ELECTRICAL NOISE

Electrical noise is important to modern communications as it determines the sensitivity and acceptability of received signals. The modern designer is concerned about minimizing the effect of noise upon his or her circuit.

Electrical noise is due to the random motion of electrons in a conductor and is characterized by a random (usually) signal of external (to your circuit) origin. Noise can come in many "colors" such as pink or white. "Color" refers to the spectral characteristics of the noise. "White" noise has a flat (or uniform) frequency dependence and is as strong at 1 GHz as it is at 1 kHz. "Pink" noise has a decreasing power spectral density (it has equal power per octave). Pink noise can be imply produced by putting a filter (−3dB/octave) on a white noise generator. Originally, noise sources were special reverse biased noise diodes (vacuum or semiconductor) but new devices have appeared for audio applications. For example, National Semiconductor manufactures a digital noise generator chip which generates a pseudo-random digital noise sequence[1].

The external origin of noise (i.e. its origin is not a function of our electronics per se) allows us to represent noise by equivalent noise generators and noiseless components. For example, the resistor is a very effective noise source and generates a noise voltage (or current) given by

$$V_n^2 = 4kTRB, \text{ or } I_n^2 = 4kTGB \qquad (1)$$

where

- k = Boltzmann's constant (1.38×10^{-38} joules/degree Kelvin)
- T = absolute temperature of the resistor (degrees Kelvin)
- B = Bandwidth of interest (Hertz)
- R, G = resistance (conductance) of the resistor

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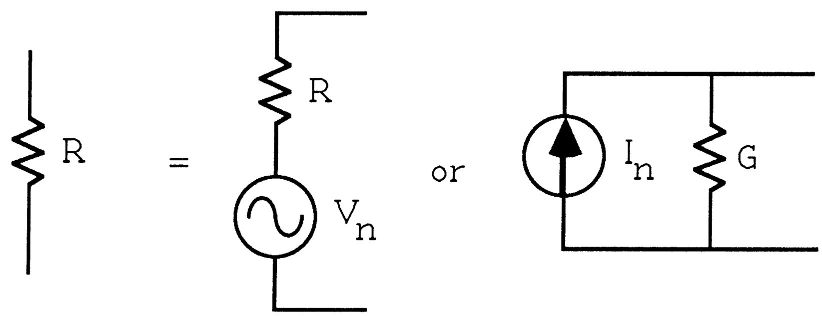
[1]MM5837 digital noise source. For a description of its use see Section 2.17.2 Pink Noise Generator in Audio/Radio Handbook by National Semiconductor, 1980, p.2-62.

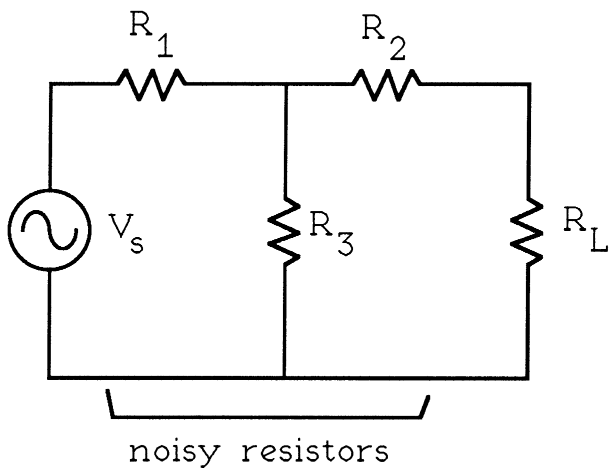
Note that the units of V_n and I_n are in volts and amperes, but the above expressions are in terms of squared quantities. This is due to the fact that noise is usually described in terms of its power spectral density which is directly proportional to the squared voltage or current. To determine the amplitude of the noise voltage one must know the noise spectral distribution and what frequency range any related circuit is operating over.

Once making the fundamental assumption that a noisy resistor, for example, can be decomposed into a noiseless resistor and an equivalent voltage, or current, noise generator one can draw an equivalent circuit for the resistor as shown below.

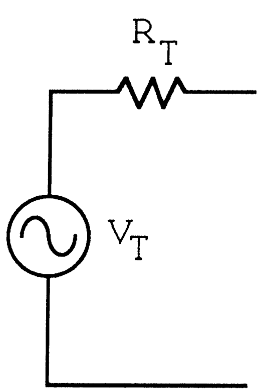


This process can be extended to networks of noisy elements. Simply replace each noisy element with an equivalent circuit containing a noise-free element and an equivalent noise generator and, then, combine the sources and passive elements separately.

If you examine the circuit below you will see that this is not as easy as it sounds. With complex circuits it is easier to combine the noisy elements first and then use an equivalent circuit for the combined circuit elements.



Another procedure to make life easier in combining noisy elements is to Thevenize the circuit. Consider the above example. Thevenize the noisy part of the circuit (i.e. V_s, R_1, R_2 and R_3) to get V_T and R_T, the Thevenin equivalent voltage and resistance. The problem is what to do with this new voltage source. Suppose we did not have a voltage source so that V_T went to zero and there would be no output voltage for this network. For a non-zero output voltage with a noise voltage source it must be in the same location as the original noise free voltage source, as shown below. This is equivalent, mathematically, to saying that the output voltage is a linear combination of statistically independent variables. For a circuit with a noise-free generator and a noise generator the two voltages would simply sum, i.e. add up in series.



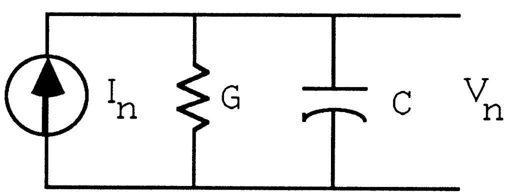
$R_T = R_2 + R_1 \| R_3$

$$V_1 = \frac{R_3}{R_1+R_3} V_s$$

As V_s goes to zero we are left with

$$V_{T_n} = \sqrt{4kTR_T B}$$

Electronic noise as described above comes from the conduction process in resistive elements. Capacitors and inductors do not contribute to a circuit's noise. In general, for any complex circuit element such as a capacitor or inductor only the resistive (real) part contributes to the noise voltage. Consider the case of a noisy resistor in parallel with a capacitor below. Note that to compute the output voltage the resistance as seen by the voltage source must be integrated over the frequency range of interest. The most common mistake in solving this type of problem is to ignore the complex element. The proper procedure is to combine the impedance (or admittance) of the circuit elements and then find the resistance (the real part) as seen by the voltage (or current) source. The result in the example shown below is not what one might have expected.



$$V_n^2 = 4kTRB$$

$$V_n^2 = 4kT\int_B R(f)\,df$$

For this circuit

$$Y_{in} = G + j\omega C \rightarrow Z_{in} = \frac{1}{G + j\omega C} = \frac{G - j\omega C}{G^2+\omega^2C^2}$$

Therefore,

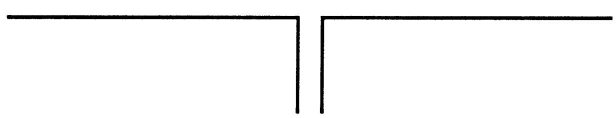
$$R(\omega) = \frac{G}{G^2+\omega^2C^2}$$

and

$$V_n^2 = 4kT\int_0^\infty \frac{G}{G^2+(2\pi f)^2C^2}\, df = \frac{kT}{C}$$

For circuit elements such as antennas which actually represent complex systems with noise sources we want a simple equivalent circuit representation. One such measure in common use is the equivalent (noise) temperature as seen at the device terminals.

Consider the case of a 75-ohm dipole antenna. The 75-ohms is the impedance of the device as measured at its terminals. If one connects a voltmeter to the antenna terminals a noise voltage of 0.1 microvolts is measured. This can be related to the temperature through equation (1) provided we realize that our meter has a finite bandwidth. Suppose that we know that the meter has a bandwidth of 10 kHz. Then the equivalent temperature T_A can be found as shown below.



75Ω dipole

V_n(measured) = 10^{-7} volts

$B = 10^4$ Hz (due to meter)

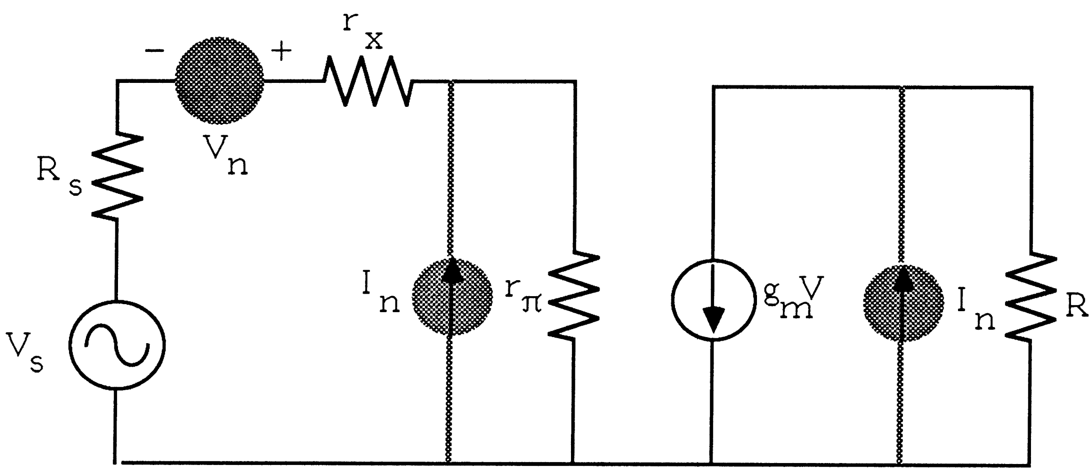
$$V_n^2 = 4kTRB$$

$$(10^{-14}) = 4(1.38\times10^{-23})T_A(10^4)(75)$$

$$T_A = 2.415 \times 10^{2}{}^{\circ}$$

$$T_A \simeq 242^{\circ}\ K$$

For an active device such as a transistor the noise behavior is very complex and based, in part, upon quantum mechanics. However, at the circuit level transistor noise, for example, can be modeled as voltage and current sources added to the input and output circuits of the device as shown below.



denotes a noise source

The physics of a particular device will determine whether the noise comes from current or voltage sources. In reality, noise voltage and current are not used to characterize real-world transistors. Instead, a parameter called noise figure (often abbreviated NF) is used. However, to understand this new parameter we must first discuss signal-to-noise ratio (or SNR as it is usually abbreviated).

Signal-to-noise ratio is defined exactly as it is written:

$$SNR = \frac{P_{SIGNAL}}{P_{NOISE}} = \frac{\frac{V_s^2}{R}}{\frac{V_n^2}{R}} = \frac{V_s^2}{V_n^2}$$

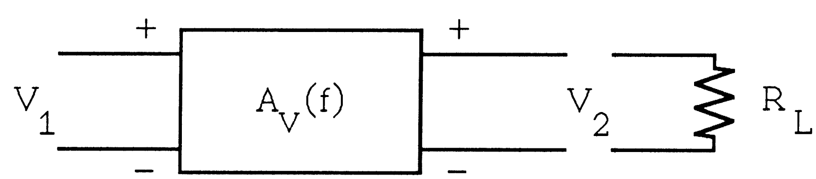
Notice how noise power expressions always result in expressions involving squared voltages or currents. SNR is often expressed in decibels for convenience

$$SNR_{db} = 10 \log_{10}\left(\frac{P_s}{P_n}\right)$$

Some reasonable numbers for SNR (in db) for commercial circuits are

- 10 db minimum for a good AM radio
- 12 db minimum for a good FM radio
- 40 db minimum for a good television
- 80-100 db for a "good" stereo receiver

SNR allows us to simply represent the "quality" of a system. However, if the noise is not spectrally uniform it is often difficult to simply express the noise. To simplify noise expressions the idea of Noise Equivalent Bandwidth (NEB) was developed. Suppose one is testing an electronic system which has colored noise and some known noise voltage at its output terminals. Assuming the gain of this system is known, we can relate the output power to the input power by squaring the input and output signals to convert them to power spectral density.



$$A_V(f) \equiv \frac{V_2}{V_1}$$

$$V_2(f) = A_V(f) V_1$$

The output power is then

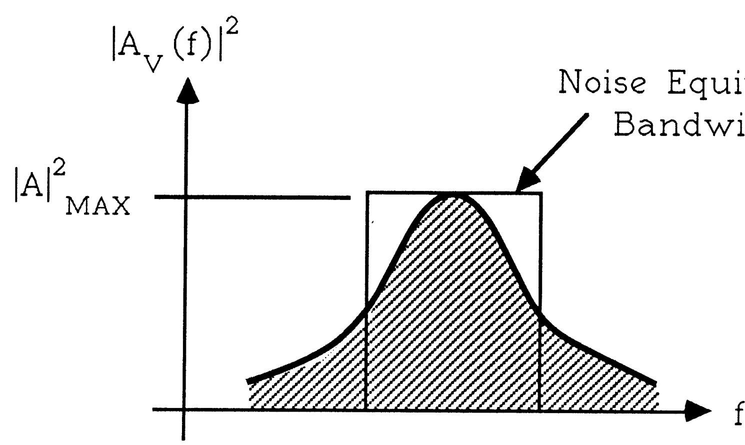
$$\frac{V_2^2(f)}{R_L} = \frac{[A_V(f)]^2 V_1^2}{R_L}$$

To relate these expressions to actual measurements (since a meter typically operates over a signal bandwidth) we must integrate over frequency to get total power, and then factor as shown below. Integrating the output voltage over frequency as an example

$$\int_0^\infty V_2^2(f)\ df = \int_0^\infty [A_V(f)]^2 V_1^2\ df = V_1^2 \int_0^\infty [A_V(f)]^2 df$$

$$\int_0^\infty V_2^2(f)\ df = V_1^2\ |A_V(f)|^2_{MAX} B$$

In the above expression we took the squared input voltage outside the integral expression for the output power because we are assuming that the signal is approximately uniform over a narrow bandwidth centered upon the frequency of maximum gain. The resulting integral of the gain over frequency can be replaced by the maximum gain multiplied by an equivalent bandwidth, i.e. a simple rectangular function as shown below.



From the above figure we can see that the noise spectra after passing through a narrow band system can be approximated by a rectangular function. The approximation becomes better as $|A_V(f)|^2$ becomes narrower. The width of the approximating rectangle is called the noise equivalent bandwidth of the system and can be mathematically defined as the total noise power divided by the maximum power

$$NEB = \frac{\int_0^\infty [A_V(f)]^2 df}{[A_V(f)]^2_{MAX}}$$

For purposes of computational simplicity one always uses a noise equivalent bandwidth when the gain is a function of frequency.

A very difficult concept and one that we will use a lot later in the course is available power. Available power is, simply expressed, the maximum power a source can deliver to a conjugately matched load. This is nothing more than the extension of the idea of a matched load to complex impedances as shown below.

$$P = vi^* = [\frac{Z_L}{Z_L+Z_S} V] [\frac{V}{Z_L+Z_S}]^* = \frac{Z_L|V|^2}{|Z_L+Z_S|^2}$$

When $Z_L = Z_S^*$

$$P_{MAX} = \frac{Z_S^* |V|^2}{(Z_L^*+Z_S)^2} = \frac{Z_S^* |V|^2}{4(Re\{Z_S\})^2}$$

Note that if one wants the real power then one must take the real part of the above expression.

$$Re\{P_{MAX}\} = \frac{R |V|^2}{4R^2} = \frac{|V|^2}{4R}$$

Similarly, if the voltage source is a voltage equivalent noise source the available power from a noise source is given by

$$P_A = \frac{4kTRB}{4R} = kTB$$

The available gain of a network is the maximum available power gain when the input and output networks are conjugately matched. This means that one must match both input and output for maximum power transfer. Under these matched conditions the available power gain is best written in terms of voltages and impedances. If the input and output are not properly matched a similar expression for power gain can be derived.

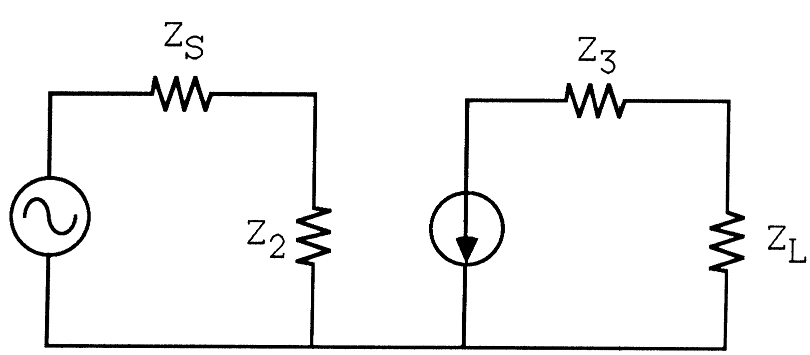
$$P_{A,S} = \frac{|V_S|^2}{4R_S} \text{ and } P_{A,O} = \frac{|V_O|^2}{4R_O}$$

$$R_S = Re\{Z_S\} \text{ and } R_L = Re\{Z_L\}$$

where the “S” denotes source and “L” denotes load.

$$G_A \equiv \frac{P_{A,O}}{P_{A,S}} = \frac{\frac{|V_O|^2}{4R_L}}{\frac{|V_S|^2}{4R_S}} = \frac{|V_O|^2 R_S}{|V_S|^2 R_L}$$

The available gain can be explicitly related to the frequency response of an amplifier or network.



Consider the network shown above. The transfer function can be written as

$$V_1 = \frac{Z_1}{Z_1+Z_2} V_S$$

$$\frac{V_O}{V_S} = \frac{V_O}{V_1} \times \frac{V_1}{V_S} = H(f) \times \frac{Z_1}{Z_1+Z_2}$$

or

$$G_A = [\frac{Z_1}{Z_1+Z_2} H(f)]^2 \frac{R_S}{R_2}$$

The equivalent temperature for a network is defined as the equivalent noise temperature corresponding to the available power. Recall that our earlier example with an antenna specified that the antenna was a 75-ohm device. If one works with available power the device impedance is included in the definition of equivalent temperature since the measurement device is matched to the source. We can write the available power of a device as

$$P_A = kTB$$

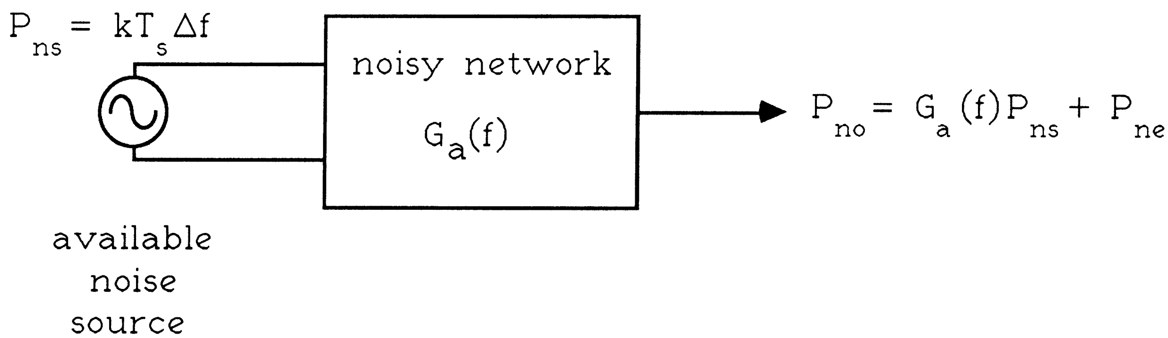
where the factor of 4 disappeared because of the matched conditions, i.e. $P=V^2/4R$ and the 4's canceled out.

With these concepts we can now quickly analyze networks. Suppose, for example, one had a noisy amplifier as shown below. The amplifier has an available power gain G_A. For a real input signal, i.e. one with a noise component, the output from the amplifier is simply the available gain times the total input power (remember these are power gains we are working with). However, if the amplifier is a real amplifier with its own internal noise sources then the output is the sum of the available gain times the input power PLUS the noise contributed by the amplifier. To illustrate this point consider the circuit shown below. The output is the sum of the gain times the input PLUS the amplifier noise. When the input power is decreased to zero the amplifier still has an output - its own noise. In practice the amplifier noise degrades the signal and can be described in terms of the amplifier's Noise Figure (NF for short). The Noise Figure is simply the ratio of the SNR at the amplifier's input to that at its output. If the amplifier has no noise the ratio is one. A noisy amplifier will always have a NF greater than one.

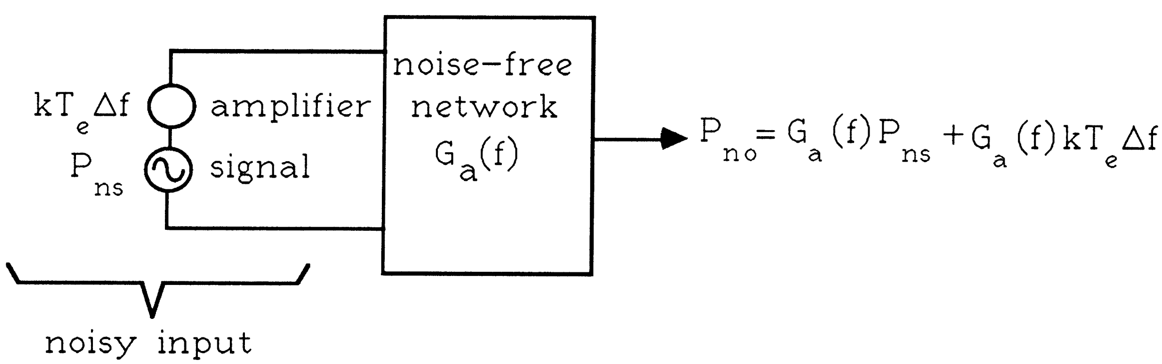
$$NF = \frac{SNR_{INPUT}}{SNR_{OUTPUT}} \qquad (2)$$

The NF is often given in decibels where NF (db) = 10 log NF.

The definition of NF as a figure of merit of an amplifier can be explicitly related to the noise added by the amplifier, the amplifier available gain and the input noise as shown below.



where P_{ne} is the noise added by the amplifier and T_e is the equivalent noise temperature of the amplifier.



The expression above can be re-written as

$$P_{no} = G_a(f)k(T_s+T_e)\Delta f$$

where we have factored the noise temperatures together.

The expression for P_{no} can be used in Equation (2) to give an expression for the NF of the noisy network.

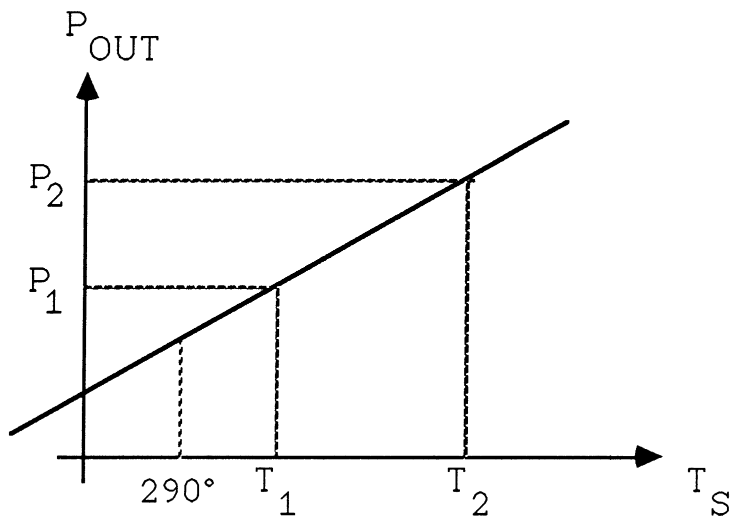
$$NF = \frac{T_S+T_e}{T_S}$$

Note that the NF IS dependent upon the input noise. Usually this is thermal noise given by kTB. To prevent ambiguity in NF measurements due to varying input source temperatures all measurements are referred to a standard temperature of 290° K. This figure was chosen because it is approximately the noise temperature seen by atmospheric radio frequency receiving antennas. An actual Noise Figure for some temperature other than 290°K can be simple related to the "standard" NF at 290°K by the relationship

$$NF_{actual}=1+(NF-1)\left(\frac{T_S}{T_S'}\right)$$

where T_S' is the "non-standard" noise temperature.

Now that we have defined relevant noise expressions let's re-examine the noisy amplifier problem. Most active networks (that we will be concerned with) are linear in the sense that their output is a linear function of the input. If the input signal is <u>set to zero</u> but the <u>source remains connected</u> to the amplifier input the amplifier output will contain the amplifier noise PLUS the amplifier input source noise. The input/output relationship for this case can be summarized by the graph shown below.



The horizontal axis indicates the noise temperature of the source impedance. The y-axis is in watts (power) and represents the total output power of the amplifier. The above graph then represents the input/output relationship of the amplifier and is a linear graph because the amplifier is linear. The y-intercept represents the noise power added by the amplifier; the slope of the line is the power gain, i.e. kG_aB. This method of looking at a noisy amplifier is very powerful in practice[2]. For example, if one simply measures the output power of an amplifier for two different inputs, i.e. source temperatures, a linear curve can be fit to the two points to give the added noise of the amplifier and, indirectly, the Noise Figure of the amplifier. In the published literature the amplifier is more properly referred to as the Device Under Test, or DUT for short.

The above graph can be re-drawn to explicitly show those elements related to calculating the Noise Figure. With reference to the re-drawn figure below, the NF is the ratio of the total output power to the height of the small triangle (the excess noise) at 290°K.

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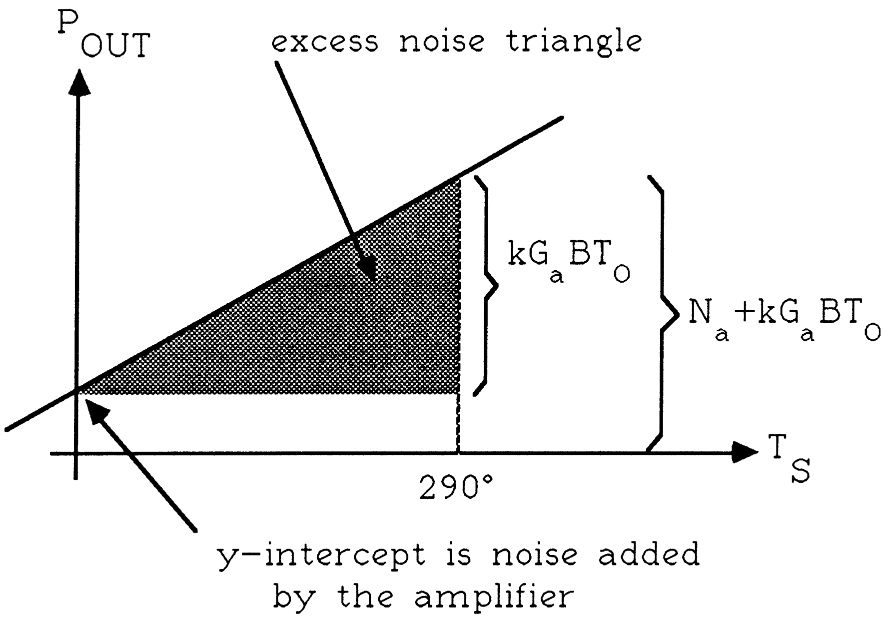
[2]See for example: Kuhn, Nick, "Getting Started With Automatic Noise Figure Measurements", Microwave System News, January 1983, p.120–136.



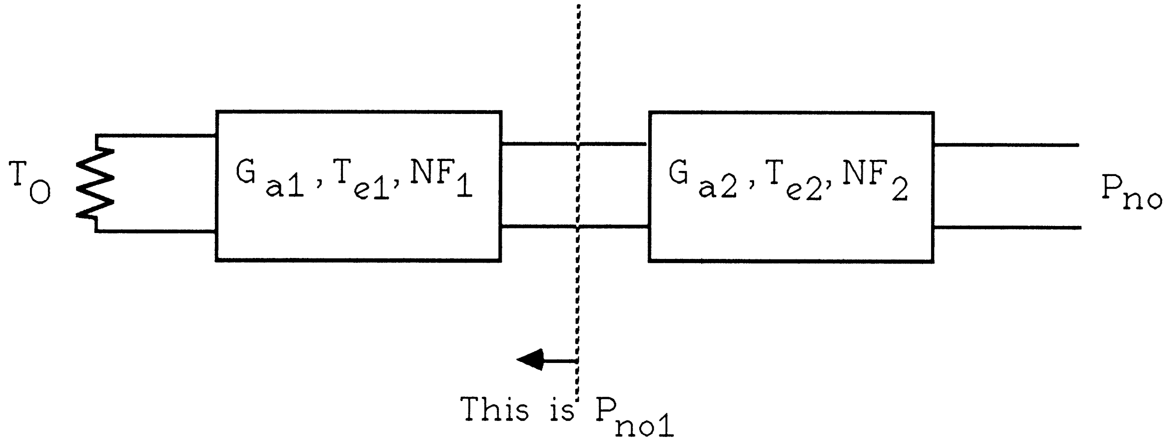
From the graph the NF is then

$$NF = \frac{N_a + kG_aBT_O}{kG_aBT_O}$$

where N_a is the y-intercept (the amplifier noise), G_a is the available gain of the amplifier, B is the amplifier bandwidth and k is Boltzmann's constant. The input noise is assumed to be at the standard noise temperature, $T_O = 290°K$.

The above definitions can be easily extended to cascaded amplifiers. Calculate the output power of the first amplifier using the definition of Noise Figure. The output power from the first amplifier is then the input to the second amplifier. The second amplifier amplifies the output of the first amplifier and adds its internal noise to the amplified signal as shown below. Note that the effective Noise Figure of the cascaded system is primarily determined by the first amplifier; hence, the importance of a low-noise front end.

EXAMPLE 1: RECEIVER FRONT END CALCULATION



$$P_{NO1} = G_{A1}k(T_0+T_{e1})\Delta f$$

Taking this as the input to G_{A2}

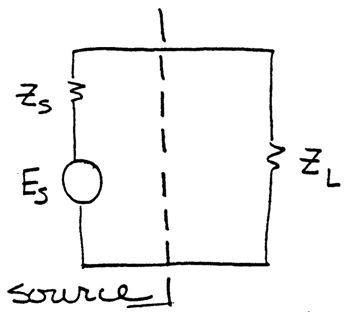
$$P_{NO} = G_{A2}P_{NO1} + G_{A2}(kT_{e2}\Delta f)$$

where the first term is the effective source term and the second is the network noise:

$$P_{NO} = G_{A2}G_{A1}kT_0\Delta f + G_{A2}G_{A1}kT_{e1}\Delta f + G_{A2}kT_{e2}\Delta f$$

Notice that the noise of the first amplifier is amplified by $G_{A2}G_{A1}$ whereas the noise of the second amplifier is only amplified by G_{A2} showing that the noise in the amplifier output is indeed dominated by the noise introduced by the first stage, by a factor of G_{A1}. This also holds true for multi-stage amplifiers.

"Systems" noise model



max P_{LOAD} when $Z_L = Z_S^*$

$$P_{LOAD} = \frac{\mathrm{Re}(Z_L)|E_S|^2}{|2\,\mathrm{Re}(Z_L)|^2}$$

$$= \frac{|E_S|^2}{4\,\mathrm{Re}(Z_L)}$$

$$P_{LOAD} = v i^*$$

$$= \left(\frac{Z_L}{Z_L + Z_S} E_S\right)\left(\frac{E_S}{Z_L + Z_S}\right)^*$$

$$= \frac{Z_L^* |E_S|^2}{|Z_L + Z_S|^2}$$

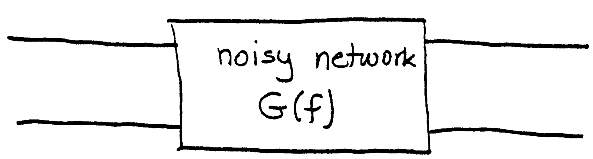
$\therefore$ the available power from the source is $\dfrac{|E_S|^2}{4R_S}$

available noise power is $\dfrac{|V_n|^2}{4R_S} = \dfrac{4kT\Delta f\, R_S}{4R_S}$

$$= kT\Delta f$$

signal power, P_{si}

noise power, P_{ni}



P_{so}, signal power output

P_{no}, noise power output
(sum of amplified input noise power and network noise power

noise factor $\quad F \triangleq \dfrac{P_{no}}{G(f)P_{ni}} \quad \dfrac{\text{total output noise power}}{\text{amplified input noise power (ideal amplifier)}}$

Noise

$$\frac{S}{N} \triangleq \frac{\text{average signal power}}{\text{average noise power}}$$

noise can be both { random (fundamental limit to receivers)
{ non-random (power supply, cross-coupling, etc.)

the non-random can eventually be limited, the random cannot

Noise is usually expressed as a power spectral density $p(f)$ so that the total noise power is

$$P = \int p(f)\,df$$

Thermal noise in resistors

$$E_n^2 = 4kTR\,\Delta f$$

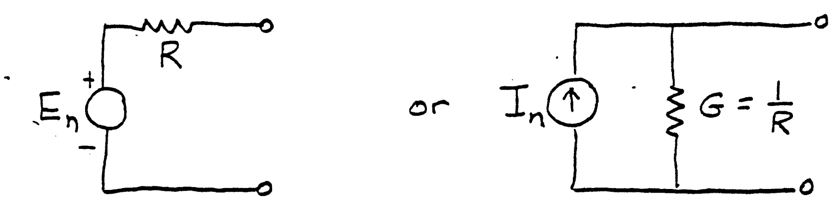
$$I_n^2 = 4kTG\,\Delta f$$

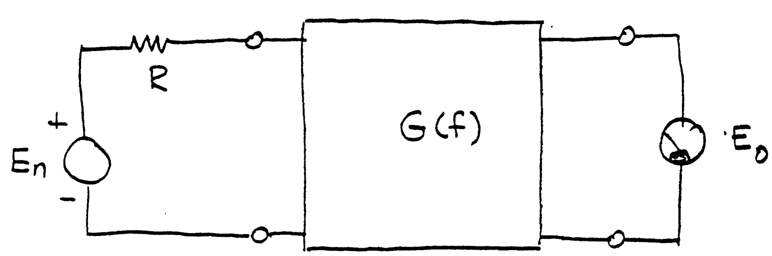
T = temperature, °K

k = Boltzmann's constant, 1.38×10^{-23} watts · sec/°K

Δf = bandwidth

equivalent circuits

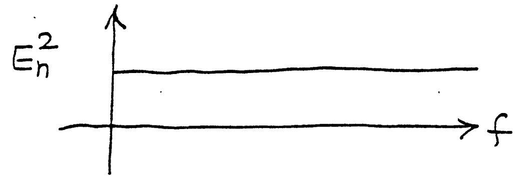




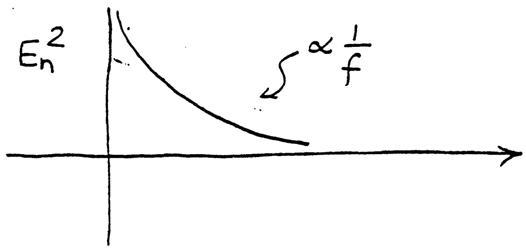
$$E_o^2 = \int_0^\infty 4kTR\,G(f)\,df$$

← the transfer function of the network.

G(f) is often used to get "colored" noise.



is uniform, or "white", for a thermal noise source

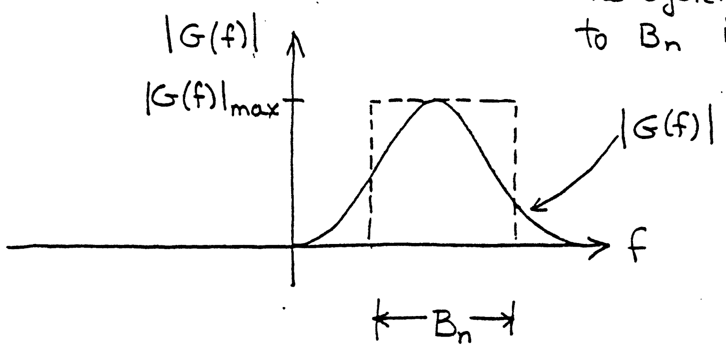


"pink" noise

We can re-write the integral as

$$E_o^2 = 4kTR \underbrace{\int_0^\infty G(f)\, df}$$

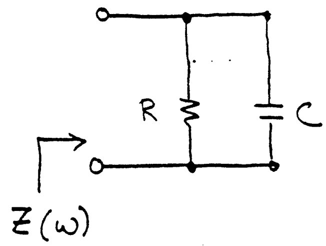
this is called the noise bandwidth B_n of the system since we will make it equivalent to B_n if $|G(f)|_{max} = 1$



$$B_n \triangleq \frac{\int |G(f)|\, df}{|G(f)|_{max}}$$

Note that G is the transfer function between the square magnitude of the input and output voltages.

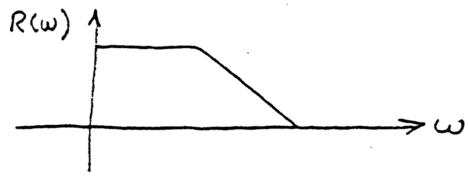
Examples:

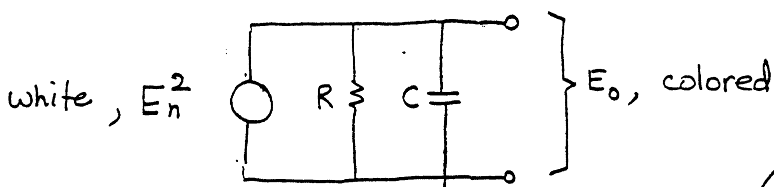


$$Z(\omega) = R \parallel \frac{1}{j\omega C} = \frac{R\left(\frac{1}{j\omega C}\right)}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC} = R(\omega) + jX(\omega)$$

$$= \frac{R}{1+\omega^2 R^2 C^2} - j\,\frac{\omega R^2 C}{1+\omega^2 R^2 C^2}$$

$$\therefore \quad R(\omega) = \frac{R}{1+\omega^2 R^2 C^2}$$





NOTE: for this formula G is normalized

$$E_o^2 = 4kTR \int G(f)\,df = 4kTR \int_0^\infty \frac{1}{1+\omega^2 R^2 C^2}\,df$$

$$= 4kTR \int_0^\infty \frac{df}{1+4\pi^2 f^2 R^2 C^2} = 4kTR\left(\frac{1}{4RC}\right) = \frac{kT}{C}$$

↑ total output power

Since $E_o^2 = 4kTR\,B_n = \dfrac{kT}{C}$

$$B_n = \frac{1}{4RC} \quad \text{(angular frequency)}$$

$$= \frac{1}{8\pi RC} \quad \text{Hz}$$

We used two interpretations

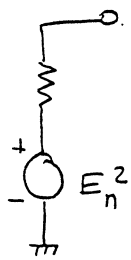
1. noise as the real part of a network impedance

2. noise originating in the resistors

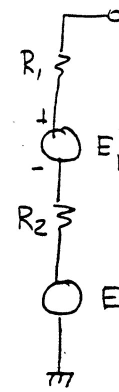
Both are equivalent

Multiple noise sources — their power, NOT their voltage adds.

series



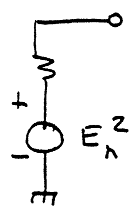
$$E_n^2 = 4kT(R_1 + R_2)\Delta f \quad \equiv$$



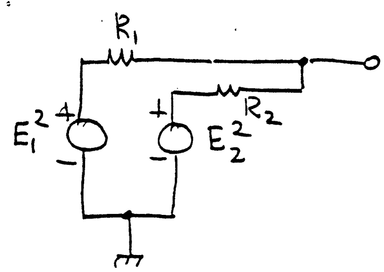
$$E_1^2 = 4kTR_1\Delta f$$

$$E_2^2 = 4kTR_2\Delta f$$

parallel

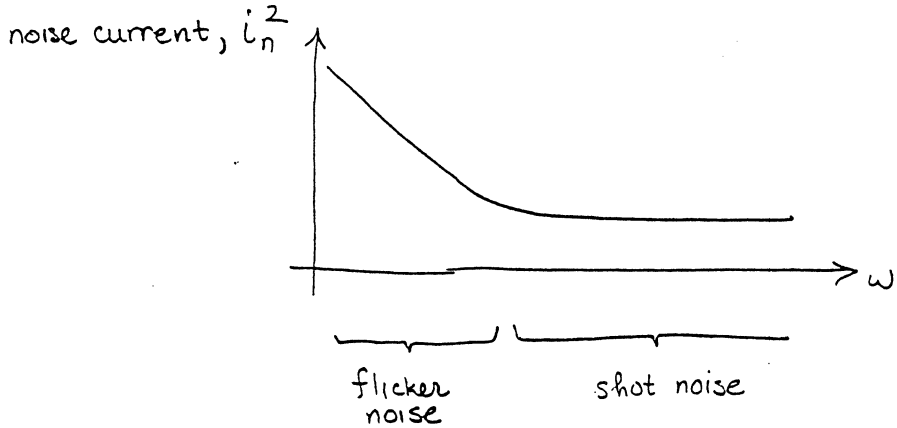


$$E_n^2 = 4kT\left(\frac{R_1 R_2}{R_1 + R_2}\right)\Delta f \quad \equiv$$

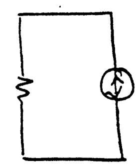


Noise in semiconductors

noise sources:
- thermal
- flicker — $\frac{1}{f}$
- shot — discrete noise

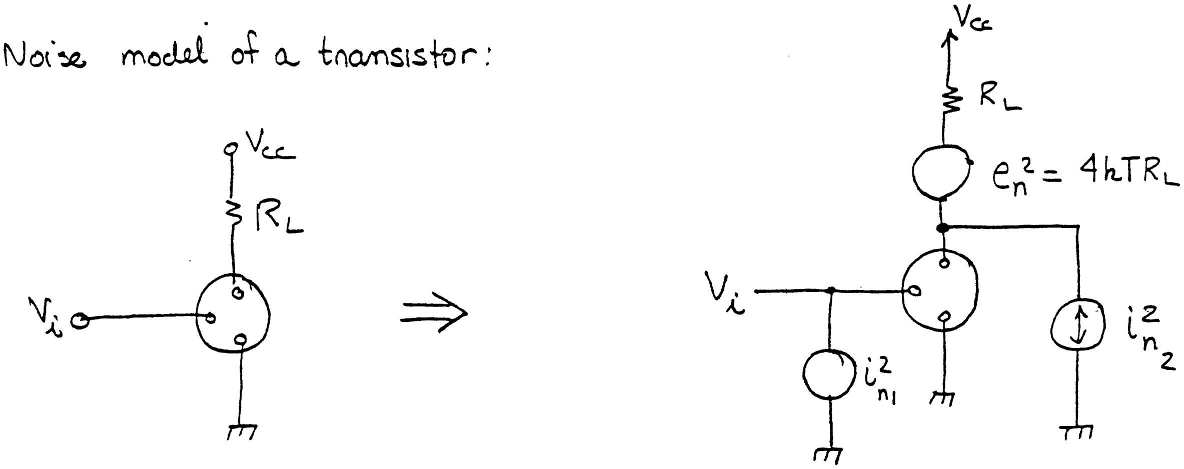


diode noise model



$$i_n^2 = 2q I_o \Delta f$$

Noise model of a transistor:



Note small letters indicate density

Usually better to refer to input of transistor

for a transistor $\quad \left|\frac{V_o}{V_i}\right| \approx g_m R_L$

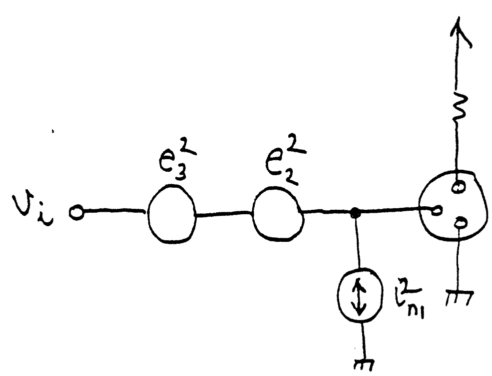
$$\therefore \; |V_i|^2 = \frac{|V_o|^2}{(g_m R_L)^2} = \frac{|i_o R_L|^2}{|g_m R_L|^2} = \frac{i_o^2}{g_m^2}$$

so i_{n2}^2 can be replaced by a voltage source $e_2^2 = \frac{i_{n2}^2}{g_m^2}$ at the input

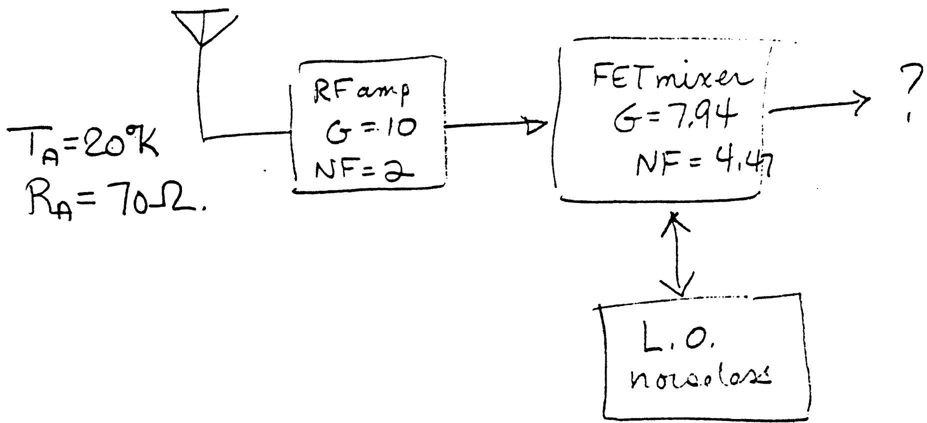
Likewise, e_n^2 can also be reflected to the input

$$|V_i|^2 = \frac{|V_o|^2}{(g_m R_L)^2} = \frac{4kTR_L}{(g_m R_L)^2} = e_3^2$$

Normally $\quad e_3 \ll e_2$



Receiver Front End calculation



assume measured at standard temperature $NF = 1 + \frac{T_e}{T_o}$.

$$T_e = T_o(NF-1) \qquad T_e = T_o(NF-1)$$
$$= 290°K(2-1) \qquad = 290°K(4.47-1)$$
$$= 290°K \qquad = 290(3.47) = 1006°K.$$

overall, effective noise temperature.

$$T_{eff} = T_{e1} + \frac{T_{e2}}{G_{a1}} = 290°K + \frac{1006°K}{10}$$

$$= 290°K + 100.6°K \cong 391°K.$$

$$NF_{eff} = NF_1 + \frac{NF_2 - 1}{G_a} = 2 + \frac{4.47-1}{10}$$

$$= 2 + \frac{3.47}{10} = 2.347$$

Since the antenna is <u>NOT</u> at 290°K we must convert to a real NF for a <u>20°</u> source.

$$NF_{actual} = 1 + (NF-1)\frac{T_o}{T_{source}}.$$

$$= 1 + (2.347-1)\frac{290°K}{20°K} \simeq 20.6$$

compare

This is poor because the antenna is so noisy

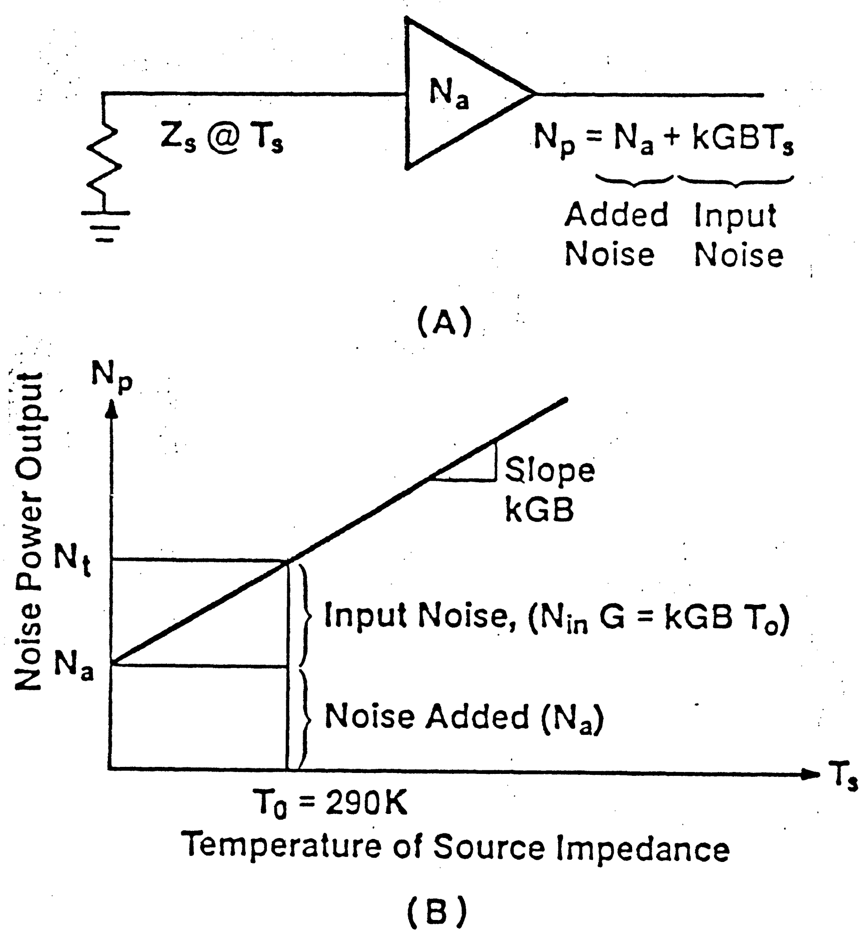
Getting Started With Automatic Noise Figure Measurements

The mystique of noise figure is removed by this succinct review of concepts and measurement principles.

By Nick Kuhn, Hewlett-Packard Co.

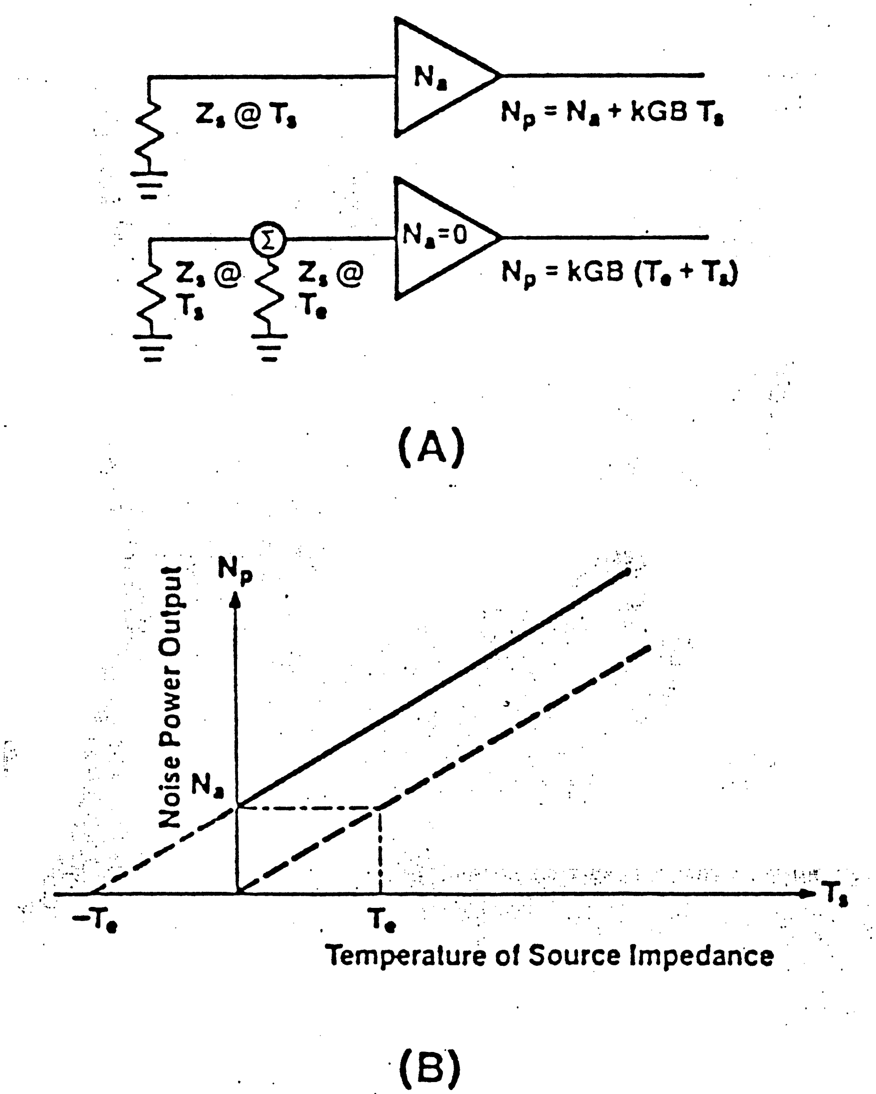
Noise figure shows the ability of a device to process weak signals (Fig. 1). Any physical source at the input of the device under test (DUT), has a finite impedance and a finite temperature, even when the signal level is reduced to insignificant levels. The finite temperature of the source implies that thermal agitation occurs. The source's finite impedance implies that there is some free charge within that source. Some of that free charge is thermally agitated within the frequency band accepted by the DUT. That energy, being transferred to the input of the DUT, is amplified and results in a certain amount of power output. Still another component of power output is a result of the noise mechanisms that occur within the DUT. Such noise mechanisms within the DUT might arise from thermal sources or from shot noise (noise caused by the corpuscular nature of current flow).

A graph of the power at the output from the DUT vs. the temperature of the source impedance that drives the DUT, would look like a straight line (Fig. 1B). When the temperature of the source is near absolute zero, a condition approached by antennas aimed into deep space or by a source at liquid helium temperatures, the noise power at the output of the receiver or amplifier under test will mostly be noise added by the DUT (N_o). As the source impedance becomes hotter it generates noise that acts as signal for the DUT. The noise is amplified and increases the power reading at the output. Terrestrial communications systems would have antennas that are aimed across the horizon and would consequently provide a source temperature corresponding to the atmosphere of about 300 Kelvins. Similar source temperatures would occur when DUTs are tested in laboratory environments because the



1. Noise figure shows the ability of a device to process weak signals. *Here (A) is the block diagram of a general device under test (DUT), shown as an amplifier, with no signal input. Output power consists of amplified input noise, due to thermal agitation in whatever source impedance is at the input, and of noise added by the DUT. Below, (B) graph is of the noise power output from a DUT along with the components that contribute to noise figure.*

Nick Kuhn is senior applications engineer at Hewlett-Packard's Stanford Park Division, 1501 Page Mill Road, Palo Alto, CA 94304; (415) 857-3387.



2. For many modern devices, a different figure of merit, the effective input noise temperature, (T_e), is commonly used. The block diagram (A) showing the general DUT with an temperature, T_e, that causes the same effect on the output power as N_a. Below, (B) is the graphical interpretation of T_e. A noise free device would have the characteristic dashed line that goes through the origin.

source impedance would be at the ambient temperature.

The slope of the power output vs. temperature line is proportional to the gain bandwidth product (GB) of the DUT. The larger the DUT's bandwidth, the more thermal agitation energy is transferred from the source within that band. The larger the gain of the DUT, the larger the input signal would be amplified to contribute power output. The slope of the DUT characteristic is also proportional to Boltzmann's constant (K = 1.38 exp −23 J/K). Boltzmann's constant is a conversion factor from one unit of energy, Kelvins (abbreviated by K), to another unit of energy, Joules (J). Boltzmann's constant times bandwidth results in the units of Watts/Kelvin.

If the gain-bandwidth product of a device were to change, the slope of the line in Figure 1(B) would change. If the noise characteristic of the device were to change, the straight line of Figure 1(B) would translate.

Although the straight line characteristic of Figure 1(B) shows the properties of the device, a graph is rather difficult to talk about. A figure of merit is more desirable. The most popular figure of merit, noise figure, was adapted in the 1940s when all communications systems were terrestrial. Noise figure relates to the height of the graph at a temperature corresponding to what excites antennas in terrestrial systems. Two hundred ninety K has been established as the standard temperature for expressing the noise figure of a DUT. Noise figure is defined as the ratio of total power output from a DUT to the power output that would occur from a DUT that adds no noise. In equation form this is given by

$$F = \frac{N_a + N_{in} G}{N_{in} G}$$

where N_a is the noise added by the DUT, N_{in} is the noise power transferred to the input of the DUT, and G is the gain of the DUT. The input noise corresponds to the source being at the standard temperature of 290K (symbolized by T_o). Under this condition, the amplified input noise is given by

$$N_{in} G = K T_o BG$$

Using Figure 1(B) to interpret noise figure, the numerator or the ratio is formed by the noise added and the amplified input noise at the X-axis position of 290 Kelvins. The denominator is formed only by the amplified input noise. Noise figure, because it is a ratio, is often expressed in dB.

$$F(dB) = 10 \log F$$

Sometimes noise figure expressed as a ratio is referred to as noise factor and then noise figure refers to the dB expression of the ratio. In almost all literature, however, when dB is meant the letters "dB" are included—when dB is omitted, the ratio is meant.

For very modern devices, a different figure of merit, the effective input noise temperature (T_e) is commonly used. T_e (Fig. 2) indicates how hot a source impedance would have to be in order to simulate the noise added by the DUT. Once that extra input temperature is included, then the device under test could be thought of as being perfect—adding no noise of its own. A source temperature of T_e would be needed in order for to have the noise output from that ideal device be equal to the noise added from the DUT. In a graph, if the straight line characteristic of the general DUT is extrapolated down to the temperature axis, the X-axis intercept turns out to be the negative of T_e.

The algebra of straight lines shows that once noise figure is known, T_e can be calculated and vice versa. The equation that relates them is

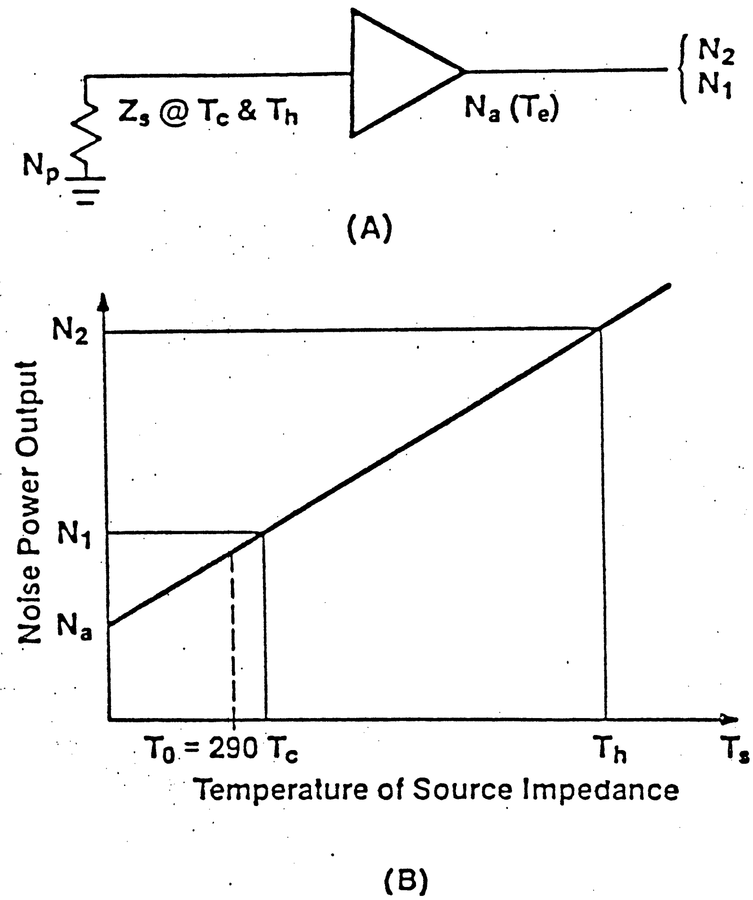
$$F = \frac{T_e + T_o}{T_o}$$

Measurement Methods

Noise properties of devices are commonly measured by means of two points along the straight line. Once those two points are measured, F or T_e can be calculated (Fig. 3). The hotter source temperature, T_h, usually corresponds to a noise source being turned on. The colder source temperature T_c, usually corresponds to the ambient temperature of the environment where the noise figure measurement is being made. The corresponding noise powers, N_2 and N_1, could be measured by a power meter but they are usually measured by the power detecting circuits of a noise figure meter. The coordinates of the two points are related to noise figure by the equation

$$F = \frac{\left(\frac{T_h}{T_o} - 1\right) - \frac{N_2}{N_1}\left(\frac{T_c}{T_o} - 1\right)}{\frac{N_2}{N_1} - 1}$$

The modern, microprocessor-based noise figure meter makes the calculations indicated in equation 5 from the coordinates of the two points—(T_h, N_2) and (T_c, N_1). Traditional noise figure meters, operating with analog circuits, utilize automatic gain control and sometimes several detectors and a non-linear meter scale in order to indicate the noise figure. All such traditional noise figure meters, because they are limited by analog circuits, had to assume that the cold temperature T_c, is equal to the standard temperature for noise figure T_o. By



3. The measurement of noise figure, or T_e, uses two different source temperatures, one colder (T_c) and the other hotter (T_h) to yield two noise power outputs, N_1 and N_2. From the temperatures and the power measurements, noise figure meters calculate the noise figure, F.

assuming that T_c is equal to T_o, equation 5 for noise figure is considerably simplified because the entire second term of the numerator disappears. A microprocessor-based noise figure meter does not have to make such an assumption.

Most measurement set-ups use an avalanche diode noise source in order to generate T_h and T_o. When a semiconductor diode is reversed biased into its avalanche region, carriers are generated in such a random fashion that noise results. The noise is equivalent to a very hot resistance. When operating in the avalanche region, furthermore, the source impedance is very close to that of a short circuit. But 50 ohms is a more ideal source impedance for testing DUTs. The avalanche diode is therefore followed by a built-in attenuator. This attenuator absorbs much of the noise output from the diode but does provide a 50 ohm source impedance. The overall effect is that commercial noise sources produce noise that closely approximates a 50 ohm source impedance at 10,000K.

When the bias power is removed from the diode, the noise output from the noise source is that associated with the ambient temperature of the built-in attenuator. This yields the T_o condition.

Commercially available noise sources are not usually specified in terms of T_h. Instead they are specified in terms of their excess noise ratio, ENR. ENR, almost always expressed in dB, is given by

$$ENR = 10 \log \left(\frac{T_h}{T_o} - 1\right)$$

Note that the quantity in parenthesis

of equation 6 is the numerator of equation 5 once the assumption is made that $T_c = T_o$. Typical values for commercially available noise sources have an ENR of about 15 dB. This means that the noise source puts out 15 dB more noise power than a resistance at a temperature of 290K.

Another buzzword that is common in noise figure measurements is the Y-factor defined by

$$Y = \frac{N_2}{N_1}$$

The Y-factor and ENR are special terms that become popular through hand calculation of equation 5.

It is obvious (Fig. 3) that an accurate knowledge of T_h (actually) its equivalent ENR) is necessary for accurate measurements. ENR usually changes by at least 0.2 dB over a 10:1 change in frequency. In order to properly measure noise figure, one microprocessor-based noise figure meter (the HP 8970A) stores a table of ENR vs. frequency. Then as each measurement is made, the noise figure meter looks up the closest two values of ENR and interpolates to find a proper value for the frequency being measured. From this value and the measured data, it uses equation 5 to find F.

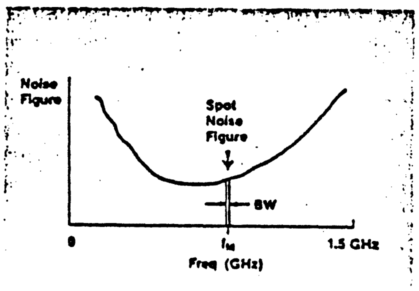
Noise figure measurements do not measure the noise characteristics of the DUT but itself. They measure the noise characteristics of the DUT combined with the noise measurement system. There is generally noise contributed by the measurement system that either must be removed or accepted as a measurement error. Such noise is frequently given the name second stage contribution—the first stage referring to the DUT. The combined noise figure that is measured can be expressed in terms of the individual noise figures by the equation

$$F_{12} = F_1 + \frac{F_2 - 1}{G_{a1}}$$

where F_{12} is the combined noise figure, F_1 is the noise figure of the DUT, F_2 is the noise figure of the measurement equipment and G_{a1} is the available gain of the DUT. The measured data, therefore, refers to F_{12} instead of the desired value, F_1. The microprocessor-based noise figure meter, however, can measure F_2 and store it in its memory. It can also measure G_{a1} by proper measurement of the slope of the straight-line characteristic. Then when it measures F_{12}, it can calculate the desired value F_1 using equation B.

In summary, the modern microprocessor-based noise figure meter corrects for three effects which, on traditional noise figure meters, were usually accepted as measurement errors. The three effects, all referred to above, are (1) T_o not being equal to 290K, (2) variation of ENR with frequency and (3) the second stage noise contribution. This last error, the removal of the second stage effect, is performed by the noise figure meter separately measuring the noise figure of the measurement system. This measurement of the noise characteristic of the measurement system is commonly referred to as calibration.

Noise figure is normally expressed as a function of frequency (Fig. 4). Sometimes the noise figure at a specific frequency is emphasized by using the term "spot noise figure." The HP 8970A directly covers the frequency range of 10 to 1500 MHz with a measurement bandwidth of 4 MHz. This corresponds to a 4 MHz spot size.



4. Noise figure is usually measured as a function of frequency or at several discreet frequencies. The noise figure meter or power measuring equipment contain the appropriate filters and circuits to measure at a specific frequency and over a certain bandwidth (usually 1 to 6 MHz).

Receiver Measurements

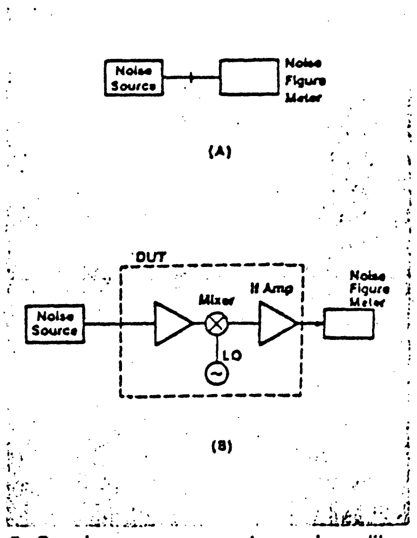
The process for measuring the noise figure of a receiver includes calibration of the measurement system noise figure and measurement with the receiver under test (Fig. 5). Calibration is performed at a frequency corresponding to the IF of the receiver. For measurement, the DUT is inserted between the noise source and the noise figure meter. The receiver under test normally includes a down converter so the receiver input frequency during measurement is higher than the IF frequency. It is very convenient to have a noise source that covers both the IF frequency (for calibration) and the receiver input frequency.

For each frequency of measurement, the receiver must be tuned to the new frequency. The noise figure meter also has to be told the frequency of measurement so it can use the proper ENR of the noise source for its calculations. Such frequency changing can take place either by manually tuning the receiver and manually operating the push buttons on the noise figure meter to the new frequency. Another way is to tune the frequency of the receiver by use of an external computer. The computer can also tell the noise figure meter the frequency of each measurement.

Microwave Amplifier Measurement

For measuring amplifiers whose output frequency is above 1500 MHz, the measurement system must include some sort of downconverter (Fig. 6). During calibration the DUT is removed and the noise source is connected directly to the input of the measurement system. Calibration of the measurement system noise figure takes place at each frequency of interest for the amplifier under test. The DUT is then inserted between the noise source and the input to the measurement system.

For manual measurements, i.e., measurements without an external controller, the microprocessor-based HP 8970A Noise Figure Meter can control suitable local oscillators on the IEEE-488 interface bus (also called GP-IB and HP-IB). Suitable local oscillators include most of the modern sweep generators and microwave synthesizers that allow operation on the interface bus. For more automated systems, an external controller will direct the frequency of the local oscillator as well as instruct the noise figure meter of the measurement frequency.



5. Receiver measurements require calibration of the noise figure meter (A); i.e., measurement and storage in memory of the measurement system noise contribution at the IF of the receiver. Then the receiver is inserted between the noise source and the noise figure meter for measurement (B).

The measurement frequency for the downconverter system (Fig. 6) is actually two frequencies—the local oscillator frequencies + and − the IF frequency that the noise figure meter is set to. This means that the measurement result is actually sort of an average of the noise figure and gain at those two sort frequencies. This is called a double sideband measurement. The measurement at those two frequencies is valid provided that the amplifier under test does not have a noise figure or gain that varies significantly over the range of those two frequencies. Small frequency variations are usually assured by setting the noise figure meter for a small IF frequency. This means that the downconverter operates at a low IF frequency, such as, 20 or 30 MHz. If such double sideband measurements are not accurate enough, techniques are available that utilize an extra filter and a high IF frequency.[1]

The Need for Automation

Because a modern noise figure meter corrects for so many measurement effects that were formerly considered as errors and is able to control a local oscillator on the interface bus, one wonders why an external controller is

Continued from page 128

ever needed. One reason was already referred to—the local oscillator of a receiver may have a tuning format that requires programming from a desktop computer. An example of such a case would be a varactor-tuned local oscillator whose frequency/voltage characteristic is not linear. A similar case that would require an external controller, is the use of an older style local oscillator. Such a local oscillator might not have a control statement that is simple enough to be controlled by the noise figure meter.

Interest is increasing in use of external controllers for noise figure measurements in order to free the technician from the most tedious parts of the measurement. When a measurement process is stored on the mass memory device of an external computer, the manufacturing operation can be assured that the procedure has not been changed in some fashion by the operator as he is connecting the equipment. The HP 8970A Noise Figure Meter can store up to ten front panel configurations in its own memory. Manufacturing operations are oftentimes interested in assuring that these front panel configurations are exactly as prescribed in a mass memory storage device for an external computer. The external computer, in this case, is used to periodically load the 8970 with the front panel configurations.

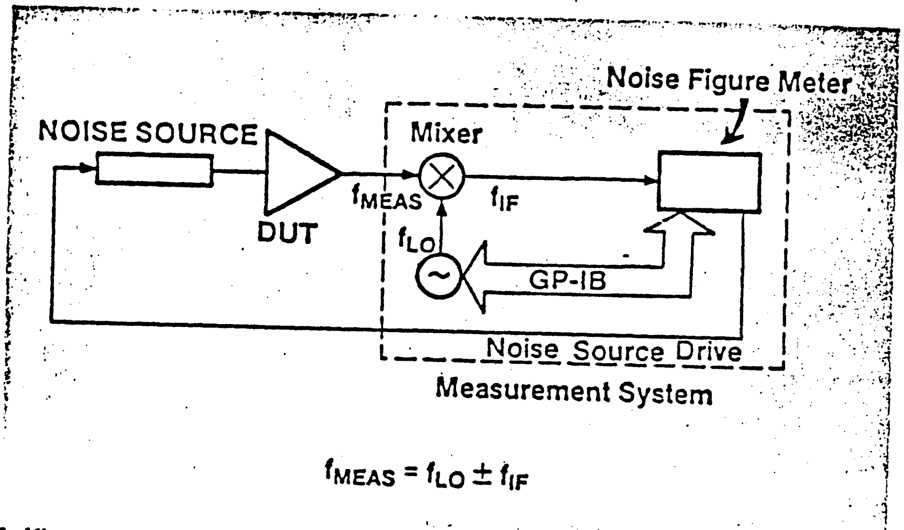
A very important case where computer control is desirable occurs when test records must be accessible in computer form. This would include the capability of compiling statistical results for overall production yield records. It also includes the case where a special report format is needed for each DUT.

An external computer may also be needed because further data processing of the measurement result will be useful. Such cases include transistor characterization, use with an automatic network analyzer for vector error correction of mismatch effects, making swept measurements that utilize physically hot and cold noise sources that are common in calibration laboratories, and the calibration of avalanche diode noise sources.

In almost all of these cases, the external computer synchronizes the measurement equipment and gathers measurement results. All the error corrections referred to above can be still performed by the microprocessor in the noise figure meter. This means that the noise figure measurement program does not need to include the algebra and storage of calibration data that is already programmed into the noise figure meter.

Measurement Subroutines

Using an external controller to control a modern noise figure meter is as simple as operating the 8970A from



6. Microwave amplifier measurements require a downconverter as part of the measurement system. This converts the microwave output frequency of the amplifier to the frequency range of the noise figure meter.

the front panel. Each push button on the front panel of the unit has two characters written along side which, when sent to it on the interface bus, perform the same operation under automatic control that the push button performs in manual control. Programming a controller, for example, to send the 8970 the character string "PR FR 135 EN," presets it to its initial measurement state, and then sets the measurement frequency to 135 MHz.

Extra care is needed whenever there is an external local oscillator included as part of the device under test or as part of the measurement system. There cannot be two controllers on the interface bus at the same time. Thus the noise figure meter cannot control the local oscillator and the computer control the noise figure meter at the same time. The computer has to control both the local oscillator and the noise figure meter and appropriately synchronize the operation of each. Two example subroutines show this synchronization. Both example subroutines are written in BASIC and assume that the noise figure meter is the HP 8970A and the controller is the HP 85. The subroutines include instructions to both the noise figure meter, whose address is 708, and to a local oscillator whose address is 719.

The first subroutine (Fig. 7) can be used for calibrating an amplifier measurement setup. The example subroutine utilizes a microwave synthesizer as a local oscillator. The frequency stepping loop for calibrating at each measurement frequency begins in line 1070 and ends at line 1160.. Steps 1080, 1090, and 1100 control the microwave synthesizer. Step 1120 commands the noise figure meter to calibrate at the specific frequency then tuned. In line 1130, the noise figure meter sends the controller its status byte. The status byte contains in its first bit the information about whether or not the calibration has been completed at this measurement frequency. Note that the actual measurement data does not have to be sent to the controller; it is stored in the noise figure meter's memory.

With steps 1060 and 1170, the measurement equipment is calibrated at the measurement frequencies three times. The 8970 performs calibration three times corresponding to three different settings of its internal gain. In this way the 8970A can set its internal gain to properly make measurements on DUTs that have low gain or high gain without saturating any internal circuits. This corresponds to the case of a power meter that has to change range in order to compensate for high gain that may occur in the equipment ahead of the power meter.

Figure 8 shows an example measurement subroutine for controlling a modern sweep oscillator from an external computer. Here the frequency loop begins in step 1410 and ends in step 1520. Lines 1430 to 1450 control the sweep oscillator. The noise figure measurement is triggered with lines

```
1000 ! CALIBRATION SUBROUTINE
1010 ! CALIBRATES THE SYSTEM.
1020 ! F1=Start Freq F2=Stop Fr
1030 ! F3=Step Size Freq' F=Fre
1040 R0=SPOLL(708)
1050 OUTPUT 708 ;"01,T1,CA"
1060 FOR A=1 TO 3 STEP 1
1070 FOR F=F1 TO F2 STEP F3
1080 IMAGE "F",5Z,"J8"
1090 OUTPUT 719 USING 1080 ; F
1100 OUTPUT 719 ;"KOL6MON703"
1110 WAIT 20
1120 OUTPUT 708 ;"T2"
1130 R0=SPOLL(708)
1140 IF BIT(R0,0)#1 THEN 1130
1150 IF F+F3>F2 AND F#F2 THEN F=F2-F3
1160 NEXT F
1170 NEXT A
1180 RETURN
```

7. Example subroutine for calibrating a microwave amplifier test setup using an HP 8672A Microwave Synthesizer as a local oscillator.

```
1300 ! *** TEST SUBROUTINE ***
1310 ! MEASURE & STORE DUT DATA
1320 ! F1=Start Freq F2=Stop Fr
1330 ! F3=Step Size Freq  F=Fre
1340 ! N=NF or Gain
1350 OPTION BASE 1
1360 DIM N(300,2)
1370 OUTPUT 708 ;"M2,01,H1,T1"
1380 OUTPUT 719 ;"IP"
1390 R0=SPOLL(708)
1400 C=0
1410 FOR F=F1 TO F2 STEP F3
1420 C=C+1
1430 IMAGE "CW",5Z,"MZ"
1440 OUTPUT 719 USING 1430 ; F
1450 WAIT 60
1460 IMAGE "FR",5D,"EN","T2"
1470 OUTPUT 708 USING 1460 ; F
1480 R0=SPOLL(708)
1490 IF BIT(R0,0)#1 THEN 1480
1500 ENTER 708 USING "11X,K,K" ; N(C,1),N(C1,2)
1510 IF F+F3>F2 AND F#F2 THEN F=F2-F3
1520 NEXT F
1530 RETURN
```

8. Example measurement subroutine that controls an HP 8350A Sweep Oscillator.

```
1900 ! CONTROLLER SUBROUTINE
1910 ! TRANSFERS CONTROL OF THE LOCAL OSCILLATOR FROM HP 85 TO 8970A AND BACK
   TO THE HP 85
1920 OUTPUT 708 ;"T0"
1930 LOCAL 708
1940 PASS CONTROL 708
1950 CLEAR
1960 DISP "**** 8970A is now in LOCAl mode and can control the LO by press-ing 4
   .1 (SP). ****"
1970 DISP
1980 DISP "When  ready to transfer control to HP 85, key in 4.0 (SP) on the8970A
    front panel."
1990 DISP "Press CONT key on the HP 85 keyboard."
2000 BEEP 20,500
2010 PAUSE
2020 ABORTIO 7
2030 REMOTE 708
2040 ENTER 708 ; B
2050 CLEAR
2060 DISP "** HP 85 IS NOW THE CONTROLLER *"
2070 RETURN
```

9. Example subroutine for switching to manual control and back to automated operation.

1460 and 1470. The noise figure meter tells the controller by means of line 1480, whether or not the measurement is completed. When it is completed, line 1500 commands the noise figure meter to send the measurement results to the controller.

When making measurements under the control of an external computer, it is best to trigger each individual measurement, and to hold the measurement result until the data is read by the computer. This prevents the noise figure meter from making a new measurement before the data is read and prevents the computer from reading old data; i.e., data from a previous measurement. The subroutines include such triggered operation. "T1" commands the 8970A not to make anymore measurements and to hold whatever measurement data it now contains. "T2" commands the 8970 to make a single measurement. After making the measurement, the noise figure meter returns to the hold (T1) state. The external computer reads the result of the single measurement before sending another "T2."

No matter how sophisticated an automatic system may be, there almost always is a need to operate the system manually. With manual operation the operator can assure that connections are properly made and the device to be tested is properly biased and operating. When the system contains a local oscillator that is controlled on the interface bus, however, going to manual or local operation means that the control of the local oscillator has to switch from the external computer to the noise figure meter. Figure 9 shows a subroutine to help the operator go from automated operation to manual operation and back again. Line 1920 places the noise figure meter back into a continuous measurement mode rather than individually triggered operations. Line 1940 tells the computer that it is no longer the controller on the bus. With line 1960 the computer displays on its CRT the instructions to the operator for telling the 8970A that it is to be the controller on the bus. Normal front panel operation is now activated.

For going back to computer operation, line 1980 contains the instructions for the operator to tell the 8970A that it will no longer be the controller on the bus. Before getting back to completely automated operation, line 2020 clears the bus of any partial messages that may have been interrupted. Line 2030 again places the 8970A in remote operation. At the end of this subroutine it may be appropriate to re-send the unit its entire front panel configuration. This would overcome the possibility that the operator may have upset the front panel setup while it was in local operation.

Although measurement programs include much more than these subroutines, the above subroutines should help avoid the trouble spots that occur in most measurement situations. ■

References

1. "Applications and Operation of the 8970A Noise Figure Meter," Hewlett-Packard Product Note 8970A-1.