Chapter 5

Two Port Electronic Control Devices, I

5.1 RESISTIVE CLASS OF ELECTRONIC CONTROL DEVICES

In the analysis and synthesis of circuits containing electronic control devices, the first consideration is a representation for each of the control devices. In this book, only electronic control devices of the resistive class are considered, i.e. the class of control devices involving resistive parameters. The present distinction is only between the types of devices in which the parameters are resistive as compared to those types in which they are reactive. The reactive types are those in which the parameters are inductive or capacitive.

The resistive class of control devices may be divided into subclasses. The subclass considered in this chapter is that whose input port is represented as an open circuit. Vacuum tube triodes, tetrodes and pentodes operating with the grid negative are among the more common examples. The circuit representations are developed as generally as possible, using specific examples as illustrations. The first example to be studied is the vacuum tube triode.

5.2 VACUUM TUBE TRIODE CHARACTERISTICS

The vacuum tube triode is considered a three element electronic control device. In addition to the plate and cathode elements, there is a third called the grid. The heater is assumed to be connected to a power supply, hence is not included in a discussion of the triode as an electronic control device. The symbol for the vacuum tube triode is given in Fig. 5-1. A distinction is sometimes made between the directly heated and the indirectly heated cathode as shown in Fig. 2-3, page 12, but this is not important in the present discussion. The electrical characteristics of the triode involve relationships between terminal voltages and currents. These form the basis for developing a representation as a circuit element.

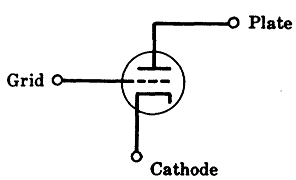

Fig. 5-1. Vacuum Tube Triode Symbol

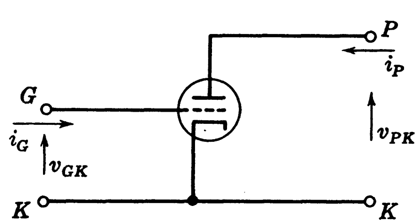

Fig. 5-2. Vacuum Tube Notation

The vacuum tube triode has three terminals, so there are at most two independent terminal voltages and two independent terminal currents (assuming no charge sources or sinks). Any one of the three elements may be employed as a common or reference element. However, in the case of the vacuum tube the cathode is usually chosen as the reference element. To be consistent with the conventions previously adopted in this book, the voltage drops from the non-reference elements to the reference element are considered along with the currents into the non-reference elements as indicated in Fig. 5-2. The notation for the voltage drops and currents is an extension of that used for diodes as listed in Appendix A.

Consider the output port of the triode, which is the plate and cathode terminal pair. Following the procedures outlined in Chapter 4, write the functional relationship for the voltages and currents so that the desired equivalent circuit is obtained. Considering instantaneous values, write

$$i_P = f(v_P, v_G) \tag{5.1}$$

This assumes the cathode temperature is in the normal operating range and that either the cathode temperature is constant or that operation is restricted to the space charge limited region of operation so that the plate current is independent of the cathode temperature. In general the relationship given in (*5.1*) is not linear, hence must be presented graphically. Because there are two independent variables, this relationship is geometrically a surface. This cannot be shown in two dimensions, so the surface must be approximated by families of curves as discussed in Chapter 4. For a vacuum tube triode, the *family of static plate characteristic curves* (often abbreviated "plate characteristics") is usually given. These curves are the intersections of the surface $i_P = f(v_P, v_G)$ and the planes $v_G =$ constants. The average plate characteristic curves for a 6C4 triode are graphed in Fig. 5-3. Since individual tubes are not identical but have characteristics which vary somewhat from tube to tube, averages of a number of tubes are plotted. Hence the characteristics of any one tube may differ somewhat from the averages, but this is not a serious problem unless a critical application is being considered. In the case of linear vacuum tube triode amplifiers, the interest is usually in the region of operation where the grid is negative with respect to the cathode, so the grid current is usually negligible.

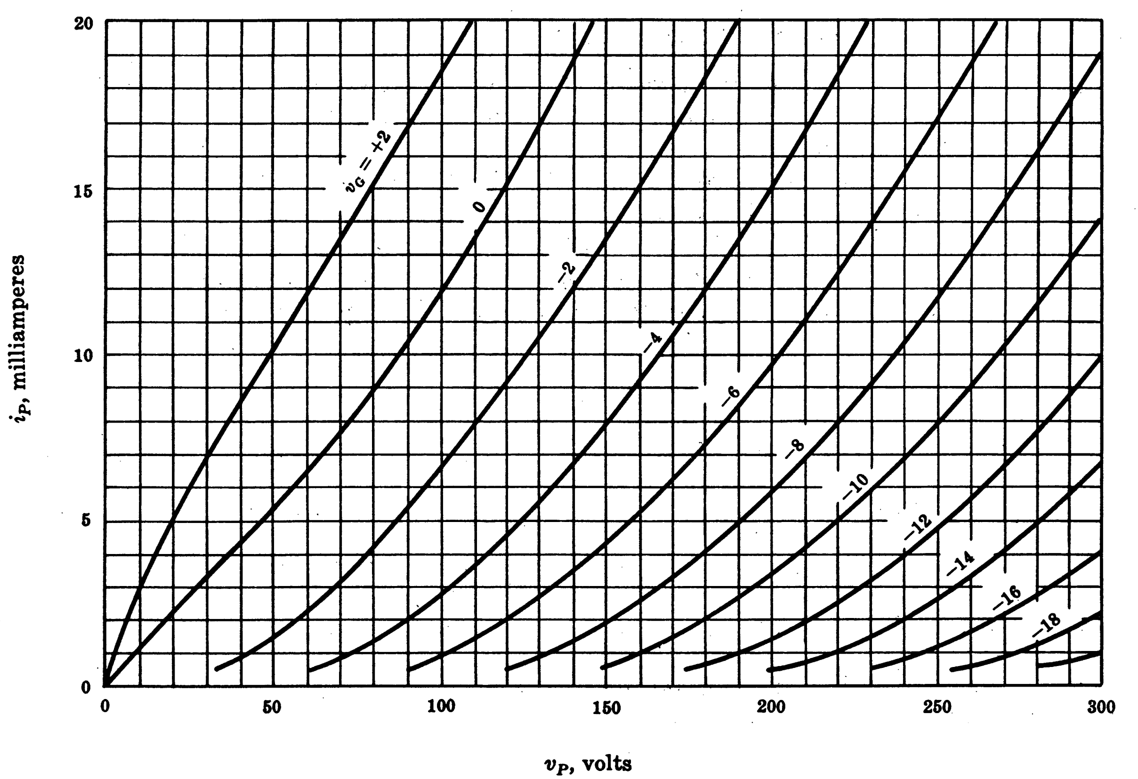

Fig. 5-3. 6C4 Static Plate Characteristics

The functional relationship of equation (*5.1*) can also be approximated by a family of curves resulting from the intersection of the surface $i_P = f(v_P, v_G)$ and the planes v_P = constants. This family of curves is called the *family of static forward transfer characteristic curves*. This is sometimes written as just the transfer characteristics; but this is done only when there is no ambiguity, since there is still another family of static forward transfer curves. The static forward transfer characteristic curves include the same information as the static plate characteristic curves. For example, the static forward transfer curves can be plotted from the static plate curves. Vertical lines are drawn on the plate characteristic curves for a chosen set of values of v_P. The points where the plate curves (v_G = a constant) intersect the particular vertical line (v_P = a constant) are used to plot one curve i_P vs. v_G. The family of static transfer characteristic curves for a 6C4 triode is shown in Fig. 5-4.

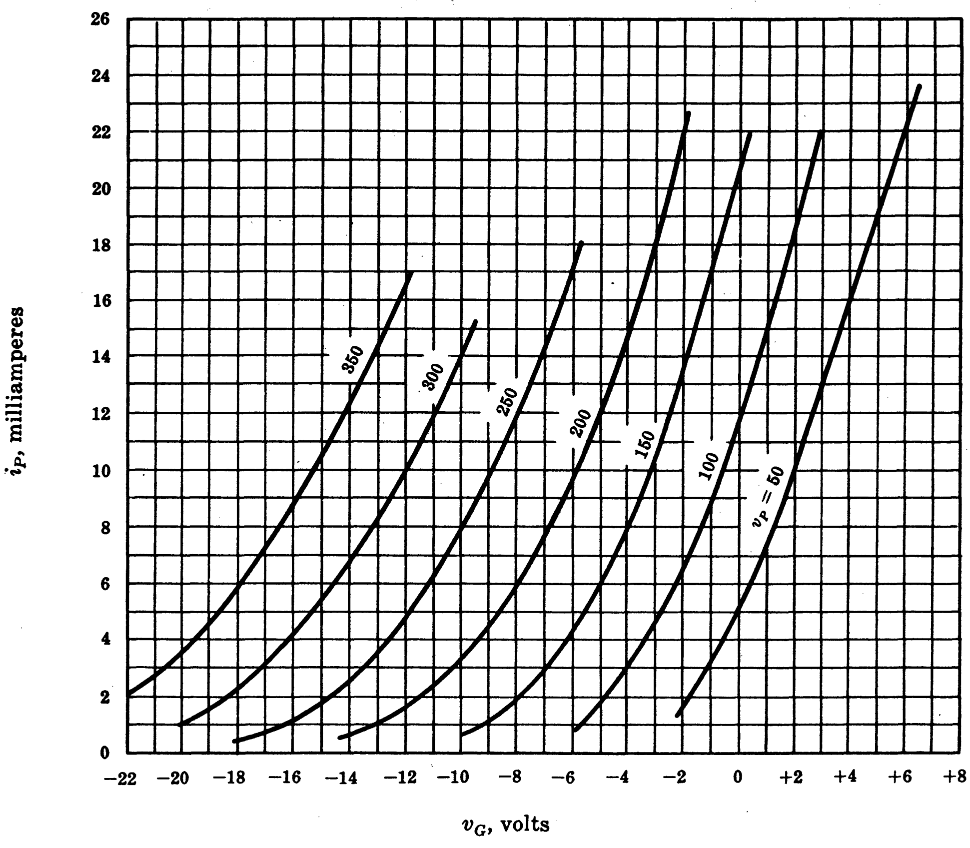

Fig. 5-4. 6C4 Static Transfer Characteristics

Consider the input port of the triode, which is the grid and cathode terminal pair. The functional relationship is

$$i_G = f(v_G, v_P) \qquad (5.2)$$

When the grid is negative with respect to the cathode, there is no appreciable grid current, so $i_G = 0$. Therefore the input port is represented as an open circuit or an infinite impedance when $v_G < 0$. When $v_G < 0$, the family of static grid characteristic curves all lie on the negative v_G axis as shown in Fig. 5-5.

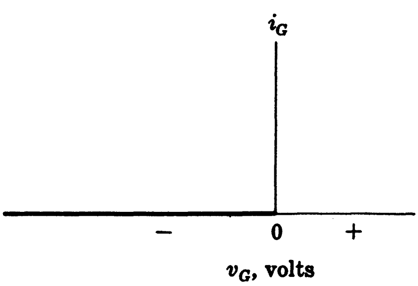

Fig. 5-5. Static Grid Characteristics

5.3 TRIODE EQUIVALENT CIRCUITS

The representation for the output port is developed by following the procedures outlined in Chapter 4. One approach involves expanding the function of equation (*5.1*) about a point (V_P, V_G) using Taylor's series.

$$i_P \;=\; f(V_P, V_G) + (v_P - V_P)\frac{\partial i_P}{\partial v_P} + (v_G - V_G)\frac{\partial i_P}{\partial v_G} + \cdots \qquad (5.3)$$

When operation is restricted to the linear range, the second and all higher derivatives are zero, so the only non-zero terms are those written in equation (*5.3*). By definition, $\partial i_P/\partial v_P = g_p$, the differential plate conductance, and $\partial i_P/\partial v_G = g_m$, the differential forward transfer conductance or mutual conductance. When the second and higher derivatives are zero, g_m and g_p are constants. Substituting g_m, g_p and $I_P = f(V_P, V_G)$ into (*5.3*),

$$i_P \;=\; I_P + (v_P - V_P)g_p + (v_G - V_G)g_m \qquad (5.4)$$

Since operation is restricted to the linear range, superposition may be applied, so the instantaneous values may be written as the average values plus the instantaneous values of the varying components. Substituting $v_P = V_P + v_p$, $v_G = V_G + v_g$, and $i_P = I_P + i_p$ into equation (*5.4*),

$$i_P \;=\; I_P + i_p \;=\; I_P + (V_P + v_p - V_P)g_p + (V_G + v_g - V_G)g_m \qquad (5.5)$$

Canceling the terms involving average values,

$$i_p \;=\; g_p v_p + g_m v_g \qquad \text{or} \qquad -i_p + g_p v_p + g_m v_g \;=\; 0 \qquad (5.6)$$

All terms of (*5.6*) contain varying components, so this is the equation for the output port of the triode when only varying components are considered within the linear operating range.

Examine each term of equation (*5.6*) (v_p and i_p references as in Fig. 5-2). The term $g_p v_p$ is a current in a linear conductor in the direction of the voltage drop v_p. The term $g_m v_g$ is independent of v_p and i_p, so it is shown as a voltage controlled, current generator. When v_g is positive this term has a + sign, so the arrow in the current generator is away from the plate node. Thus the equivalent circuit for the output port may be drawn as in Fig. 5-6.

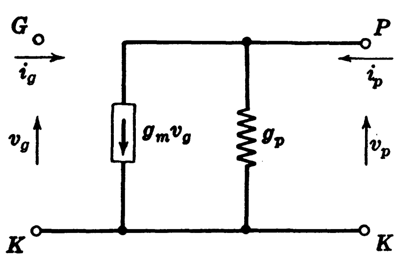

Fig. 5-6. Triode Equivalent Circuit

The input port is an open circuit (assuming $v_G < 0$), and its representation is added to complete Fig. 5-6. Fig. 5-6 exhibits an equivalent circuit for a vacuum tube triode when only varying components are considered and operation is limited to the linear range. This is sometimes called the "ac equivalent circuit". In practical situations operation is restricted to a range which is approximately linear, so this is sometimes said to involve "small signal analysis" or "linear approximation". This particular equivalent circuit contains a constant current generator circuit with a voltage controlled generator. Since $i_G = 0$, the triode plate circuit cannot be represented using current controlled generators.

A model may be developed which contains a constant voltage generator circuit in which the generator is voltage controlled. In this case, write the functional relationship as

$$v_P \;=\; f(i_P, v_G) \qquad (5.7)$$

and expand the function about an average value using Taylor's series:

$$v_P \;=\; f(I_P, V_G) + (i_P - I_P)\frac{\partial v_P}{\partial i_P} + (v_G - V_G)\frac{\partial v_P}{\partial v_G} + \cdots \qquad (5.8)$$

By definition, $\partial v_P/\partial i_P = r_p$, the differential plate resistance, and $\partial v_P/\partial v_G = -\mu$, a dimensionless quantity. It is somewhat confusing to have μ defined as the negative of a partial derivative. As will be seen later, $\partial v_P/\partial v_G$ is negative; so years ago μ was defined as the negative of the partial derivative, making μ a positive quantity. When r_p and μ are constants (linear range), superposition may be applied as before, and after substitution the final equation containing only varying components is

$$-v_p + r_p i_p - \mu v_g = 0 \tag{5.9}$$

Examine each term of this equation. v_p is the voltage rise from cathode to plate. The term $r_p i_p$ is the voltage drop across a linear resistor r_p in the direction of the current i_p. The term $-\mu v_g$ is independent of v_p and i_p, so it is shown as a voltage generator. A convention has been established showing the arrow directed opposite to the voltage drop. Thus the arrow is upward and the generator magnitude is $-\mu v_g$ as shown in Fig. 5-7. The input port is still an open circuit, so the grid terminal is shown as an open circuit. This equivalent circuit is, of course, only for varying components and linear operation. The equivalent circuit representation which is the most convenient should be used in a particular problem.

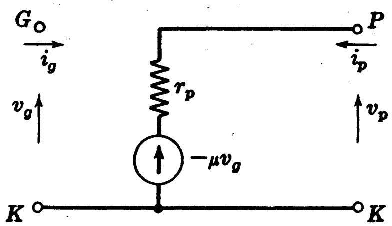

Fig. 5-7. Triode Equivalent Circuit

Since both the equivalent circuits of Fig. 5-6 and Fig. 5-7 represent the same thing, they should be equivalent to each other in some way. In the circuits of Fig. 5-6 and Fig. 5-7, v_g is independent of i_p and v_p because the reverse transfer function is zero, hence the generators are independent. This will not always be the case for vacuum tube equivalent circuits. Apply Thévenin's theorem to the circuit of Fig. 5-6 and obtain the circuit of Fig. 5-8(*a*). Since this is equivalent to the circuit of Fig. 5-7, the coefficients of the two generators in these circuits must be the same:

$$\mu = g_m/g_p = g_m r_p \tag{5.10}$$

Hence the parameters μ, r_p and g_m are not independent but are related by equation (*5.10*). From the definitions it is seen that $r_p = 1/g_p$.

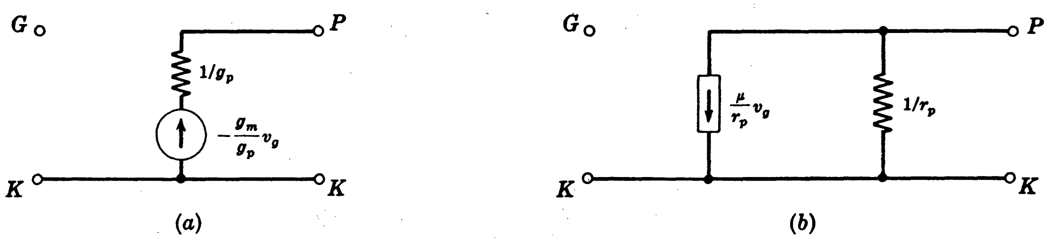

Fig. 5-8

Norton's theorem may be applied to the circuit of Fig. 5-7, giving the circuit of Fig. 5-8(*b*). Similarly, from Fig. 5-6 and Fig. 5-8(*b*), equation (*5.10*) follows: $g_m = \mu/r_p$ or $\mu = g_m r_p$.

5.4 OPERATING POINT AND TUBE PARAMETERS

The equivalent circuits which have been developed for vacuum tube triodes are for varying components only and for operation on the linear operating range. The parameters must be determined for a given operating point. The operating point or quiescent point involves the average values of the triode terminal voltages and currents. In general, the relationships between the voltages and currents are nonlinear, so the operating point is found graphically. Hence the particular circuit of interest must be examined. The circuit equations may be written in terms of the average values of the voltages and currents, and then the operating point found by plotting these equations on the static characteristics of the triode. When the operating point has been determined, the parameters may be found by graphically determining the slopes of the appropriate curves. Note that each parameter is defined as a partial derivative.

Consider the simple triode amplifier circuit of Fig. 5-9. When control devices of the resistive class are employed in amplifier circuits, they are usually placed in series with a linear resistor R_L and a constant voltage source V_{PP}. This circuit is similar to some of the diode circuits of Chapter 2; but the triode has three terminals, so there are transfer characteristics also to be considered. The characteristics of the vacuum tube triode, in this example a 6J5, are given graphically by the static plate characteristic curves of Fig. 5-10. The plate loop equation in terms of instantaneous values is

$$-V_{PP} + R_L i_P + v_P = 0 \tag{5.11}$$

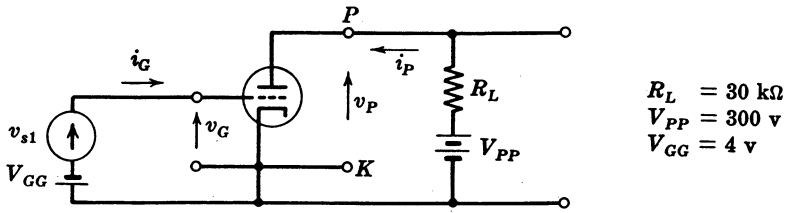

Fig. 5-9. Simple Triode Amplifier

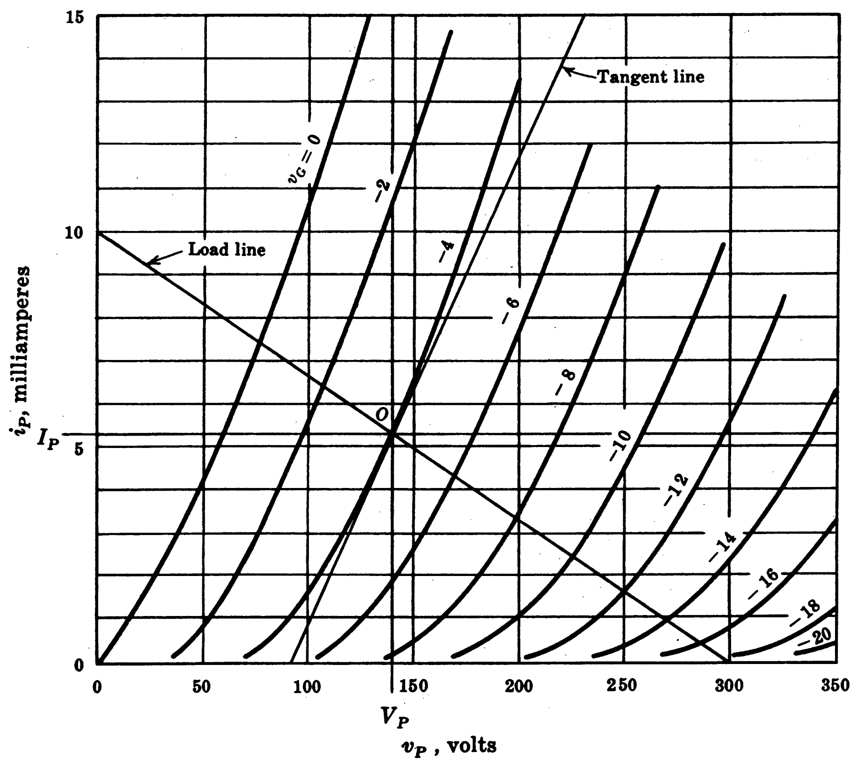

Fig. 5-10. 6J5 Plate Characteristics

Rewrite (5.11) as

$$(R_L/V_{PP})i_P + (1/V_{PP})v_P = +1 \tag{5.12}$$

which is the equation of a line in the standard intercept form and is the load line. Equation (*5.12*) is identical to equation (*2.11*), page 18, for the diode circuit of Fig. 2-13. The operating point is a particular point on the load line, so (*5.12*) is usually written in terms of the average values of i_P and v_P rather than the instantaneous values. The load line is plotted on the family of static plate characteristic curves, giving a solution to $i_P = f(v_P, v_G)$ and equation (*5.12*).

The grid loop equation in terms of average values (direction of i_G if it were not zero) is

$$+V_{GG} + 0 + V_G = 0 \qquad (5.13)$$

The signal source v_{si} will usually have an average value of zero and so is written as zero in (*5.13*). This is done to emphasize the fact that the average value must be included if it is not zero. By (*5.13*),

$$V_G = -V_{GG} \qquad (5.14)$$

The average value of the grid to cathode voltage drop is usually negative and is often called the "grid bias." The value of the grid supply voltage V_{GG} is chosen to give the desired operating point in a simple circuit of this type.

Substituting the numerical values of the circuit elements given in Fig. 5-9 into equations (*5.12*) and (*5.14*),

$$(30{,}000/300)I_P + (1/300)V_P = +1 \qquad \text{and} \qquad V_G = -4 \text{ volts}$$

The load line is plotted on the 6J5 static plate characteristic curves of Fig. 5-10. The operating point is found by locating the point $V_G = -4$ volts on the load line. This is the intersection of the plate characteristic curve $v_G = -4$ volts and the load line. The values $V_P = 140$ volts and $I_P = 5.3$ ma are then read from the graph.

The triode parameters are now determined as the slopes of the appropriate curves at the given operating point. The differential plate conductance g_p is found in the same way as it was for the diodes of Chapter 2. A straight line is drawn tangent to the curve $v_G = -4$ volts at the operating point and then the slope of this line is determined by taking large increments. This is the differential plate conductance because along the $v_G = -4$ volts curve, i_P is a function of v_P only. If the differential plate resistance r_p is desired, the reciprocal of g_p is used. For the example given, the slope of the tangent line is

$$g_p = \Delta i_P/\Delta v_P$$
$$= 0.015/(229-91) = 109\ \mu\text{mho}$$

The differential forward transfer conductance g_m is found by first plotting the static forward transfer characteristic curve which includes the given operating point. Draw the vertical line $v_P = V_P = 140$ volts. Plot the points which are the intersections of the $v_P = V_P$ line and the v_G = constants curves as shown in Fig. 5-11. A smooth curve (static transfer characteristic curve) is drawn through the plotted points, and the operating point is then located on this curve. The slope of the static transfer characteristic curve at the operating point is g_m. Since v_P is a constant along this curve, i_P is a function of v_G only

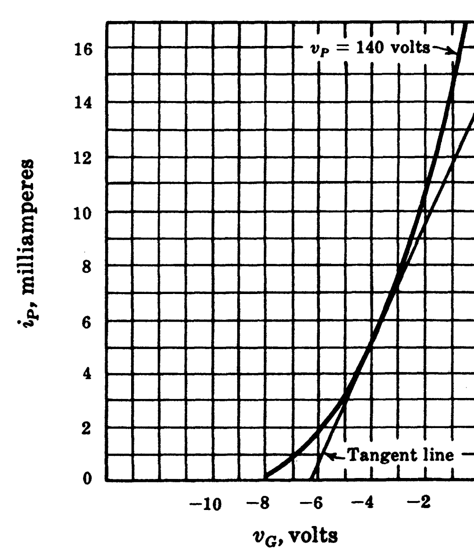

Fig. 5-11. 6J5 Static Transfer Characteristic Curve

along this curve. A line is drawn tangent to the transfer characteristic curve at the operating point and the slope of the tangent line is determined by taking large increments. For this example,

$$g_m = \Delta i_P/\Delta v_G = 0.0142/[0-(-6.5)] = 2190\ \mu\text{mhos}$$

If the equivalent constant voltage generator circuit is to be used, the parameter μ can be calculated knowing g_p and g_m. However, since μ involves a forward transfer characteristic, it may be determined graphically by plotting the appropriate transfer characteristic curve and then finding the slope at the operating point. Since by definition $\mu = -\partial v_P/\partial v_G$, the curve for $i_P = I_P =$ 5.3 ma is plotted. This curve is plotted in the second quadrant because $v_G < 0$, as shown in Fig. 5-12. The entire family of these curves is often called the *family of constant current curves*. This terminology is used for vacuum tube characteristic curves only and was probably adopted to avoid ambiguity, since there are two families of static forward transfer characteristic curves. The slope of the transfer characteristic curve at the operating point is found graphically as before. From Fig. 5-12, v_P decreases as v_G increases; hence $\partial v_P/\partial v_G$ is negative. However, by definition μ is positive. For this example,

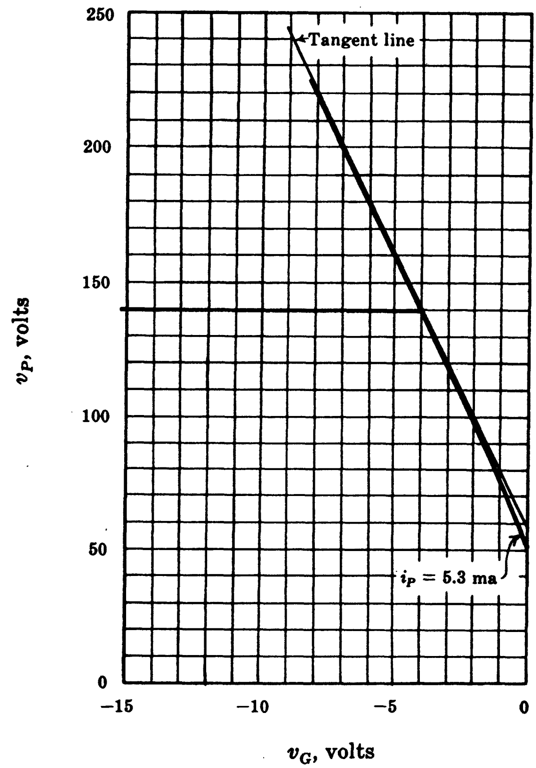

Fig. 5-12. 6J5 Constant Current Curve

$$\mu = -\Delta v_P/\Delta v_G = -(260-59)/(-10.0-0) = 20.1$$

The value of μ calculated using g_m and g_p is

$$\mu = g_m/g_p = (2190\times 10^{-6})/(109\times 10^{-6}) = 20.1$$

The two values of μ are usually somewhat different because of the poor accuracy when small graphs are used.

In this example the parameters were determined by plotting the appropriate curve and then finding the slope of the curve at the operating point. This emphasizes that the parameters are defined as partial derivatives. Since the characteristics are usually an average of a number of tubes and are not those of a particular tube, the parameters are estimates. Also to be considered is the problem of accuracy of the graphical determinations. For these reasons the parameters are sometimes estimated by taking incremental values of the appropriate voltages and currents. For the example given, g_p could have been estimated by choosing two points on the $v_G = -4$ volts static plate characteristic curve and finding the slope of the line through these points. If the points are close together, the accuracy is limited by the ability to read the graph to a sufficient number of significant figures. If the points are chosen farther apart, they may not be on the linear portion of the curve. However, in many practical problems a sufficient degree of accuracy may be achieved. If this procedure is followed it is not necessary to plot the transfer characteristic curves, since the incremental values can be taken directly from the plate characteristics by choosing points along the curves $v_P = $ a constant, $v_G = $ a constant, or $i_P = $ a constant.

The parameters of individual electronic control devices may be determined for an established operating point by some direct measurement procedure employing electronic instruments. In many design problems the values of the parameters given in handbooks are sufficiently accurate even though the operating point may not be the same.

5.5 SIMPLE TRIODE AMPLIFIER

Equivalent circuit representations have been developed for the triode and methods have been presented for evaluating the triode parameters from characteristic curves. The equivalent circuit for an amplifier considering only varying components is now drawn. Since operation is limited to the linear range, superposition may be applied and then only varying components of the voltages and currents are considered in the equivalent circuit. Hence the dc power supplies are represented as short-circuits, since the terminal voltage of an ideal dc source is constant and so the varying component is zero. The ac equivalent circuit for the amplifier of Fig. 5-9 is shown in Fig. 5-13. After the equivalent circuit has been drawn, the circuit equations may be written and solved. For the example under consideration, assume the problem is to determine the output signal voltage when the input signal voltage is given. Signal voltages at various points in amplifiers will be rather arbitrarily numbered in order, beginning at the input terminal. The grid loop and plate nodal equations for the circuit of Fig. 5-13 are respectively

$$-v_{s1} + v_g = 0 \quad \text{or} \quad v_g = v_{s1} \tag{5.15}$$

$$g_m v_g + g_p v_p + G_L v_p = 0 \tag{5.16}$$

from which, using $v_p = v_{s2}$, we obtain

$$v_{s2} = \frac{-g_m}{g_p + G_L} v_{s1}$$

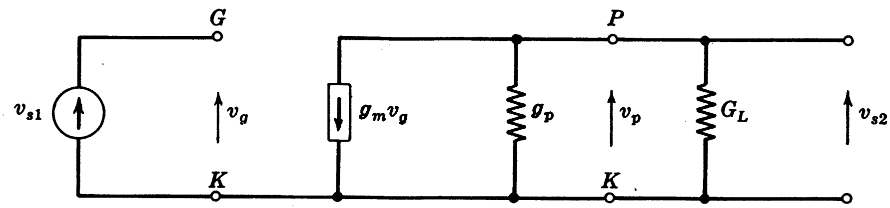

Fig. 5-13. Amplifier Equivalent Circuit

By definition, the voltage gain is

$$A_v = v_{s2}/v_{s1} = -g_m/(g_p + G_L) \tag{5.17}$$

For example, assume that $v_{s1} = 2 \sin \omega t$; then

$$v_{s2} = \{-2190 \times 10^{-6}/[(109 + 33.3) \times 10^{-6}]\} v_{s1}$$
$$= -15.4\, v_{s1} = -30.8 \sin \omega t = 30.8 \sin(\omega t + \pi) \text{ volts}$$

In general, the voltage gain is complex and is often expressed in polar form. The voltage gain is denoted A_v where the A indicates amplification and the subscript v specifies voltage amplification. In this example the voltage gain has magnitude 15.4 and a negative sign. The negative sign indicates a shift in phase of 180° or π radians between the input and output signal voltages.

A graphical solution to the simple amplifier circuit of Fig. 5-9 may provide additional insight into amplifiers containing resistive electronic control devices. A load line for the circuit of Fig. 5-9 is plotted on the 6J5 static plate characteristic curves of Fig. 5-14. The instantaneous grid to cathode voltage drop v_G may be considered point by point and the instantaneous values of i_P and v_P may be found on the graph. If linear operation is assumed, then the varying components of the voltages and currents are all sinusoidal when a sinusoidal signal voltage is applied to the input. The instantaneous value of v_G may be sketched. First draw a line perpendicular to the load line at the operating point. Knowing v_{s1}, the maximum and minimum values of v_G may be located on the load line. For the present example,

$$v_G = V_G + v_g = -4 + 2 \sin \omega t \text{ volts}$$

When $\sin \omega t = 1$, $v_{G(max)} = -2$ volts; when $\sin \omega t = -1$, $v_{G(min)} = -6$ volts. Lines may be drawn perpendicular to the load line indicating $v_{G(max)}$ and $v_{G(min)}$ and then the sine wave may be sketched.

When $v_G = -2$: $i_P = i_{P(max)} = 6.4$ ma, $v_P = v_{P(min)} = 107$ volts

When $v_G = -6$: $i_P = i_{P(min)} = 4.3$ ma, $v_P = v_{P(max)} = 170$ volts

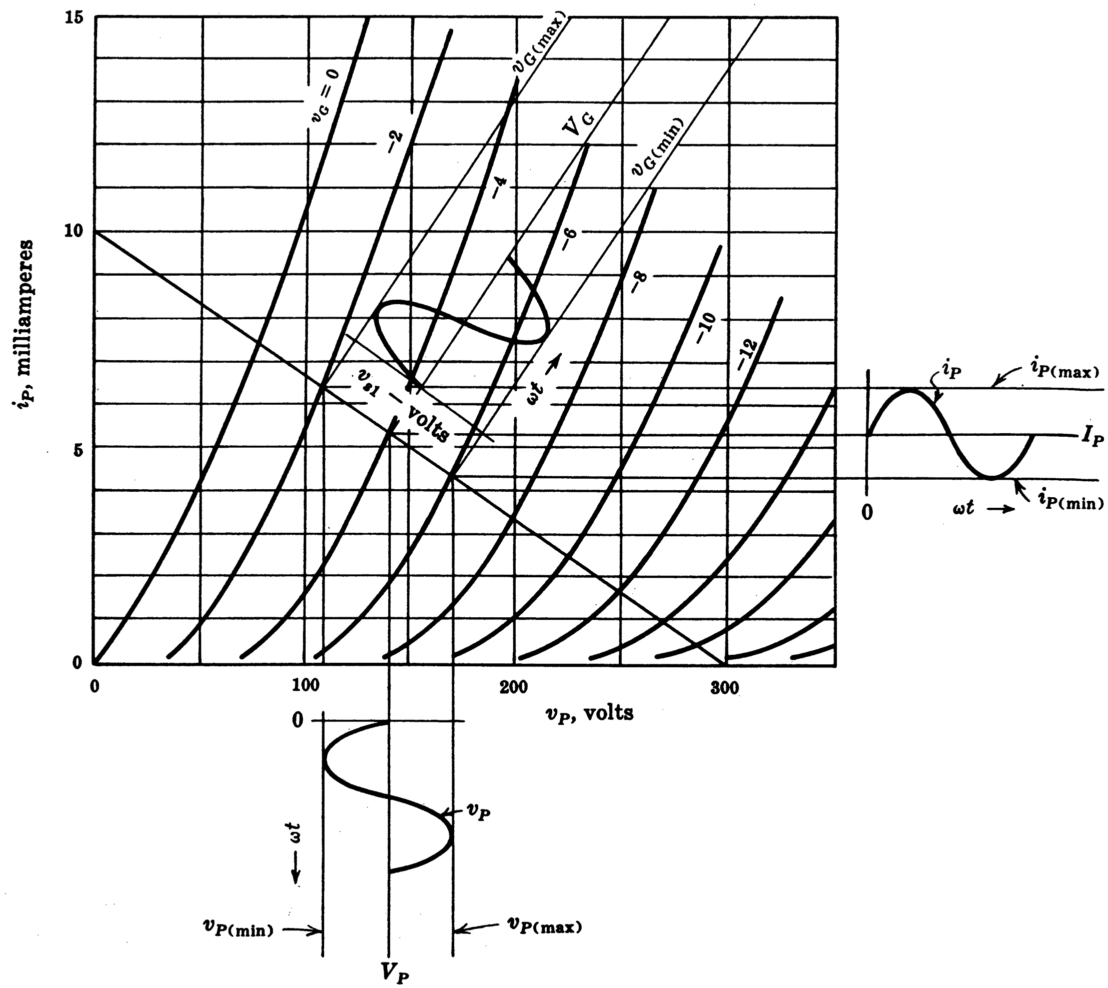

Fig. 5-14. Graphical Solution

The instantaneous values of i_P and v_P may be sketched as shown in Fig. 5-14. As may be seen on this graph, i_p is in phase with v_g but v_p is shifted in phase by π radians. The magnitude of the varying components may be calculated assuming linear operation and sinusoidal variations by taking the difference between the maximum and minimum values, giving the peak-to-peak values of the varying components. For this example,

$$v_p = \tfrac{1}{2}(v_{P(\min)} - v_{P(\max)}) \sin \omega t = \tfrac{1}{2}(107 - 170) \sin \omega t = -31.5 \sin \omega t \text{ volts}$$

$$i_p = \tfrac{1}{2}(i_{P(\max)} - i_{P(\min)}) \sin \omega t = \tfrac{1}{2}(6.4 - 4.3)10^{-3} \sin \omega t = 1.05 \times 10^{-3} \sin \omega t \text{ amps}$$

and the voltage gain $A_v = v_{s2}/v_{s1} = v_p/v_g = -15.7$. The value of A_v calculated from the equivalent circuit was -15.4.

It is of interest to consider the instantaneous resistance looking between the plate and the cathode terminals of the triode. For the triode, $v_{P(\max)}$ occurs when i_P is minimum and $v_{P(\min)}$ occurs when i_P is maximum. This is different from the diode circuits of Chapter 2, but the concept of instantaneous resistance is the same. When $v_G = -2$, the instantaneous plate resistance is

$$r_{P(\min)} = v_{P(\min)}/i_{P(\max)} = 107/0.0064 = 16{,}750\ \Omega$$

When $v_G = -6$, the instantaneous plate resistance is

$$r_{P(\max)} = v_{P(\max)}/i_{P(\min)} = 170/0.0043 = 39{,}500\ \Omega$$

Thus in this example the instantaneous plate resistance varies over a range of about 2.4 to 1 for the given input voltage. An equivalent circuit for the circuit of Fig. 5-9 in terms of instantaneous values is shown in Fig. 5-15. The equations for the circuit of Fig. 5-15 will be nonlinear because r_P is nonlinear. However, this circuit illustrates the principle of voltage amplification when resistive control devices are included.

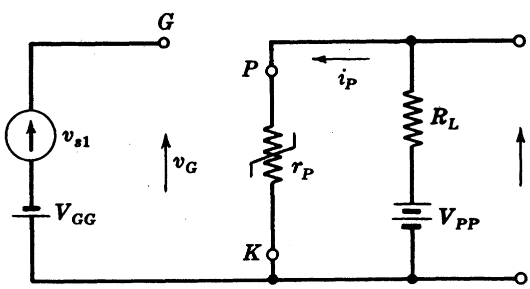

Fig. 5-15. Equivalent Circuit — Instantaneous Values

Since V_{PP} and R_L are constant, the voltage drop v_P will be a function of r_P. Also, r_P is a function of v_G. Hence v_P will decrease as r_P decreases (v_G increases) and v_P will increase as r_P increases (v_G decreases). The instantaneous plate resistance r_P is the resistive parameter of this subclass of electronic control device. In general, in electronic amplifiers of this type the electronic control device (nonlinear resistance) is in series with a constant voltage power supply and a linear resistor.

5.6 CATHODE BIAS

The triode amplifier circuit of Fig. 5-9 includes two power supplies. The grid supply may be eliminated and improved performance obtained by using what is called "cathode bias". The same simple amplifier circuit with cathode bias is shown in Fig. 5-16. An average value of grid to cathode voltage is required to establish the desired operating point, which is $V_G = -4$ v for this example. The instantaneous plate current $i_P = I_P + i_p$ is out of the cathode, so when a resistor is

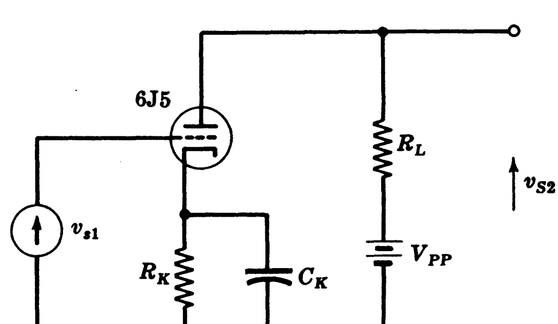

Fig. 5-16. Cathode Bias Amplifier

placed in series from cathode to ground (reference), the voltage drop across the cathode resistor R_K is

$$v_{KGnd} = R_K i_P = R_K I_P + R_K i_p \tag{5.18}$$

The average value of v_{KGnd} is $R_K I_P$ which has the desired polarity, since the cathode is positive with respect to the ground and the average value of the grid to ground voltage is zero. There is also the varying voltage from cathode to ground $R_K i_p$ when only a resistor is used, hence a rather large capacitor C_K is added from the cathode to ground as shown in Fig. 5-16. Considering only varying components, the varying voltage is

$$v_{kgnd} = Z_K i_p \tag{5.19}$$

where Z_K is the impedance of the parallel combination of the capacitor and resistor and is chosen so that the varying voltage is so small that it may be neglected. This will be given more consideration later.

The problem of choosing circuit parameters for a given amplifier is not a simple one, since all of the parameters must be specified and the criteria for optimizing the design must be considered. For this reason the present discussion is mostly concerned with the analysis of given circuits. To add cathode bias to the circuit of Fig. 5-9, the value of R_K may be determined approximately by substituting the values of V_G and I_P into equation (*5.18*).

$$V_{KGnd} = R_K I_P = 4 = R_K(0.0053), \qquad R_K = 4/0.0053 = 750\ \Omega$$

In practice the nearest manufactured value is used, which in this case is 680 Ω. However, since this is an academic problem, a value of 700 Ω will be taken so that the operating point remains approximately the same. The value of R_L is also not an available value but was chosen to make the i_P intercept an even 10 ma. Writing the plate loop equation for the circuit of Fig. 5-16 in terms of average values,

$$-V_{PP} + R_L I_P + V_P + R_K I_P = 0 \qquad \text{or} \qquad \frac{R_L + R_K}{V_{PP}} I_P + \frac{1}{V_{PP}} V_P = +1 \tag{5.20}$$

Because of the cathode resistance R_K, the i_P intercept of the load line is slightly different from that given by equation (*5.12*). If R_L is large compared to R_K, then $R_L + R_K$ is approximately equal to R_L. In this example the difference between 30,000 Ω and 30,700 Ω is not significant. However, this will not always obtain. The small difference in the slope of the load line accounts for the difference between the instantaneous plate to ground voltage and the instantaneous plate to cathode voltage.

The grid loop equation in terms of average values is

$$0 + V_G + R_K I_P = 0 \tag{5.21}$$

where the first zero indicates that the average value of the external generator is zero. The operating point is the point which satisfies equations (*5.20*) and (*5.21*) and the relationship for the tube $i_P = f(v_P, v_G)$. The load line is a solution for $i_P = f(v_P, v_G)$ and equation (*5.20*). Equation (*5.21*) may also be plotted on the plate characteristic curves as shown in Fig. 5-17. Values of V_G are chosen so that they lie on v_G = a constant curves and then the corresponding values of I_P are calculated using equation (*5.21*). For example, let $V_G = -2$; then $I_P = 2/700 = 2.86$ ma. A smooth curve is drawn through all the plotted points as shown in Fig. 5-17 below; this curve is called the *grid curve*. The operating point is the intersection of the plate load line and the grid curve as shown in Fig. 5-17. For this circuit,

$$V_P = 135 \text{ volts}, \quad I_P = 5.3 \text{ ma}, \quad \text{and} \quad V_G = -3.75 \text{ volts}$$

V_G is read from the graph by interpolating linearly between the two adjacent v_G = a constant curves. The tube parameters may be found as before. One parameter is determined by plotting the appropriate forward transfer characteristic curve and finding the slope at the operating point. Since there is no v_G = a constant curve through the operating point, the slope of the v_G = a constant curve nearest the operating point may be used because the slope should be constant over the linear operating range. An alternative would be to sketch the $v_G = V_G$ curve by linear interpolation and then find the slope of this curve at the operating point.

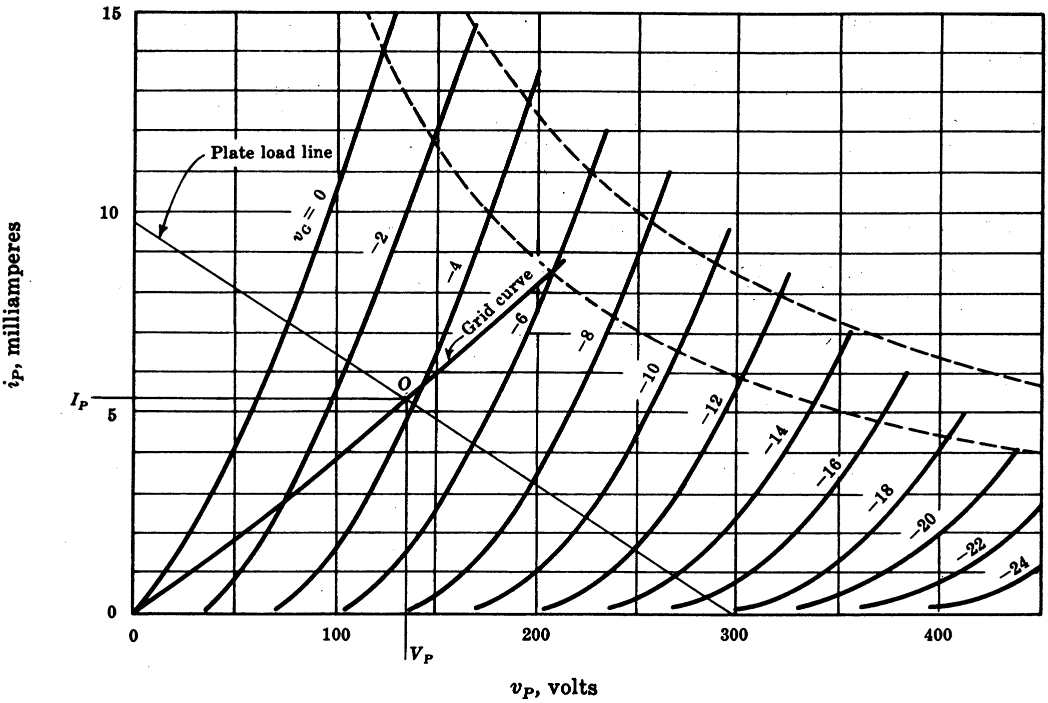

Fig. 5-17. 6J5 Static Plate Characteristics

In Fig. 5-18, equation (*5.21*) is plotted on a graph of i_P vs. v_G; it is a straight line through the origin with slope $-1/R_K$. The plate load line may be transferred from the static plate characteristic curves to the graph of i_P vs. v_G by plotting enough points from the load line to define the curve. The intersections of the plate load line and the static plate characteristics are plotted point by point. For example, when $v_G = 0$, $i_P = 7.3$ ma. The plate load line is not a straight line on the i_P vs. v_G graph. This curve is used frequently enough so that it has been called the dynamic transfer characteristic curve. The dynamic transfer curve takes into account the particular tube in a particular circuit. It differs from the static transfer characteristic curve in that v_P is not constant but varies depending on the current i_P. The static transfer characteristic curve for $v_P = V_P$ will intersect the dynamic transfer characteristic curve at the operating point.

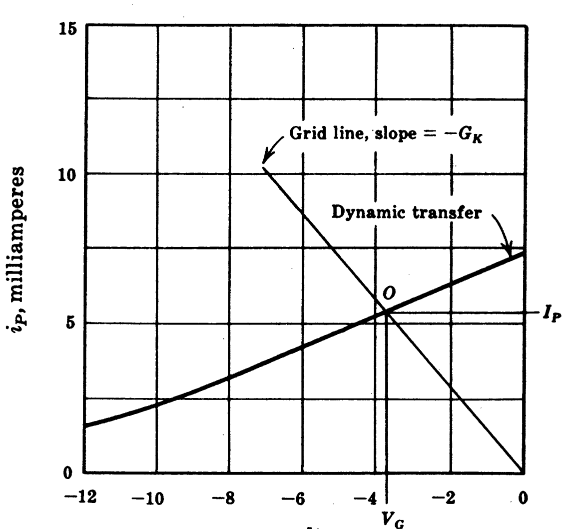

Fig. 5-18. Dynamic Transfer Characteristic Curve

Another advantage of cathode bias is that it tends to stabilize the operating point. If something should occur which would tend to increase the plate current, the "bias" voltage V_G would decrease (become more negative), causing the plate current to decrease. This may be seen by noting the points along the load line which are the intersections with the static plate characteristic curves.

The amplifier circuit (with cathode bias) of Fig. 5-16 is very similar to the amplifier circuit of Fig. 5-9. Considering only varying components in the frequency range where Z_K is very small, the equivalent circuit is drawn as in Fig. 5-13. The same equivalent circuit is used to represent the triode and the operating point is nearly the same as that for the circuit of Fig. 5-9, so the triode parameters are approximately the same. Hence for all practical purposes the voltage gain of the amplifier with cathode bias including a cathode capacitor, is the same when the frequency of the input signal is high.

5.7 VOLTAGE GENERATOR EQUIVALENT CIRCUIT

The voltage gain for the amplifier circuit of Fig. 5-13 was found to be

$$A_v = v_{s2}/v_{s1} = -g_m/(g_p + G_L) \qquad (5.17)$$

In some cases it is more convenient to represent the amplifier by using a constant voltage equivalent circuit as shown in Fig. 5-19. The grid loop equation is still

$$-v_{s1} + v_g = 0 \quad \text{or} \quad v_g = v_{s1} \qquad (5.22)$$

and the plate loop equation is

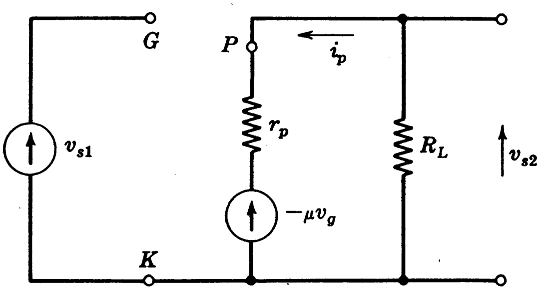

Fig. 5-19. Amplifier Equivalent Circuit

$$R_L i_p + r_p i_p - \mu v_g = 0 \qquad (5.23)$$

From the two equations, $i_p = \mu v_{s1}/(R_L + r_p)$. Then $v_{s2} = -R_L i_p$ and

$$A_v = -\mu R_L/(R_L + r_p) \qquad (5.24)$$

To show that equations (*5.24*) and (*5.17*) are the same, divide the numerator and denominator of (*5.24*) by $R_L r_p$ and obtain (*5.17*).

5.8 UNBYPASSED CATHODE

Consider the amplifier circuit of Fig. 5-16 without the cathode bypass capacitor C_K. The equivalent circuit is shown in Fig. 5-20 for this case. The equivalent circuit for the triode tube is drawn first, and then the remainder of the circuit is sketched considering only varying components. Here the constant voltage equivalent circuit is employed to represent the tube.

To find the voltage gain $A_v = v_{s2}/v_{s1}$, write the grid loop and plate loop equations

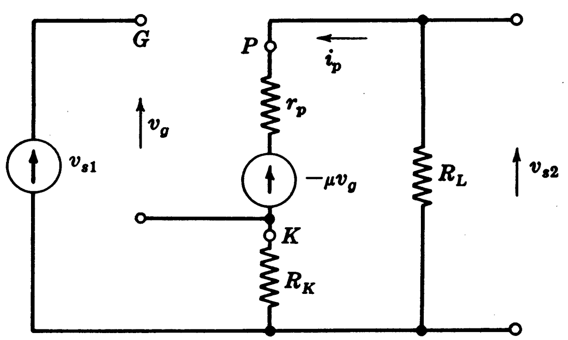

Fig. 5-20. Unbypassed Cathode Amplifier Circuit

$$-v_{s1} + v_g + R_K i_p = 0 \qquad (5.25)$$

$$R_L i_p + r_p i_p - \mu v_g + R_K i_p = 0 \qquad (5.26)$$

and solve for $i_p = \mu v_{s1}/[R_L + r_p + (1+\mu)R_K]$. Then $v_{s2} = -R_L i_p$ and

$$A_v = -\mu R_L/[R_L + r_p + (1+\mu)R_K] \qquad (5.27)$$

Note that when the cathode is unbypassed (capacitor omitted) the denominator of the voltage gain equation has the additional term $(1+\mu)R_K$, hence the gain is lower.

When a capacitor is in the cathode circuit, (5.25) would be written as

$$-v_{s1} + v_g + Z_K i_p = 0 \qquad \text{or} \qquad v_g = v_{s1} - Z_K i_p \tag{5.28}$$

The magnitude of the impedance Z_K is the highest at the lowest frequency, hence C_K should be chosen so that $Z_K i_p$ is very small compared to v_{sl} for the lowest frequency. This will be examined in more detail when overall frequency response is considered.

Using the values for the present example ($\mu = 20.1$, $r_p = 9{,}180\ \Omega$, $R_L = 30{,}000\ \Omega$, $R_K = 700\ \Omega$), we find $A_v = -11.2$. The voltage gain without the cathode capacitor has magnitude 11.2 as compared to 15.4 with the capacitor.

5.9 *RC* COUPLING

In the amplifier circuits of Fig. 5-9 and 5-16, the output signal voltage is the tube plate to ground voltage. Hence the output voltage includes an average value which is positive with respect to ground. If amplifier stages are cascaded, this creates problems in establishing the operating point for the following amplifier tube. A simple capacitor-resistor high pass filter is added to the amplifier circuit to eliminate the average value at the output as shown in Fig. 5-21. The coupling capacitor C_C charges so that the average voltage drop across it equals the average plate to ground voltage. The resistor across the output terminals is denoted R_G because this is frequently the grid resistance of the next amplifier stage. For the present, consider only signal frequencies which are high enough so that the impedance of the coupling capacitor is negligibly small. Then the equivalent circuit has the added resistor R_G as shown in Fig. 5-22. The grid loop equation is the same as equation (5.22). The plate nodal equation is now

$$+g_m v_g + g_p v_{s2} + G_L v_{s2} + G_G v_{s2} = 0 \tag{5.29}$$

Substitute $v_g = v_{s1}$ and solve for the voltage gain

$$A_v = \frac{-g_m}{g_p + G_L + G_G} \tag{5.30}$$

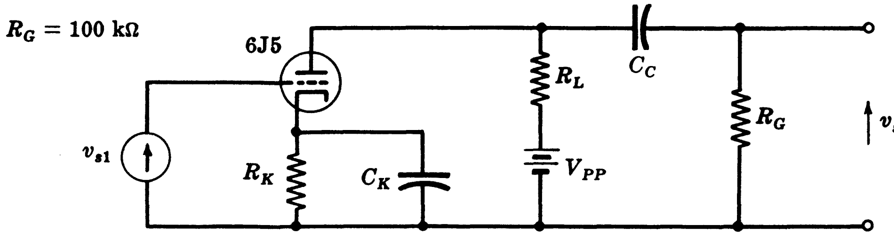

Fig. 5-21. Amplifier with Coupling Network

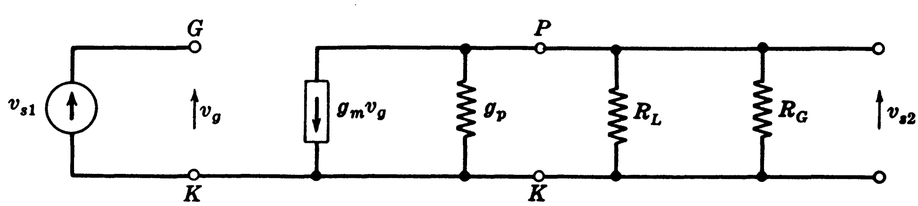

Fig. 5-22. Equivalent Circuit

From Fig. 5-21 and 5-22 it is seen that the effective resistance external to the tube plate is no longer R_L, but is the parallel combination of R_L and R_G. It is necessary to consider the "ac" load line as distinguished from the "dc" load line. The dc load line involves the resistance external to the tube in the plate loop considering only average values. This has R_L and R_K in series as shown before. However, for varying components (ac signals), the resistance external to the tube is the parallel combination of R_L and R_G (with the cathode bypass capacitor in the circuit). The ac load line is drawn through the operating point with slope $-(G_L + G_G)$. This is shown in Fig. 5-23 for the circuit of Fig. 5-21, using the same values for the circuit elements as before but adding $R_G = 100{,}000\ \Omega$. An easy way to draw the ac load line is to first draw a line with slope $-(G_L + G_G)$, and then draw the line through the operating point which is parallel to this line.

A dynamic transfer characteristic curve may now be drawn to include the effect of R_G in the circuit. To distinguish the two dynamic transfer characteristic curves, the one involving points along the dc load line is said to be based on the dc load line, while the one involving points along the ac load line is said to be based on the ac load line.

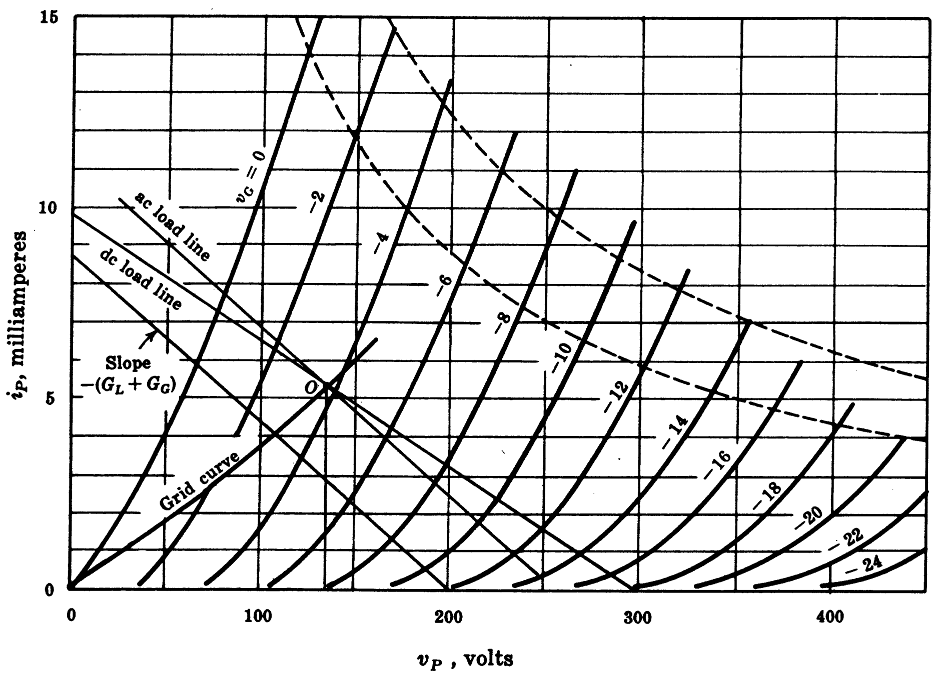

Fig. 5-23. 6J5 Static Plate Characteristics

The ac load line is used when varying signals are considered. The transfer characteristic curves for this example are shown in Fig. 5-24. The static transfer and the two dynamic transfer curves all pass through the operating point.

The linear operating range may be determined from the dynamic transfer characteristic curve based on the ac load line. The linear operating range is that range of v_G and i_P for which the dynamic transfer characteristic curve based on the ac load line is a straight line or approximately a straight line. For the present example, this curve is approximately a straight line for the range $-8 < v_G < 0$.

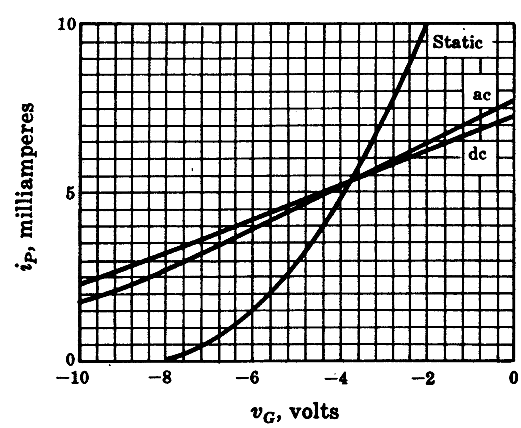

Fig. 5-24. Transfer Characteristic Curves

Since the amplifier circuit of Fig. 5-21 includes two capacitors, it is in general frequency dependent. Shunt and stray circuit capacitances have not been shown. In this chapter it is assumed that the *mid-band* range of frequencies is involved. This is the range of frequencies where the circuit parameters are all resistive, i.e. coupling and bypass capacitors have negligibly small impedances and shunt capacitances have negligibly large impedances. This will be discussed further in Chapter 7.

5.10 CATHODE FOLLOWER AMPLIFIER

The reference terminal for the vacuum tube is the cathode terminal, so the equivalent circuit has been developed with the cathode as the reference element. The simple amplifier circuits considered in previous sections have the cathode as the common terminal, with the input signal applied to the grid and the output signal taken from the plate. Circuits having the plate or grid as common elements are also frequently encountered. The common plate circuit is usually called a *cathode follower*. The simple cathode follower circuit shown in Fig. 5-25 has the input signal connected to the grid and the output terminals are the cathode and ground. The plate to ground circuit has only the dc power supply, hence the plate is at ground potential considering only varying components.

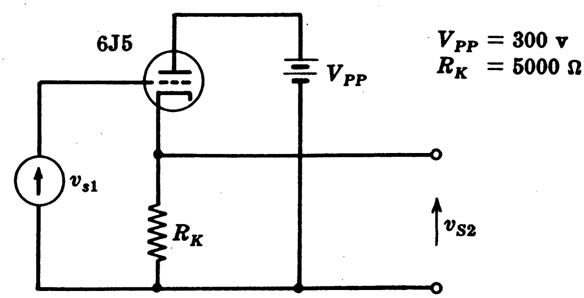

Fig. 5-25. Simple Cathode Follower Circuit

The operating point may be found by first writing the plate loop equation in terms of average values,

$$-V_{PP} + V_P + R_K I_P = 0 \tag{5.31}$$

which plots as a straight line on the static plate characteristic curves, with intercepts $v_P = V_{PP}$ and $i_P = V_{PP}/R_K$. Using the values given in Fig. 5-25, the i_P intercept is $300/5000 = 60$ ma, which is not on the graph. Since two points are needed to locate a line, let $I_P = 15$ ma in (*5.31*) and then find the corresponding value $V_P = 300 - (5000)(0.015) = 225$ v. The dc load line is drawn through the two points as shown in Fig. 5-26.

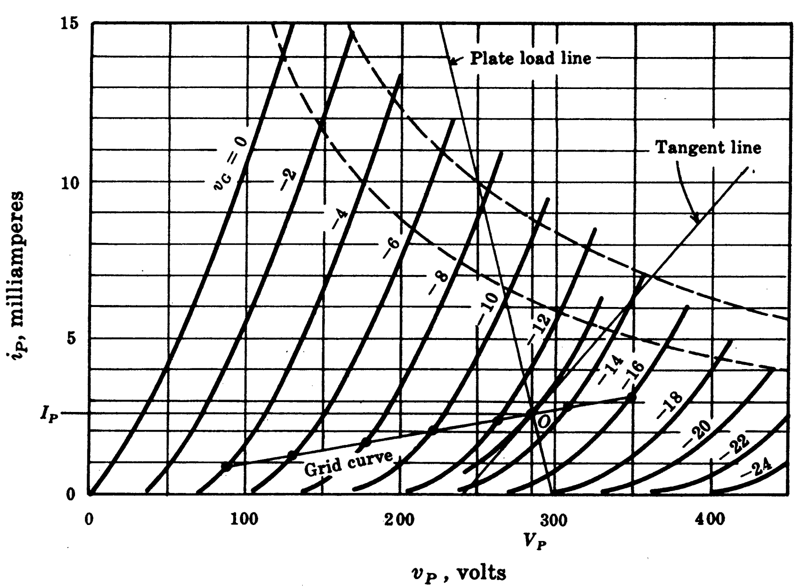

Fig. 5-26. 6J5 Static Plate Characteristics

Since the amplifier circuit of Fig. 5-21 includes two capacitors, it is in general frequency dependent. Shunt and stray circuit capacitances have not been shown. In this chapter it is assumed that the *mid-band* range of frequencies is involved. This is the range of frequencies where the circuit parameters are all resistive, i.e. coupling and bypass capacitors have negligibly small impedances and shunt capacitances have negligibly large impedances. This will be discussed further in Chapter 7.

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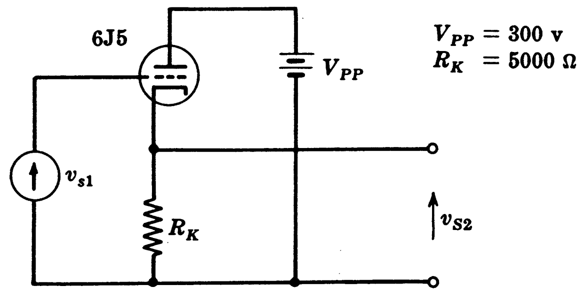


Fig. 5-25. Simple Cathode Follower Circuit

The operating point may be found by first writing the plate loop equation in terms of average values,

$$-V_{PP} + V_P + R_K I_P = 0 \tag{5.31}$$

which plots as a straight line on the static plate characteristic curves, with intercepts $v_P = V_{PP}$ and $i_P = V_{PP}/R_K$. Using the values given in Fig. 5-25, the i_P intercept is $300/5000 = 60$ ma, which is not on the graph. Since two points are needed to locate a line, let $I_P = 15$ ma in (*5.31*) and then find the corresponding value $V_P = 300 - (5000)(0.015) = 225$ v. The dc load line is drawn through the two points as shown in Fig. 5-26.

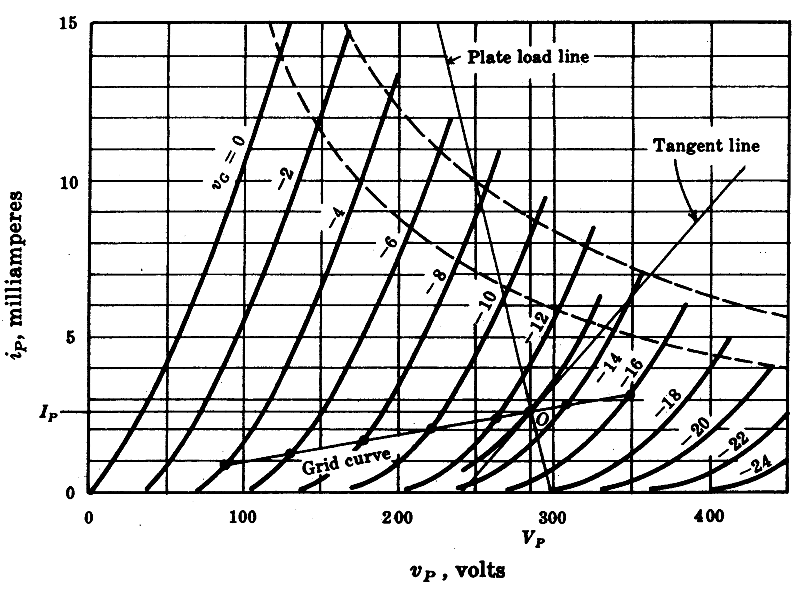


Fig. 5-26. 6J5 Static Plate Characteristics

The grid loop equation in terms of average values is

$$0 + V_G + R_K I_P = 0 \tag{5.32}$$

A sufficient number of points may be plotted and the curve representing the grid equation may be drawn on the plate characteristics as shown in Fig. 5-26. Values of v_G are chosen so that they fall on the v_G = a constant curves, and then the corresponding values of i_P are calculated. The operating point is the intersection of the two curves. For this example, $V_P = 287$ v, $I_P = 2.6$ ma, and $V_G = -13$ v.

The parameters for the tube may be found by using the procedures previously outlined. Here the slopes of the two adjacent v_G = a constant curves are not the same, so a $v_G = -13$ v curve is sketched by interpolating between the $v_G = -12$ v and $v_G = -14$ v curves as shown in Fig. 5-26. The slope of this curve is found at the operating point and is

$$g_p = 0.01/(413-240) = 57.8\ \mu\text{mho}$$

The static transfer characteristic curve for this operating point is plotted in Fig. 5-27. The slope of the line tangent to the static transfer characteristic curve of Fig. 5-27 at the operating point is

$$g_m = 0.0064/[-10-(-15.2)] = 1230\ \mu\text{mho}$$

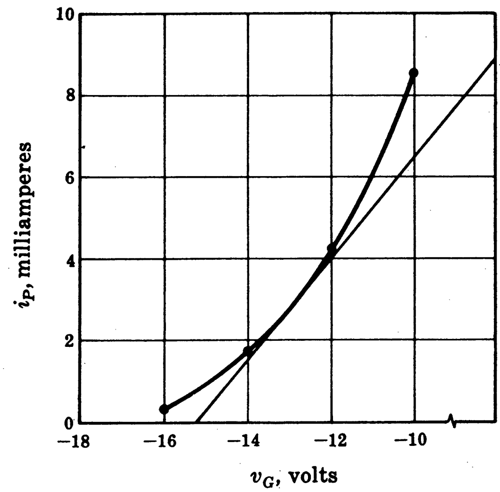


Fig. 5-27. Static Transfer Characteristic Curve

The equivalent circuit for the simple cathode follower circuit of Fig. 5-25 is shown in Fig. 5-28(*a*) with the tube indicated by a constant current generator representation. This circuit is redrawn in Fig. 5-28(*b*). The voltage gain for this circuit is found by writing the grid loop and cathode nodal equations

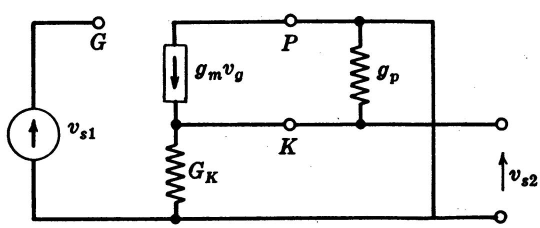
$$-v_{s1} + v_g + v_{s2} = 0 \tag{5.33}$$

$$-g_m v_g + g_p v_{s2} + G_K v_{s2} = 0 \tag{5.34}$$

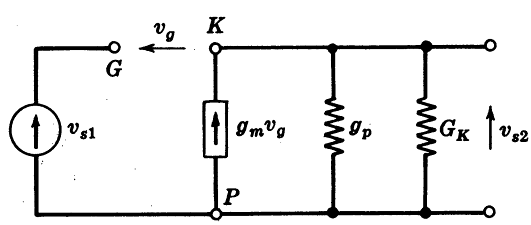
and solving to obtain

$$A_v = v_{s2}/v_{s1} = g_m/(g_p + G_K + g_m) \tag{5.35}$$

For this example, $A_v = \dfrac{1230 \times 10^{-6}}{(58+200+1230)\times 10^{-6}} = 0.828.$



(*a*) Cathode Follower Equivalent Circuit



(*b*) Cathode Follower Equivalent Circuit

Fig. 5-28

The voltage gain is positive, since there is no phase shift between the input and output signal voltages in a cathode follower. The output signal voltage is taken from the cathode terminal, so it is equal to $R_K i_p$. The varying component of the plate current is in phase with the grid voltage, so the output signal voltage is in phase with the input signal voltage. Since all quantities in equation (*5.35*) are positive, the voltage gain is always less than 1.

If g_m is very large compared to $g_p + G_k$, the voltage gain will approach 1. It is, however, possible to have a power gain (output signal power greater than input signal power) greater than 1. Even though the voltage gain is less than 1, there are many applications where cathode followers are useful because of the high input impedance and low output impedance. This will be discussed in Chapter 7.

The cathode follower circuit of Fig. 5-25 has the disadvantage that the triode is operated at a low average plate current. This is not a desirable operating point in terms of linear operating range or of the values of the tube parameters. Practical cathode followers are usually designed so that the operating point involves a higher average plate current. The linear operating range is frequently not a problem, since the actual value of v_g is small compared to v_{s1} because of the rather large voltage drop across the cathode resistor. This is illustrated in Problems 5.1 and 5.2.

5.11 GROUNDED GRID AMPLIFIER

Common grid or *grounded grid triode amplifier* circuits are often of the tuned amplifier type, but they will be illustrated by considering an untuned amplifier circuit as shown in Fig. 5-29. Here the grid is grounded and the input signal is applied to the cathode. The plate current is through the external generator, so the external input circuit must be capable of carrying this current (dc current path). A cathode resistor is included in the circuit to establish the desired operating point, and is shown bypassed with a cathode capacitor. The plate circuit is the same as that shown for the common cathode amplifier circuit of Fig. 5-21.

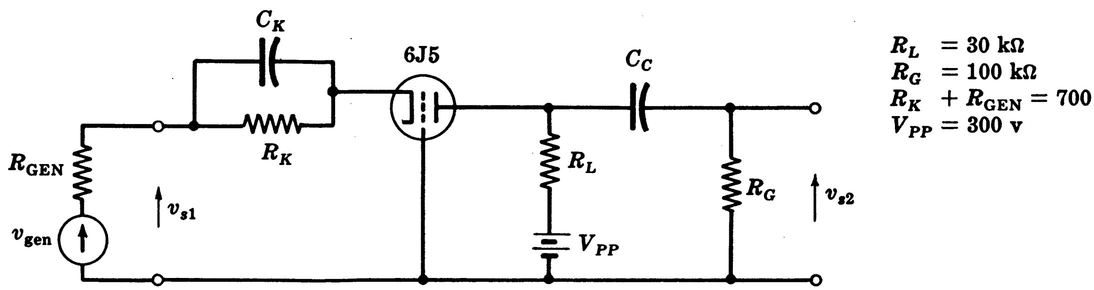


Fig. 5-29. Common Grid Amplifier

The operating point for the triode is found by writing the plate-cathode loop equation in terms of average values,

$$-V_{PP} + R_L I_P + V_P + R_K I_P + R_{GEN} I_P = 0 \tag{5.36}$$

where R_{GEN} is the dc resistance of the input generator circuit and it is assumed that the average value of the generator voltage is zero. Equation (*5.36*) is written in the standard intercept form as

$$\frac{R_L + R_K + R_{GEN}}{V_{PP}} I_P + \frac{1}{V_{PP}} V_P = +1 \tag{5.37}$$

Entering the numerical values listed in Fig. 5-29,

$$\frac{30{,}700}{300} I_P + \frac{1}{300} V_P = +1 \tag{5.38}$$

The grid-cathode loop equation in terms of average values is

$$V_G + R_K I_P + R_{GEN} I_P = 0 \tag{5.39}$$

Solved Problems

5.1. The cathode follower circuit of Fig. 5-25, page 84, has been changed to that of Fig, 5-43 which allows the tube to be biased for a more favorable operating point. Choose the resistor R_1 so that V_G is approximately -6 volts. (*a*) Find V_P and I_P for this value of R_1. (*b*) Determine g_m and g_p graphically.

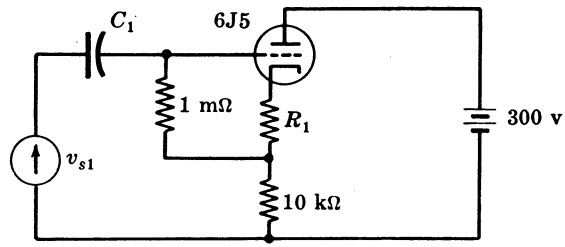


Fig. 5-43

Since $V_G = -6$ volts,. the voltage drop across R_1 must be $+6$ volts in the direction of i_P. The current in the 1 megohm grid resistor is negligible because the grid is negative. First, determine the dc plate load line by writing the plate loop equation in terms of average values:

$$-300 + V_P + 6 + 10{,}000 I_P = 0 \quad \text{or} \quad V_P + 10{,}000 I_P = 294 \qquad (5.75)$$

which may be plotted using the v_P intercept 294 and the point $i_P = 20$ ma. The corresponding value of v_P for $i_P = 20$ ma is found by substituting into (*5.75*):

$$v_P = 294 - (10{,}000)(0.02) = 94 \text{ volts}$$

The dc plate load line is now plotted as in Fig. 5-44. The intersection of the $v_G = -6$ volts plate characteristic curve and the dc load line is the operating point for these conditions. From the graph, $I_P = 9.6$ ma and $V_P = 198$ volts.

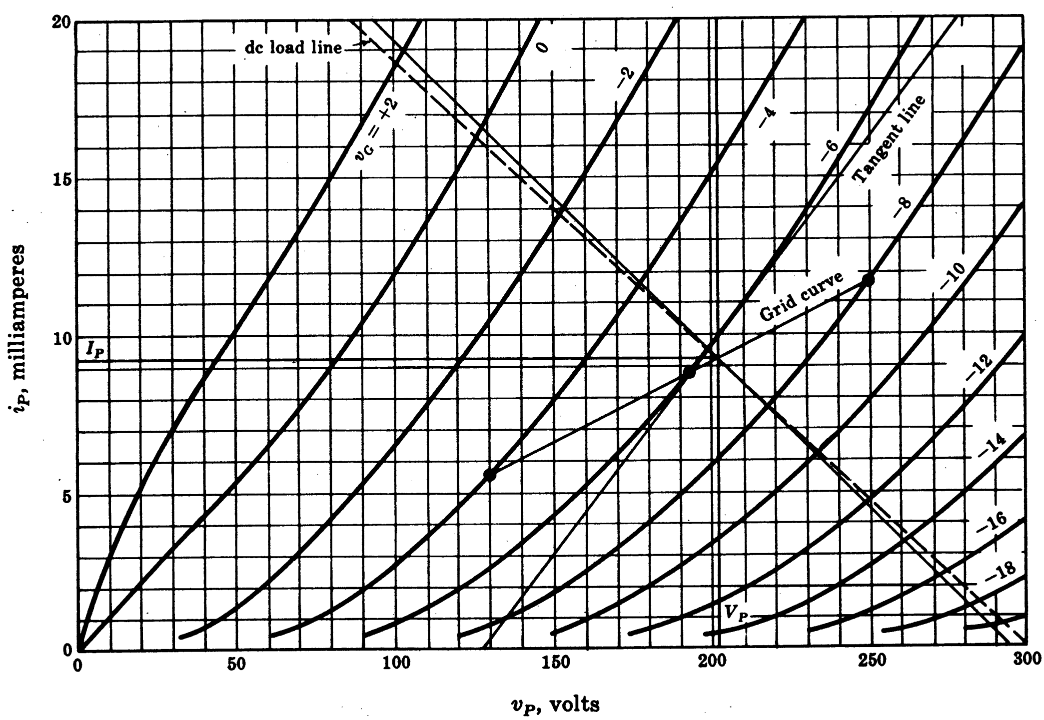


Fig. 5-44. 6C4 Static Plate Characteristics

R_1 may now be calculated knowing the current in R_1 and the voltage drop: $R_1 = 6/0.0096 = 625\ \Omega$. Since this is not a standard value, let $R_1 = 680\ \Omega$. The dc load line using this value of R_1 will not be appreciably different near the operating point, as may be shown by considering the plate loop equation using $R_1 = 680\ \Omega$.

$$V_P + 10{,}680 I_P = 300$$

Two points on the dc load line are $i_P = 0$, $v_P = 300$; and for $i_P = 20$ ma,

$$v_P = 300 - (10{,}680)(0.02) = 86.4 \text{ volts}$$

This dc load line is shown as a dashed line in Fig. 5-44. The grid loop equation based on average values is

$$0 + V_G + 680 I_P = 0$$

Plot the grid curve point by point on the static plate characteristic curves of Fig. 5-44. For $v_G = -6$ v, $I_P = 6/680 = 8.83$ ma; for $v_G = -8$ v, $I_P = 8/680 = 11.77$ ma; for $v_G = -4$ v, $I_P = 4/680 = 5.88$ ma. The curve through these three points is sufficiently close to a straight line, so a line may be drawn through these points, giving a portion of the grid curve. The operating point is the intersection of the grid curve and the dc load line shown as a dashed line in Fig. 5-44.

(a) The final values are: $V_G = -6.3$ volts, $I_P = 9.3$ ma, $V_P = 203$ volts.

(b) To find g_p, take the slope of the nearest v_G = a constant curve, which is $v_G = -6$ volts. The slope of the line tangent to the plate characteristic curve is $g_p = (0.02 - 0)/(279 - 127) = 131$ μmho.

To find g_m, plot the static transfer characteristic curve and then find the slope at the operating point as shown in Fig. 5-45: $g_m = 0.015/[-3.4 - (-10.6)] = 2080$ μmho.

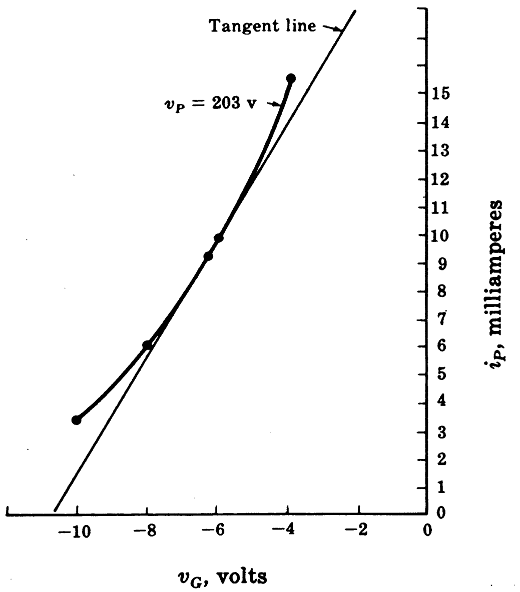


Fig. 5-45. Static Transfer Characteristic Curve

5.2. Given the cathode follower amplifier circuit of Fig. 5-46 and the 6J5 static plate characteristics of Fig. 5-47. Solve this circuit graphically for the open circuit transfer characteristics in terms of instantaneous values.

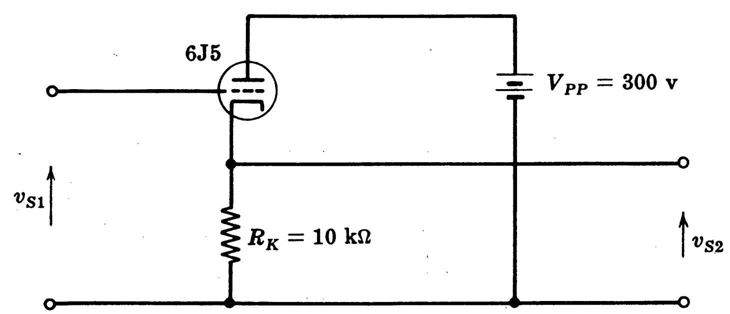


Fig. 5-46

The plate loop equation in terms of instantaneous voltages and currents is

$$-V_{PP} + v_P + R_K i_P = 0$$

Substituting numerical values,

$$(1/300)v_P + (10{,}000/300)i_P = +1$$

which is plotted on the static plate characteristics as in Fig. 5-47, using the v_P intercept and the $i_P = 15$ ma point. The corresponding value of v_P is $v_P = 300 - (10{,}000)(0.015) = 150$ volts. Any pair of instantaneous values (i_P, v_P) must lie on the load line graphed in Fig. 5-47 below.

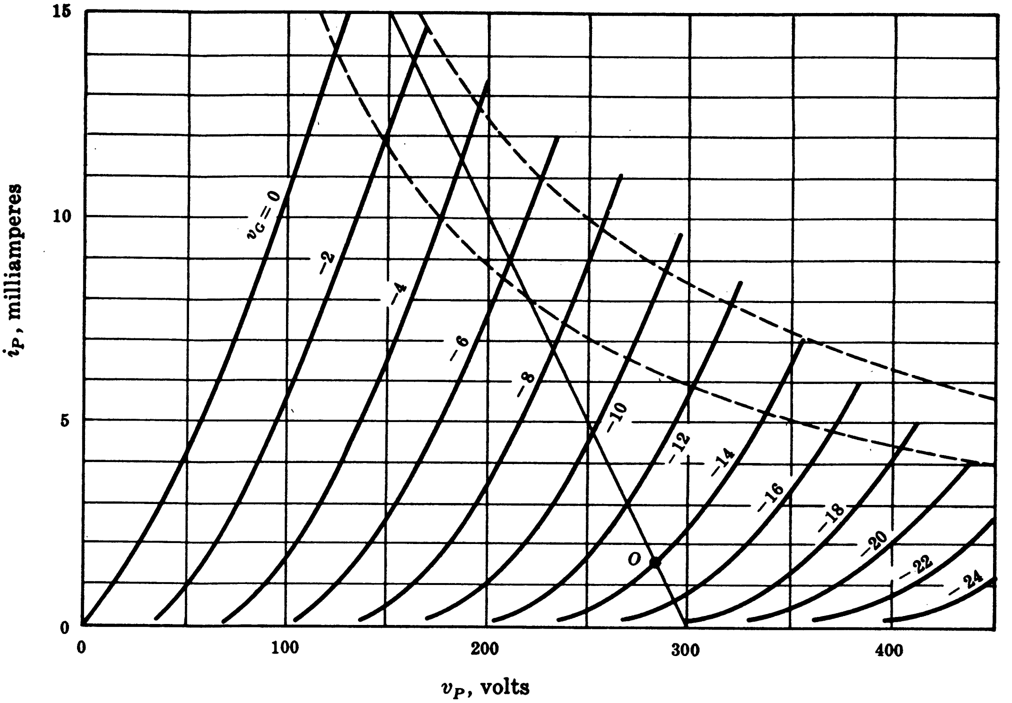


Fig. 5-47. 6J5 Static Plate Characteristics

The input and output loop equations in terms of instantaneous values are

$$-v_{S1} + v_G + R_K i_P = 0 \quad \text{or} \quad v_{S1} = v_G + R_K i_P \tag{5.76}$$

$$-v_{S2} + R_K i_P = 0 \quad \text{or} \quad v_{S2} = R_K i_P \tag{5.77}$$

A table is made using instantaneous values as shown below. The values of i_P for given values of v_G are taken from the intersections of the static plate characteristic curves and the load line. For each value of i_P the value of $R_K i_P$ can be computed, giving the instantaneous output voltage v_{S2}. Using equation (5.76), the corresponding value of the instantaneous input voltage v_{S1} is calculated. The curve showing v_{S2} as a function of v_{S1} is plotted as shown in Fig. 5-48. This curve indicates that the linear operating range is considerably extended if v_{S1} has an average value greater than zero. If V_{S1} (average value of v_{S1}) is zero, the operating point is that shown as O in Fig. 5-48. If V_{S1} is in the order of 50 to 75 volts, the linear operating range is extended and the tube is biased at a more favorable operating point.

v_G volts	i_P ma	$R_K i_P = v_{S2}$ volts	$v_{S1} = v_G + R_K i_P$ volts
−2	13.8	138	136
−4	11.4	114	110
−6	8.9	89	83
−8	6.7	67	59
−10	4.6	46	36
−12	2.9	29	17
−14	1.5	15	1
−16	0.5	5	−21

TABLE 5.1

The operating point for the given circuit was found by locating the point $V_{S1} = 0$ on the load line. The corresponding value of V_{S2} is 15 volts. Substitute into (5.77) to find $I_P = V_{S2}/R_K = 15/10{,}000 = 1.5$ ma. This point is located on the load line of Fig. 5-47, giving

$$V_G = -14 \text{ volts}$$

and $$V_P = 284 \text{ volts}$$

The small signal voltage gain is the slope of the transfer characteristic curve at the operating point. For this circuit,

$$A_v = \partial v_{S2}/\partial v_{S1} = 100/[110-(-20)] = 0.77$$

The accuracy is limited because the operating point is not on the straight line portion of the transfer characteristic curve and the plotted points are relatively far apart.

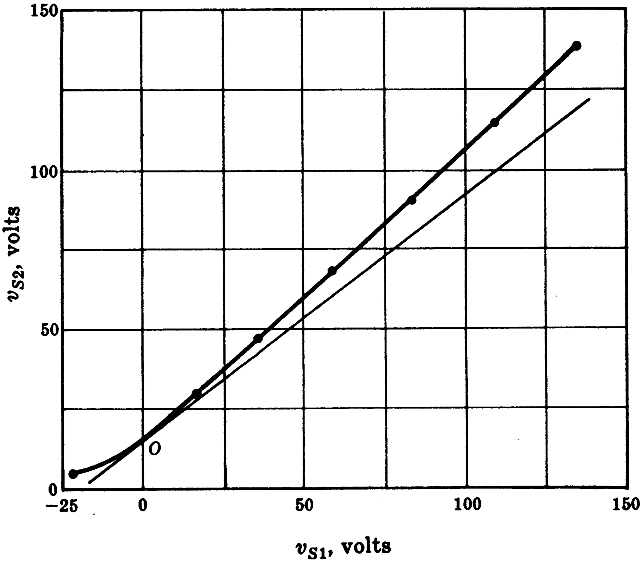


Fig. 5-48. Overall Cathode Follower Transfer Characteristics

5.3. Given the parallel combination of two tubes shown in Fig. 5-49. The tubes are operating on the linear portion of the operating range and their parameters are

T_1: $\mu_1 = 30$, $r_{p1} = 12$ kΩ

T_2: $\mu_2 = 60$, $r_{p2} = 50$ kΩ

Find the effective μ and r_p for the parallel combination.

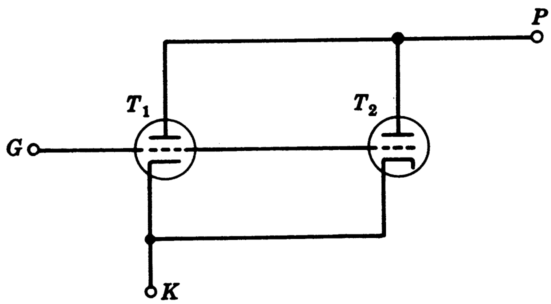


Fig. 5-49

Draw the ac equivalent circuit for the two tubes in parallel as shown in Fig. 5-50. The equation for the current loop "i" is

$$+60v_g + 50{,}000i + 12{,}000i - 30v_g = 0 \quad \text{or} \quad i = -484 \times 10^{-6}\, v_g$$

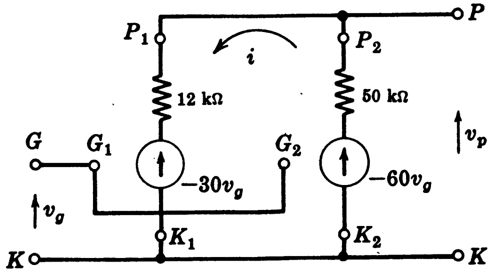


Fig. 5-50

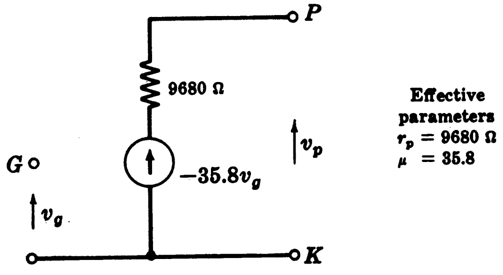


Fig. 5-51. Equivalent Circuit for the Two Tubes in Parallel

The two generators are independent because the generated voltage is independent of v_p and i_p. In the triode, the reverse transfer function is zero when the grid is negative. Thévenin's theorem may be used. The open-circuit voltage drop is $v_p = -50{,}000i - 60v_g = 24.2v_g - 60v_g = -35.8v_g$. The effective internal impedance (generator impedance zero) is the parallel combination:

$$R_{eff} = (12{,}000)(50{,}000)/62{,}000 = 9680\ \Omega$$

5.4. Given the circuit of Fig. 5-52. When $R_L = R_K$, the two output signal voltages are of equal magnitude but 180° out of phase. This circuit is a *split-load phase inverter* and is employed as a driver for a push-pull amplifier stage. Derive the equations for the two signal output voltages in terms of the input signal v_{s1}.

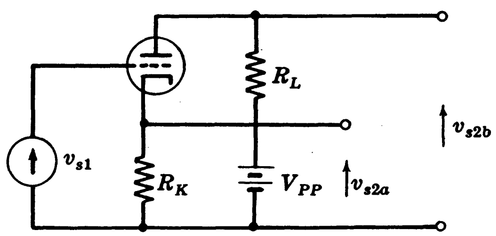


Fig. 5-52

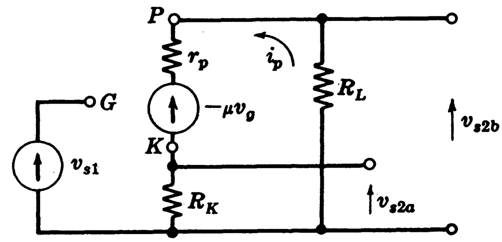


Fig. 5-53

First draw an equivalent circuit representation considering varying components only. In this case the equivalent voltage generator circuit is most convenient for representing the triode as shown in Fig. 5-53.

The grid loop and plate loop equations are

$$-v_{s1} + v_g + R_K i_p = 0 \qquad \text{and} \qquad R_L i_p + r_p i_p - \mu v_g + R_K i_p = 0$$

from which $i_p = \dfrac{+\mu v_{s1}}{R_L + r_p + R_K + \mu R_K}$. Note on the circuit diagram of Fig. 5-53 that

$$v_{s2a} = R_K i_p = \frac{\mu R_K}{R_L + r_p + (\mu+1)R_K} v_{s1}, \qquad v_{s2b} = -R_L i_p = \frac{-\mu R_L}{R_L + r_p + (\mu+1)R_K} v_{s1}$$

Thus when $R_L = R_K$, then v_{s2a} and v_{s2b} are the same in magnitude but opposite in phase, giving the desired output signals.

5.5. Given the triode amplifier circuit of Fig. 5-54. The power supply was poorly designed and has considerable ripple. Under normal operating conditions the instantaneous terminal voltage of the power supply is $v_{PP} = 250 + 2 \sin 377t$ volts. The tube parameters are $\mu = 30$ and $r_p = 20$ kΩ. If $v_{s1} = 1.0 \sin 2000\pi t$ volts, find the instantaneous value of the varying component of the output voltage v_{s2}.

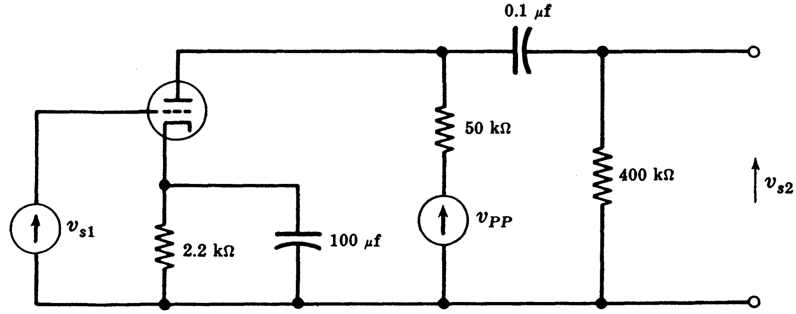


Fig. 5-54

Assume the tube is operating on the linear portion of the operating range so that superposition may be applied and an equivalent circuit used to represent the tube. First check to see if the cathode is effectively bypassed at 60 cps by computing the reactance of the capacitor at this frequency.

$$X_{C_K} = 1/\omega C_K = 1/[(377)(100 \times 10^{-6})] = 26.5\ \Omega$$

The cathode impedance may be assumed negligibly small.

Draw the equivalent circuit considering varying components, assuming the cathode is grounded as in Fig. 5-55. Apply superposition and first consider the 1000 cps signal frequency. The 0.1 μf coupling capacitor has reactance $X_{C_C} = 1/[(2000\pi)(0.1 \times 10^{-6})] = 1590\ \Omega$, which is negligible compared to the 400 kΩ and the effective resistance looking to the left. The circuit considering the 1000 cps signal is shown in Fig. 5-56.

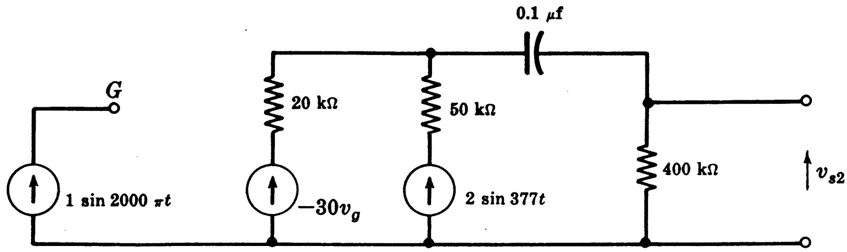


Fig. 5-55. Equivalent Circuit

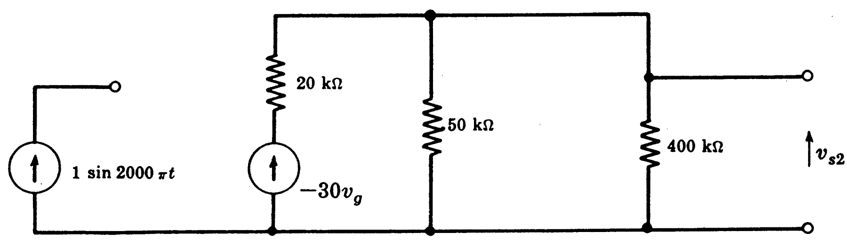


Fig. 5-56. Equivalent Circuit for $f = 1000$ cps

The 1000 cps component of the voltage v_{s2} is more conveniently found by using the constant current representation for the tube as shown in Fig. 5-57.

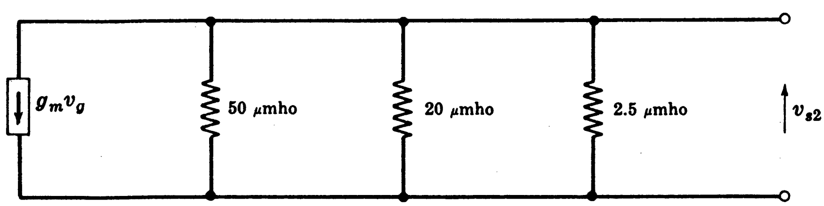


Fig. 5-57. Equivalent Circuit for $f = 1000$ cps

The plate node equation is, using $g_m = \mu/r_p = 30/20{,}000 = 1500\ \mu$mho,

$$1500 \times 10^{-6}\, v_g + 72.5 \times 10^{-6}\, v_{s2} = 0 \quad \text{or} \quad v_{s2} = -20.65 v_g = -20.65 \sin 2000\pi t \text{ volts}$$

This is the 1000 cps component of v_{s2}.

Consider the 60 cps ripple frequency. The equivalent circuit is shown in Fig. 5-58. The 60 cps component for v_{s2} may be found either by writing two loop equations and solving or by applying Thévenin's theorem to the circuit on the left of the terminals a and b. Use Thévenin's theorem and open the circuit at terminal a. Assume the current i of Fig. 5-58 and write the loop equation:

$$-2 \sin 377t + 50{,}000i + 20{,}000i = 0$$

or $\quad i = (2/70{,}000) \sin 377t$

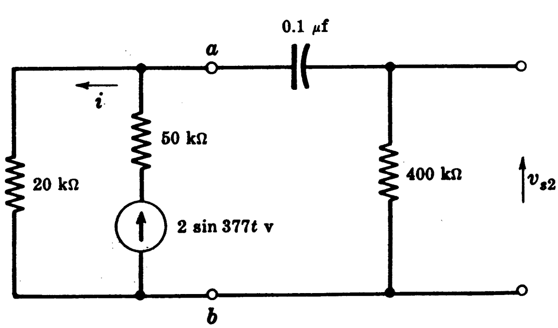


Fig. 5-58. Equivalent Circuit for $f = 60$ cps

The open circuit voltage $v_{ab} = 20{,}000i = 0.572 \sin 377t$ volts. The effective resistance $R_{ab} = (20{,}000)(50{,}000)/(70{,}000) = 14{,}300\ \Omega$.

The circuit may be redrawn as shown in Fig. 5-59. The loop equation is

$$-0.572 \sin 377t + 14{,}300i + \frac{-j}{377(0.1 \times 10^{-6})} i + 400{,}000i = 0$$

from which $$i = \frac{0.572}{414{,}300 - j26{,}500} \sin 377t = \frac{0.572}{414{,}350\angle{-3.68^\circ}} \sin 377t$$

and $$v_{s2} = 400{,}000i = 0.552 \sin(377t + 0.064) \text{ volts}$$

which is the 60 cps component of v_{s2}. Adding the two components of v_{s2} gives

$$v_{s2} = 20.65 \sin(2000\pi t + \pi) + 0.552 \sin(377t + 0.064) \text{ volts}$$

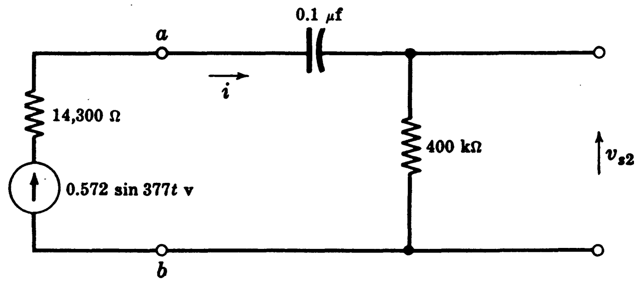


Fig. 5-59. Equivalent Circuit for $f = 60$ cps

5.6. Given the pentode amplifier circuit of Fig. 5-60 and the static plate characteristics for the 6AU6 in Fig. 5-61. (a) Determine the operating point for the circuit as given, assuming $V_{G2} = 150$ volts. (b) Find V_{G1}, V_P and I_P. (c) Choose R_2 so that $V_{G2} = 150$ volts. (d) Estimate g_m and g_p graphically. Calculate the approximate value of μ. (e) Compute the voltage gain $A_v = v_{s2}/v_{s1}$.

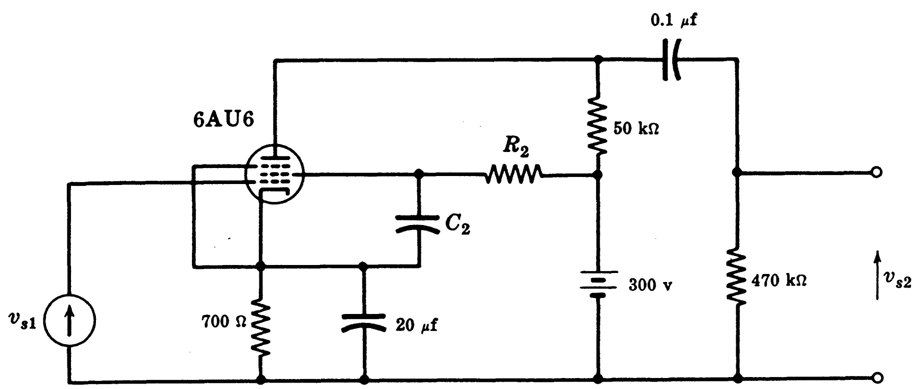


Fig. 5-60. Pentode Amplifier

Assume $V_{G2} = 150$ volts; this is the value for which the static plate characteristic curves of Fig. 5-61 below are plotted. The plate loop equation based on average values is

$$-300 + 50{,}000I_P + V_P + 700(I_P + I_{G2}) = 0$$

If I_{G2} is not small compared to I_P, it may not be neglected. As a first approximation, neglect I_{G2}. Then the equation becomes

$$(50{,}700/300)I_P + (1/300)V_P = +1$$

which is plotted on the plate characteristic curves of Fig. 5-61 using the intercepts $v_P = 300$ v and $i_P = 5.92$ ma.

5.8. **A triode vacuum tube is to be used in a circuit which has a relay in the plate circuit. This relay is to be operated whenever a positive input voltage of an appropriate value is applied between the grid and ground. The current in the input circuit must be negligibly small because the source is a very high impedance. A relay is available which operates when the relay coil current is 3 ma. The relay coil resistance is 10,000 Ω. Design a circuit which will satisfy these requirements using a single 250 v dc power supply.**

Since the plate current must be greater than 3 ma to operate the relay, a 6C4 tube will meet the plate current requirement. The tube must be operating near plate current cutoff (i_P near zero) when the grid to ground voltage is zero. Cathode bias requires a sufficient magnitude of current in the cathode resistor to produce the desired magnitude of V_G. To provide sufficient current in the cathode resistor, a second resistor R_1 may be added from the cathode terminal to the power supply as shown in Fig. 5-67.

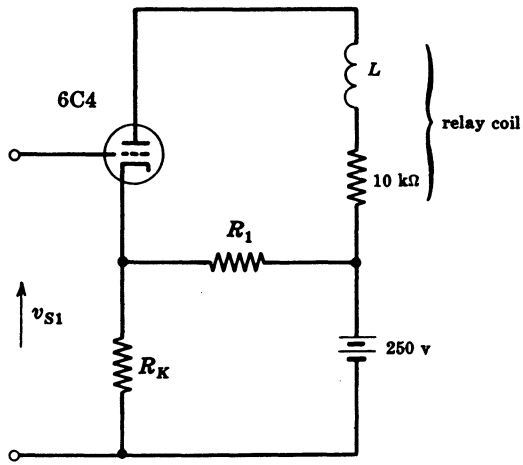


Fig. 5-67

As a starting point, plot the dc load line neglecting R_K. The equation of the load line is then

$$(10{,}000/250)i_P + (1/250)v_P = +1 \qquad (5.82)$$

Since the 25 ma intercept is not on the graph, use the point $i_P = 20$ ma and find the corresponding voltage v_P:

$$(10{,}000)(0.02) + v_P = 250 \quad \text{or} \quad v_P = 50 \text{ v}$$

From the plate characteristic curves of Fig. 5-68 it is seen that if $v_G = -18$ v, then $i_P \cong 0.3$ ma. This should be a sufficiently small value so that the relay remains unoperated unless a positive input voltage is applied. If $v_G = -8$ v, then $i_P = 5.5$ ma which should assure reliable operation of the relay. For the cathode to ground voltage of 18 v, a 2200 ohm resistor could be used if the current is

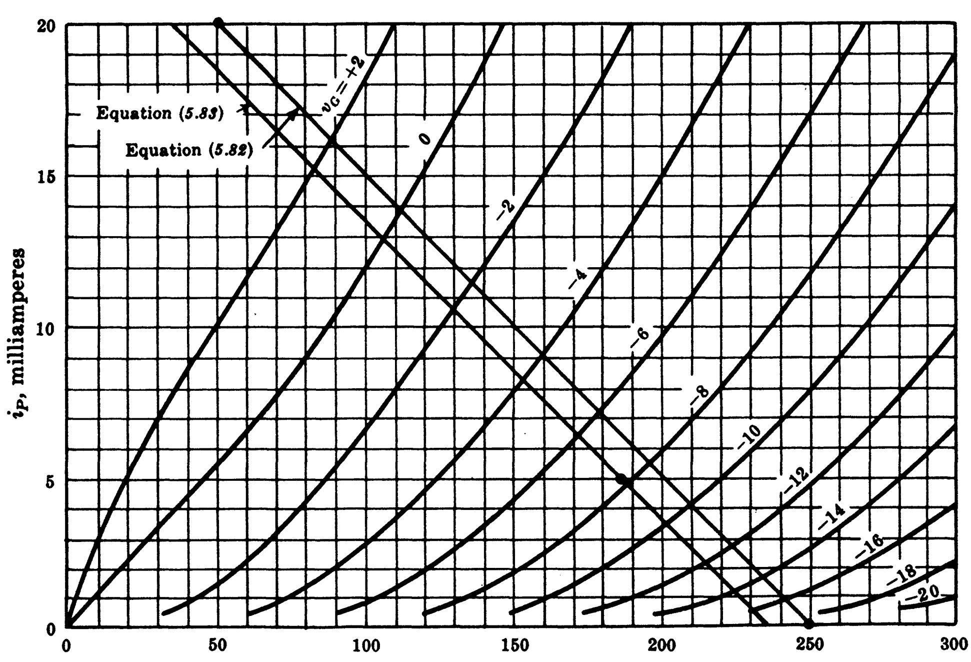


Fig. 5-68. Static Plate Characteristics 6C4

18/2200 = 8.2 ma. Since under these conditions the tube plate current is about 0.3 ma, the current in the resistor R_1 must be about 8 ma. The value of R_1 should then be $R_1 = (250-18)/0.008 = 29{,}000\ \Omega$; since this is not a standard value, use 33,000 Ω. The current in R_1 may be approximated by solving the loop equation assuming $i_P = 0.3$ ma:

$$-250 + 33{,}000i + 2200(i + 0.0003) = 0 \quad \text{or} \quad i = 7.1 \text{ ma}$$

The voltage from cathode to ground is $v_{KGnd} = 2200(7.1 + 0.3)10^{-3} = 16.3$ volts. Using this value, the plate loop equation is

$$-250 + 10{,}000i_P + v_P + 16.3 = 0 \quad \text{or} \quad (10{,}000/234)i_P + (1/234)v_P = +1 \qquad (5.83)$$

This line is also plotted on the plate curves of Fig. 5-68. The zero input plate current is approximately 0.5 ma. The grid to ground voltage required for reliable operation of the relay (assuming $i_P = 5$ ma) can be approximated. When the positive voltage is applied to the grid the plate current increases, so the load line shown is only an approximation. The $i_P = 5$ ma line intersects the load line at $v_G = -7.7$ volts. Using $v_G = -7.7$ volts and $i_P = 5$ ma, the input voltage may be approximated:

$$-v_{S1} + (-7.7) + 2200(0.005 + 0.0071) = 0 \quad \text{or} \quad v_{S1} = 18.9 \text{ v}$$

which should provide reliable operation of the relay.

This problem emphasizes that a tube cannot be biased to cutoff by using conventional cathode bias.

Supplementary Problems

5.9. Given the circuit of Fig. 5-69 and the static plate characteristic curves for a 6J5 tube in Fig. 5-70 below.

(a) Find the operating point. Determine V_G, V_P and I_P.

(b) Calculate the static plate conductance G_P.

(c) For the given operating point, find g_p and g_m. Graph any transfer characteristic curves needed.

(d) Find μ graphically, plotting any needed curves. Taking the values of g_p and g_m from part (c), calculate μ and compare with the value determined graphically. Explain any differences.

(e) Draw the equivalent circuit considering only average values. Include the values of the circuit elements.

(f) Construct the equivalent circuit considering only varying components ("ac" equivalent circuit), using a voltage generator circuit and then a current generator circuit for the triode. Include the values for the circuit elements.

(g) Compute the voltage gain A_v. Do not substitute into the formula but write the necessary equations and solve them.

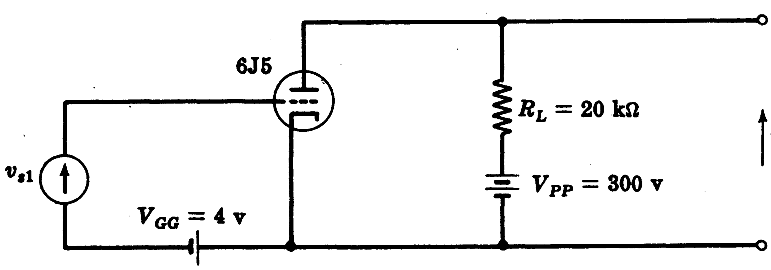


Fig. 5-69

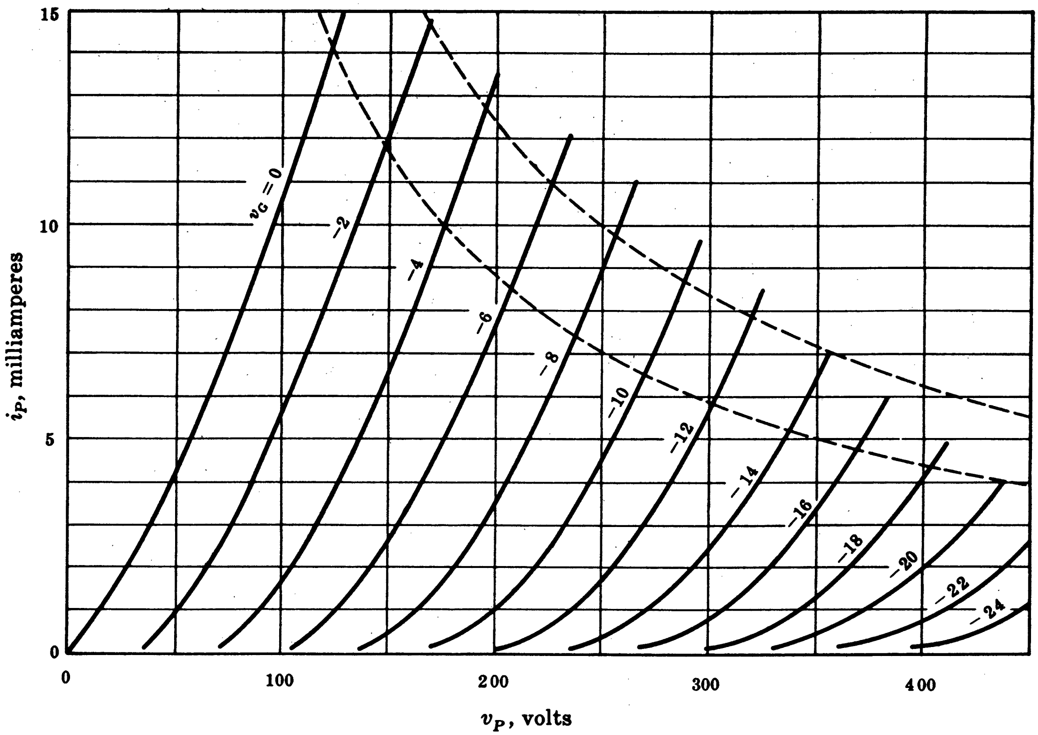


Fig. 5-70. 6J5 Plate Characteristics

5.10. Given the circuit of Fig. 5-71 and the static plate characteristic curves for a 6AV6 tube in Fig. 5-72 below.

(a) Determine V_G, V_P and I_P.

(b) Find μ and r_p for the given operating point, plotting any needed curves.

(c) Plot the dynamic transfer characteristic curve based on the dc load line and the one based on the ac load line, assuming C_K large.

(d) Assuming C_K is large so that the impedance from cathode to ground is negligible, construct the ac equivalent circuit and compute the voltage gain $A_v = v_{s2}/v_{s1}$.

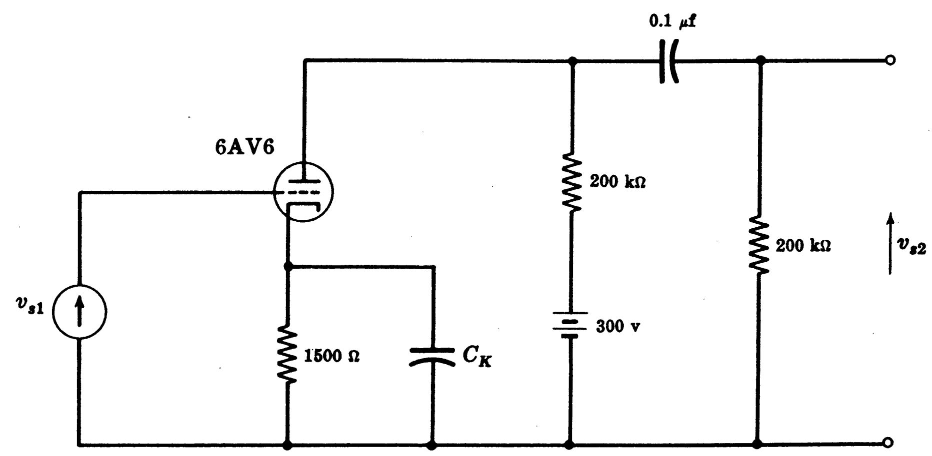


Fig. 5-71

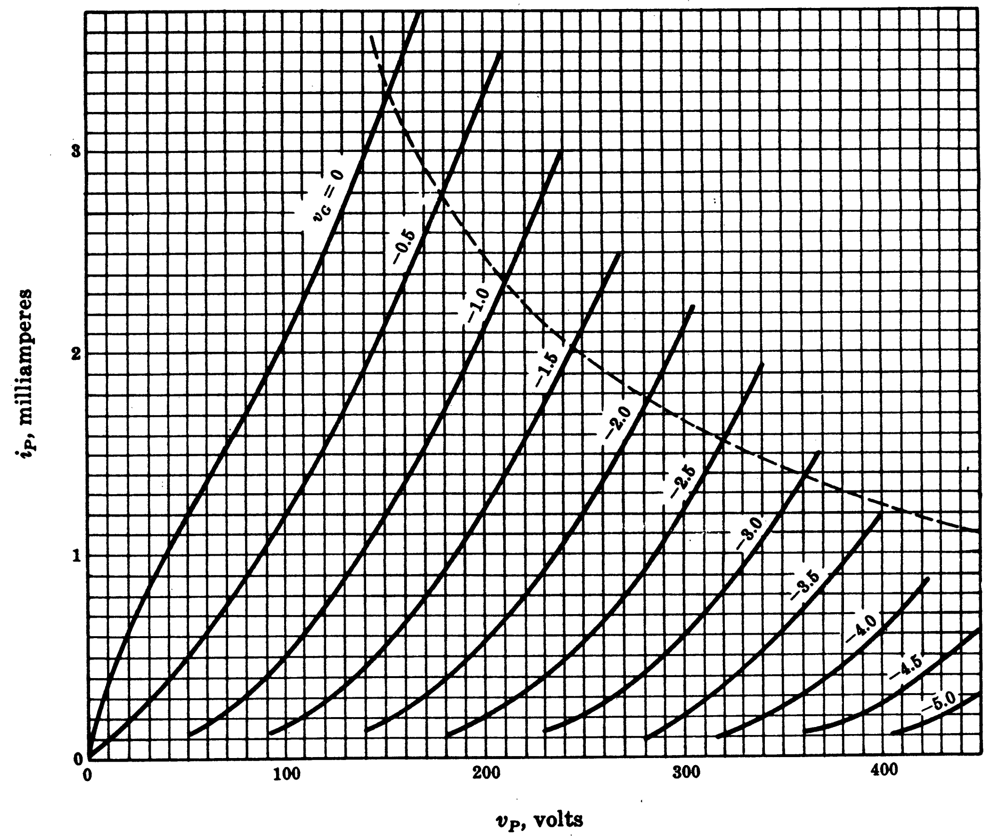


Fig. 5-72. 6AV6 Plate Characteristics

5.11. Given the circuit of Fig. 5-73 and using the characteristic curves of Fig. 5-74, find V_{PP} and V_{GG}. Take $V_P = 150$ volts and $V_G = -4$ volts.

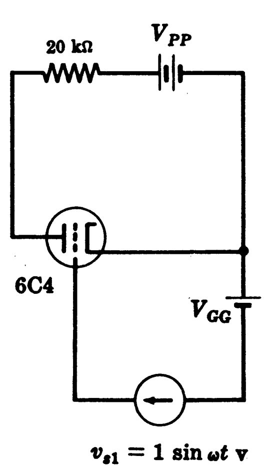


Fig. 5-73

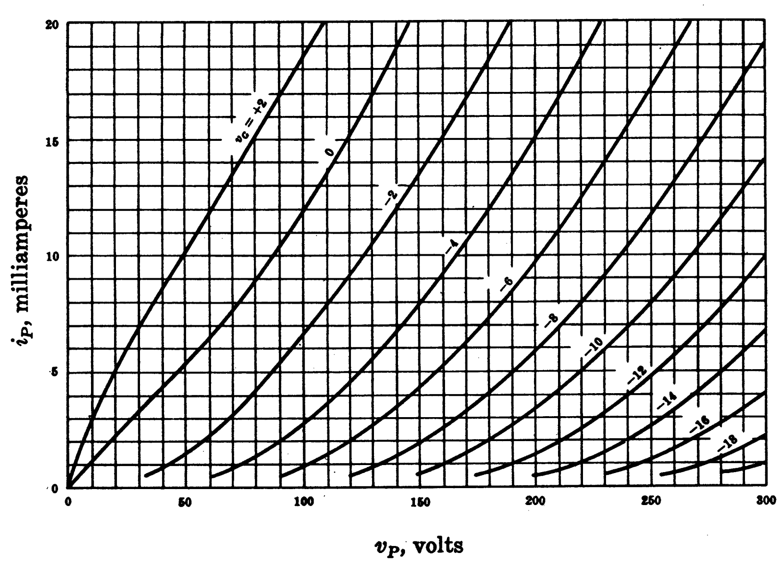


Fig. 5-74. 6C4 Static Plate Characteristic Curves

5.12. Given the circuit of Fig. 5-75 and the plate characteristics of Fig. 5-72. (*a*) Find V_P, I_P and V_G. (*b*) Calculate R_P (static plate resistance). (*c*) Draw the ac equivalent circuit.

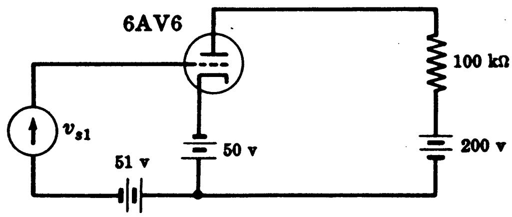


Fig. 5-75

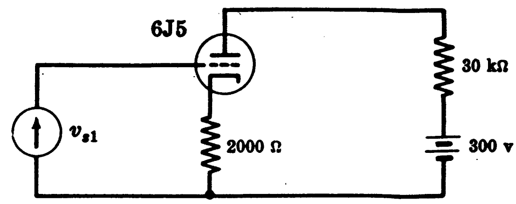


Fig. 5-76

5.13. Given the circuit of Fig. 5-76 and the plate characteristics of Fig. 5-70. (*a*) Find V_P, V_G and I_P. (*b*) Draw the ac equivalent circuit.

5.14. Given the circuit of Fig. 5-77 and the transfer 6C4 characteristic curves of Fig. 5-4, page 70. Find V_P, I_P and V_G. (*Hint.* Use the plate equation even though it does not plot as a straight line on a i_P vs. v_G graph.)

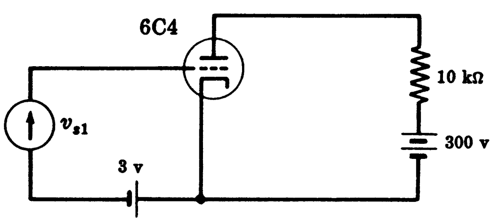


Fig. 5-77

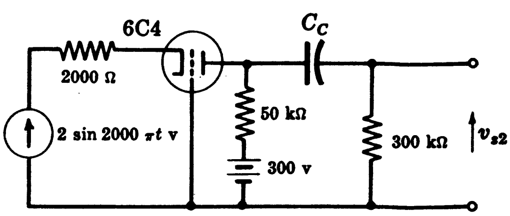


Fig. 5-78

5.15. Given the circuit of Fig. 5-78 and the characteristic curves of Fig. 5-74. (*a*) Find V_P, I_P and V_G. (*b*) Find μ and r_p for this operating point. (*c*) Draw the ac equivalent circuit assuming C_C is large so that the impedance is negligibly small. (*d*) Determine the signal output voltage v_{s2}.

5.16. Refer to the circuit of Fig. 5-79 and the plate characteristic curves of Fig. 5-74. (*a*) Find V_P, I_P and V_G. (*b*) Determine g_m and g_p for the given operating point.

5.17. Given the circuit of Fig. 5-69 (Problem 5.9, page 107) and the plate characteristic curves for a 6J5 tube in Fig. 5-70, page 108. The signal voltage $v_{s1} = 1 \sin \omega t$ volts. (*a*) Graphically find the instantaneous value of the varying component of the plate current i_p and the instantaneous value of the varying component of the plate voltage v_p. (*b*) Using the value of v_p from part (*a*), calculate the voltage gain A_v and compare this value with the one calculated in Problem 5.9(*g*).

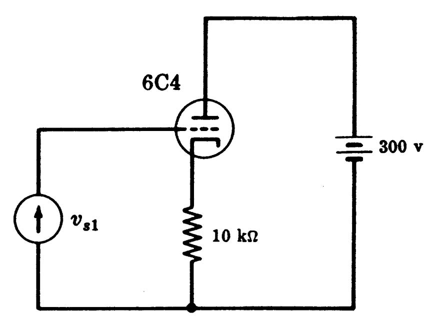


Fig. 5-79

5.18. Refer to the circuit of Fig. 5-80 and the plate characteristics of Fig. 5-74. (*a*) Find I_P, V_P and V_G. (*b*) Find g_m and g_p. (*c*) Plot the ac load line and the dynamic transfer characteristic curve based on the ac load line. (*d*) Determine graphically the maximum amplitude of v_{s1} so that operation is approximately linear. (*e*) Calculate the voltage gain A_v. (*f*) Calculate the power dissipated in the cathode resistor.

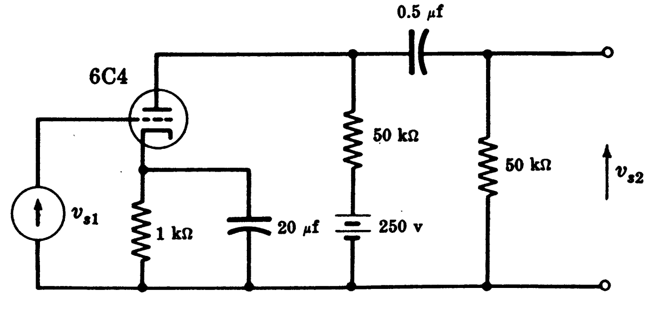


Fig. 5-80

5.46. Given the circuit of Fig. 5-103 and the static plate characteristics of one triode section of a 6SL7 tube in Fig. 5-104. The milliammeter in the plate circuit reads $I_P = 1.48$ ma. What is the value of V_{GG}?

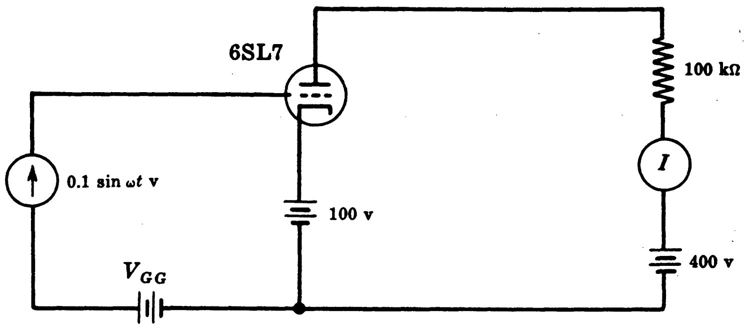


Fig. 5-103

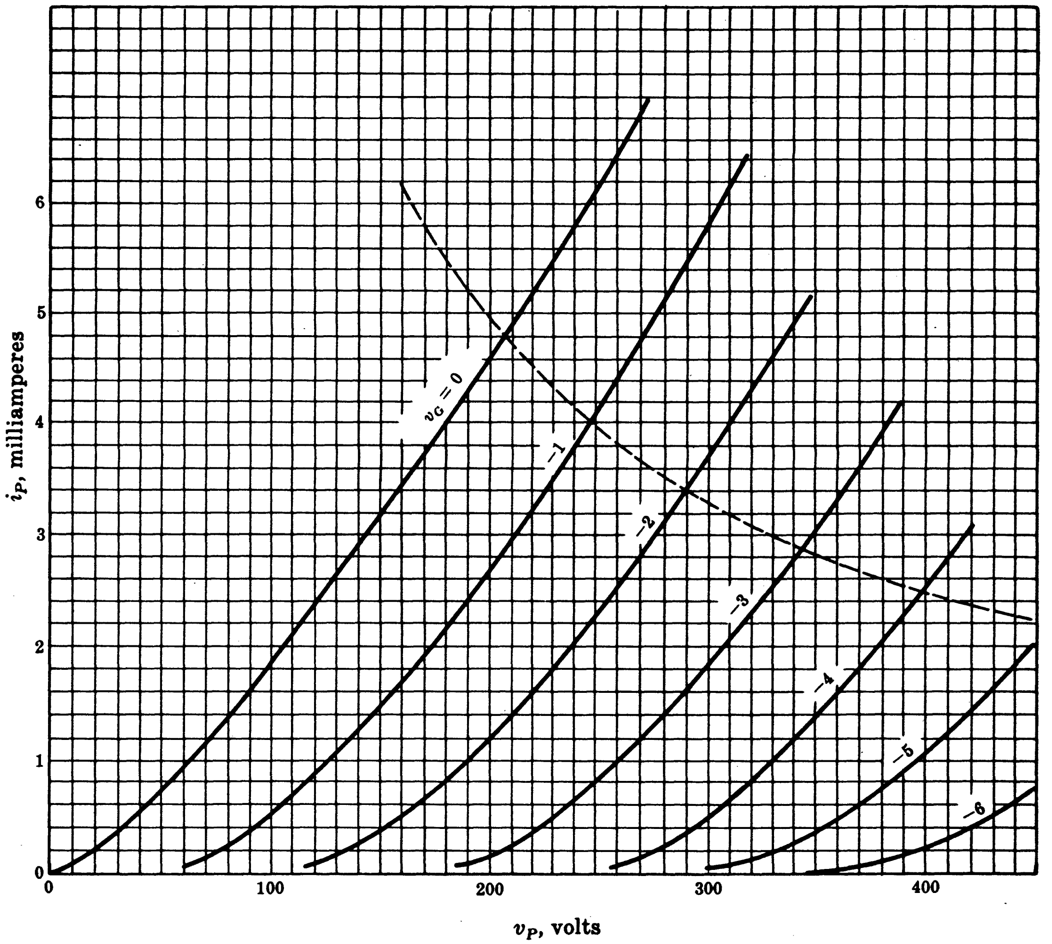


Fig. 5-104. 6SL7 Plate Characteristics

5.47. Given the circuit of Fig. 5-105 below and the 6J5 plate characteristic curves in Fig. 5-70, page 108.

(*a*) The voltage V_G is measured to be −3.0 volts. What is the value of R_K?

(*b*) For the above conditions, find V_P and I_P.

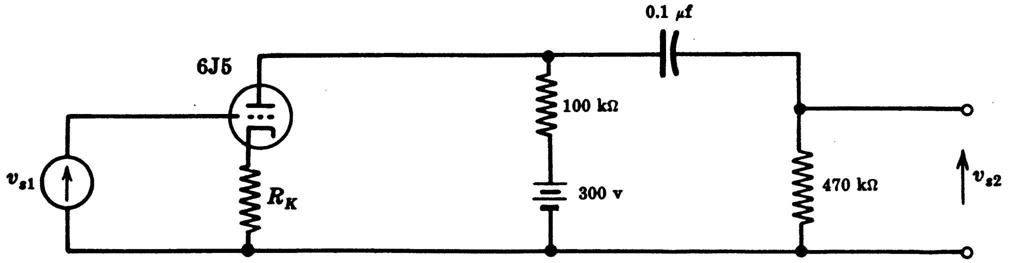


Fig. 5-105

5.48. Given the following equation for the plate voltage of a triode vacuum tube operating on the linear operating range: $v_P = r_p i_P - \mu v_G + k$, where k is a constant. The grid bias for plate current cutoff (plate current approaches zero) is approximately equal to $-v_P/\mu$. Show that this is a valid approximation and indicate the assumptions involved.

5.49. Given the circuit of Fig. 5-106. The two tubes are identical, with $\mu = 20$ and $r_p = 10$ kΩ. Find the overall voltage gain $A_v = v_{s3}/v_{s1}$ (considering varying components only).

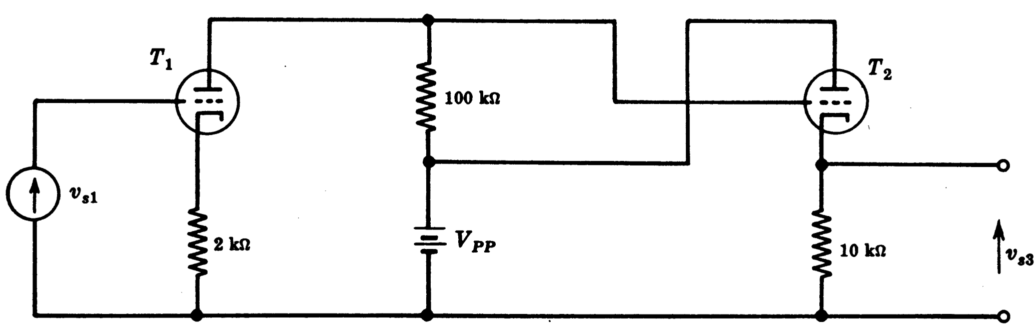


Fig. 5-106

5.50. Two triode tubes are to be connected in parallel. The parameters of the two tubes are

T_1: $g_{m1} = 3000$ μmho, $r_{p1} = 6700$ Ω $\qquad$ T_2: $g_{m2} = 1900$ μmho, $r_{p2} = 8500$ Ω

Find the effective values of the parameters μ, r_p and g_m for the parallel combination.

5.51. Given the circuit of Fig. 5-107 and the parameters for the pentode tube: $g_m = 5100$ μmho, $r_p = 500$ kΩ. Assuming linear operation and mid-band frequencies, calculate (*a*) the voltage gain $A_v = v_{s2}/v_{s1}$ and (*b*) the value of μ for the pentode. (*c*) Assuming the tube parameters remain the same but that R_L is changed from 100 kΩ to 2 kΩ, find the voltage gain.

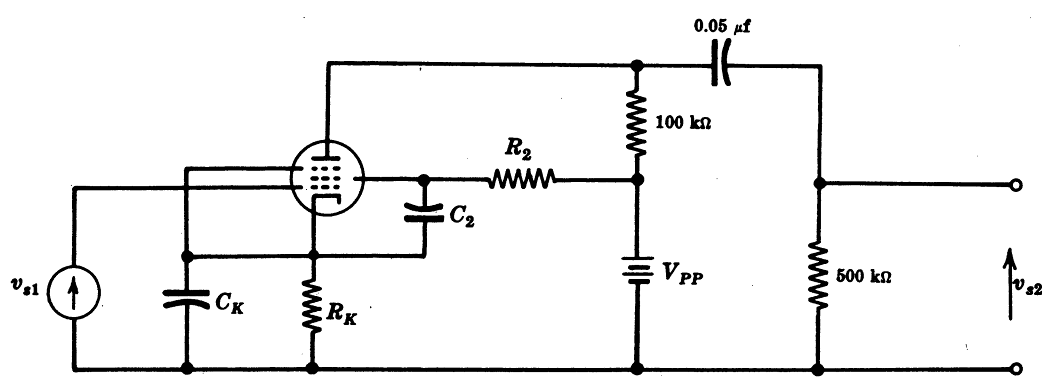


Fig. 5-107