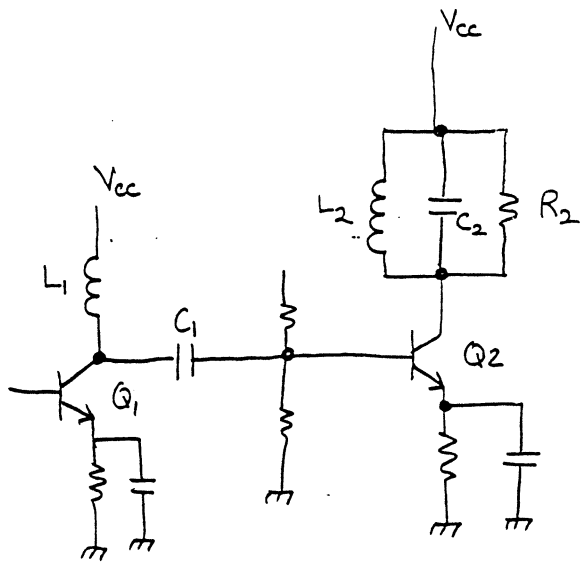


design of a small signal r.f. amplifier:



from data sheets Q_1 & Q_2 are identical:

$$y_{ie} = 10 + j10 \text{ m}\Omega$$

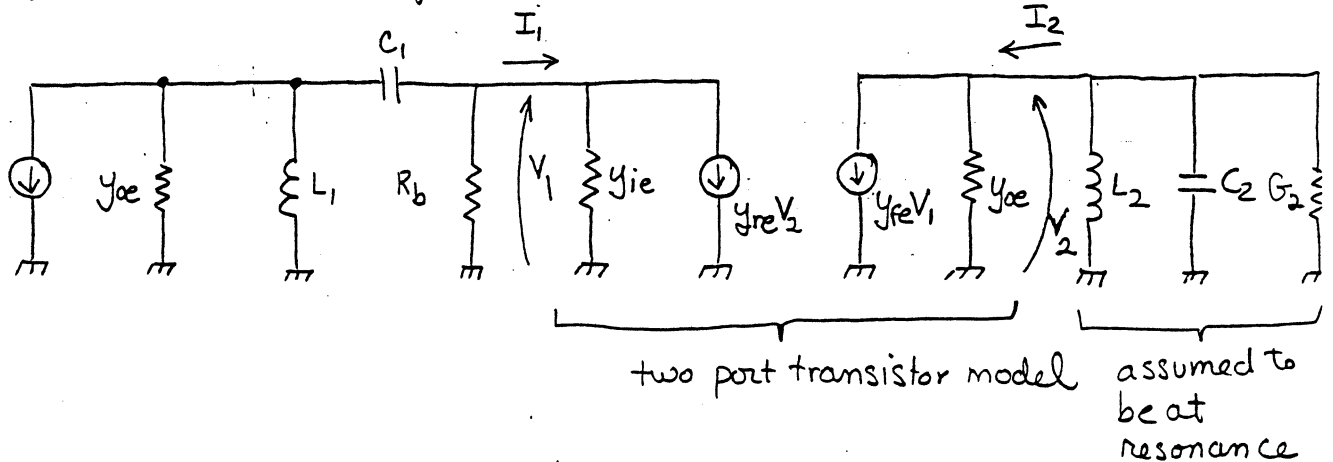
$$y_{re} = -j0.001 \text{ m}\Omega$$

$$y_{fe} = 10 \times 10^{-3}$$

$$y_{oe} = 0.1 \times 10^{-3}$$

design for $f_0 = 50 \text{ MHz}$
 $\text{BW} = 0.5 \text{ MHz}$

First, draw small signal equivalent circuit:



$$Z_{in} \rightarrow$$

$$Z_{in} = \frac{V_1}{I_1} = \frac{V_1}{y_{ie} V_1 + y_{re} V_2}$$

at resonance: $y_{fe} V_1 + g_{oe} V_2 = -G_2 V_2$

$$\therefore V_2 = -\frac{y_{fe}}{g_{oe} + G_2}$$

at resonance.

$$\therefore Z_{in} = \frac{V_1}{y_{ie}V_1 - \frac{y_{re}y_{fe}}{g_{oe} + G_2} V_1} = \frac{1}{y_{ie} - \frac{y_{re}y_{fe}}{g_{oe} + G_2}}$$

$$\text{or } Y_{in} = y_{ie} - \frac{y_{re}y_{fe}}{g_{oe} + G_2}$$

$$= (10 + j10) \times 10^{-3} + \frac{(10 \times 10^{-3})(-j.001 \times 10^{-3})}{0.1 \times 10^{-3} + 10 \times 10^{-3}}$$

use complex number calculator

$$Y_{in} \approx (10 + j10) \text{ mS} + \frac{j.001 \text{ mS}}{\text{negligible effect of load}}$$

$$Z_{in} = \frac{1}{Y_{in}} = \frac{1}{10 + j10 \text{ mS}} = 50 - j50 \Omega$$

find C_{in} ;
at 50 MHz

$$\frac{1}{j\omega_0 C_{in}} = -j50$$

$$C_{in} = \frac{1}{50 \cdot 2\pi (50 \times 10^6)} = 63.7 \text{ pf}$$

What is A_v at resonance?

$$A_v = \frac{V_2}{V_1}$$

$$I_1 = y_{ie}V_1 + y_{re}V_2$$

$$I_2 = y_{fe}V_1 + y_{oe}V_2$$

$$\text{but } I_2 = -\left(j\omega C_2 + \frac{1}{j\omega L_2} + G_2\right) V_2$$

$$\therefore y_{fe} V_1 + y_{oe} V_2 = \left(-j\omega C_2 - \frac{1}{j\omega L_2} - G_2 \right) V_2$$

$$V_1 = \frac{\left(-j\omega C_2 - \frac{1}{j\omega L_2} - G_2 - y_{oe} \right) V_2}{y_{fe}}$$

$$A_v = \frac{V_2}{V_1} = \frac{V_2}{\left(-j\omega C_2 - \frac{1}{j\omega L_2} - G_2 - y_{oe} \right) \frac{V_2}{y_{fe}}}$$

$$= - \frac{y_{fe}}{j\omega C_2 + \frac{1}{j\omega L_2} + G_2 + y_{oe}}$$

$$g_{oe} + jb_{oe} = g_{oe} + \underbrace{j\omega_0 C_{oe}}_{\text{at resonance}}$$

$$= - \frac{y_{fe}}{\underbrace{(G_2 + g_{oe}) + j(\omega_0 C_2 + \omega_0 C_{oe} - \frac{1}{\omega_0 L_2})}_{\text{goes to zero at resonance}}}$$

$$\omega_0^2 = \frac{1}{L_2(C_2 + C_{oe})}$$

at resonance

$$A_v = - \frac{y_{fe}}{G_2 + g_{oe}} \quad \leftarrow \quad y_{fe} = |y_{fe}| e^{j\theta}$$

Since I am only interested in gain use the magnitude

$$|A_v| = - \frac{|y_{fe}|}{G_2 + g_{oe}}$$

to find bandwidth I need to understand how the y-parameters vary with frequency.

For narrow band circuits the y-parameters are usually simple functions of frequency and can be expanded in a Taylor series about ω_0 , i.e.

$$y_{ie} = g_{ie} + jb_{ie}$$

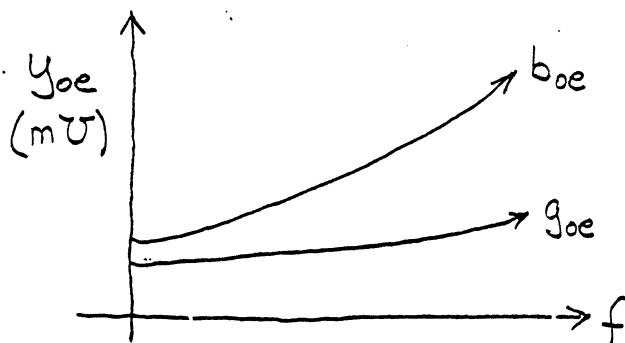
$$\approx g_{ie} + jb_{ie} \Big|_{\omega=\omega_0} + j(\omega-\omega_0) \frac{\partial b_{ie}}{\partial \omega} \Big|_{\omega=\omega_0}$$

In general, b_{ie} will be capacitive at high frequencies and g_{ie} will be independent of frequency (approximately)

For the transistor amplifier

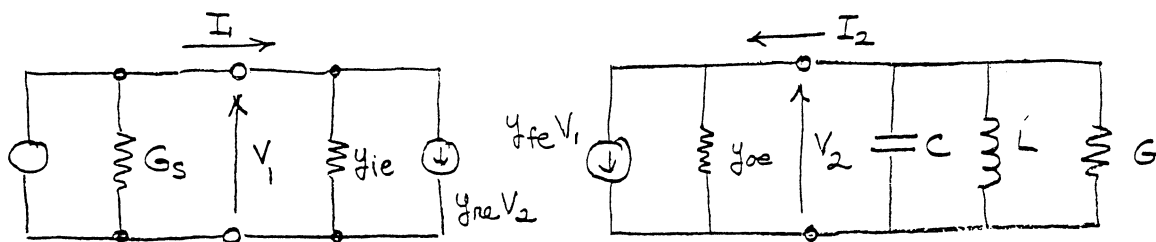
$$A_v(f) = - \frac{y_{fe}}{\underbrace{(g_2 + g_{oe})}_{\text{independent of } f} + j(\omega C_2 + \underbrace{b_{oe} - \frac{1}{\omega L_2}}_{\text{only this term is a sharp function near resonance}})}$$

this term is not involved in resonance



introduction to stability

consider the small signal tuned amplifier shown below

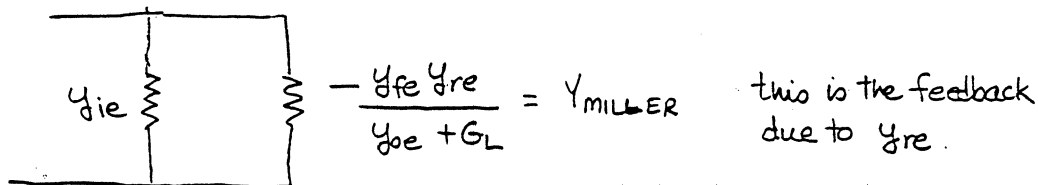


input admittance can be shown to be

$$Y_{IN} = y_{ie} - \frac{y_{fe} y_{re}}{y_{oe} + G_L}$$

this was the basic result of our last homework problem

input then looks like



examine this feedback term



y_{fe} is usually real (for $f < f_T$)

$y_{re} \cong -j\omega C_{re}$ (due to capacitive nature of feedback)

the negative sign is due to the phase inversion between the input and output

$$Y_{MILLER} = - \frac{g_{fe} (-j\omega C_{re})}{y_{oe} + Y_L} = \frac{+ j\omega C_{re} g_{fe}}{y_{oe} + Y_L}$$

if $f < f_0$ (the resonant frequency of the tank) Y_L is inductive

$f > f_0$ Y_L is capacitive

$$\text{for } f < f_0 \quad Y_{oe} + Y_L = R - jB$$

$$Y_{\text{MILLER}} = \frac{j\omega C_{re} g_{fe}}{R - jB} \frac{R + jB}{R + jB} = - \frac{\omega C_{re} g_{fe} B + j\omega R C_{re} g_{fe}}{R^2 + B^2}$$

note $Y_{\text{MILLER}} < 0$ for $f < f_0$

this can give rise to $\text{Re}(Y_{in}) < 0$ and will result in oscillation. One way to keep an amplifier from oscillating is to add a suitable positive conductance at the input terminals.

stability : what happens to Y_{IN} with different loads

$$Y_{IN} = y_{11} - \frac{y_{12} y_{21}}{y_{22} + Y_L}$$

$$g_{IN} = g_{11} \frac{(g_{22} + g_L)^2 + (b_{22} + b_L)^2 - (g_{22} + g_L) \operatorname{Re}(y_{12} y_{21}) - \underbrace{(b_{22} + b_L) \operatorname{Im}(y_{12} y_{21})}_{\rightarrow y_{12} y_{21}}}{(g_{22} + g_L)^2 + (b_{22} + b_L)^2}$$

consider two cases

I. $g_L = \infty$ a short circuit

$$g_{IN} \rightarrow g_{11} \frac{g_L^2 - g_L \operatorname{Re}(y_{12} y_{21})}{g_L^2} \rightarrow g_{11}$$

simply require $g_{11} > 0$ for stability

II. $g_L = 0$ open circuit

complicated expression

examine $g_{IN, MIN}$

minimize $g_{IN, MIN}$ by taking $\frac{\partial g_{IN}}{\partial b_L}$ (since $g_L = 0$ don't need it)

$$\text{result is } g_{11} g_{22}^2 - g_{22} \operatorname{Re}(y_{12} y_{21}) - \frac{\operatorname{Im}(y_{12} y_{21})}{4g_{11}} > 0$$

if this is satisfied, $g_{IN, MIN} > 0$

III. general case of $Y_L = g_L + jb_L$

minimize g_{in} by differentiating $\frac{\partial g_{in}}{\partial g_L}$, $\frac{\partial g_{in}}{\partial b_L}$

$$\text{we get } \begin{cases} g_L = 0 \\ b_L = -b_{22} + \frac{\Im m(y_{12}y_{21})}{2g_{11}} \end{cases}$$

as worse case

$$\text{evaluate } g_{in, \min} = \frac{\left[g_{22} - \frac{\text{Re}(y_{12}y_{21})}{2g_{11}} \right]^2 - \frac{|y_{12}y_{21}|^2}{4g_{11}^2}}{(g_{22})^2 + \frac{[\Im m(y_{12}y_{21})]^2}{4g_{11}^2}}$$

$$\text{for stability } g_{22} - \frac{\text{Re}(y_{12}y_{21})}{2g_{11}} > \frac{|y_{12}y_{21}|}{2g_{11}}$$

re-write as

$$\frac{|y_{12}y_{21}|}{2g_{11}g_{22} - \text{Re}(y_{12}y_{21})} < 1$$

Linivill stability factor
usually labeled C .

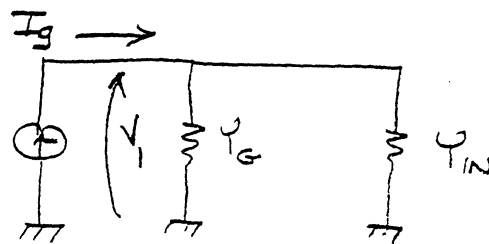
For $0 < C < 1$ a device is unconditionally stable

$C \geq 1$ device is conditionally stable

in terms of C

$$A_{MAG} = 2 \frac{1 - \sqrt{1 - C^2}}{C^2} G_{\infty}$$

$$\text{where } G_{\infty} = \frac{|y_{21}|^2}{4g_{11}g_{22} - 2\text{Re}(y_{12}y_{21})}$$



$$I_g = Y_G V_1 + Y_{IN} V_1 \\ = (Y_G + Y_{IN}) V_1$$

$$\therefore A_T = 4g_G g_L \frac{|V_2|^2}{|Y_G + Y_{IN}|^2 |V_1|^2}$$

$$A_T = \frac{4g_G g_L}{|Y_G + Y_{IN}|^2} \frac{|y_{21}|^2}{|y_{22} + Y_L|}$$

Usually we want to maximize \$A_T\$. How?

$$A_T = f(g_G, b_G, g_L, b_L)$$

take partial derivatives with respect to these variables and set all derivatives equal to zero.

$$A_{T, \max} = A_{\text{MAG}} = \frac{|y_{21}|^2}{2g_{11}g_{22} - \text{Re}(y_{12}y_{21}) + \sqrt{(2g_{11}g_{22} - \text{Re}(y_{12}y_{21}))^2 - |y_{12}y_{21}|^2}}$$

the necessary terminations are

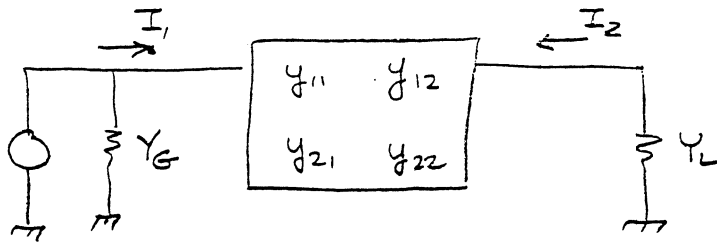
$$g_{L0} = \frac{1}{2g_{11}} \sqrt{(2g_{11}g_{22} - \text{Re}(y_{12}y_{21}))^2 - |y_{12}y_{21}|^2}$$

$$b_{L0} = -b_{22} + \frac{\text{Im}(y_{12}y_{21})}{2g_{11}}$$

$$g_{G0} = \frac{g_{11}}{g_{22}} g_{L0}$$

$$b_{G0} = -b_{11} + \frac{\text{Im}(y_{12}y_{21})}{2g_{22}}$$

p. 358 notes amplifier gains in terms of y-parameters



$$Y_{IN} = y_{11} - \frac{y_{12} y_{21}}{y_{22} + Y_L}$$

$$Y_{OUT} = y_{22} - \frac{y_{12} y_{21}}{y_{11} + Y_G}$$

$$\left. \begin{aligned} I_1 &= y_{11} V_1 + y_{12} V_2 \\ I_2 &= y_{21} V_1 + y_{22} V_2 \end{aligned} \right\} \text{from definitions}$$

but $I_2 = -V_2 Y_L$

$$-V_2 Y_L = y_{21} V_1 + y_{22} V_2$$

$$\frac{V_2}{V_1} = \frac{-y_{21}}{y_{22} + Y_L}$$

simple expression for A_p

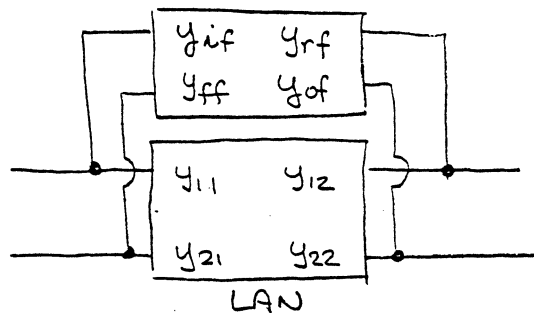
$$A_p = \frac{P_L}{P_{IN}} = \frac{|V_2|^2 g_L}{|V_1|^2 g_{IN}} = \left| -\frac{y_{21}}{y_{22} + Y_L} \right|^2 \frac{g_L}{g_{11} - \operatorname{Re} \left\{ \frac{y_{12} y_{21}}{y_{22} + Y_L} \right\}}$$

$$A_T = \frac{P_L}{P_{AS}} = \frac{|V_2|^2 g_L}{\frac{|I_g|^2}{4g_G}} = 4g_G g_L \left| \frac{V_2}{I_g} \right|^2$$

get I_g by examining input circuit.

unilateralized amplifier :

- use an external feedback network to make a composite $y_{12} = 0$
- increases input-output isolation

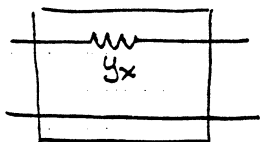


since devices are in parallel.

$$\text{composite } [Y] = \underbrace{\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}}_{\text{LAN}} + \underbrace{\begin{bmatrix} y_{if} & y_{rf} \\ y_{ff} & y_{of} \end{bmatrix}}_{\text{feedback}} = \underbrace{\begin{bmatrix} y_{11} + y_{if} & y_{12} + y_{rf} \\ y_{21} + y_{ff} & y_{22} + y_{of} \end{bmatrix}}_{\text{composite}}$$

for unilateralization want $y_{12} + y_{rf} = 0$

typical feedback element

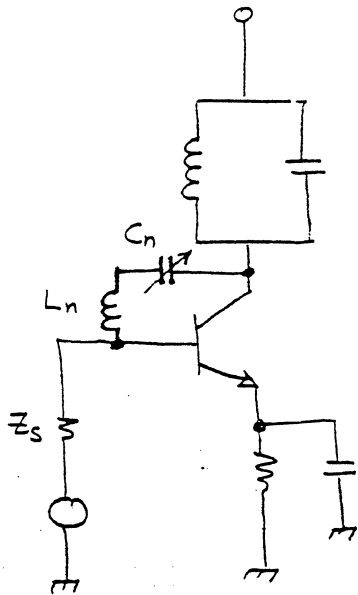


$$[Y]_{\text{feedback}} = \begin{bmatrix} y_x & -y_x \\ -y_x & y_x \end{bmatrix}$$

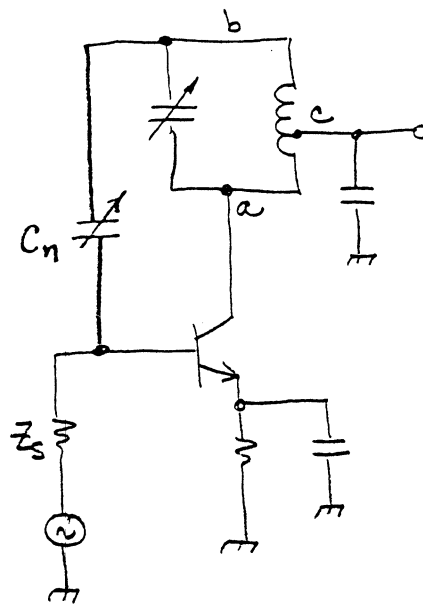
neutralization

just cancel $\Im y_{12}$, composite for neutralization
 real part remains, usually amplifier is stable with
 just resistive y_{12} , composite

examples of neutralization



tune L_n - C_n to below resonance (inductive) to cancel capacitive feedback from collector to base.

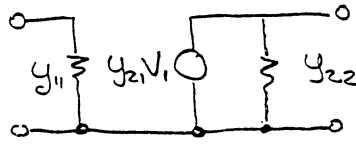


This is a clever circuit. center-tap of transformer (c) provides ground reference. Points (a) and (c) are 180° out of phase $\Rightarrow C_n$ looks like its phase is inverted, i.e. an inductance to the base of the transistor

few remaining topics

unilateral amplifiers

$y_{12} = 0$ (no feedback)



$$A_{MAG} = \frac{|y_{21}|^2}{2g_{11}g_{22} - \text{Re}(y_{12}y_{21}) + \sqrt{[2g_{11}g_{22} - \text{Re}(y_{12}y_{21})]^2 - |y_{12}y_{21}|^2}}$$

if $y_{12} = 0$

$$A_{MAG} \rightarrow \frac{|y_{21}|^2}{2g_{11}g_{22} + \sqrt{[2g_{11}g_{22}]^2}} = \frac{|y_{21}|^2}{4g_{11}g_{22}}$$

conjugate matching : $\Gamma_L = y_{22}^*$

$$A_P = \frac{|y_{21}|^2 g_L}{|y_{22} + \Gamma_L|^2 g_{in}} \rightarrow \frac{|y_{21}|^2 g_{22}}{4g_{22}^2 g_{in}} = \frac{|y_{21}|^2}{4g_{22} g_{in}} \equiv G_{oo}$$

$2g_{22}$ since imaginary parts cancel

G_{oo} is the Linvill figure of merit for a transistor

$$g_{in} = g_{11} - \text{Re} \left[\frac{y_{12}y_{21}}{y_{22} + \Gamma_L} \right] = g_{11} - \frac{\text{Re}(y_{12}y_{21})}{2g_{22}}$$

$$= \frac{2g_{11}g_{22} - \text{Re}(y_{12}y_{21})}{2g_{22}}$$

$$G_{oo} = \frac{|y_{21}|^2}{4g_{22} [2g_{11}g_{22} - \text{Re}(y_{12}y_{21})]}$$

maximum
unilateral
gain

Stern stability factor

✖

Suppose we evaluate a transistor for which $C > 1$

$$C = \frac{|y_{12} y_{21}|}{2g_{11} g_{22} - \operatorname{Re}(y_{12} y_{21})}$$

We can make this transistor stable by increasing the product $2g_{11} g_{22}$ until $C < 1$. This can be done by external loading, i.e. adding g_L and g_G to get

$$g'_{11} = g_{11} + g_G$$

$$g'_{22} = g_{22} + g_L$$

Note: adding g_L and g_G means lowering R_L and R_G so that the stability criteria becomes:

$$\frac{|y_{12} y_{21}|}{2(g_{11} + g_G)(g_{22} + g_L) - \operatorname{Re}(y_{12} y_{21})} < 1$$

cross-multiplying:

$$2(g_{11} + g_G)(g_{22} + g_L) - \operatorname{Re}(y_{12} y_{21}) > |y_{12} y_{21}|$$

$$2(g_{11} + g_G)(g_{22} + g_L) > |y_{12} y_{21}| + \operatorname{Re}(y_{12} y_{21})$$

$$K \equiv \frac{2(g_{11} + g_G)(g_{22} + g_L)}{|y_{12} y_{21}| + \operatorname{Re}(y_{12} y_{21})} > 1$$

This is defined to be the Stern stability factor and the terminated device is stable as long as $K > 1$.

Typical values might be $K = 2$ for a good amplifier

Sensitivity

A final factor is the degree of interaction between the input and output tuned circuits. We define this sensitivity to be

$$S = \frac{\left| \frac{\Delta Y_{IN}}{Y_{IN}} \right|}{\left| \frac{\Delta Y_{OUT}}{Y_{OUT}} \right|} = \frac{\text{fractional change in input}}{\text{fractional change in output}}$$
$$= \frac{\Delta Y_{IN}}{\Delta Y_{OUT}} \frac{Y_{OUT}}{Y_{IN}} = \frac{dY_{IN}}{dY_{OUT}} \frac{Y_{OUT}}{Y_{IN}}$$

We can use our prior results that

$$Y_{IN} = y_{11} - \frac{\text{Re}(y_{12}y_{21})}{y_{22} + Y_L}$$

$$Y_{OUT} = y_{22} - \frac{\text{Re}(y_{12}y_{21})}{y_{11} + Y_G}$$

The result of evaluating S is

$$S = \frac{|y_{12}y_{21}| |Y_L|}{|y_{22} + Y_L| |y_{11}(y_{22} + Y_L) - y_{12}y_{21}|}$$

A good amplifier will have $S \leq 0.3$. We can always lower S by adding resistance (loading) to the LAN, i.e. let

$$y'_{11} = y_{11} + g'_G$$

$$y'_{22} = y_{22} + g'_L$$

This can get complicated to analyze as adding g'_G and g'_L will change C so a change in loading will change C , S and A_{mag} .

Design with unconditionally stable device

$$[Y] = \begin{bmatrix} 8 + j6.8 & 0 - j0.1 \\ 53 - j22 & 0.4 + j1.5 \end{bmatrix}$$

\uparrow y_f

Evaluate Linvill stability factor

$$C = \frac{|y_f y_r|}{2g_i g_o - \text{Re}(y_f y_r)}$$

$$y_f y_r = (-j0.1)(53 - j22) = -2.2 - j5.3 = 5.74 \angle -112.5^\circ$$

$$\therefore C = \frac{5.74}{2(8)(0.4) - (-2.2)} = \frac{5.74}{8.6} = 0.67 < 1$$

\therefore device is unconditionally stable

MAG is the maximum available gain ($y_r = 0$
source & load matched)

~~$$G_T = \frac{4G_s G_L |y_f|^2}{|(y_i + Y_s)(y_o + Y_L) - y_f y_r|^2}$$~~

~~if $y_i = Y_s^*$ $y_o = Y_L^*$ $y_r = 0$~~

~~$$G_T = \frac{4G_s G_L |y_f|^2}{|(2g_i)(2g_o) - 0|^2} = \frac{4G_s G_L |y_f|^2}{16(g_i g_o)^2}$$~~

but $g_i = G_s$, $g_o = G_L$ for conjugate matching

MAG $G_T = \frac{|y_f|^2}{4g_i g_o} =$

$$G_A = \frac{|y_f|^2 G_s}{\text{Re}[(y_i y_o - y_f y_r + y_o y_s)(y_i + y_s)^*]} \leftarrow \text{use other equation}$$

$$= \frac{|y_f|^2 G_s}{\text{Re}[\underbrace{y_o (y_i + y_s)(y_i + y_s)^*}_{\text{for conjugate match}}]}$$

for conjugate match

$$y_i = G_s - jB_s$$

$$y_s = G_s + jB_s$$

$$\therefore y_i + y_s = 2G_s$$

$$= \frac{|y_f|^2 G_s}{\text{Re}[y_o (2G_s)(2G_s)]} = \frac{|y_f|^2}{4 G_s g_o} = \frac{|y_f|^2}{4 g_i g_o}$$

\downarrow
 g_i

$$\therefore G_{MAG} = \frac{|53 - j22|^2}{4(8)(0.4)} = \frac{|57.4|^2}{12.8} = \frac{3294.8}{12.8} = 257$$

How could we neutralize? Need a feedback element

$$-y_x + (-j0.1 \text{ mS}) = 0$$

$$\therefore y_x = -j0.1 \text{ mS}$$

this is an inductance.

composite y parameters $[Y]_c = \begin{bmatrix} 8 + j6.8 & -j0.1 & -j0.1 & +j0.1 \\ 53 - j22 + j0.1 & 0.4 + j1.5 & -j0.1 & \end{bmatrix}$

$$= \begin{bmatrix} 8 + j6.7 & 0 \\ 53 - j21.9 & 0.4 + j1.4 \end{bmatrix}$$

note this is neutralized and unilateralized

since $y_f = 0$

$$Y_{in} = 8 + j6.7$$

$$Y_{out} = 0.4 + j1.4 \quad 346$$

to maximize gain conjugate match at input & output

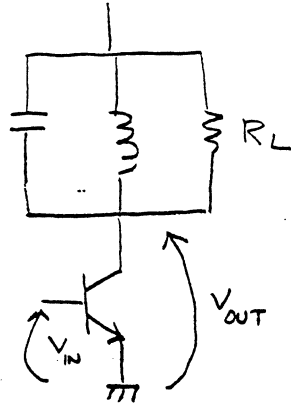
$$\text{i.e. } Y_S = 8 - j6.7$$

$$Y_L = 0.4 - j1.4$$

maximum transducer gain

$$G_{T, \max} = \frac{|y_{f,c}|^2}{4 g_{i,c} g_{o,c}} = \frac{|53 - j21.9|^2}{4(8)(0.4)} = \frac{|57.35|^2}{12.8} = 25$$

pole-zero analysis of tuned amplifiers



The gain for such a circuit might be written as

$$\text{at resonance } A_v(\omega_0) = -g_m V_{in} R_L$$

$$\text{off resonance } A_v(s) = -g_m V_{in} \frac{1}{G + sC + \frac{1}{sL}} \quad \text{where } G = \frac{1}{R_L}$$

in general, an amplifier transfer function can be written as

$$\frac{s}{(s-s_1)(s-s_2)}$$

with a single zero at $s=0$ and poles at $s=s_1, s_2$

If we solve $A_v(s)$ for s_1 and s_2 we get

$$s_1, s_2 = -\frac{G}{2C} \pm \sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}$$

If we recognize

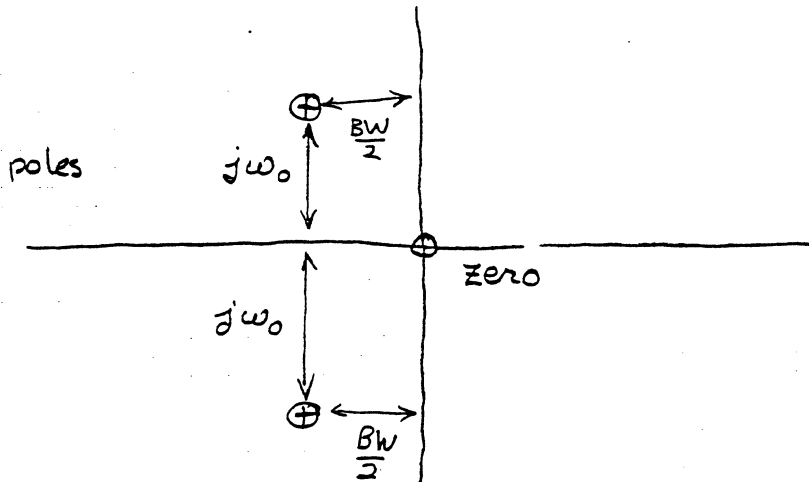
$$\text{Bandwidth } BW = \frac{1}{R_L C} \quad \text{and} \quad \omega_0^2 = \frac{1}{LC}$$

then, by substitution,

$$s_1, s_2 = -\frac{BW}{2} \pm \sqrt{\left(\frac{BW}{2}\right)^2 - \omega_0^2}$$

if we have a high-Q circuit where $\frac{\omega_0}{BW} \gg 1$

then $s_1, s_2 \approx -\frac{BW}{2} \pm j\omega_0$



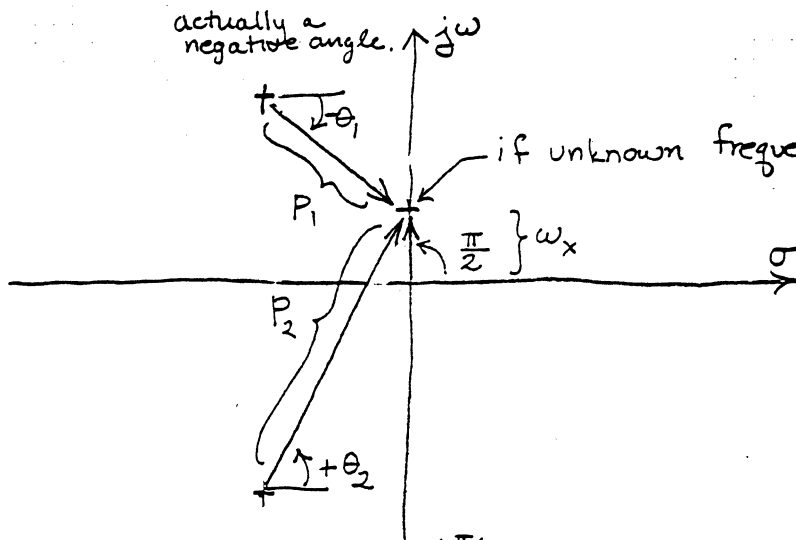
If our original transfer function can be re-written

$$A_v(s) = K \frac{Z_1}{(s-s_1)(s-s_2)}$$

where K is a gain constant,

the zero. Then, from the pole-zero diagram.

and Z_1 is



$$\begin{aligned} Z_1 &= \omega_x e^{j\pi/2} \\ s-s_1 &= P_1 e^{j\theta_1} \\ s-s_2 &= P_2 e^{j\theta_2} \end{aligned}$$

Then $A_v(j\omega_x) = K \frac{\omega_x e^{j\pi/2}}{P_1 e^{j\theta_1} P_2 e^{j\theta_2}} = \frac{K \omega_x}{P_1 P_2} e^{j(\pi/2 - \theta_1 - \theta_2)}$

Notice that as ω_x approaches ω_0 (one of the poles)
 P_1 (or P_2) will decrease in magnitude increasing $A_v(s)$
 and near a pole

$$A_v = \frac{k\omega_x e^{j(\frac{\pi}{2} - \theta_1 - \theta_2)}}{P_1 P_2} \approx \frac{k\omega_0 e^{j(\frac{\pi}{2} - \theta_1 - \frac{\pi}{2})}}{P_1 2\omega_0} = \frac{k}{2P_1} e^{-j\theta_1}$$

This is called the single pole approximation since everything
 about the circuit is described by the single pole.

HOW TO ANALYZE TUNED RF AMPLIFIERS

FORMULAS:

Linville stability factor:

$$C = \frac{|y_{12}y_{21}|}{2g_{11}g_{22} - \operatorname{Re}\{y_{12}y_{21}\}}$$

Stern stability factor:

$$K = \frac{2(g_{11} + G_S)(g_{22} + G_L)}{|y_{12}y_{21}| + \operatorname{Re}\{y_{12}y_{21}\}}$$

Optimum terminations for MAG:

$$g_{LO} = \frac{1}{2g_{11}} \sqrt{2g_{11}g_{22} - [\operatorname{Re}\{y_{12}y_{21}\}]^2 - |y_{12}y_{21}|^2}$$

$$g_{SO} = \frac{g_{11}}{g_{22}} g_{LO}$$

$$b_{LO} = -b_{22} + \frac{\operatorname{Im}\{y_{12}y_{21}\}}{2g_{11}}$$

$$g_{SO} = \frac{g_{11}}{g_{22}} g_{LO}$$

$$b_{SO} = -b_{11} + \frac{\operatorname{Im}\{y_{12}y_{21}\}}{2g_{22}}$$

Transducer gain (general case):

$$G_T = \frac{4G_S G_L |y_{21}|^2}{|(y_{11} + Y_S)(y_{22} + Y_L) - y_{12}y_{21}|^2}$$

Maximum available gain (unilateralized and conjugate matched):

$$\operatorname{MAG} = \frac{|y_{21}|^2}{4g_{11}g_{22}}$$

