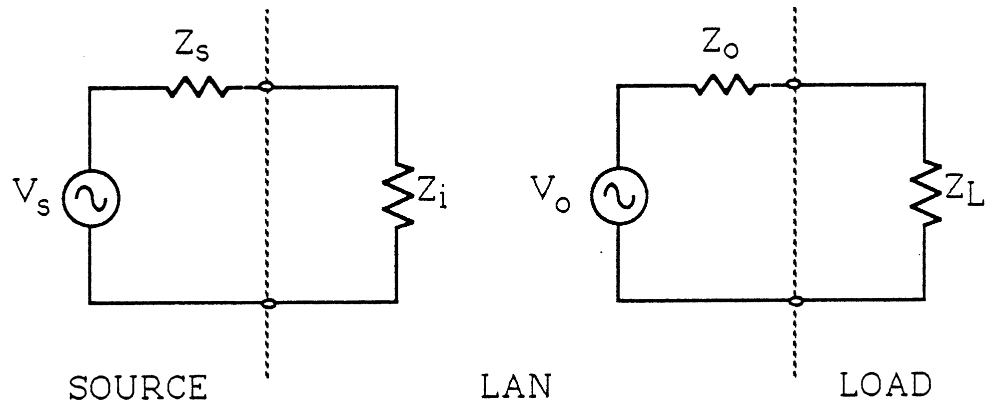
TWO-PORT POWER GAINS:



NOTE
$z_S=R_S+jX_S$, $y_S=G_S+jB_S$, $z_L=R_L+jX_L$, $y_L=G_L+jB_L$, etc.

AVAILABLE GAIN

$$G_A = \frac{P_{A,OUTPUT}}{P_{A,SOURCE}} = \frac{\frac{V_O^2}{4R_L}}{\frac{V_S^2}{4R_S}} = \frac{V_O^2 R_S}{V_S^2 R_L}$$

and in terms of the two-port parameters

$$G_A = \frac{|y_f|^2 G_S}{|y_i+y_S|^2 \ \mathrm{Re}\{y_O - \frac{y_f y_r}{y_i+y_S}\}}$$

POWER GAIN

$$G_P = \frac{P_L}{P_{IN}} = \frac{|y_f|^2 G_L}{|y_O+y_L|^2 \ \mathrm{Re}\{y_i - \frac{y_f y_r}{y_O+y_L}\}}$$

TRANSDUCER GAIN

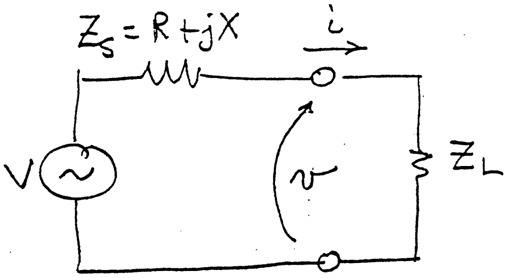
$$G_T = \frac{P_L}{P_{A,SOURCE}} = \frac{4\,|y_f|^2\;G_S G_L}{|(y_i+y_S)(y_O+y_L)-y_f y_r|^2}$$

INSERTION POWER GAIN

$$G_I = \frac{P_L}{P_{L,DELIVERED}} = \frac{|y_f|^2\;|y_S+y_L|^2}{|(y_i+y_S)(y_O+y_L)-y_f y_r|^2}$$

Available power (p. 39, 40)

available power – maximum power a source can deliver to a conjugately matched load.



for conjugate matching $Z_L = Z_S^*$

in general

$$P_{LOAD} = v i^*$$

$$= \left[\frac{Z_L}{Z_L + Z_S} V\right]\left[\frac{V}{Z_L + Z_S}\right]^*$$

$$P_{LOAD} = \frac{Z_L |V|^2}{|Z_L + Z_S|^2}$$

maximum power occurs when $Z_L = Z_S^*$

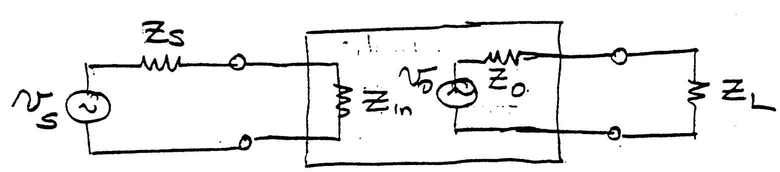
$$P_{MAX,\,LOAD} = \frac{Z_S^* |V|^2}{|Z_S^* + Z_S|^2} = \frac{Z_S^* |V|^2}{4 |\mathrm{Re}\ Z_S|^2}$$

if Z_S is only real

$$P_{max,\,LOAD} = \frac{R|V|^2}{4 R^2} = \frac{|V|^2}{4R}$$

this the the available power from a voltage source

available gain of a network is simply the ratio of available power from a device to the available power of the source. Note That this assumes the device input is conjugately matched to The source and the device output is conjugately matched to the load.



$Z_S = Z_{in}^*$ $\qquad$ $Z_o = Z_L^*$

$$\text{available gain } G_a = \frac{P_{A,output}}{P_{A,source}} = \frac{(v_o)^2/4R_L}{(v_S)^2/4R_S} = \frac{(v_o)^2}{(v_S)^2}\frac{R_S}{R_L}$$

note that the available gain is the square of the voltage gain times the ratio of input to output impedances. We can imagine an amplifier with a voltage gain of 1 (say an emitter follower) but a large power gain since typically $R_S \sim$ several kΩ, $R_L \sim$ several hundred ohms for a emitter follower.

Because the input and output impedances of any two-port device can be calculated as functions of the four matrix parameters and the source impedance the available gain can also be written

$$G_A = \frac{|y_f|^2 G_S}{|y_i + Y_S|^2 \, \mathrm{Re}\left\{ y_o - \frac{y_f y_r}{y_i + Y_S} \right\}}$$

where $Y_S = \frac{1}{Z_S} = G_S + jB_S$.

The operating power gain is what we customarily think of as power gain, the ratio of power output to power input, i.e.

$$G_P = \frac{P_L}{P_{IN}}$$

if the input circuit is matched $P_{IN} = P_{AV,S}$

$$G_P = \frac{|y_f|^2 G_L}{|y_o + Y_L|^2 \, \mathrm{Re}\left\{ y_i - \frac{y_f y_r}{y_o + Y_L} \right\}}$$

The transducer power gain is defined as the ratio of load power to available power from the source.

$$G_T = \frac{P_L}{P_{av,s}}$$

In general, this is the most useful gain to design engineers because it includes the effects of both source and load mismatching.

$$G_T = \frac{4|y_f|^2 G_S G_L}{|(y_i + Y_S)(y_o + Y_L) - y_f y_r|^2}$$

The final gain is the least useful, insertion power gain. This is defined to be

$$G_I = \frac{P_L}{P_{DL}}$$

where P_{DL} is the power delivered to the load when it is directly connected to the source.

$$G_I = \frac{|y_f|^2 |Y_S + Y_L|^2}{|(y_i + Y_S)(y_o + Y_L) - y_f y_r|^2}$$

Example: consider a device with measured y-parameters

$y_i = 10 + j0$ $\qquad$ $y_r = 1.5 + j0$

$y_f = 150 + j0$ $\qquad$ $y_o = 10 + j0$

Assume that the terminations are

$Y_S = 20 + j0$ $\qquad$ $(Z_S = 50\Omega)$

$Y_L = 10 + j0$ $\qquad$ $(Z_L = 100\Omega)$

$$G_T = \frac{4\,|y_f|^2\,|Y_S+Y_L|^2}{|(y_i+Y_S)(y_o+Y_L)-y_f y_r|^2}$$

$$= \frac{4\,|150|^2\,(20+10)^2}{|(10+20)(10+10)-(1.5)(150)|^2} = 128 \qquad (21.07\text{db})$$

$$G_A = \frac{|y_f|^2\,G_S}{|y_i+Y_S|^2\,Re\left\{y_o - \dfrac{y_f y_r}{y_i+Y_S}\right\}}$$

$$= \frac{|150|^2\cdot 20}{|10+20|^2\,Re\left\{10 - \dfrac{(150)(1.5)}{10+20}\right\}} = 200 \qquad (23.01\text{db})$$

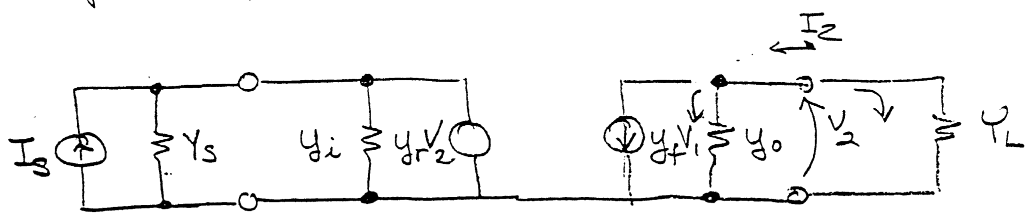
$$G_p = \frac{|y_f|^2\,G_L}{|y_o+Y_L|^2\,Re\left\{y_i - \dfrac{y_f y_r}{y_o+Y_L}\right\}}$$

$$= \frac{|150|^2\,10}{(10+10)^2\,Re\left\{10 - \dfrac{(150)(1.5)}{10+10}\right\}}$$

= not defined since it is negative.

This can often happen when mismatches occur.

proof of gain expressions



$$Y_{in} = y_i - \frac{y_f y_r}{y_o + Y_L}$$

$$Y_{OUT} = y_o - \frac{y_f y_r}{y_i + Y_s}$$

where derived ? similar to p. 142, etc.

$$G_T = \frac{\text{actual power delivered to load}}{\text{power available from source}}$$

The power available from the source is easy to calculate

$$P_{a,s} = \frac{I_s^2}{4G_s} \quad \left(\text{the reciprocal of } \frac{V_s^2}{4R_s}\right)$$

the actual power delivered to the load P_L is much more complicated

$$P_{OUT} = |V_2|^2 G_L$$

to find V_2

$$V_1 = \frac{I_s}{Y_s + Y_{in}} = \frac{I_s}{Y_s + y_i - \dfrac{y_f y_r}{y_o + Y_L}} = \frac{I_s\,(y_o + Y_L)}{(Y_s + y_i)(y_o + Y_L) - y_f}$$

but $\quad V_2 = -\dfrac{I_2}{Y_L}$

$$I_2 = \underbrace{y_f V_1}_{\text{current generator}} - \underbrace{y_o \frac{I_2}{Y_L}}_{\text{current thru } y_o}$$

V_2

solving for $V_2 \qquad V_2 = -\dfrac{y_f V_1}{Y_L + y_o}$

Expression for output node is

$$I_2 = y_f V_1 + y_o V_2$$

$$I_2 = y_f V_1 + y_o\left(-\frac{y_f V_1}{Y_L + y_o}\right) = \left(y_f - \frac{y_o y_f}{Y_L + y_o}\right) V_1$$

$$I_2 = \frac{y_f Y_L + \cancel{y_f y_o} - \cancel{y_o y_f}}{Y_L + y_o} V_1 = \frac{y_f Y_L}{Y_L + y_o} V_1$$

$$I_2 = \frac{y_f Y_L}{\cancel{(Y_L + y_o)}} \frac{I_S \cancel{(y_o + Y_L)}}{\{(Y_S + y_i)(y_o + Y_L) - y_f y_r\}}$$

$$I_2 = \frac{y_f Y_L I_S}{(Y_S + y_i)(y_o + Y_L) - y_f y_r}$$

$$P_{OUT} = \frac{|I_2|^2}{G_L} = \frac{1}{G_L} \frac{|y_f|^2 G_L^2 I_S^2}{|(Y_S + y_i)(y_o + Y_L) - y_f y_r|^2}$$

$$P_{OUT} = \frac{I_S^2 G_L |y_f|^2}{|(Y_S + y_i)(y_o + Y_L) - y_f y_r|^2}$$

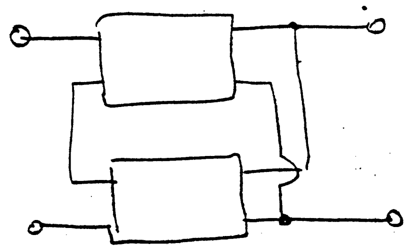
$$G_T = \frac{P_{OUT}}{P_{a,S}} = \frac{\cancel{I_S^2} G_L |y_f|^2}{|(Y_S + y_i)(y_o + Y_L) - y_f y_r|^2} \frac{4 G_S}{\cancel{I_S^2}}$$

$$G_T = \frac{4 G_S G_L |y_f|^2}{|(Y_S + y_i)(y_o + Y_L) - y_f y_r|^2}$$

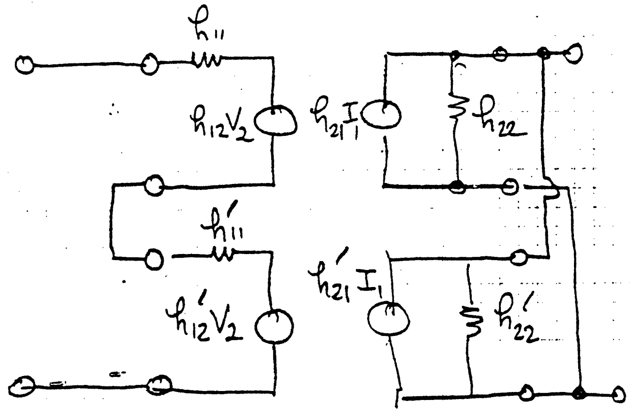
The significance of two port networks is that they allow uc to interconnect networks quickly and easily.

Example 1

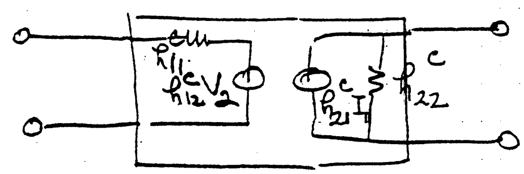
input in series



output in parallel



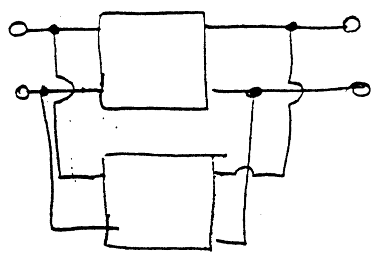
composite network

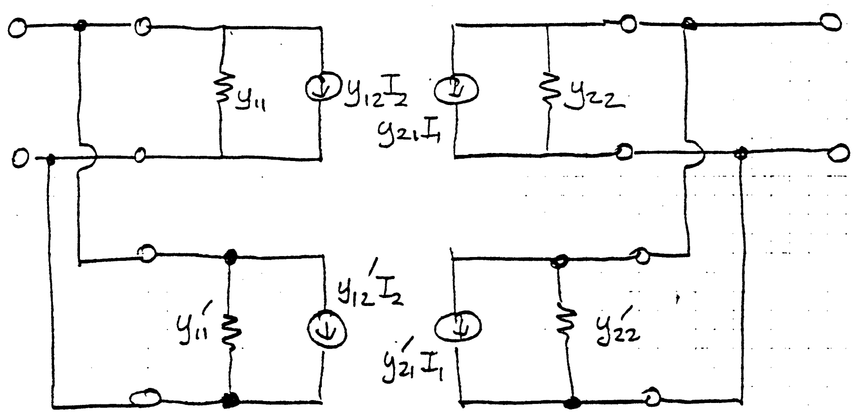


where $h_{11}^c = h_{11} + h_{11}'$ $\qquad h_{12}^c = h_{12} + h_{12}'$

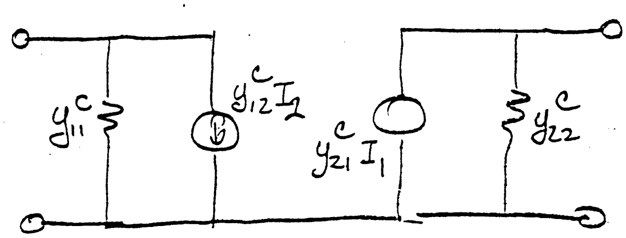
$h_{21}^c = h_{21} + h_{21}'$ $\qquad h_{21}^c = h_{22} + h_{22}'$

y- parameters two - networks in parallel





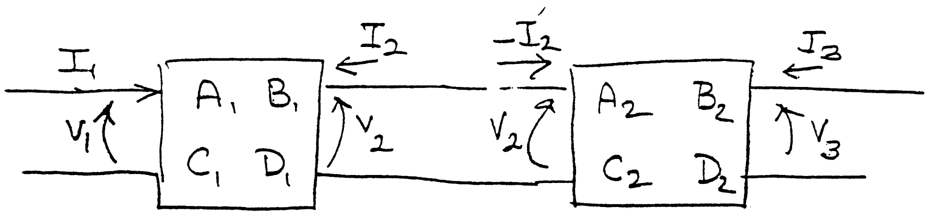
this becomes



where $y_{11}^c = y_{11} + y_{11}'$ $\quad y_{12}^c = y_{12} + y_{12}'$

$y_{21}^c = y_{21} + y_{21}'$ $\quad y_{22}^c = y_{22} + y_{22}'$

ABCD or transmission parameters are good for cascaded networks.



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

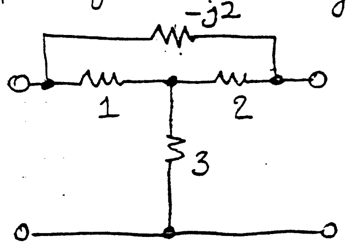
$$\begin{bmatrix} V_2' \\ I_2' \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix}$$

but $$\begin{bmatrix} V_2' \\ I_2' \end{bmatrix} = \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix}$$

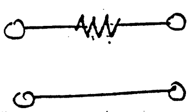
Note that the minus sign on I_2 is chosen so we can simply multiply matrices.

Example of determining 2 port parameters. (find y-parameters)

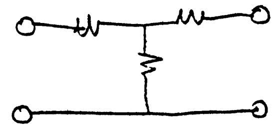


This is very difficult to solve in this form because of algebraic complexity.

Easier to solve using what we have just learned. Rule is to decompose into something we recognize. Such as.



and



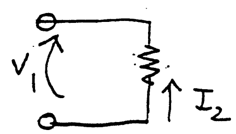
in parallel.

Thus, find y-parameters of each network and add together.

by definition $I_1 = y_{11} V_1 + y_{12} V_2$
$I_2 = y_{21} V_1 + y_{22} V_2$

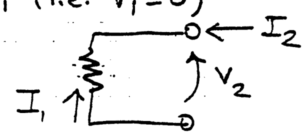
for the impedance short output $(V_2 = 0)$ to get $y_{11} = \frac{I_1}{V_1} = \frac{1}{-j2} = j\frac{1}{2}$

$y_{21} = \frac{I_2}{V_1}$



$I_2 = -I_1 \Rightarrow y_{21} = -\frac{I_1}{V_1} = -j\frac{1}{2}$

shorting V_1 (i.e. $V_1 = 0$)

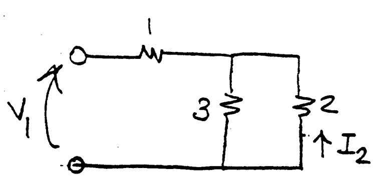


$I_1 = y_{12} V_2$
$I_2 = y_{22} V_2$

$\therefore \; y_{22} = \frac{I_2}{V_2} = j\frac{1}{2}$

$y_{12} = \frac{I_1}{V_2} = -\frac{I_2}{V_2} = -j\frac{1}{2}$

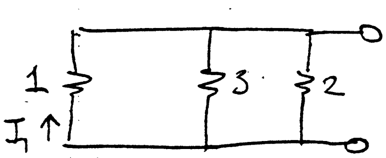
for the resistor network:



as before $y_{11} = \frac{I_1}{V_1} = \frac{1}{1 + 2\|3} = \frac{5}{11}$

voltage divider

$y_{21} = \frac{I_2}{V_1} = \frac{-\left(\frac{2\|3}{1+2\|3} V_1\right)\left(\frac{1}{2}\right)}{V_1} = -\frac{1}{2}\frac{6}{11}$
$= -\frac{3}{11}$



$y_{22} = \frac{I_2}{V_2} = 1 + \frac{1}{2} + \frac{1}{3} = \frac{6+3+2}{6} = \frac{11}{6}$

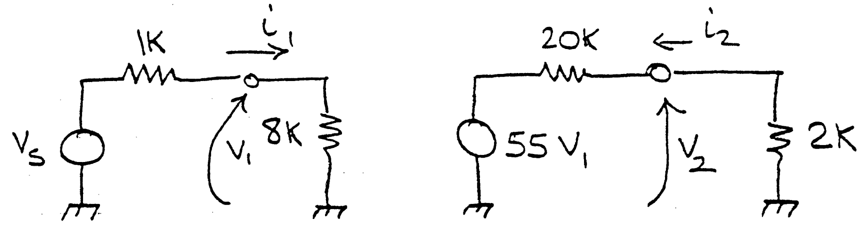
$y_{12} = \frac{I_1}{V_2} = -1$

The composite parameters are then

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \underbrace{\begin{bmatrix} j\frac{1}{2} & -j\frac{1}{2} \\ -j\frac{1}{2} & j\frac{1}{2} \end{bmatrix}}_{\text{parallel capacitor}} + \underbrace{\begin{bmatrix} \frac{5}{11} & 1 \\ -\frac{3}{11} & \frac{11}{6} \end{bmatrix}}_{\text{resistor network}}$$

$$= \begin{bmatrix} \frac{5}{11}+j\frac{1}{2} & 1-j\frac{1}{2} \\ -\frac{3}{11}-j\frac{1}{2} & \frac{11}{6}+j\frac{1}{2} \end{bmatrix}$$

Example: An i.f. amplifier has an input impedance of 8K, an output impedance of 20K and an open (no load) voltage gain of 55. The i.f. amplifier is driven by a voltage source of 1K output impedance and the i.f. amplifier drives a 2K load. Specify A_T, A_A, A_P, A_{MAG}



no matching is assumed.

$$A_P = \frac{P_{LOAD}}{P_{IN}} = \frac{\left(55V_1 \frac{2}{22K}\right)^2/2K}{V_1^2/8K} = (5)^2 \frac{8K}{2K} = 100$$

$$A_P = 100 \quad (\text{or } 10 \log 100 = 20 \text{ db gain})$$

only defined for conjugate matching

$$A_A = \frac{P_{A,X}}{P_{A,S}} = \frac{(55V_1)^2/4 \cdot 20K}{(V_s)^2/4 \cdot 1K} = \left(\frac{55V_1}{V_s}\right)^2 \frac{1}{20}$$

but $\frac{V_1}{V_s} = \frac{8K}{8K+1K} = \frac{8}{9}$ $\Rightarrow A_A = \left(55 \cdot \frac{8}{9}\right)^2 \frac{1}{20} = 119.5$

$= 20.8$ db

$$A_{MAG} = \frac{P_{A,X}}{P_{IN}} = \frac{(55V_1)^2/4 \cdot 20K}{(V_1)^2/8K} = \frac{(55)^2}{4}\frac{8}{20} = 302.5$$

can never be obtained

$= 24.8$ db

how much from source gets to load. (power in practice)

$$A_T = \frac{P_L}{P_{AS}} = \frac{(5V_1)^2/2K}{V_s^2/4 \cdot 1K} = 25\left(\frac{V_1}{V_s}\right)^2 \frac{4}{2} = 50\left(\frac{8}{9}\right)^2$$

$= 39.50$

(15.97 db)

Example #1

Consider the common base transistor amplifier whose y-parameters are given by

$$Y_{cB} = \begin{bmatrix} 100\,m\mho & 0 \\ -j91\,m\mho & 0 \end{bmatrix}$$

(a) Add a capacitor (feed back) between the input and output terminals. What are the composite parameters if $y_c = j1.0\,m\mho$.

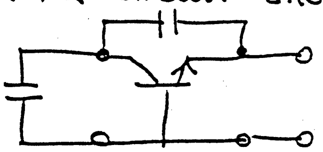
$$[Y_c] = \begin{bmatrix} 100+y_f & -y_f \\ -91-y_f & +y_f \end{bmatrix} = \begin{bmatrix} 100+j1 & -j1 \\ -91-j1 & j1 \end{bmatrix}$$

(b) If this composite amplifier is connected to a source admittance $Y_S = j10\,m\mho$, what is Y_{OUT}?

$$Y_{OUT} = y_{22} - \frac{y_{12}\,y_{21}}{y_{11}+y_s} = j1 - \frac{(-j1)(-91-j1)}{(100+j1)+j10} = j1 + \frac{(j1)(-91-j1)}{100+j11}$$

$$= j1 + \frac{1-j91}{100+j11} = j1 + (-.089 - j.9) = -.089 + j0.1$$

(c) Draw the final circuit showing Y_c and Y_s. Don't show bias components.

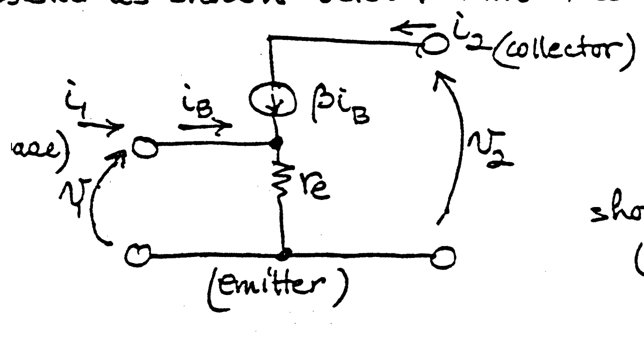


Y_{OUT}

(d) Suggest a use for this circuit! OSCILLATOR

Exampl

A common model for a transistor consists of a current generator and an emitter resistor as shown below. Find the y-parameters for this model.



by definition $i_1 = y_{11}v_1 + y_{12}v_2$

$i_2 = y_{21}v_1 + y_{22}v_2$

shorting the output

$(\beta+1)i_1 r_e = v_1$

$i_1 = \dfrac{v_1}{(\beta+1)r_e} \Rightarrow y_{11} = \dfrac{1}{(\beta+1)r_e}$

$i_2 = \beta i_1 = \dfrac{\beta}{(\beta+1)r_e}v_1 \Rightarrow y_{21} = \dfrac{\beta}{(\beta+1)r_e}$

shorting the input $i_2 \to 0$ as long as $i_B = 0$ and a current source has infinite impedance

$i_1 = 0 = y_{12}v_2 \Rightarrow y_{12} = 0$

$i_2 = 0 = y_{22}v_2 \Rightarrow y_{22} = 0$

final result:

$$Y_{CE} = \begin{bmatrix} \dfrac{1}{(\beta+1)r_e} & 0 \\ \dfrac{\beta}{(\beta+1)r_e} & 0 \end{bmatrix}$$