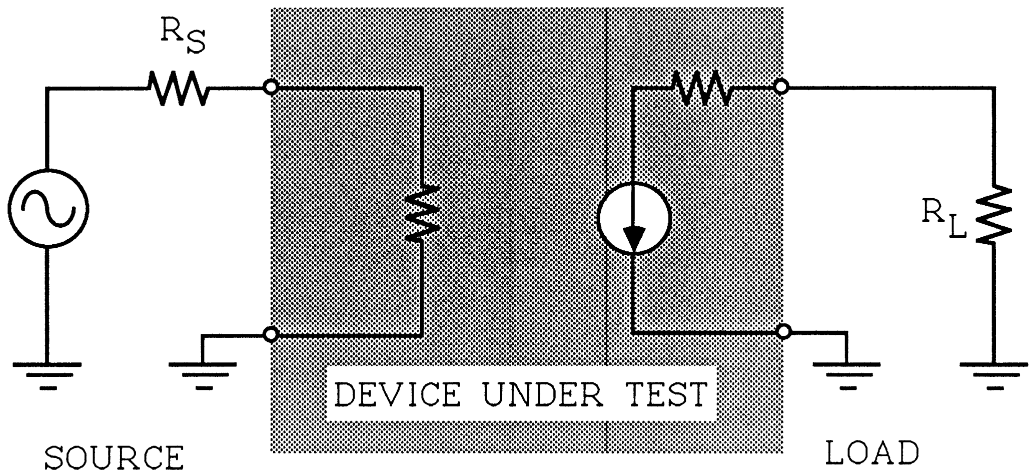
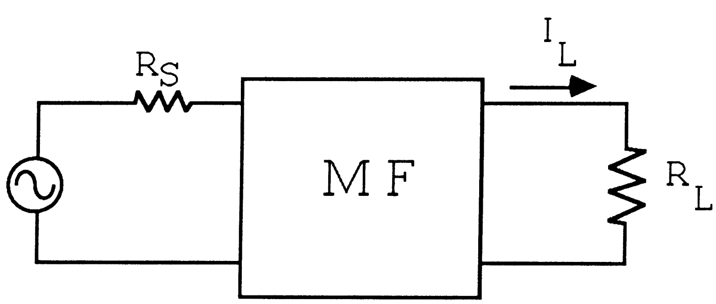
GAIN EXPRESSIONS

The previous discussion of noise introduced the idea of available power and available gain. When one is concerned with source and load impedances and possible impedance mismatching between source, amplifier and load: several useful definitions of gain may be introduced: power gain, available gain, maximum available gain and transducer gain. These may be defined by considering the source-amplifier (also called the Device Under Test or DUT in RF literature) load circuit shown below. Note that the DUT is characterized by internal input and output impedances and the source and load are also represented as simple impedances.



There is, as we know from our discussion on noise, a maximum of power transferred from one circuit to another when they are conjugately matched. To illustrate this point, we introduce the fictitious concept of Matching Factor (or MF). The MF is simply the ratio of the actual power transferred from one device to another to the available power from the source. For the circuit shown below this may be simply calculated.



$$MF = \frac{P_L}{P_{AS}} = \frac{R_L\left(\frac{V_S}{R_S+R_L}\right)^2}{\frac{V_S^2}{4R_S}} = \frac{4R_SR_L}{(R_S+R_L)^2}$$

Note that if the load impedance is the conjugate of the source impedance the MF becomes one (1) as we might have expected.

Returning to our original source-DUT-load circuit we may now define the four gains referred to at the beginning of this section:

POWER GAIN is the ratio of the power delivered to the load to the power input to the DUT. This is the ratio of the two load powers of the individual input and output circuits. Note that NO matching is ASSUMED.

AVAILABLE GAIN is the ratio of available DUT power to available source power. Note that the available power was defined as the maximum power capable of being delivered by a source to a load under conditions of conjugate matching and that AVAILABLE GAIN is only defined under conditions of CONJUGATE MATCHING.

MAXIMUM AVAILABLE GAIN is the ratio of the available power from the DUT to the input power. This gain definition ASSUMES DUT-load conjugate matching but no input conjugate matching. This is the device gain in actual practice.

TRANSDUCER GAIN is the ratio of power delivered to the load to the source's available power. This definition ASSUMES source-DUT input conjugate matching but makes no assumptions about the DUT-load matching. This is simply a measure of how much power from the source gets to the load.

The definitions can be summarized in algebraic form as:

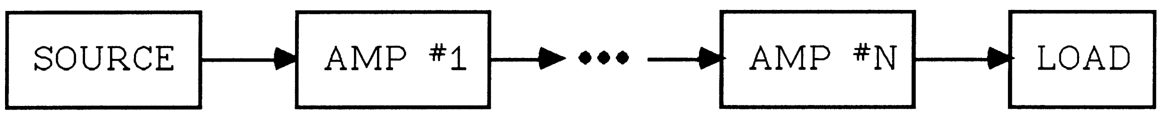
$$A_P = \frac{P_L}{P_{IN}} = \frac{\text{power to load}}{\text{power into DUT}}$$

$$A_A = \frac{P_{AX}}{P_{AS}} = \frac{\text{power available from DUT}}{\text{power available from source}}$$

$$A_{MAG} = \frac{P_{AX}}{P_{IN}} = \frac{\text{power available from DUT}}{\text{power to DUT}}$$

$$A_T = \frac{P_L}{P_{AS}} = \frac{\text{power to load}}{\text{power available from source}}$$

These definitions are of great interest to the systems designer as he/she will often want to do system power analysis. Suppose one considers a cascaded network of amplifier stages as shown below.



Each amplifier is shown with a specified power gain, but the problem is how to specify the transducer gain of the overall network (with the given data since that expression will tell one what power is actually being delivered to the load. If we start our calculations at the load we must have a specified load power and a specified power input to the last amplifier. This last quantity could be inferred from the given power gain of the amplifier. This process can be repeated backwards to represent the power gains of each amplifier stage (known quantities); however, when we reach the input (first) amplifier stage we must recall that we want the overall transducer gain. As transducer gain is the ratio of power delivered to the load to the available source power we must know (or specify) the ratio of load power to available source power for the first amplifier. This is actually the transducer gain of the first amplifier; hence, the overall system transducer gain is the product of the transducer gain of the first amplifier times the power gains for all succeeding amplifier stages.

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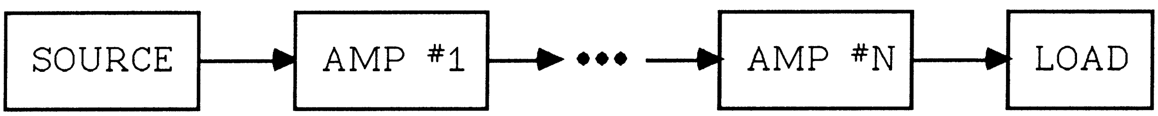

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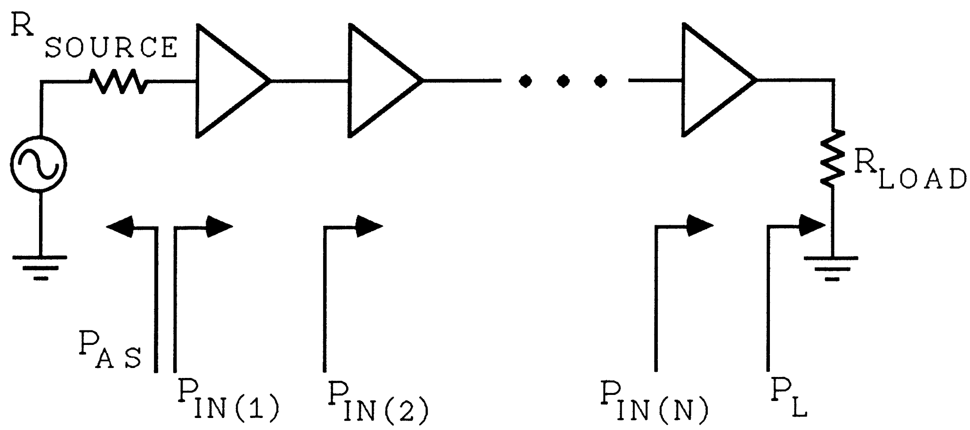
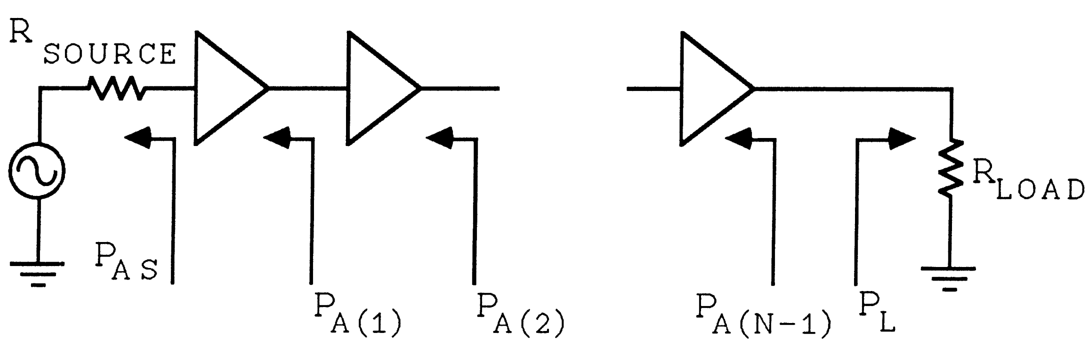


$$A_T = \frac{P_L}{P_{AS}} = \frac{P_{(1)}}{P_{AS}} \times \frac{P_{(2)}}{P_{(1)}} \times \frac{P_{(3)}}{P_{(2)}} \times \ldots \frac{P_{(n)}}{P_{(n-1)}} \times \frac{P_L}{P_{(n)}}$$

$$A_T = A_{T1} \times A_{P1} \times A_{P2} \times \ldots \times A_{Pn}$$

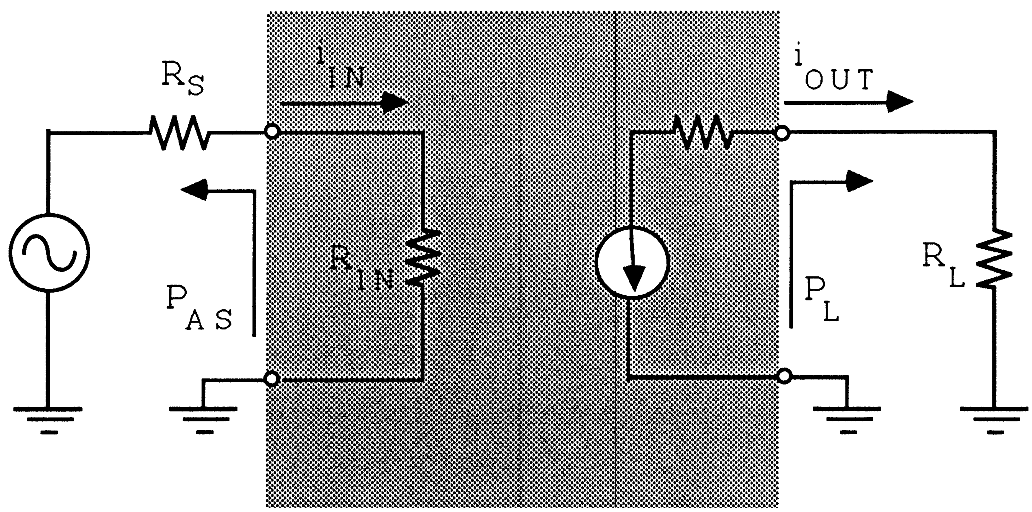
In the previous example we started from the load and worked backwards to the input. We could have just as easily started with the input and worked forward to the load. Starting from the source we must know the source available power since that is required for the overall transducer gain specification. If we know the impedance of each amplifier stage we can specify the available gain of each stage. For the last amplifier stage we thus know the available input power; however, to calculate (or specify) transducer gain we need the load power. For the last power amplifier the transducer gain is the ratio of load power to available source power. Thus, the overall transducer gain is the product of the available gains for the first (n-1) power amplifiers and the transducer gain of the n-th power amplifier as shown below.



$$A_T = \frac{P_L}{P_{AS}} = \frac{P_{A1}}{P_{AS}} \times \frac{P_{A2}}{P_{A1}} \times \ldots \times \frac{P_{An}}{P_{A(n-1)}} \times \frac{P_L}{P_{An}}$$

$$A_T = A_{A1} \times A_{A2} \times \ldots \times A_{A(n-1)} \times A_T$$

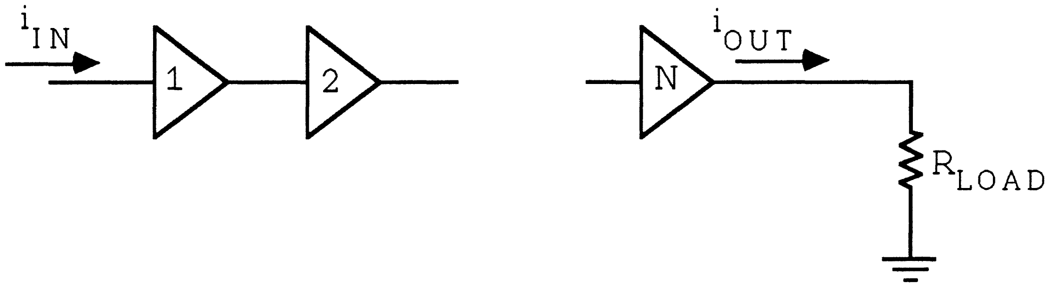
In all previous gain examples we worked with powers derived from voltages; it is just as correct to work with powers derived from currents. Consider the same source-DUT-load network that we started with. The power available from the source can be written in terms of the current flowing in the input circuit; similarly, the output power can be written in terms of the output current as shown below.



$$P_L = (i_{OUT})^2 R_L \qquad P_{AS} = \frac{V_S^2}{4R_S} = \frac{i_{IN}^2 (R_S + R_{IN})^2}{4R_S}$$

The DUT transducer gain is then given by the ratio of the load power to the source's available power as before, but can now be related to the current gain of the DUT.

$$A_T = \frac{P_L}{P_{IN}} = \frac{i_{OUT}^2 R_L 4P_S}{(i_{IN})^2 (R_S + R_{IN})^2} = \frac{4R_L R_S}{(R_S + R_{IN})^2} \left(\frac{i_{OUT}}{i_{IN}}\right)^2$$



Current gain is easy to measure and, because it includes the effects of source and load resistances, gives a very simple expression for cascaded gains. To illustrate this point reconsider the expression for the transducer gain of a system of cascaded power amplifiers. The overall transducer gain can be written as the product of current gains for all amplifiers except the first and the last. This implicitly includes all impedance matching between amplifiers. However, to specify transducer gain we must look at what terms we want at the input and output.

$$A_T = \frac{4R_S R_L}{(R_S+R_{IN})^2}\left(\frac{i_{OUT}}{i_{IN}}\right)^2 = \frac{4R_S R_L}{(R_S+R_{IN})^2}\left(\frac{i_1}{i_{IN}}\right)^2\left(\frac{i_2}{i_1}\right)^2 \cdots \left(\frac{i_{OUT}}{i_{n-1}}\right)^2$$

Recall that the overall transducer gain is the ratio of load power to available source power. The first amplifier stage must be characterized by a ratio of output current to available source power. The derivation of this expression is shown below and this ratio is often denoted by the symbol U. Note that this expression is valid as long as the input impedance of the first amplifier is independent of the impedances of later amplifiers, i.e. loading effects or Miller effects can be neglected.

$$U = \frac{4R_S}{R_S+R_{IN}}\left(\frac{i_1}{i_{IN}}\right)^2$$

For the output amplifier we must specify the load power for a given input current. This factor is often denoted M and must be an explicit function of the load impedance.

$$M = R_L \left(\frac{i_{OUT}}{i_{n-1}}\right)^2$$

The major use of the gain relationships we have defined above is to simply and succinctly define the interrelationships between power gains and impedance matching in complex systems such as radios. For example, we can now write the overall transducer gain of a chain of power amplifiers as

$$A_T = U \times A_2^2 \times A_3^2 \times \ldots \times A_{n-1}^2 \times M$$

where A_2 is the current gain of amplifier #1, A_3 is the current gain of amplifier #3, and so forth.