EECS 311 - SPRING 2003

EXAM– 3/22/03

NAME: SOLUTIONS CWRUnet e-mail address:____________

IMPORTANT INFORMATION:

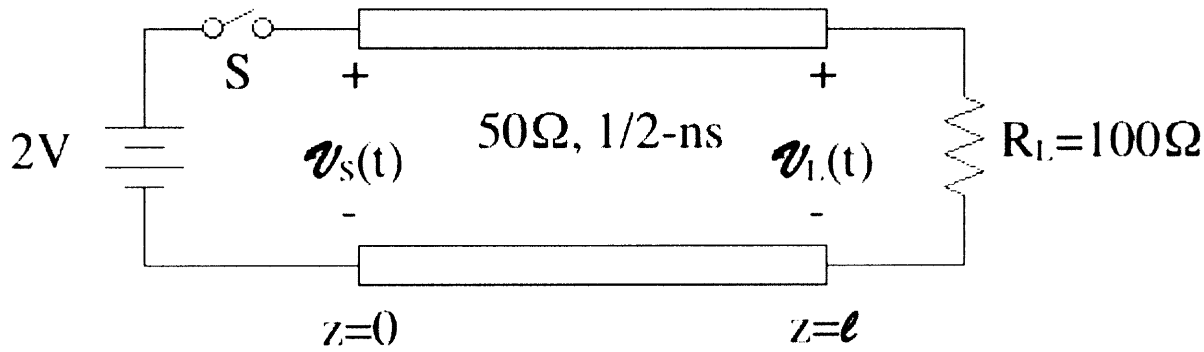
1. All questions are worth the same.
2. Exam is open book, open notes. Calculators are allowed.

		Possible	
1.		20	Pulses on transmission lines
2.		20	Charged connected transmission lines.
3.		20	Reactive Load
4.		20	Unknown Load
5.		20	Smith chart impedance matching
SCORE		100	

Name: ____________________ CWRUnet: ____________________

1. PULSES ON TRANSMISSION LINES

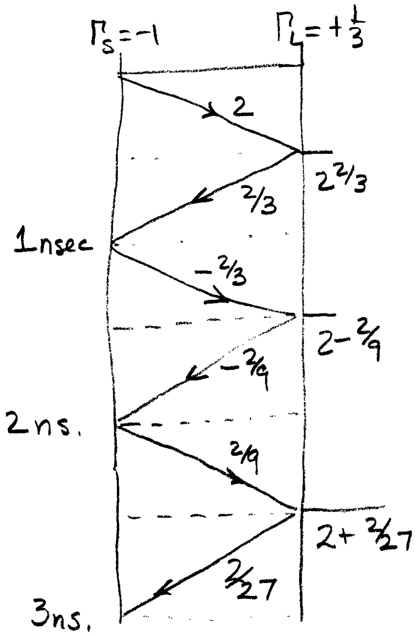
The switch S is closed at t=0 in the transmission line system shown below.



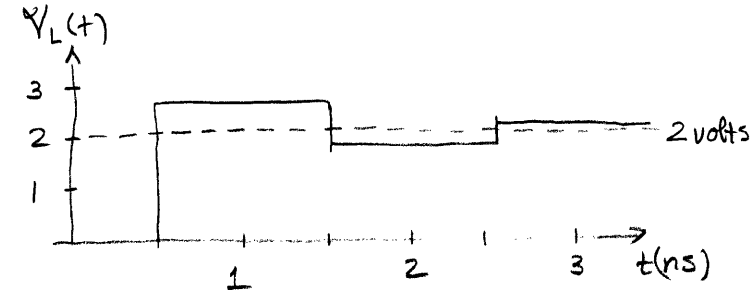
(a) Sketch $\mathcal{V}_L(t)$. Be sure to put units on your graph.
(b) Sketch $\mathcal{I}_L(t)$. Again be sure to indicate units on your graph.
(c) What are the final values of $\mathcal{V}_L$ and $\mathcal{I}_L$?

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = \frac{100-50}{100+50} = \frac{50}{150} = \frac{1}{3}$$

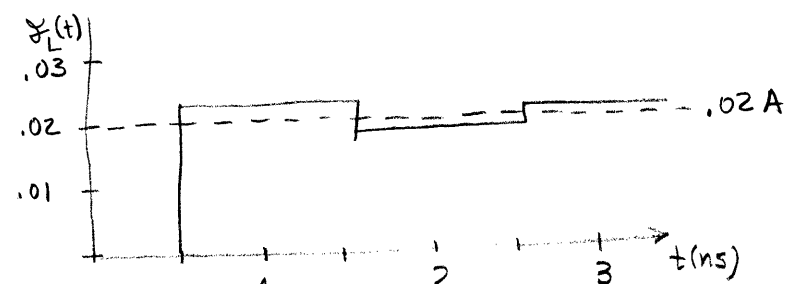
$$\Gamma_S = \frac{R_S - Z_0}{R_S + Z_0} = \frac{-Z_0}{Z_0} = -1$$



(a)



(b)



(c) $\mathcal{V}_L(\infty) = 2$ volts

$\mathcal{I}_L(\infty) = .02$ Amps.

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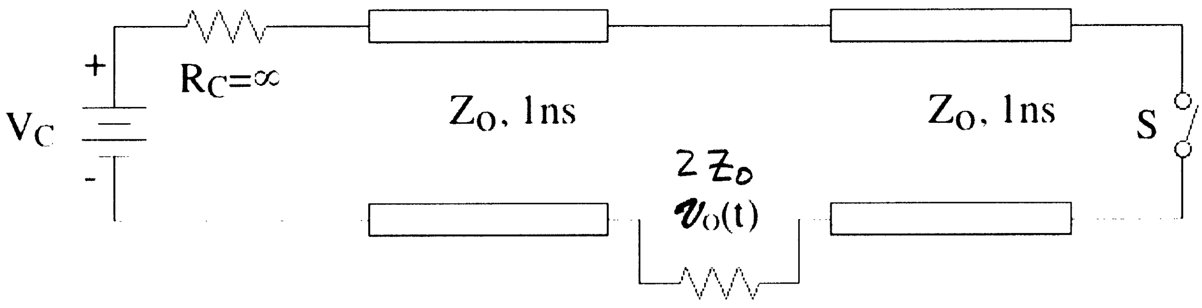
2. CHARGED LINE

Transmission lines are often used to generate high voltage pulses. A Blumlein pulse generator is shown below.



Assume the switch S is closed at S=0. Sketch the behavior of the voltage v_O as a function of time.

at $t=0$ S closes forcing $V=0$

This creates a wave $V^- = -V_c$ since line is charged to V_c

at $t=T$ part of wave gets reflected

$$\Gamma = \frac{(2Z_0+Z_0)-Z_0}{(2Z_0+Z_0)+Z_0} = \frac{2Z_0}{4Z_0} = \frac{1}{2}$$

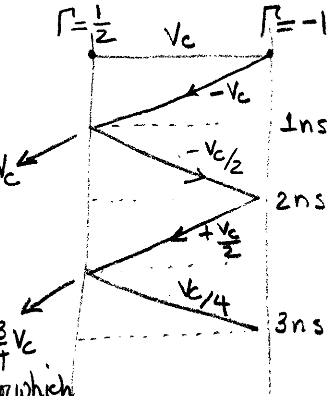
Voltage $1+\Gamma = 1+\frac{1}{2} = \frac{3}{2}$ gets transmitted

$\frac{3}{2}(-V_c) = -\frac{3}{2}V_c$

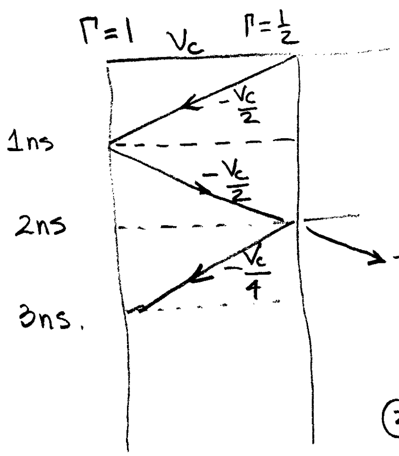
$\frac{1}{3}$ across Z_0 is $-\frac{1}{2}V_c$ launched into 2nd line

$\frac{2}{3}$ across $2Z_0$ is $-V_c$ across $2Z_0$

this is $V_0(t)$

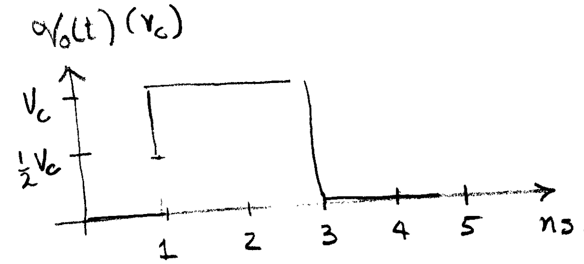


$+\frac{3}{4}V_c$ of which $+\frac{1}{4}$ goes into left cable.



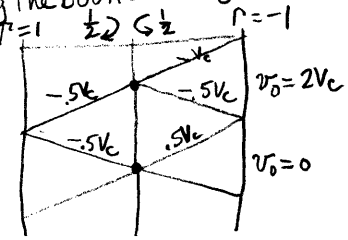
$-\frac{3}{4}V_c$ of which $-\frac{1}{4}$ goes into right cable.

(1) These two voltages cancel at $t=3$ns.



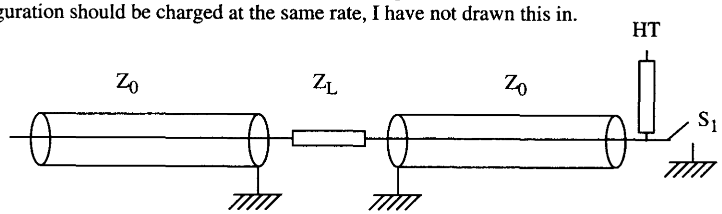
(2) The pulses into the cables also cancel the previous pulses for $t>3$nsec.

Combining the bounce diagrams.



The following explains how the Blumlein configuration for transmission lines achieves the charge voltage into a matched load for a pulse length equal in length to the double transit time of one of two lines.

In this configuration two lines are charged and drive a load with an impedance twice that of a single line.
Note that to avoid the charging current flowing though the load both sides of the configuration should be charged at the same rate, I have not drawn this in.

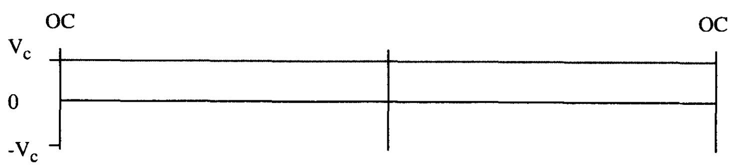


The matched condition is $Z_L = 2\,Z_0$

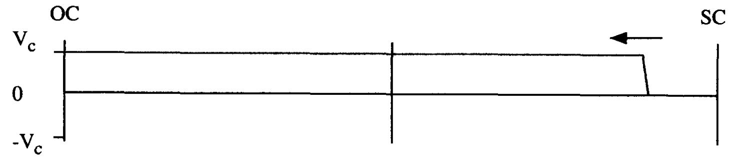
Reflected Voltage = R
Transmitted Voltage = T
Incident line impedance = Z_0
Terminating line impedance = Z_T

$$T = \frac{2\,Z_T}{Z_0 + Z_T} \qquad R = \frac{Z_0 - Z_T}{Z_0 + Z_T}$$

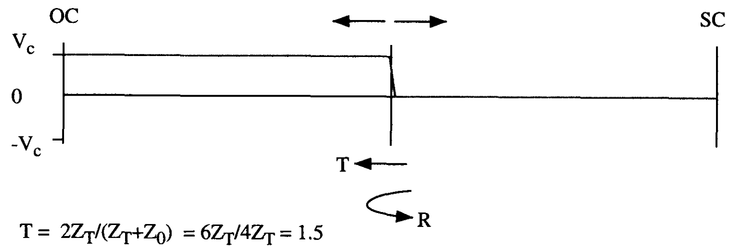
The Initial State
OC = Open Circuit, SC = Short Circuit V_c = charge voltage



The switch S_1 closes



The edge reaches the load

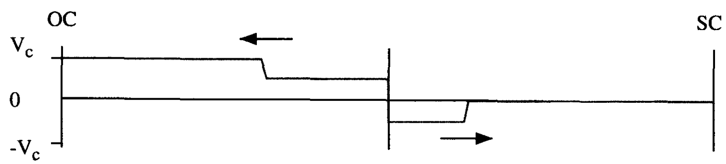


$T = 2Z_T/(Z_T+Z_0) = 6Z_T/4Z_T = 1.5$

$R = (Z_T-Z_0)/(Z_T+Z_0) = 2\,Z_0/\,4\,Z_0 = 0.5$

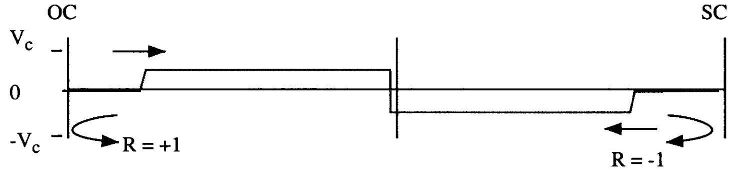
Of the tranmitted pulse this is shared between the load and the left line in the ratio 2 : 1 (The ratio of their impedances)
Hence the voltages on the left of the line as a $-V_c/2$ edge $+V_c = V_c/2$
On the right of the line the voltage is $-V_c/2$ as the initial $-V_c$ edge is reflected with magnitude 0.5.

So we have



The voltage across the load is V_c

The edges reflect from the ends.



When the edges return to the load we have to consider the contribution to the load voltage from each and the splitting of the two edges into four.

from the left
Incident $-V_c/2$ T = 1.5 R= 0.5 Voltage into right line = $-V_c/4$
Voltage into lenft line = $-V_c/4$
Voltage into load = $-V_c/2$

from the right
Incident $V_c/2$ T = 1.5 R= 0.5 Voltage into right line = $V_c/4$
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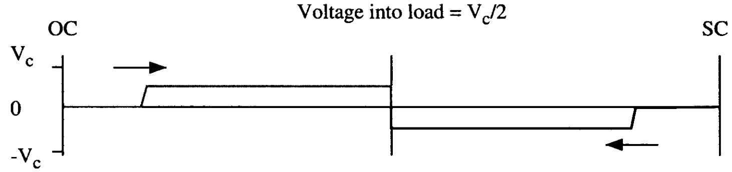
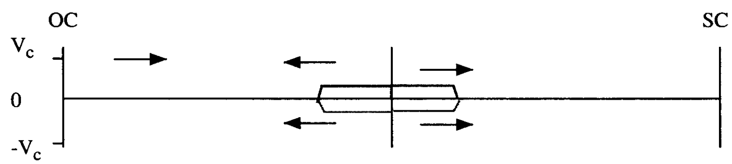
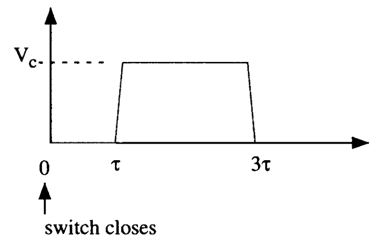


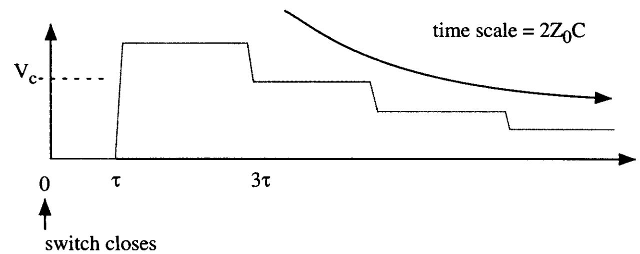


These all cancel out leaving no charge on the line.

The voltage pulse across the load is therefore



It is similarly shown that if $Z_L > 2Z_0$ The voltage on the load follows:-



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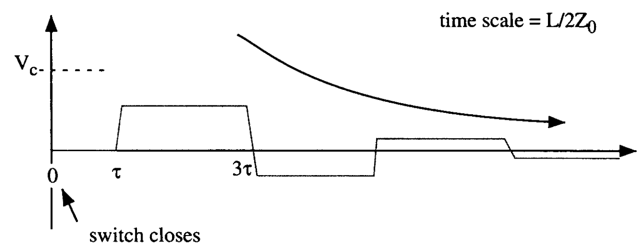

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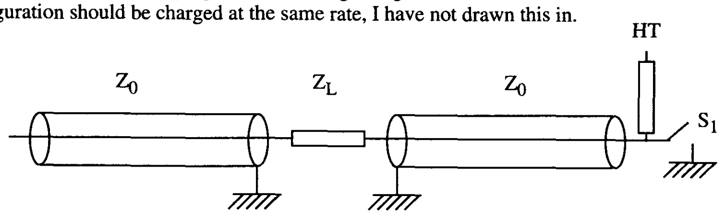

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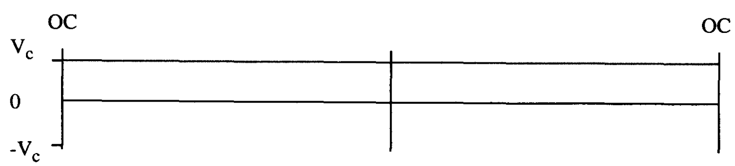

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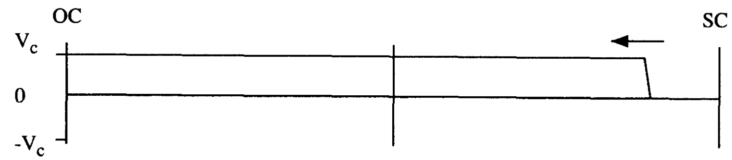

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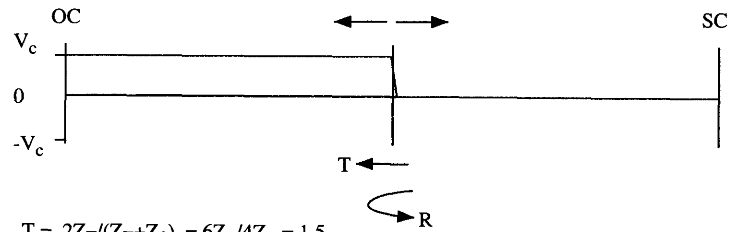

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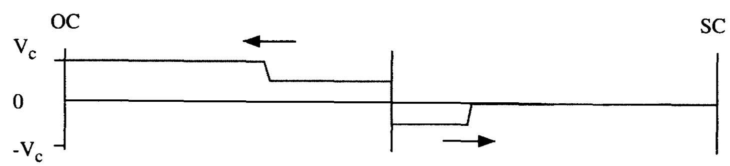

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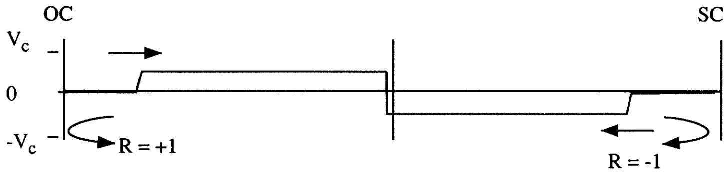

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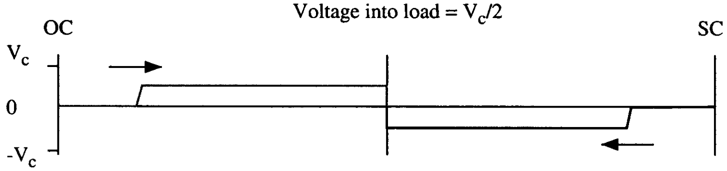

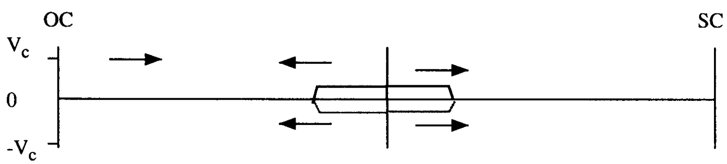

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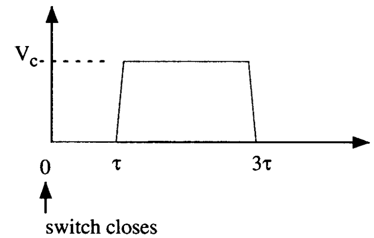

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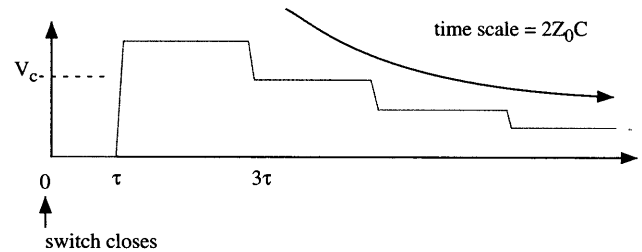

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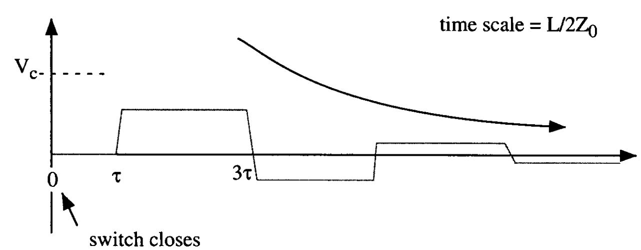

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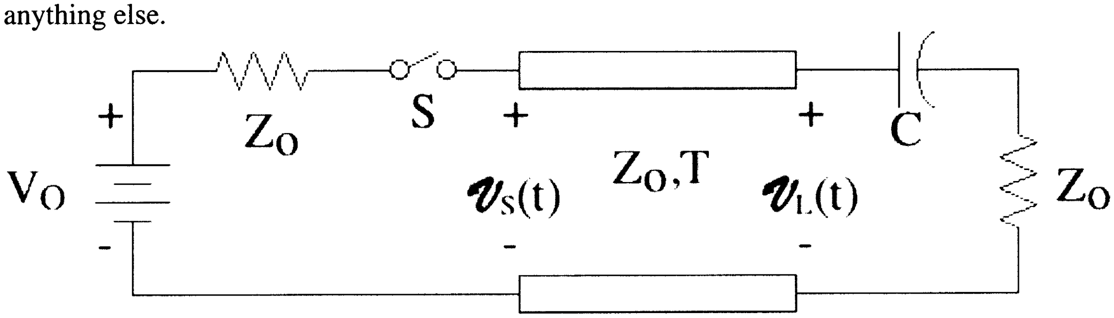
3. REACTIVE LOAD

In the system shown below the switch is closed at t=0. The voltage across the capacitor is initially zero. Write the equation for the voltage wave V^- at $z=\ell$. You don't have to do anything else.



for t>T the load voltage and current are

$$\mathcal{V}_L(t) = \mathcal{V}_1^+(\ell,t) + \mathcal{V}_1^-(\ell,t) \qquad (1)$$

$$\mathcal{I}_L(t) = \mathcal{I}_1^+(\ell,t) + \mathcal{I}_1^-(\ell,t) = \frac{\mathcal{V}_1^+(\ell,t)}{Z_0} - \frac{\mathcal{V}_1^-(\ell,t)}{Z_0} \qquad (2)$$

where $\mathcal{V}_1^+(\ell,t) = \frac{V_0}{2}$, the wave launched into the transmission line

The B.C. presented by the load is $\mathcal{V}_L(t) = \frac{1}{C}\int \mathcal{I}_L(t')\,dt' + \mathcal{I}_L(t) Z_0$ (3)

differentiate this to get

$$\frac{d\mathcal{V}_L(t)}{dt} = \frac{1}{C}\mathcal{I}_L(t) + Z_0 \frac{d\mathcal{I}_L(t)}{dt}$$

substitute

$$\frac{d}{dt}\left[\frac{V_0}{2} + \mathcal{V}_1^-(\ell,t)\right] = \frac{1}{C}\left[\frac{V_0}{2Z_0} - \frac{\mathcal{V}_1^-(\ell,t)}{Z_0}\right] + Z_0\frac{d}{dt}\left[\frac{V_0}{2Z_0} - \frac{\mathcal{V}_1^-(\ell,t)}{Z_0}\right]$$

$$\frac{d\mathcal{V}_1^-(\ell,t)}{dt} = \frac{V_0}{2Z_0C} - \frac{\mathcal{V}_1^-(\ell,t)}{CZ_0} - \frac{d\mathcal{V}_1^-(\ell,t)}{dt}$$

$$\frac{2\,d\mathcal{V}_1^-(\ell,t)}{dt} + \frac{\mathcal{V}_1^-(\ell,t)}{CZ_0} = \frac{V_0}{2Z_0C}$$

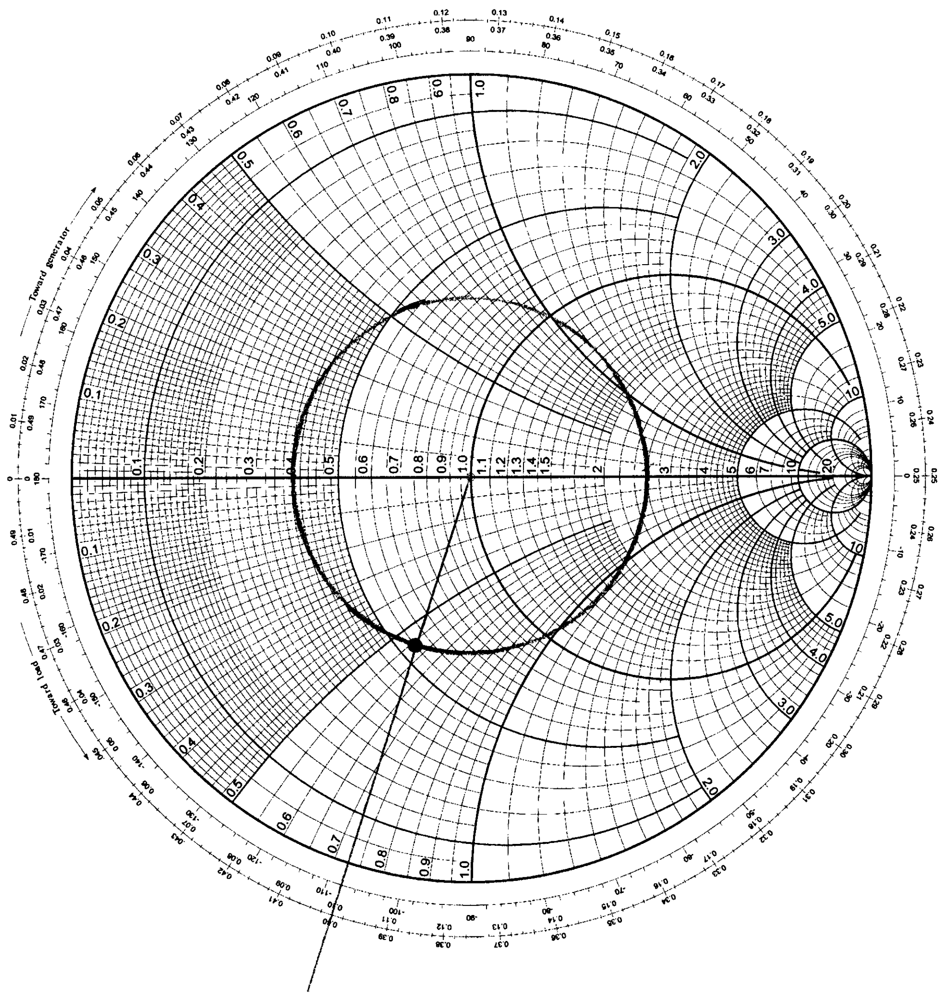
$$\frac{d\mathcal{V}_1^-(\ell,t)}{dt} + \frac{\mathcal{V}_1^-(\ell,t)}{2CZ_0} = \frac{V_0}{4Z_0C}$$

GRADING: kept $\mathcal{V}_c$ in expression −4
errors with $\mathcal{V}_c$ −3
Errors in setting up B.C.'s −6
did not get beyond equations (1) − (3) −10.

Name: ___________________ CWRUnet: ___________________

4. UNKNOWN LOAD

Standing wave measurements on a transmission line of characteristic impedance 100Ω indicate an SWR of 2.8 and a voltage minimum at a distance of 0.1λ from the load. Determine the value of the load impedance.



$\bar{z}_L = .54 - j.56$

$Z_L = 54 - j56\,\Omega$

GRADING: sign of reactance −1

location of minimum −3

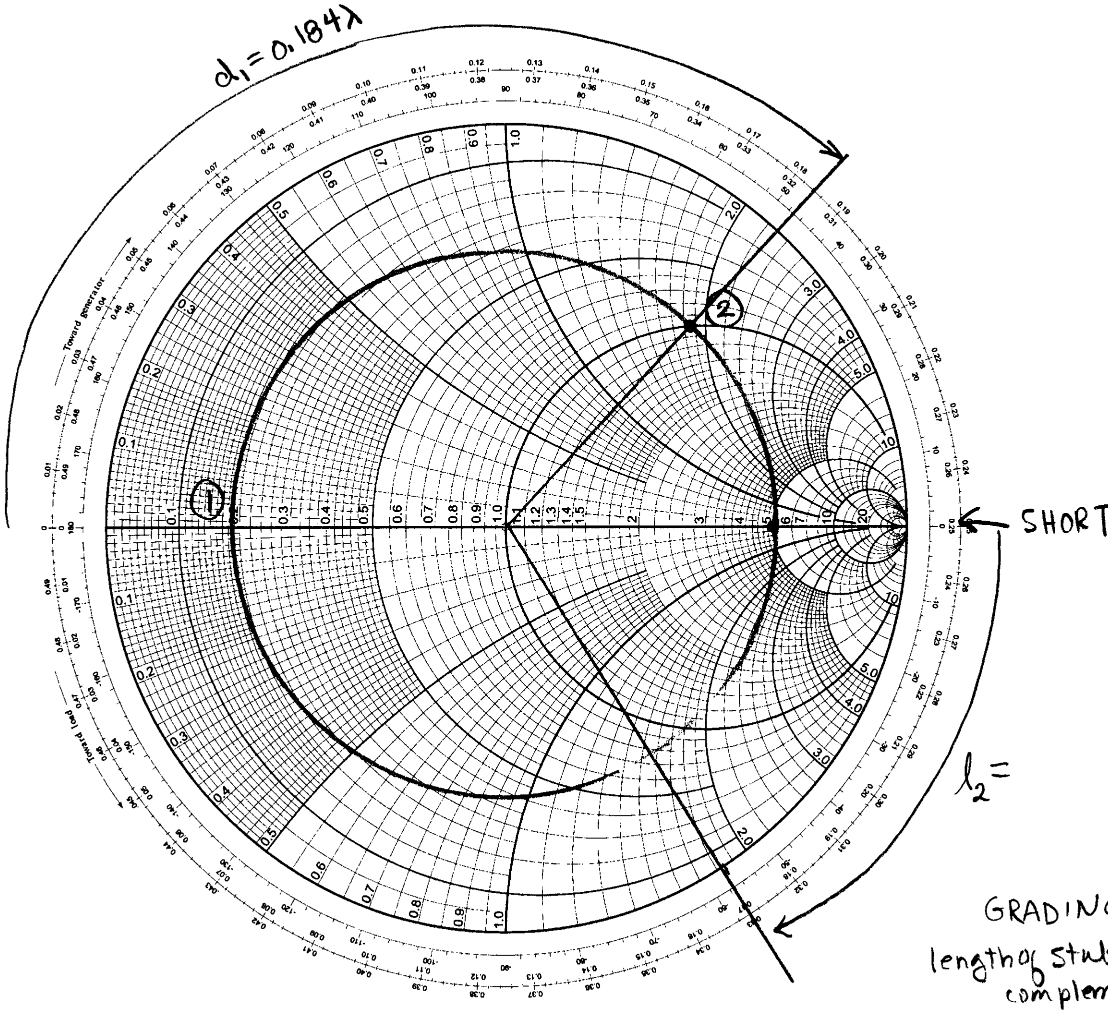
Name: ____________________ CWRUnet: ____________________

5. STUB IMPEDANCE MATCHING

A 100Ω transmission line is terminated in a 500Ω resistive load. You want to use a shorted (parallel) stub to match this load to the transmission line.

(a) What is the distance d_1 that the stub should be placed from the load?

(b) What is the length ℓ_2 of the stub?



GRADING:

length of stub complement −1

totally wrong −4

wrong θ $\theta = \pm\cos^{-1}(-\rho)$ −2

① $\bar{z}_L = \frac{500}{100} = 5$. Convert to admittance chart.

$\bar{Y}_L = 0.2$

② move along constant ρ circle to reach constant g circle

$d_1 = 0.184\lambda$ (a)

③ The corresponding admittance is $y = 1 + j1.85$

④ stub must have value $-j1.85$.

⑤ Start at short and move to -1.85 giving $\ell_2 = .08\lambda$