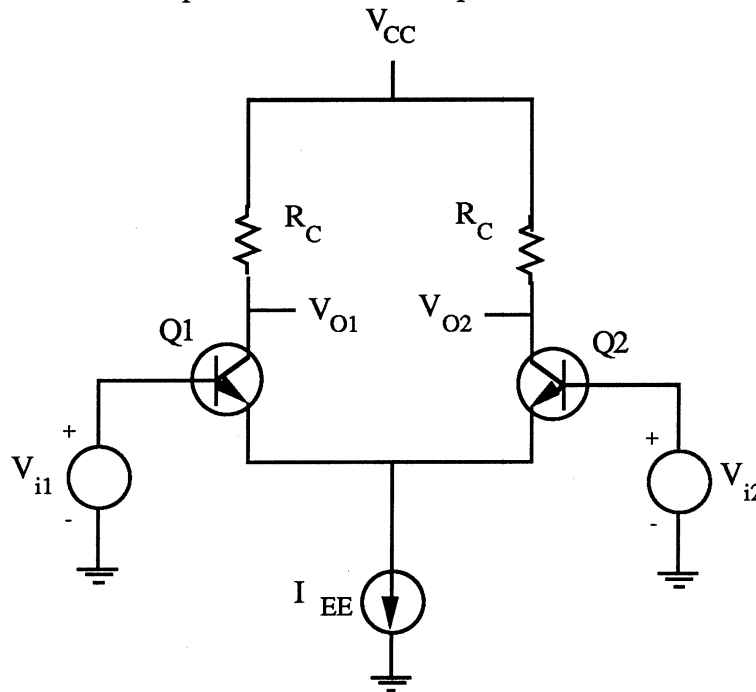


## DIFFERENTIAL AMPLIFIER MIXER SUPPLEMENT

The differential amplifier to be used in Lab #6 functions as a mixer because it essentially multiplies the RF and LO input voltages together. To provide you with some guidelines for mixer the following document develops a simple mixer model for Lab #6 and suggests some values for your mixer circuit.

To begin with consider the simplified differential amplifier circuit shown below.



Several important assumptions about this circuit must be made for a simple analysis to yield meaningful results. First, the emitter current source which produces  $I_{EE}$  is assumed to have infinite parallel resistance, i.e. the emitter current source is a perfect current source. Second, the output impedances of Q1 and Q2 are also assumed to be infinite, i.e. to be perfect current sources. Finally, the base resistances of Q1 and Q2 are assumed to be negligible. These assumptions will yield a model that can roughly predict the mixer performance but is not adequate to accurately predict small-signal mixer performance.

The first step of the analysis is to apply Kirchoff's Voltage Law to the base loop formed by transistors Q1 and Q2. Specifically,

$$V_{i1} - V_{BE1} + V_{BE2} - V_{i2} = 0 \quad (1)$$

We can use the Ebers-Moll equations (this is what most of the above assumptions affect) to write a general expression for the emitter current of Q1 and Q2.

$$I_C = \alpha_F I_{ES} \left( e^{\frac{V_{BE}}{V_T}} - 1 \right) - I_{CS} \left( e^{\frac{V_{BC}}{V_T}} - 1 \right) \quad (2)$$

where  $I_C$  is the collector current,  $\alpha_F$  is the common base current gain,  $V_{BE}$  is the base emitter voltage,  $V_{BC}$  is the base collector voltage,  $I_{ES}$  is the emitter saturation current and  $I_{CS}$  is the collector saturation current. Basically, the first term represents the diode equation for the forward biased base-emitter junction and the second term represents the diode equation from the reverse base-collector junction. The second term represents a reverse biased diode and is very small; consequently, we will ignore it in the following calculations. The common base current gain  $\alpha_F$  is related to the more normally used common emitter current gain  $\beta$  by

$$\alpha = \frac{\beta}{\beta+1} \quad (3)$$

Neglecting the current contribution from the reverse biased junction and assuming that the exponential is much greater than one we can re-write Equation (2) as

$$I_C \approx \alpha_F I_{ES} e^{\left(\frac{V_{BE}}{V_T}\right)} \quad (4)$$

Equation (4) can be solved for  $V_{BE}$  to give

$$V_{BE} \approx V_T \ln\left(\frac{I_C}{\alpha_F I_{ES}}\right) \quad (5)$$

The term  $\alpha_F I_{ES}$  is effectively a constant saturation current which we can define as  $I_S$ . Using this definition we can write Equation (5) as

$$V_{BE} \approx V_T \ln\left(\frac{I_C}{I_S}\right) \quad (6)$$

The two base-emitter voltages can then be written as

$$V_{BE1} \approx V_T \ln\left(\frac{I_{C1}}{I_{S1}}\right) \quad (7a)$$

and

$$V_{BE2} \approx V_T \ln\left(\frac{I_{C2}}{I_{S2}}\right) \quad (7b)$$

Substituting Equation (7) into Equation (1) we get

$$V_{i1} - V_T \ln\left(\frac{I_{C1}}{I_{S1}}\right) + V_T \ln\left(\frac{I_{C2}}{I_{S2}}\right) - V_{i2} = 0 \quad (8)$$

If we assume that  $I_{S1}=I_{S2}=I_S$  and solve for the differential voltage  $V_{i1}-V_{i2}$  we get

$$V_{i1} - V_{i2} = V_T \ln\left(\frac{I_{C1}}{I_S}\right) - V_T \ln\left(\frac{I_{C2}}{I_S}\right) = V_T \ln\left(\frac{I_{C1}}{I_{C2}}\right) \quad (10)$$

which can be inverted to give

$$\frac{I_{C1}}{I_{C2}} = e^{\frac{V_{i1}-V_{i2}}{V_T}} \equiv e^{\frac{\Delta V_i}{V_T}} \quad (11)$$

where  $\Delta V_i$  is  $V_{i1}-V_{i2}$ .

The sum of the emitter currents of Q1 and Q2 must be equal to the bias current  $I_{EE}$ , i.e.

$$I_{EE} = -(I_{E1}+I_{E2}) = \frac{1}{\alpha_F}(I_{C1}+I_{C2}) \quad (12)$$

Equations (11) and (12) may be combined to give

$$I_{C1} = \frac{\alpha_F I_{EE}}{1 + e^{\frac{-\Delta V_i}{V_T}}} \quad (13a)$$

and

$$I_{C2} = \frac{\alpha_F I_{EE}}{1 + e^{\frac{\Delta V_i}{V_T}}} \quad (13b)$$

These equations are the fundamental differential amplifier mixer equations. We can make two observations about Equation (13). First, that the collector currents saturate to  $\alpha_F I_{EE}$  as a function of the differential input voltage; second, that making  $I_{EE}$  larger will make  $I_C$  larger.

The output voltage is  $V_{CC}$  less the collector current times the collector load resistance, i.e.

$$V_{O2} = V_{CC} - I_{C2} R_C = V_{CC} - \frac{\alpha_F I_{EE}}{1 + e^{\frac{-\Delta V_i}{V_T}}} R_C \quad (14)$$

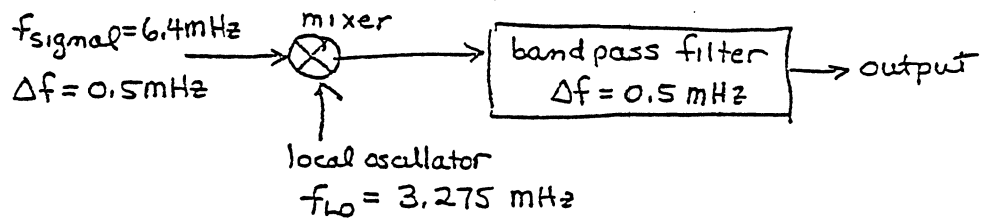
The ac output voltage  $\Delta V_{O2}$  is then, approximately,

$$\Delta V_{O2} \approx -\frac{\alpha_F I_{EE} R_C}{1 + e^{\frac{-\Delta V_i}{V_T}}} = -\frac{\alpha_F I_{EE} R_C}{1 + \left(1 - \frac{\Delta V_i}{V_T}\right)} = -\frac{\alpha_F I_{EE} R_C}{2 \left(1 - \frac{\Delta V_i}{2V_T}\right)} = -\frac{\alpha_F I_{EE} R_C}{2} \left(1 + \frac{\Delta V_i}{2V_T}\right) \quad (15)$$

Note that the first term contains only  $I_{EE}$  whereas the second term contains the product term  $I_{EE} \Delta V_i$  which is the desired mixer term. Examination of the second term reveals that the mixer term is proportional to  $I_{EE}$ ,  $R_C$  and  $\Delta V_i$ . Unfortunately, the first term is also proportional to  $I_{EE}$  and  $R_C$  and making these terms too large will lead to saturation of the mixer output by the unwanted first term. In practice, optimizing your mixer output will require a careful adjustment of the bias current and load resistance.

The current  $I_{EE}$  is actually an ac current source controlled by the local oscillator. A mixer design I tested used a relatively small nominal  $I_{EE}$  of about 1mA and collector resistances of 1k $\Omega$ . [Actually, one collector load resistance was 1k $\Omega$  and the other was a 42IF104 i.f. transformer.] The local oscillator voltage was approximately 500 mV and the rf input voltage was estimated to be about 100 $\mu$ V. The voltage out of the i.f. amplifier was on the order of one volt.

Consider the mixer-local oscillator system shown below



(a) what are the 2,1 and 1,2 spurious frequencies in the mixer output

1,2 spurs  $2(3.275) \pm 6.4 = 6.55 \pm 6.4 = 12.95, 0.15$

2,1 spurs  $2(6.4) \pm 3.275 = 12.8 \pm 3.275 = 9.525, 16.075$

(b) should I choose the i.f. filter (bandwidth  $\Delta f = 0.5 \text{ MHz}$ ) to be centered at the sum or difference frequency? Why?

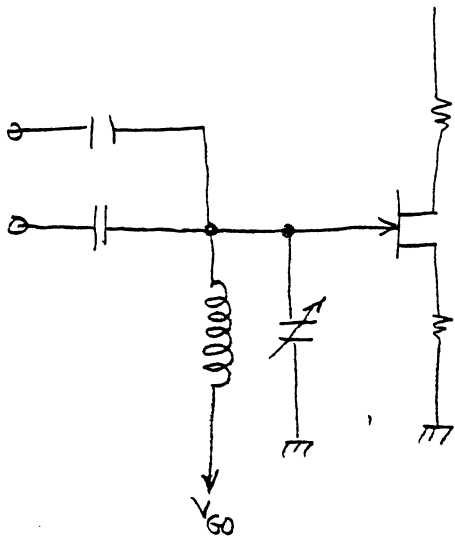
sum =  $6.4 + 3.275 = 9.675$

difference =  $6.4 - 3.275 = 3.125$

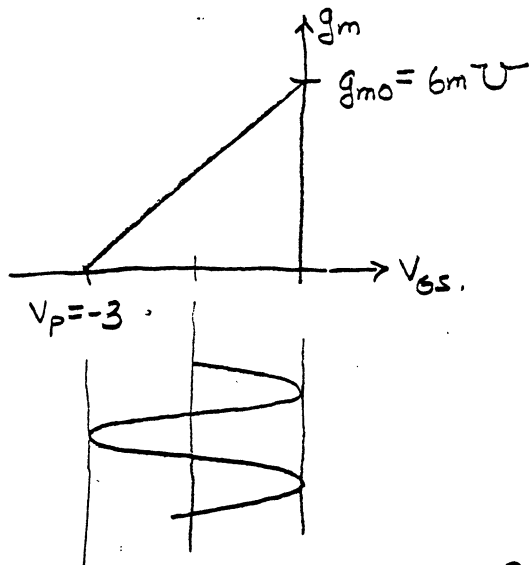
Cannot choose sum because it overlaps the 2,1 spur at 9.525.  
 Therefore choose the difference, but 3.125 is within 0.5 MHz of  $f_{\text{LO}} = 3.275$ .

$\therefore$  choose sum because we want to maximize rejection of the strongest signal, i.e. that at  $f_{\text{LO}}$ .

7.6-1



(a)



$$g_c = \frac{I_{IF}}{V_{RF}} = \frac{g_{m0} V_{Lo}}{2|V_p|}$$

for a sinusoidal L.O.  
and maximum voltage swing  
we pick  $V_{Lo} = 1.5$  volts peak.  
 $= 1.06$  volts rms.

$$g_c = \frac{(6 \times 10^{-3})(1.5)}{2(3)} = 1.5 \text{ mS}$$

(b) for  $V_{RF} = 1$  volt rms, what is difference-frequency current,

simple method:

$$I_{IF} = g_c V_{RF} = (1.5 \times 10^{-3})(1 \text{ volt rms}) = 1.5 \text{ mA. } \underline{\underline{\text{RMS}}}$$

providing  $V_{Lo}$  does not change to keep  $0 < V_{gs} < 3$  volts.

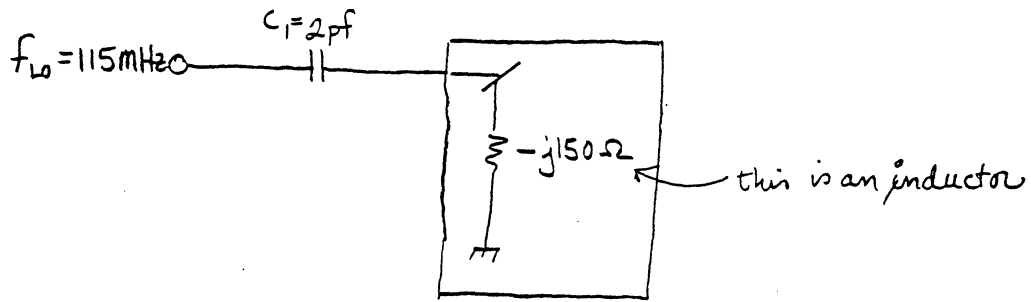
If this is not so then 1 volt rms = 1.4 volts peak

$$V_{Lo} \cong 0.1 \text{ volts peak.}$$

$$g_c' = \frac{(6 \times 10^{-3})(0.1)}{2(3)} = 0.1 \text{ mS.}$$

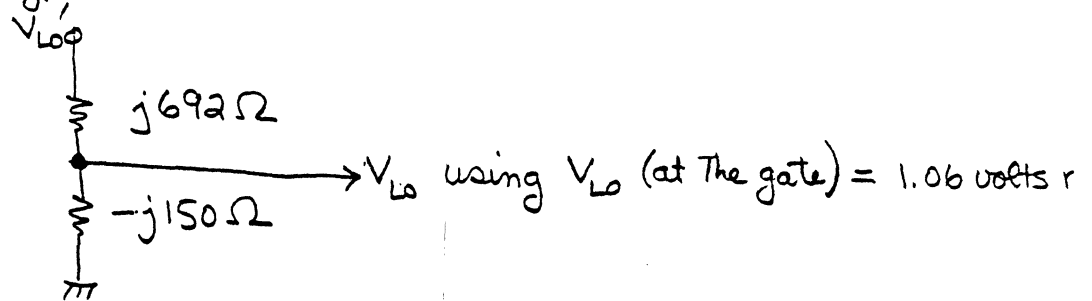
$$I_{IF} = g_c' V_{RF} = (0.1 \times 10^{-3})(1) \cong 0.1 \text{ mA}$$

(c)



$$\text{at } 115 \text{ MHz} \quad X_C = \frac{1}{2\pi(115 \times 10^6)(2 \times 10^{-12})} = \frac{1}{1.445 \times 10^{-3}} = 692 \Omega$$

voltage divider



$$V_{Lo} = \frac{-j150}{j692 - j150} V'_{Lo} = \frac{-j150}{+j542} V'_{Lo} = -0.277 V'_{Lo}$$

neglecting the phase factor:  $V'_{Lo} = \frac{1}{0.277} V_{Lo} = 3.6 V_{Lo}$

$$\therefore V'_{Lo} = 3.83 \text{ volts rms.}$$

The total capacitance needed is then 43.9 pf.

The total capacitance in the RLC tank is then

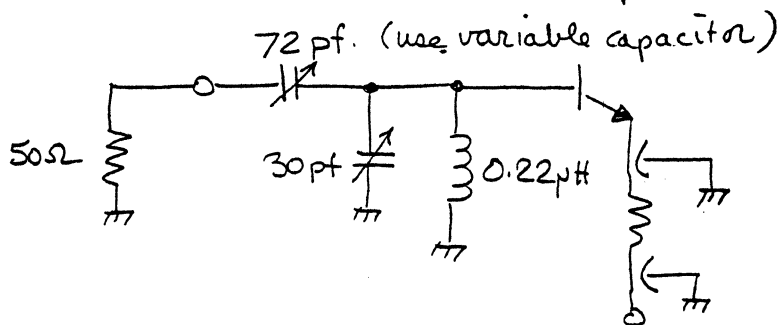
$$49.3 \text{ pf} + 50.5 \text{ pf} = 99.8 \text{ pf}.$$

To resonate

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2\pi \times 80 \times 10^6)^2 (99.8 \times 10^{-12})} = 0.28 \text{ pH}.$$

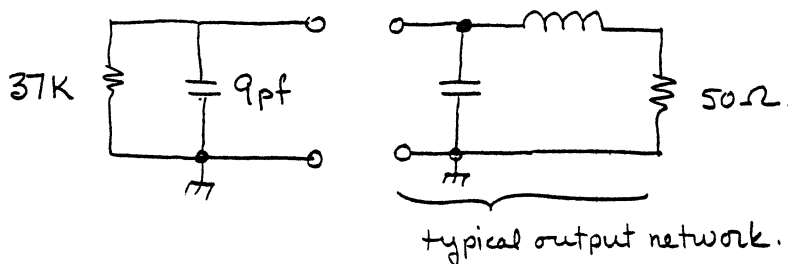
This is a small, but reasonable value.

For a practical circuit we pick 0.22 pH as the nearest standard L value and add a 30 pf variable capacitor to tune its reactance



Output circuit

$$(Y_{22})_e = (0.027 + j0.28) \times 10^{-3} \text{ U}$$



for RLIC circuit  $Q_t^2 = \frac{R_t}{R} - 1 = \frac{37K}{50} - 1 = 739 \quad \therefore Q_t = 27.2.$

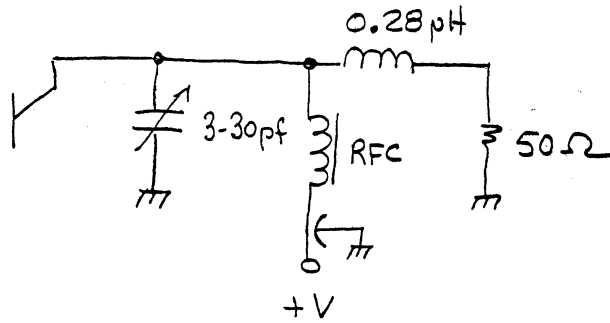
$$Q_t = \frac{\omega_0 L}{R} \text{ or } L = \frac{R Q_t}{\omega_0} = \frac{(50)(27.2)}{31.45 \times 10^6} = 43.3 \text{ pH}$$

convert back to series circuit to get desired C.

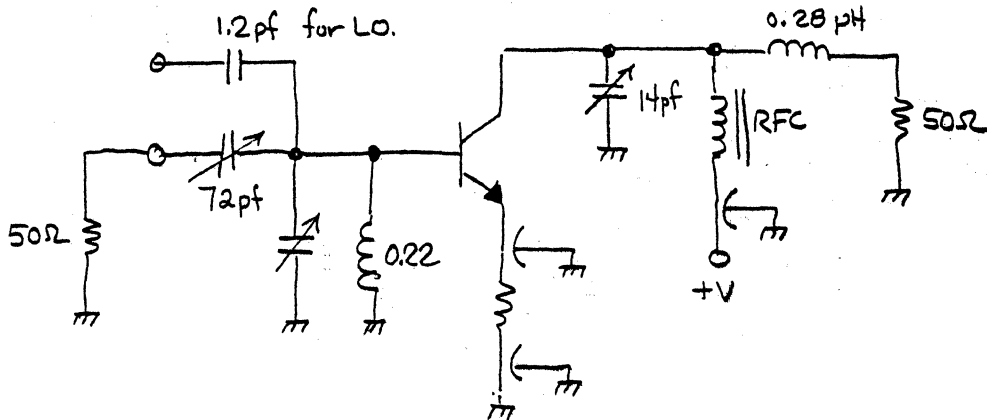
series to parallel conversion  $L_p = \left(\frac{Q_t^2 + 1}{Q_t^2}\right) L_s = \frac{739 + 1}{739} L_s \approx L_s = 43.3 \text{ pH}$

to resonate:  $C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi \times 5 \times 10^6)^2 (43.3 \times 10^{-6})} = 23.4$

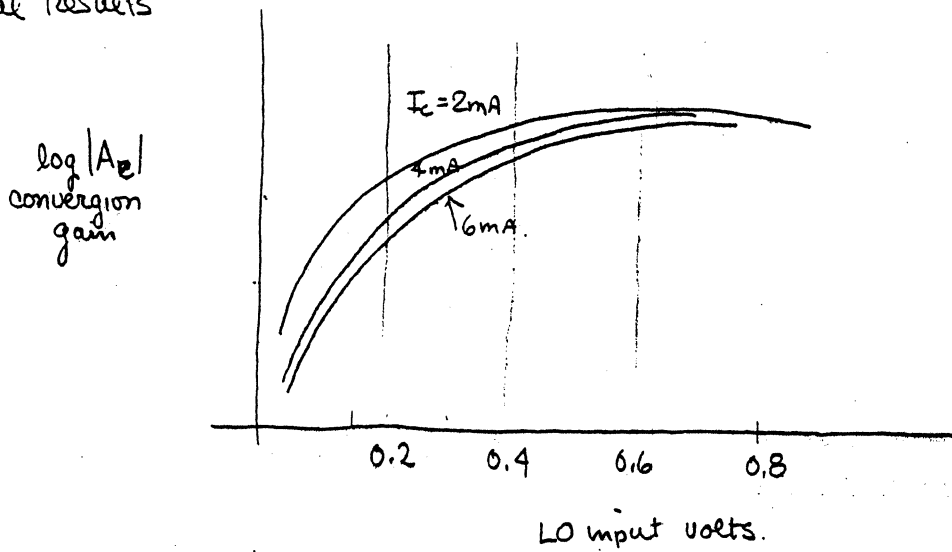
∴ add  $C = 23.4 - 9 \text{ pf} = 14.4 \text{ pf}$  in parallel at output



Total circuit:



actual results



LO input is low so as not to alter  $Y_{in}$  admittance seen by source.

