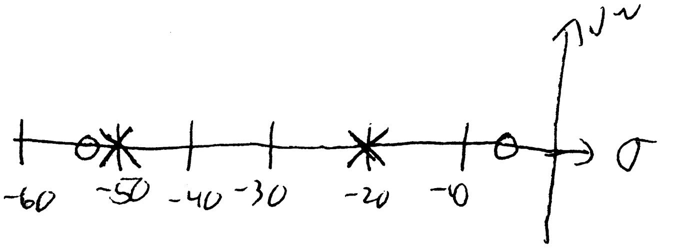
(24)

EECS 245 HMWK8 sol'n

9-8 (a) From Table 9-2, pg 442 & linearity principle

$\mathcal{L}[\delta(t)] = 1 \quad \mathcal{L}[5e^{-50t}u(t)] = \frac{5}{s+50} \quad \mathcal{L}[-15e^{-20t}u(t)] = \frac{-15}{s+20} \therefore$

$$\boxed{F(s) = 1 + \frac{5}{s+50} - \frac{15}{s+20} = \frac{(s^2+60s+350)}{(s+50)(s+20)} = \frac{(s+6.548)(s+53.45)}{(s+50)(s+20)}}$$



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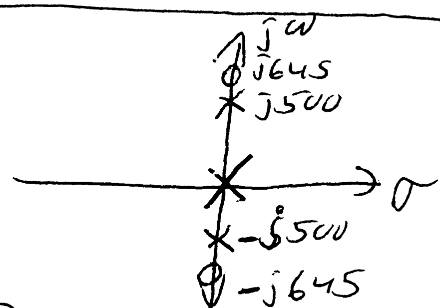
(b) $\mathcal{L}[25u(t)] = \frac{25}{s} \quad \mathcal{L}[-10\cos(500t)u(t)] = \frac{-10s}{s^2+500^2} \therefore$

$$\boxed{F(s) = \frac{25}{s} - \frac{10s}{s^2+500^2} = \frac{15(s^2+645.5^2)}{s(s^2+500^2)} = \frac{15(s+j645.5)(s-j645}{s\cdot(s+j500)(s-j500)}}$$



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9-14 (a) $f(t) = \frac{3A}{T}\cdot t\cdot u(t) - \frac{3A}{T}\cdot t u(t-\frac{T}{3}) + Au(t-\frac{T}{3}) - Au(t-T$

$$\boxed{f(t) = \frac{3A}{T}tu(t) - \frac{3A}{T}(t-\frac{T}{3})(u(t-\frac{T}{3})) - A\cdot u(t-T)}$$

(b) $\mathcal{L}[\frac{3A}{T}\cdot tu(t)] = \frac{3A}{Ts^2}$ shifting by $t-\frac{T}{3}$ will multiply by $e^{-\frac{T}{3}s}$;

$\mathcal{L}[\frac{3A}{T}\cdot(t-\frac{T}{3})\cdot u(t-\frac{T}{3})] = \frac{3A}{Ts^2}(e^{-T/3s}) . \mathcal{L}[Au(t)] = \frac{A}{s} \therefore$

$\mathcal{L}[Au(t-T)] = \frac{A}{s}(e^{-T\cdot s}) \therefore$

2

$$\boxed{F(s) = \frac{3A}{Ts^2} - \frac{3A}{Ts^2}(e^{-sT/3}) - \frac{A}{s}(e^{-Ts})}$$

(c) $F(s) = \int_0^{T/3} \frac{3At}{T} e^{-st}\,dt + \int_{T/3}^{T} A e^{-st}\,dt$

$= \frac{3A}{T} e^{-st}\left[-\frac{1}{s^2} - \frac{t}{s}\right|_0^{T/3} + \frac{-A}{s} e^{-st}\Big|_{T/3}^{T}$

$= \frac{3A}{T}\left[e^{-sT/3}\left(\frac{-1}{s^2} - \frac{T}{3s}\right) + 1/s^2\right] - \frac{A}{s}\left(e^{-sT} - e^{-sT/3}\right)$

1

$= \frac{3A}{Ts^2} + e^{-sT/3}\left[\frac{-3A}{Ts^2} - \frac{A}{s} + \frac{A}{s}\right] - \frac{A}{s} e^{-st}$

$\boxed{F(s) = \frac{3A}{Ts^2} - \frac{3A}{Ts^2}\left(e^{-sT/3}\right) - A/s\, e^{-st}}$ ✓

Now, wasn't the other way a lot easier???

$\boxed{9\text{-}18(a)}$

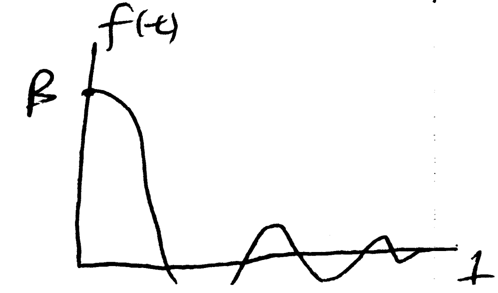
$F(s) = \frac{B(s+2\alpha)}{(s+\alpha)^2 + B^2} = \frac{B(s+\alpha)}{(s+\alpha)^2 + B^2} + \frac{\alpha B}{(s+\alpha)^2 + B^2}$

$\boxed{f(t) = e^{-\alpha t} u(t)\left[B\cos(Bt) + \alpha \sin(Bt)\right]}$

2

*Note: The secret to doing these types of problems is knowing your 9-2 table, pg 442. Then, you need to re-arrange your expression algebraically until everything matches a term in the table. Just remember, $\frac{1}{s+\alpha} \neq \frac{1}{s} + \frac{1}{\alpha}$

$f(t)$:



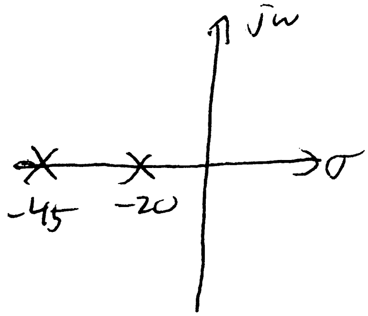
$\boxed{9\text{-}21\,(a)}$

$$F(s)=\frac{900}{s^2+65s+900}=\frac{900}{(s+20)(s+45)}=\frac{K_1}{s+20}+\frac{K_2}{s+45}$$

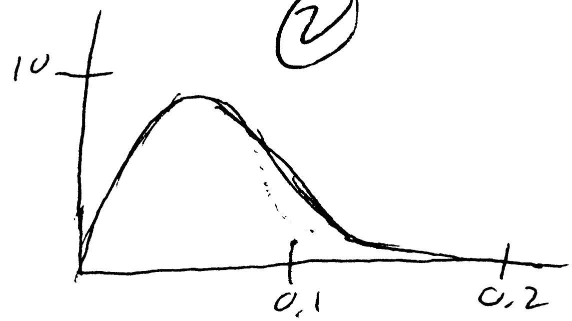
$$K_1=\frac{900}{(-20+45)}=36 \qquad K_2=\frac{900}{(-45+20)}=-36 \therefore$$

$$F(s)=\frac{36}{(s+20)}-\frac{36}{s+45} \rightarrow \boxed{f(t)=36\left(e^{-20t}-e^{-45t}\right)u(t)}$$

Pole-zero:



f(t) →



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$\boxed{9\text{-}26\,(b)}$

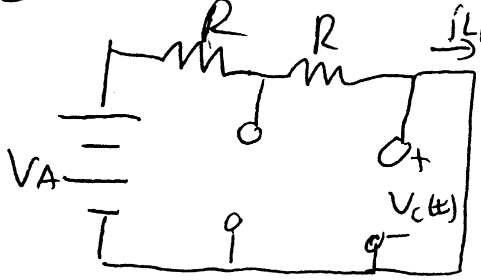
$$F(s)=\frac{s+1}{(s^2-2s+1)}=\frac{s+1}{(s-1)^2}=\frac{K_1}{(s-1)^2}+\frac{K_2}{(s-1)}$$

$$\frac{K_1(s-1)+K_2(s-1)^2}{(s-1)^2(s-1)}=\frac{s+1}{(s-1)^2} \therefore K_1+K_2(s-1)=s+1 \therefore \quad K_2=1, \; K_1=2 \therefore$$

$$F(s)=\frac{2}{(s-1)^2}+\frac{1}{(s-1)} \rightarrow \boxed{f(t)=\left[2te^{t}+e^{t}\right]u(t)}$$

2

9-39 First, let's find the initial conditions. DC ckt:

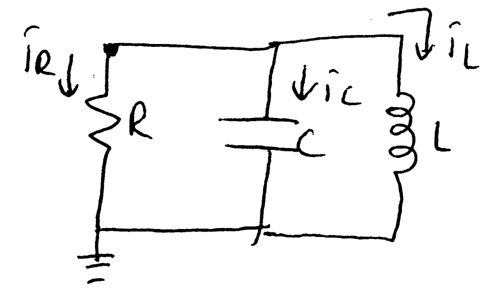


$$\underline{V_C(0)=0}$$

$$i_L(0)=\frac{V_A}{2R}=\frac{10}{2000}\qquad \underline{i_L(0)=5\times10^{-3}A}$$

$$\underline{\frac{d}{dt}I_L(0)=\frac{1}{L}V_C(0)=0}$$

At $t=0$, V_A is shorted out:



$i_R+i_C+i_L=0$ (KCL at gnd)

$$i_R=V_R/R=V_L/R=\frac{L}{R}\frac{di_L}{dt}\quad \left(V_L=L\frac{di_L}{dt}\right)$$

$$i_C=C\frac{dv_C}{dt}=C\frac{dV_L}{dt}=LC\frac{di_L^2}{dt^2}\ldots$$

diffeqn: $LC\frac{d^2}{dt^2}i_L+\frac{L}{R}\frac{d}{dt}i_L+i_L=0$ 2

$$\boxed{10^{-7}\frac{d^2}{dt^2}i_L+2\cdot\times10^{-4}\frac{d}{dt}i_L+i_L=0,\quad V_C(0)=0\quad i_L(0)=5mA\quad \frac{d}{dt}I_L(0)=\frac{1}{L}V_C(0)=0}$$

(b) From differentiation property,

$$10^{-7}(s^2I_L(s)-5\cdot10^{-3}s)+2\cdot10^{-4}(s\cdot I_L(s)-5\cdot10^{-3})+I_L(s)=0$$

$$I_L(s)\left[10^{-7}s^2+2\times10^{-4}s+1\right]=(10^{-7})(5\times10^{-3}s)+(2\times10^{-4})(5\times10^{-3})$$

$$I_L(s)=\frac{(5\times10^{-3})(s+2000)}{s^2+2000s+10^7}=\frac{1}{200}\left(\frac{s+2000}{(s+1000)^2+3000^2}\right)$$

$$s^2+2000s+10,000,000$$
$$s^2+2,000s+1,000,000+9,000,000$$
$$(s+1,000)^2+3,000^2$$

$$I_L(s)=\frac{1}{200}\left[\frac{s+1,000}{(s+1,000)^2+3000^2}+\frac{1}{3}\cdot\frac{3,000}{(s+1,000)^2+3,000^2}\right]$$

$$\boxed{i_L(t)=\frac{1}{200}\left(e^{-1,000t}u(t)\right)\left(\cos(3,000t)+\tfrac{1}{3}\sin(3,000t)\right)}$$ 2

$\boxed{9\text{-}47(a)}$ Both the initial & final value theorems apply; this is a proper rational function w/ negative poles.

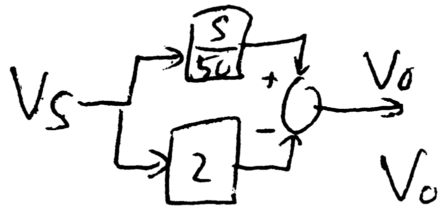
$$sF(s) = \frac{s^2+15s+12}{s^2+6s+12} \quad f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{1+\cancel{15/s}^{\to 0}+\cancel{12/s^2}^{\to 0}}{1+\cancel{6/s}_{\to 0}+\cancel{12/s^2}_{\to 0}} = \boxed{1 = f(0)}$$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \frac{12}{12} = \boxed{1 = \lim_{t\to\infty} f(t)}$$

2

$\boxed{9\text{-}53}$

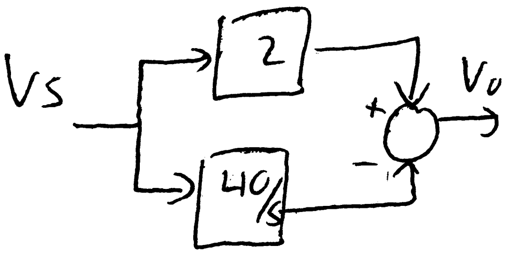
(a) $V_o(s) = \mathcal{L}\left(-2V_s(t) + \frac{1}{50}\frac{d}{dt}V_s(t)\right) = -2V_s(s) + \frac{s}{50}V_s(s) = \left(\frac{s-100}{50}\right)V_s(s)$



The time differentiation translates to the $s/50$ block in the diagram. Since $V_o(s) = \left(\frac{s-100}{50}\right)V_s(s)$, V_o has all of the poles/zeros that V_s does, plus an additional zero at $s=100$. Thus, the differentiation adds a zero, or in otherwards, to retrieve the input from the output, a pole must be added at $s=100$

2

(b) $V_o(t) = \mathcal{L}^{-1}\left[2\cdot V_s(s) - \frac{40}{s}V_s(s)\right] = 2V_s(t) - 40\int_0^t V_s(x)dx$



As seen from the inverse transform, the time-domain signal is integrated, shown by $40/s$ block. $V_o(s) = \frac{2s-40}{s}$. Thus, integration adds a pole at $s=0$, and a zero at $s=20$.

This is an important analysis if Vs/Vo are high frequency. The low-frequency poles & zeros added will alter the circuit's performance dramatically.

2