FEEDBACK AMPLIFIERS AND OSCILLATORS

This section will address one of the most important electrical engineering concepts, that of feedback, for rf applications. The two-port models that we have used for r.f. analysis are particularly well suited for feedback analysis and will be used throughout.

REVIEW OF BASIC FEEDBACK THEORY

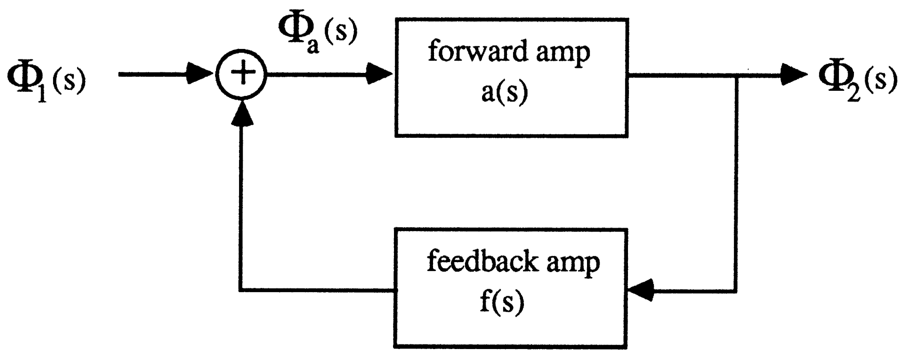


Figure 1 - Basic feedback amplifier

Consider the basic feedback amplifier shown in Figure 1. The basic elements of the amplifier are the difference operator (symbolized by the added), the forward amplifier with transfer function a(s), and the feedback amplifier with transfer function f(s). Note that most feedback analysis takes place in the frequency domain. Using the terminology of Figure 1, the open loop (forward) gain a(s) and the feedback gain f(s) can be written in terms of the loop variables as

$$a(s) = \frac{\Phi_2}{\Phi_a}$$

and

$$f(s) = \frac{\Phi_f}{\Phi_2}$$

The overall amplifier gain (also known as the closed loop gain function) can be written as

$$A(s) = \frac{\Phi_2(s)}{\Phi_1(s)} = \frac{a(s)}{1 + a(s)f(s)}$$

An important observation that we will use later is the the only units retained in this expression are in the numerator, the denominator is dimensionless, i.e. the term a(s) has dimensions but the term a(s)f(s) has no dimensions. This latter term, a(s)f(s) is very significant in understanding feedback amplifiers and is defined to be the return ratio T(s)

$$T(s) = a(s)f(s)$$

An approximation that will be repeatedly used in this section is that if

$|T(j\omega)| >> 1$

then

$$A(s) \approx \frac{1}{f(s)}$$

The denominator of the closed loop gain is called the characteristic equation of the feedback system and specifies the effect of feedback on the system. This term is often called the return difference, or often simply the feedback function

$$F(s) = 1+T(s) = 1+a(s)f(s)$$

This term is usually expressed for engineering analysis in decibels as

$$20 \log(|F(j\omega)|) = 20 \log(1+a(j\omega)f(j\omega))$$

If the magnitude of the feedback term F(s) is greater than 1 this is termed negative feedback since it reduces the closed loop gain; if the magnitude of F(s) is less than one this increases the closed loop gain and is called positive feedback.

APPLICATIONS OF FEEDBACK IN ELECTRONICS

We have already seen some applications of feedback in electronics. The description of the biasing of a class B transistor amplifier is a feedback process. If a resistor divider network is used the feedback is positive as a function of temperature. If a current mirror is used the the bias as a function of temperature is described by a negative feedback process. In this case we have used feedback to minimize the sensitivity of the bias point to temperature changes. The improvement of the gain sensitivity with respect to the change of some circuit parameter is one of the major electronic applications of feedback. The second major application of feedback in electronics is to reduce distortion. Other applications include improving the bandwidth of an amplifier and controlling its impedance. This latter is exactly related to the Miller feedback effect in amplifiers. These latter two points will be derived in following sections. Let's examine the first two applications in a little more detail since we will not really use them any further in this course.

The variation in the gain (or other parameter) of an electronic circuit as a function of circuit parameters, temperature changes, β variations and the like is called sensitivity. In fact, PSpice can perform a sensitivity analysis on an electronic circuit by computing the partial derivatives of a parameter of interest (such as gain) with respect to every other circuit parameter. Such tables allow engineers to determine the critical components in their designs and intelligently assign resistor tolerances for example. Starting with the expression for the closed loop gain

$$A(s) = \frac{a(s)}{1 + a(s)f(s)}$$

we can differentiate it to arrive at

$$dA = -\frac{da}{(1 + af)^2}$$

If we take the ratio of these two results we can express the fractional change in the closed loop amplifier gain as a function of changes in the forward gain a(s)

$$\left|\frac{dA}{A}\right| = \left|\frac{1}{1+af}\right|\left|\frac{da}{a}\right|$$

This formula is a very basic result. If the feedback is negative, i.e.

$$|1+af| > 1$$

the variation in A will be much smaller than the variation in a. As an example, feedback could be used to reduce the variations in closed loop amplifier gain A due to the presence of manufacturing tolerances in an operational amplifier's gain a. The formal definition of sensitivity S is the ratio of the fractional variation of some circuit parameter to the fractional change in another circuit parameter.

$$S_a^A = \frac{\frac{dA}{A}}{\frac{da}{a}}$$

Note that in general, because electronic circuits are strong functions of frequency, the sensitivity S can also be a function of frequency.

Distortion reduction is a straightforward application of feedback theory. Consider the feedback amplifier shown below

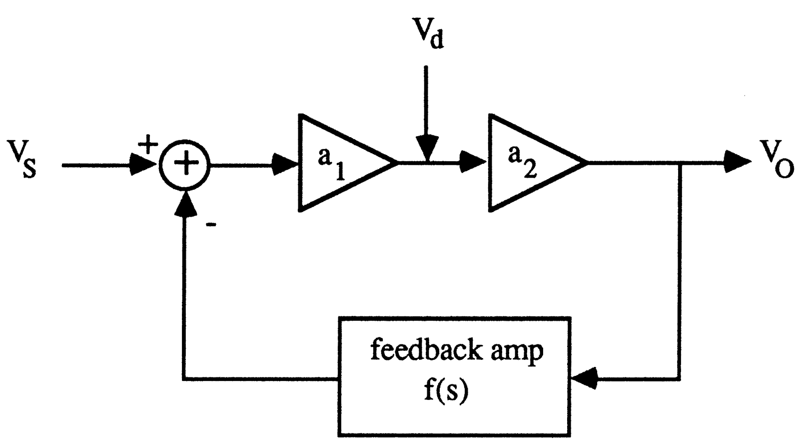


Figure 2 - Feedback amplifier used to diminish distortion added by amplifier a_1

where a_1 and a_2 represent the gain functions of two amplifiers in the forward gain path. Suppose that V_d represents some form of distortion added by amplifier a1. The output voltage V_O can be easily written using superposition, i.e.

$$V_O = V_S \frac{a_1 a_2}{1 + a_1 a_2 f} + V_d \frac{a_2}{1 + a_1 a_2 f}$$

Note that the input voltage V_S passes through both a_1 and a_2 and sees their product as the forward gain. On the other hand, the distortion V_d only sees a_2 in its forward path. Both voltage inputs see the same closed loop denomination since T(s) is the product of all forward and feedback gains. If the magnitude of the open loop gain T(s) is large, the above expression for V_O reduces to

$$V_O \approx V_S \frac{1}{f} + V_d \frac{1}{a_1 f}$$

The immediate observation is that the distortion is reduced by a factor of $1/a_1$ compared to the normal gain of the amplifier. The basic result is that we can improve the performance of amplifier a_1 as far as distortion by applying some form of feedback.

Before closing this section, feedback does not possess only advantages for its use. It achieves its performance advantages by reducing the overall gain of the circuit. Furthermore, the very use of feedback makes the potential for oscillation much higher. This latter point means that electronic circuits using feedback must be carefully designed to prevent oscillation (unless you are designing an oscillator).

FEEDBACK CLASSIFICATION USING TWO-PORTS

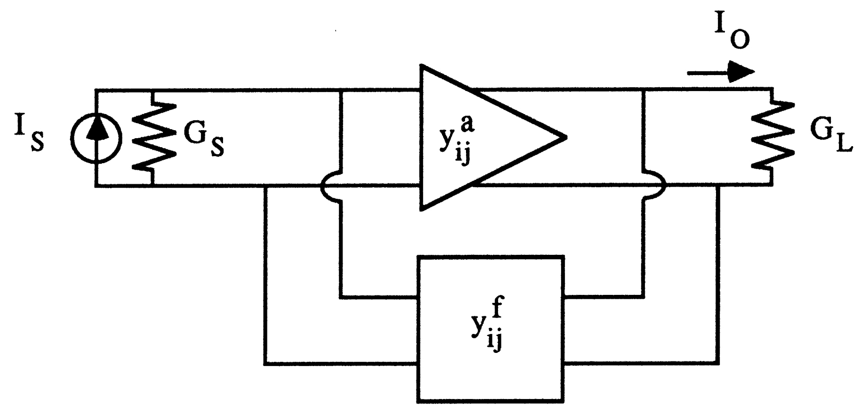
Feedback amplifier analysis is very straight forward using two-port models for the forward and feedback amplifiers. The ease with which two-ports are interconnected simplifies the modeling of the various types of feedback connections. In general, the forward amplifier is a real amplifier, i.e. an active two-port. The feedback amplifier is usually not an active amplifier. Rather, it is usually a passive network represented by a two-port. To simplify our analysis of feedback using two-port models for the forward and feedback amplifiers we will make two fundamental assumptions:

1. The feedback transmission through the feedback network is negligible compared to the forward transmission through the amplifier.

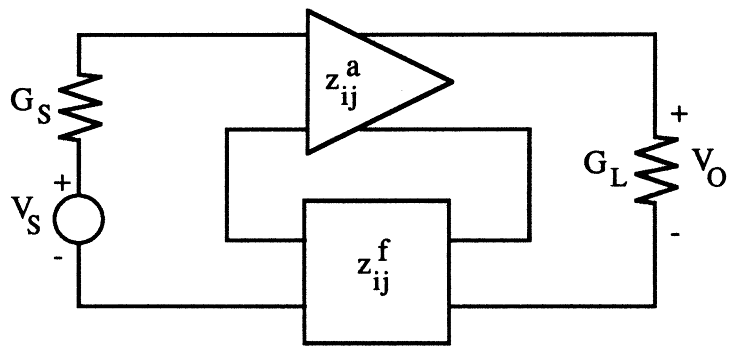
2. The reverse transmission through the forward amplifier is negligible compared to the reverse transmission through the feedback network.

Although sounding very complex these assumptions have a very simple interpretation. All two-ports have coupling between the input and output ports. For example, the load impedance would have no effect on the input impedance if there were no coupling between the input and output. What the two above assumptions are saying is that we will neglect any reverse gains by ignoring all reverse voltage and current sources in the models we will use.

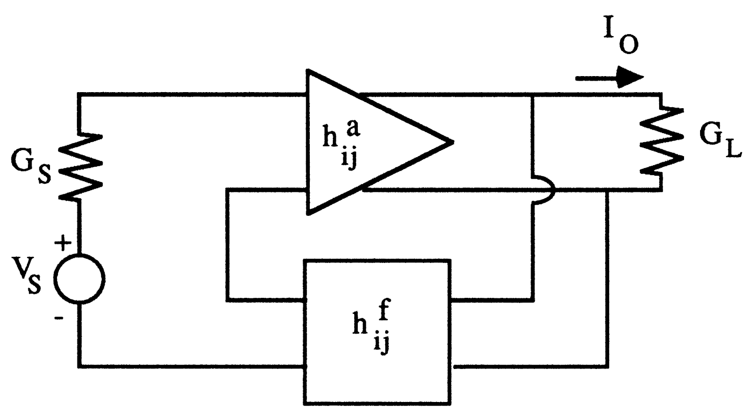
There are four fundamental types of feedback amplifier connections based upon how the actual feedback connections are made. These are shunt-shunt, series-series, series-shunt and shunt-series and may be interpreted as being based upon whether the forward and feedback amplifiers are voltage or current amplifiers. Furthermore, each type of connection will use a different set of possible two-port parameters, i.e. h-, y-, g- or z-parameters as shown in Figure 3.



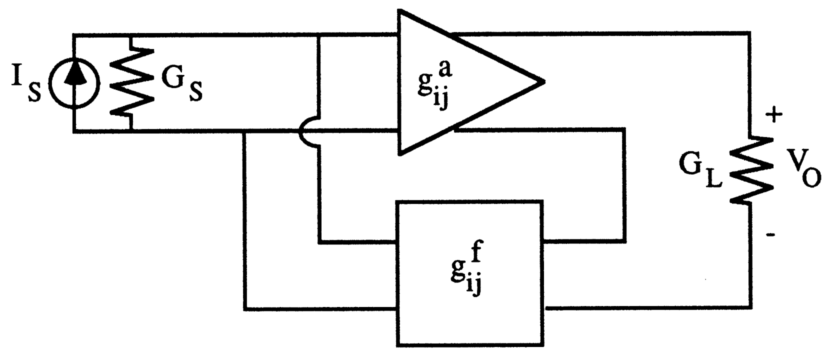
(a) shunt-shunt feedback amplifier configuration modeled using y-parameter two-ports



(b) series-series feedback amplifier configuration modeled using z-parameter two-ports



(c) series-shunt feedback amplifier configuration modeled using h-parameter two-ports



(d) shunt-series feedback amplifier configuration modeled using g-parameter two-ports

Figure 3 - Modeling of feedback amplifier configurations using two-port models

SERIES-SERIES CONNECTED TWO-PART FEEDBACK AMPLIFIER

The series-series feedback amplifier configuration shown in Figure 3(a) will be examined in detail. The first step is to replace the forward and feedback amplifiers by a complete y-parameter representation using the superscripts "a" and "f" to denote the parameters for the forward and feedback amplifiers respectively.

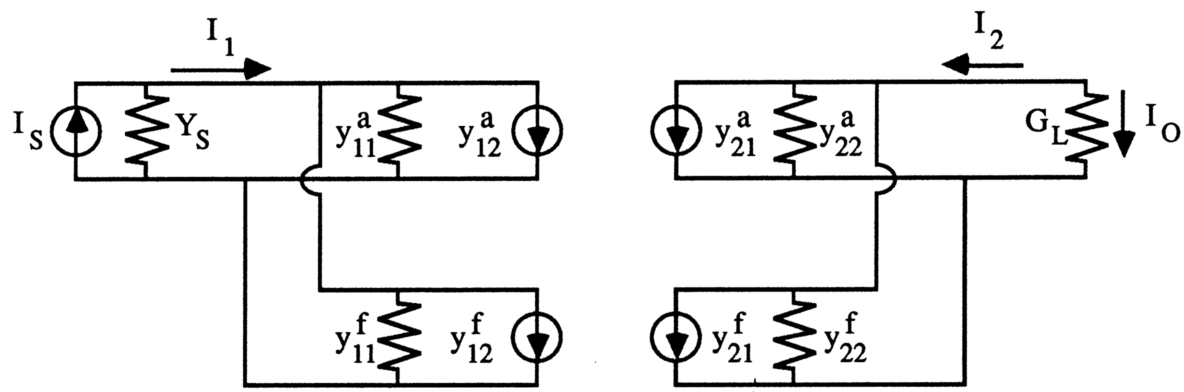


Figure 4 - Detailed shunt-shunt feedback amplifier modeled using y-parameter two-ports

Figure 4 details the shunt-shunt feedback amplifier modeled using y-parameter networks. Note the directions of all currents and the use of I1 and I2 to denote the directions of the currents flowing into the interconnected two-ports. As we know the y-parameters of two-port networks connected in parallel (shunt-shunt) simply adding giving us the overall y-parameters denoted by a superscript "T" for "total."

$$y^T_{11} = y^f_{11} + y^a_{11}$$

$$y^T_{12} = y^f_{12} + y^a_{12}$$

$$y^T_{21} = y^f_{21} + y^a_{21}$$

$y^T_{22} = y^f_{22} + y^a_{22}$

The current gain for the composite network can be immediately written using the composite parameters as

$$\frac{I_O}{I_S} = \frac{-I_2}{I_1} = \frac{-y^T_{21}G_L}{\left(y^T_{11}+G_S\right)\left(y^T_{22}+G_L\right) - y^T_{12}y^T_{21}}$$

where the sign conventions regarding the output current I_O in Figure 3(a) were observed. This can be easily re-written in the form of the classic feedback amplifier equation by dividing the numerator and denominator of the above equation by the first term in the denominator, i.e.

$$A_I = \frac{I_O}{I_S} = \frac{\dfrac{-y^T_{21}G_L}{\left(y^T_{11}+G_S\right)\left(y^T_{22}+G_L\right)}}{1 - \dfrac{y^T_{12}y^T_{21}}{\left(y^T_{11}+G_S\right)\left(y^T_{22}+G_L\right)}}$$

We can immediately identify the numerator as the forward gain a(s) and the second term in the denominator as the return ratio T(s):

$$a_i(s) = \frac{-y^T_{21}G_L}{\left(y^T_{11}+G_S\right)\left(y^T_{22}+G_L\right)}$$

$$T(s) = a(s)f(s) = -\frac{y^T_{12}y^T_{21}}{\left(y^T_{11}+G_S\right)\left(y^T_{22}+G_L\right)}$$

which allows the identification of the feedback gain as

$$f(s) = \frac{y^T_{12}}{G_L}$$

Note that, although the expressions are algebraically unwieldy, their interpretation in terms of feedback amplifiers is straight forward. The only item of note is the interpretation of a(s) which is indicated by the subscript "i" to emphasize that a(s) is, in this example, current gain. To this point we have used no approximations. Now use the approximations that the forward amplifier has no reverse component and the feedback amplifier has no forward component of gain. This means that we are assuming that

$y^a_{21} >> y^f_{21}$

and

$y^a_{12} << y^f_{12}.$

This simply eliminates any interaction between the input and output ports of the individual two-port networks. These approximations can be used in our expression for current gain with the result that

$$\frac{I_O}{I_S} \approx \frac{\dfrac{-y^a_{21}G_L}{\left(y^T_{11}+G_S\right)\left(y^T_{22}+G_L\right)}}{1-\dfrac{y^f_{12}y^a_{21}}{\left(y^T_{11}+G_S\right)\left(y^T_{22}+G_L\right)}}$$

Since most amplifiers we will be concerned with are used to amplify voltage, not current, let's convert our gain expression into one that will give the output voltage. This is simply done by dividing the gain expression by G_L.

$$\frac{V_O}{I_S} = \frac{\dfrac{I_O}{G_L}}{I_S} \approx \frac{\dfrac{-y^a_{21}}{\left(y^T_{11}+G_S\right)\left(y^T_{22}+G_L\right)}}{1-\dfrac{y^f_{12}y^a_{21}}{\left(y^T_{11}+G_S\right)\left(y^T_{22}+G_L\right)}}$$

This changes the gain expression into the final form we will use. Our old expressions for a(s), f(s) and T(s) can now be re-written as

$$a_z(s) = \frac{-y^T_{21}}{\left(y^T_{11}+G_S\right)\left(y^T_{22}+G_L\right)} = \left.\frac{V_O}{I_S}\right|_{f=0}$$

$$f(s) = y^f_{12}$$

and

$$T(s) = -a_z(s)f(s) = \frac{-y^T_{21}y^f_{12}}{\left(y^T_{11}+G_S\right)\left(y^T_{22}+G_L\right)}$$

There have been two major effects of this re-writing of the desired gain expression: first, the feedback term has become simply the reverse admittance of the feedback two-port; second, a(s) is no longer dimensionless and now has the units of resistance. More technically, a(s) is said to have a transresistance since it is not really a circuit component. Furthermore, note that a(s) is simply the feedback function evaluated for f=0, i.e. without feedback. This means that once we write the forward gain and find the reverse admittance of the feedback network, we can immediately write the expression for the closed-loop gain function. At this point we have determined the voltage and current gain of the network. The input and output impedances are simply obtained from the "total" two-port network parameters as

$$Y_{in} = \left(y^T_{11} + G_S\right) - \frac{y^T_{12}y^T_{21}}{y^T_{22} + G_L}$$

which can be re-written as

$$Y_{in} = \left(y_{11}^T + G_S\right)\left(1 - \frac{y_{12}^T y_{21}^T}{\left(y_{22}^T + G_L\right)\left(y_{11}^T + G_S\right)}\right)$$

Applying the approximations that the forward amplifier has no reverse admittance and the feedback amplifier has no forward admittance, the input impedance becomes

$$Y_{in} \approx \left(y_{11}^T + G_S\right)\left(1 - \frac{y_{12}^f y_{21}^a}{\left(y_{11}^T + G_S\right)\left(y_{22}^T + G_L\right)}\right)$$

which can be written using the return ratio T(s) as

$$Y_{in} \approx \left(y_{11}^T + G_S\right)\left(1 + T(s)\right)$$

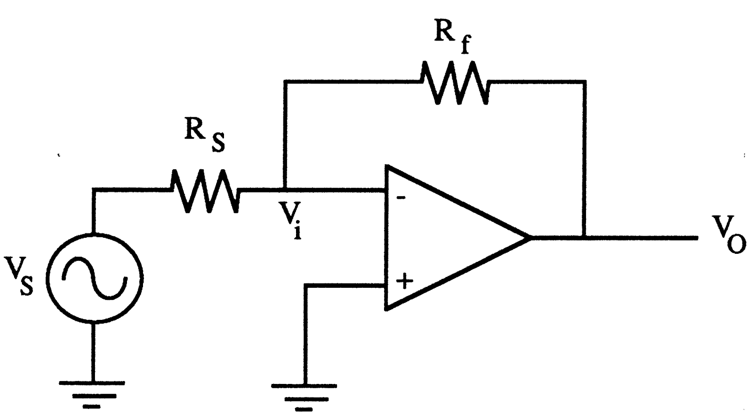
Note that this expression includes the admittance of the source G_S. To get just the amplifier input admittance we would need to subtract G_S giving

$$Y_{in,amplifier} \approx \left(y_{11}^T + G_S\right)\left(1 + T(s)\right) - G_S$$

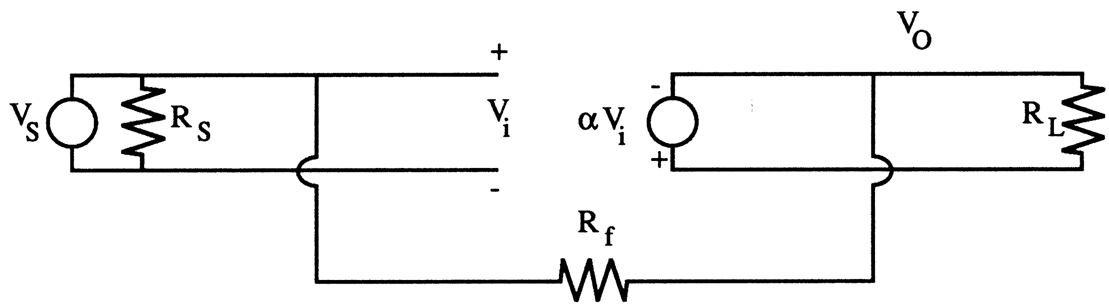
Notice that since T(s)>0 (typically) the input admittance of a shunt-shunt connected feedback amplifier increases with feedback.

EXAMPLES OF SHUNT-SHUNT FEEDBACK AMPLIFIERS

The conventional operational-amplifier circuit shown below can be examined using the two-port formalism for a shunt-shunt feedback amplifier developed previously. The operational amplifier circuit is shown in Figure 5. The two-port equivalent circuit is shown in Figure 6.



(a) Basic operational amplifier circuit



(b) Equivalent circuit of operational amplifier circuit

Figure 5 - Operational amplifier feedback amplifier

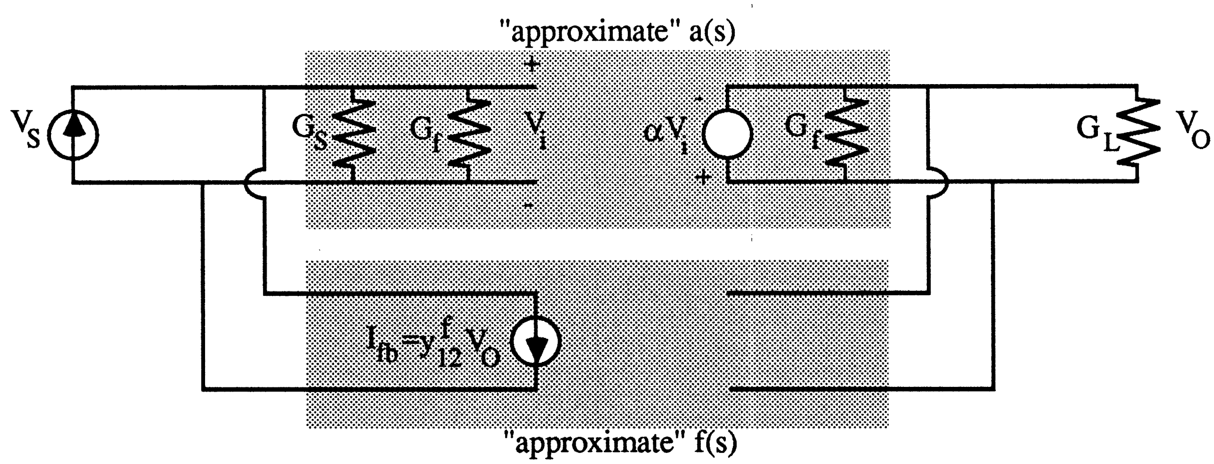


Figure 6 - Approximate feedback amplifier representation of Figure 5 modeled as an interconnection of two-port networks

The greatest difficulty in analyzing circuits such as shown in Figure 5 is drawing the approximate two-port interconnect feedback amplifier as shown in Figure 6. Note that all impedances have been drawn as part of the forward amplifier. The only feedback element is the reverse admittance. The reverse admittance of the forward amplifier and the forward admittance of the feedback amplifier have been neglected as per our assumptions. Furthermore, since it was not specified the input and output impedances of the operational-amplifier itself were assumed to be infinite. Once drawn as in Figure 6 the actual analysis is reasonably straight forward. By inspection the output voltage is

$$V_O = -\alpha V_i$$

or

$$V_O = -\alpha \frac{I_S}{G_S + G_f}$$

The forward gain is recognizable as

$$a_Z = \frac{V_O}{I_S} = \frac{-\alpha}{G_S + G_f}$$

where it has the expected "units" of transresistance. The feedback term f is immediately calculated from the y-parameter definitions for a series resistor to be

$$f = y^f_{12} = \frac{I_1}{V_2} = \frac{I_{fb}}{V_O} = -\frac{1}{R_f} = -G_f$$

The return ratio T=af is given as

$$T = a_Z f = -\frac{\alpha G_f}{G_S + G_f}$$

With these identifications the closed loop gain can be immediately written as

$$A(s) = \frac{a(s)}{1 + a(s)f(s)} \approx \frac{1}{f(s)} = -R_f$$

Note that this is not the standard expression for the gain of the operational-amplifier shown in Figure 5 because the above gain expression is the ratio of output voltage to input current. The conventional gain expression is simply the ratio of output voltage to input voltage. The above A(s) can be converted into this more standard form by recognizing that

$$I_S = \frac{V_S}{R_S}$$

and, after substituting this result into the expression for A(s), we get

$$\frac{V_O}{V_S} = -\frac{R_f}{R_S}$$

the standard operational-amplifier voltage gain expression. We could also have written the expressions for G_{in} and G_{out} as

$$G_{in} = (G_i + G_f + G_S)(1 + T(s))$$

and

$$G_{out} = (G_O + G_f)(1 + T(s))$$

Note that this increases the input and output admittances, decreasing the input and output impedances, because of the feedback.

ADVANCED ASPECTS OF FEEDBACK AMPLIFIERS

A commercially available internally-compensated operational amplifier typically has a single pole frequency response of the form

$$a(s) = \frac{a_O}{1 + \frac{s}{p_O}}$$

The feedback network for an operational amplifier is typically a simple resistor without any frequency dependence so that

$$f = f_o$$

giving the closed loop frequency dependent gain of the operational amplifier

$$A(s) = \frac{a(s)}{1 + a(s)f(s)} = \frac{\dfrac{a_o}{1 + \dfrac{s}{p_o}}}{1 + \dfrac{a_o f_o}{1 + \dfrac{s}{p_o}}}$$

This can be re-arranged into a slightly more useful form

$$A(s) = \frac{\dfrac{a_o}{1 + a_o f_o}}{1 + \dfrac{s}{p_o(1 + a_o f_o)}}$$

The closed-loop frequency response retains the single pole of the forward gain characteristic, but, if p_o was the location of the original operational amplifier pole, the pole has now been shifted to $p_o(1+a_of_o)$. Since the pole represents the frequency of the breakpoint for the low-pass transfer function shown the bandwidth as indicated by the breakpoint (i.e. the -3db point) has been increased by the loop gain. This is an important characteristic showing that the gain-bandwidth product relationship is observed. The frequency dependent characteristics for the commercial 741 operational amplifier are shown in Figure 7.

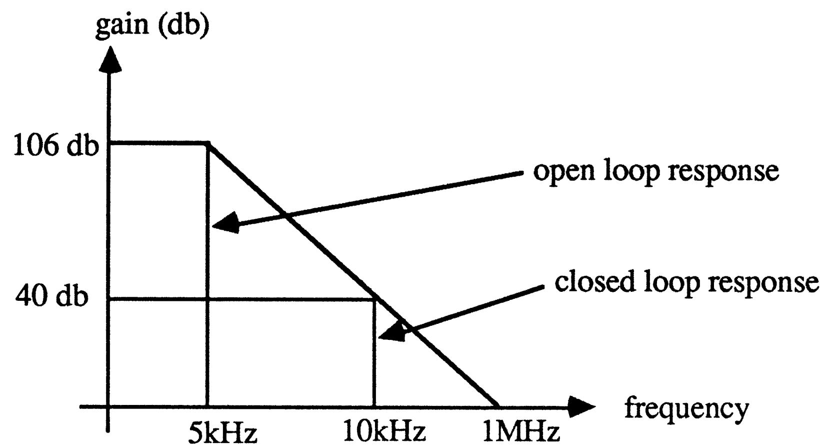


Figure 7 - Gain vs. Frequency characteristics of 741 operational amplifier

In general, feedback can be used to extend the frequency response of an amplifier at the sacrifice of gain. The situation becomes more complex with multi-stage amplifiers exhibiting multiple poles. Such situations require a careful study of the pole locations and can often be approximated by the simple single-pole models described above.

STABILITY CONSIDERATIONS

The use of feedback as described above introduces the possibility of oscillation into any system. A classic theorem states that a feedback system will remain stable (i.e. not oscillate) if all roots (i.e. poles) of the characteristic equation of the transfer function lie in the left hand complex plane (They cannot be on the $j\omega$ axis.). Stability can only be determined when all poles are considered. This means that the characteristic equation cannot be simplified using dominant pole techniques. This usually means that the (usually very complex) characteristic equation must be solved by computer methods. This is an ideal way for theoretical systems engineers to analyze complex systems but is often ignored by practical electrical and control engineers. Such engineers simply measure the open loop gain of their system (by the way the open loop characteristics of an electrical circuit can be calculated using PSpice with some precautions). Typical open loop amplifier gain and phase responses are shown in Figure 7. A system will oscillate if the feedback is exactly in phase and amplitude with the input signal. For the typical negative feedback system we have been discussing this means that the return ratio $T(j\omega)$ must equal ± 1 causing the denominator of the closed loop transfer function to become zero. This can be expressed in the concise oscillation requirement that

$$|T(j\omega)| = 1$$

and

$$\angle T(j\omega) = \pm 180°$$

The frequency at which the open loop gain of the amplifier drops to 1 (0 db) is known as the gain crossover frequency ω_g. The frequency at which the phase shift of the open loop amplifier crosses 180 degrees is known as the phase crossover frequency ω_p. These are illustrated in Figure 8. If $\omega_p=\omega_g$ the condition that $T(j\omega)=\pm 1$ is met and the amplifier will oscillate. Engineers specify amplifiers by how far the crossover point frequencies are from each other. The difference between the amplifier gain and 0 db at the phase crossover frequency is known as the gain margin G_m; the difference between 180 degrees and the amplifier phase shift at the gain crossover frequency is known as the phase margin Φ_m. Rules of thumb for stable amplifier design are that G_m > 10db and Φ_m>60 degrees. These rules ensure stability and allow for good transient response.

REFERENCES

Mohammed Ghausi, Electronic Devices and Circuits: Discrete and Integrated, Holt,Rinehart and Winston, 1985. Chapter 10 Feedback Amplifiers and Oscillators.

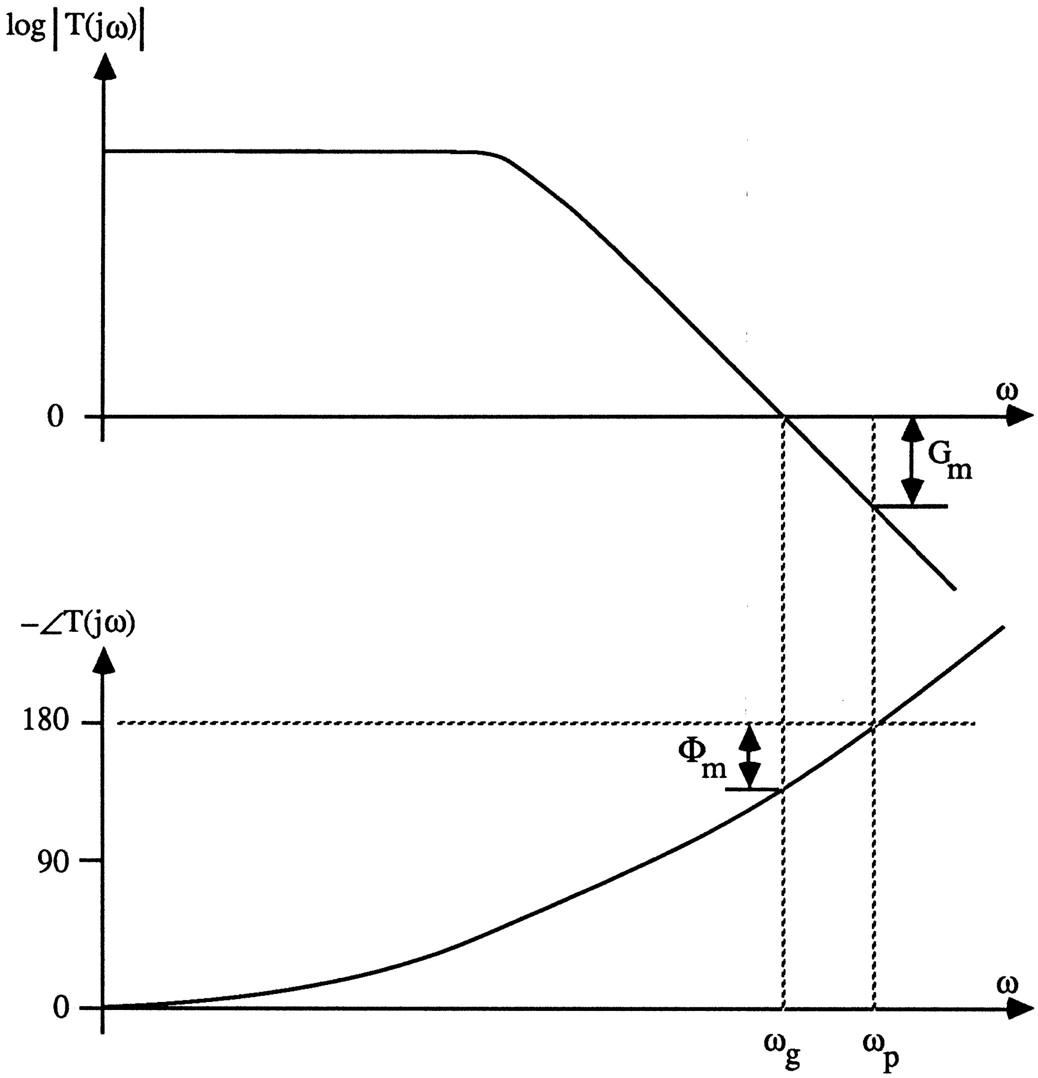
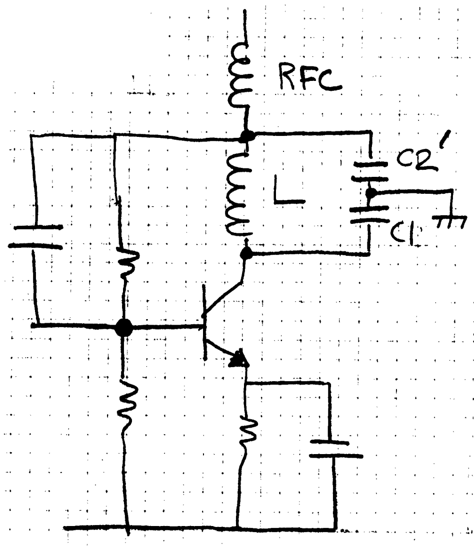


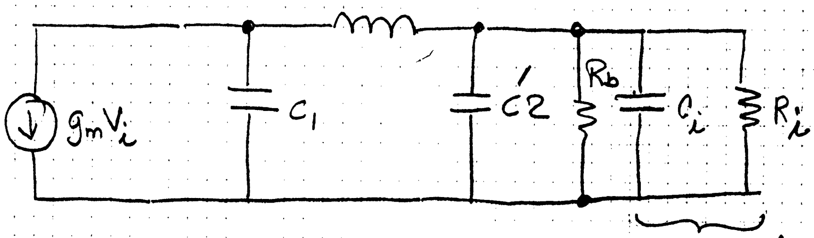
Figure 8 - Gain and phase relationships for a stable feedback amplifier

Simplified LC oscillator



Colpitts oscillator

① draw small signal equivalent circuit



define $R_1 = R_i \parallel R_b$

$C_2 = C_2' + C_i$

② open loop gain $T(s) = \dfrac{-g_m R_1}{sR_1C_1 + (1 + sR_1C_2)(1 + s^2LC_1)}$

$$= \frac{-g_m R_1}{1 + s(R_1C_2 + R_1C_1) + s^2LC_1 + s^3LC_1R_1C_2}$$

③ $= 1\angle 0°$

for no imaginary part in denominator

$$j\omega(R_1C_2 + R_1C_1) + (j\omega)^3 LC_1R_1C_2 = 0$$

$$\cancel{R_1}C_2 + \cancel{R_1}C_1 = \omega^2 LC_1\cancel{R_1}C_2$$

$$\omega^2 = \frac{1}{L}\left(\frac{C_1 + C_2}{C_1C_2}\right)$$

from gain consideration

$$\frac{-g_m R_1}{1-\omega^2 L C_1} = 1$$

$$1-\omega^2 L C_1 = -g_m R_1$$

$$-g_m R_1 = 1 - \frac{1}{L}\,\frac{C_1+C_2}{C_1C_2}\,LC_1 = 1 - \frac{C_1+C_2}{C_2}$$

$$\frac{C_1+C_2}{C_2} = 1 + g_m R_1$$

if $R_b \gg R_i$ Then $R_1 \approx R_i = r_\pi = \frac{\beta_0}{g_m}$

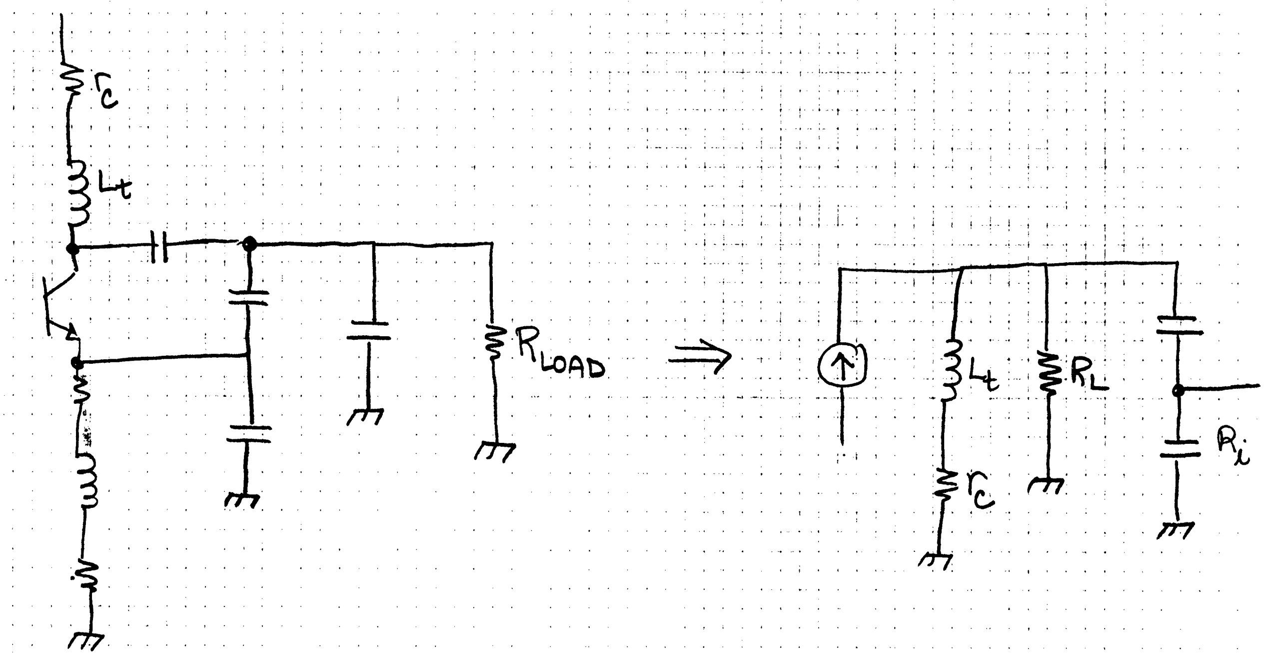
$$\therefore \quad 1 + \frac{C_1}{C_2} = 1 + \cancel{g_m}\frac{\beta_0}{\cancel{g_m}}$$

pick $\beta_0 \gtrsim \frac{C_1}{C_2}$ for oscillation

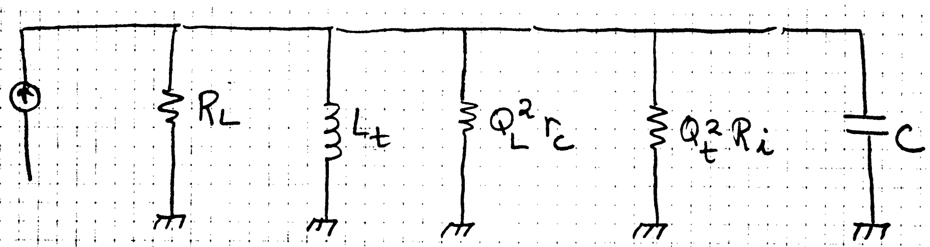
$$\omega = \frac{1}{\sqrt{L\,(C_1 \| C_2)}}$$

how to get power out of oscillator.

look at collector load circuit for a Colpitts oscillator



the output is a tapped capacitor network which we can re-write



at ω_0 the susceptances cancel so the circuit reduces to an amplifier with a load resistance R_o of

$$R_o = R_L \parallel Q_L^2 r_c \parallel Q_t^2 R_i$$

for maximum power transfer to R_L we should have

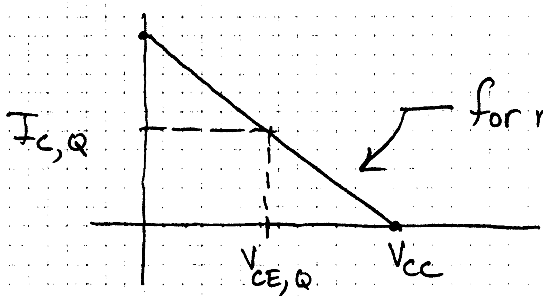
$$R_L = Q_L^2 r_c \parallel Q_t^2 R_i$$

i.e. "conjugate" matching at resonance

In general, oscillator can have an output power of

$$P_L = I_{C,Q}^2 \frac{R_L}{8}$$

where $I_{C,Q}$ is the optimium bias current



for maximum power transfer slope is

$$-\frac{1}{R_0} = -\frac{2}{R_L}$$

i.e. matched

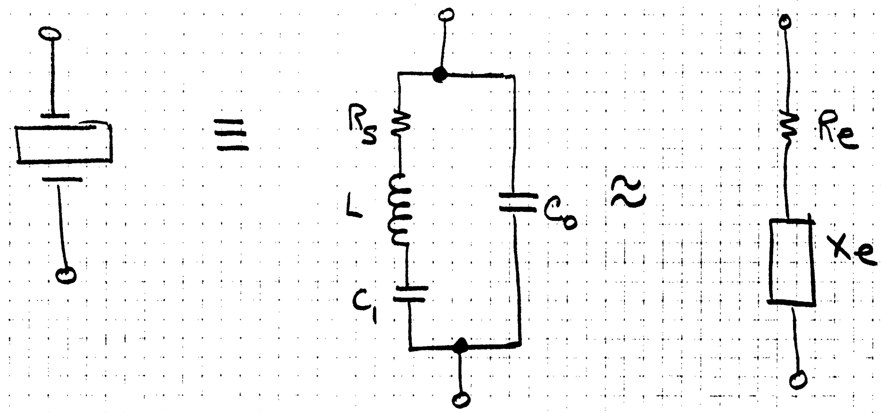
maximum output power

$$P_L = \left(\frac{\Delta I_C}{\sqrt{2}}\right)\left(\frac{\Delta V_{CE}}{\sqrt{2}}\right) = \frac{I_{C,Q}}{\sqrt{2}} \; \frac{\frac{I_{C,Q} R_o}{2}}{\sqrt{2}} = \frac{I_{C,Q}^2 R_o}{4}$$

$$= \frac{I_{C,Q}^2 R_L}{8}$$

Colpitts crystal oscillator

crystal — used for best oscillators (most stable).



Q ~ several hundred thousand

for a typical 10-MHz crystal

$$Q = 1.5 \times 10^5$$

$$\frac{C_o}{C_1} = 300$$

$$L = 12\,\text{mH}$$

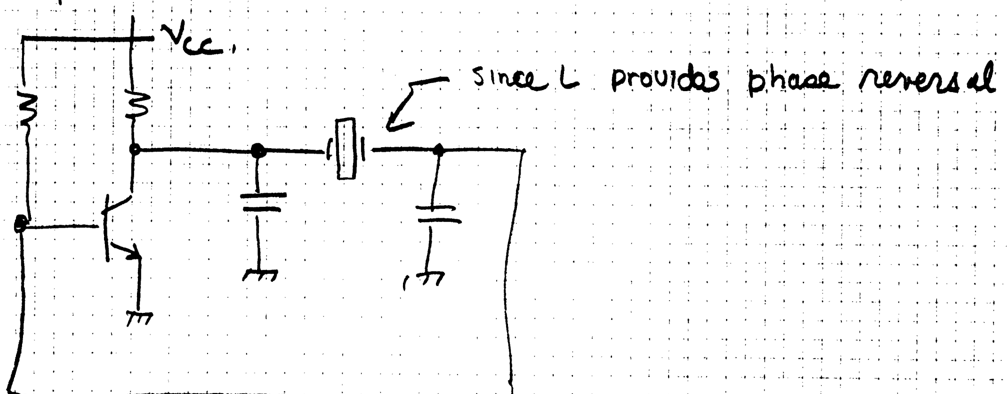
$$C_1 = 10\,\text{pf}$$

$$R_S = 5\,\Omega$$

$$\omega_0^2 \approx \frac{1}{LC_1}$$

$$R_s \approx \frac{\omega_0 L}{Q}$$

crystal colpitts



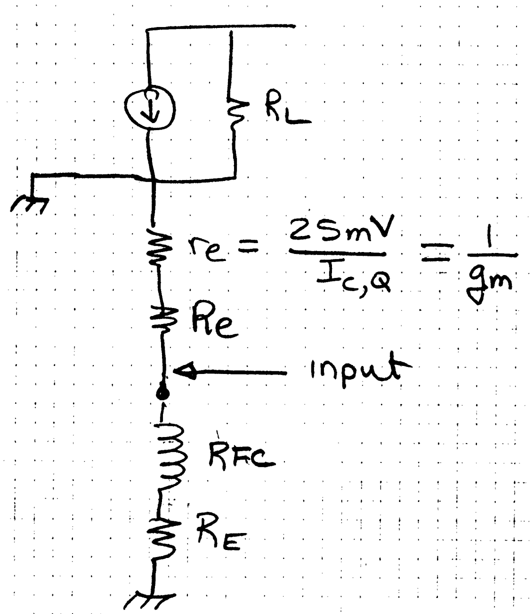
A procedure to design a Colpitts oscillator might be

1. determine output power required
2. choose a R_L
3. select $I_{c,Q}$ to give desired output power
4. bias transistor to $I_{c,Q}$
5. choose tank capacitor C.
   Note: circuit Q should be large (>50) to prevent frequency drift
6. calculate L to be resonant at ω_0
7. select C_1, C_2 to match R_i to the collector circuit

this process will require variable capacitors or inductors to "fine tune"

Note: what is R_i for oscillator?

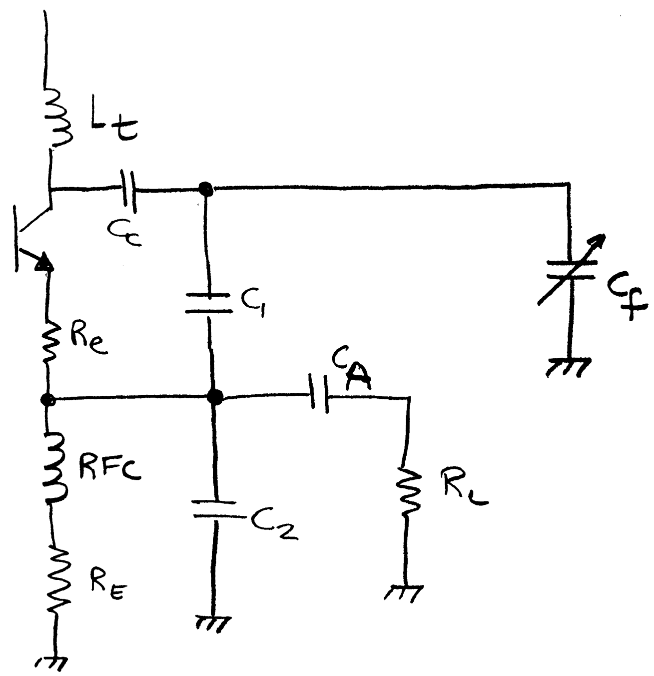
using tee model



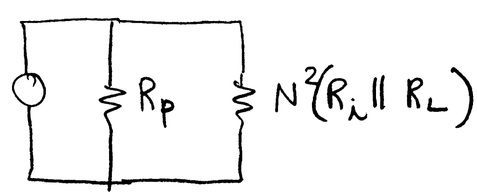
$R_i = r_e + R_e$ since base is at ac ground and R_L is usually quite large.

How to couple an oscillator to a low resistance load

For $R_L \lesssim 1000\,\Omega$ feedback to sustain oscillation is difficult because of phase shift with R_L connected as a collector load. In this situation we can instead put the load in parallel with the emitter as shown below



This circuit is <u>not</u> as efficient because more power is dissipated in the coil resistance. The second problem is that if R_L is too small oscillations will stop. R_L and R_i are now in parallel They are reflected by the C1/C2 divider into $N^2(R_i \| R_L)$ into the collector circuit. The results are



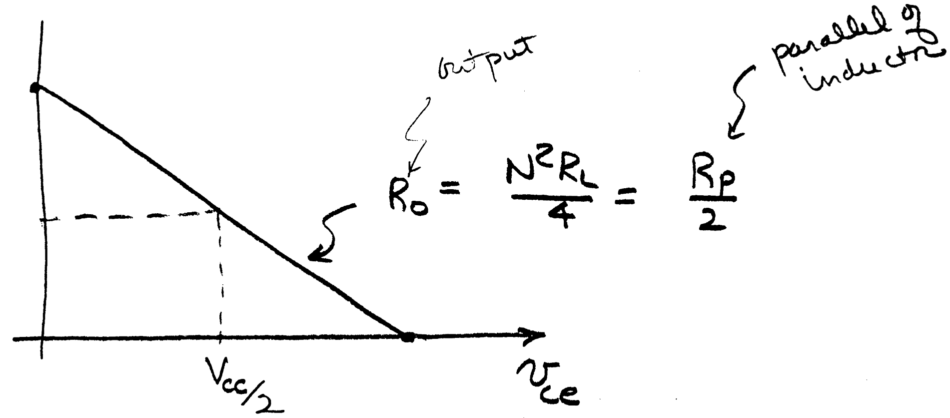
The matching condition is now $R_p = N^2 \dfrac{R_i R_L}{R_i + R_L}$

Note the problem. If half the power goes into $N^2(R_i \| R_L)$ only a fraction of this power makes it to R_L. If $R_i = R_L$ then

$$R_p = N^2 \frac{R_L}{2}$$

or $N = \sqrt{\dfrac{2R_p}{R_L}}$

This means, by the way, that only 1/4 of the output power goes to R_L.



$$P_L = \left(\frac{I_{c,Q}}{\sqrt{2}}\right)^2 R_o = \frac{I_{c,Q}^2}{2}\,\frac{R_p}{2}$$

for rms

$$I_{c,Q} = 4\sqrt{P_L/R_p}$$

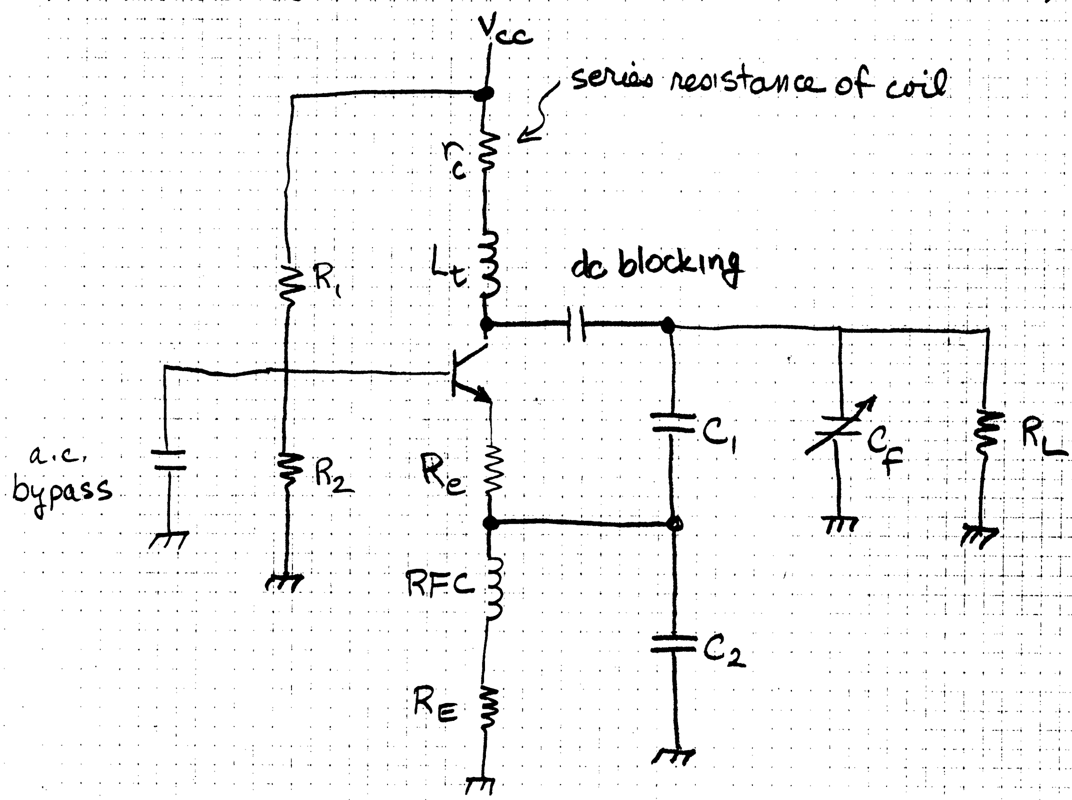
$$V_{cc} = 2 I_{c,Q} R_o = 2 I_{c,Q} \frac{R_p}{2} = I_{c,Q} R_p$$

Same design procedure

1. determine output power required
2. choose an R_L
3. select $I_{c,Q}$ to give output power $\quad I_{c,Q} = 4\sqrt{P_L/R_p}$
4. bias transistor to give $I_{c,Q}$
5. choose tank capacitance C. Choose a large circuit Q (say 50) to prevent frequency drift.
6. calculate L to be resonant at ω_o
7. select C_1, C_2 to match $R_i \| R_L$ to the collector circuit

Colpitts oscillator

slightly different (uses feedback to emitter)



R_L = load

C_f = frequency tuning

C_1, C_2 = feedback network

R_e – stabilizes circuit must be large enough swamp out any input inductance.

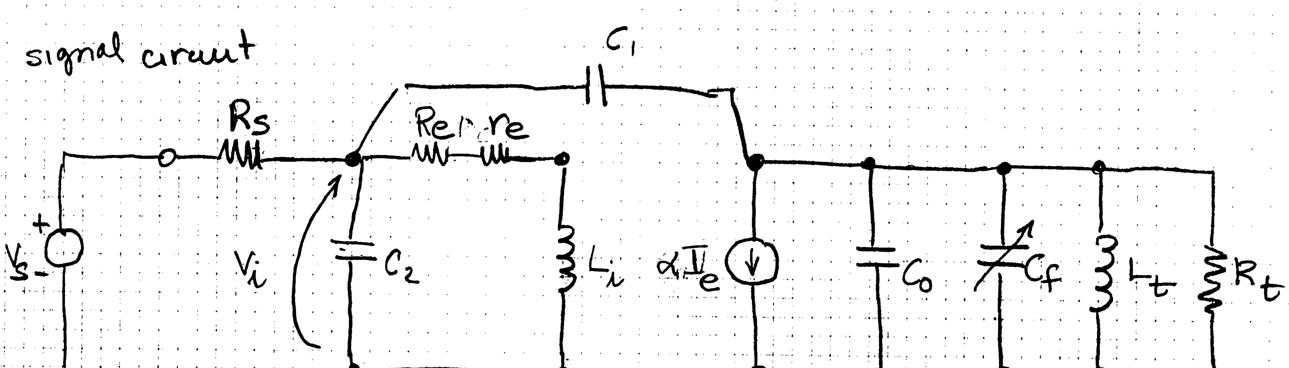
R_1, R_2, R_E – bias resistors

L_t – tank inductance

Note that the Colpitts oscillator is common base

1. this allows the emitter and collector currents to be in phase
2. the oscillator can operate up to the α-cutoff frequency of the transistor – the point at which its α begins to drop.

small signal circuit

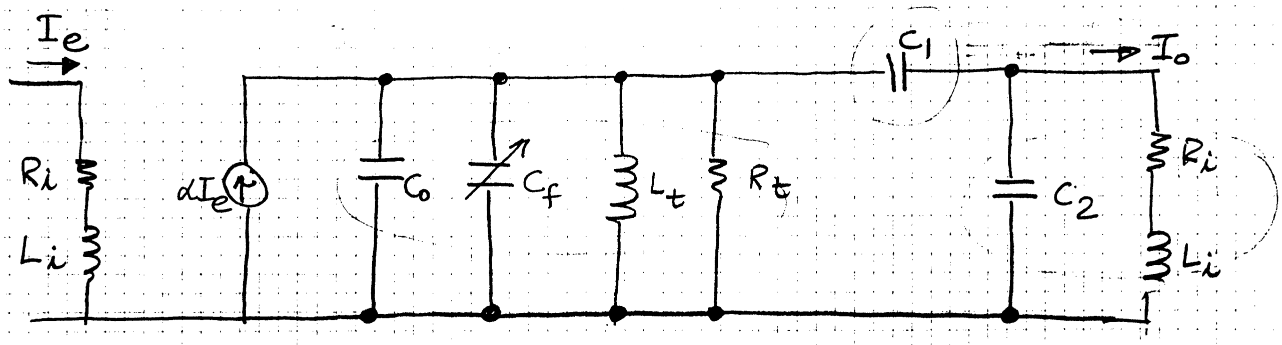


R_e is external resistor to swamp out L_i

r_e is transistor's emitter resistance.

$R_i = R_e + r_e$

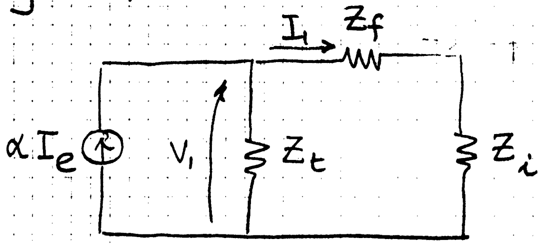
we can re-arrange slightly to get more useful circuit to analyze.



$R_i \gg j\omega L_i$
To swamp L_i out R_i typicall is several hundred ohms

what is the open loop gain $T(s) = +\dfrac{I_o}{I_e}$

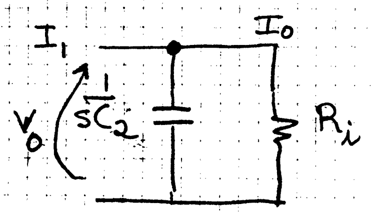
simplify output



$$I_1 = \frac{V_1}{Z_f+Z_i} = \frac{\alpha I_e \left[Z_t \| (Z_f+Z_i)\right]}{Z_f+Z_i} = \alpha I_e \frac{\dfrac{Z_t \cancel{(Z_f+Z_i)}}{Z_t+Z_f+Z_i}}{\cancel{Z_f+Z_i}}$$

$$I_1 = \alpha I_e \frac{Z_t}{Z_t+Z_f+Z_i}$$

but I_1 further divides to give I_o.



$$I_o = \frac{V_o}{R_i} = \frac{I_1 \left(\frac{1}{sC_2} \| R_i\right)}{R_i} = \frac{I_1}{R_i} Z_i$$

$$T(s) = \frac{I_o}{I_e} = \frac{\alpha I_e \dfrac{Z_t}{Z_i + Z_f + Z_t} \dfrac{Z_i}{R_i}}{I_e} = \frac{\alpha}{R_i} \frac{Z_i Z_t}{Z_i + Z_f + Z_t}$$

$$= \frac{\alpha}{R_i} \frac{1}{\dfrac{1}{Z_t} + \dfrac{Z_f}{Z_i Z_t} + \dfrac{1}{Z_i}} = \frac{\alpha}{R_i} \frac{1}{Y_t + Y_i + Y_i Y_t Z_f}$$

$$= \frac{\alpha}{R_i} \frac{1}{s(C_o + C_f) + \dfrac{1}{sL_t} + \dfrac{1}{R_t} + sC_2 + \dfrac{1}{R_i} + \left(sC_2 + \dfrac{1}{R_i}\right)\left[s(C_o+C_f) + \dfrac{1}{sL_t} + \dfrac{1}{R_t}\right]\dfrac{1}{sC}}$$

$$= \frac{\alpha}{R_i} \frac{1}{s(C_o+C_f) + \dfrac{1}{sL_t} + \dfrac{1}{R_t} + sC_2 + \dfrac{1}{R_i} + \dfrac{C_2}{C_1} s(C_o+C_f) + \dfrac{C_2}{C_1}\dfrac{1}{sL_t} + \dfrac{C_2}{C_1}\dfrac{1}{R_t} + \dfrac{1}{R_i C_1}(C_o+C_f) + \dfrac{1}{s^2 R_i L_t C_1} + \dfrac{1}{sC_1 R_i R_t}}$$

$$= \frac{\alpha}{R_i} \frac{1}{\underbrace{\dfrac{1}{R_t} + \dfrac{1}{R_i} + \dfrac{C_2}{C_1}\dfrac{1}{R_t} + \dfrac{1}{R_i C_1}(C_o+C_f)}_{\text{real}} + \underbrace{s(C_o+C_f) + sC_2 + s\dfrac{C_2}{C_1}(C_o+C_f)}_{\text{imaginary}} + \underbrace{\dfrac{1}{sL_t} + \dfrac{C_2}{C_1}\dfrac{1}{sL_t} + \dfrac{1}{sC_1 R_i R_t}}_{\text{imaginary}} + \underbrace{\dfrac{1}{s^2 R_i L_t C_1}}_{\text{real}}}$$

for $\angle T(s) = 0°$ imaginary part of denominator $= 0$.

$$s(C_o+C_f) + sC_2 + s\frac{C_2}{C_1}(C_o+C_f) + \frac{1}{sL_t} + \frac{C_2}{C_1}\frac{1}{sL_t} + \frac{1}{sC_1 R_t R_i} = 0$$

$$j\omega\left[C_o + C_f + C_2 + \frac{C_2}{C_1}(C_o+C_f)\right] = +\frac{j}{\omega}\left[\frac{1}{L_t} + \frac{C_2}{C_1}\frac{1}{L_t} + \frac{1}{C_1 R_t R_i}\right]$$

$$\omega^2 = \underbrace{\frac{1}{L_t}\left(1+\frac{C_2}{C_1}\right)\frac{1}{C_o+C_f+C_2+\frac{C_2}{C_1}(C_o+C_f)}}_{\text{this is actual tank circuit resonant frequency}} + \underbrace{\frac{1}{C_1 R_t R_i}\;\frac{1}{C_o+C_f+C_2+\frac{C_2}{C_1}(C_1}}_{\text{this is loading term.}}$$

$$\frac{1}{L_t}\;\frac{C_1+C_2}{C_1\left[C_o+C_f+C_2+\frac{C_2}{C_1}(C_o+C_f)\right]} = \frac{1}{L_t}\;\frac{C_1+C_2}{C_1(C_o+C_f)+C_1C_2+C_2(C_o+C_f)}$$

$$\frac{1}{L_t}\;\frac{C_1+C_2}{(C_1+C_2)(C_o+C_f)+C_1C_2} = \frac{1}{L_t\left(C_o+C_f+\frac{C_1C_2}{C_1+C_2}\right)}$$

$$\frac{1}{R_iR_t}\;\frac{1}{C_1(C_o+C_f)+C_1C_2+C_2(C_o+C_f)} = \frac{1}{R_iR_t\;(C_o+C_f)(C_1+C_2)+C_1C_2}$$

$$= \frac{C_1+C_2}{R_iR_t\left[C_o+C_f+\frac{C_1C_2}{C_1+C_2}\right]}$$

Note that as R_i or R_t get bigger the loading term decreases

$$\Rightarrow \quad \omega_o^2 \approx \frac{1}{L_t\left(C_o+C_f+\frac{C_1C_2}{C_1+C_2}\right)}$$

for $\quad T(s) = 1\angle 0^\circ$

$$\frac{\alpha}{R_i}\;\frac{1}{\dfrac{1}{R_t}+\dfrac{1}{R_i}+\dfrac{C_2}{C_1}\dfrac{1}{R_t}+\dfrac{1}{R_i}\dfrac{C_o+C_f}{C_1}-\dfrac{1}{\omega^2 R_i L_t C_1}} = 1.$$

$$\frac{\alpha}{R_i} = \frac{1}{R_t}+\frac{1}{R_i}+\frac{C_2}{C_1}\frac{1}{R_t}+\frac{1}{R_i}\frac{C_o+C_f}{C_1}-\frac{1}{\omega^2 R_i L_t C_1}$$

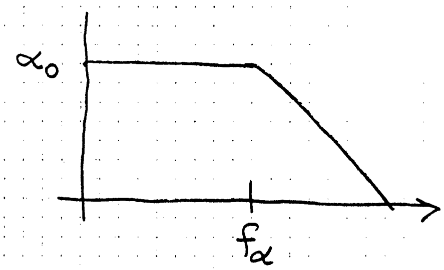
$$\alpha = \frac{R_i}{R_t}+1+\frac{C_2}{C_1}\frac{R_i}{R_t}+\left(\frac{C_o+C_f}{C_1}\right)-\frac{1}{\omega^2 L_t C_1}$$

$$= \frac{R_i}{R_t}\left(1+\frac{C_2}{C_1}\right)+\underbrace{\left(1+\frac{C_o+C_f}{C_1}\right)-\frac{1}{\omega^2 L_t C_1}}$$

$$= \frac{R_i}{R_t}\left(1+\frac{C_2}{C_1}\right)+1+\frac{C_o+C_f}{C_1}-\frac{L_t\left[C_o+C_f+\frac{C_1C_2}{C_1+C_2})\right]}{L_t C_1}$$

$$= \frac{R_i}{R_t}\left(1+\frac{C_2}{C_1}\right)+1+\cancel{\frac{C_o+C_f}{C_1}}-\cancel{\frac{C_o+C_f}{C_1}}-\frac{C_2}{C_1+C_2}$$

$$= \frac{R_i}{R_t}\left(1+\frac{C_2}{C_1}\right)+1-\frac{C_2}{C_1+C_2}$$



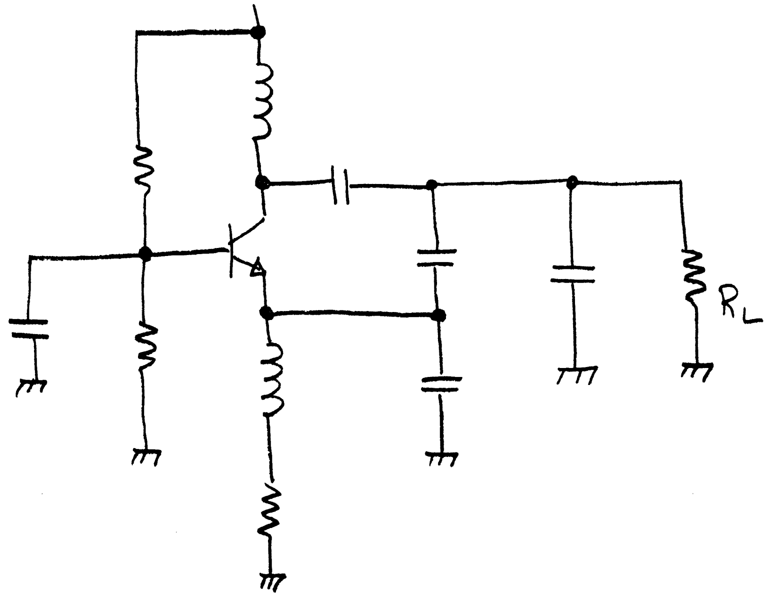
but since $\alpha \approx 1$ as long as we are below f_α

$$\frac{R_i}{R_t}\left(1+\frac{C_2}{C_1}\right) = \frac{C_2}{C_1+C_2}$$

$$\frac{R_i}{R_t}\left(\frac{C_1+C_2}{C_1}\right) = \frac{C_2}{C_1+C_2}$$

$$\frac{R_i}{R_t} = \frac{C_1 C_2}{(C_1+C_2)^2}$$ condition for oscillation

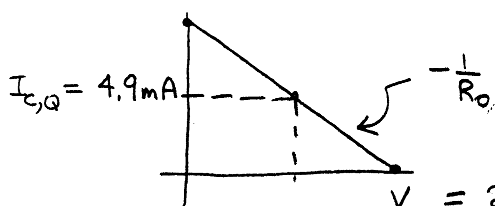
oscillator design example:



requirements are 15 mW to be delivered to $R_L = 5000\,\Omega$ @ 10 MHz

$$P_L = \frac{I_{c,Q}^2 R_L}{8}$$

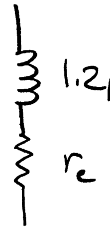
$$I_{c,Q} = \sqrt{\frac{8P_L}{R_L}} = \sqrt{\frac{8(15\times10^{-3})}{5\times10^{3}}} \approx 4.9\,\text{mA}$$



where $R_o = \frac{R_L}{2}$ (assumes matching)

$$V_{cc} = 2\,I_{c,Q} R_o = 2(4.9\,\text{mA})\left(\frac{5k}{2}\right) \approx 24.5 \text{ volts}$$

Experience allows us to use what is immediately available. In this case, we have a 1.2µH inductor with a Q of 150. (Real oscillators like high Q coils).



$$Q_L = \frac{\omega_o L}{r_c}$$

$$r_c = \frac{2\pi(10\times10^{6})(1.2\times10^{-6})}{150} = 0.503\,\Omega$$

parallel resistance

$$\approx Q_L^2 r_c = 11.3\text{k}\,\Omega$$

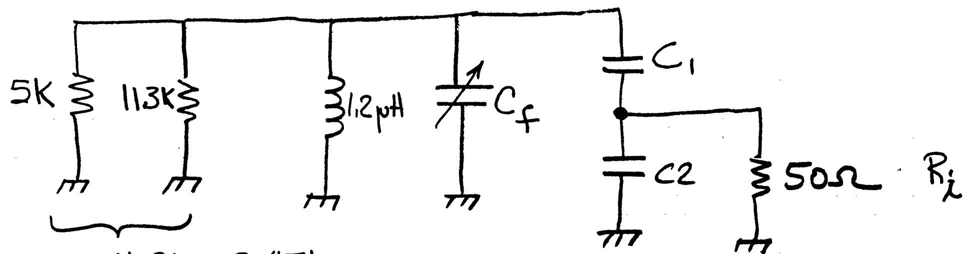
What is input impedance of transistor?

$$r_e \approx \frac{25\,mV}{I_{c,Q}} \simeq 5.1\,\Omega$$

We will arbitrarily pick $R_e = 44.9\,\Omega$ so

$$R_i = r_e + R_e = 50\,\Omega$$

Open loop circuit looks like



$5K \parallel 11.3K = 3.47k$

Note what we are doing here, the 11.3K is not being neglected and the final tank is now $\frac{3.47k}{2} = 1.74$

(Technically we need to rebias but we will include the 11.3k as part of the matching network

For matching

$$\frac{1}{R_L} = \frac{1}{5\times10^3} + \frac{1}{N^2\,50}$$

$$\frac{1}{N^2(50)} = \left(\frac{1}{5} - \frac{1}{11.3}\right)\times10^{-3} = 111.5\times10^{-6}$$

$$N^2 = 179.36$$

$$N = 13.4$$

From formulas in Ch. 3 of KBR

pick $C_{tot} = \frac{1}{\omega_0^2 L} = \frac{1}{(2\pi\times10\times10^6)^2\,(1.2\times10^{-6})} = 211\,pf$

How many capacitors are present?

C_o for the transistor

C_s is the feedback network, C_1 in series with C_2

C_f is the parallel tuning capacitor

for a 2N3866 $\quad C_o \approx 3\,pf$

pick $C_f \approx 16\,pf$ (3–30 pf variable)

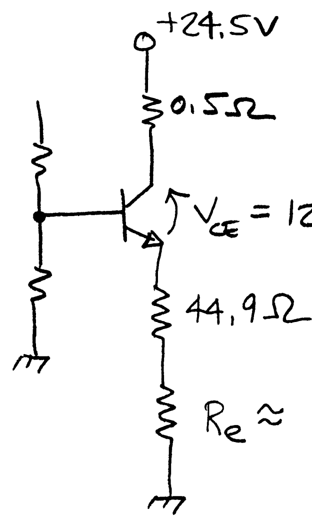
$C_s = 211 - 19 = 192\,pf$

Solving for C_1 and C_2

$$C_1 = \frac{NC_s}{N-1} = \frac{13.4}{13.4-1}(192) = 207.5\,pf$$

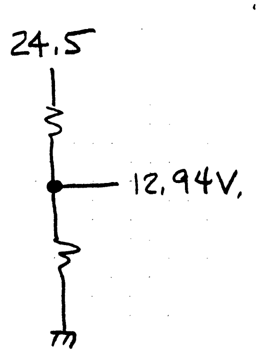
$$C_2 = NC_s = 13.4\,(192) = 2573\,pf$$

finish design with bias resistors and R_E



$$R_e \approx \frac{12.25 - 4.9\,mA(0.5 + 44.9\,\Omega)}{4.9\,mA} = 2455\,\Omega$$

$$\therefore\ V_B = (4.9\,mA)(2455 + 44.9) + 0.7 \cong 12.94 \text{ volts.}$$



$$\frac{4.9\,mA}{\beta} = \frac{4.9\,mA}{50} = 98\,\mu A$$

pick $I_d = 10\,I_b = 980\,\mu A$

$$R_1 = \frac{24.5 - 12.94}{980\,\mu A} = 11.79k$$

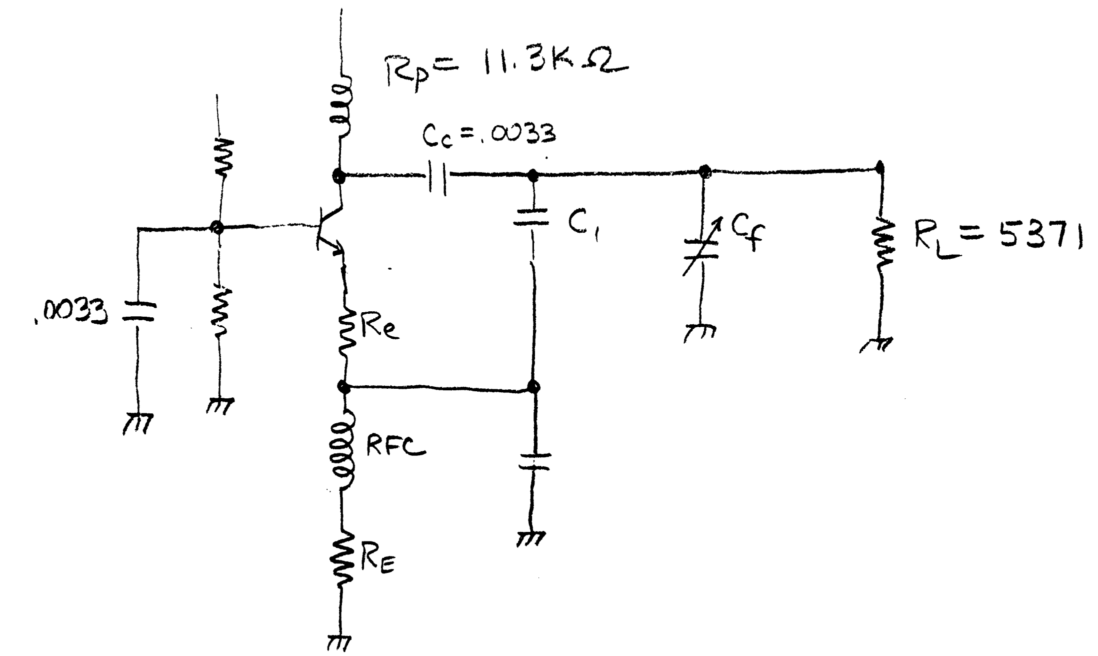
$$R_2 = \frac{12.94k}{980\,\mu A} = 13.2k$$

Pick bypass capacitors for low X_c at 10 MHz.

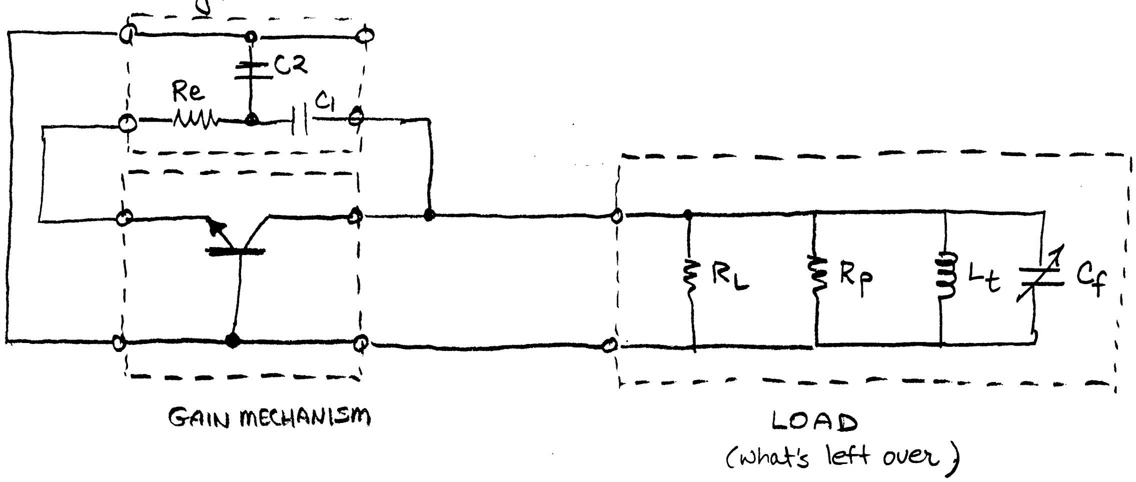
$$X_c\,(0.003\,\mu f) = \frac{1}{2\pi\,10\times10^6 \times 3\times10^{-3}\times10^{-6}} \cong 5\,\Omega$$ (towers

self resonance at least 15 MHz so OK.

Problem 5.5-2



Redraw as small-signal circuit



(a) Given the transistor parameters as:

L_i negligible.

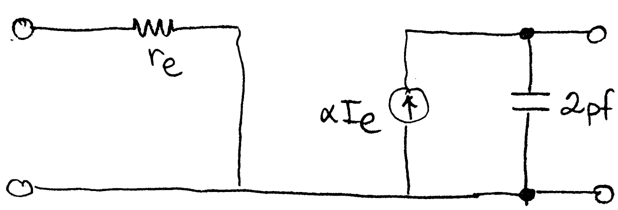
$r_e = 5.3\Omega$

$\alpha = 0.99$

$f = 10 mHz$

$C_o = 2pf.$

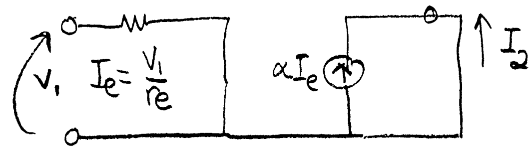
calculate the y-parameters



2

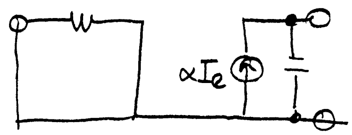
y-parameters:
$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

if $V_2 = 0$ (shorted output)
$$I_1 = y_{11} V_1 \qquad \therefore \quad y_{11} = \frac{I_1}{V_1} = \frac{V_1/re}{V_1} = \frac{1}{re}$$
$$I_2 = y_{21} V_1 \qquad y_{21} = \frac{I_2}{V_1} = \frac{-\alpha \frac{V_1}{re}}{V_1} = -\frac{\alpha}{re}$$



$$\therefore \quad I_2 = -\alpha I_e = -\alpha \frac{V_1}{re}$$
$$I_1 = \frac{V_1}{re}.$$

if $V_1 = 0$ (shorted input)



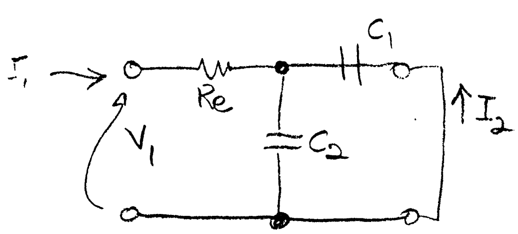
$$: \quad I_1 = y_{12} V_2 \qquad \therefore \quad y_{12} = \frac{I_1}{V_2} = 0$$
$$I_2 = y_{22} V_2 \qquad y_{22} = \frac{I_2}{V_2} = j\omega C_0.$$

$$\therefore \quad [Y_t] = \begin{bmatrix} \frac{1}{re} & 0 \\ -\frac{\alpha}{re} & j\omega C_0 \end{bmatrix}$$

Is this transistor stable?

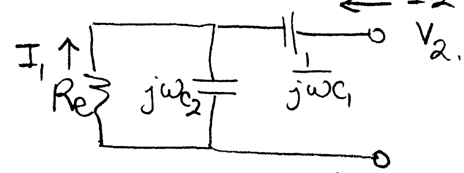
$$C = \frac{|y_{12} y_{21}|}{2 g_{11} g_{22} - Re(y_{12} y_{21})} = \frac{0}{0.}$$

(b) Calculate the y-parameters of the feedback network.



$$y_{11} = \frac{I_1}{V_1} = \frac{I_1}{\left(Re + \frac{1}{j\omega C_1} \| \frac{1}{j\omega C_2}\right) I_1} = \frac{1}{Re + \frac{\frac{1}{j\omega C_1} \frac{1}{j\omega C}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C}}}$$
$$= \frac{1}{Re + \frac{\left(\frac{1}{j\omega C_1 C_2}\right) C_1 C_2}{\left(\frac{1}{C_1} + \frac{1}{C_2}\right) C_1 C_2}} = \frac{1}{Re + \frac{\frac{1}{j\omega}}{(C_2 + C_1)}}$$
$$= \frac{1}{Re + \frac{1}{j\omega (C_1 + C_2)}} \quad \checkmark$$

$$y_{21} = \frac{I_2}{V_1} = \frac{-\overbrace{(V_1 y_{11})}^{I_1}}{V_1}\;\overbrace{\frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)}}^{\text{current divider}} = -\,y_{11}\;\frac{\frac{1}{C_2}\,(C_1C_2)}{\left(\frac{1}{C_1}+\frac{1}{C_2}\right)(C_1C_2)} = -\underbrace{\left(\frac{C_1}{C_1+C_2}\right)y_{11}}_{\text{different}}$$



$$y_{22} = \frac{I_2}{V_2} = \frac{1}{\frac{1}{j\omega C_1} + \left(\frac{R_e \frac{1}{j\omega C_2}}{R_e + \frac{1}{j\omega C_2}}\right)\frac{j\omega C_2}{j\omega C_2}} = \frac{1}{\frac{1}{j\omega C_1} + \frac{R_e}{1 + j\omega C_2 R_e}}$$

$$= \frac{1}{\frac{1 + j\omega C_2 R_e + j\omega C_1 R_e}{(j\omega R_e C_1)(1 + j\omega C_2 R_e)}} = \frac{(j\omega R_e C_1)(1 + j\omega C_2 R_e)}{1 + j\omega R_e (C_1 + C_2)}$$

$$y_{12} = \frac{I_1}{V_2} = \frac{-(V_2 y_{22})}{V_2}\;\frac{\frac{1}{j\omega C_2}}{R_e + \frac{1}{j\omega C_2}} = -\,y_{22}\;\frac{1}{1 + j\omega R_e C_2}$$

$$Y_f = \begin{bmatrix} \dfrac{1}{R_e + \frac{1}{j\omega (C_1+C_2)}} & -y_{22}\,\dfrac{1}{1+j\omega R_e C_2} \\[2ex] -\left(\dfrac{C_1}{C_1+C_2}\right) y_{11} & \dfrac{j\omega C_1 (1 + j\omega C_2 R_e)}{1 + j\omega R_e (C_1 + C_2)} \end{bmatrix}$$

Numerically. $R_e = 44\,\Omega$ $\quad \omega = 2\pi \times 10 \times 10^6 = 6.28 \times 10^7$ $\quad C_1 = 218\,\text{pf}$, $C_2 = 2842\,\text{pf}$

$$y_{11} = \frac{1}{R_e + \frac{1}{j\omega (C_1 + C_2)}} = \frac{1}{44 - j\frac{1}{(6.28\times 10^7)(218 + 2842)\times 10^{-12}}}$$

$$y_{11} = \frac{1}{44 - j\,5.20} = (22.4 + j\,2.65)\ \text{m}\mho$$

$$y_{21} = -\,y_{11}\left(\frac{C_1}{C_1 + C_2}\right) = -y_{11}\left(\frac{218}{218 + 2842}\right) = -0.0712\,y_{11} = -1.59 - j\,.188$$

$$y_{22} = \frac{1}{\frac{1}{j\omega C_1} + R_e \| \frac{1}{j\omega C_2}} = \frac{1}{\frac{1}{j\omega C_1} + \frac{R_e \frac{1}{j\omega C_2}}{R_e + \frac{1}{j\omega C_2}}} = \frac{1}{\frac{1}{j\omega C_1} + \frac{R_e}{1 + j\omega C_2 R}}$$

$$y_{22} = \frac{(j\omega C_1)(1+j\omega C_2 R_e)}{(1+j\omega C_2 R_e) + j\omega C_1 R_e}$$ 4

$$y_{22} = \frac{j\omega\; C_1(1+j\omega C_2 R_e)}{1+j\omega\, R_e\,(C_1+C_2)} = \frac{j\omega C_1 - \omega^2 C_1 C_2 R_e}{1+j\omega\, R_e(C_1+C_2)}$$

$$= \frac{-(6.28\times10^7)^2(218\times10^{-12})(2842\times10^{-12})\,44 \;+\; j\,(6.28\times10^7)(218\times10^{-12}}{1+j(6.28\times10^7)(44)(2842+218)\times10^{-12}}$$

$$\begin{array}{r} 2842 \\ 218 \\ \hline 3060 \end{array}$$

$$= \frac{-.1075 + j.0137}{1+j8.455} = 0.115 + j12.72 \quad \text{millimhos}$$

$$y_{12} = -\,y_{22}\big/_{1+j\omega R_e C_2} = -\frac{0.115+j12.72\ \ m\mho}{1+j(6.28\times10^7)(44)(2842\times10^{-12})}$$

$$= -\frac{(0.115+j12.72)\,m\mho}{1+j7.85} = -1.596 - j.189$$

(c) Composite parameters

in $m\mho$ in $m\mho$

$$[Y_c] = [Y_t] + [Y_f] = \begin{bmatrix} 188.7 & 0 \\ -186.8 & j.125 \end{bmatrix} + \begin{bmatrix} 22.4+j2.65 & -1.596-j.18 \\ -1.59-j.188 & 0.115+j12.7 \end{bmatrix}$$

$$= \begin{bmatrix} 211.4+j2.65 & -1.596-j.189 \\ -188.4-j.188 & 0.115+j12.85 \end{bmatrix}$$

Now what is C.

$$C = \frac{|y_{12}\,y_{21}|}{2g_{11}g_{22} - Re(y_{12}y_{21})}$$

$y_{12}y_{21} = 300.65 + j35.90\times10^{-6}$

$Re(y_{12}y_{21}) - 300.65\times10^{-6}$

$|y_{12}y_{21}| = 302.8\times10^{-6}$

$$= \frac{302.8\times10^{-6}}{2(181.4\times10^{-3})(.115\times10^{-3}) - 300.65\times10^{-6}}$$

$$= \frac{302.8\times10^{-6}}{41.72\times10^{-6} - 300.65\times10^{-6}} = -\frac{302.0}{258.9} \simeq -1.17$$

potentially unstable.

(d) Our load from page (1) is given by

$$G_L = \frac{1}{R_L} + \frac{1}{R_p} = \frac{1}{5371} + \frac{1}{11300}$$

$$= 0.186 + .0885 \cong 0.275 \text{ m}\mho$$

$$K = \frac{2(g_i + G_s)(g_o + G_L)}{|y_f y_r| + \mathrm{Re}\,|y_f y_r|}$$

from before $|y_f y_r| = 302.8 \times 10^{-6}$

$\mathrm{Re}\,|y_f y_r| = 300.65 \times 10^{-6}$

no input

$$K = \frac{2(211.4 + 0)(0.115 + .275) \times 10^{-6}}{(302.8 \times 10^{-6}) + (300.65 \times 10^{-6})}$$

$$= \frac{2(211.4)(.390)}{302.8 + 300.65} = \frac{164.9}{603.45} = 0.273$$

potentially unstable!

5-5.3

Colpitts oscillator design

$P_L = 30\,mW$

$R_L = 4000$ @ 10 mHz.

$L = 1.5\,\mu H \quad Q_c = 160 \Longrightarrow R_p = Q_c \omega_o L \cong 5280\,\Omega$

transistor: $\boxed{\begin{array}{l}\beta = 50 \\ C_o = 4\,pf\end{array}}$

Design

What is the rms output voltage?

$$P_o = \frac{V_o^2}{R_L} \quad \therefore \quad V_o = \sqrt{P_o R_L} = \sqrt{(30\times10^{-3})(4\times10^{3})}$$

$$= 10.95 \text{ volts.}$$

$$(V_o)_{peak} \cong 15.5 \text{ volts.}$$

We can use the basic equation for gain in a very interesting way:

Eqn 5-12 of the text (the gain equation for an oscillator)

$$g_i - \omega^2 L_t (C_b g_i + C_a g_t - C_1 \alpha g_i) = 0$$

$$\omega^2 L_t C_1 \alpha g_i = \omega^2 L_t (C_b g_i + C_a g_t) - g_i$$

$$\alpha = \frac{\omega^2 L_t C_b g_i + \omega^2 L_t C_a g_t - g_i}{\omega^2 L_t C_1 g_i}$$

$$= \frac{C_b}{C_1} + \frac{g_t C_a}{g_i C_1} - \frac{1}{\omega^2 L_t C_1}$$

$$\alpha = \frac{C_1 + C_o + C_f}{C_1} + \frac{R_i}{R_t}\left(\frac{C_1 + C_2}{C_1}\right) - \frac{1}{\omega^2 L_t C_1}$$

$$\therefore \alpha = 1 + \frac{C_o + C_f}{C_1} + \frac{R_i}{R_t}\left(1 + \frac{C_2}{C_1}\right) - \frac{1}{\omega^2 L_t C_1}$$

if $\quad \omega_0^2 \approx \dfrac{1}{L_t\left[C_f + C_o + \dfrac{C_1 C_2}{C_1 + C_2}\right]}$

$$\frac{1}{\omega_0^2 L_t C_1} = \frac{C_f + C_o + \dfrac{C_1C_2}{C_1+C_2}}{C_1} = \frac{C_f + C_o}{C_1} + \frac{C_2}{C_1 + C_2}$$

$$\therefore \quad \alpha = 1 + \cancel{\frac{C_o + C_f}{C_1}} + \frac{R_i}{R_t}\left(1 + \frac{C_2}{C_1}\right) - \cancel{\frac{C_f + C_o}{C_1}} - \frac{C_2}{C_1 + C_2}$$

$$= 1 - \frac{C_2}{C_1 + C_2} + \frac{R_i}{R_t}\left(1 + \frac{C_2}{C_1}\right) = \frac{1}{1 + \frac{C_2}{C_1}} + \frac{R_i}{R_t}\left(1 + \frac{C_2}{C_1}\right)$$

This form is interesting because $1 + \frac{C_2}{C_1} = \frac{C_1 + C_2}{C_1}$ which is

$$\frac{1}{N} = \frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} = \frac{\frac{1}{C_2} \; C_1C_2}{\left(\frac{1}{C_1} + \frac{1}{C_2}\right) C_1C_2} = \frac{C_1}{C_1 + C_2}$$ The divider step-up ratio

This gives up

$$\alpha = \frac{1}{N} + \frac{R_i}{R_t}(N)$$

or $\quad N\alpha = 1 + \dfrac{R_i}{R_t} N^2$

$$N^2 - \frac{R_t}{R_i}\alpha N + \frac{R_t}{R_i} = 0$$

R_t is the tank resistance $= R_L \| R_p = (4000)\|(5280) = 2276$

this is a design decision $\rightarrow$ $R_i = R_e + r_e$

circuit (R_e) device (r_e)

$r_e \approx h_{ib}$ for this transistor which is less than 10Ω for most transist.

This creates large problems for the feedback network as it phase shifts the feedback voltag if $R_i \lesssim X_{C_2}$ (say 10–20 Ω).

So we usually want Re large enough that it doesn't load the network. Good values are, say, $R_i \geq 100\,\Omega$.

$$\therefore\ N^2 - \left(\frac{2276}{100}\right)\alpha N + \frac{2276}{100} = 0.$$

since $\alpha \approx 1$ for a transistor (actually $\alpha = \frac{\beta}{\beta+1} = \frac{50}{50+1} = 0.98$)

$$N^2 - 22.31\,N + 22.76 = 0$$

$$N = \frac{+22.31 \pm \sqrt{497.9 - 91.04}}{2} = \frac{22.31 \pm \sqrt{406.86}}{2}$$

$$= \frac{22.31 \pm 20.17}{2} = \frac{42.48}{2},\ \frac{2.14}{2} = 21.24,\ 1.07$$

this really says that

$$1.07 \leq N \leq 21.24$$

for maximum power transfer

$$N = \sqrt{\frac{(5.371)(11.3)}{(100)(11.3 - 5.371)}} = \sqrt{102.4}$$

$N = 10.1$

In this range we have enough feed back to oscillate. Select $N = 20$ since this will give us a high oscillator Q.

Now, re-evaluate for closed loop.

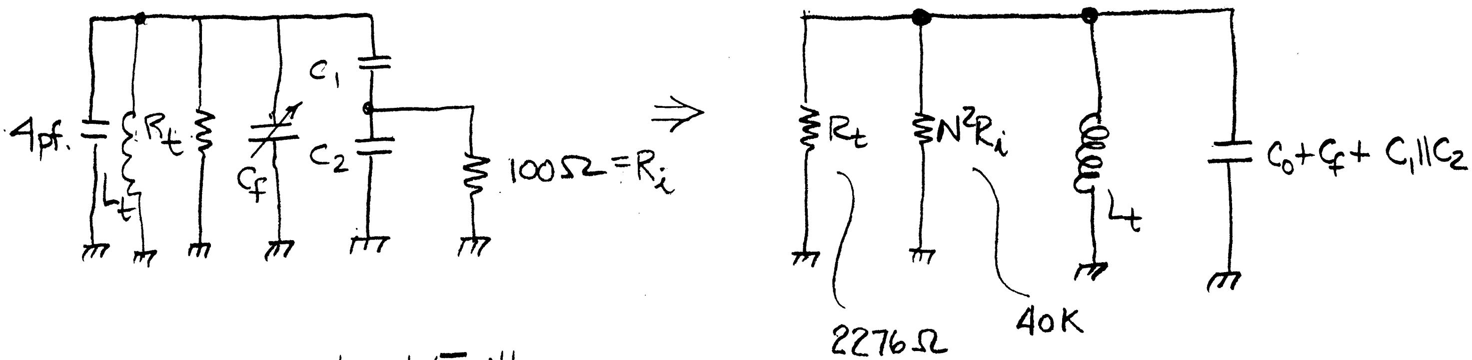
$$R_L = R_p \parallel N^2 R_i$$

You see, without the information on N, we cannot complete the amplifier loading.

$$N^2 R_i = (20)^2 (100\,\Omega) = 40K.$$

$$R_L = \frac{(2.276K)(40K)}{(2.276 + 40)K} = \frac{(2.276)(40)K}{42.276} = 2.15K.$$

Note that the effects of loading are almost negligible, but the effect of N upon oscillator gain is crucial but not critical. As long as $1.07 \leq N \leq 21.24$ the circuit will work.



$L = 1.5\ \mu H$

Neglecting loading of the resonance frequency

$$\omega_o^2 = \frac{1}{L_t\left[C_o + C_f + C_1 \| C_2\right]}$$

$$\therefore\ C_o + C_f + C_1\|C_2 = \frac{1}{\omega_o^2 L_t} = \frac{1}{(2\pi \times 10 \times 10^6)^2 (1.5\times 10^{-6})} \doteq \frac{1}{5.92 \times 1C}$$

$$= 168.8\ pf.$$

$C_o = 4\,pf$ so $\quad C_f + C_1\|C_2 = (168.8 - 4) = 164.4\,pf.$

We also know that

$$\frac{C_1}{C_1 + C_2} = \frac{1}{N} = \frac{1}{20} \quad \therefore \quad 20C_1 = C_1 + C_2$$

$$19C_1 = C_2.$$

$$C_1 \| C_2 = \frac{C_1 C_2}{C_1 + C_2} = \frac{C_1(19C_1)}{C_1 + 19C_1} = \frac{19C_1^2}{20C_1} = \frac{19}{20}C_1$$

$$\therefore\ C_f + \frac{19}{20}C_1 = 164.4\,pf.$$

We will pick C_f to be a 3-30pf trimmer cap and adjust it to mid-value, i.e. 15 pf.

$$\therefore\ \frac{19}{20}C_1 = 164.4 - 15\,pf = 149.4\,pf$$

$$C_1 = \frac{20}{19}(149.4\,pf) \cong 157\,pf. \qquad C_2 = 2988\,pf.$$

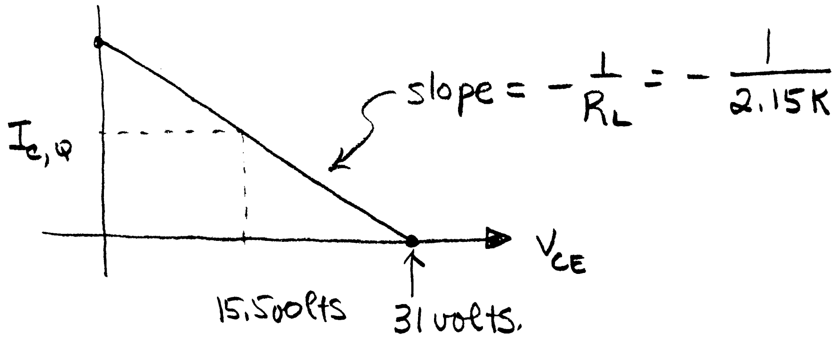
Do our values satisify the neglect of loading?

is $L_t \ll R_t R_i (C_1 + C_2) = (2.15\times 10^3)(100)(157 + 2988)\times 10^{-12} = 676\ \mu H.$

$\therefore$ loading of ω_o can be neglected.

To this point we have calculated everything but the resistor values.
Recall $R_i = 100\Omega$. So neglecting r_e we pick $R_e = 100\Omega$.

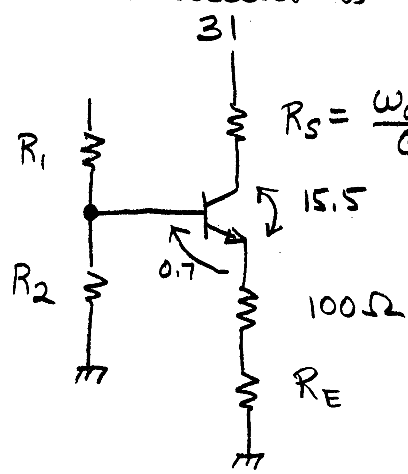
Let's get the collector current value



By inspection $\quad 2I_{c,Q} = \frac{1}{2.15K}\, 31.0$ volts.

$$I_{c,Q} = \frac{15.5}{(2.15)} \text{ mA} = 7.20 \text{ mA}$$

The d.c. collector circuit is



$$R_S = \frac{\omega_o L}{Q} = \frac{(6.28\times10^7)(1.5\times10^{-6})}{160} = 0.59\,\Omega.$$

$$\therefore R = \frac{15.5 \text{ volts}}{7.20 \text{ mA}} = 2.15K.$$

$$\therefore R_E = 2153 - 0.59 - 100 \cong 2.2K$$

$$V_E = (2.2K)(7.20\text{ mA}) = 15.84 \text{ volts}$$
$$V_B = 15.84 + 0.7 = 16.54 \text{ volts}$$

using $\quad i_B = \frac{7.20\text{mA}}{50} = 144\mu A$

$i_D = 10\, i_B = 1.440$ mA $\quad$ Then $\quad R_2 = \frac{16.54 \text{ volts}}{1.44 \text{ mA}} = 11.48K.$

$\therefore$ choose $R_2 = 10K$

$$R_1 = \frac{31.0 - 16.54}{1.44} = \frac{14.46}{1.44} \cong 10K$$

$\therefore$ choose $R_1 = 10K$.

The power dissipated in the transistor is then

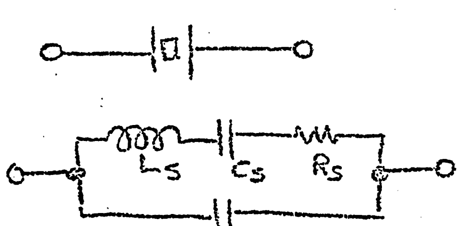
$$P_{DC} = (15.5 \text{ volts})(3.60 \text{ mA}) = 55.8 \text{ mW}$$

$$P_{AC} = \frac{\Delta I_C \, \Delta V_{CE}}{2} = \frac{(7.2 \text{ mA})(15.5 \text{ v.})}{2} = 55.8 \text{ mW}$$

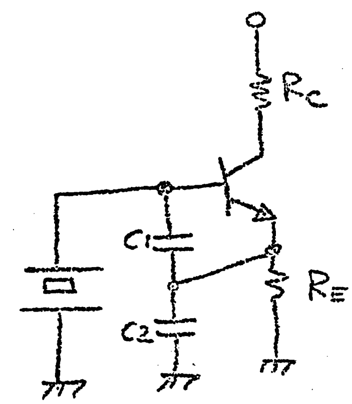
$\therefore$ We need a transistor capable of dissipating at least 120 mW.

Example #2

It is desired to analyze a crystal controlled oscillator. A crystal is a very high Q (~100,000) mechanical resonant circuit with the electrical equivalent circuit shown below.



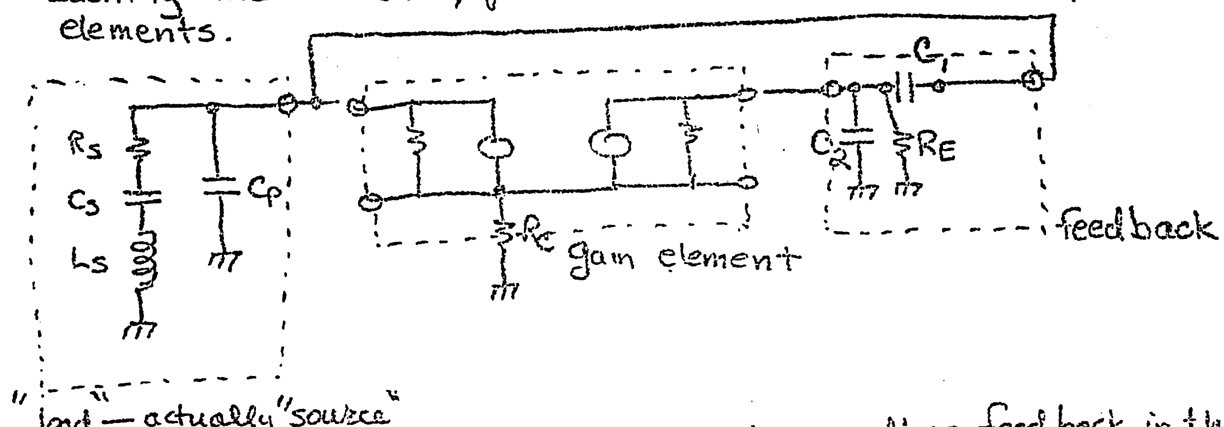
$L_s = 100\ \mu H$
$R_s = 20\ \Omega$
$C_s = 2.53\ pf$
$C_p = 5\ pf$



For the common collector shown at the right;

$R_C = 470\ \Omega$
$R_E = 1000\ \Omega$
$C_1 = 470\ pf$
$C2 = 1000\ pf$

(a) Draw the small signal equivalent circuit. Identify the feedback, gain and load elements.

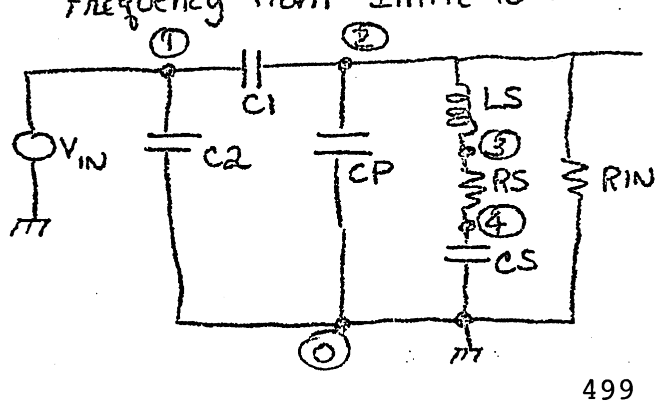


(b) Discuss why one should analyze current or voltage feedback in this circuit.

CURRENT feedback. | For CC amplifier input and output voltage are in phase
IMPOSSIBLE TO HAVE VOLTAGE GAIN FOR A CC AMPLIFIER | input and output current out of phase

To satisfy Barkhausen criteria want in-phase feedback.

(c) Write a SPICE program to analyze the feedback — "load" circuit. You should print and/or plot the feedback gain and phase as a function of frequency from 1MHz to 20MHz, Assume RIN for the transistor is 15k.




```
TITLE   FEEDBACK NETWORK
RIN  2  Ø   15K
RS   3  4   2Ø
C1   1  2   47ØPF
C2   1  Ø   1ØØØ PF
CP   2  Ø   5 PF
CS   4  Ø   2.53 PF
LS   2  3   100 UH
*END OF COMPONENTS
VIN  1  Ø   AC  1,Ø
.AC DEC 10  1MEG  20MEG
.PRINT IM(2,Ø), IP(2,Ø)
```