

Example 3-7

Identify (a) the normalized impedance value  $\bar{z} = 0.5 + j0.5$  and (b) the normalized admittance value  $\bar{y} = 1 + j2$  in the combined ZY-Smith Chart and find the corresponding values of normalized admittance and impedance.

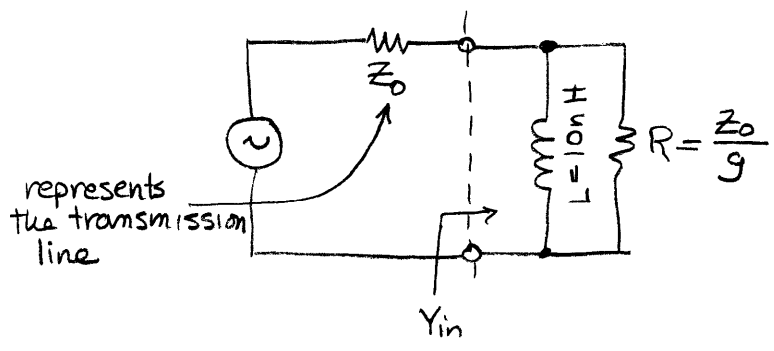
These are gotten from the Smith Chart by switching between the impedance and admittance charts.

For  $\bar{z}$  the corresponding admittance is  $1 - j1$

For  $\bar{y}$  the corresponding impedance is  $0.2 - j0.4$

We can use the Smith Chart to include lumped circuit elements.

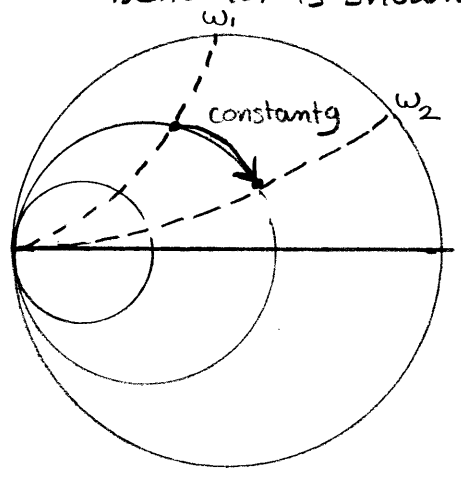
Parallel connection of R & L Elements.



where we have normalized  $Y_{in}$  to  $Z_0$ .

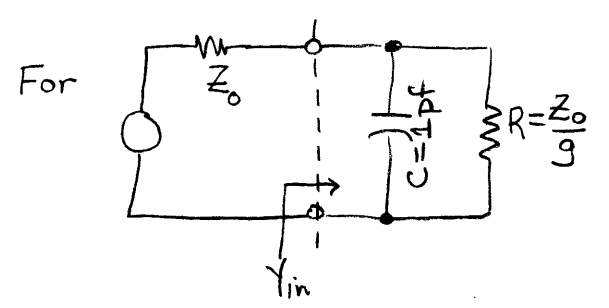
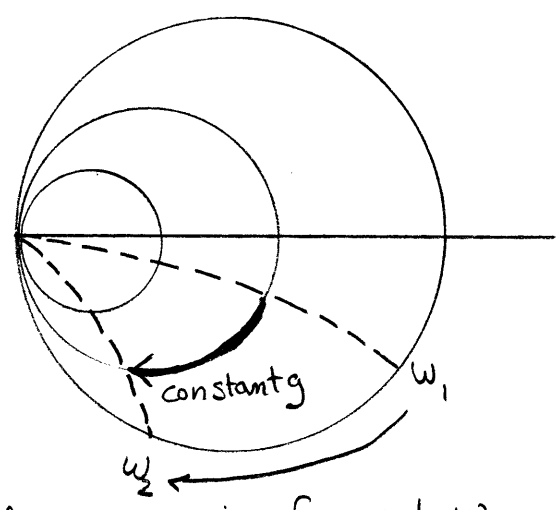
In general  $Y_{in} = g - j \frac{Z_0}{\omega L}$

Notice that as  $\omega$  increases  $b$  changes, but  $g$  remains constant. The overall behavior is shown below.



As  $\omega$  increases from  $\omega_1$  to  $\omega_2$  the conductance  $g$  remains constant, and  $b = -\frac{Z_0}{\omega L}$  decreases.

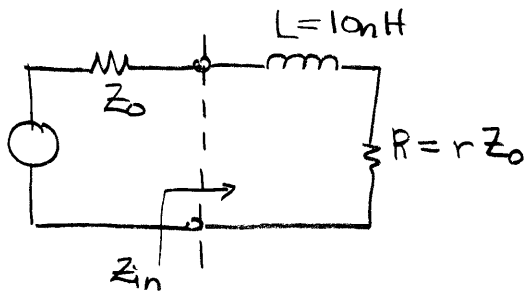
Parallel connection of R & C elements.



$Y_{in} = g + j Z_0 \omega C$

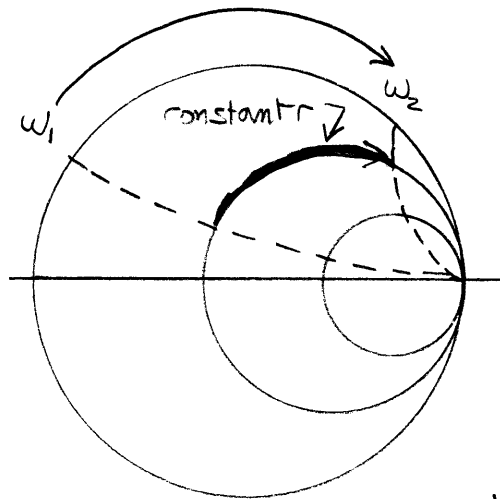
As  $\omega$  increases from  $\omega_1$  to  $\omega_2$  again the conductance  $g$  remains constant and  $b = Z_0 \omega C$  increases.

Series connection of R & L elements.



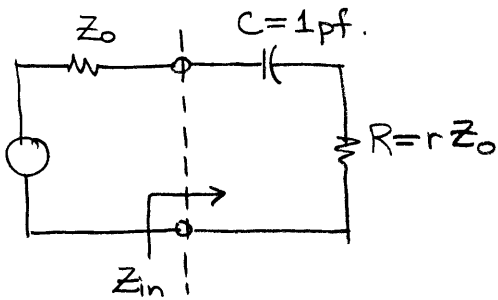
$$Z_{in} = r + j\omega \frac{L}{Z_0}$$

Normalized



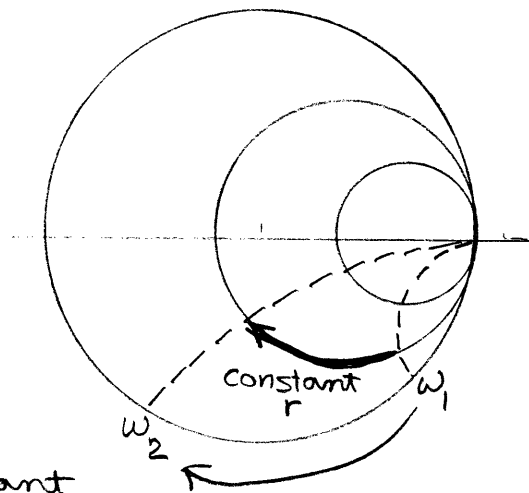
As  $\omega$  increases  $r$  remains constant but  $x$  travels an arc.

Series connection of R & C elements:

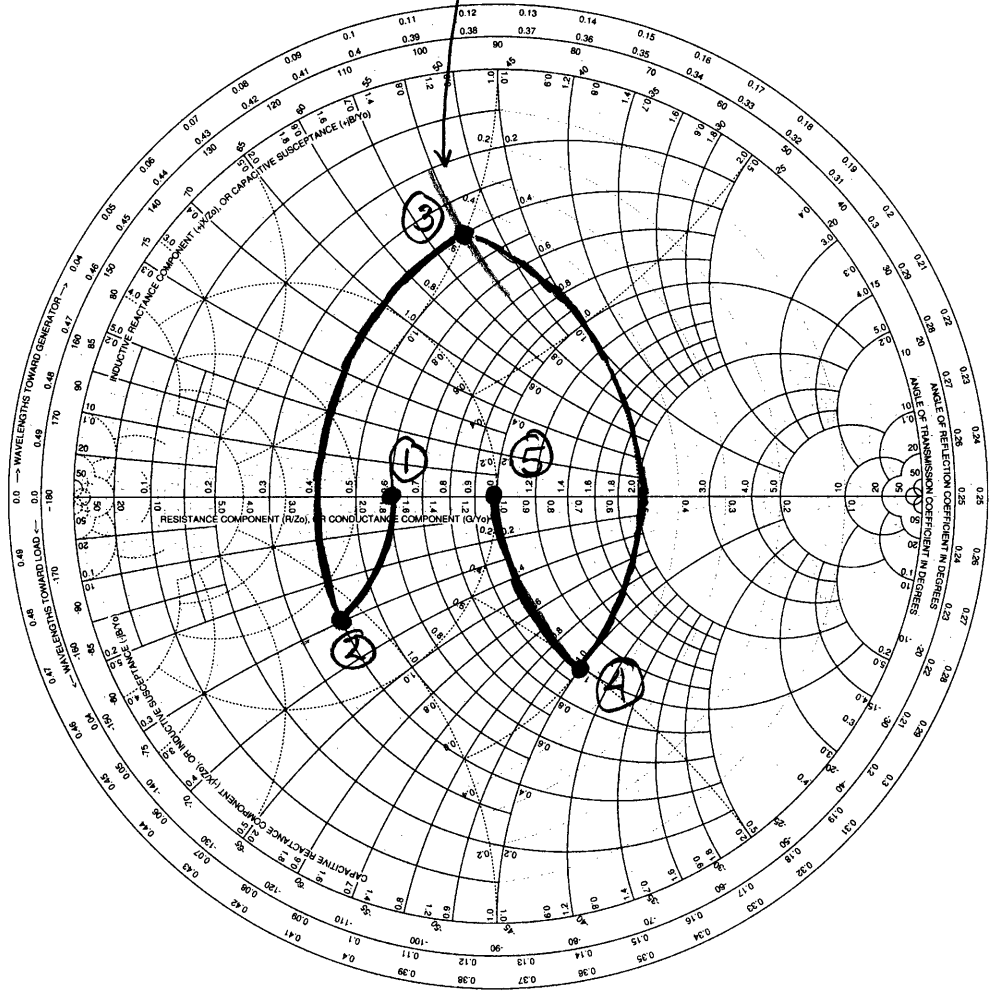


$$Z_{in} = r - j\omega C Z_0$$

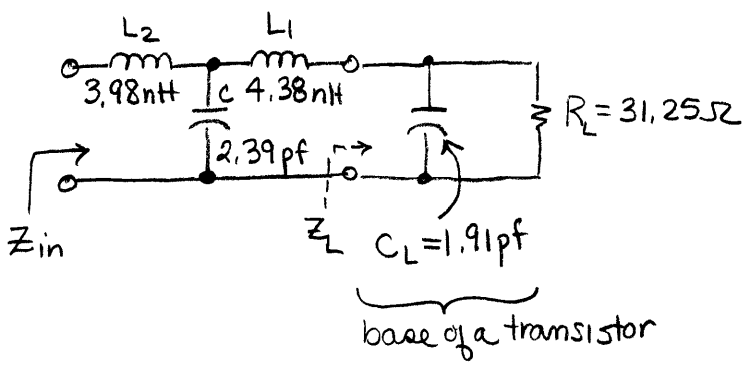
As  $\omega$  increases  $r$  remains constant but  $x$  decreases along an arc.



$\bar{z} = 0.4 + j0.8$   
 $\bar{y} = 0.5 - j1$



Example: T-network @ 2GHz



① start with load resistor  
 Assume  $Z_0 = 50 \Omega$

$g = \frac{50}{31.25} = 1.6$  (no b)

② Now add  $C_L$

$\omega L C_L = 2\pi(2 \times 10^9)(1.91 \times 10^{-12})$

$\omega L C_L = .024$

The normalized bis then  
 $b = \omega L C_L Z_0 = .024(50) = 1.2$

This describes a circle of constant  $g$  until we reach  $b = 1.2$

③ Reading the  $z$ -chart this point is at  $0.4 - j0.3$ . Point ②

Adding a series  $L_1$  will keep this same  $z$  but will add a reactance

$$+ \frac{\omega L_1}{Z_0} = + \frac{2\pi(2 \times 10^9)(4.38 \times 10^{-9})}{50} = +1.1$$

Adding  $+1.1$  to  $-0.3$  gives  $+0.8$  on a constant  $r = 0.4$  circle.

$Z = 0.4 + j0.8$

④ Now we switch back to admittance to add the parallel C. The admittance is  $y = 0.5 - j1$

$$b = \omega C Z_0 = 2\pi(2 \times 10^9)(2.39 \times 10^{-12})(50) = +1.50$$

$$\text{Adding } (0.5 - j1) + (+j1.50) = 0.5 + j0.5$$

This is point ④.

⑤ We convert back to Z. Reading from the chart

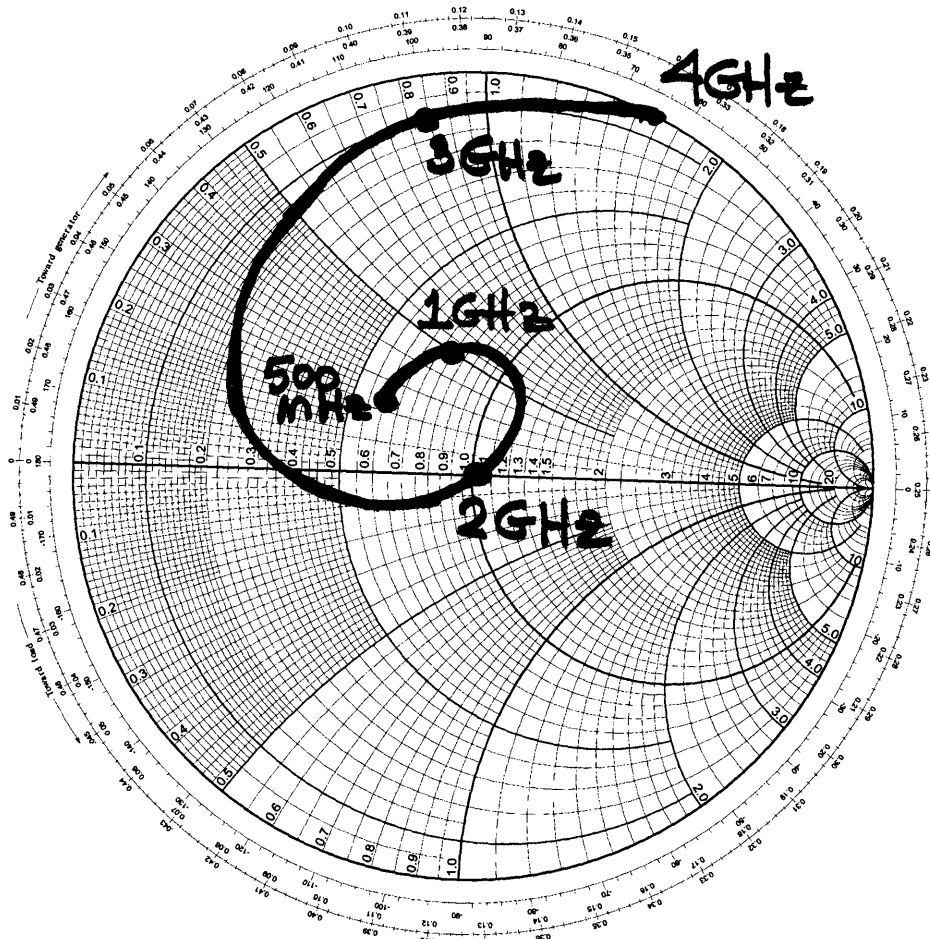
$$z = 1 - j1$$

We compute the impedance associated with  $L_2$  and add.

$$+ \frac{\omega L_2}{Z_0} = + \frac{2\pi(2 \times 10^9)(3.98 \times 10^{-9})}{50} = +1.00$$

$-1 + 1$  gives zero and takes us to ⑤

$$z = (1 - j1) + (j1) = 1 \quad (\text{A match})$$

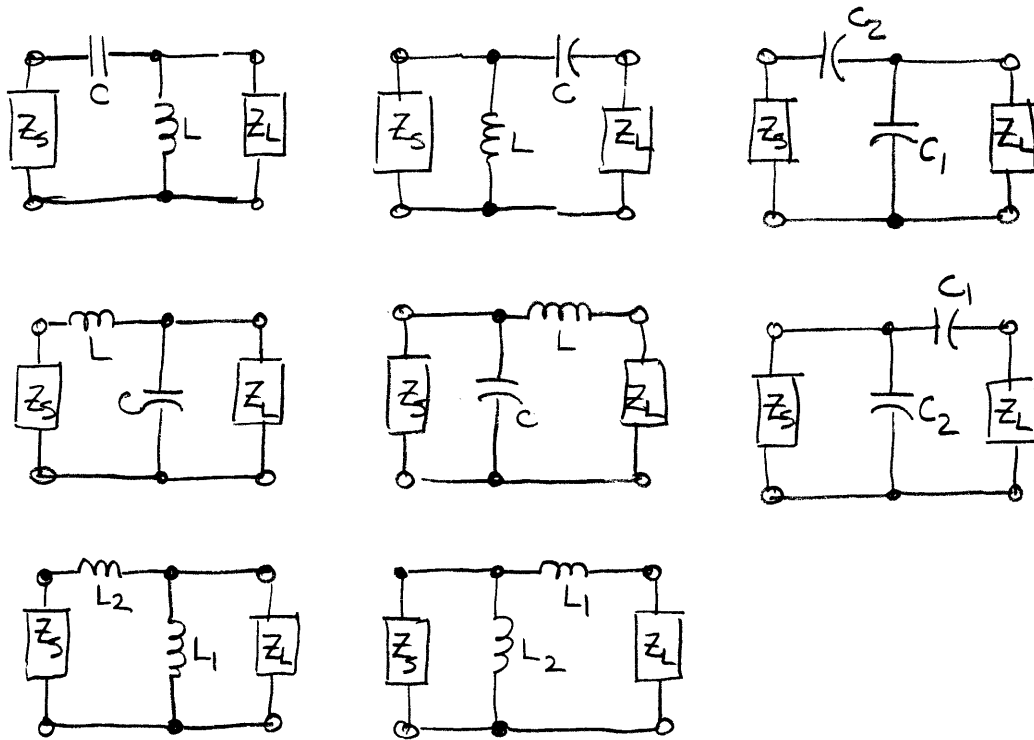


A plot of  $Z_{in}$  as a function of  $f$  is interesting and given by most RF/microwave CAD programs.

(see MATLAB program fig 3.18.m)

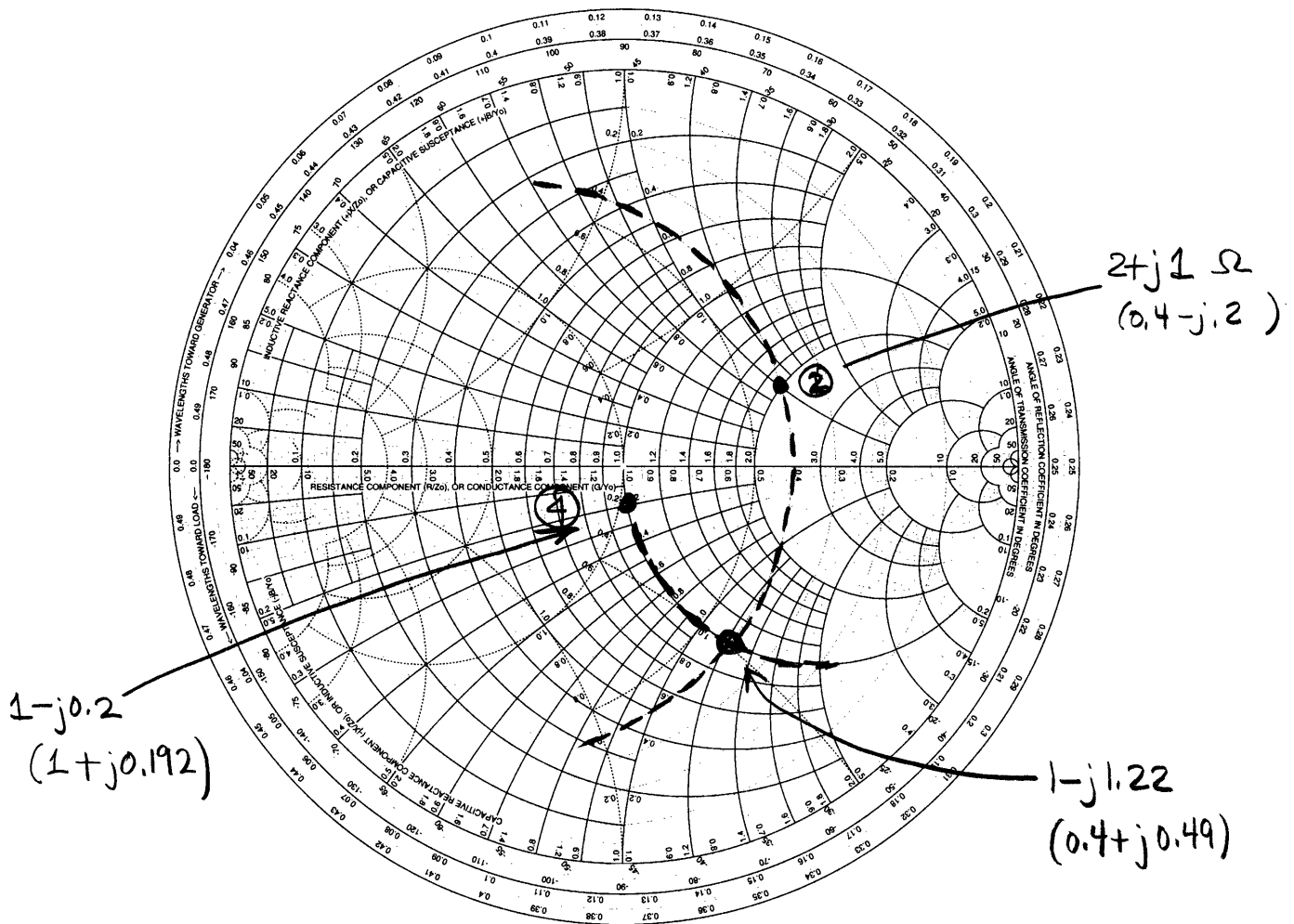
# Ch. 8 Matching and Biasing Networks

L-section (two-component) networks.

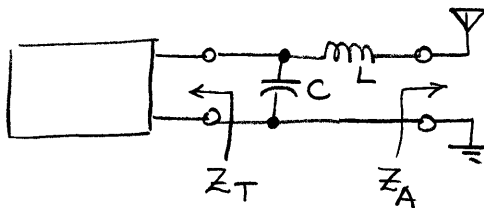


L-networks perform impedance matching at a single frequency.

8-2 Design an L-type network to match a transmitter with  $Z_T = 150 + j75 \Omega$  to an antenna with impedance  $Z_A = 75 + j15 \Omega$



8-2



Select  $Z_0 = 75 \Omega$

- ① Normalize impedances.  

$$\bar{Z}_T = \frac{Z_T}{Z_0} = \frac{150 + j75}{75} = 2 + j1$$

$$\bar{Z}_A = \frac{Z_A}{Z_0} = \frac{75 + j15}{75} = 1 + j0.2$$
- ② Plot  $\bar{Z}_T$
- ③ Add a shunt capacitor by using a constant conductance circle. Draw  $g = 0.4$  circle going thru  $Z_T$ .
- ④ Plot  $\bar{Z}_A^*$  (for matching) =  $1 - j0.2$
- ⑤ Add a series inductor by drawing constant  $Z$  circle through  $\bar{Z}_A^*$
- ⑥ Circles intersect giving normalized impedance of ell network. Reading from chart this is  $\bar{Z}_{TC} \approx 1 - j1.22$



The values of  $L$  and  $C$  are given by the changes in  $b$  along the arcs.

The value of the impedance change from  $1-j0.2$  to  $1-j1.22$  is  $j1.02$

The value of the inductor is

$$X_L = \frac{\omega L}{Z_0} \Rightarrow L = \frac{X_L Z_0}{\omega} = \frac{(1.02)(75)}{2\pi(2 \times 10^9)}$$

$$L = 6.09 \times 10^{-9} = 6.09 \text{ nH}$$

Similarly, the value of the susceptance is given by the change from  $0.4-j0.2$  to  $0.4+j0.49$ .

The change in susceptance is  $0.49 - (-0.2) = 0.69$ .

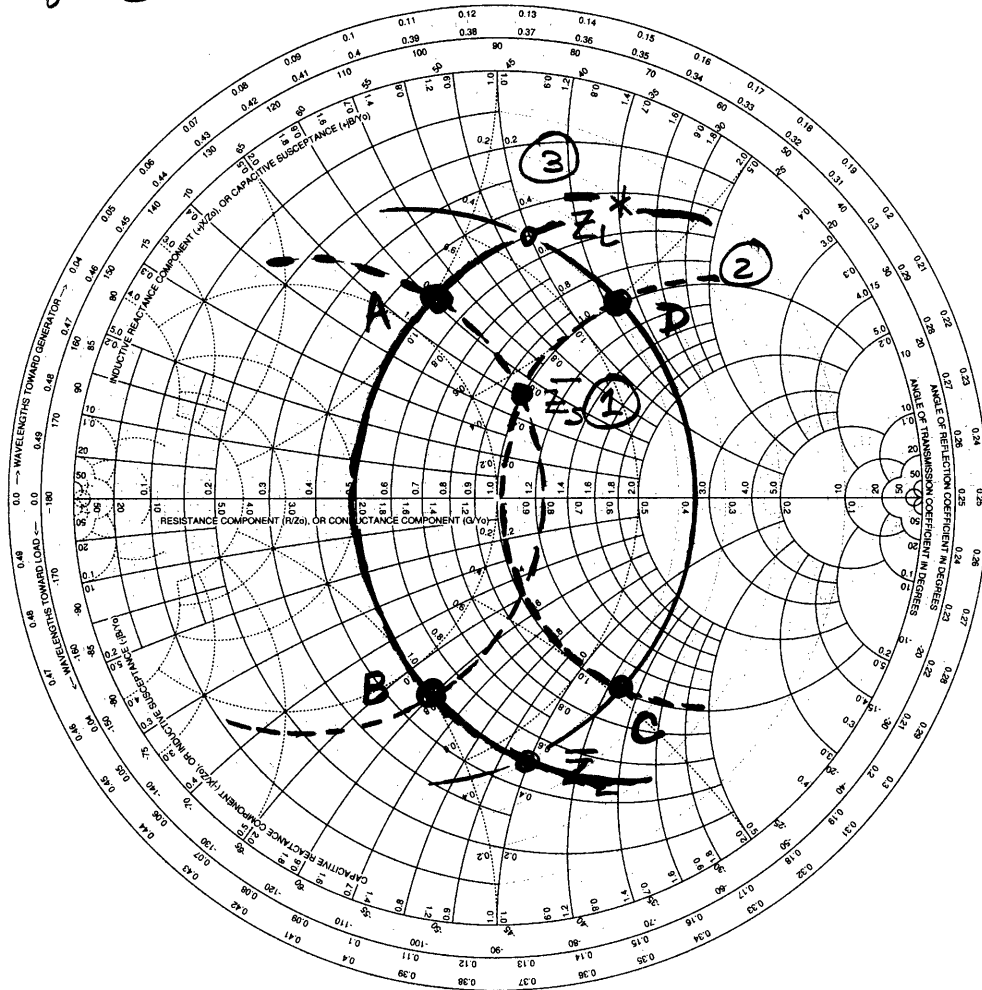
The value of the capacitor is

$$b_C = \omega C Z_0 \Rightarrow C = \frac{b_C}{\omega Z_0} = \frac{0.69}{2\pi(2 \times 10^9)(75)}$$

$$C = 0.73 \times 10^{-12}$$

$$C = 0.73 \text{ pf.}$$

Example 8-3. Using the Smith Chart, design all possible configurations of discrete two-element matching networks that match the source impedance  $Z_S = 50 + j25 \Omega$  to the load  $Z_L = 25 - j50 \Omega$ . Assume a characteristic impedance  $Z_0 = 50 \Omega$  and an operating frequency of  $f = 2 \text{ GHz}$



① Normalize the source and load impedances

$$\bar{Z}_S = \frac{50 + j25}{50} = 1 + j0.5$$

$$\bar{Z}_L = \frac{25 - j50}{50} = 0.5 - j1$$

② Plot circles of constant R and constant G that pass through the source impedance.

③ Locate  $\bar{Z}_L^*$  on Smith Chart

④ Plot circles of constant R and constant G that pass through  $\bar{Z}_L^*$ . The intersections of these circles represent all possible L-section matching networks

$$Z_A = 0.5 + j0.6$$

$$Y_A = 0.8 - j1$$

$$Z_B = 0.5 - j0.6$$

$$Y_B = 0.8 + j1$$

$$Z_C = 1 - j1.2$$

$$Y_C = 3 + j0.5$$

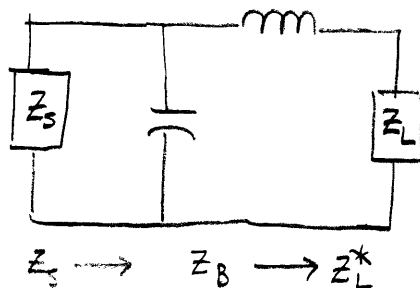
$$Z_D = 1 + j1.2$$

$$Y_D = 3 - j0.5$$

Each of these points represents a different filter

Consider  $Z_B = 0.5 - j0.6$

To achieve this we rotate clockwise through  $\bar{Z}_S$ , i.e., add a parallel  $C$  to  $B$ . We then go clockwise from  $B$  to  $\bar{Z}_L^*$ . This filter then corresponds to



The values of the components can be read (approximately) from the chart.  $Z_S \rightarrow Z_B$  is a parallel susceptance of

$$jb = jb_B - jbs = j1.0 - j(-0.4) = j1.4 \Omega$$

but  $j\omega C(Z_0) = jb = j1.4$

$$C = \frac{1.4}{2\pi(2 \times 10^9)(50)} = 2.228 \text{ pf.}$$

Moving from  $Z_B$  to  $Z_L^*$  gives the series impedance

$$Z_B = 0.5 - j0.6$$

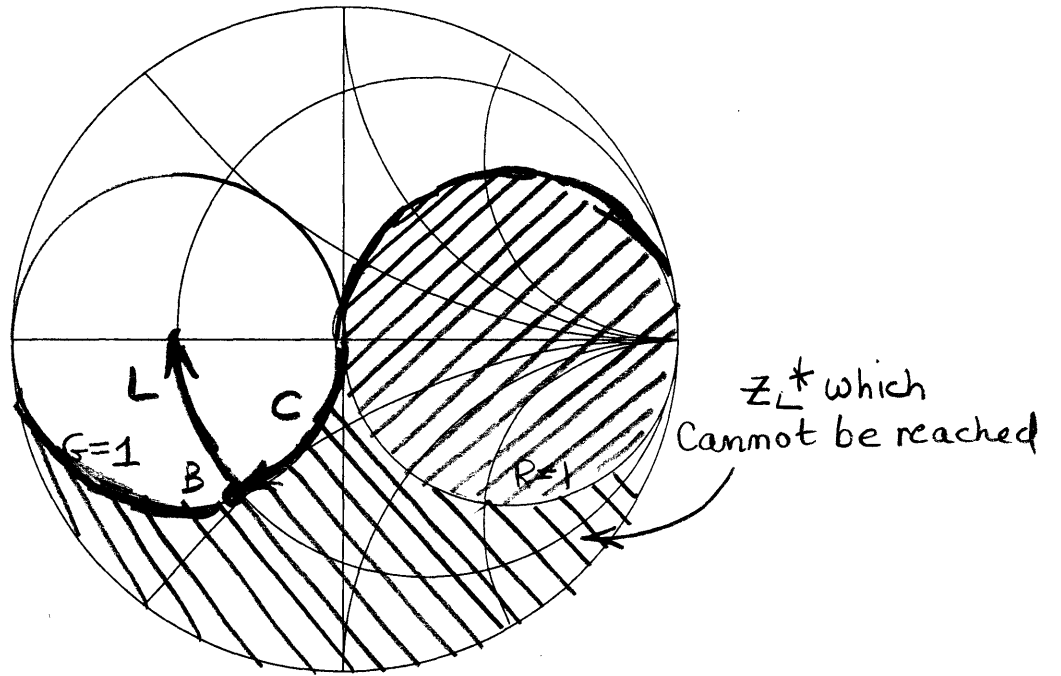
$$Z_L^* = 0.5 + j1$$

$$Z_L^* - Z_B = j1 - (-j0.6) = j1.6 = j\frac{\omega L}{Z_0}$$

Solving for  $L = \frac{1.6 Z_0}{\omega} = \frac{1.6(50)}{2\pi(2 \times 10^9)} = 6.37 \text{ nH}$

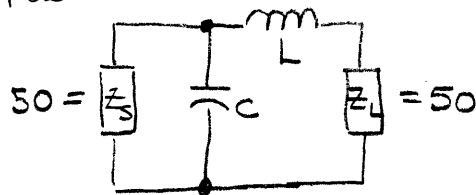
The other filters can be solved similarly

8.1.2. Forbidden Regions, Frequency Response, and Quality Factor



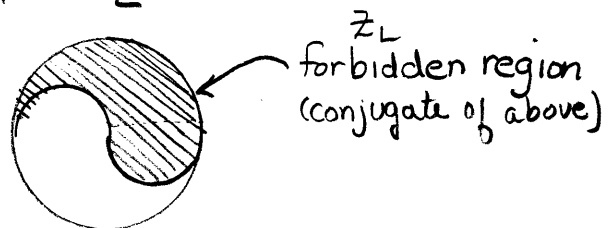
An L-network cannot perform impedance matching between any arbitrary load and source impedance

Consider  $Z_s = Z_0 = 50$  and this L network

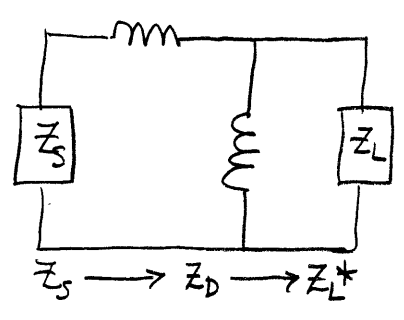
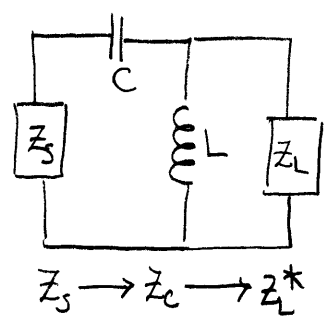
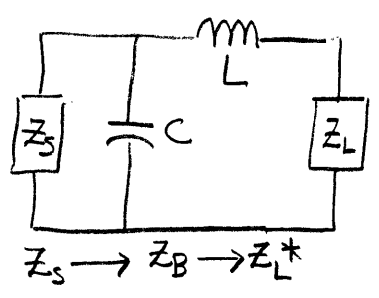
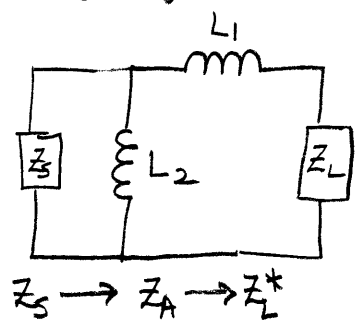


- ① Start at  $\bar{Z}_s = 1$
- ② Putting a C in parallel with  $R_s$  produces motion to B in the clockwise direction from  $\bar{Z}_s$  on a  $G=1$  circle
- ③ L causes a clockwise motion from,

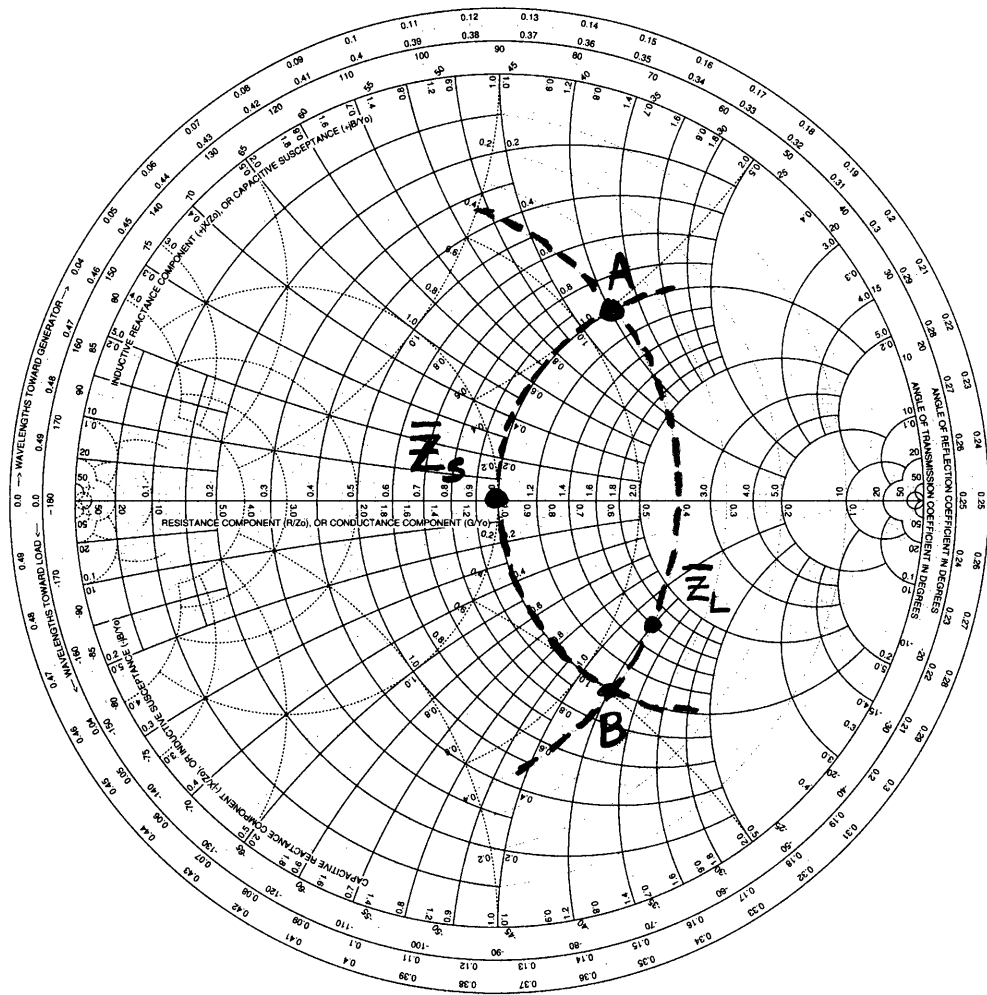
Note that I cannot reach any point within the  $R=1$  circle nor can I reach any point below the  $G=1$  circle with a positive Z. This is the shaded region. The conjugate of this represents the impedances I cannot match at  $\bar{Z}_L$  and is called the forbidden region for this network.



The four types of networks for Example 8-3 are



These are all resonant networks but they have different frequency responses and Q's.



Example: Design a matching network that transforms a complex load, consisting of resistance  $R_L = 80\Omega$  in series with a capacitance  $C_L = 2.65\text{ pf}$ , into a  $50\Omega$  input impedance at  $f_0 = 1\text{ GHz}$ .

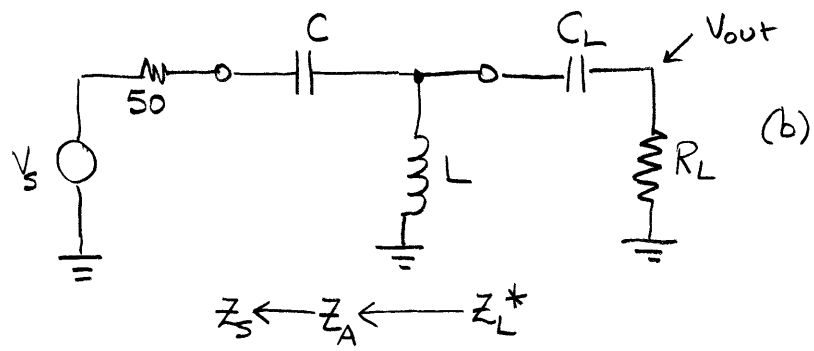
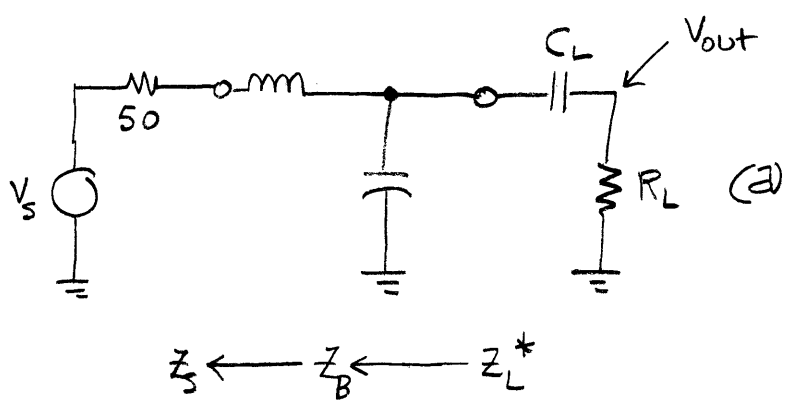
$$\frac{1}{j\omega C} = -j \frac{1}{2\pi(1 \times 10^9)(2.65 \times 10^{-12})} = -j60$$

$$\bar{Z}_L = \frac{80 - j60}{50} = 1.6 - j1.2$$

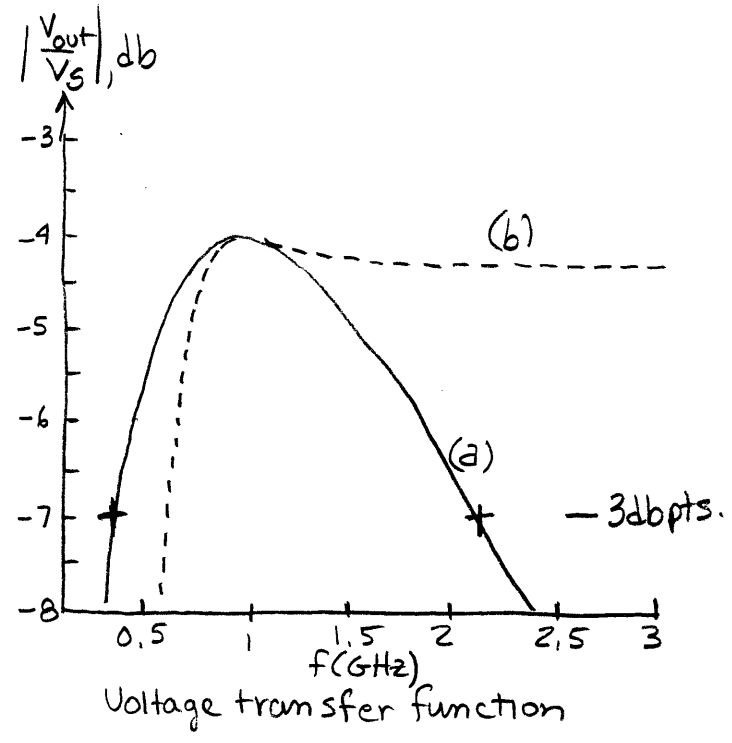
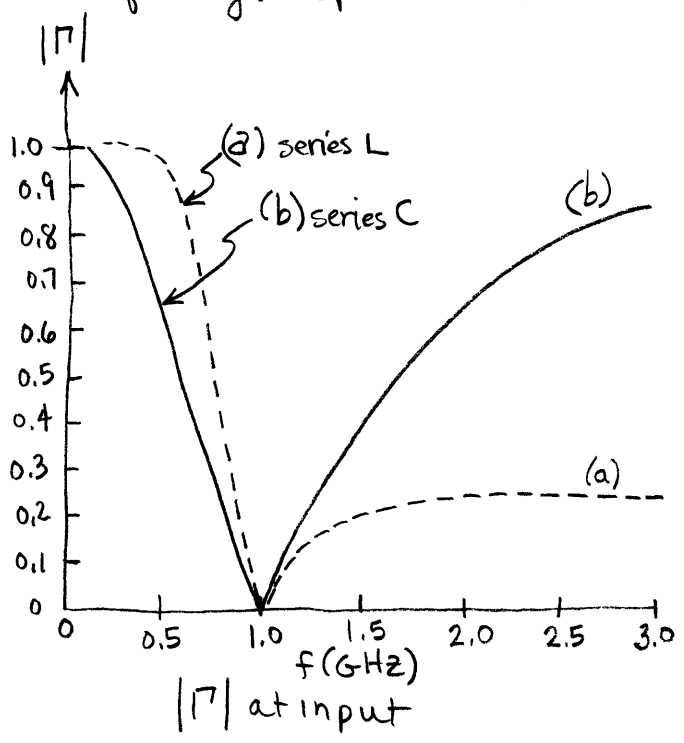
$$\bar{Z}_S = \frac{50}{50} = 1 \quad \bar{Z}_S^* = 1$$

We plot  $\bar{Z}_S$  and  $\bar{Z}_L$  and draw the circles of constant conductance and resistance through them. These are the dashed lines. There are two possible filters which go thru A or B

These two filters are

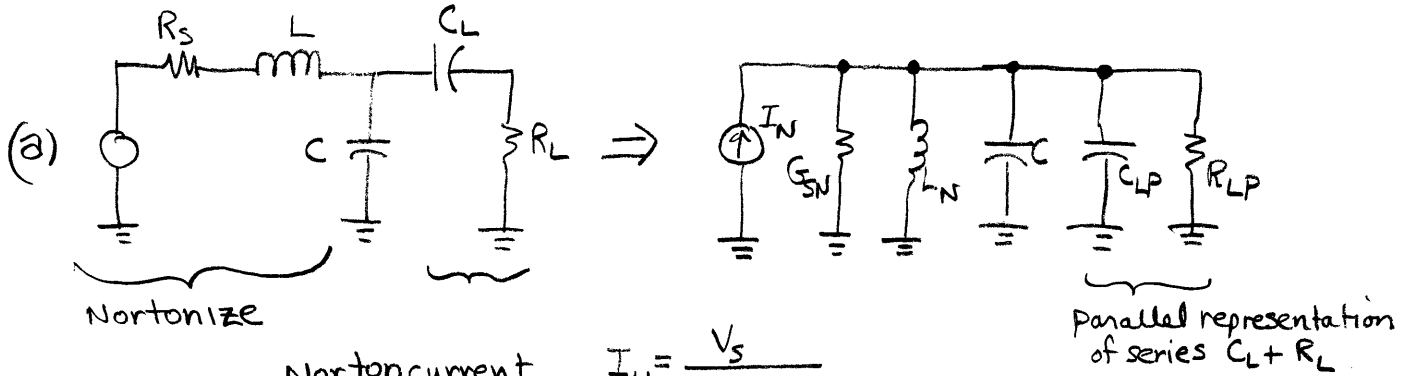


Although they have the same impedance transformation at  $f = 1 \text{ GHz}$  their frequency response is different.



We want to talk about the Q of the circuit,  $Q_L = \frac{f_0}{BW}$

We will approximate the frequency response near  $f_0$ .



Norton current  $I_N = \frac{V_s}{R_s + j\omega L}$

Norton conductance  $G_{SN} + \frac{1}{j\omega L_N} = \frac{1}{R_s + j\omega L}$  Thevenin Resistance

$Z_L = R_L + \frac{1}{j\omega C_L}$        $Y_L = \frac{1}{R_{LP}} + j\omega C_{LP}$

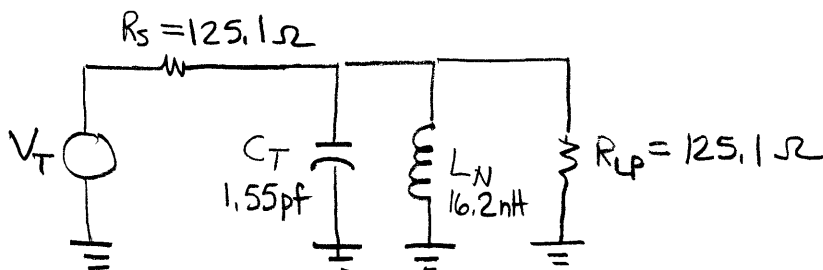
$\frac{1}{R_{LP}} + j\omega C_{LP} = \frac{1}{R_L + \frac{1}{j\omega C_L}} = \frac{1}{\frac{j\omega R_L C_L + 1}{j\omega C_L}} = \frac{j\omega C_L}{1 + j\omega R_L C_L} \cdot \frac{1 - j\omega R_L C_L}{1 - j\omega R_L C_L}$

$\frac{1}{R_{LP}} + j\omega C_{LP} = \frac{j\omega C_L + \omega^2 R_L C_L^2}{1 + \omega^2 R_L C_L}$

$\frac{1}{R_{LP}} = \frac{\omega^2 R_L C_L^2}{1 + \omega^2 R_L C_L}$        $R_{LP} = \frac{1 + \omega^2 R_L C_L}{\omega^2 R_L C_L^2}$

$j\omega C_{LP} = \frac{j\omega C_L}{1 + \omega^2 R_L C_L}$        $C_{LP} = \frac{C_L}{1 + \omega^2 R_L C_L}$

Combine the parallel C's and Thevenize  $I_N$  &  $G_{SN}$





The  $Q$  of this circuit is given by

$$Q_L = \frac{f_0}{BW} = \omega_0 R_T C = \frac{R_T}{X_C} = 0.61$$

$\uparrow$   
 $R_T = R_S \parallel R_{LP} \sim 62.54 \Omega$

The bandwidth is then

$$BW = \frac{f_0}{Q_L} = \frac{1 \text{ GHz}}{0.61} = 1.63 \text{ GHz.}$$

From the calculated frequency response

$$f_{-3\text{db}, \text{min}} \approx 0.4 \text{ GHz}$$

$$f_{-3\text{db}, \text{max}} \approx 2.2 \text{ GHz.}$$

which alternatively gives  $2.2 - 0.4 = 1.8 \text{ GHz}$   
which is fair agreement.

How to estimate Q from the Smith Chart.

At any point  $Z_s = R_s + jX_s$  or  $Y_p = G_p + jB_p$

$\therefore$  at each mode of the filter  $Q_n = \frac{|X_s|}{R_s} = \frac{|B_p|}{G_p}$

$$Q_L = \frac{Q_n}{2}$$

This is true for any L-type network.

$Q_L$  includes source and load resistances;  $Q_n$  only includes the source or load resistance

For the general Smith Chart

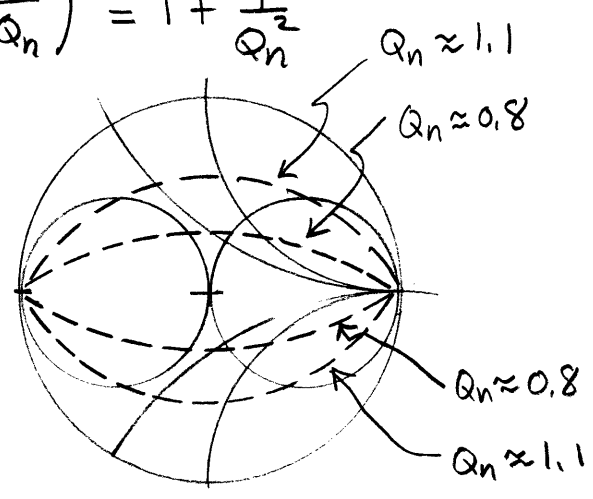
$$z = r + jx = \frac{1 - u^2 - v^2}{(1-u)^2 + v^2} + j \frac{2v}{(1-u)^2 + v^2}$$

where  $\Gamma = u + jv$

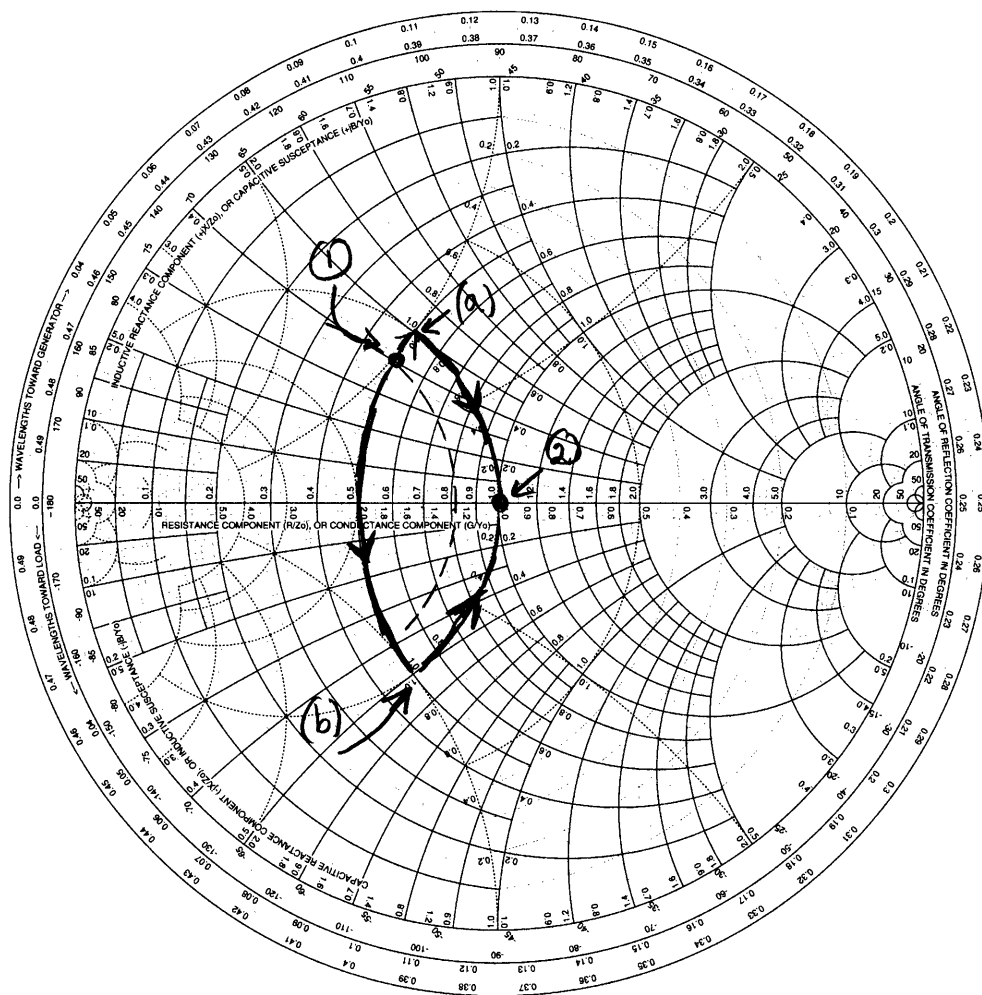
$$Q_n = \frac{|x|}{r} = \frac{\left| \frac{2v}{(1-u)^2 + v^2} \right|}{\frac{1 - u^2 - v^2}{(1-u)^2 + v^2}} = \frac{2v}{1 - u^2 - v^2}$$

These describe new circles given by

$$v^2 + \left(u \pm \frac{1}{Q_n}\right)^2 = 1 + \frac{1}{Q_n^2}$$



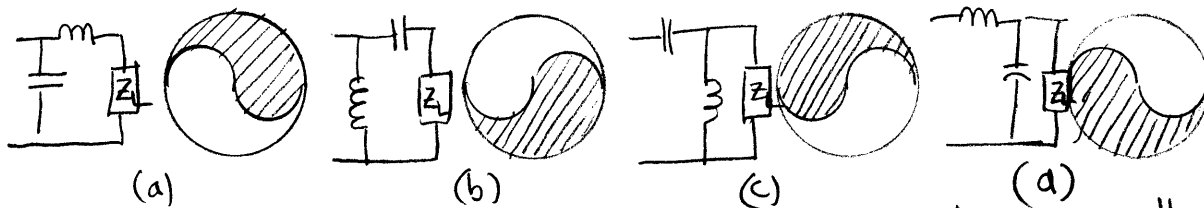
Example 8-4



Using the forbidden regions in Figure 8-7, design two L-type networks that match a  $Z_L = 25 + j20 \Omega$  load impedance to a  $50 \Omega$  source at  $1 \text{ GHz}$ . Determine the loaded quality factors of these networks from the Smith Chart and compare them to the bandwidth obtained from their frequency response. Assume that the load consists of a resistance and inductance connected in series.

① Plot  $\bar{Z}_L = \frac{25 + j20}{50} = 0.5 + j0.4$ . We want to get to  $\bar{Z}_{in} = 1$  ②

③ We check the forbidden regions in Figure 8-7 as shown below for  $Z$



We see that networks (a) and (b) have  $\bar{Z}_L$  within the allowed region

④ Two networks are possible. These lie along the  $\bar{z} = 0.5$  and  $\bar{y} = 1$  circles. We can travel

(a)  $\bar{z}_L = 0.5 + j0.4 \rightarrow \bar{z} = 0.5 + j0.5, \bar{y} = 1 - j1 \rightarrow \bar{y} = 1$

(b)  $\bar{z}_L = 0.5 + j0.4 \rightarrow \bar{z} = 0.5 - j0.5, \bar{y} = 1 - j1 \rightarrow \bar{y} = 1$

Both will give acceptable filters

(a) requires adding a series inductance followed by a shunt capacitance

(b) requires adding a series capacitance followed by a series inductance.

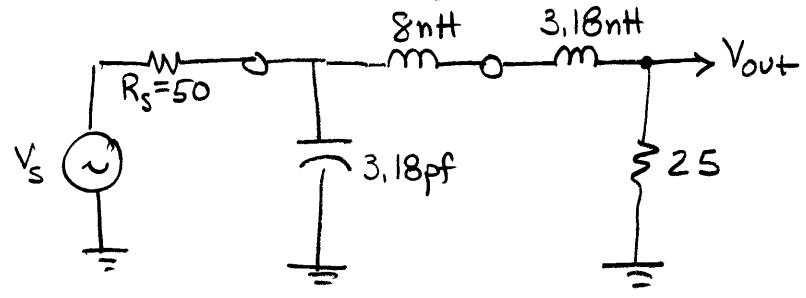
The values can be readily computed. For (a) the series impedance change is  $+0.1\Omega$  or  $\frac{\omega L}{50} = 0.1$ . Solving for L

$$L = \frac{(0.1)50}{\omega} = \frac{0.1(50)}{2\pi f} = \frac{0.1(50)}{2\pi(1 \times 10^9)} = 7.96 \times 10^{-10} \approx 8 \text{ nH}$$

The required parallel susceptance is 1. or  $\omega C = \frac{1}{50}$  Solving for C

$$C = \frac{1}{\omega \cdot 50} = \frac{1}{2\pi(1 \times 10^9) \cdot 50} = 3.18 \times 10^{-12} \approx 3.18 \text{ pf}$$

This network is shown below

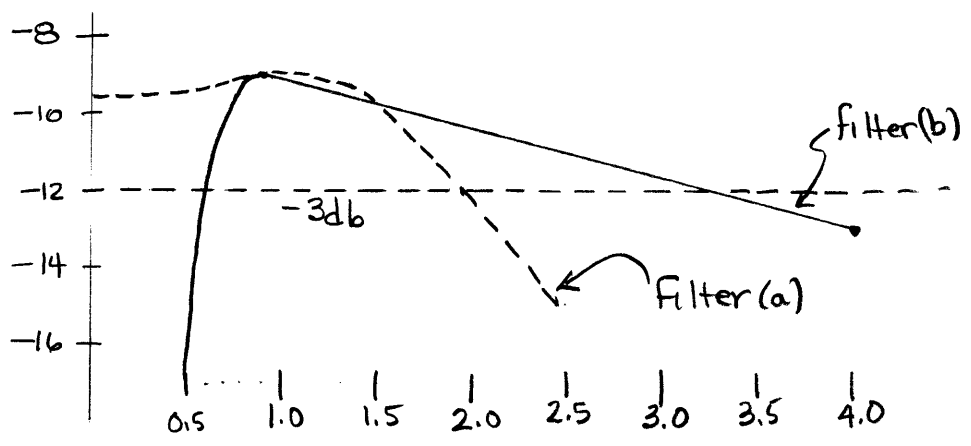


The calculations for (b) are very similar.

The Q is estimated as  $Q_n = \frac{|x_s|}{R_s} = \frac{|\pm 0.5|}{0.5} = 1$  for both filters.

$$\therefore BW = \frac{f_0}{Q_L} = \frac{f_0}{Q_n/2} = \frac{1 \times 10^9}{1/2} = 2 \text{ GHz}$$

The actual plotted frequency response is



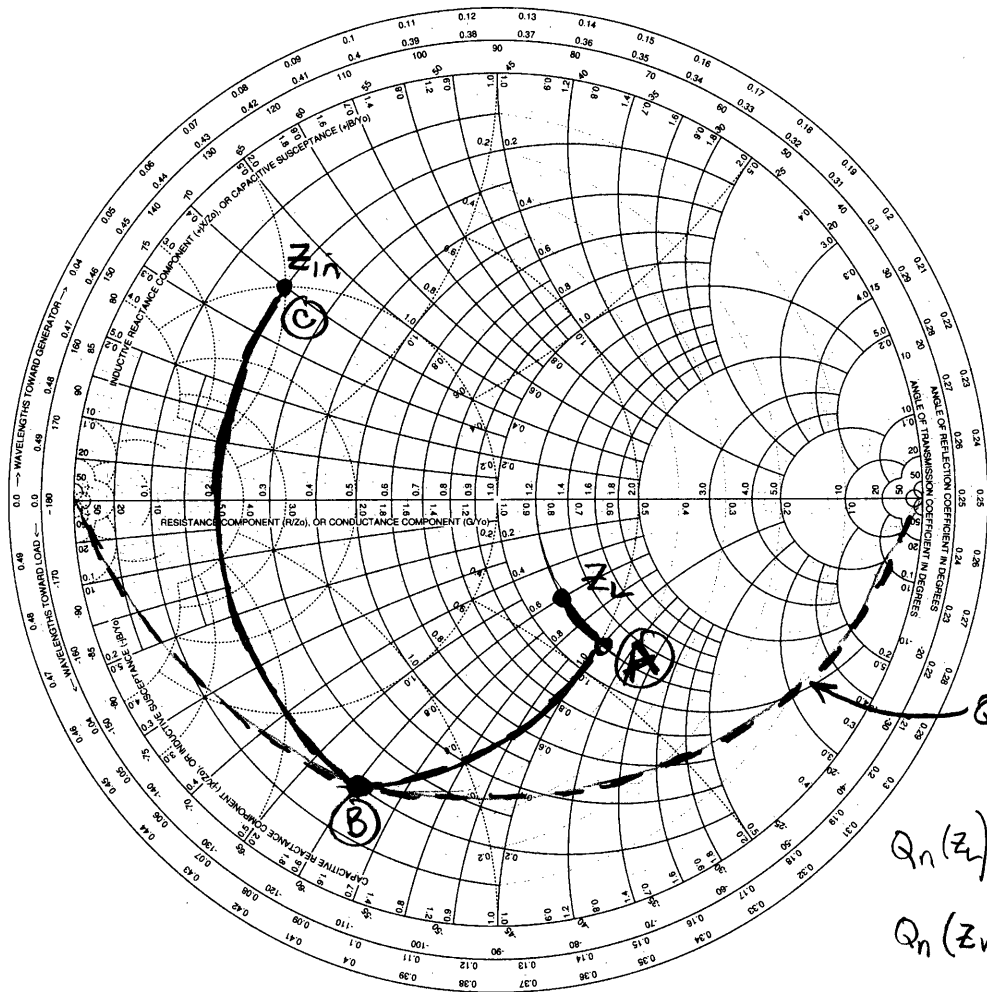
The actual BW for filter (b) is just under 2 GHz.

Filter (a) has no lower cutoff so it doesn't fit the estimates well

NOTE:

T-network - used to match two low impedance values when a high intermediate Q is needed

Pi-network somewhat similar



$Q_n = 3$  circle

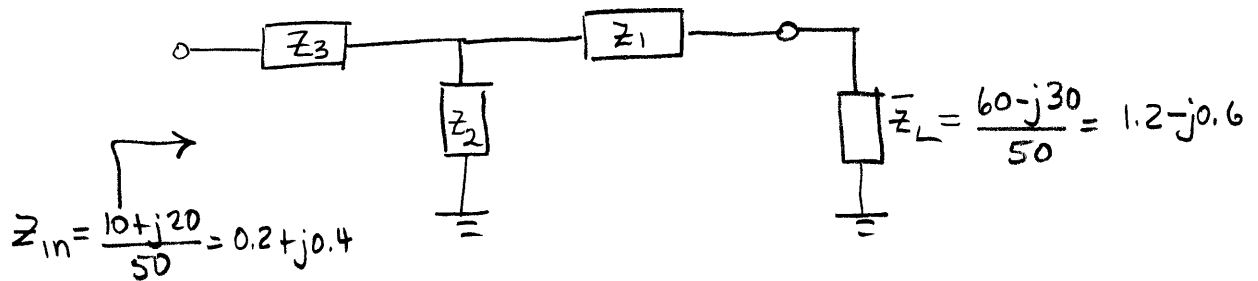
$$Q_n(z_L) = \frac{30}{60} = \frac{1}{2}$$

$$Q_n(z_{in}) = \frac{20}{10} = 2$$

Example 8-5 Design of a T matching Network

Design a T-type matching network that transforms a load impedance  $Z_L = 60 - j30 \Omega$  into a  $Z_{in} = 10 + j20 \Omega$  input impedance and has a maximum nodal Q of 3. Compute the values for the matching network components, assuming that matching is required at  $f = 1 \text{ GHz}$ .

General T network (Normalize to  $50 \Omega$ )



Only one possible path is shown in the Smith Chart.

It adds a negative series impedance (a capacitor) to the load. It then adds a positive parallel susceptance (another capacitor) to the  $Z_1 - Z_L$  combination. Finally it adds a positive series impedance (an inductor) to get the desired  $Z_{in}$ .

$$\bar{Z}_L \rightarrow Z_A \text{ is } 1.2 - j0.6 \rightarrow 1.2 - j1.0$$

$$\therefore \text{therefore } \frac{-1}{\omega C} = -0.4(50)$$

$$C = \frac{1}{2\pi(1 \times 10^9)(0.4)(50)} = 7.95 \times 10^{-12} = 7.95 \text{ pf.}$$

$Z_1$  is a 8 pf capacitor.

$Z_A \rightarrow Z_B$  is what actually sets the  $Q_n$ . Many such paths are possible.

$$Y_A \rightarrow Y_B \text{ is } 0.5 + j0.4 \rightarrow 0.5 + j1.5$$

$$\therefore \omega C = \frac{1.1}{50}$$

$$C = \frac{1.1}{2\pi(1 \times 10^9)(50)} = 3.50 \times 10^{-12} = 3.5 \text{ pf.}$$

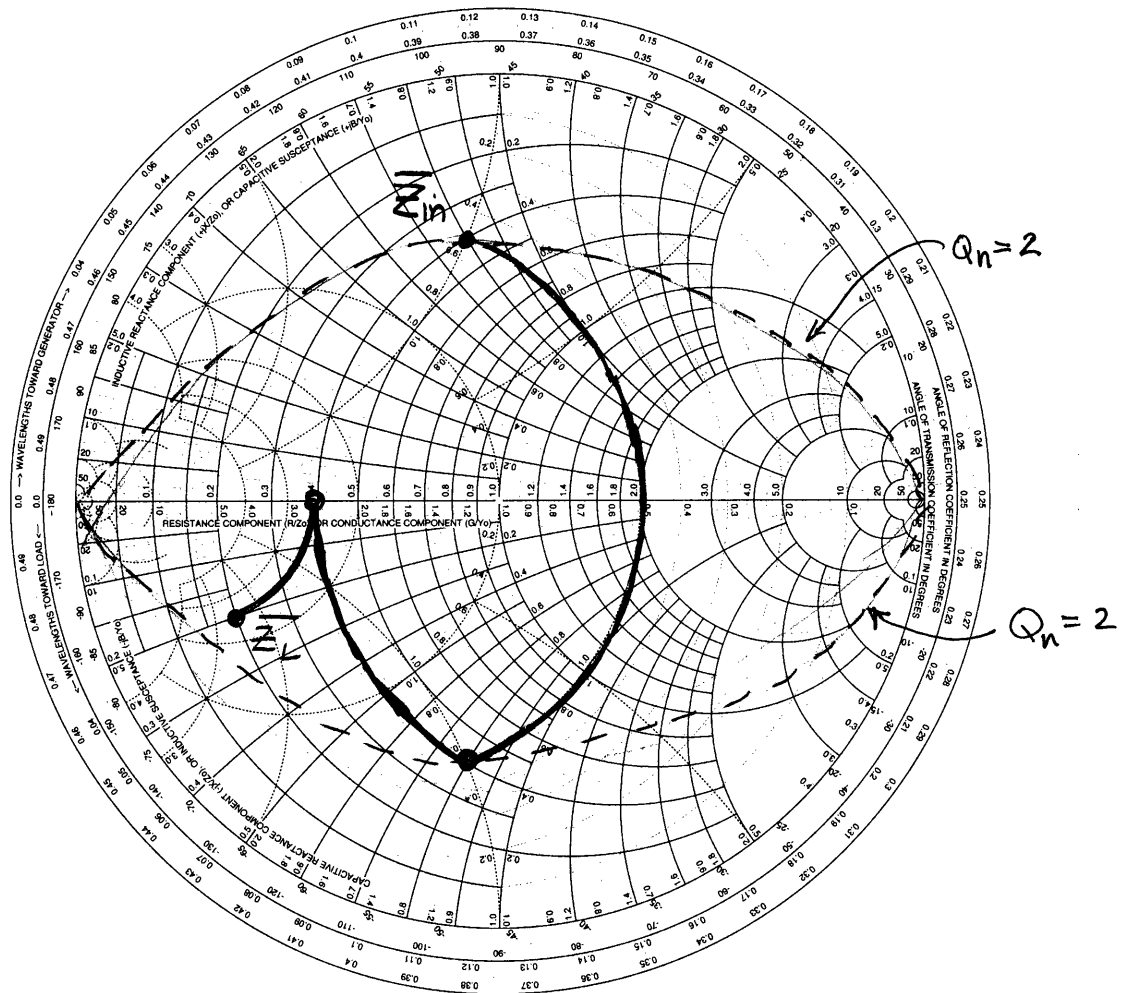
$Z_B \rightarrow Z_C$  completes the filter design.

$$Z_B \rightarrow Z_C \text{ is } 0.2 - j0.6 \rightarrow 0.2 + j0.4$$

$$\therefore \omega L = (0.6 + 0.4) 50$$

$$L = \frac{(1)(50)}{2\pi(1 \times 10^9)} = 7.95 \times 10^{-9} = 7.95 \text{ nH.}$$

## 8-6 Design of a Pi-type matching network



For a broadband amplifier it is required to develop a Pi-type matching network that transforms a load impedance of  $Z_L = 10 - j10 \Omega$  into an input impedance of  $Z_{in} = 20 + j40 \Omega$ . The design should involve the lowest possible nodal quality factor. Find the component values, assuming that matching should be achieved at a frequency of  $f = 2.4 \text{ GHz}$ .

$$\bar{Z}_{in} = \frac{20 + j40}{50} = 0.4 + j0.8 \quad \bar{Z}_L = \frac{10 - j10}{50} = 0.2 - j0.2$$

The  $Q_n$  cannot be lower than the  $Q$  at either the given  $\bar{Z}_{in}$  or  $\bar{Z}_L$ ,

$$\text{The } Q_n \text{ at the input is } Q_n = \frac{|X|}{R} = \frac{0.8}{0.4} = 2$$

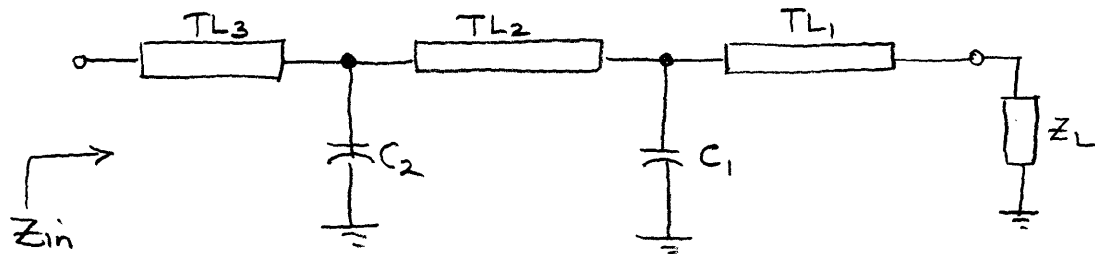
The  $Q_n = 2$  circles define the limits for matching

A solution is shown above. The component values can be readily calculated.



## 8.2 Microstrip Line Matching Networks.

The common approach at mid-GHz frequencies is to use transmission line segments in series and parallel capacitors.

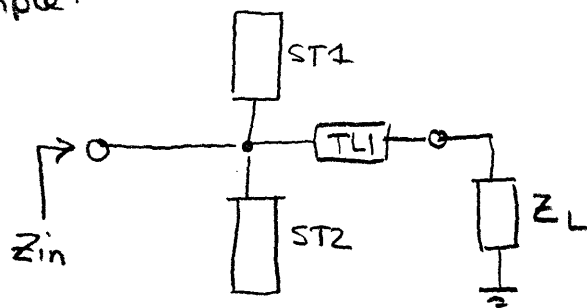


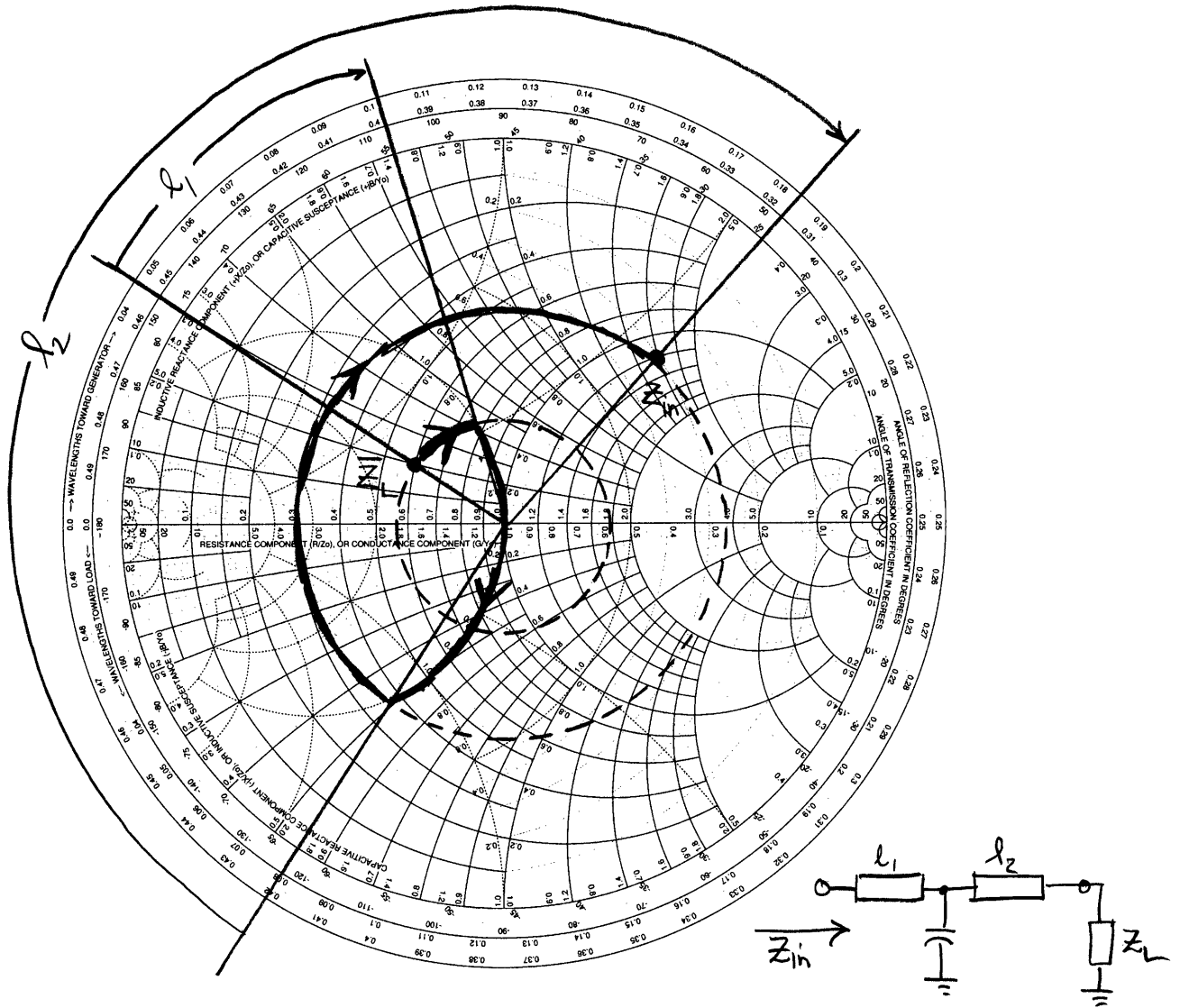
Avoid inductors at these frequencies because they tend to have higher losses than capacitors. As the frequencies increase the capacitors will also be replaced by transmission lines. Part of the consideration is that the capacitors will be smaller than the transmission lines. Also, C<sub>1</sub> and C<sub>2</sub> can be varactors allowing electronic tuning of the impedance/frequency response.

This circuit also lends itself to good layout on printed circuit boards.

Some PClayouts lend themselves to a balanced stub

For example.





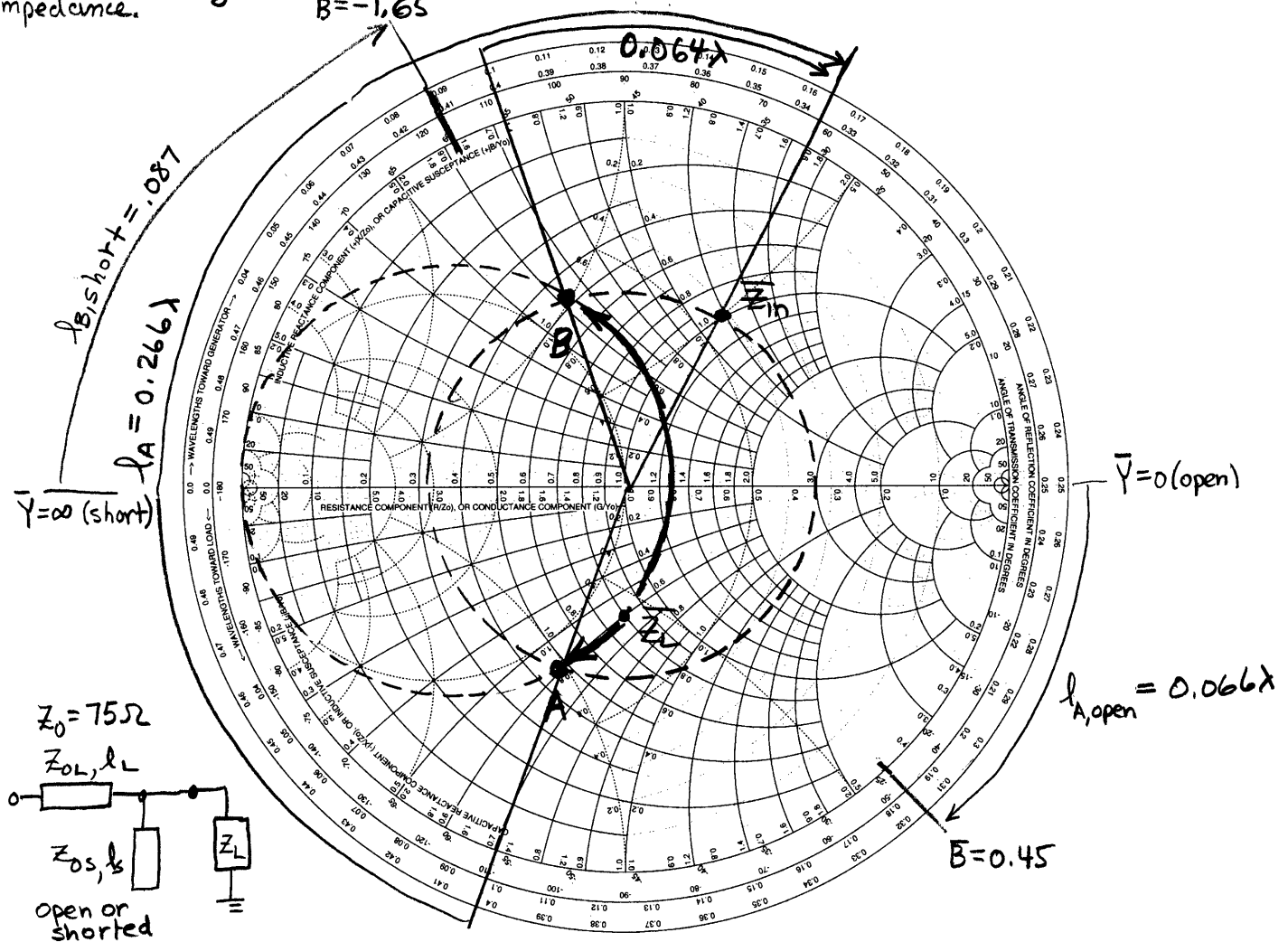
### Example 8-7

Design a matching network that transforms the load  $Z_L = (30 + j10)\Omega$  to an input impedance  $Z_{in} = (60 + j80)\Omega$ . The matching network should contain only two series transmission lines and a shunt capacitance. Both transmission lines have a  $50\Omega$  characteristic line impedance, and the frequency at which matching is desired is  $f = 1.5\text{ GHz}$ .

- ①  $\bar{Z}_L = \frac{30 + j10}{50} = 0.6 + j0.2$ .
- ② Plot the constant SWR ( $\rho$ ) circle for the load.
- ③ Plot  $\bar{Z}_{in} = \frac{60 + j80}{50} = 1.2 + j1.6$  and the corresponding SWR circle.
- ④ Transition from the inner circle to the outer circle using a parallel C (i.e. susceptance). We arbitrarily chose the  $\bar{G} = 1$  circle.
- ⑤ Determine the value of C and the electrical lengths of the lines.

NOTE: The input impedance is VERY dependent on the positioning of the capacitor.

Example B-8: For a load impedance of  $Z_L = 60 - j45$ , design two single stub matching networks that transform the load to a  $Z_{in} = 75 + j90$  input impedance.



① Plot both  $\bar{Z}_{in}$  and  $\bar{Z}_L$ .  $\bar{Z}_L = \frac{60 - j45}{75} = 0.8 - j0.6$

$$\bar{Z}_{in} = \frac{75 + j90}{75} = 1 + j1.2$$

② Plot the SWR ( $\rho$ ) circle through  $\bar{Z}_{in}$ .

③ Plot the  $\bar{G} = 0.8$  circle through  $\bar{Z}_L$ .

④ Add a stub next to the load. The stub susceptance should take you from  $\bar{Z}_L$  to either A or B on the  $\bar{G} = 0.8$  circle. The values of the stub susceptance can be read from the chart as 0.45 to get to A and -1.65 to get to B.

⑤ The corresponding length of the line (NOT the stub) to get the desired  $\bar{Z}_{in}$  are (from A to  $\bar{Z}_{in}$ )  $l_A = 0.266\lambda$ . The corresponding length for the B solution (from B to  $\bar{Z}_{in}$ ) is  $l_B = 0.064\lambda$ .

⑥ The stubs can be either open or shorted. For the A solution  $\bar{B} = 0.45$ . For an open we start at  $\bar{Y} = 0$  (open) and move clockwise until we reach  $\bar{B} = 0.45$ . The distance is  $l_{A,open} = 0.066\lambda$ . For the B solution you can start at  $\bar{Y} = \infty$  (short) and move clockwise to  $\bar{B} = 0.45$ . This is simply  $0.066\lambda + 0.250\lambda = 0.316\lambda$ .

## Summary of Results

A solution: series transmission line  $0.266\lambda$

$\bar{B} = 0.45$  { open stub  $0.066\lambda$   
shorted stub  $0.316\lambda$

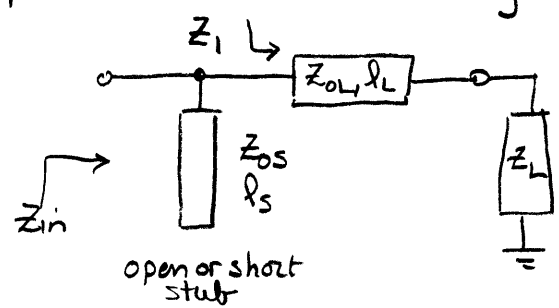
B solution series transmission line  $0.064\lambda$

$\bar{B} = -1.65$  { open stub  $0.337\lambda$   
shorted stub  $0.087\lambda$

← smallest solution

Note that the shorted stub for the B solution is much shorter than the open since it is negative.

Example 8-9. Using the circuit shown below choose the characteristic impedance of the stub and transmission line such that the load impedance  $Z_L = 120 - j20$  is transformed into the input impedance  $Z_{in} = 40 + j30$ . Assume that the length of the transmission line is  $l_L = 0.25\lambda$  and the stub's  $l_s = 0.375\lambda$ . Furthermore, determine whether a short-circuit or an open-circuit stub is necessary.



$$\frac{1}{Z_{in}} = .016 - j0.012$$

The impedance  $Z_1$  of the transmission line + the load is given by the quarter-wave transformer equation

$$Z_1 = \frac{Z_{0L}^2}{Z_L} \leftarrow \text{to be determined}$$

$Z_L \leftarrow Z_L = 120 - j20$

$$Y_{in} = Y_1 + jB_s = \frac{1}{Z_1} + jB_s = \frac{Z_L}{Z_{0L}^2} + jB_s$$

susceptance of the stub,

$$G_{in} + jB_{in} = \frac{R_L + jX_L}{Z_{0L}^2} + jB_s$$

$$G_{in} = \frac{R_L}{Z_{0L}^2} \quad \text{and} \quad B_{in} = \frac{X_L}{Z_{0L}^2} + B_s$$

Solving for  $Z_{0L}$  gives

$$\begin{cases} .016 = \frac{120}{Z_{0L}^2} \\ Z_{0L} = \sqrt{\frac{120}{.016}} \end{cases}$$

Then solve for  $B_s$

$$\begin{cases} B_s = B_{in} - \frac{X_L}{Z_{0L}^2} = -.012 - \frac{(-20)}{(86.6)^2} \\ B_s = -.012 + .00267 = -.009333 \\ Z_s = \frac{1}{jB_s} = +j107.14 \end{cases}$$

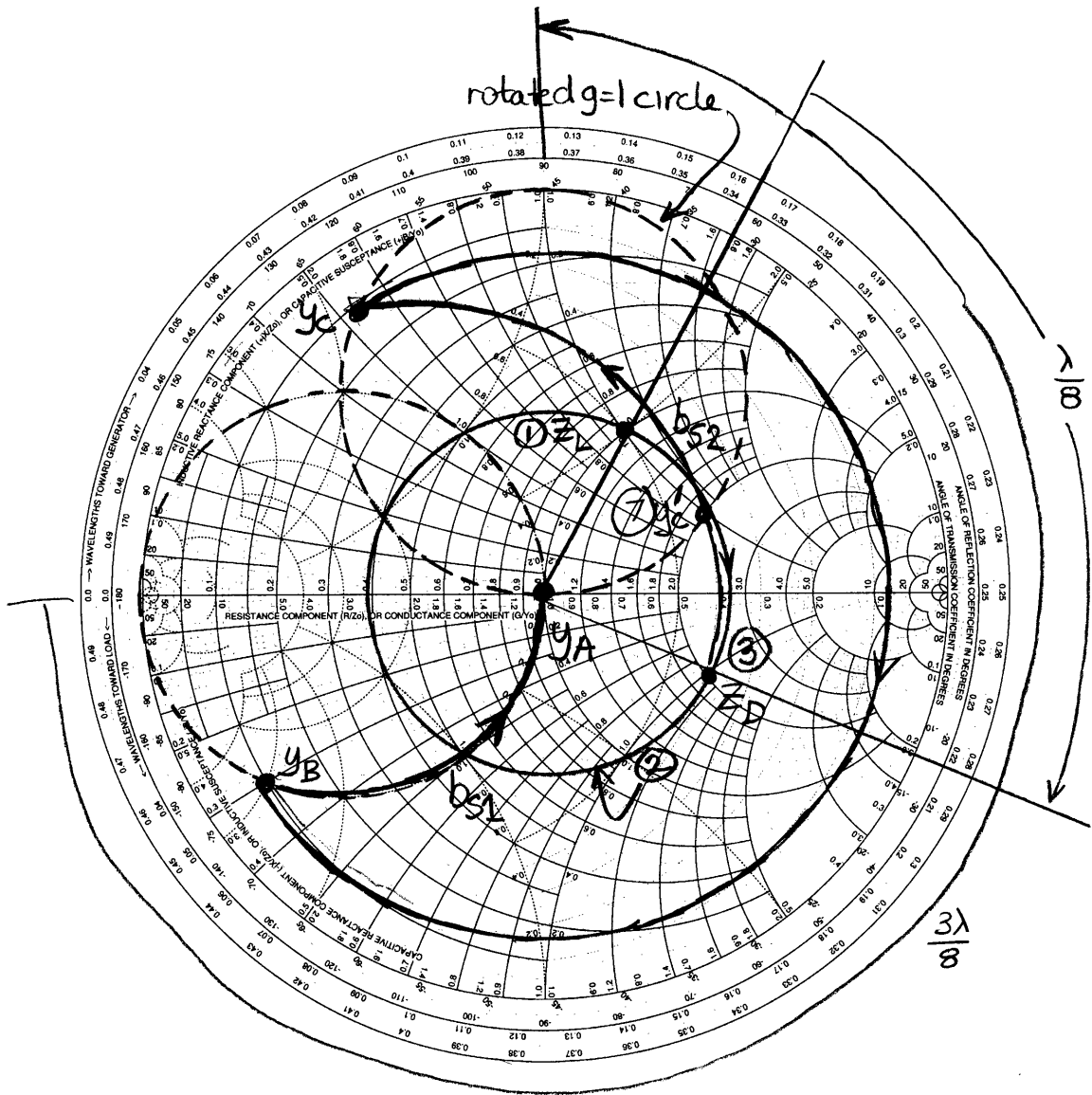
$Z_{0L} = 86.6 \Omega$

For  $l = 0.375\lambda$  the input impedance is given by since  $\tan(\frac{3}{4}\lambda) = -1$

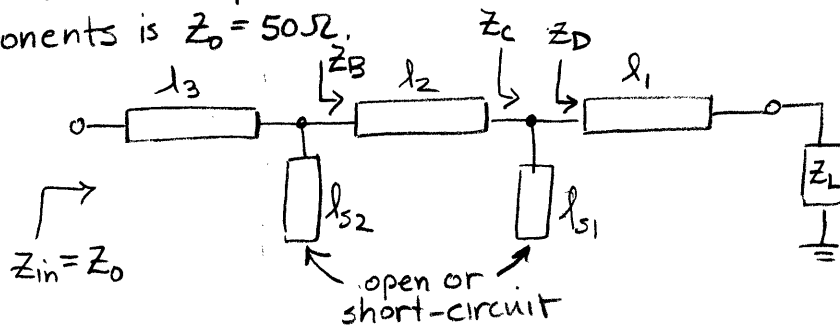
$$Z_{in}(0.375\lambda) = Z_0 \frac{Z_L + jZ_0 \tan(\frac{3}{4}\lambda)}{Z_0 + jZ_L \tan(\frac{3}{4}\lambda)} = Z_0 \frac{Z_L - jZ_0}{Z_0 - jZ_L}$$

if  $Z_L = 0$  (a short)  $Z_{in} = -jZ_0$   
 if  $Z_L = \infty$  (an open)  $Z_{in} = +jZ_0$

$\Rightarrow$  use an open since  $Z_{in} > 0$ , Choose  $Z_0 = 107.14 \Omega$ .

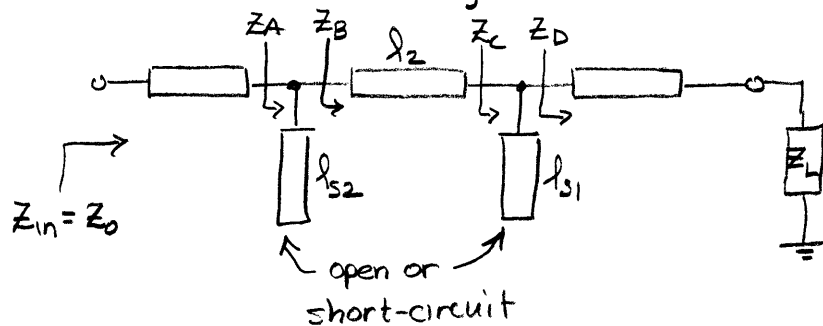


Example 8-10. It is assumed that in the double-stub matching network shown above the lengths of the transmission lines are  $l_3 = l_2 = \frac{3\lambda}{8}$  and  $l_1 = \frac{\lambda}{8}$ . Find the lengths of the short-circuit stubs that match the load impedance  $Z_L = 50 + j50$  to a  $50\Omega$  input impedance. The characteristic line impedance for all components is  $Z_0 = 50\Omega$ .



- ① Plot  $Z_L$
- ② draw constant SWR circle
- ③ transform by  $\frac{\lambda}{8}$  to  $Z_D$
- ④ rotate on constant  $g$  circle to intersect  $y_c$  circle. This is stub  $l_{s1}$ .
- ⑤ transform by  $l_2 = \frac{3\lambda}{8}$  to  $g=1$  circle. This is  $y_B$
- ⑥ stub  $l_{s2}$  transforms to  $y_A$  - a match.
- ⑦ Note There are two possible solutions. Another is at  $y_c'$ . This detailed solution is not shown.

## 8.2.3. Double Stub Matching Networks



A general problem of a single-stub matching network is that the length of the transmission line varies. This creates special problems for variable tuners. A double stub tuner allows one to use a fixed length transmission line for  $l_2$ .

In general,  $l_2$  is fixed - usually  $\frac{1}{8}$ ,  $\frac{3}{8}$  or  $\frac{5}{8} \lambda$ . The stubs are open or shorted.

Discussion. Assume  $l_2 = \frac{3}{8} \lambda$  and start at  $Z_{in}$ .

Assume  $Z_{in} = Z_0$ .

Since  $Z_{in} = Z_0$   $Y_A = 1$

When you add the parallel susceptance

$$Y_B = Y_A - j b_{s2}$$

For  $l_2 = \frac{3}{8} \lambda$  the  $g=1$  circle is rotated by

$$2\beta l_2 = 2 \frac{2\pi}{\lambda} \cdot \frac{3}{8} \lambda = \frac{3\pi}{2} \quad (270^\circ) \text{ towards the load (ccw)}$$

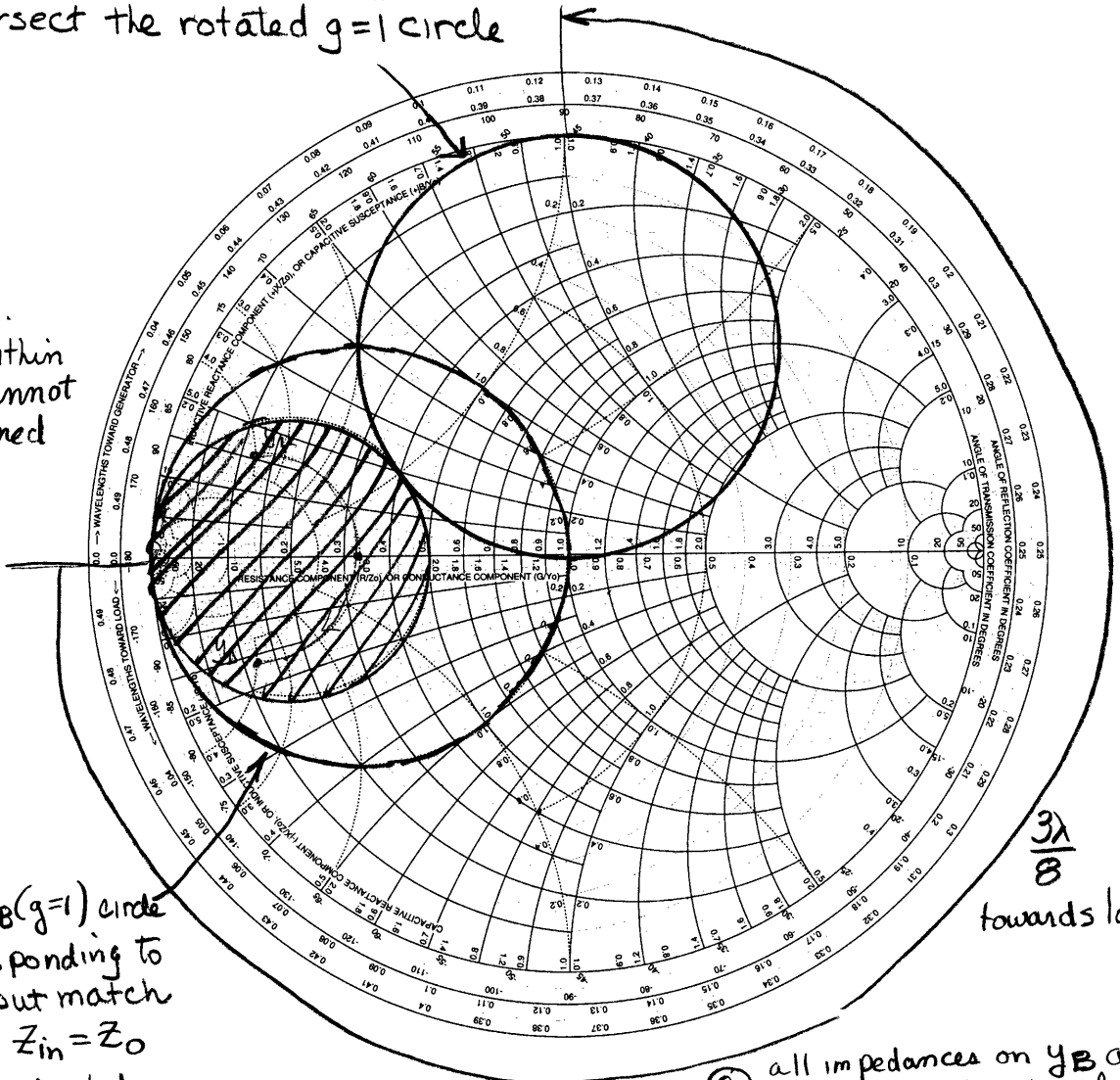
The admittance  $Y_c$  needs to lie on this rotated  $g=1$  circle for matching.

$l_{s1}$  is varied to transform  $Y_D$  to the  $g=1$  circle.

③ The transformed load  $Z_c$  MUST intersect the rotated  $g=1$  circle

Forbidden region

$y_D$  points within this circle cannot be transformed onto the  $y_c$  circle



①  $y_B(g=1)$  circle corresponding to the input match since  $Z_{in} = Z_0$ . The input stub can put you anywhere on the  $g=1$  circle.

② all impedances on  $y_B$  are transformed by the  $l_2$  transmission line [Here assumed to be  $\frac{3}{8}\lambda$ ]

④ For a solution to exist we must be able to transform  $y_D$  to the rotated  $g=1$  circle. Any point inside the  $g=2$  circle CANNOT be transformed by a susceptance to the  $y_c$  circle.