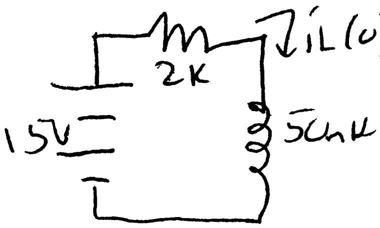
EECS 245 HMWK #9 Solution

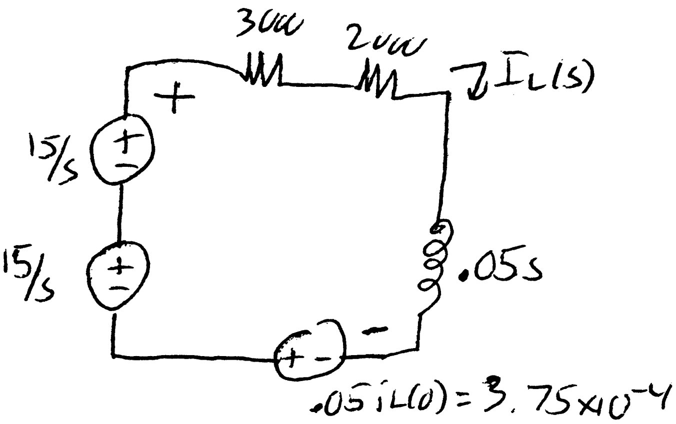
$\boxed{10.15}$

With the switch closed, the circuit reduces to that shown.

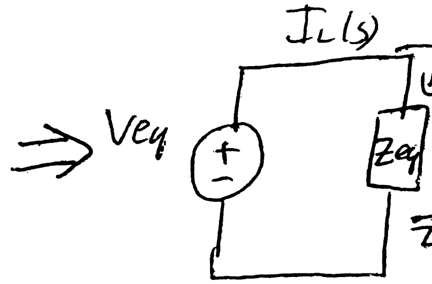


$i_L(0) = \frac{15V}{2000} = \underline{7.5mA}$

s-domain circuit:



$\Rightarrow$



$V_{eq} = \frac{15}{s} + \frac{15}{s} + 3.75\times10^{-4}$

$= \frac{30 + s(3.75\times10^{-4})}{s}$

$Z_{eq} = 3{,}000 + 2{,}000 + .05s$

$= 5{,}000 + .05s$

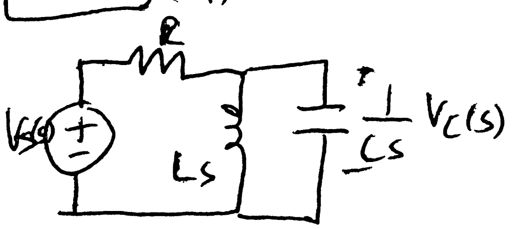
$$I_L(s) = \frac{V_{eq}}{Z_{eq}} = \frac{30 + s(3.75\times10^{-4})}{s(.05s + 5{,}000)} = \frac{600 + .0075s}{s(s + 100{,}000)} = \frac{K_1}{s} + \frac{K_2}{s + 100{,}000}$$

$K_1 s + 100{,}000 K_1 + K_2 s = 600 + .0075s \quad K_1 = \frac{3}{500} \quad K_2 = \frac{3}{2{,}000} \therefore$

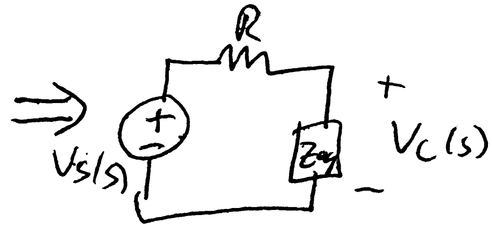
$$\boxed{I_L(s) = s\left(\frac{3}{500}\right) + \left(\frac{3}{2{,}000}\right)\left(\frac{1}{s + 100{,}000}\right)}$$

$$\boxed{i_L(t) = \frac{3}{500}\left(1 + \frac{1}{4}e^{-100{,}000t}\right) A}$$ $\leftarrow$ standard inverse Laplace using table 9-2.

$\boxed{10.21}$ (a)



$\Rightarrow$



$Z_{eq} = Ls \,//\, \frac{1}{Cs}$

$= \frac{Ls\left(\frac{1}{Cs}\right)}{Ls + 1/Cs}$

$= \frac{Ls}{LCs^2 + 1}$

$\Rightarrow$

'0.21 Cont

By voltage division, $V_C(s) = \dfrac{V_s(s)\, Z_{eq}}{R + Z_{eq}}$

$'_C(s) = V_s(s) \dfrac{\frac{Ls}{LCs^2+1}}{R + \frac{Ls}{LCs^2+1}}$

$$\boxed{V_C(s) = V_s(s)\left[\frac{Ls}{RLCs^2+R+Ls}\right]}$$

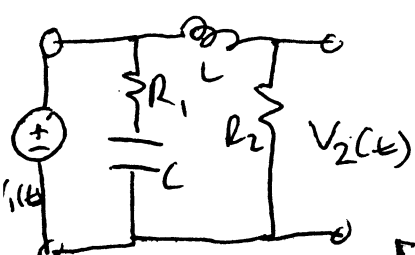
b) $\mathcal{L}[100u(t)] = \frac{100}{s}$ ∴ $V_C(s) = \left(\dfrac{s \quad .05}{1.25\times10^{-5}s^2 + .05s + 5{,}000}\right)\left(\dfrac{100}{s}\right)$

$'_C(s) = \dfrac{4\times10^5}{s^2 + 4000s + 4\times10^8}$ Solving $s^2 + 4{,}000s + 4\times10^8$

$S = -2{,}000 \pm j20{,}000$

$\boxed{\text{Natural poles: } -2{,}000 \pm j20{,}000}$ $\boxed{\text{Forced-poles: none}}$ * The $S=0$ pole caused by the step function is cancelled by a natural zero

1.1 (a) $T_V(s) = \dfrac{V_2(s)}{V_1(s)}$ By simple voltage division, $V_2(s) = \dfrac{R_2}{Ls+R_2} V_1(s)$

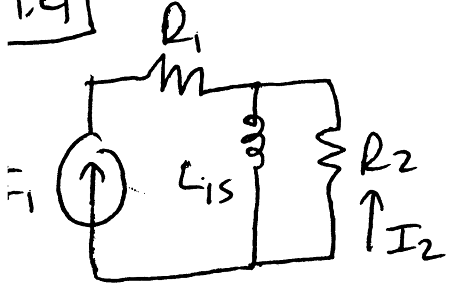


$$\boxed{T_V(s) = \frac{R_2}{Ls+R_2}}$$

(b) $T_V(s) = \dfrac{2000}{.1s + 2000} = \dfrac{20{,}000}{s + 20{,}000}$ ∴

$\boxed{T_V(s) \text{ has a pole at } S = -20{,}000}$

1.4



(a) By current division, $I_2 = \dfrac{-L_1s}{L_1s + R_2} \cdot I_1(s)$ ∴

$$\boxed{T_i(s) = \frac{I_2(s)}{I_1(s)} = \frac{-L_1s}{L_1s + R_2}}$$

(L_2s is shorted out by the short over $V_2(t)$)

(b) $T_i(s) = \dfrac{-.4s}{.4s + 2{,}000} = \dfrac{-s}{s + 5{,}000}$

$\boxed{T_i(s) \text{ has a zero at } s=0 \text{ and a pole at } s=-5{,}000}$

11.27 Just find the transfer functions $T(s) = sG(s)$ by eqn 11-15

(a) $g(t) = -u(t)e^{-2{,}000t}$ $T(s) = sG(s) = s\,\mathcal{L}\{g(t)\}$ $\boxed{T(s) = \frac{-s}{s+2{,}000}}$

(b) $T(s) = sG(s) = s\,\mathcal{L}\{u(t) - u(t)e^{-2{,}000t}\} = s\left(\frac{1}{s} - \frac{1}{s+2{,}000}\right)$

$\boxed{T(s) = \frac{2{,}000}{s+2{,}000}}$

11.28 $T(s) = H(s)$ by eqn. 11-14

(a) $\boxed{H(s) = \frac{-1{,}000}{s+2{,}000} = T(s)}$ (b) $H(s) = 1 - \frac{2{,}000}{s+2{,}000}$ $\boxed{T(s) = \frac{s}{s+2{,}000}}$

11.37 $G(s) = \mathcal{L}[e^{-1{,}000t}]u(t) = \frac{1}{s+1{,}000}$ $T(s) = sG(s) = \frac{s}{s+1{,}000}$

$x(t) = 5\cos 2{,}000t$ by eqn 11-21:

Amplitude $= 5\,|T(j\cdot 2{,}000)| = 5\left|\frac{j\cdot 2{,}000}{j\cdot 2{,}000 + 1{,}000}\right|$ $\boxed{\text{Amplitude} = 4.472}$

phase angle $= \phi + \theta = 0 + \text{angle}\left(\frac{j\cdot 2{,}000}{j2{,}000+1{,}000}\right)$ $\boxed{\phi = 26.565°}$

11.40 $h(t) = [400e^{-100t} - 400e^{-5{,}000t}]u(t)$ $H(s) = T(s) = 400\left(\frac{1}{s+100} - \frac{1}{s+5{,}000}\right)$

$T(s) = \frac{1.96\times10^6}{(s+100)(s+5{,}000)}$ Amplitude $= 5\left|\frac{1.96\times10^6}{(700j+100)(700j+5{,}000)}\right| = \boxed{2.745}$

$\phi = \text{angle}\left|\frac{1.96\times10^6}{(700j+100)(700j+5{,}000)}\right|$ $\boxed{\phi = -89.84°}$