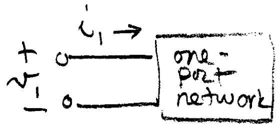
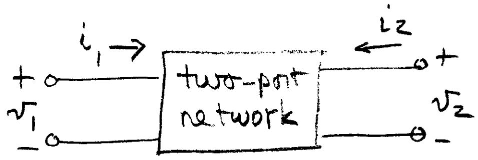
4.1 Basic Definitions





Consider writing the following voltage current relationships between voltage and current.

$$v_1 = z_{11} i_1 + z_{12} i_2$$

$$v_2 = z_{21} i_1 + z_{22} i_2$$

In matrix form

$$[V] = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = [Z][I]$$

Note that each element in this matrix can be found by measuring the voltage v_i while port i is being driven by current i_i and all other currents are zero.

For example, let $i_2 = 0$, then the matrix equations reduce to

$$v_1 = z_{11} i_1 \qquad \text{or} \qquad z_{11} = \frac{v_1}{i_1}$$

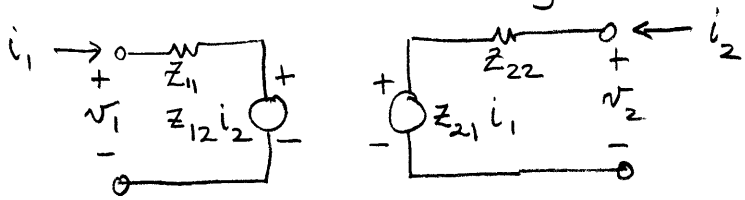
$$v_2 = z_{21} i_1 \qquad \qquad z_{21} = \frac{v_2}{i_1}$$

Similarily, if $i_1 = 0$

$$v_1 = z_{12} i_2 \qquad \text{or} \qquad z_{12} = \frac{v_1}{i_2}$$

$$v_2 = z_{22} i_2 \qquad \qquad z_{22} = \frac{v_2}{i_2}$$

Z-parameters can be represented by the following equivalent circuit



Very commonly rf engineers use y-parameters instead of z-parameters.

Y parameters are defined by the following matrix equation

$$[I] = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = [Y][V]$$

We can find the various y-parameters by <u>shorting</u> one of the outputs sequentially, i.e.

if $v_1 = 0$

$$i_1 = y_{12} v_2 \qquad y_{12} = \frac{i_1}{v_2}$$

$$i_2 = y_{22} v_2 \qquad y_{22} = \frac{i_2}{v_2}$$

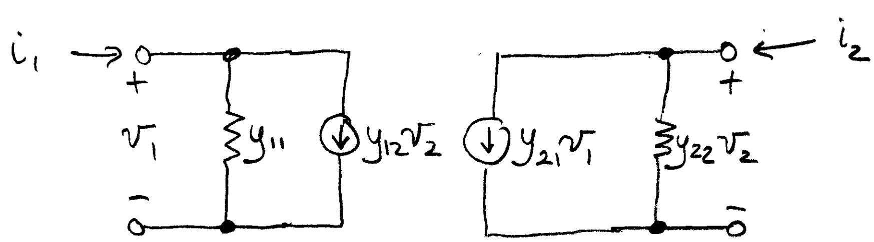
if $v_2 = 0$

$$i_1 = y_{11} v_1 \qquad y_{11} = \frac{i_1}{v_1}$$

or

$$i_2 = y_{21} v_1 \qquad y_{21} = \frac{i_2}{v_1}$$

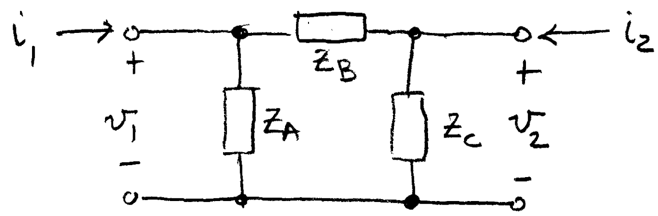
The y-parameters can be electrically modeled by the following equivalent circuit



NOTE: $[Z] = [Y]^{-1}$

Example: z-parameters of Pi-network.

For the pi-network shown below with generic impedances Z_A, Z_B and Z_C find the impedance and admittance matrices.



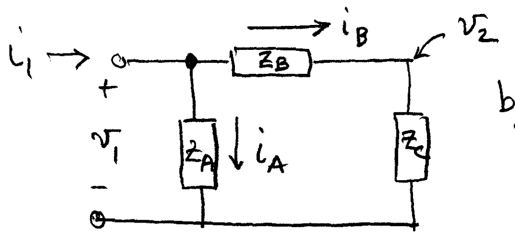
to find z_{ij} drive the input with i_1 and let the output open, i.e., $i_2 = 0$

$$\therefore \quad \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

if $i_2 = 0$ $\quad v_1 = z_{11} i_1 \quad$ or $\quad z_{11} = \dfrac{v_1}{i_1}$

$v_2 = z_{21} i_1$

$$z_{21} = \frac{v_2}{i_1}$$



by inspection $\quad v_2 = \dfrac{Z_C}{Z_B + Z_C} v_1$

$$i_A = \frac{v_1}{Z_A}$$

$$i_B = \frac{v_1}{Z_B + Z_C}$$

$$z_{11} = \frac{v_1}{i_1} = \frac{v_1}{\dfrac{v_1}{Z_A} + \dfrac{v_1}{Z_B + Z_C}} = \frac{(Z_B + Z_C)(Z_A)}{Z_A + Z_B + Z_C} \qquad i_1 = i_A + i_B = \frac{v_1}{Z_A} + \frac{v_1}{Z_B + Z_C}$$

$$z_{21} = \frac{v_2}{i_1} = \frac{\dfrac{Z_C}{Z_B + Z_C} v_1}{\dfrac{v_1}{Z_A} + \dfrac{v_1}{Z_B + Z_C}} = \frac{\dfrac{Z_C}{Z_B + Z_C}}{\dfrac{Z_B + Z_C + Z_A}{Z_A (Z_B + Z_C)}} = \frac{Z_A Z_C}{Z_B + Z_C + Z_A}$$

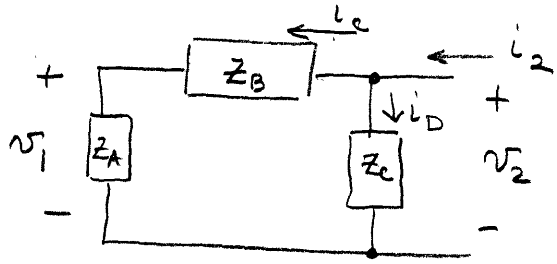
We get the remaining z parameters by repeating the process for $i_1 = 0$.

if $i_1 = 0$ (input open)

$$v_1 = z_{12}\, i_2 \qquad \text{or} \qquad z_{12} = \frac{v_1}{i_2}$$

$$v_2 = z_{22}\, i_2 \qquad\qquad z_{22} = \frac{v_2}{i_2}$$

For the given circuit



by inspection

$$v_1 = \frac{Z_A}{Z_A + Z_B}\, v_2$$

$$i_c = \frac{v_2}{Z_A + Z_B}$$

$$i_D = \frac{v_2}{Z_C}$$

$$i_2 = i_c + i_D$$

using these results in the above formulas gives

$$z_{12} = \frac{v_1}{i_2} = \frac{\dfrac{Z_A}{Z_A + Z_B}\, v_2}{\dfrac{v_2}{Z_A + Z_B} + \dfrac{v_2}{Z_C}} = \frac{\dfrac{Z_A}{\cancel{Z_A + Z_B}}}{\dfrac{Z_C + Z_A + Z_B}{\cancel{(Z_A + Z_B)}\, Z_C}}$$

$$z_{12} = \frac{Z_C Z_A}{Z_A + Z_B + Z_C}$$

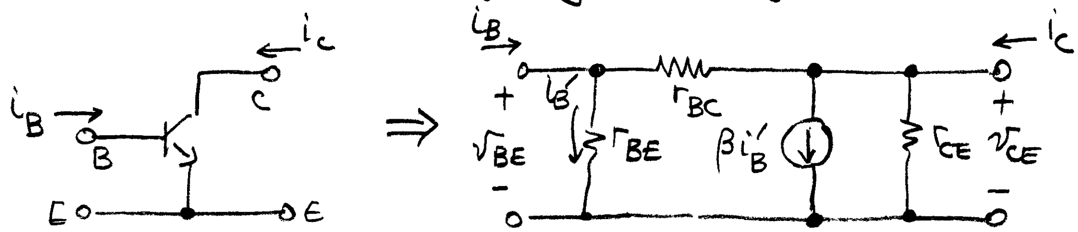
$$z_{22} = \frac{v_2}{i_2} = \frac{v_2}{\dfrac{v_2}{Z_A + Z_B} + \dfrac{v_2}{Z_C}} = \frac{1}{\dfrac{1}{Z_A + Z_B} + \dfrac{1}{Z_C}} = \frac{Z_C (Z_A + Z_B)}{Z_A + Z_B + Z_C}$$

$$z = \begin{bmatrix} \dfrac{Z_A (Z_B + Z_C)}{Z_A + Z_B + Z_C} & \dfrac{Z_A Z_C}{Z_A + Z_B + Z_C} \\[2ex] \dfrac{Z_A Z_C}{Z_A + Z_B + Z_C} & \dfrac{Z_C (Z_A + Z_B)}{Z_A + Z_B + Z_C} \end{bmatrix}$$

You can find the Y parameters the same way.

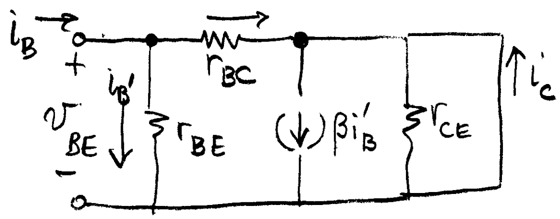
Example 4.2

Describe the common-emitter BJT transistor in terms of its h (hybrid) parameters for the low-frequency, small signal model shown below.



$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

① to evaluate h_{11} short the output, i.e. $v_2 = v_{CE} = 0$



$$i_{R_{BE}} = \frac{v_{BE}}{r_{BE}}$$

$$i_{R_{BC}} = \frac{v_{BE}\ (\text{since } v_{CE}=0)}{r_{BC}}$$

$$i_B = \frac{v_{BE}}{r_{BE}} + \frac{v_{BE}}{r_{BC}} = v_{BE}\,\frac{r_{BE}+r_{BC}}{r_{BE}r_{BC}}$$

$$i_c = -i_{r_{BC}} + \beta i_B' = -\frac{v_{BE}}{r_{BC}} + \beta\frac{v_{BE}}{r_{BE}}$$

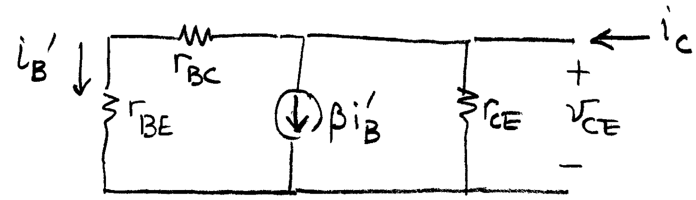
from matrix if $v_2 = 0$

$$v_1 = h_{11}\, i_1 \Rightarrow h_{11} = \frac{v_1}{i_1} = \frac{v_{BE}}{i_B}$$

$$i_2 = h_{21}\, i_1 \Rightarrow h_{21} = \frac{i_2}{i_1} = \frac{i_c}{i_B}$$

$$h_{11} = \frac{v_{BE}}{i_B} = \frac{v_{BE}}{v_{BE}\left(\frac{r_{BE}+r_{BC}}{r_{BE}r_{BC}}\right)} = \frac{r_{BE}r_{BC}}{r_{BE}+r_{BC}}$$

$$h_{21} = \frac{i_c}{i_B} = \frac{v_{BE}\left(\frac{-r_{BE}+\beta r_{BC}}{r_{BC}\,r_{BE}}\right)}{v_{BE}\left(\frac{r_{BE}+r_{BC}}{r_{BE}\,r_{BC}}\right)} = \frac{-r_{BE}+\beta r_{BC}}{r_{BE}+r_{BC}} = \frac{\beta r_{BC}-r_{BE}}{r_{BE}+r_{BC}}$$



$$i_B' = \frac{v_{CE}}{r_{BC}+r_{BE}}$$

$$i_{R_{CE}} = \frac{v_{CE}}{r_{CE}}$$

$$\beta i_B' = \beta \frac{v_{CE}}{r_{BC}+r_{BE}}$$

$$i_c = \frac{v_{CE}}{r_{BC}+r_{BE}} + \beta \frac{v_{CE}}{r_{BC}+r_{BE}} + \frac{v_{CE}}{r_{CE}}$$

$$v_{BE} = \frac{r_{BE}}{r_{BE}+r_{BC}} v_{CE}$$

from matrix if $i_1 = i_B = 0$

$$v_1 = h_{12} v_2 \qquad h_{12} = \frac{v_1}{v_2}$$

$$i_2 = h_{22} v_2 \qquad h_{22} = \frac{i_2}{v_2}$$

$$h_{12} = \frac{v_1}{v_2} = \frac{\frac{r_{BE}}{r_{BE}+r_{BC}} v_{CE}}{v_{CE}} = \frac{r_{BE}}{r_{BE}+r_{BC}}$$

$$h_{22} = \frac{i_2}{v_2} = \frac{(1+\beta)\frac{v_{CE}}{r_{BC}+r_{BE}} + \frac{v_{CE}}{r_{CE}}}{v_{CE}} = \frac{(1+\beta)}{r_{BC}+r_{BE}} + \frac{1}{r_{CE}}$$

$$h_{22} = \frac{(1+\beta) r_{CE} + r_{BC} + r_{BE}}{r_{CE}(r_{BC}+r_{BE})}$$

$$[h] = \begin{bmatrix} \frac{r_{BE} r_{BC}}{r_{BE}+r_{BC}} & \frac{r_{BE}}{r_{BE}+r_{BC}} \\ \frac{\beta r_{BC} - r_{BE}}{r_{BE}+r_{BC}} & \frac{(\beta+1)}{r_{BC}+r_{BE}} + \frac{1}{r_{CE}} \end{bmatrix} \approx \begin{bmatrix} r_{BE} & 0 \\ \beta & \frac{1}{r_{CE}} + \frac{\beta}{r_{BC}} \end{bmatrix}$$

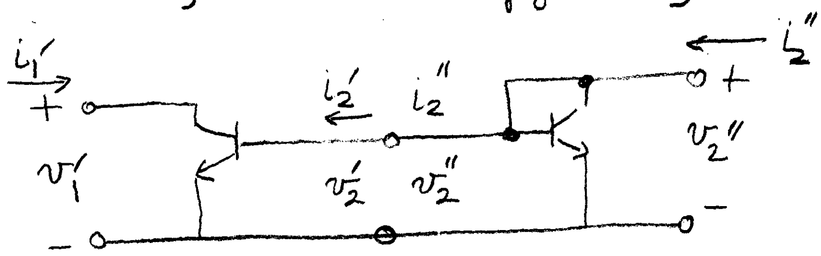
for real transistors $\beta \gg 1$, $r_{BC} \gg r_{BE}$

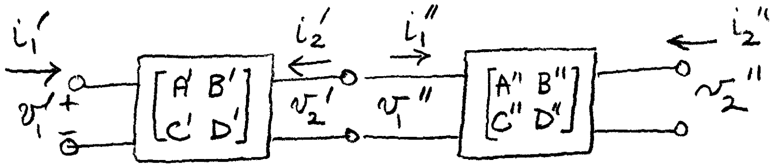
The hybrid network representation is a very popular way to characterize the BJT, and h-parameter coefficients are reported in many data sheets.

Define the chain or ABCD parameters as

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

Cascading networks analyzed using ABCD matrices





For the first network

$$\begin{bmatrix} v_1' \\ i_1' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} v_2' \\ -i_2' \end{bmatrix}$$

For the second network

$$\begin{bmatrix} v_1'' \\ i_1'' \end{bmatrix} = \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} v_2'' \\ -i_2'' \end{bmatrix}$$

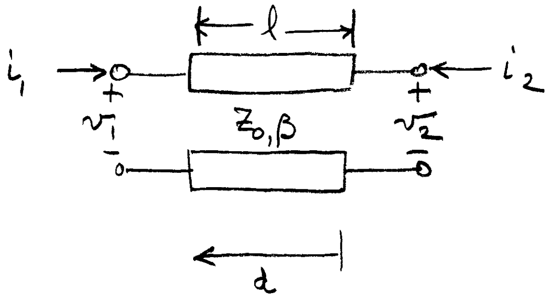
The key thing about cascading networks is that $i_1'' = -i_2'$
and $v_1'' = v_2'$

Then

$$\begin{bmatrix} v_1' \\ i_1' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} v_2' \\ -i_2' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} v_1'' \\ i_1'' \end{bmatrix}$$

$$\begin{bmatrix} v_1' \\ i_1' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} v_1'' \\ i_1'' \end{bmatrix}$$

Example 4-6 ABCD-matrix coefficient representation of a transmission line section.



$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

if we short port 2, i.e. $v_2 = 0$ (a shorted line)

$$v_1 = -B\, i_2 \qquad B = -\frac{v_1}{i_2}$$

$$i_1 = -D\, i_2 \qquad D = -\frac{i_1}{i_2}$$

If we open port 2, i.e., $i_2 = 0$ (an open line)

$$v_1 = A\, v_2 \qquad A = \frac{v_1}{v_2}$$

$$i_1 = C\, v_2 \qquad C = \frac{i_1}{v_2}$$

For a open-line $\quad V(d) = 2V^+ \cos(\beta d)$

$\quad I(d) = \dfrac{2jV^+}{Z_0} \sin(\beta d)$ current defined as towards load

For a short line $\quad V(d) = 2jV^+ \sin(\beta d)$

$\quad I(d) = \dfrac{2V^+}{Z_0} \cos(\beta d)$ current defined as towards load.

using these results for a transmission line.

$$A = \left. \frac{v_1}{v_2} \right|_{i_2 = 0} = \frac{2V^+ \cos(\beta \ell)\big|_{d=\ell}}{2V^+ \cos(\beta \cdot 0)\big|_{d=0}} = \cos \beta \ell$$

$$B = -\left. \frac{v_1}{i_2} \right|_{v_2 = 0} = \frac{2jV^+ \sin(\beta \ell)}{\frac{2V^+}{Z_0} \cos(\beta \cdot 0)} = jZ_0 \sin(\beta \ell)$$

$$C = \left.\frac{i_1}{v_2}\right|_{i_2=0} = \frac{2j\frac{V^+}{Z_0}\sin(\beta l)}{2V^+\cos(\beta\cdot 0)} = jY_0\sin(\beta l)$$

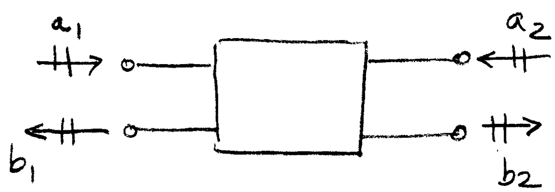
$$D = -\left.\frac{i_1}{i_2}\right|_{v_2=0} = -\frac{\frac{2V^+}{Z_0}\cos(\beta l)}{2\frac{V^+}{Z_0}\cos(\beta\cdot 0)} = -\cos(\beta l)$$

$\therefore$ For a transmission line

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta l & jZ_0\sin\beta l \\ jY_0\sin\beta l & -\cos\beta l \end{bmatrix}$$

4.4 Scattering parameters

Almost all microwave engineers use S-parameters (scattering parameters) at microwave frequencies because it is very difficult to achieve a true open or short at r.f./microwave frequencies. Furthermore, you do not want to introduce large reflection coefficients which can lead to oscillations and/or destroy a semiconductor device.



define a <u>NORMALIZED</u> incident power wave

$$a_1 = \frac{1}{2\sqrt{Z_o}}(V_1 + Z_o I_1)$$ where I_1, V_1 are at the input

I_2, V_2 are at the output

$$a_2 = \frac{1}{2\sqrt{Z_o}}(V_2 + Z_o I_2)$$

and a reflected normalized power

$$b_1 = \frac{1}{2\sqrt{Z_o}}(V_1 - Z_o I_1)$$

$$b_2 = \frac{1}{2\sqrt{Z_o}}(V_2 - Z_o I_2)$$

If we solve these equations for V and I we get

$$V_1 = \sqrt{Z_o}(a_1 + b_1) \quad (1)$$

$$V_2 = \sqrt{Z_o}(a_2 + b_2) \quad (2)$$

$$I_1 = \frac{1}{\sqrt{Z_o}}(a_1 - b_1) \quad (3)$$

$$I_2 = \frac{1}{\sqrt{Z_o}}(a_2 - b_2) \quad (4)$$

These look like strange definitions but consider these in terms of traveling waves and power

If you simply examine (1) & (3) a_1 is simply the forward traveling wave and b_1 is the backward traveling wave.

$$a_1 = \frac{V_1^+}{\sqrt{Z_0}} = \sqrt{Z_0}\, I_1^+$$

$$b_1 = \frac{V_1^-}{\sqrt{Z_0}} = -\sqrt{Z_0}\, I_1^-$$

Note that $V^+ = I_1^+ Z_0$ and $V_1^- = -Z_0 I_1^-$

$$a_2 = \frac{V_2^+}{\sqrt{Z_0}} = \sqrt{Z_0}\, I_2^+$$

$$b_2 = \frac{V_2^-}{\sqrt{Z_0}} = -\sqrt{Z_0}\, I_2^-$$

The S-parameters are closely related to power.

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \begin{matrix} \leftarrow \text{incident \& reflected at port \#1} \\ \leftarrow \text{incident \& reflected at port \#2} \end{matrix}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \left.\frac{\text{reflected power wave at port \#1}}{\text{incident power wave at port \#1}}\right| \text{no input power at port \#2}$$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} = \left.\frac{\text{transmitted power wave at port \#1}}{\text{incident power wave at port \#2}}\right| \text{no input power at port \#1}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} = \left.\frac{\text{transmitted power wave at port \#2}}{\text{incident power wave at port \#1}}\right| \text{no input power at port \#2}$$

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} = \left.\frac{\text{reflected power wave at port \#2}}{\text{incident power wave at port \#1}}\right| \text{no input power at port \#1}$$

these conditions
are true when input (a_1)
or output (a_2) are
matched to port impedance.

Recall that $P_{AV} = \frac{1}{2} \operatorname{Re}\{V I^*\}$

At the input port

$$P_1 = \frac{1}{2} \operatorname{Re}\{V_1 I_1^*\} = \frac{1}{2} \operatorname{Re}\left\{ V^+ (1+\Gamma_{in}) \frac{V^+}{Z_0} (1-\Gamma_{in}) \right\}$$

$$P_1 = \frac{1}{2} \frac{|V^+|^2}{Z_0} \left(1 - |\Gamma_{in}|^2\right) = P_1^+ + P_1^-$$

using our definitions that

$$V_1 = \sqrt{Z_0}\,(a_1 + b_1)$$

$$I_1 = \frac{1}{\sqrt{Z_0}} (a_1 - b_1)$$

we can also compute the power at port #1 as

$$P_1 = \frac{1}{2} \operatorname{Re}\{V_1 I_1^*\} = \frac{1}{2} \operatorname{Re}\left\{ \sqrt{Z_0}\,(a_1 + b_1) \frac{1}{\sqrt{Z_0}} (a_1^* - b_1^*) \right\}$$

$$P_1 = \frac{1}{2} \operatorname{Re}\left\{ |a_1|^2 - |b_1|^2 \right\} = \frac{1}{2}\left\{|a_1|^2 - |b_1|^2\right\} = \frac{1}{2} |a_1|^2 \left\{ 1 - \frac{|b_1|^2}{|a_1|^2} \right\}$$

but $a_1 = \dfrac{V_1^+}{\sqrt{Z_0}}$ so we can re-write this equation as

$$P_1 = \frac{1}{2} \frac{|V_1^+|^2}{Z_0} \left\{ 1 - \left|\frac{b_1}{a_1}\right|^2 \right\} = \frac{1}{2} \frac{|V_1^+|^2}{Z_0} \left\{ 1 - |S_{11}|^2 \right\}$$

Comparing these two expressions for P_1 we quickly see that

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = S_{11}$$

This allows us the write the SWR at port #1 as

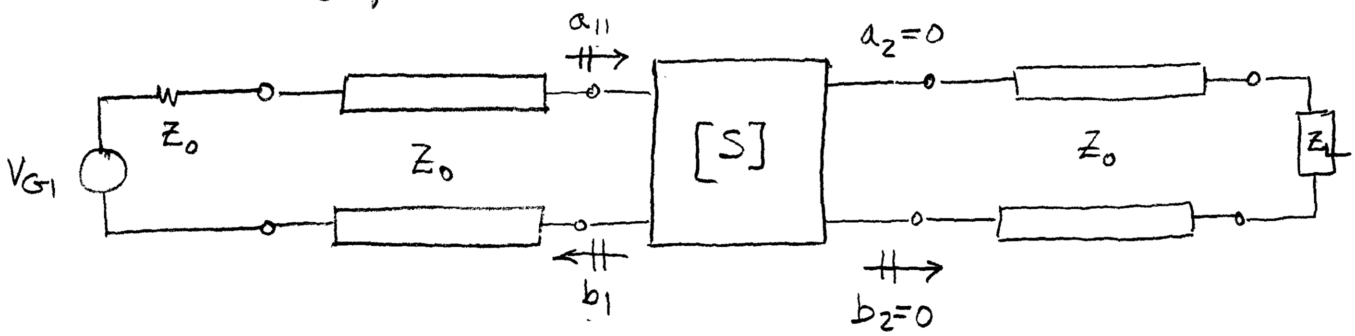
$$S = \frac{1 + |S_{11}|}{1 - |S_{11}|}$$

Note also that $\frac{1}{2}|a_1|^2 = \frac{1}{2} \frac{|V_1^+|^2}{Z_0} = P_{incident}$

You can do the same analysis at the output to get at port #2

$$P_2 = \frac{1}{2}\left\{ |a_2|^2 - |b_2|^2 \right\} = \frac{|a_2|^2}{2} \left(1 - |\Gamma_{out}|^2\right).$$

4.4.2. Meaning of s-parameters



the output Z_L is matched to Z_o so that no V_2^+ is created at the output.

under the about output matched conditions $a_2=0$.

Since $S_{11} = \Gamma_{in} = \dfrac{Z_{in}-Z_o}{Z_{in}+Z_o}$ since $S_{11} = \left.\dfrac{b_1}{a_1}\right|_{b_2=0}$

we have just developed a method for measuring S_{11}

also since $a_2=0$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{b_2=0} = \left.\frac{\dfrac{V_2^-}{\sqrt{Z_o}}}{\dfrac{1}{2\sqrt{Z_o}}(V_1+Z_oI_1)}\right|_{I_2^+=V_2^+=0}$$

Note that V_2^+, I_2^+ are going <u>into</u> the output just like a two-port

substituting $V_1 = V_{G1} - Z_oI_1$

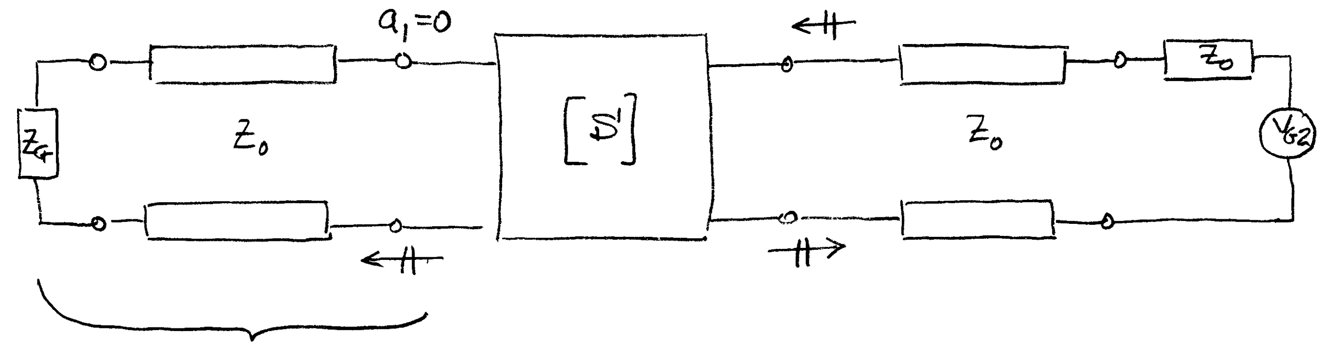
$$S_{21} = \frac{2V_2'}{V_{G1}-Z_oI_1+Z_oI_1} = \frac{2V_2'}{V_{G1}} = \frac{2V_2}{V_{G1}}$$

This is the forward <u>voltage</u> gain G of the network

$$G_o = |S_{21}|^2 = \left|\frac{V_2}{V_{G1}/2}\right|^2$$

is the forward <u>power</u> gain

You can measure S_{22} and S_{12} by matching at port #1 and using a generator at port #2.



line is matched to the impedance Z_G to make sure no V_1^- is created at the load

The results at the output are identical to the input

$$S_{22} = \Gamma_{out} = \frac{Z_{out} - Z_0}{Z_{out} + Z_0} \qquad \text{since} \quad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

Since $a_1 = 0$

$$S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0} = \left.\frac{\frac{V_1^-}{\sqrt{Z_0}}}{\frac{1}{2\sqrt{Z_0}}\left(V_2 + Z_0 I_2\right)}\right|_{I_1^+ = V_1^+ = 0}$$

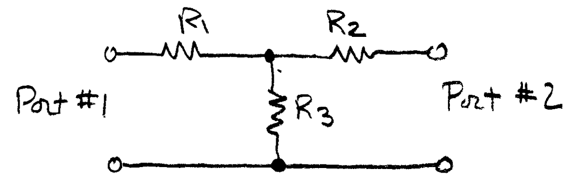
Note again that I_1^+, V_1^+ are going into the input just like a two-port

Canceling terms and substituting $V_2 = V_{G2} - Z_0 I_2$.

$$S_{12} = \frac{2V_1^-}{V_{G2} - Z_0 I_2 + Z_0 I_2} = \frac{2V_1^-}{V_{G1}} = \frac{2V_1}{V_{G1}}$$

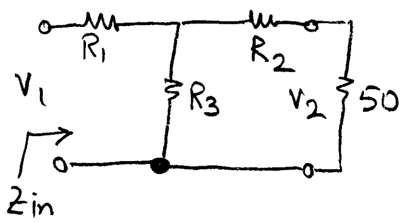
This is the reverse voltage gain.

Example 4-7 Find the s-parameters and the resistive elements for the 3dB attenuator network shown below assuming that the network is placed in a transmission line section with a characteristic line impedance of $Z_0 = 50\Omega$



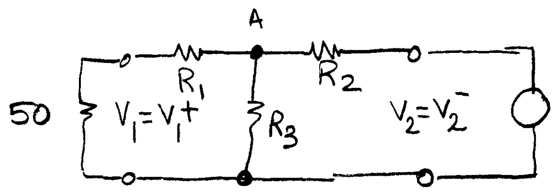
The attenuator must be matched to the line impedance Z_0

For a terminated output $(z = 50\Omega)$ we have



$$Z_{in} = R_1 + \frac{R_3(R_2+50)}{R_3 + R_2 + 50} = 50$$

For the terminated input $(Z = 50\Omega)$ we have the same circuit



if matched
no reflection

For the voltage relationship we have the output voltage given by

$$V_2 = \underbrace{\frac{R_3 \| (R_2+50)}{R_3 \| (r_2 + 50) + R_2}}_{\text{voltage at A}} \cdot \underbrace{\frac{50}{R_1 + 50}}_{\text{voltage at } 50\Omega \text{ input termination}} V_1$$

For 3 db attenuation (no input power at port #2)

$$S_{21} = \left.\frac{V_2}{V_1}\right|_{I_2^+ = V_2^+ = 0} = \frac{1}{\sqrt{2}} = \frac{2V_2}{V_{G1}}$$

This is where we bring in the s-parameters.

This gives two equations.
Since we want the network to be symmetric $R_1 = R_2$.
leaving two equations in two unknowns.

These can be solved to give

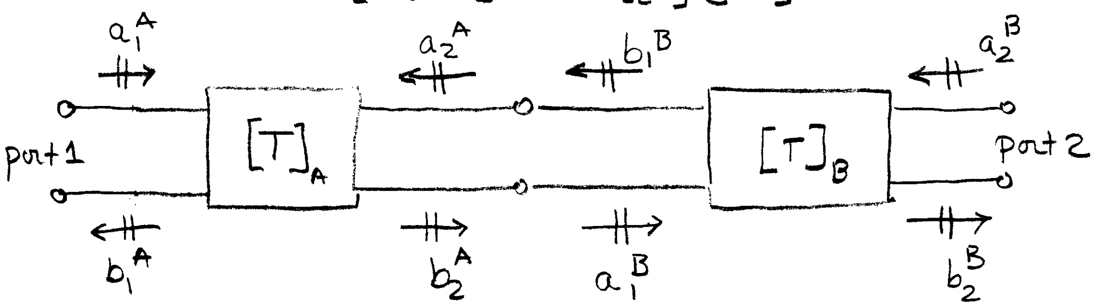
$$R_1 = R_2 = \frac{\sqrt{2}-1}{\sqrt{2}+1} Z_0 = 8.58\Omega$$

$$R_3 = 2\sqrt{2} Z_0 = 141.4\Omega.$$

4.4.3. Chain Scattering Matrix

We can re-arrange the S matrix to group input and output terms together. This is for cascading networks just like the ABCD matrix.

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} b_2 \\ a_2 \end{bmatrix}$$



Network A is described by
$$\begin{bmatrix} a_1^A \\ b_1^A \end{bmatrix} = \begin{bmatrix} T_{11}^A & T_{12}^A \\ T_{21}^A & T_{22}^A \end{bmatrix} \begin{bmatrix} b_2^A \\ a_2^A \end{bmatrix}$$

inputs outputs

Network B is described by
$$\begin{bmatrix} a_1^B \\ b_1^B \end{bmatrix} = \begin{bmatrix} T_{11}^B & T_{12}^B \\ T_{21}^B & T_{22}^B \end{bmatrix} \begin{bmatrix} b_2^B \\ a_2^B \end{bmatrix}$$

since
$$\begin{bmatrix} b_2^A \\ a_2^A \end{bmatrix} = \begin{bmatrix} a_1^B \\ b_1^B \end{bmatrix}$$
Note change in direction reverses a and b.

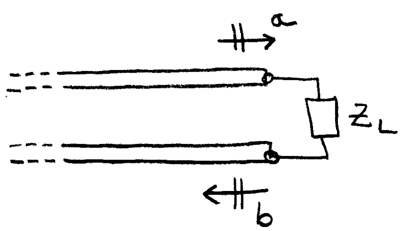
we can rewrite as

$$\begin{bmatrix} a_1^A \\ b_1^A \end{bmatrix} = \begin{bmatrix} T_{11}^A & T_{12}^A \\ T_{21}^A & T_{22}^A \end{bmatrix} \begin{bmatrix} T_{11}^B & T_{12}^B \\ T_{21}^B & T_{22}^B \end{bmatrix} \begin{bmatrix} b_2^B \\ a_2^B \end{bmatrix}.$$

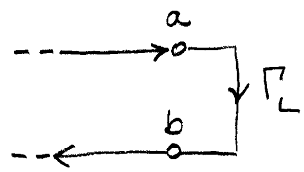
T_{ij} and S_{ij} can be algebraically related.

S_{ij} can also be related to Z_{ij} parameters

4.4.5. Signal Flow Chart Modeling



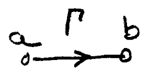
conventional transmission line representation



equivalent signal flow representation

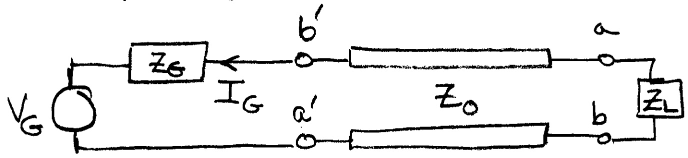
source node a which launches a wave

sink node b which receives a wave

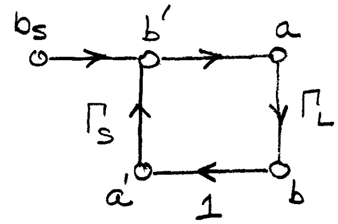


branch which connects source and sink $b = \Gamma a$

Simple system



signal generator



equivalent signal flow chart

Note: Don't interpret variables as static voltages but waves propagating either L→R or R→L

by inspection

$$b' = b_s + a'\Gamma_s$$

giving the source as $b_s = b' - a'\Gamma_s'$

We can compare this with the wave based equivalent representation.

$$\underbrace{V_S^+ + V_S^-}_{\text{at input of line}} = \underbrace{V_G}_{\text{generator}} + \underbrace{Z_G\left[\frac{V_S^+}{Z_o} - \frac{V_S^-}{Z_o}\right]}_{\text{voltage drop across } Z_G}$$

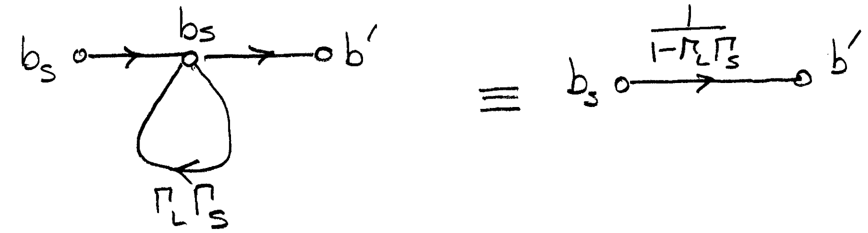
Note that $a' = \Gamma_L b'$

so that $b' = b_s + a'\Gamma_s = b_s + \Gamma_L \Gamma_s b'$

$\therefore \quad b'(1 - \Gamma_L\Gamma_s) = b_s$

$$b' = \frac{b_s}{1 - \Gamma_L \Gamma_s}$$

In signal flow chart terms this can be reduced to a single branch



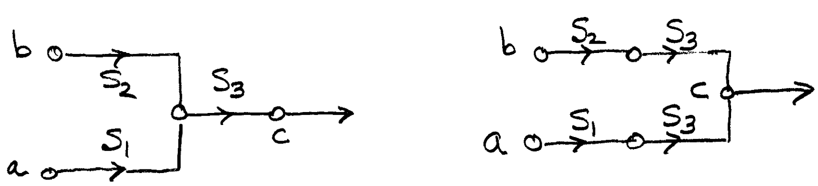
<u>Basic signal flowchart elements.</u>

node assignment

branch

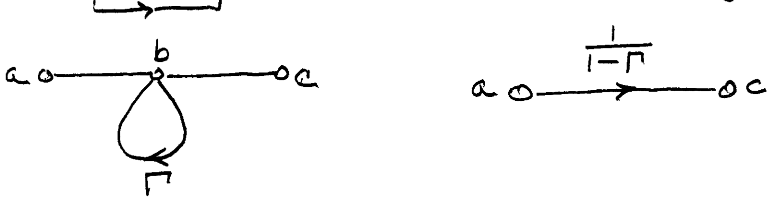
series connection

splitting of branches

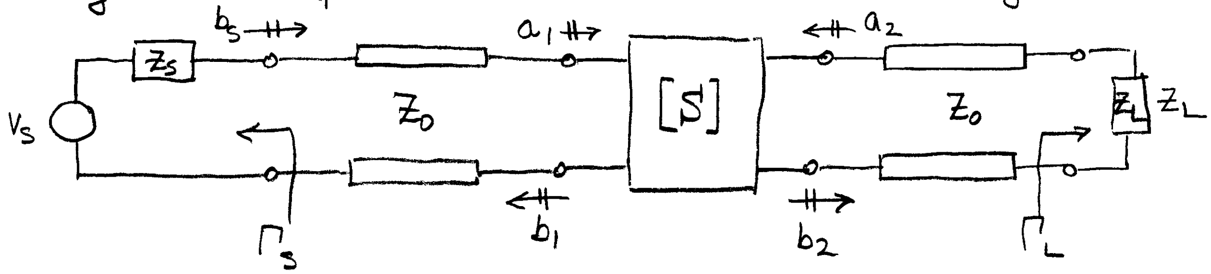


parallel connection

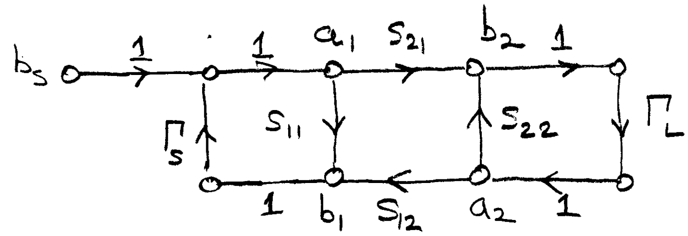
self-loop



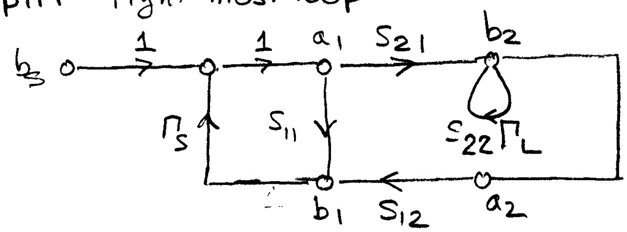
Example 4-8 For the network shown below find the ratio of a_1/b_s. Assume unity for the multiplication factor of the transmission line segments.



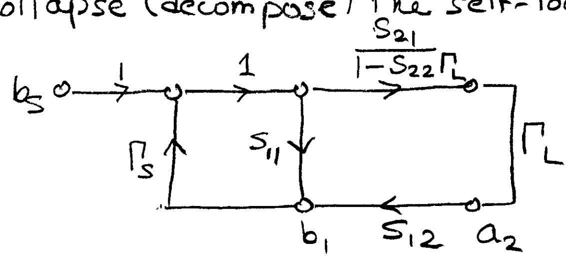
Initial signal flow chart



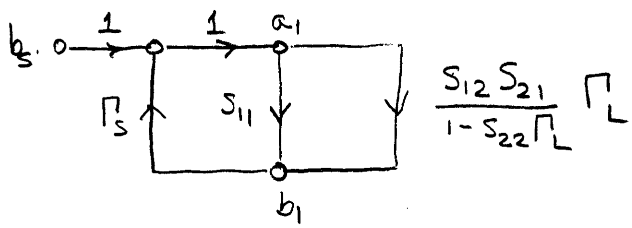
step 1: split right-most loop



step 2: collapse (decompose) the self-loop between b_2 and a_2

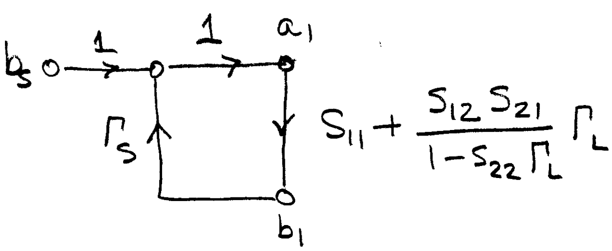


step 3:

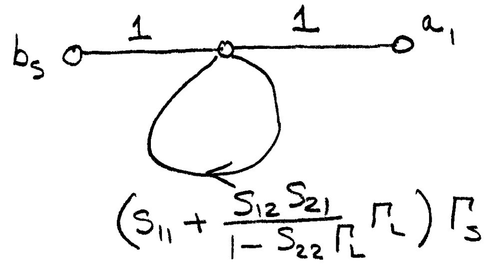


then combine series elements

and parallel elements



step 4: split loop into a self-loop



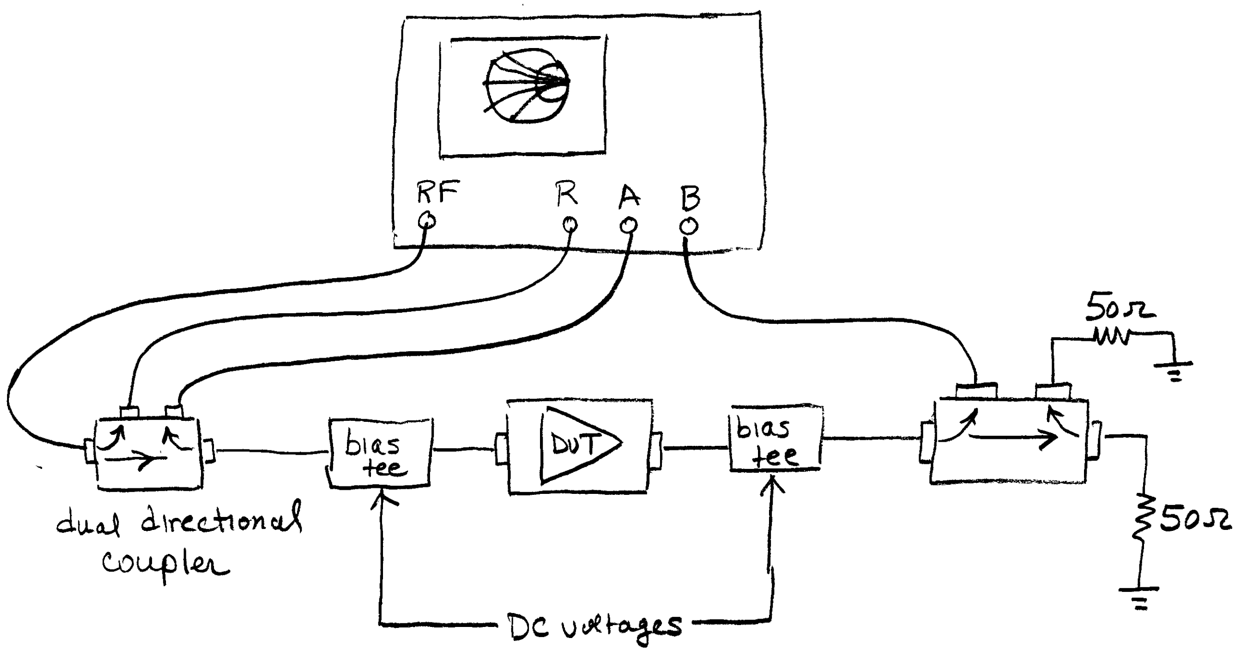
step 5: decompose the self-loop

$$b_s \xrightarrow[\dfrac{1}{1-\left(S_{11}+\dfrac{S_{12}S_{21}}{1-S_{22}\Gamma_L}\Gamma_L\right)\Gamma_S}]{} a$$

$$\therefore \quad a_1 = \frac{1}{1-\left(S_{11}+\dfrac{S_{12}S_{21}}{1-S_{22}\Gamma_L}\Gamma_L\right)\Gamma_S} \cdot b_S .$$

$$\frac{a_1}{b_s} = \frac{1-S_{22}\Gamma_L}{1-(S_{11}\Gamma_S + S_{22}\Gamma_L + S_{12}S_{21}\Gamma_S) + S_{11}S_{22}\Gamma_S\Gamma_L}$$

4.4.7. Practical Measurement of S-parameters.

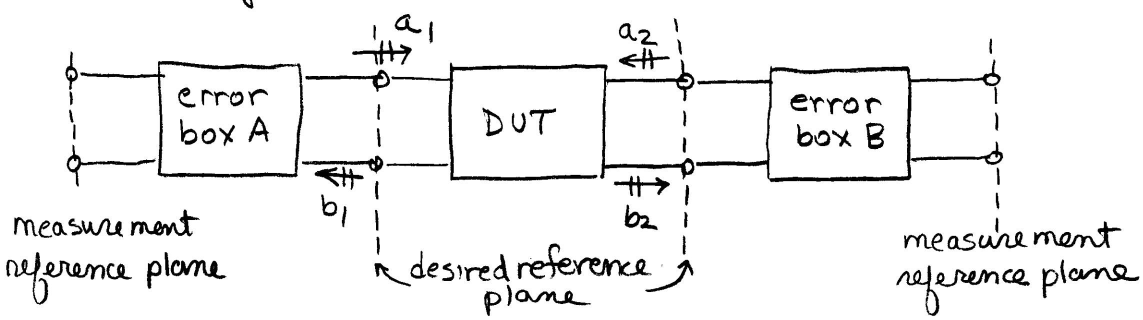


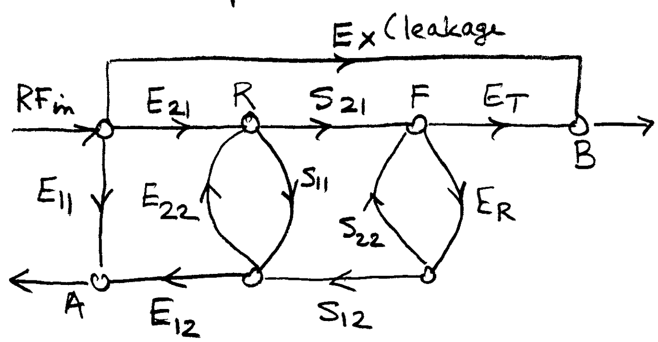
Basic concept is relatively simple. The ratio A/R gives S_{11}; S_{21} comes from B/R.

You can measure S_{12} and S_{22} by reversing the DUT.

Real system is much more complex because of cable lengths, impedances, non-ideal external components, etc.

Practical system for measurement



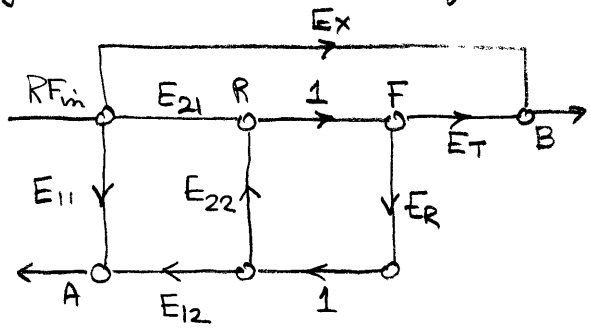


A lot of research involves using a computer and three known loads (open, short, and matched) to estimate $E_{11}, E_{12}, E_{22}, E_X, E_R$ and E_T.

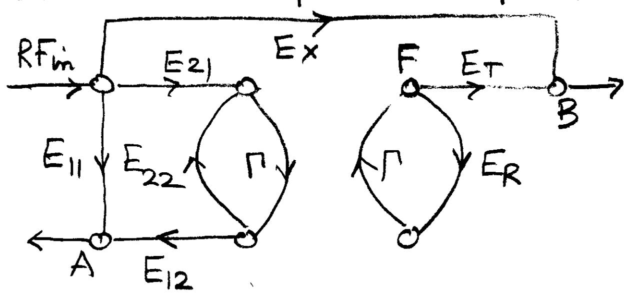
Another popular method is the Through–Reflect–Line (TRL) method.

Through: directly connect ports 1 and 2 of the DUT

(a through short, not an end short)

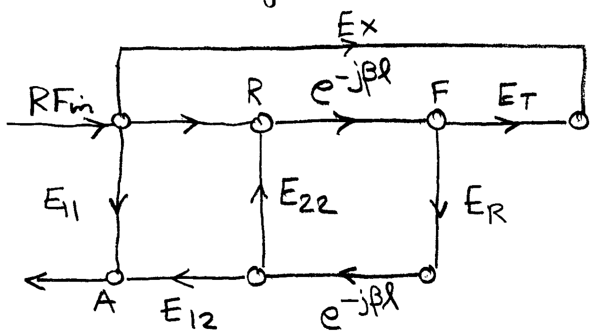


Reflect: use a load with high reflectivity and the same reflection coefficient for both input and output ports of the DUT.



Through: Connect ports 1 and 2 by a transmission line matched to the impedance of the error boxes.

(matched short)



See: G. F. Engen & C. A. Hoer, "Thru-Reflect-Line: An Improved Technique for Calibrating the Dual Six-Port Automatic Network Analyzer," IEEE Trans. Microwave Theory and Techniques, Vol. MTT-27, pp. 987–998, 1979.