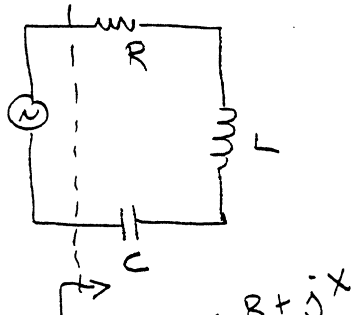
series RLC circuit



$$= R + jX$$

$$Z_{in}(j\omega) = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

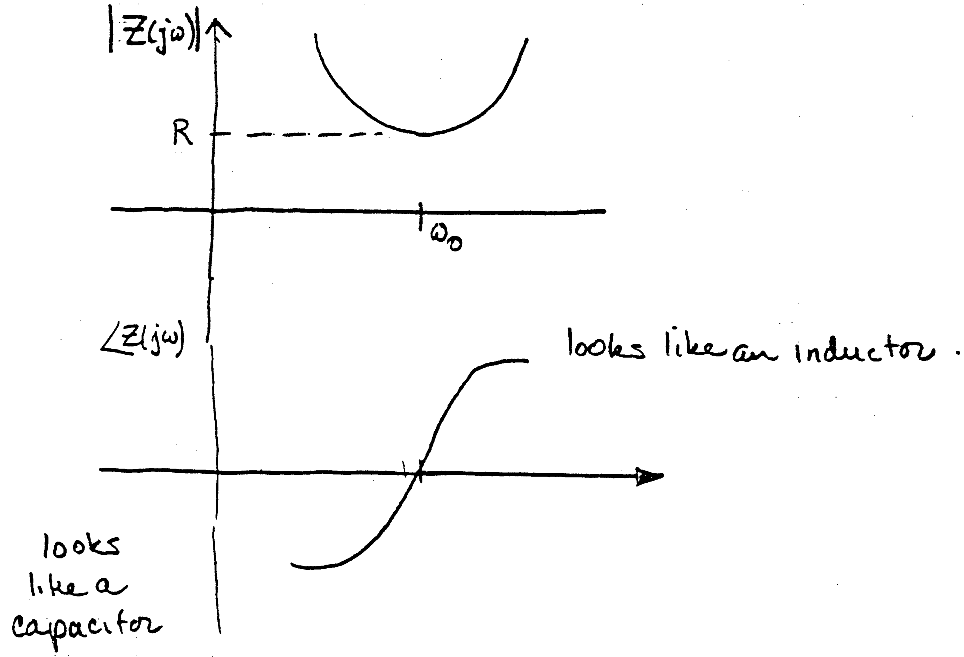
at ω_o $\qquad \omega_o L - \frac{1}{\omega_o C} = 0$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

from an admittance viewpoint

multiply top & bottom

$$Y(j\omega) = G + jB = \frac{1}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{R - j\left(\omega L - \frac{1}{\omega C}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$



just like for filters (3db point) we define

$$Z = R + j(\pm R) \qquad \text{i.e. } X = R$$

at $|Z| = \sqrt{2}\, R$

$$I = \frac{I_{resonance}}{\sqrt{2}} \qquad ,\text{etc.}$$

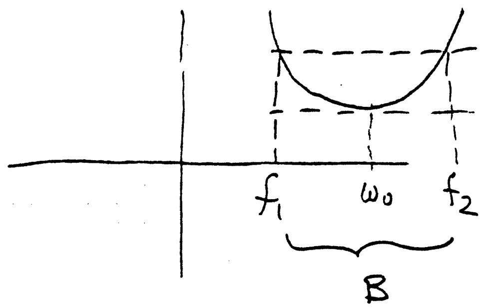
in terms of power

at resonance $\quad P = I I^{*} R = |I_{max}|^2 R = P_{res.}$

at $X = \pm R$ $\quad P = \left|\frac{I_{max}}{\sqrt{2}}\right|^2 R = \frac{P_{res}}{2}$

there is a phase factor

half power points



you can see immediately that B is an indicator of how narrow the resonance is.

We use $Q = \frac{f_0}{B}$ to denote the selectivity of the circuit

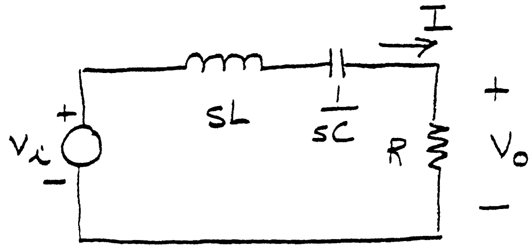
More generally $\quad Q = 2\pi \dfrac{\text{maximum instaneous energy stored in the circuit}}{\text{energy dissipated per cycle.}}$

for circuits with complex impedances

$$Q = \frac{\text{reactance (inductive)}}{\text{resistance.}} = \frac{\omega_0 L}{R}$$

use inductive by convention
we can also use cap we will see later

Proof that $Q = \frac{f_0}{B}$ is found in class notes transfer function of a series RLC circuit



$$I(s) = \frac{V_i(s)}{sL + \frac{1}{sC} + R} = \frac{V_i(s)\left(\frac{s}{L}\right)}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$V_o(s) = I(s)R = \frac{V_i(s)\ s\frac{R}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

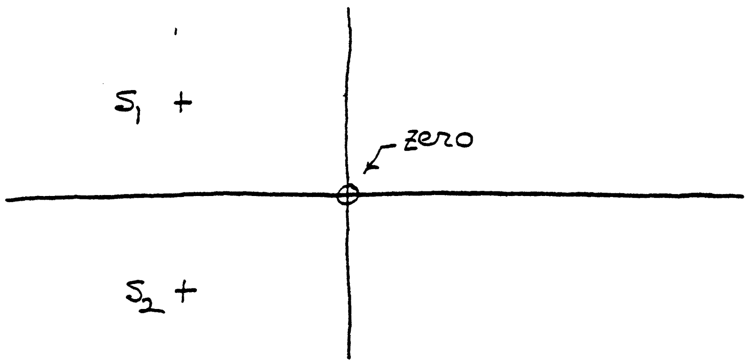
$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$ characteristic equation

$$s_{1,2} = -\frac{R}{2L} \pm \frac{\sqrt{(R/L)^2 - 4/LC}}{2}$$

in most r.f. networks $\frac{4}{LC} > \frac{R^2}{L^2}$ or $R^2 < \frac{4L}{C}$

(this is known as underdamping)

so that $s_{1,2}$ are a complex conjugate pair



Then $V_o(s)$ can be written

$V_o(s) = T(s)V_i(s)$ where $T(s) = \dfrac{RC\ s}{s^2/\omega_0^2 + \frac{2\zeta}{\omega_0}s + 1}$

$\uparrow$
transfer function

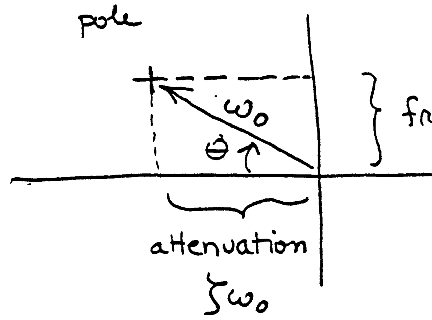
where $\omega_0 = \dfrac{1}{\sqrt{LC}}$ undamped natural frequency

$\zeta = \dfrac{R}{2}\sqrt{\dfrac{C}{L}}$ damping ratio

there is a reason for this. If $V_i(s)$ is a unit step

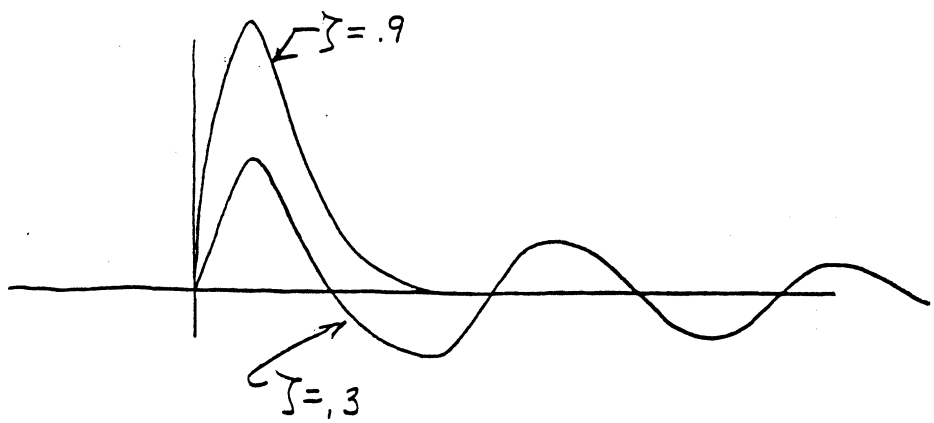
The output is

$$V_o(t) = \frac{2\zeta}{\sqrt{1-\zeta^2}}\, e^{\underbrace{-\zeta\omega_0 t}_{\text{damping}}} \sin\left(\underbrace{\omega_0\sqrt{1-\zeta^2}}_{\text{frequency of ringing}}\, t\right) u(t)$$



} frequency $\omega_0\sqrt{1-\zeta^2}$

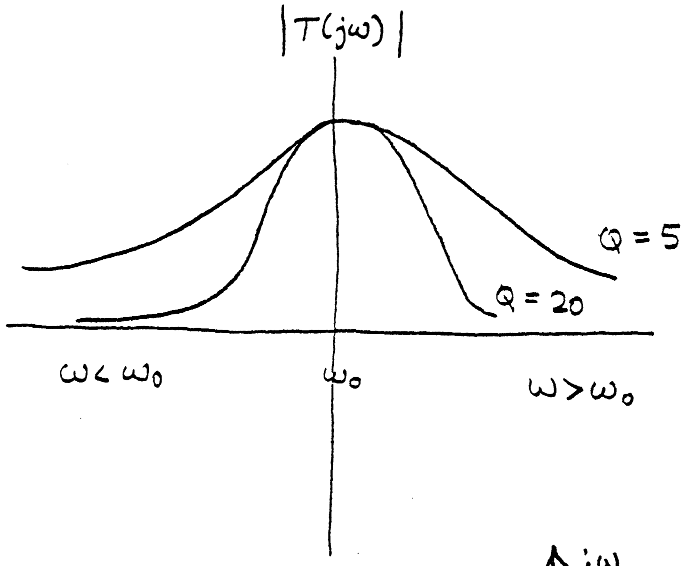
where $\zeta = \cos\theta$

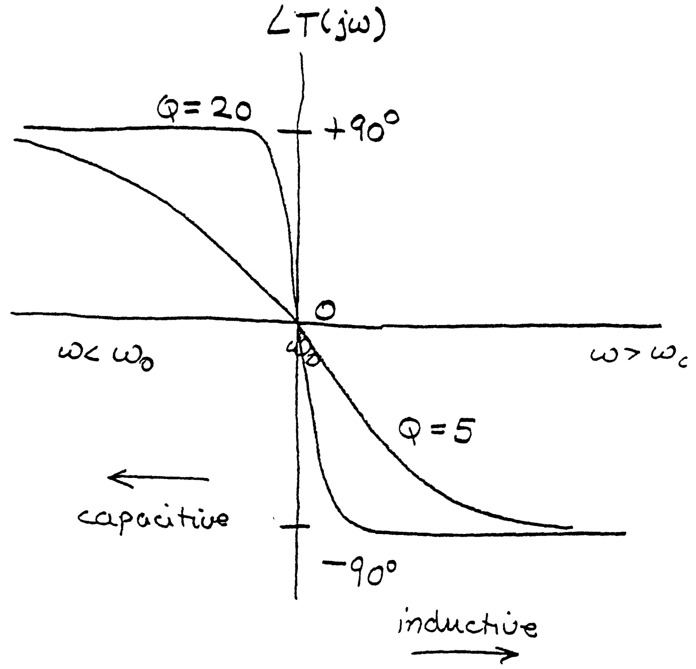


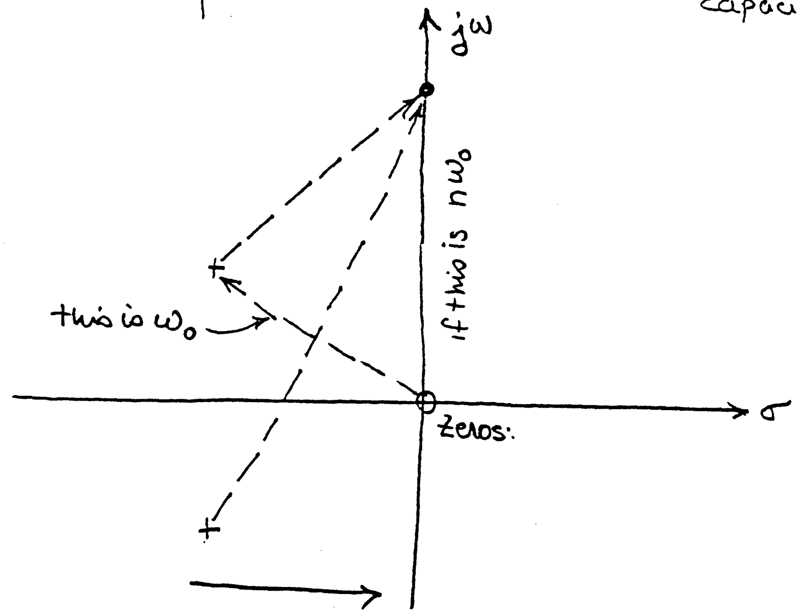
As ζ increases the transient increases but decays faster.

For steady state let $s = j\omega$

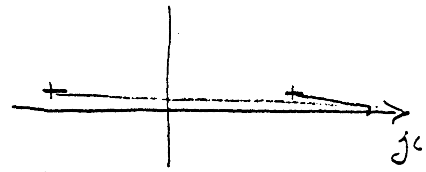
$$T(j\omega) = \frac{1}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$







close to axis for high Q circuit



poles close to axis so projections are essentially good estimates of their frequency.

without proof:

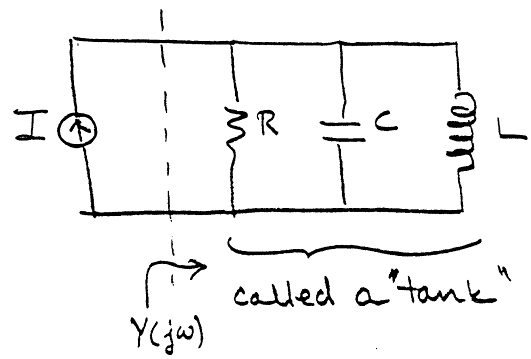
$$\text{Gain}(\omega) = \frac{\prod(\omega-\omega_z)}{\prod(\omega-\omega_p)} B = \frac{n\omega_0}{(n-1)\,\omega_0\,(n+1)\,\omega_0} B$$

$$= \frac{n}{n^2-1}\,\frac{1}{\omega_0}\,B = \frac{n}{n^2-1}\,\frac{1}{Q}$$

attenuation at harmonics

$$\frac{G(n\omega_0)}{G(\omega_0)} = \frac{n}{n^2-1}\,\frac{1}{Q}$$

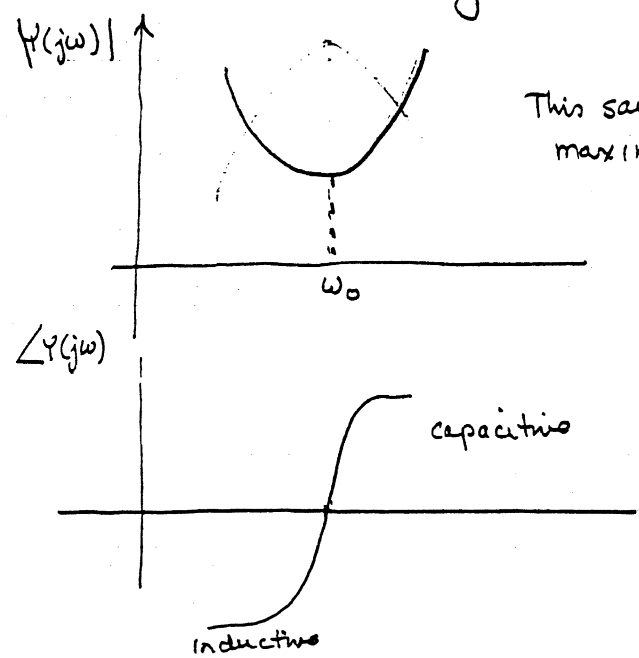
parallel RLC circuit



$$Y(j\omega) = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = \underbrace{\frac{1}{R}}_{\equiv G} + j\underbrace{\left(\omega C - \frac{1}{\omega L}\right)}_{= B} = G + jB$$

at ω_0 $B = 0$

this circuit behaves differently at resonance.



This says that <u>impedance</u> is a maximum at ω_0

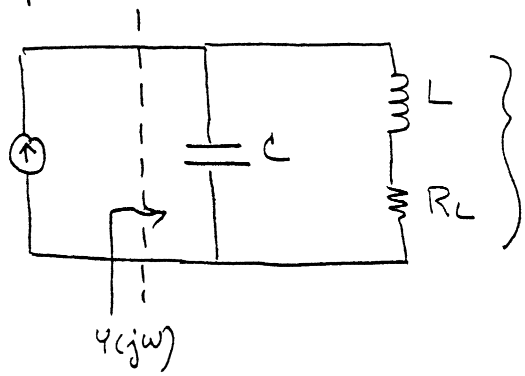
As before $Q = \dfrac{f_0}{B}$

However now define Q in terms of reactance (actually admittance)

$$Q = \frac{\omega_0 C}{G}$$

These give us a very useful result $\dfrac{f_0}{B} = \dfrac{\omega_0 C}{G}$ or $B = \dfrac{1}{2\pi RC}$

more complex circuit



$$Z_1 = j\omega L + R_L$$

$$\frac{1}{Z_1} = \frac{1}{R_L + j\omega L} = \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

by inspection $\quad Y(j\omega) = j\omega C + \frac{1}{Z_1} = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$

$$Y(j\omega) = \frac{R}{R^2 + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)$$

at resonance $B \to 0 \quad \therefore \quad \omega_0 C - \frac{\omega_0 L}{R^2 + \omega_0^2 L^2} = 0$

$$R^2 + \omega_0^2 L^2 = \frac{\omega_0 L}{\omega_0 C} = \frac{L}{C}$$

$$\therefore \ \omega_0^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right) = \frac{1}{LC} - \left(\frac{R}{L}\right)^2$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

resonance frequency is lowered by add resistance.

what we do here is note that

$$Y(j\omega_0) = \frac{R_L}{R_L^2 + (\omega_0 L)^2}$$

$$Z(j\omega_0) = \frac{R_L^2 + (\omega_0 L)^2}{R_L^2} = R_L\left[1 + \left(\frac{\omega_0 L}{R_L}\right)^2\right]$$

this is NOT the Q of the circuit but looks like a Q. We call it Q_t for transformation Q

$$Z(j\omega_0) = R_t = R_L\left[1 + Q_t^2\right]$$

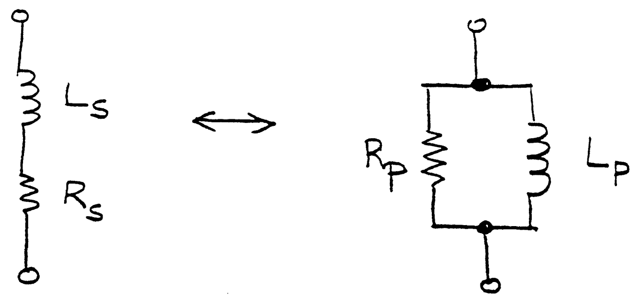
the resistance at resonance is apparantly increased by the Q of the LR branch.

The bandwidth of this circuit is

$$B = f_2 - f_1 = \frac{1}{2\pi R_t C}$$

Powerful technique to handle unknown configurations

series $\longleftrightarrow$ parallel conversions



$$Z = R_s + j\omega L_s$$

$$\begin{aligned} Z &= j\omega L_p \parallel R_p \\ &= \frac{j\omega L_p R_p}{R_p + j\omega L_p} \\ &= \frac{j\omega L_p R_p (R_p - j\omega L_p)}{R_p^2 + \omega^2 L_p^2} \\ &= \frac{(\omega L_p)^2 R_p + j\omega L_p R_p^2}{R_p^2 + \omega^2 L_p^2} \end{aligned}$$

equating real and imaginary parts:

$$R_s = \frac{(\omega L_p)^2 R_p}{R_p^2 + \omega^2 L_p^2} \qquad j\omega L_s = \frac{j\omega L_p R_p^2}{R_p^2 + \omega^2 L_p^2}$$

Now make some definitions

$$X_s = \omega L_s \qquad X_p = \omega L_p$$

$$Q_s = \frac{X_s}{R_s} = \frac{\omega L_s}{R_s} \qquad Q_p = \frac{R_p}{X_p} \Big\} \text{ always inverted for parallel circuits}$$

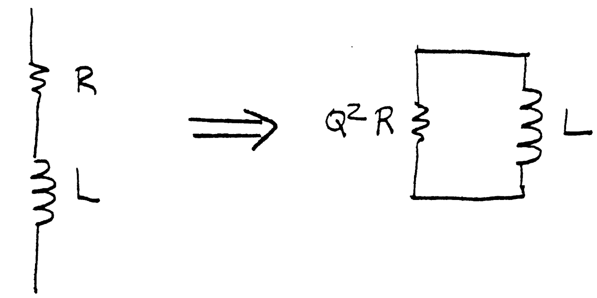
then

$$R_s = \frac{R_p}{1 + Q_p^2} \qquad R_p = R_s (1 + Q_s^2)$$

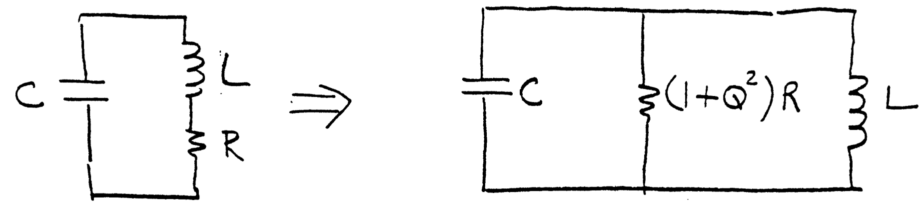
$$L_s = L_p \frac{Q_p^2}{Q_p^2 + 1} \qquad L_p = L_s \left(\frac{Q_s^2 + 1}{Q_s^2} \right)$$

note: $Q_s = Q_p$

if $Q \gg 1$



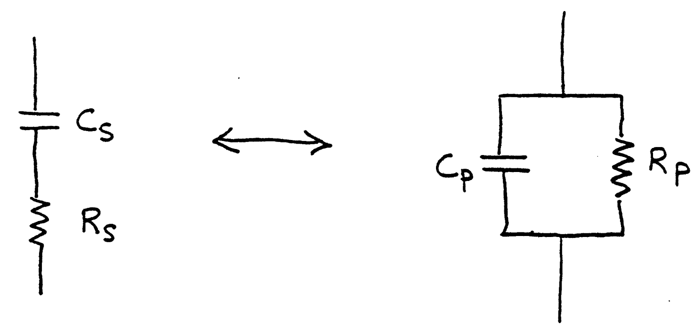
Example: high Q "complicated" circuit



at resonance $R_t = (1+Q^2)R$

$$B = \frac{1}{2\pi R_t C}$$

Similar results for capacitor circuits



$$X_s = \frac{1}{\omega R_s} \qquad X_p = \frac{1}{\omega C_p}$$

$$Q_s = \frac{X_s}{R_s} = \frac{1}{\omega R_s C_s} \qquad Q_p = \frac{R_p}{X_p} = \frac{R_p}{\frac{1}{\omega C_p}} = \omega R_p C_p$$

$$R_s = \frac{R_p}{1+Q_p^2} \qquad R_p = R_s(1+Q_s^2)$$

$$C_s = C_p \frac{Q_p^2+1}{Q_p^2} \qquad C_p = C_s \frac{Q_s^2}{Q_s^2+1}$$

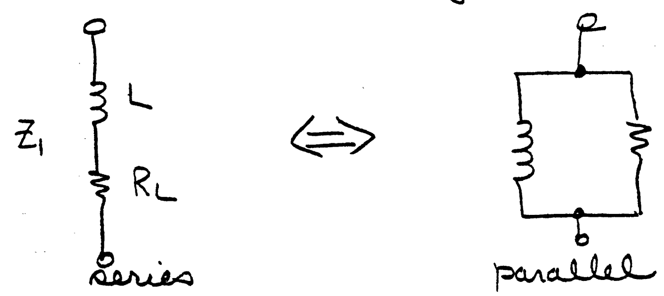
$$Z(j\omega_0) = R_t = R_L\left[1+Q_t^2\right]$$

The resistance at resonance is increased by the "Q" of the L-R branch.

What is the bandwidth of this circuit

$$B = f_2 - f_1 = \frac{1}{2\pi R_t C}$$

Much more powerful general way to solve this circuit



$$Z = R_s + j\omega L_s$$

$$Z = j\omega L_p \,\|\, R_p = \frac{j\omega L_p R_p}{R_p + j\omega L_p}$$

$$= \frac{j\omega L_p R_p\,(R_p - j\omega L_p)}{R_p^2 + \omega^2 L_p^2}$$

$$= \frac{(\omega L_p)^2 R_p + j\omega L_p R_p^2}{R_p^2 + \omega^2 L_p^2}$$

equate real & imaginary parts.

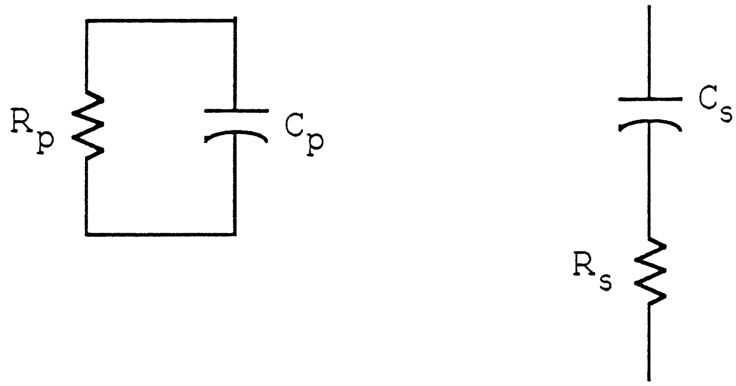
$$R_s = \frac{(\omega L_p)^2 R_p}{R_p^2 + \omega^2 L_p^2} \qquad j\omega L_s = \frac{j\omega L_p R_p^2}{R_p^2 + \omega^2 L_p^2}$$

make some definitions

$$X_s = \omega L_s \qquad X_p = \omega L_p$$

$$Q_s = \frac{X_s}{R_s} = \frac{\omega L_s}{R_s} \qquad Q_p = \frac{R_p}{X_p}$$ } always inverted for parallel circu

PARALLEL-SERIES CONVERSION FOR RC NETWORKS



DEFINE	
$X_p=1/(\omega C_p)$	$Xs=1/(\omega C^s)$
$Q_p=R_p/X_p$	$Qs=X_s/R_s=1/(\omega R_s C_s)$

PARALLEL EQUIV OF SERIES	SERIES EQUIV OF PARALLEL

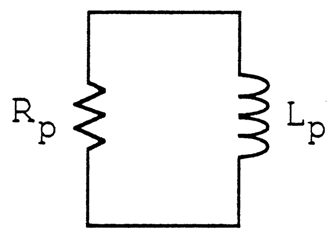
EXACT FORMULA

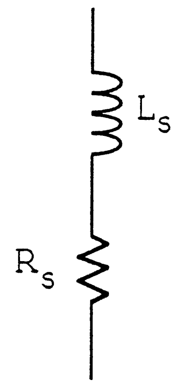
$R_{pe}=R_s(1+Q_s2)$	$R_{se}=R_p/(1+Q_p2)$
$X_{pe}=X_s(1+Q_s2)/Q_s2$	$X_{se}=X_pQ_p2/(1+Q_p2)$
$C_{pe}=C_sQ_s2/(1+Q_s2)$	$C_{se}=C_p(Q_p2+1)/Q_p2$

APPROXIMATE FORMULA

if $Q_s \geq 10$	if $Q_p \geq 10$
$R_{pe} \simeq R_sQ_s2$	$R_{se} \simeq R_p/Q_p2$
$X_{pe} \simeq X_s$	$X_{se} \simeq X_p$
$C_{pe} \simeq C_s$	$C_{se} \simeq C_p$

PARALLEL-SERIES CONVERSION FOR RL NETWORKS





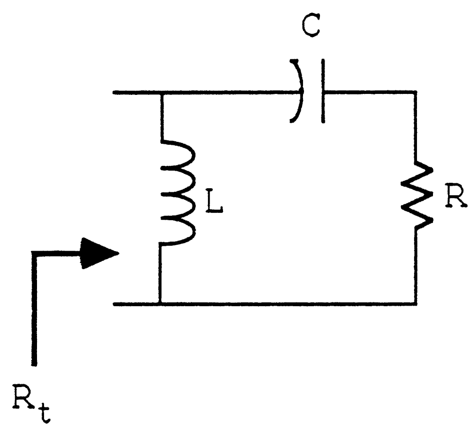
DEFINE	
$X_p=\omega L_p$	$Xs=\omega L_s$
$Q_p=R_p/X_p$	$Qs=X_s/R_s=\omega L_s/R_s$

PARALLEL EQUIV OF SERIES	SERIES EQUIV OF PARALLEL
EXACT FORMULA	
$R_{pe}=R_s(1+Q_s2)$	$R_{se}=R_p/(1+Q_p2)$
$X_{pe}=X_s(1+Q_s2)/Q_s2$	$X_{se}=X_pQ_p2/(1+Q_p2)$
$L_{pe}=L_s(1+Q_s2)/Q_s2$	$L_{se}=L_pQ_p2/(Q_p2+1)$
APPROXIMATE FORMULA	
if $Q_s \geq 10$	if $Q_p \geq 10$
$R_{pe} \simeq R_sQ_s2$	$R_{se} \simeq R_p/Q_p2$
$X_{pe} \simeq X_s$	$X_{se} \simeq X_p$
$L_{pe} \simeq L_s$	$L_{se} \simeq L_p$

IMPEDANCE MATCHING NETWORKS

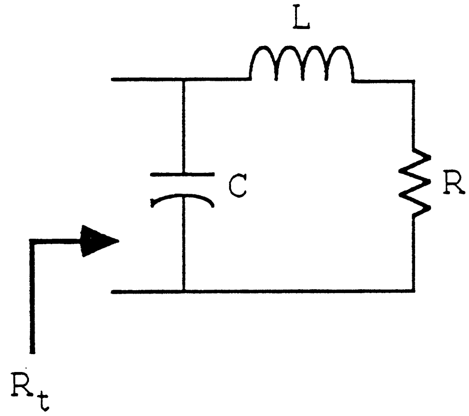
Note that each set of analysis/design formulas has two columns labeled "Exact" and "Approximate" The approximate column is valid when the Q_t of the network is ≥ 10 (the are about 10% accurate when $3 \leq Q_t \leq 10$). This type of network is known as a L network because the basic output element looks like a letter "L" turned on its side. Note that because these networks only have two resonant elements you cannot simultaneously design for a specific bandwidth, resonant frequency and impedance transformation ratio; you can only do that by adding reactive components to the network.

DESIGN FORMULAS FOR RC "L" NETWORK



Quantity	Exact	Approximate
ω_o	$=\sqrt{\dfrac{1}{LC-R^2C^2}}$	$\simeq \dfrac{1}{\sqrt{LC}}$
Q_t	$=\dfrac{1}{\omega_o C R}=\dfrac{R_t}{\omega_o L}$	$\simeq \dfrac{\omega_o L}{R}$
$\omega_o L$	$=\dfrac{1}{\omega_o C}\dfrac{Q_t^2+1}{Q_t^2}$	$\simeq \dfrac{1}{\omega_o C}$
R_t	$=\dfrac{L}{CR}=\dfrac{Q_t}{\omega_o C}= R(Q_t^2+1)$	$\simeq Q_t^2 R = \omega_o L Q_t$
B		$\simeq \dfrac{1}{2\pi C R_t}=\dfrac{R}{2\pi L}=\dfrac{f_o}{Q_t}$

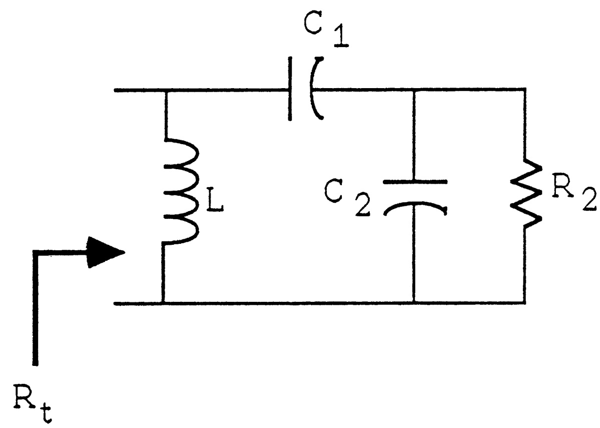
DESIGN FORMULAS FOR RL "L" NETWORK:



Quantity	Exact	Approximate
ω_o	$= \sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$	$\simeq \frac{1}{\sqrt{LC}}$
Q_t	$= \frac{\omega_o L}{R} = \omega_o C R_t$	$\simeq \frac{1}{\omega_o C R}$
$\omega_o L$	$= \frac{1}{\omega_o C} \frac{Q_t^2}{Q_t^2 + 1}$	$\simeq \frac{1}{\omega_o C}$
R_t	$= \frac{L}{RC} = \frac{Q_t}{\omega_o C} = R\,(Q_t^2 + 1)$	$\simeq Q_t^2 R = \omega_o L Q_t$
B		$\simeq \frac{f_o}{Q_t} = \frac{R}{2\pi L} = \frac{1}{2\pi C R_t}$

DESIGN FORMULAS FQR TAPPED-CAPACITOR MATCHING NETWORK

For $Q_t \simeq \frac{f_o}{B} \geq 10$,



(1) $C \simeq \frac{1}{2\pi B R_t}$

(2) $L \simeq \frac{1}{\omega^2 C}$

(3) $Q_t \simeq \frac{f_o}{B}$

(4) $N = \sqrt{\frac{R_t}{R_2}}$

(5) $\frac{Q_t}{N} \simeq Q_p$

Formulas for $Q_p \geq 10$

(6) $Q_p = \frac{Q_t}{N}$

(7) $C_2 = N\,C$

(8) $C_1 = \frac{C_2}{N-1}$

$Q_p < 10$

(6) $Q_p = \sqrt{\frac{Q_t^2 + 1}{N^2} - 1}$

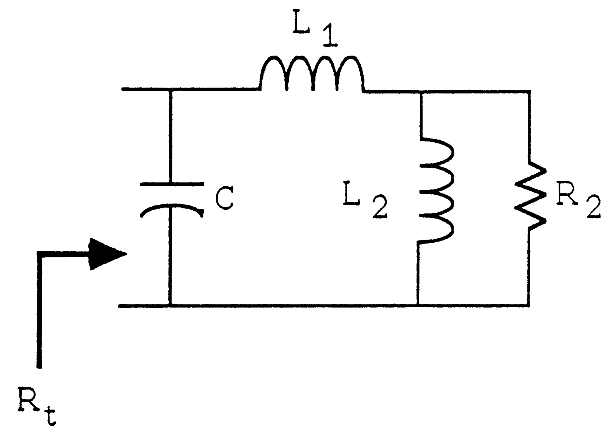
(7) $C_2 = \frac{Q_p}{\omega_o R_2}$

(8) $C_{se} = C_2 \frac{Q_p^2 + 1}{Q_p^2}$

(9) $C_1 = \frac{C_{se}\ C}{C_{se} - C}$

DESIGN FORMULAS FOR TAPPED-INDUCTOR MATCHING NETWORK

For $Q_t \simeq \frac{f_o}{B} \geq 10$,



(1) $C \simeq \dfrac{1}{2\pi B R_t}$

(2) $L \simeq \dfrac{1}{\omega^2 C}$

(3) $Q_t \simeq \dfrac{f_o}{B}$

(4) $N = \sqrt{\dfrac{R_t}{R_2}}$

(5) $\dfrac{Q_t}{N} \simeq Q_p$

Formulas for $Q_p \geq 10$

(6) $Q_p = \dfrac{Q_t}{N}$

(7) $L_2 = \dfrac{L}{N}$

(8) $L_1 = (N - 1)\, L_2$

$Q_p < 10$

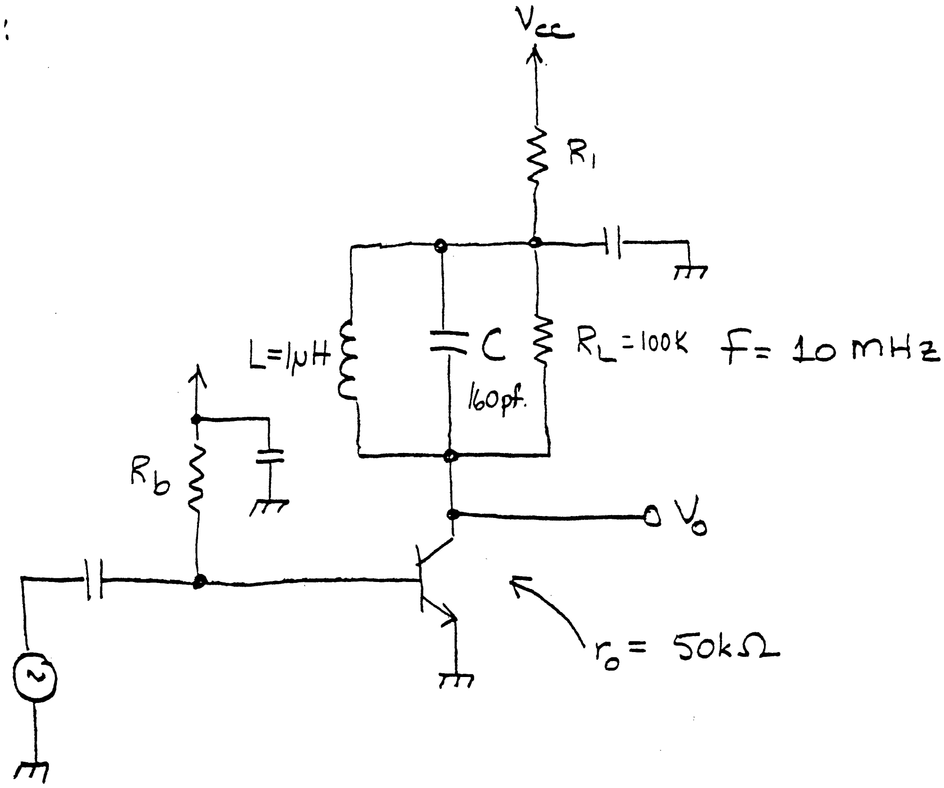
(6) $Q_p = \sqrt{\dfrac{Q_t^2 + 1}{N^2} - 1}$

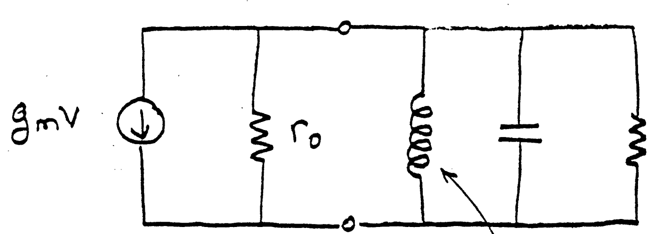
(7) $L_2 = \dfrac{R_2}{\omega_o Q_p}$

(8) $L_{se} = L_2 \dfrac{Q_p^2}{Q_p^2 + 1}$

(9) $L_1 = L - L_{se}$

Example:





$L = 1\mu H,\ Q_\mu = 100$

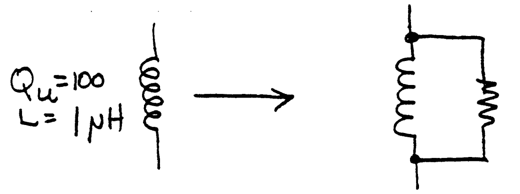
parallel RLC circuit

$$Q_p = \frac{R_p}{\omega_o L} \longrightarrow \frac{R_p}{\omega_o \frac{1}{\omega_o^2 C}} = \omega_o R_p C$$

$$\omega_o^2 = \frac{1}{LC}$$

using the above result $\quad B = \dfrac{\omega_o}{Q} = \dfrac{\omega_o}{\omega_o R_p C} = \dfrac{1}{R_p C}$

$$B = \frac{1}{(r_o \parallel R_L)\, C}$$ is this really correct



$Q_u = \dfrac{R_p}{\omega_o L}$ so $100 = \dfrac{R_p}{2\pi(10^7)(10^{-6})}$

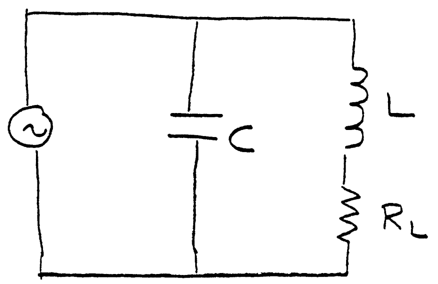
$\therefore\ R_p = 2\pi\ 1000$

$= 6280\,\Omega$

$R_p \parallel r_o \parallel R_L = 6280 \parallel 50k \parallel 100K = 9.13\,k\Omega$

$$B = \frac{1}{(9.13\times10^3)(160\times10^{-12})} = 0.865\times10^6 \text{ rad/sec.}$$

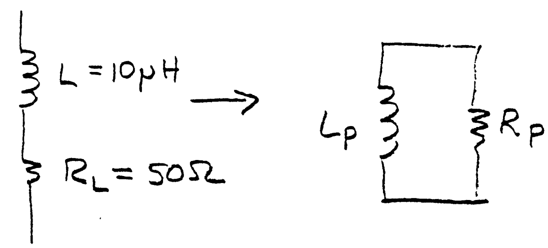
Example:



$C = 1000\ pf$

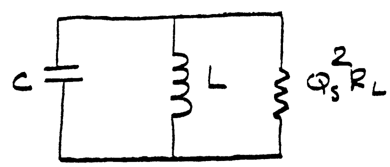
$L = 10\mu H$

$R_L = 50\Omega$



$Q_s = \dfrac{\omega L_s}{R_s}$ what do we do since we don't know ω.

Assume $Q_s \geqslant 10$ then circuit looks like



$$\omega_0 \approx \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(10\times10^{-6})(1000\times10^{-12})}}$$

10^7 Hz

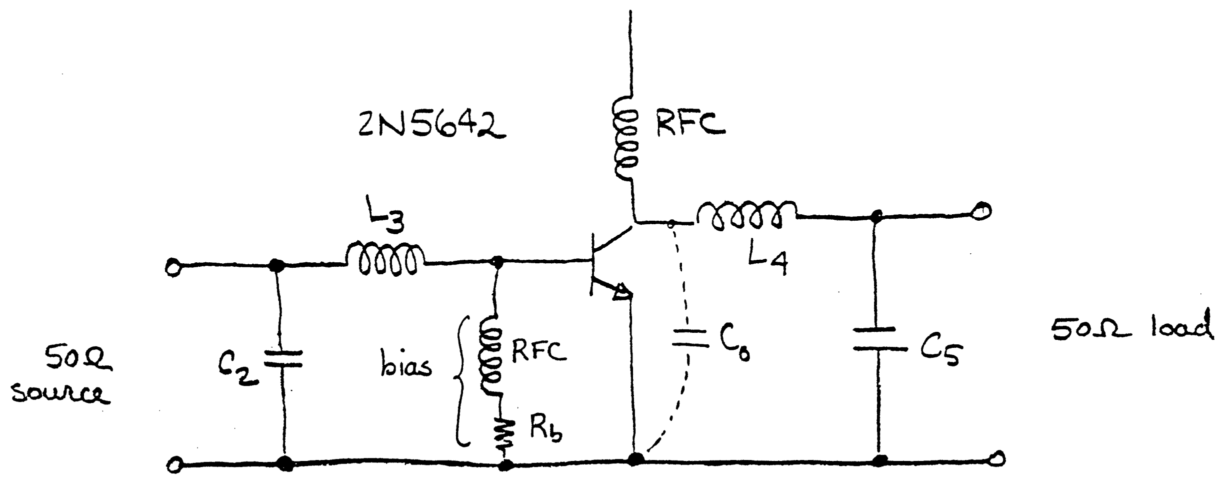
$$Q_p = \frac{R_p}{\omega L_p} = \frac{Q_s^2 R_s}{\omega L_p}$$

but $Q_s = Q_p$

$$Q_s = \frac{\omega L_p}{R_s} = \frac{(10^7)(10\times10^{-6})}{50} = 20$$

so approximation was valid

real circuit

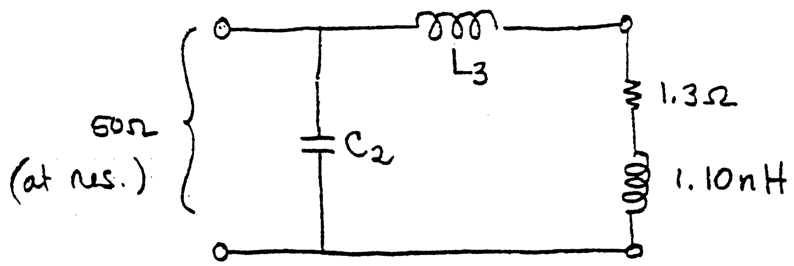


from data sheet for transistor

Z_o @ 175 MHz $= 12.2 - j13.55$ (capacitive)

Z_{in} @ 175 MHz $= 1.3 + j1.22$ (inductive)

look at input circuit



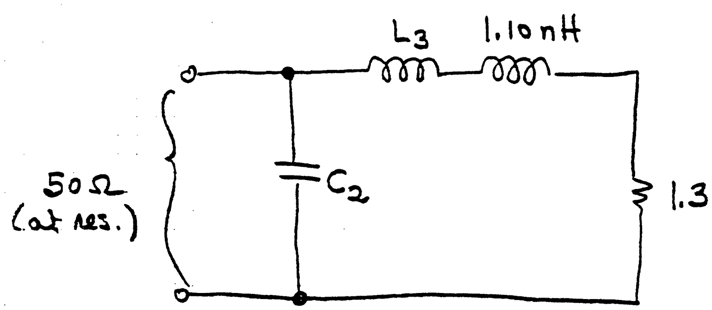
$\omega L = 1.22\Omega$

$2\pi(175 \times 10^6)\ L = 1.22$

$L = 1.10 \times 10^{-9}$ H

$Q = \frac{\omega L}{R} = \frac{1.22}{1.3} = 0.93$

low Q



this is resonant RL‖C

[for $Q_t \geq 10$ use approximate formula]

① find Q_t

$R_t = R(Q_t^2 + 1)$

$50 = 1.3\,(Q_t^2 + 1)$

$Q_t^2 = \frac{50}{1.3} - 1 = 37.46$

$Q_t \simeq 6.12$

∴ use exact formula

② using Q_t find C

$$Q_t = \omega_o C R_t$$

$$6.12 = 2\pi(175\times10^6)\, C\, (50)$$

$$C = 111.3\times10^{-12} = 111.3 \text{ pf.}$$

③ solve for L

$$Q_t = \frac{\omega_o L}{R}$$

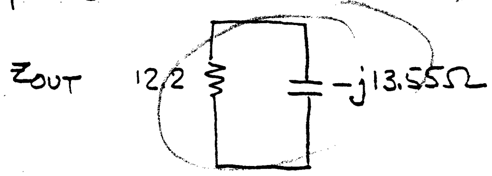
$$6.12 = \frac{2\pi\,(175\times10^6)\,L}{1.3}$$

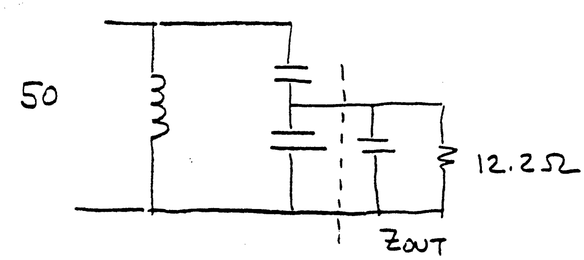
$$L = 7.24\times10^{-9} = 7.24 \text{ nH}$$

$\therefore$ use $L_3 = 7.24 - 1.10 \text{ nH} = 6.14 \text{ nH}$

this is incorrect.

Output circuit





$$-j13.55 = \frac{1}{j2\pi(175\times10^6)C}$$

$$C = \frac{1}{2\pi(175\times10^6)(13.55)} = 67.1 \text{ pf}$$

Do by capacitive divider $N = \sqrt{\frac{50}{12.2}} = 2.02$

check for max Q_p $\quad Q_p = \omega_o R_2 C_2 = 2\pi(175\times10^6)(12.2)(67.1\times10^{-9})$

$= 900.$

$Q_t \overset{?}{=} N Q_p$

$\therefore$ pick $Q_t = 10$ i.e. 17.5 MHz bandwidth.

$Q_t = \frac{f_o}{B}$ $\quad B = \frac{f_o}{Q} = 17.5\text{MHz}$

$nF_{n}pf$

$$C = \frac{1}{2\pi R_t B} = \frac{1}{2\pi(50)(17.5\times10^6)} = 181.9 \text{ pf.}$$

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{\underbrace{(2\pi\ 175\times10^6)^2}_{1.209\times10^{18}}(181.9\times10^{-9})} = \frac{1}{219.9\times10^6}$$

$$= 4.54\times10^{-9} \text{ Henrys}$$

$$Q_p \cong \frac{Q_t}{N} = \frac{10}{2.02} = 4.95$$

$C_2 \cong NC = 49.5$ pf   but   C of transistor $= 67.1$ pf

how can we make work.

① change Q_t

② use inductor to cancel out part of C_{OUT}.

work backwards.   pick $C_2 = 100$pf

$N = 2.02$

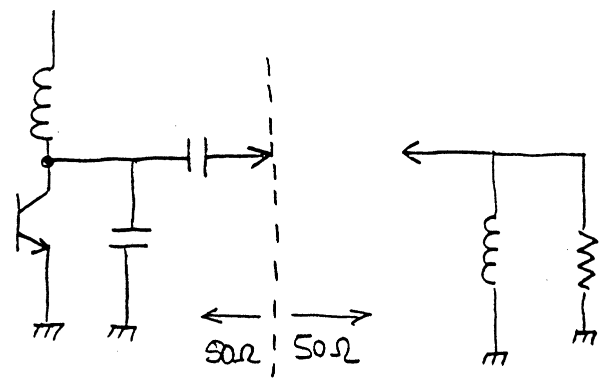
$$\therefore\ C = \frac{C_2}{N} = \frac{100}{2.02} = 49.5 \text{pf}$$

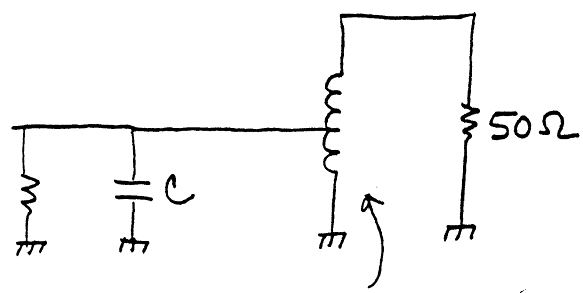
$$C_1 = \frac{C_2}{N-1} = \frac{49.5 \text{ pf}}{2.02-1} = 48.5 \text{ pf}$$

$$B = \frac{1}{2\pi R_t C} = \frac{1}{2\pi(50)(49.5\times10^{-12})} = 64.31 \text{ MHz}$$

$$\therefore \text{ restricted to } Q_t = \frac{175}{64.31} = 2.7$$

Use a more sophisticated design to get a more realistic value of L.





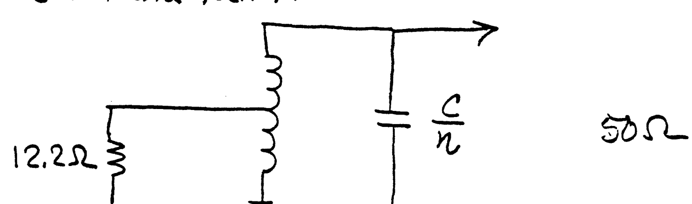
since L goes as turns ratio.

need at least 10:1 to get reasonable value of R

$10 \times .0167 = 0.17 \mu H$ ← L goes as n.

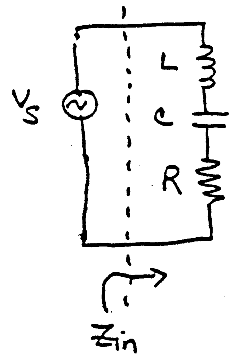
impedance steps down to $\frac{50}{n^2} = \frac{50}{100} = 0.5$

put in standard form:



Now use either a tapped inductor or an autotransformer.

What is the voltage across the capacitor (at resonance) in the series R-L-C circuit shown below.

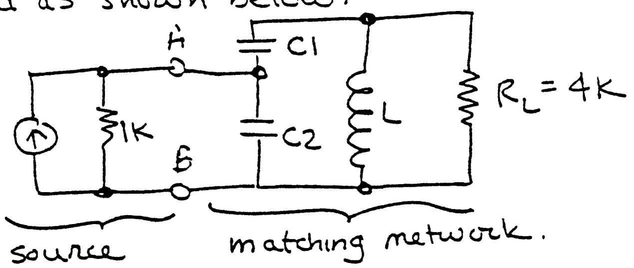


At resonance Z_{in} is purely resistive, i.e. $Z_{in}(\omega_o) = R$.

$$\Rightarrow I_{resonance} = I_{resistor} = \frac{V_S}{R}.$$

$$V_{capacitor} = (I_{resistor}) Z_{capacitor} = \frac{V_s}{R}\left(\frac{-j}{\omega C}\right) = -j\frac{V_s}{\omega RC}$$

A current source with an internal impedance of 1000 Ω is used to drive a tapped capacitor impedance matching network terminated into a 4000 Ω load as shown below.



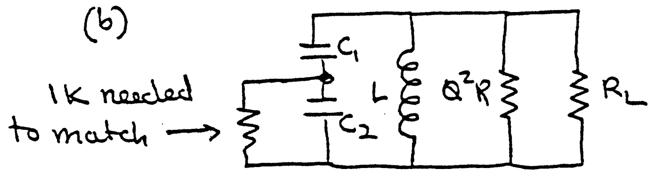
a) For maximum power transfer to the load at resonance what impedance should the network show between points A and B?

b) Draw the impedance matching network including the resistance of the inductor.

c) For $\omega_0 = 10^6$, $B = 10^5$ radians/sec, $Q_L = 50$ find L, C_1 and C_2. Is R negligible?

(a) in simplest form the network must show 1000 Ω there.

(b)



(c) assume Q^2R is negligible at first

$$C = \frac{1}{2\pi B R_t} = \frac{1}{10^5(4\times10^3)} = 0.25\times10^{-8} = 2500\text{ pf}$$

$$L = \frac{1}{\omega_0^2 C} = \frac{1}{(10^6)^2(.25\times10^{-8})} = 4\times10^{-4} = 400\ \mu H$$

$$R = \frac{\omega_0 L}{Q_L} = \frac{(10^6)(400\times10^{-6})}{50} = 8\,\Omega$$

$$Q_L^2 R = (50)^2\, 8\,\Omega = 20K.$$

we can re-calculate this using $R_t = 20K \parallel 4K = 3.33K$ error in these values is about $.6/4 = 15\%$.

2nd iteration:

$$c' = \frac{1}{2\pi B R_t'} = \frac{1}{(2\pi B)(3.33\times10^3)} = \frac{1}{10^5 \cdot 3.33\times10^3} = 3003\text{ pf}$$

$$L' = \frac{1}{\omega_0^2 C'} = \frac{1}{(10^6)^2(3.0\times10^{-9})} = 333\ \mu H$$

$$R' = \frac{\omega_0 L'}{Q_L} = \frac{(10^6)(333\times10^{-6})}{50} = 6.66\,\Omega$$

$$Q_L^2 R = (50)^2\, 6.66 = 16.65K$$

$$R_t'' = 16.65K \parallel 4K = 3.23K$$

error is now about $\frac{3.33-3.23}{3.23} \cong \frac{0.1}{3.23}$ or about 3%.