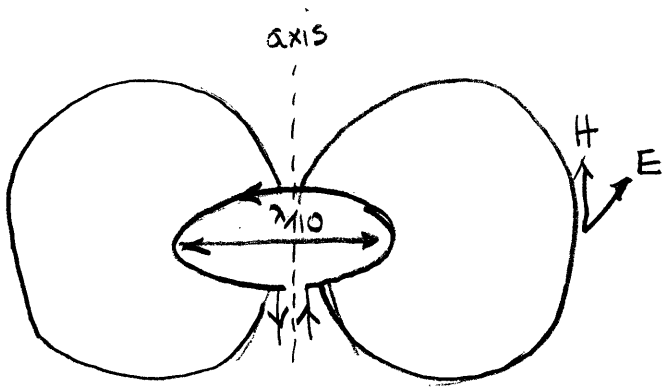
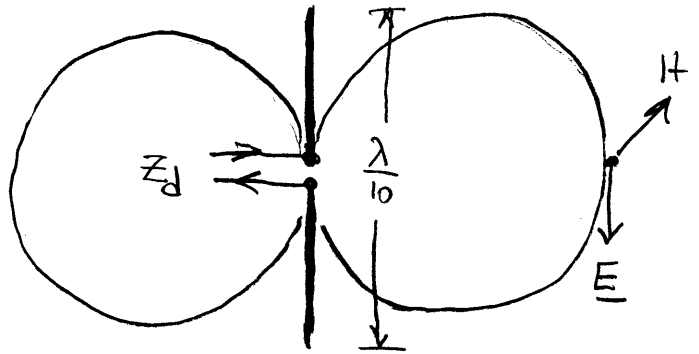


5.9 Antenna types (23 covered here)



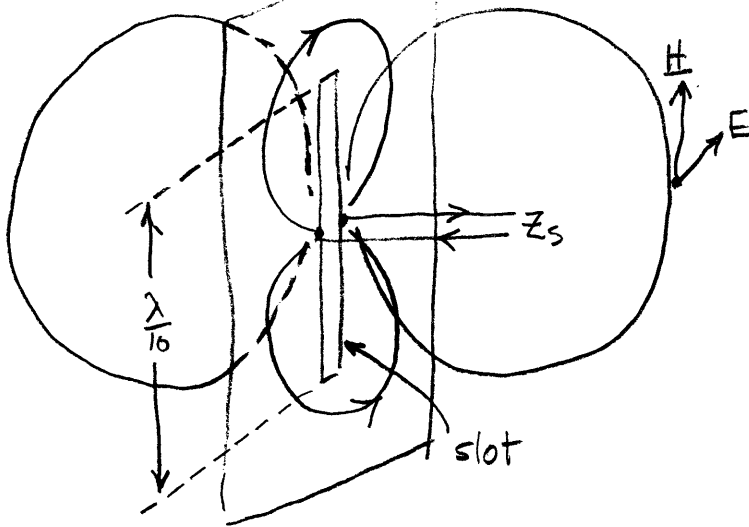
small loop  
D = 1.5



short dipole  
D = 1.5

field patterns are identical  
except that E and H are interchanged

slot antennas



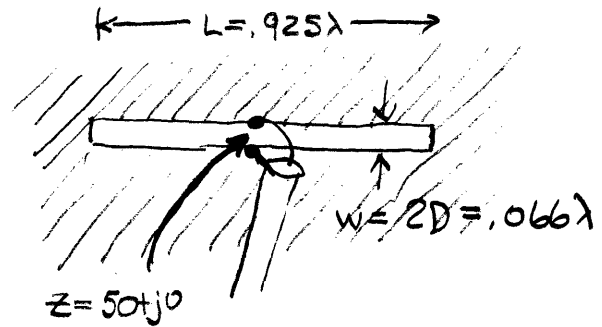
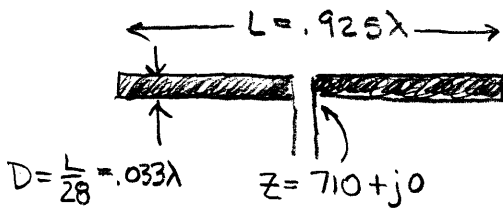
field pattern same as electric  
dipole of same length but  
E & H are interchanged

$$Z_d Z_s = \frac{Z_0^2}{4} \leftarrow \text{free space impedance}$$

$$Z_s = \frac{Z_0^2}{4Z_d}$$

Example 5-14

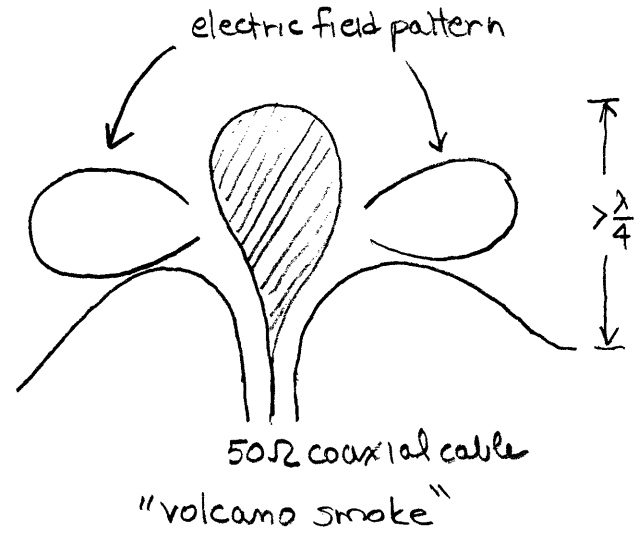
A  $\lambda$ -long cylindrical dipole and its complementary slot are compared. The actual length  $L = 0.925\lambda$ . The dipole cylinder has a diameter  $D = \frac{L}{28} = .033\lambda$  and a terminal impedance  $Z_d = 710 + j0 \Omega$ . The complementary slot has a width  $w = 2D$ . Find the terminal impedance  $Z_s$  of the slot.



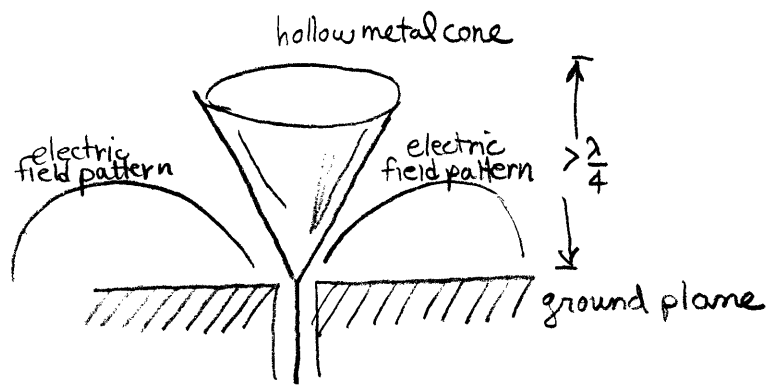
This is pretty simple

$$\text{for the slot } Z_s = \frac{Z_0^2}{4Z_d} = \frac{(377)^2}{4(710)} = 50 \Omega$$

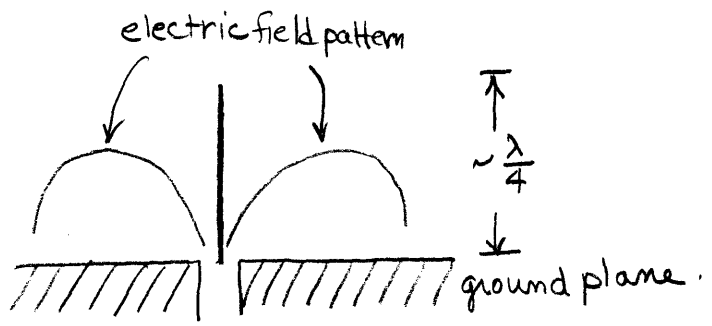
Opened out coaxial antennas



$D \approx 4$   
broadband due to gradual taper

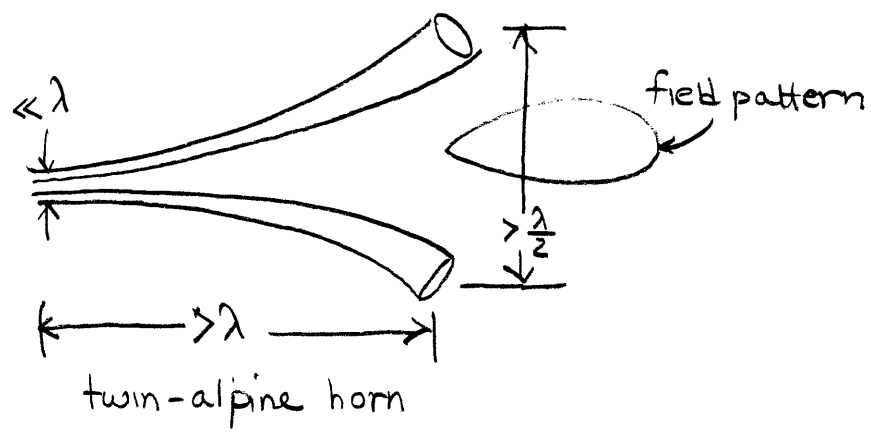


$D \approx 3.5$   
intermediate bandwidth

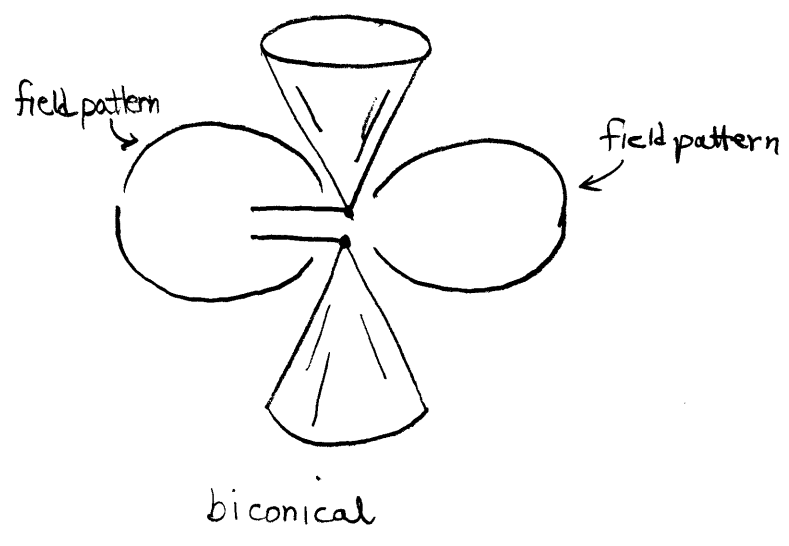


narrow band  
 $D \approx 3.3$

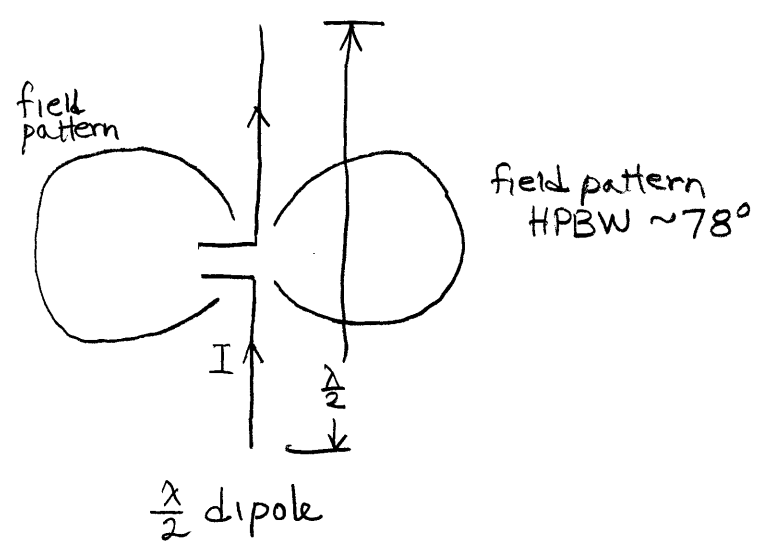
opened out two-conductor antennas



$D > 2$   
broadband

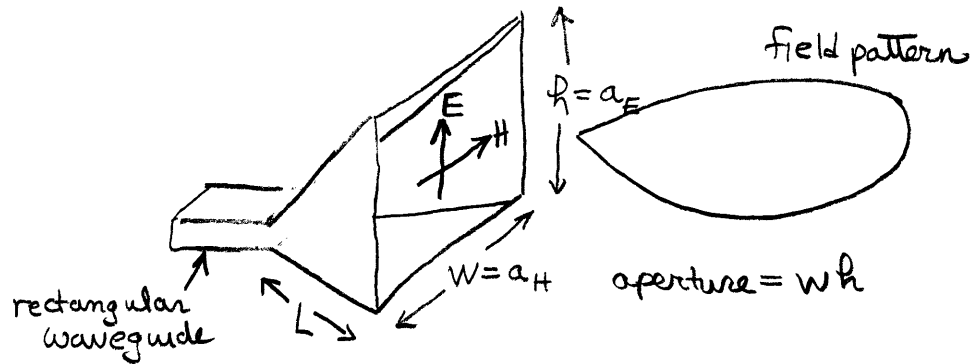


$D \sim 1.75$   
intermediate  
bandwidth



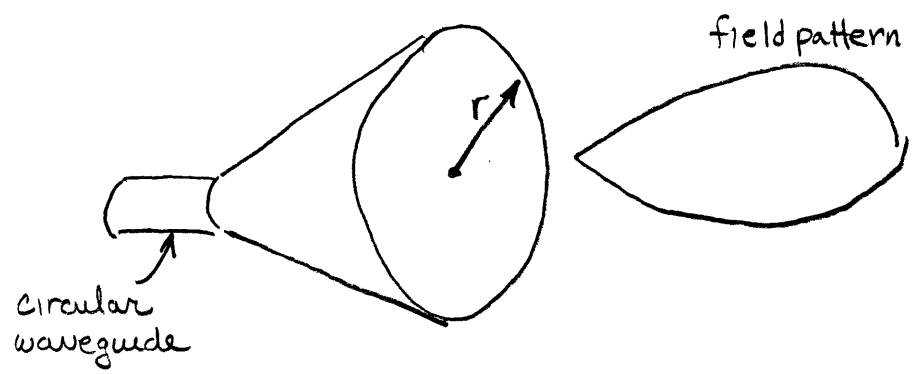
narrow band  
 $D \sim 1.64$

Opened out waveguide antennas



$$D \sim 7.5 \frac{wh}{\lambda^2}$$

Rectangular (pyramidal) horn



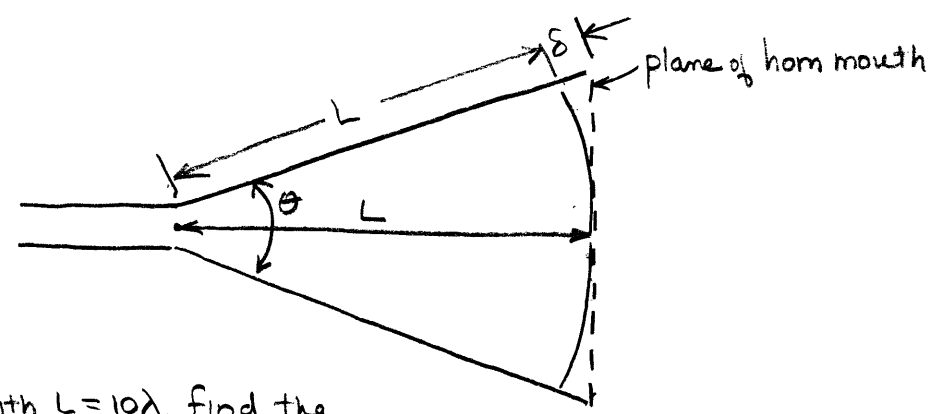
$$D \sim 6.5 \frac{\pi r^2}{\lambda^2}$$

### Example 5-15 Optimum Pyramidal horn

Ideally the phase of the field across the horn mouth should be a constant. This requires a very long horn. However, for practical convenience the horn should be as short as possible.

An optimum horn is a compromise in which the difference in the path length  $\delta$  along the edge and the center of the horn is made  $0.25\lambda$  or less in the E-plane. However, in the H-plane,  $\delta$  can be larger since the field goes to zero at the horn edges (boundary condition  $E_{tan} = 0$  satisfied). From the figure below the horn flare angle  $\theta$  is given by

$$\theta = 2 \cos^{-1} \left( \frac{L}{L + \delta} \right).$$

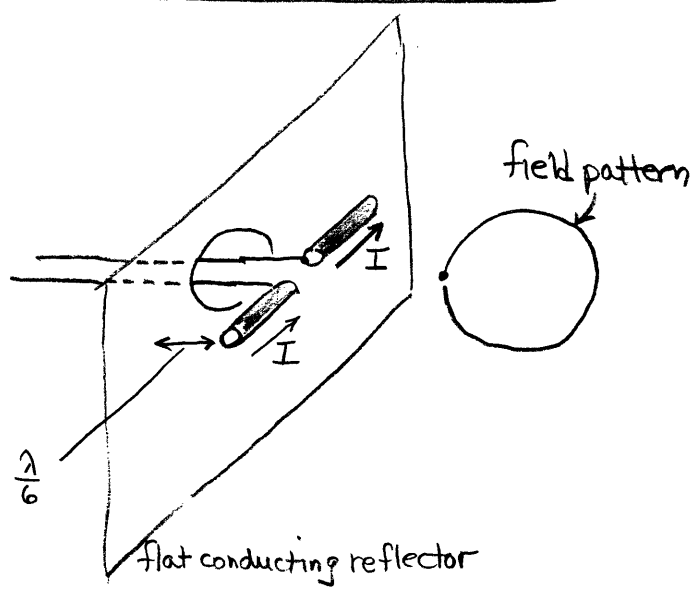


For a horn with  $L = 10\lambda$ , find the largest flare angle for which  $\delta = 0.25\lambda$ .

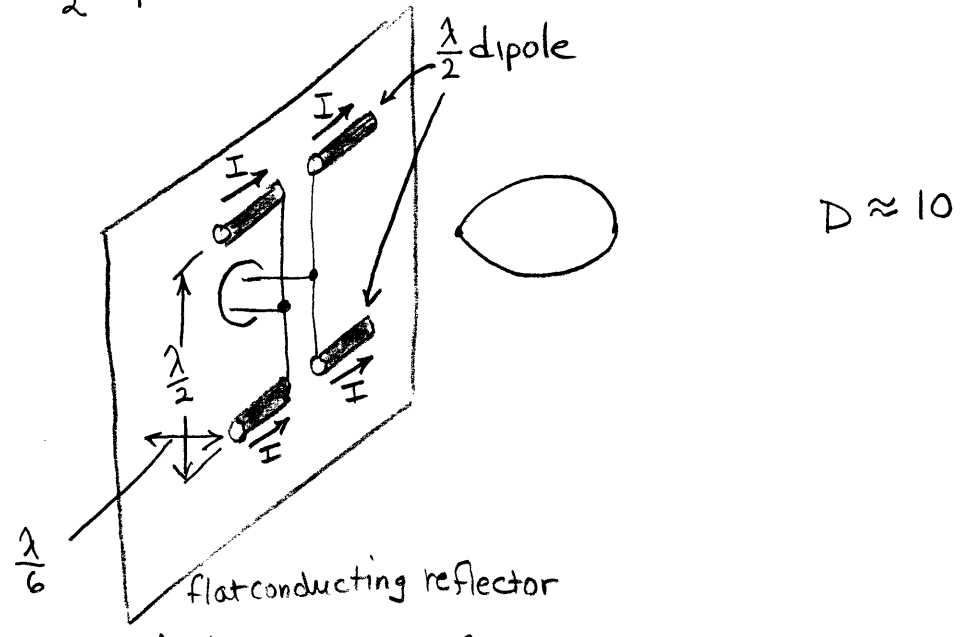
$$\theta = 2 \cos^{-1} \left( \frac{10\lambda}{10\lambda + 0.25\lambda} \right) = 2 \cos^{-1} \left( \frac{10}{10.25} \right)$$

$$\theta = 25.36^\circ$$

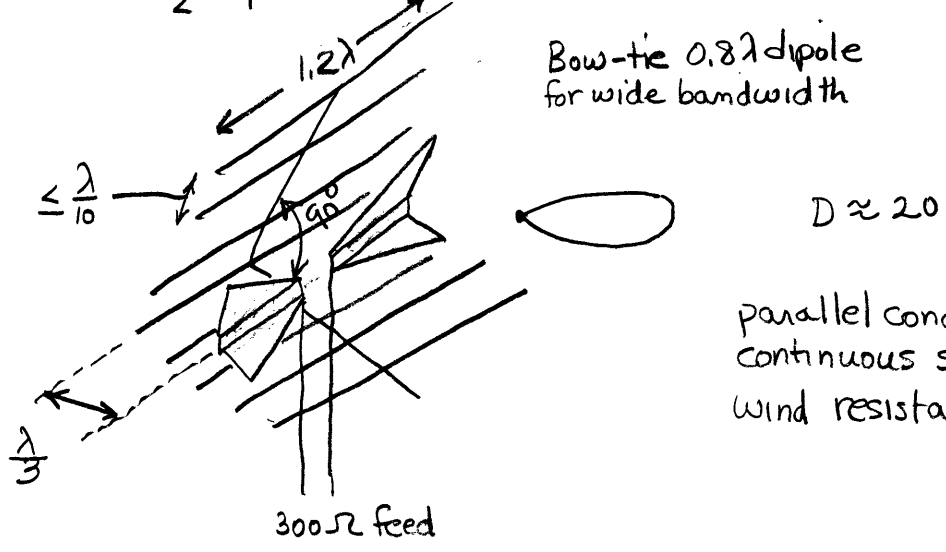
# Flat-sheet reflector antennas



$\frac{\lambda}{2}$  dipole with reflector



two  $\frac{\lambda}{2}$  dipoles with reflector



Example 5-16 Power received by square-corner reflector

A US channel 35 (599 MHz) TV station produces a field strength of  $1 \mu\text{V/m}$  at a square-corner receiving antenna with optimum dimensions for this channel. Find the power delivered to the receiver assuming it is matched to the antenna.

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/s}}{599 \times 10^6 / \text{s}} = 0.501 \text{ meters.}$$

From notes  $D = 20$

The effective aperture can then be calculated from  $D = \frac{4\pi A_e}{\lambda^2}$

$$A_e = \frac{D\lambda^2}{4\pi} = \frac{(20)(.501)^2}{4\pi} = 0.4 \text{ m}^2$$

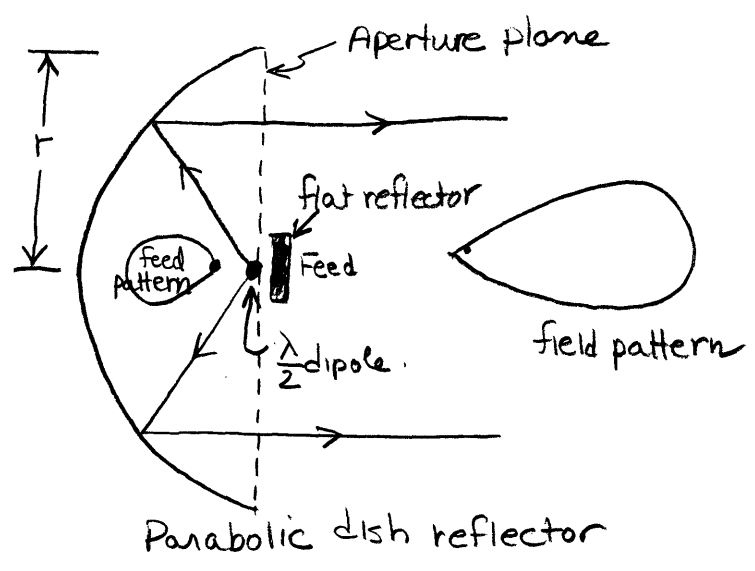
The received power is then:

$$\frac{E^2}{Z_0} A_e = \frac{(10^{-6} \frac{\text{V}}{\text{m}})^2}{(377)} 0.4 \text{ m}^2 = 1.06 \times 10^{-15} \text{ watts.}$$

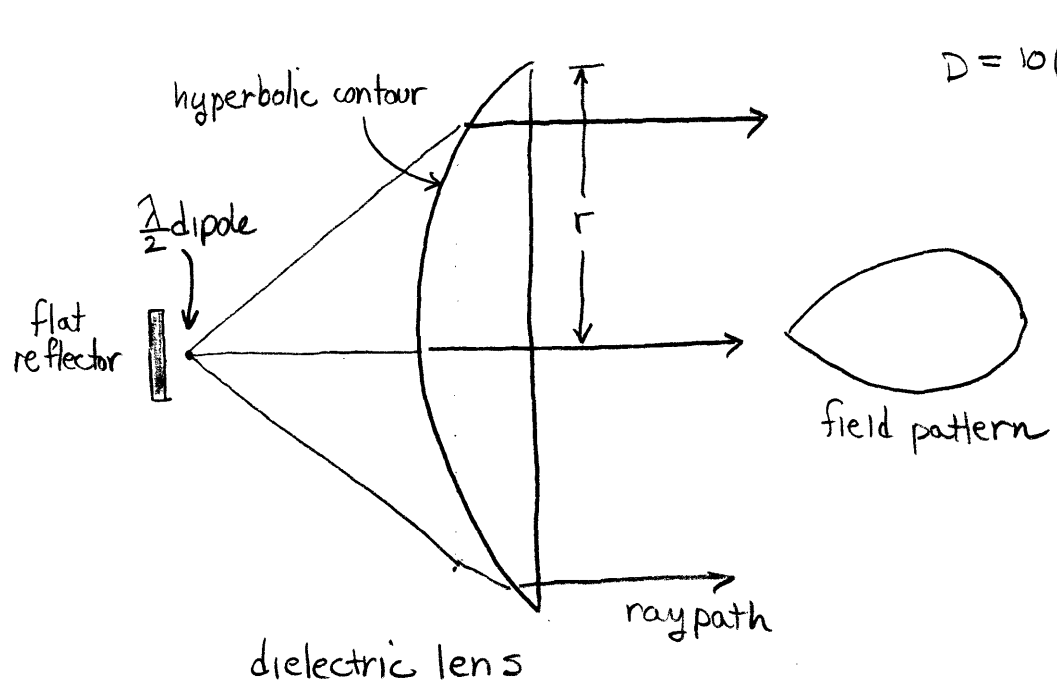
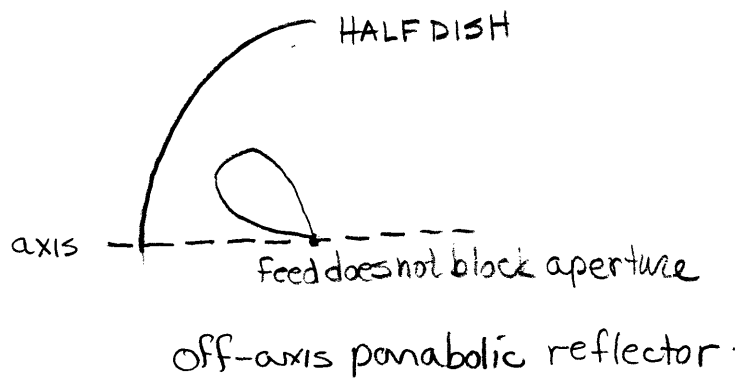
the Poynting vector



aperture antennas



$$D = 10 \left( \frac{r}{\lambda} \right)^2$$



$$D = 10 \left( \frac{r}{\lambda} \right)^2$$

### Example 5-17

The directivity (or gain) of a parabolic dish antenna depends on many factors:

1. The pattern of the feed antenna. If its pattern is too broad and spills over the edge of the dish, the gain is reduced. On the other hand, if the pattern is too narrow, the dish is not fully "illuminated" by the feed and the aperture is not fully utilized.
2. The accuracy of the dish surface relative to an ideal parabola. For example, if the surface departs a distance  $\delta = \frac{\lambda}{4}$  (or 90 electrical degrees) from the parabolic curve, the reflected field is phase shifted  $180^\circ$  which reduces the aperture efficiency.
3. The feed system blocks the center of the dish further reducing efficiency. By using only part of a full dish blocking is avoided. The feed is still on the axis of the parabola but no longer blocks the aperture being used.
4. Many other factors are also involved. The aperture efficiency therefore varies widely depending on the specific design.

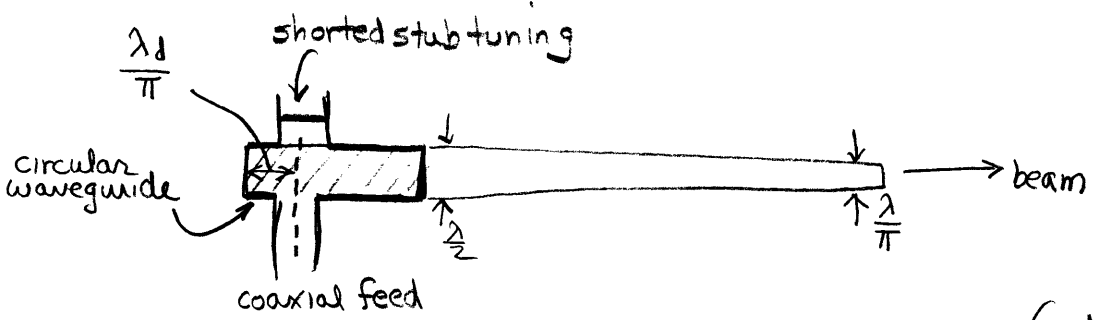
Assuming an aperture efficiency of 70%, what is the directivity of a parabolic dish antenna as a function of its radius.

$$D = 4\pi \frac{A_e}{\lambda^2} \quad \leftarrow \text{replace by } \underbrace{0.7}_{\text{aperture efficiency}} \underbrace{(\pi r^2)}_{\text{area of aperture}}$$

$$D = 4\pi \frac{0.7(\pi r^2)}{\lambda^2}$$

$$D = 27.63 \left(\frac{r}{\lambda}\right)^2$$

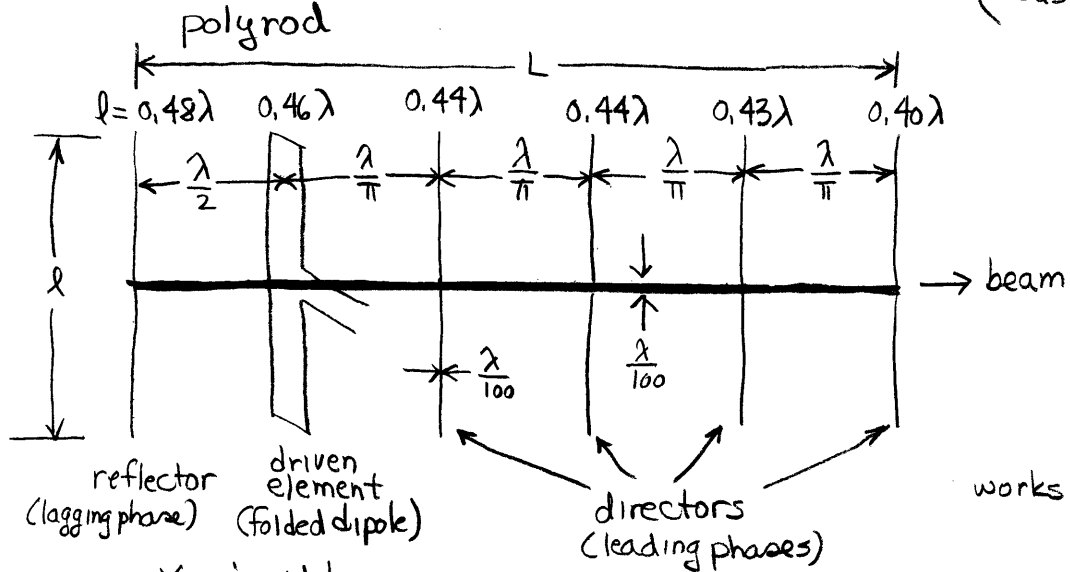
End-fire antennas: Polyrod, Yagi-Uda, Helical



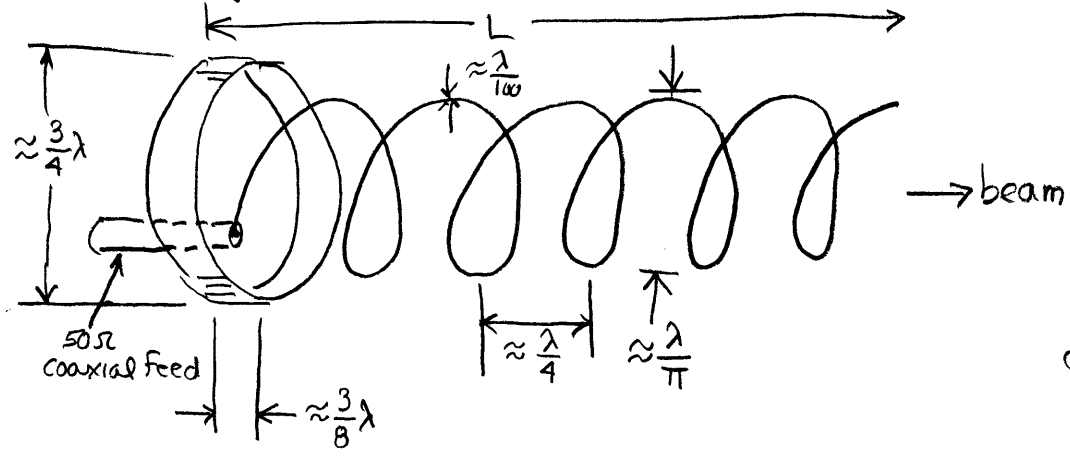
$D \approx 6 \frac{L}{\lambda}$

15% bandwidth

(rods in eyes operate this way)

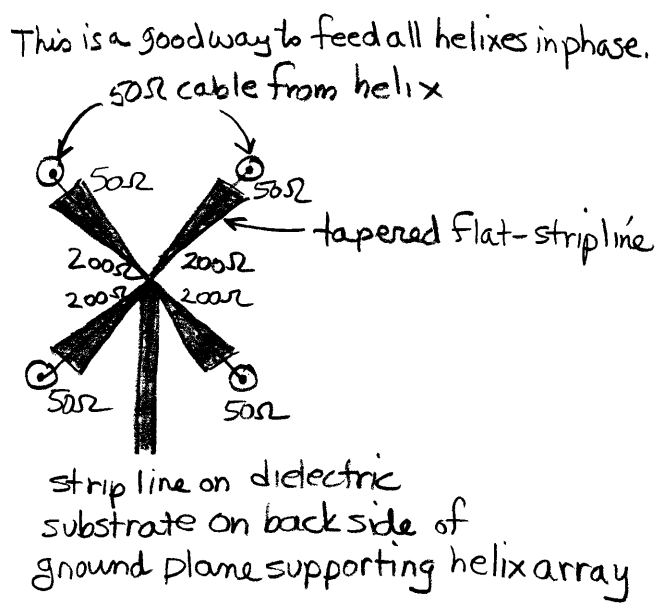
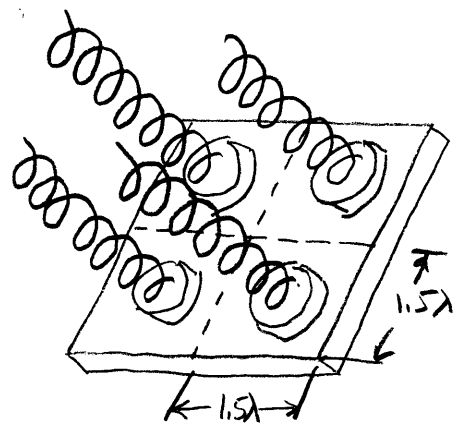


Yagi-Uda



Example 5-18

The figure below shows an array of four right-handed axial-mode helical antennas for communication with satellites. Since the fields hug the helixes, there is minimal coupling or "cross talk" between adjacent helixes and the terminal impedance of each helix is approximately  $50\Omega$  in the array, the same as when used alone.



Determine (a) the best spacing based on the effective apertures of the helixes, (b) the directivity of the array, and (c) connections for feeding all helixes equally and in phase.

The directivity of an axial-mode helix with circumference  $\lambda$  at the center frequency is approximately

$$D = 12 n S_x \quad \begin{matrix} n = \# \text{ of turns} \\ S_x = \text{spacing between turns in } \lambda \end{matrix}$$

For  $n=10$ ,  $S_x = 0.236$

$$D = (12)(10)(0.236) = 28.3$$

The effective aperture of each helix is then

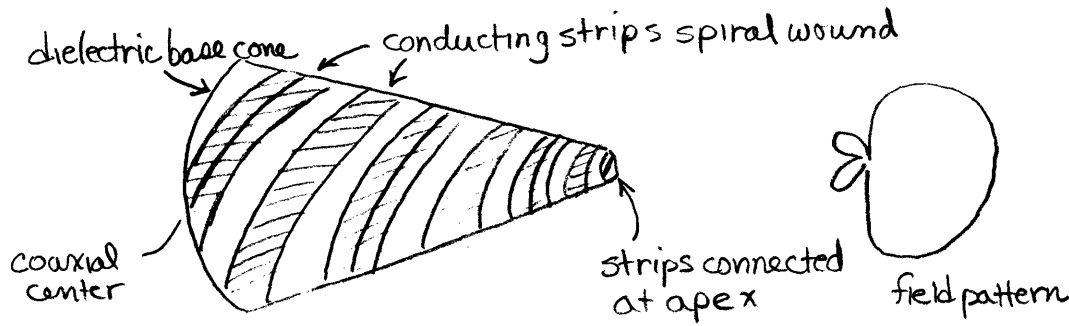
$$D = 4\pi \frac{A_e}{\lambda^2}$$

$$A_e = \frac{D\lambda^2}{4\pi} = \frac{(28.3)}{4\pi} \lambda^2 = 2.25\lambda^2$$

This corresponds to a square of side  $\sqrt{2.25\lambda^2} = 1.5\lambda$ . Use this spacing. At  $1.5\lambda$  spacing the antennas do not interact and the total aperture is  $4A_e$

$$D = 4\pi \frac{A_e}{\lambda^2} = 4\pi \frac{(4)(2.25\lambda^2)}{\lambda^2} = 113.1 \quad (20.5 \text{ dBi})$$

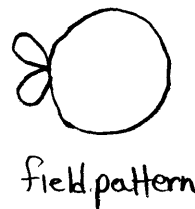
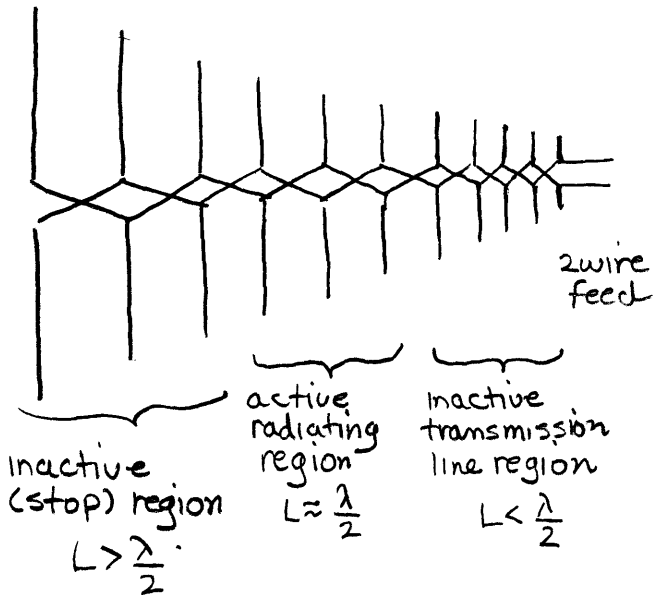
Broad-bandwidth antennas conical spiral, log-periodic, 3-in-1



circularly polarized  
 $D \approx 3$   
 bandwidth 7:1

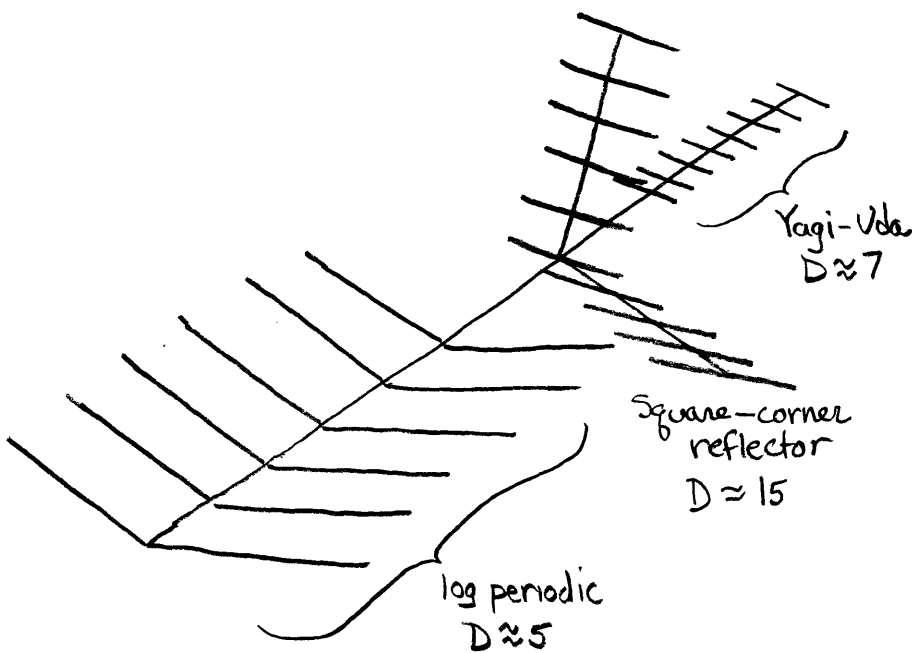
CONICAL SPIRAL

bandwidth limits: low - base  $\frac{\lambda}{2}$  diameter  
 high - apex  $\frac{\lambda}{4}$  diameter



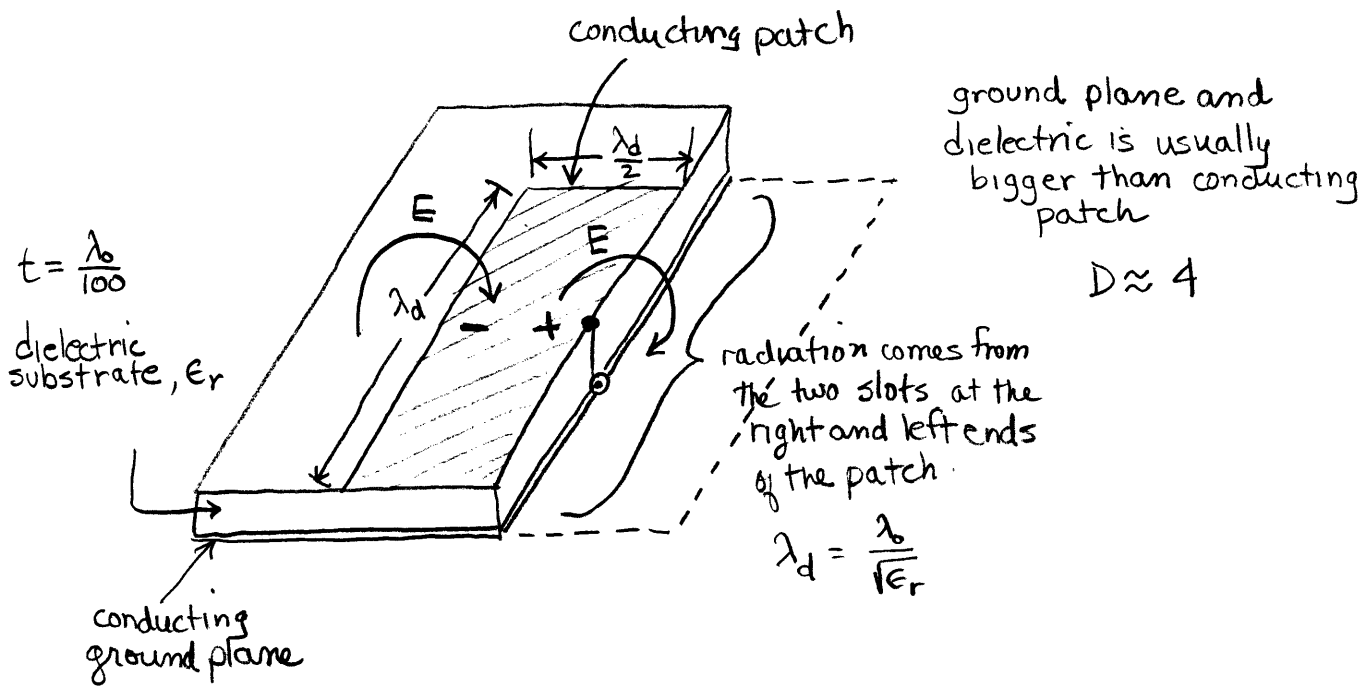
$D \approx 5$

LOG PERIODIC



covers a 16:1 bandwidth  
 VHF-TV, FM, UHF-TV

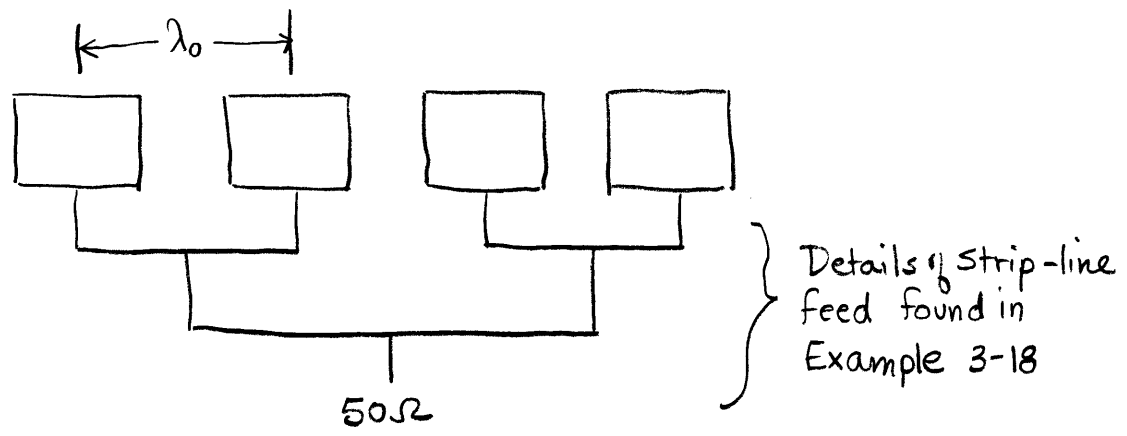
# Patch antennas



$Z_{slot} \approx 50 \Omega$

Example 5-19 Four-patch array.

Find (a) the directivity and (b) beam area of the four-patch array shown below.



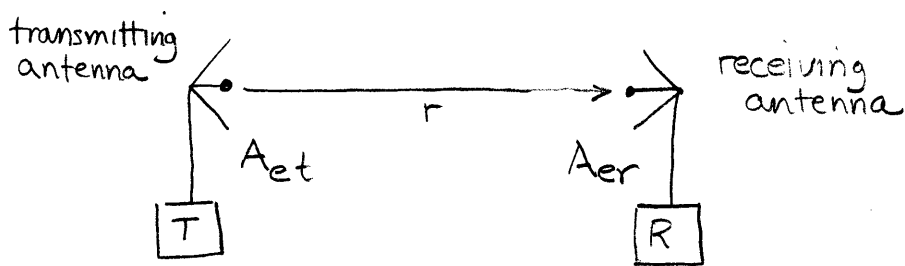
Assuming the dimensions of each patch are  $\lambda$  by  $\frac{\lambda}{2}$

$$A_e = 4 \left( \lambda \times \frac{\lambda}{2} \right) = 2\lambda^2$$

$$D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi (2\lambda^2)}{\lambda^2} = 8\pi \text{ (14dB)}_i$$

$$\Omega_A = \frac{4\pi}{D} = \frac{4\pi}{25} = 0.5 \text{ steradians}$$

## 5.10 Radio Link and Friis Formula



Begin with an isotropic transmitter. The received power at the receiving antenna from the isotropic receiver is

$$P_r = S A_{er}$$

$\uparrow$                        $\nwarrow$   
 Poynting vector      receiving antenna aperture

$$S = \frac{P_t}{4\pi r^2}$$

However, the transmitting antenna actually has a directivity

$$D = \frac{4\pi A_{et}}{\lambda^2}$$

which multiplies the received power by  $D$

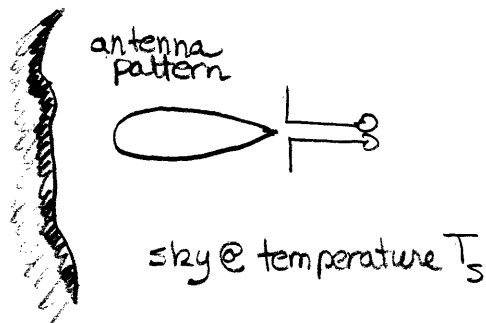
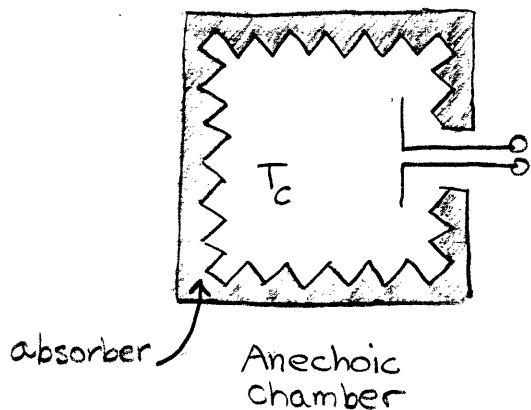
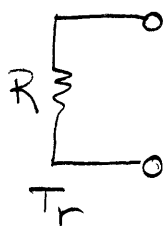
$$\therefore P_r = D S A_{er} = \frac{4\pi A_{et}}{\lambda^2} \frac{P_t}{4\pi r^2} A_{er}$$

$$\frac{P_r}{P_t} = \frac{A_{et} A_{er}}{\lambda^2 r^2}$$

This is the Friis transmission formula for a direct path.



5.11 Antenna temperature, S/N ratio, and remote sensing



Nyquist relation

$$p = kT_r$$

$p = \text{power} / \text{bandwidth}$

$T_r = \text{absolute temp}$

$k = \text{Boltzmann's const.}$

lossless antenna radiation resistance  $R$

noise power at antenna leads  $= p$  if  $T_c = T_r$

The same antenna is pointed at the sky and we receive same  $p$   
we say antenna has noise temp.  $T_A = \text{sky temp. } T_s$

This may be calibrated by switching the receiver between a calibrated resistor at temp.  $T_r$  and the antenna.

The total power available at the receiver is

$$P = kT_A B$$

↑  
receiver bandwidth

assuming that the antenna field pattern encompasses only that region of the sky (i.e., neglect any side or back lobes)

The flux density at the receiver antenna is then

$$S = \frac{P}{A_e} = \frac{kT_A}{A_e}$$

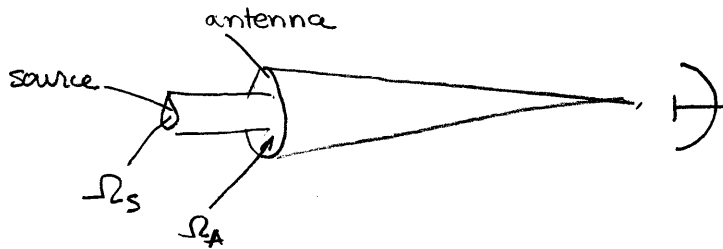
Corrections:

1. multiple sources  
 $S = \frac{k \Delta T_A}{A_e}$  ← differential temp. as antenna moves on and off a source

2. small sources  
 $T_s = \frac{\Omega_A}{\Omega_s} \Delta T_A$  must be corrected for the small size  
 $\Omega_A = \text{antenna solid angle}, \Omega_s = \text{source solid angle}$

### Example 5-20 Mars temperature

The incremental antenna temperature for the planet Mars measured with the U.S. Naval Research Laboratory 15-m radio telescope at 31.5 mm wavelength was found to be  $0.24^\circ\text{K}$ . Mars subtended an angle of  $0.005^\circ$  at the time of measurement. The antenna HPBW =  $0.116^\circ$ . Find the average temperature of Mars at 31.5 mm wavelength.



$$T_s = \frac{\Omega_A}{\Omega_s} \Delta T_A = \frac{(0.116)^2}{\pi \frac{(0.005)^2}{4}} (0.24) = 164^\circ\text{K}$$

$$\Omega_s = \pi \frac{d^2}{4} \quad \text{since the } .005 \text{ represents the angular diameter of Mars.}$$

An infrared measurement made at the same time gave  $T = 250^\circ\text{K}$  for the sunlit planet.

The minimum possible measured temperature by an antenna is  $3^\circ\text{K}$ , the cosmic "background" temperature.

We have previously neglected polarization.

If the source is unpolarized the receiving antenna will only receive  $\frac{1}{2}$  of the transmitted power. The antenna can be linearly or circularly polarized.

$\therefore S = \frac{2k \Delta T_A}{A_e}$       the source flux must be twice as large since the receiving antenna only picks up  $\frac{1}{2}$  the incident power

The signal/noise ratio for a radio link

$$\frac{S}{N} = \frac{\frac{P_t A_{et} A_{er}}{r^2 \lambda^2}}{k T_{sys} B}$$

← received power

← electronic system noise

## 5-22 Downlink S/N

The criterion of detectability of a signal is the S/N. For a transmitter power of 1 watt and an isotropic antenna, the S/N of a lossless line of sight radio link is given by

$$\frac{S}{N} = \frac{\lambda^2}{16\pi^2 r^2 k T_{\text{sys}} B}$$

For a Clarke-orbit geostationary satellite C-band transponder downlink to an earth station, the transponder power = 5W, distance = 36,000 km,  $\lambda = 7.5\text{cm}$ , and antenna gain = 30dB.

If the earth station antenna has 38dB gain and the earth station receiver a system temperature of 100K, find the earth station S/N. The bandwidth  $B = 30\text{MHz}$  for FM TV signals.

For the isotropic antenna

$$\frac{S}{N} = \frac{\lambda^2}{16\pi^2 r^2 k T_{\text{sys}} B} = \frac{(0.075)^2}{16\pi^2 (36 \times 10^6)^2 (1.38 \times 10^{-23})(100)(30 \times 10^6)}$$

$$\frac{S}{N} = 6.64 \times 10^{-7} = -61.8\text{dB}$$

Now, the actual link is using 5W, or  $10 \log \left( \frac{5\text{W}}{1\text{W}} \right) = +7\text{dB}$  power

The satellite has a 30dB antenna giving  $30\text{dB} + 7\text{dB} = +37\text{dB}$   
 compared to the isotropic antenna link

The overall link is then

$$\frac{S}{N} (\text{down link}) = \underbrace{+38\text{dB}}_{\text{earth antenna}} + \underbrace{+37\text{dB}}_{\text{satellite transponder}} - \underbrace{61.8\text{dB}}_{\text{reference design}} = +13.2\text{dB}$$

NOTE: 10dB would be regarded as minimally acceptable.

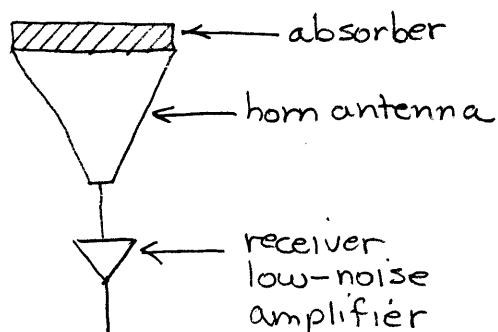
### Example 5-21 Horn absorber

If a perfect absorber with the impedance of space ( $= 377\Omega/\text{square}$ ) is placed so as to completely cover the front D of a horn antenna, it will ideally produce an antenna temperature equal to the absorber's thermometer measured temperature (See Figure below). If the absorber is completely shielded from the outside (open only to the horn) and is cooled, it can provide calibration temperatures for the radio telescope. Thus, if cooled to liquid helium temperature it will give a  $4.1^\circ\text{K}$  calibration.

Equivalently, if the absorber temperature can be controlled so that when the absorber is in front of the horn the radio telescope response is the same as with the absorber removed, the temperature of the object or region being observed by the radio telescope is equal to the absorber temperature.

This type of null measurement was used on the Cosmic Background Explorer (COBE) satellite to remeasure the  $3^\circ\text{K}$ , 4 GHz sky background originally discovered by Penzias and Wilson. The temperature has now been evaluated more accurately as  $2.73^\circ\text{K}$ . This temperature is believed to be from the remnant of the primordial Big Bang and is the lowest possible antenna temperature for a sky-scanning antenna.

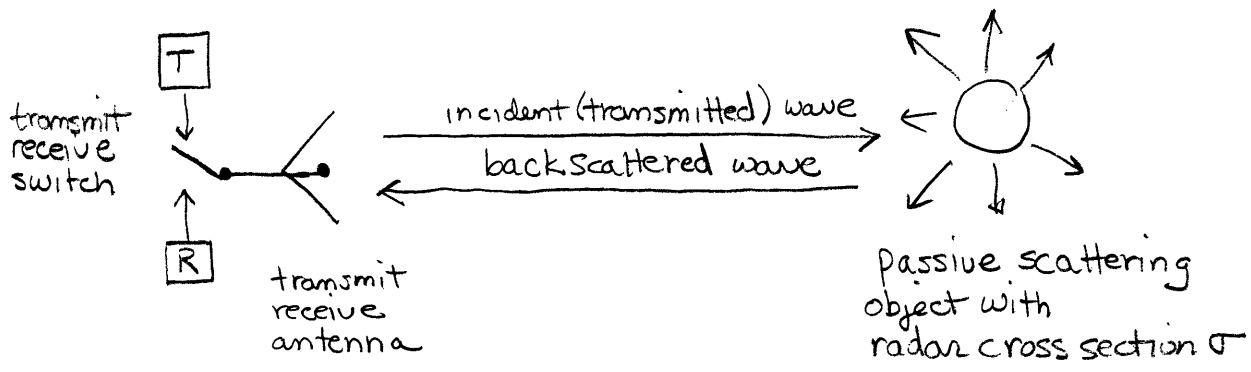
If a narrow-beam 4-GHz antenna looking at essentially empty sky at the zenith has an antenna temperature of  $4.73^\circ\text{K}$ , how much is due to side lobes or antenna loss?



$$\text{Solution: } 4.73 - 2.73 = 2^\circ\text{C}$$

↑ this is what the antenna should be measuring  
 ↑ this is what it actually measures.

### 5-12 Radar and radar cross-section



The power actually hitting the object is

$$P_{\text{intercepted}} = \frac{P_t A_{et}}{r^2 \lambda^2} \sigma \leftarrow \sigma \text{ is radar cross-section and is essentially an effective aperture}$$

where we used the Friis formula in the forward direction

If the object scatters isotropically ( $D=1$ ) its effective gain (directivity) is  $\frac{\lambda^2}{4\pi}$  since  $A_e = \frac{D\lambda^2}{4\pi}$

Going in the other direction the power back to the antenna is

$$P_r(\text{by antenna}) = \frac{A_{et}}{r^2 \lambda^2} P_{\text{backscattered}}$$

$$= \frac{A_{et}}{r^2 \lambda^2} \left( P_{\text{intercepted}} \frac{\lambda^2}{4\pi} \right)$$

$$P_r(\text{by antenna}) = \frac{A_{et}}{r^2 \lambda^2} \frac{P_t A_{et} \sigma^2 \lambda^2}{r^2 \lambda^2 4\pi}$$

$$\frac{P_r(\text{by antenna})}{P_t} = \frac{A_{et}^2 \sigma}{4\pi r^4 \lambda^2} \quad \text{This is the radar equation.}$$

This can also be used to define  $\sigma$

$$\sigma = \frac{P_r(\text{by antenna}) 4\pi r^4 \lambda^2}{P_t A_{et}^2} = \frac{S_r}{S_{inc} / 4\pi r^2} = \frac{\text{scattered power}}{\text{incident power density}}$$

Pulse radar — uses a T/R switch

$$d = \frac{1}{2} c \Delta t$$

where  $\Delta t$  = time between transmission of the pulse and reception of its echo

$d$  = distance to object

Doppler radar — transmitter & receiver continuously using a circulator

$$v = \frac{1}{2} \frac{\Delta f}{f_0} c$$

where  $\Delta f$  = change in frequency  $\Delta f$  of the echo with respect to the transmitted frequency  $f_0$

$v$  = velocity of object

(coherent) Pulse doppler weather radar

phase of echo  $\phi = 2\beta r = 2 \left( \frac{2\pi}{\lambda} \right) r = \frac{4\pi r}{\lambda}$

if the source is moving you get

$$\phi(t) = \frac{4\pi}{\lambda} (r + v_r t)$$

↑  
velocity component in direction of  $r$

Differentiating gives the Doppler shift in radians/sec

$$\Delta\omega = \frac{d\phi}{dt} = \frac{4\pi}{\lambda} v_r$$

Converting to linear frequency

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{2v_r}{\lambda}$$

If the radar is emitting pulses every  $T$  seconds the maximum frequency shift that can be measured is given by Nyquist requirement as

$$\Delta f_{\max} = \frac{1}{2T}$$

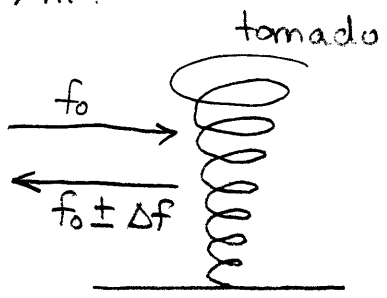
If the radar emits  $N$  pulses the lowest frequency that can be measured is that which corresponds to the measurement period

$$\Delta f_{\min} = \frac{1}{NT}$$

Example 5-23 Weather radar

For an X-band (10GHz) weather radar, find

- (a) the minimum pulse repetition (PRF = 1/PRI) which may be used to unambiguously measure the wind velocity in a tornado with a wind speed of 350 km/hr.
- (b) At this PRF, how many pulses must be sampled to resolve in frequency two portions of the tornado with a differential velocity of 1 km/hr?



Using the doppler shift relationship

$$\Delta f = \frac{2V_r}{\lambda} = \frac{2 \left( \frac{350 \times 10^3 \text{ m}}{3600 \text{ sec}} \right)}{\frac{c}{f}} = \frac{2 \left( \frac{3.5 \times 10^5 \text{ m}}{3.6 \times 10^3 \text{ s}} \right)}{\frac{3 \times 10^8 \text{ m/s}}{10 \times 10^9 / \text{s}}}$$

$$\Delta f = 6.5 \text{ kHz}$$

To measure this requires a T of

$$\Delta f_{\text{max}} = \frac{1}{2T}$$

$$T = \frac{1}{2\Delta f_{\text{max}}} = \frac{1}{2(6.5 \times 10^3 / \text{sec})} = 7.7 \times 10^{-5} \text{ sec} = 77 \mu\text{sec}$$

$$\text{PRF} = \frac{1}{T} = \frac{1}{7.7 \times 10^{-5}} \approx 13 \text{ kHz}$$

The difference in frequency from the two portions is

$$\Delta f_1 - \Delta f_2 = \frac{2V_{r1}}{\lambda} - \frac{2V_{r2}}{\lambda} = \frac{2(V_{r1} - V_{r2})}{\lambda} = \frac{2 \left( \frac{1000 \text{ m}}{3600 \text{ s}} \right)}{\left( \frac{3 \times 10^8 \text{ m/s}}{10 \times 10^9 / \text{s}} \right)}$$

$$\Delta f_1 - \Delta f_2 = 18.5 \text{ kHz}$$

This must satisfy

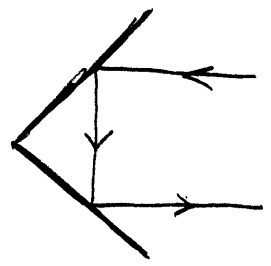
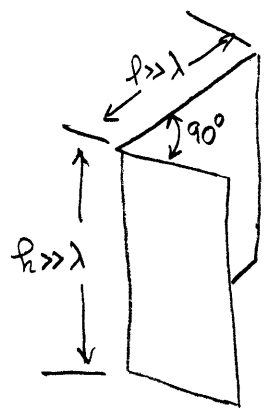
$$\Delta f_{\text{min}} = \frac{1}{NT}$$

to given

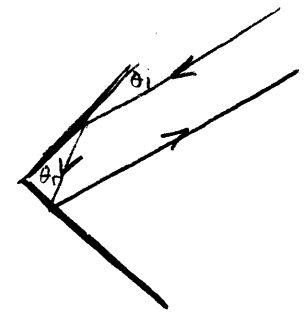
$$N = \frac{1}{\Delta f_{\text{min}} T} = \frac{1}{(18.5 \times 10^3)(7.7 \times 10^{-5})} = 702 \text{ pulses}$$



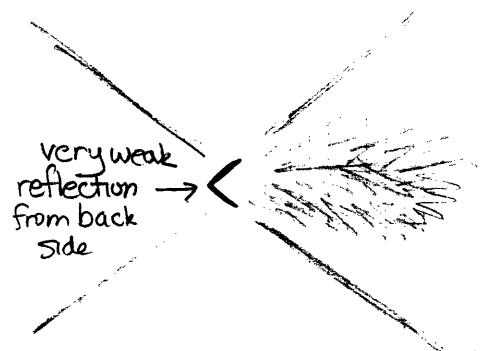
### The corner reflector



at  $45^\circ$  incidence  
the reflection is parallel  
to the original wave

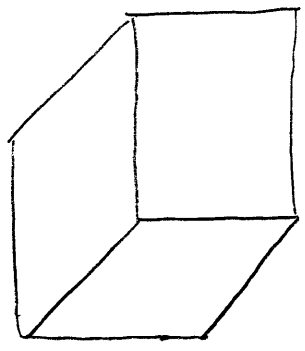


this same parallel  
reflection occurs for  
all other angles  
since  $\theta_i = \theta_r$



strong broad reflection  
from front side

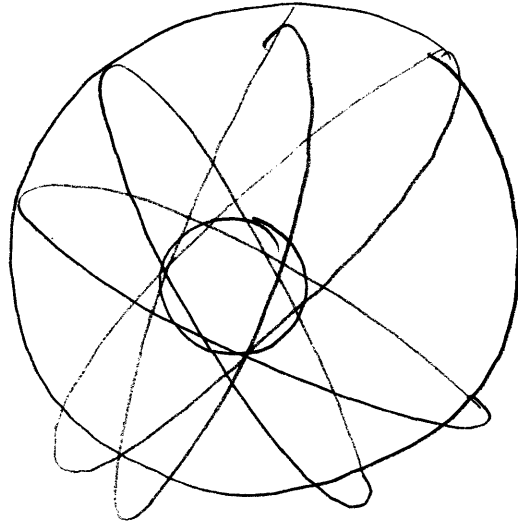
strong reflections from flat sides



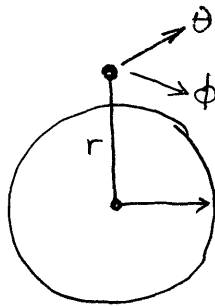
trihedral corner reflector  
provides 3-D reflections parallel  
to incident wave

### 5-13 Global Positioning Satellites

The satellites transmit position and time continuously at 1.23GHz and 1.58GHz



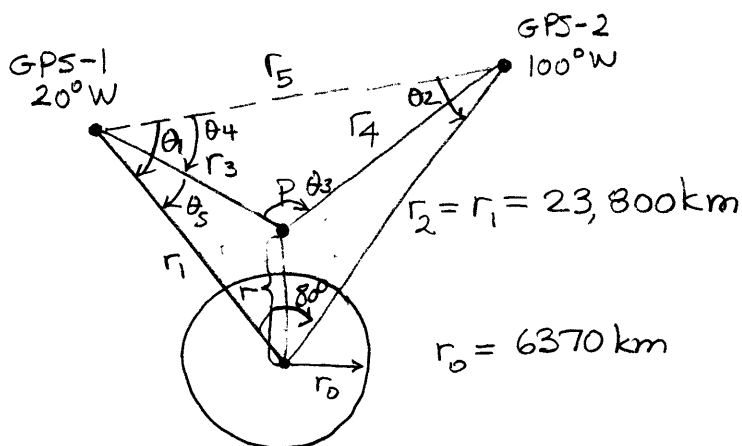
24 satellites  
in six transpolar  
orbits at 20,006 km  
above earth



### Example 5-24

Consider a GPS receiver at point P above the earth's equator with two GPS satellites, 1 and 2, also above the equator at different longitudes as shown below. The receiver detects a pulse from GPS-1 with code that informs the receiver it was sent at a time  $\Delta t_1$  earlier from a position at radius  $r_1$  and longitude  $20^\circ$  W. Simultaneously GPS-2 sends a pulse coded to tell the receiver it was sent  $\Delta t_2$  earlier from a radius  $r_2$  and longitude  $100^\circ$  W.

If  $\Delta t_1 = 64$  ms and  $\Delta t_2 = 60$  ms, find the longitude and elevation of the GPS receiver at point P.



$$r_3 = c \Delta t_1 = (3 \times 10^8)(64 \times 10^{-3}) = 19,200 \text{ km}$$

$$r_4 = c \Delta t_2 = (3 \times 10^8)(60 \times 10^{-3}) = 18,000 \text{ km}$$

using law of cosines

$$r_5^2 = r_1^2 + r_2^2 - 2r_1r_2 \cos 80^\circ = (23800)^2 + (23800)^2 - 2(23800)^2 \cos 80^\circ$$

$$r_5^2 = 9.3616 \times 10^8$$

$$r_5 = 30596.7 \text{ km.}$$

Now using law of sines

$$\frac{r_5}{\sin 80^\circ} = \frac{r_1}{\sin \theta_1} = \frac{r_2}{\sin \theta_2}$$

$$\theta_1 = \theta_2 = \sin^{-1} \left( \frac{r_1 \sin 80^\circ}{r_5} \right) = \sin^{-1} \left( \frac{23800 \sin 80^\circ}{30596.7} \right)$$

$$\theta_1 = \theta_2 = \sin^{-1}(.7660) = 50.0^\circ$$

Again using law of cosines

$$r_5^2 = r_3^2 + r_4^2 - 2r_3r_4 \cos \theta_3$$

$$\cos \theta_3 = \frac{r_3^2 + r_4^2 - r_5^2}{2r_3r_4} = \frac{(19200)^2 + (18000)^2 - (30596.7)^2}{2(19200)(18000)} = -0.3523$$

$$\theta_3 = 110.62^\circ$$

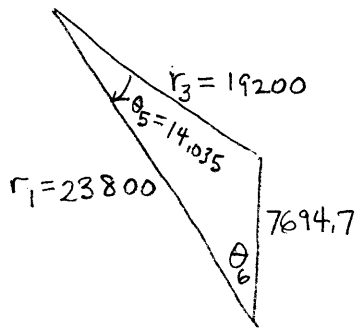
Now, we use law of sines again

$$\frac{r_5}{\sin(110.62^\circ)} = \frac{r_4}{\sin \theta_4}$$

$$\sin \theta_4 = \frac{r_3 \sin(110.62^\circ)}{r_5} = \frac{(18000) \sin(110.62^\circ)}{30596.7} = 0.5506$$

$$\theta_4 = 33.409^\circ$$

$$\text{This gives } \theta_5 = \theta_1 - \theta_4 = 50.0^\circ - 33.409^\circ = 16.591^\circ$$



using law of cosines

$$r^2 = r_1^2 + r_3^2 - 2r_1 r_3 \cos \theta_5$$

$$r^2 = (23800)^2 + (19200)^2 - 2(23800)(19200) \cos(16.591^\circ)$$

$$r^2 = 9.3508 \times 10^8 - 8.7587 \times 10^8 = 5.92088 \times 10^7$$

$$r = 7694.7 \text{ km.}$$

$$\text{elevation} = 7694.7 - 6370 = 1324.7 \text{ km.}$$

The difference between this and the book's answer is probably due to math errors on my part.

$$\frac{\sin 14.035}{7694.7} = \frac{\sin \theta_6}{19200.}$$

$$\sin \theta_6 = \frac{19200 \sin(14.035)}{7694.7} = 0.605$$

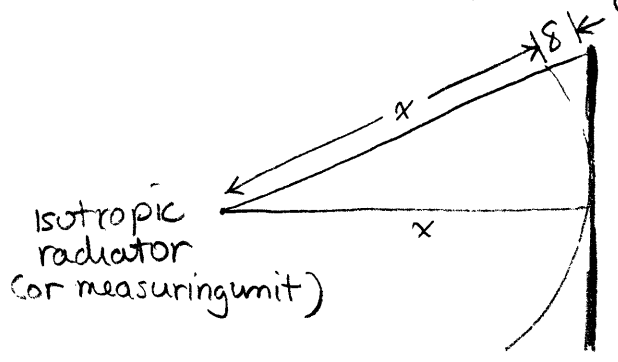
$$\theta_6 = 37.24^\circ$$

The latitude is then  $20^\circ + 37.24^\circ = 57.24^\circ \text{W}$

This is due to the difference in height.

# 5-14 Far Field & Near Field

In the far field the path lengths from all parts of the antenna are nearly in phase. Alternatively we might require that the wavefront will depart by no more than a specified distance  $\delta$  if the antenna is replaced by an isotropic radiator.



In general, you need a very long distance  $x$  to make measurements on large antennas. Two options are (1) to use a celestial source to do antenna measurements, or (2) to do near-field measurements right at the antenna and determine the far-field pattern by a Fourier transform.

In option (1) we measure the received signal strength as the source moves across the sky in front of the antenna.

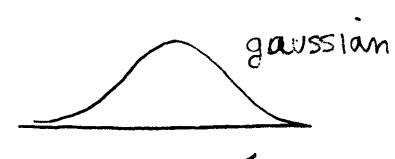
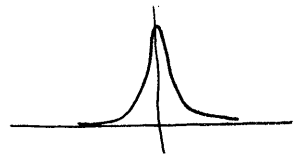
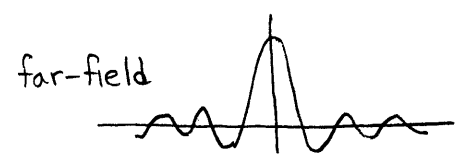
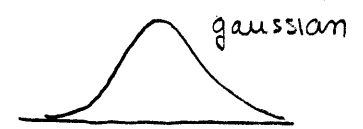
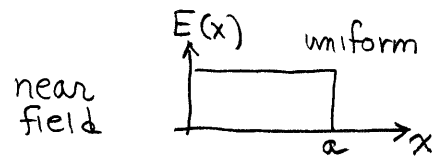
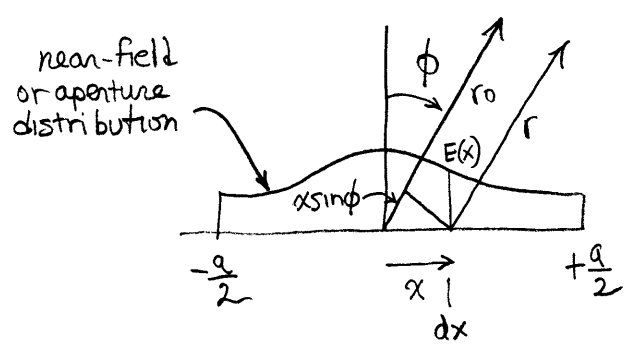
In option (2)

$$E(\phi) = \int_{-\frac{a_x}{2}}^{+\frac{a_x}{2}} E(x_x) e^{j2\pi x_x \sin \phi} dx_x$$

where  $E(x_x)$  = electric field measured across aperture

and  $x_x = \frac{x}{\lambda}$

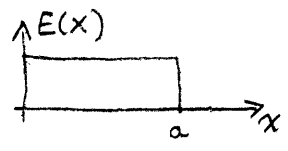
This is a Fourier transform.



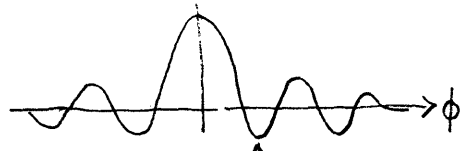
no side lobes.

Figure 5-56

Near field or aperture distribution



far-field pattern

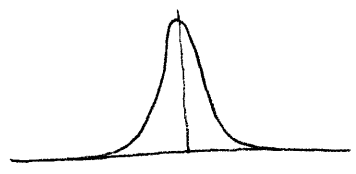
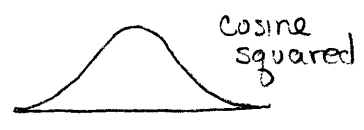


first side lobe level

Far-field

HPBW

Side-lobe level

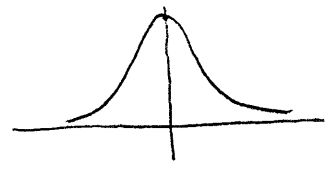
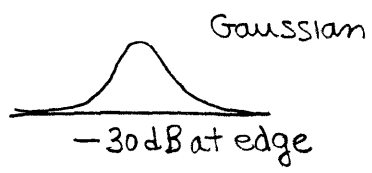


$$\frac{57^\circ}{a_\lambda}$$

$$-13\text{dB}$$

$$\frac{83^\circ}{a_\lambda}$$

$$-32\text{dB}$$

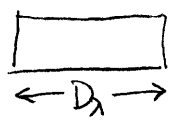


$$\frac{85^\circ}{a_\lambda}$$

$$-49\text{dB}$$

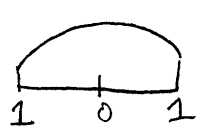
Figure 5-57

uniform



$$\frac{58^\circ}{D_\lambda}$$

$$-18\text{dB}$$

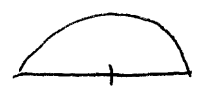


$$E(r) = 1 - \frac{2r^2}{3}$$

tapered to  $\frac{1}{3}$  at edge

$$\frac{66^\circ}{D_\lambda}$$

$$-23\text{dB}$$



$$E(r) = 1 - r^2$$

tapered to 0 at edge

$$\frac{73^\circ}{D_\lambda}$$

$$-25\text{dB}$$



$$E(r) = (1 - r^2)^2$$

tapered to 0 at edge

$$\frac{84^\circ}{D_\lambda}$$

$$-31\text{dB}$$

Example 5-25 Far field measurement

If the antenna dimension  $y = 20\text{m}$  and  $\delta = \frac{\lambda}{10}$ , find the required distance  $x$  at a frequency of 12 GHz.

$$\text{From the geometry } (x + \delta)^2 = x^2 + \left(\frac{y}{2}\right)^2$$

$$x^2 + 2\delta x + \delta^2 = x^2 + \frac{y^2}{4}$$

For  $x \gg \delta$  we get

$$2\delta x = \frac{y^2}{4}$$

$$\text{or } x = \frac{y^2}{8\delta}$$

For this frequency  $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{12 \times 10^9} = 0.025\text{m}$

The required far-field measurement distance is then

$$x = \frac{(20\text{m})^2}{8(0.025\text{m})} = 2 \times 10^4 = 20\text{km}$$

Example 5-26 100m dish for deep-space communication

The aperture distribution of a 100-m-diameter dish antenna is tapered to one-third at the edge or about 10db down. At 10 GHz, what is (a) its HPBW, (b) gain from beam width, (c) gain from effective aperture, and (d) first-side-lobe level.

From Fig. 5-57 HPBW =  $\frac{66^\circ}{D_x}$  for a  $\frac{1}{3}$  (10db) taper.

$$\text{HPBW} = \frac{66^\circ}{\left(\frac{100\text{m}}{.03\text{m}}\right)} \cong 0.02^\circ \quad (\text{since } \lambda = .03\text{m})$$

Recall that  $G \cong \frac{41,000}{(\text{HPBW})^2} = 1.05 \times 10^8$  (80 dBi)

From aperture

$$G = \frac{4\pi A_p \epsilon_{ap}}{\lambda^2} = \frac{4\pi \pi (50\text{m})^2 (0.725)}{(.03)^2}$$

this aperture efficiency was not given in the problem

$$G = 8.0 \times 10^7 \quad (79 \text{ dBi})$$

Table 5-57 gives the first side lobe level as -23dB

## 5-15 Cellular Systems

## Example 5-27

A cell-tower system operates at 850 MHz for communication with hand-held phones. The cell-tower antenna array has  $360^\circ$  horizontal and  $10^\circ$  HPBW vertical coverage. However, the particular antenna in use has  $90^\circ$  horizontal HPBW coverage and, therefore, 4 times more gain. Find (a) tower power and (b) hand-held power to provide a SNR = 40 dB for voice and high-speed data 1 MHz bandwidth at 12 km distance. The cell-tower receiver temperature  $T = 30^\circ\text{K}$  and the hand-held  $T = 100\text{K}$ .

The SNR for a radio link comes from the Friis transmission formula as

$$P_t = \frac{S}{N} \frac{r^2 \lambda^2 kTB}{A_e r A_{e_t}}$$

For 850 MHz  $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{850 \times 10^6} = 0.35\text{m}$

Assuming the hand-held receiver uses an isotropic antenna

$$A_{e_r} = \frac{D \lambda^2}{4\pi} = \frac{\lambda^2}{4\pi} \quad \text{since } D=1$$

For the tower antenna

$$D_t = \frac{41000}{(360^\circ)(10^\circ)} = 11.4$$

The corresponding tower aperture is

$$A_{e_t} = \frac{D_t \lambda^2}{4\pi} = 0.9 \lambda^2$$

This is for the given cell-phone tower. I am not sure what the second antenna is for. If it replaces the original antenna it will change  $D_t$  and all subsequent calculations.

$$\text{SNR} = 40\text{dB} = 10 \log_{10} \frac{P_2}{P_1} \quad \therefore \frac{P_2}{P_1} = 10^4$$

From the tower to the hand-held  
The required transmitter power is then receiver temp

$$P_t = \frac{(10^4) (12,000)^2 \lambda^2 (1.38 \times 10^{-23}) (100) (1 \times 10^6)}{\left(\frac{\lambda^2}{4\pi}\right) (0.9 \lambda^2)}$$

$$P_t = 0.227 \text{ watts}$$

In the opposite direction the only thing that changes is the receiver temperature so

$$P_r = 0.3 \times 0.227 = 0.068 \text{ watts}$$



## 5-16 Absorption by atmosphere and foliage

For frequencies  $> 50$  GHz the atmosphere becomes highly attenuating due to water vapor and  $O_2$  absorption. Weather such as rain (scattering by the raindrops) also causes attenuation.

A radio link operating through an absorbing medium sees a temperature

$$T = T_a (1 - e^{-\tau})$$

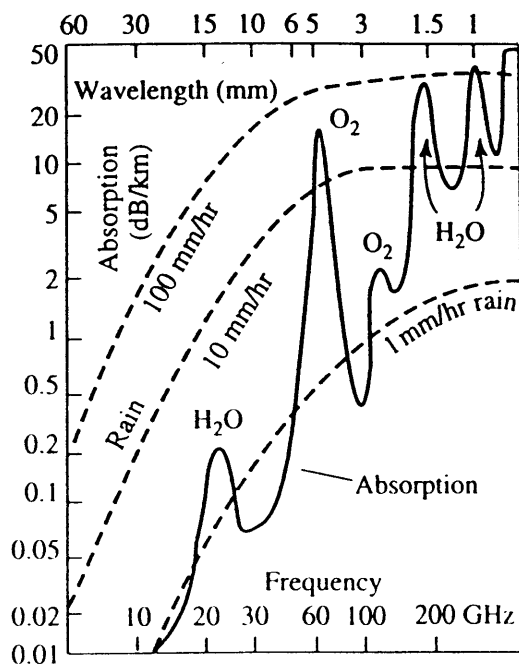
where

$T_a$  = atmospheric temperature

$\tau = x\alpha$  is the absorption coefficient

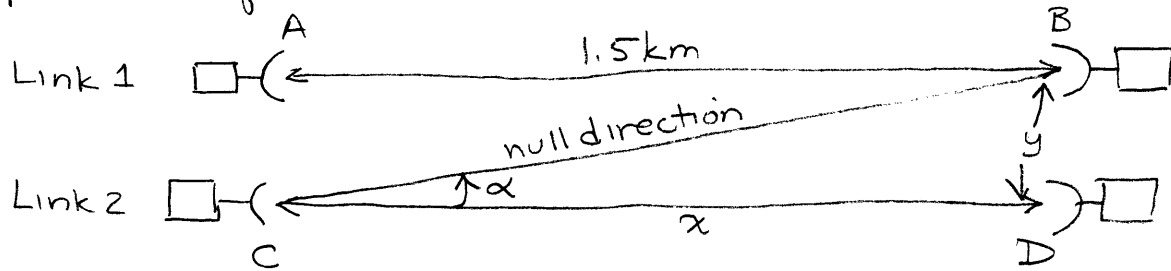
$x$  = distance through absorber, m

$\alpha$  = absorption coefficient,  $Np/m$



## Example 5-28

Two links each 1.5 km long are to be operated parallel to each other in close proximity as shown below. They are to have a SNR = 50 dB and a cross-talk isolation of 50 dB. For a bandwidth = 5 MHz, power = 1 W, parabolic dish diameter = 50 cm, aperture efficiency = 60%, frequency = 55 GHz, receiver temperature  $T = 53$  K, and atmospheric temperature =  $20^\circ\text{C}$ , what is the minimum permissible separation of the two links?



From Fig 5-59 attenuation at 55 GHz = 2 dB/km.

We need to convert this to an exponent (nepers/km).

$$1 \text{ Neper} = 10 \log e = 4.34 \text{ dB.}$$

$$\alpha = 2 \frac{\text{dB}}{\text{km}} \times \frac{\text{Neper}}{4.34 \text{ dB}} = 0.46 \frac{\text{Np}}{\text{km}}$$

$$\gamma = 0.46 \frac{\text{Np}}{\text{km}} \times 1.5 \text{ km} = 0.69$$

The atmospheric temperature is  $273 + 20 = 293$  K

The link then sees a temperature

$$T = 293 (1 - e^{-0.69}) = 147^\circ \text{K}$$

This adds to the receiver temperature

$$T = 53 + 147 = 200^\circ \text{K}$$

The effective apertures of the antennas are

$$A_{er} = A_{et} = (0.6) \pi r^2 = (0.6) \pi \left(\frac{1}{4} \text{m}^2\right) = 0.12 \text{m}^2$$

Now we use the Friis formula to calculate link SNR

$$\begin{aligned} \frac{S}{N} &= \frac{P_t A_{et} A_{er}}{r^2 \lambda^2 k T B} = \frac{(1 \text{W})(0.12 \text{m}^2)(0.12 \text{m}^2)}{(1.5 \times 10^3 \text{m})^2 \left(\frac{3 \times 10^8}{55 \times 10^9}\right)^2 (1.38 \times 10^{-23})(200)(5 \times 10^6)} \\ &= 1.56 \times 10^{10} \quad (102 \text{dB}) \end{aligned}$$

Since we only need 50 dB this gives us a margin of  $102 - 50 = 52$  dB.

The spacing between the two links is dependent upon the antenna pattern. For a uniform circular aperture Kraus introduces the formula

$$BWFN = \frac{140^\circ}{D_x} = \frac{140^\circ}{\frac{0,5m}{5,45 \times 10^{-3}m}} = 1,526^\circ$$

since  $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{55 \times 10^9} = 5,45 \times 10^{-3}$  meters

From our figure  $\alpha = \frac{1,526^\circ}{2} = 0,713^\circ$

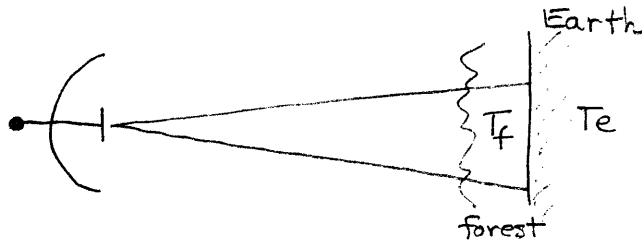
You want to then space antenna B and D by

$$y = x \tan \alpha = (1,5 \text{ km}) \tan (0,713^\circ) = 20,0 \text{ meters}$$

You should also do the same thing for antennas A and C by the same amount.

### Example 5-29 Forest absorption

An earth-resource satellite's passive remote sensing antenna directed at the Amazon River Basin measures a nighttime temperature  $T_A = 21^\circ\text{C}$ . If the earth temperature  $T_e = 27^\circ\text{C}$  and the Amazon forest temperature  $T_f = 15^\circ\text{C}$ , find the forest absorption coefficient  $\mathcal{J}_f$ .



The incremental satellite temperature

$$\Delta T_A = T_f (1 - e^{-\mathcal{J}_f}) + T_e e^{-\mathcal{J}_f}$$

this is the temp of the radar beam going thru the forest      this is the temperature of the earth thru the forest.

$$\Delta T_A = T_f - T_f e^{-\mathcal{J}_f} + T_e e^{-\mathcal{J}_f}$$

$$\frac{\Delta T_A - T_f}{T_e - T_f} = e^{-\mathcal{J}_f}$$

$$e^{-\mathcal{J}_f} = \frac{21 - 15}{27 - 15} = 0.5$$

$$\mathcal{J}_f = 0.69$$