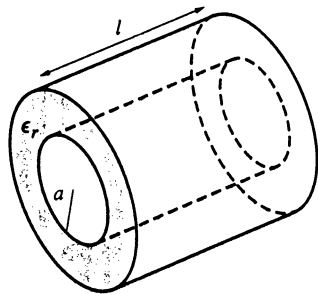


FIGURE 4.69 Coaxial capacitor. Problem 4-32.

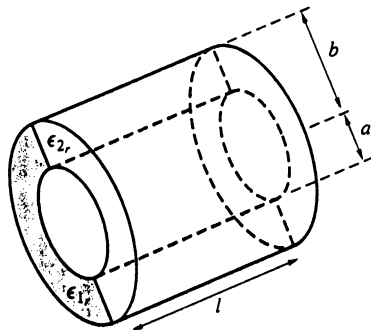


FIGURE 4.70 Coaxial capacitor with two dielectrics. Problem 4-33.

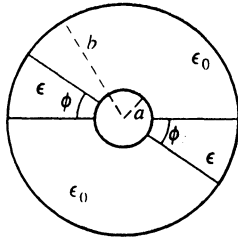


FIGURE 4.71 Coaxial capacitor with spacers. Problem 4-34.

- 4-34. Capacitor with spacers.** The cross-sectional view of an air-filled coaxial capacitor with spacers made out of material with permittivity ϵ is shown in Figure 4.71. (a) Find the capacitance of this coaxial line in terms of ϵ , a , b , and ϕ . (b) If the spacers are to be made out of mica ($\epsilon = 6\epsilon_0$), determine the angle ϕ such that only 10% of the total energy stored by the capacitor is stored in the spacers. (c) Consider the capacitor without the spacers (i.e., $\phi = 0$). For a given potential difference V_0 between the inner and outer conductors and for a given fixed value of b , determine the inner radius a for which the largest value of the electric field is a minimum.
- 4-35. Earth capacitor.** Consider the earth to be a large conducting sphere. (a) Find its capacitance (the earth's radius is $\sim 6.371 \times 10^6$ m). (b) Find the total charge and energy stored on the earth (take the electric field on the surface of the earth to be $100 \text{ V}\cdot\text{m}^{-1}$). (c) Find the maximum charge and energy that can be stored on the earth.
- 4-36. A coaxial capacitor with variable permittivity.** Consider a coaxial capacitor with two concentric metal cylinders of radii a and b ($a < b$), filled with a dielectric material whose permittivity ϵ varies linearly from ϵ_a at $r = a$ to ϵ_b at $r = b$. (a) Find the capacitance per unit length. (b) Find the numerical value of the capacitance if the radii are 2 mm and 6 mm and the relative dielectric constant varies from 2.25 to 8.5, respectively.
- 4-37. Planar charge.** A surface charge distribution $\rho_s(x, z)$ exists on the xz plane, with no charge anywhere else (i.e., $\rho = 0$ for $|y| > 0$). Which of the following potential functions are valid solutions for the electrostatic potential in the half-space $y > 0$, and what is the corresponding charge distribution $\rho_s(x, z)$ on the xz plane?

$$\Phi_1 = e^{-y} \cosh x$$

$$\Phi_2 = e^{-y} \cos x$$

$$\Phi_3 = e^{-\sqrt{2}y} \cos x \sin x$$

$$\Phi_4 = \sin x \sin y \sin z$$

- 4-38. Parallel power lines.** An isolated pair of parallel power lines a distance of d_1 apart have a potential difference of V_{AB} and are located a distance h above a pair of telephone wires, as shown in Figure 4.72. The parameter values are $d_1 = 1$ m, $a = 2$ cm, $V_{AB} = 440$ V, $h = 60$ cm, and $d_2 = 15$ cm. (a) Find the direction and magnitude of the electric field at points 1 and 2. Take the midpoint between the power lines as the origin of your coordinate system. (b) Determine the potential difference Φ_{12} between points 1 and 2.

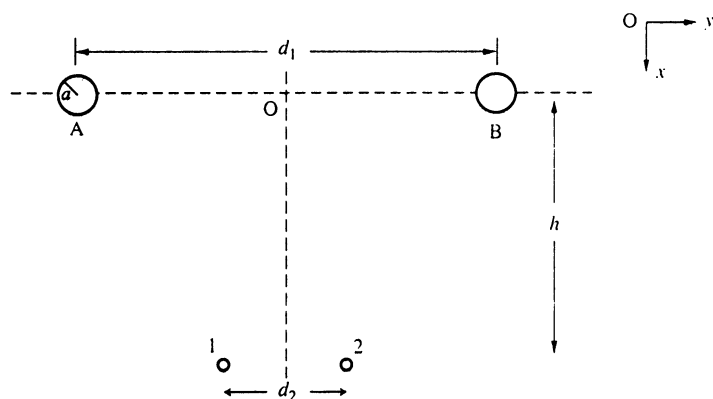


FIGURE 4.72 Parallel power lines above telephone lines.
Problem 4-38.

- 4-39. Field under high-voltage line.** Many 60 Hz high-voltage transmission lines operate at an rms alternating voltage of 765 kV. (a) What is the peak electric field at ground level under such a line if the wire is 12 m above the ground? (b) What is the peak potential difference between the head and feet of a 6-ft tall person? (c) Is the field sufficient to ignite a standard (110 V) fluorescent lamp of 2 ft length?
- 4-40. High-voltage dc transmission line.** A high-voltage dc transmission line consists of a single 25-mm-diameter cylindrical conductor supported at an average height of 15 m above the ground. What is the electric field \mathbf{E} at the ground level (a) directly under the conductor, (b) at a ground distance of 25 m, and (c) at a ground distance 50 m perpendicular to the line? Assume the line to be operating at 500 kV, and the ground to be flat and perfectly conducting.
- 4-41. Thundercloud fields.** A typical thundercloud can be modeled as a capacitor with horizontal plates with 10 km^2 area separated by a vertical distance of 5 km. Just before a large lightning discharge, the upper plate may have a total positive charge of up to 300 C, with the lower plate having an equal amount of negative charge. (a) Find the electrostatic energy stored in the cloud just before a discharge. (b) What is the potential difference between the top and bottom plates? (c) What is the average electric field within the cloud? How does this value compare to the dielectric breakdown field of dry air ($3 \text{ MV}\cdot\text{m}^{-1}$)?
- 4-42. Two conducting spheres.** Consider a pair of small conducting spheres with radii a , b , small compared to the separation distance d between their centers (i.e., $a, b \ll d$). (a) Determine the electrostatic energy stored by this configuration, assuming that the spheres with radii a and b carry charges of Q and $-Q$, respectively. Your answer should depend on d . State all assumptions. (b) Repeat part (a) assuming that the spheres with radii a and b carry charges of $+Q$ and $+2Q$, respectively.