<File: Final Exam S'02>

EECS 245 - SPRING 2002

FINAL EXAM– 5/3/02

NAME: ________________ CWRUnet e-mail address:____________

Solutions

51 took exam

IMPORTANT INFORMATION:

1. All questions are worth the same.
2. Exam is closed book, closed notes. Calculators are allowed.
3. There are tables of transistor data and Laplace transform properties at the end of this exam.

		Possible
1.		25
2.		25
3.		25
4.		25
5.		25
6.		25
7.		25
8.		25
SCORE		200

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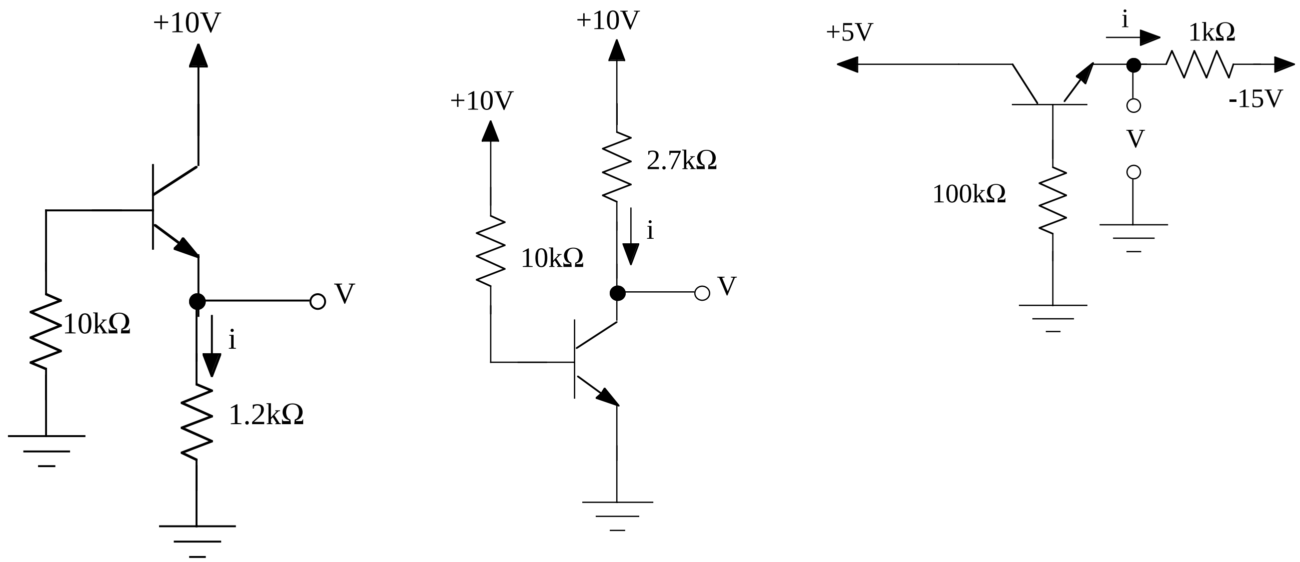
1. BJTS (large signal)

Find I and V in the circuits shown below. For all transistors, assume that β =100 and $|V_{BE}|$=0.7 volts in both the active and saturation regions.



(a) (b) (c)

SOLUTIONS:

(a) By inspection V_{BE} is NOT forward biased. That means that this transistor is cutoff. Since the transistor is cutoff i=0 and V also is zero.

(b) This one appears to have the base emitter junction forward biased. Calculating the base current gives $-10 + i_B(10k\Omega) + V_{BE} = 0$ or $i_B = \dfrac{10-0.7}{10k\Omega} = 0.93mA$. This would give a collector current of $i_C = \beta i_B = 100(0.93mA) = 93mA$ which is quite large. The maximum collector current is $i_{C,MAX} = \dfrac{10 - V_{CE,sat}}{2.7k\Omega} = \dfrac{10-0.2}{2.7k\Omega} = 3.63mA$ so this transistor is clearly saturated. In this saturated mode V=0.2 volts and i=3.63mA.

(c) This amplifier is drawn funny but is a perfectly reasonable amplifier. Doing KVL around the base circuit gives $i_B(100k\Omega) + V_{BE} + (\beta+1)i_B(1k\Omega) - 15V = 0$. Solving for i_B gives

$i_B = \dfrac{15-0.7}{100k\Omega + (\beta+1)1k\Omega} = \dfrac{14.3}{201k\Omega} = 0.071mA$. Under these conditions

$i_C = \beta i_B = 100(0.071mA) = 7.11mA$. The maximum current is given by

$i_{C,MAX} = \dfrac{5+15-0.2}{1k\Omega} = \dfrac{19.8}{1k\Omega} = 19.8mA$ so the transistor is clearly active. Since

$i_E = (\beta+1)i_B = (100+1)(0.071mA) = 7.171mA$ we have

$V = i_E R_E - 15 = (7.171mA)(1k\Omega) - 15 = 7.171 - 15 = -7.83 volts$

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GRADING

(a) V and I were worth 4 points each.

The most common errors were (1) not detecting that the transistor was in cutoff and (2) V=10 instead of V=0 since transistor is cutoff

(b) V and I were worth 4 points each.

–6 for not detecting that transistor was saturated.

-1 for using 0 instead of 0.2 volts for saturated transistor

The most common error was similar to that in (a) — saying the output V was 10 instead of 0 or 0.2 volts.

(c) V was worth 5 points and I was worth 4 points.

Many of you thought this was cutoff since there was no explicit base power supply. This circuit is very similar to that used in lab 9a where you had to bias the transistor modulator using a – supply on the emitter.

Some of the more common errors were

-6 for not including R_E in your calculations

-3 for not including the $\beta+1$ term for RE in your calculations

-2 for not calculating V including the –15 volt supply, i.e., V=7.17-15=-7.83 volts

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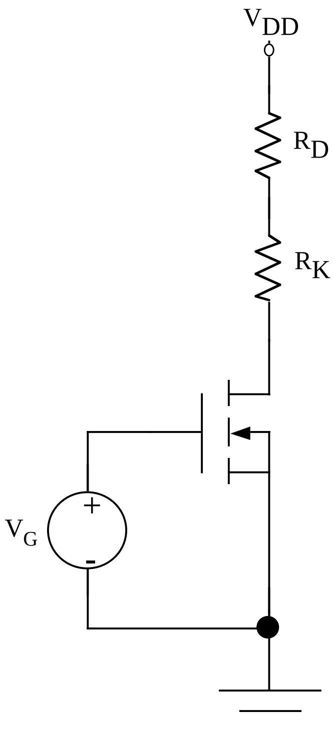
2. MOSFETS (DC CHARACTERISTICS)

The amplifier circuit shown below uses a n-channel MOSFET with k=2.5 mA/V^2 and V_T=2 volts. The circuit parameters are R_K=1kΩ, R_D=10kΩ, and V_{DD}=18 volts.

(a) For the circuit shown below, determine V_{DS}, V_{GS} and V_G for the transistor to operate at I_D=1.5 mA.



ANSWER: These are straight forward problems using $I_D = K\left(V_{GS} - V_{TR}\right)^2$ Using the given values $1.5mA = 2.5\frac{mA}{V^2}\left(V_{GS} - 2\right)^2$ which can be solved to give

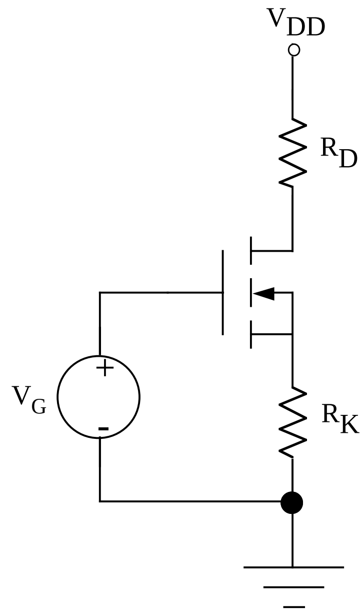
$$V_{GS} = \sqrt{\frac{1.5}{2.5}} + 2 = 2.77\ volts$$

For the drain circuit

$$V_{DS} = V_{DD} - I_D\left(R_D + R_K\right)$$
$$= 18\ volts - 1.5mA\left(10k\Omega + 1k\Omega\right)$$
$$= 1.5\ volts$$

By inspection, V_G=V_{GS}=1.5 volts.

(b) For the re-arranged circuit shown below, determine V_{DS}, V_{GS} and V_G for the transistor to operate at I_D=1.5 mA.



ANSWER:
The values of V_{GS} and V_{DS} do NOT change at all. However, using KVL we get $V_G = V_{GS}+V_S = V_{GS}+I_DR_K$ = 2.77 volts + 1.5mA(1kΩ) = 2.77 + 1.5 volts = 4.27 volts.

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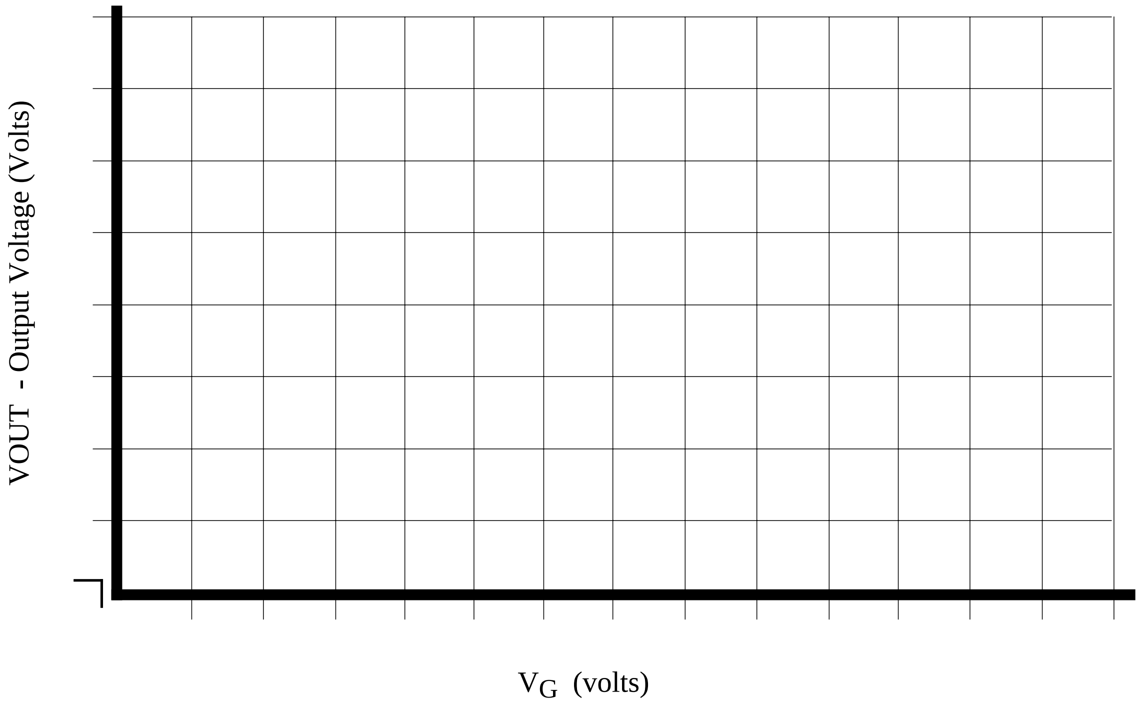
The following questions refer to the circuit and transistor parameters of part (a) where $V_{OUT}=V_D$.

(c) Determine the voltage transfer curve V_{OUT} vs. V_G.



ANSWER:

For the given transistor parameters $I_D = 2.5\frac{mA}{V^2}\left(V_{GS} - 2\right)^2$ and

$V_{OUT} = V_{DD} - I_D\left(R_D + R_K\right) = 18\ volts - I_D\left(11k\Omega\right)$. For the given parameters it is easiest to construct a table of values to plot.

V_{GS}	I_D	V_{OUT}
<2.0 volts	0 mA	18 volts
2.2 volts	0.1 mA	16.9 volts
2.4 volts	0.4mA	13.6 volts
2.5 volts	0.625 mA	11.125 volts
2.75 volts	1.40 mA	2.53 volts
2.80 volts	1.60 mA	0.4 volts
3.0 volts	2.5 mA	0

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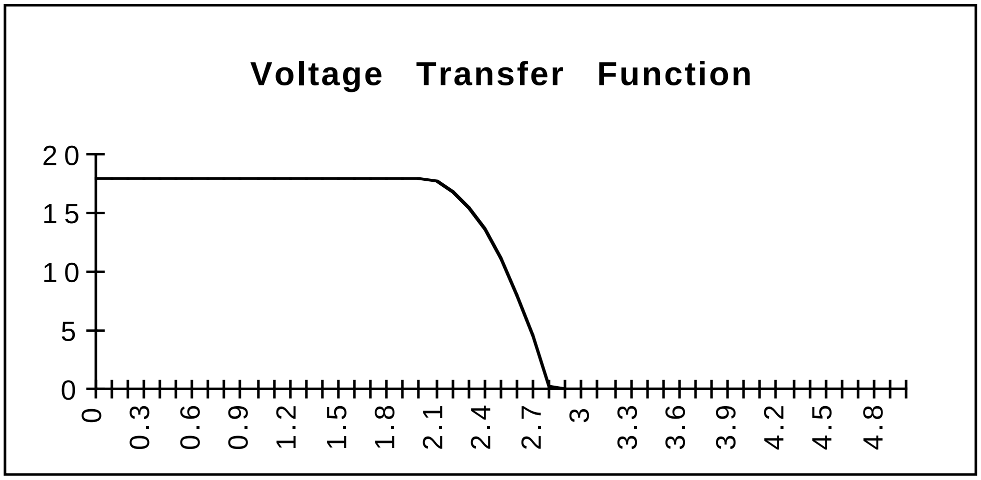
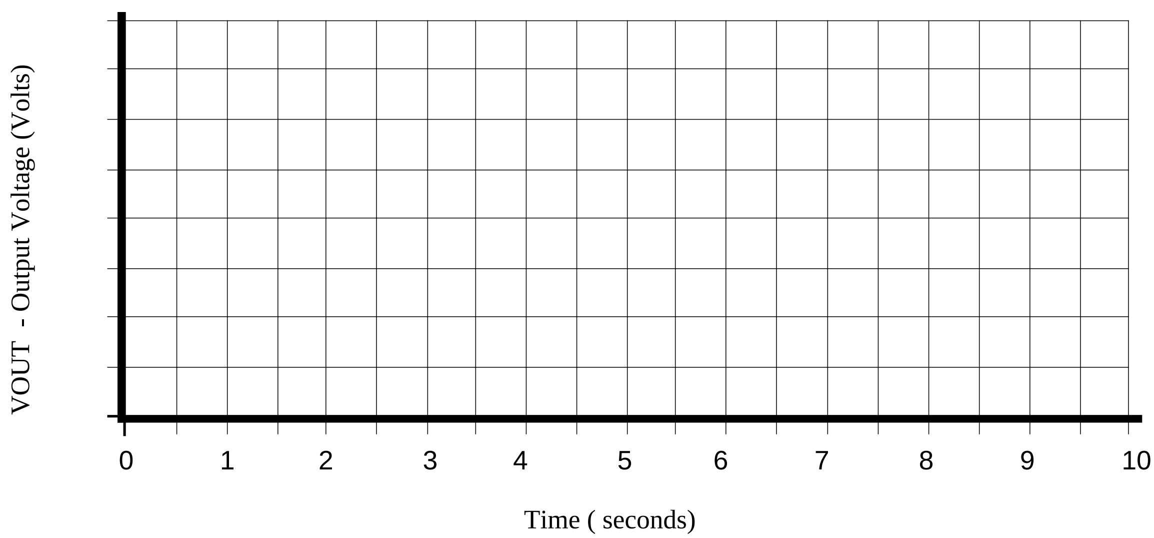


(d) Using your graph from (c) estimate the voltage gain of the circuit, i.e. $\Delta V_{OUT}/\Delta V_G$, for V_G=2.4 volts?

The gain is hard to estimate accurately but is is about

$$\frac{\Delta V_{out}}{\Delta V_{in}} \approx \frac{11.125 - 16.9}{2.5 - 2.2} = \frac{-5.775}{0.3} = -19.25$$

(e) Sketch the output of the circuit for a linear ramp that rises from zero to three volts in 10 seconds.



ANSWER

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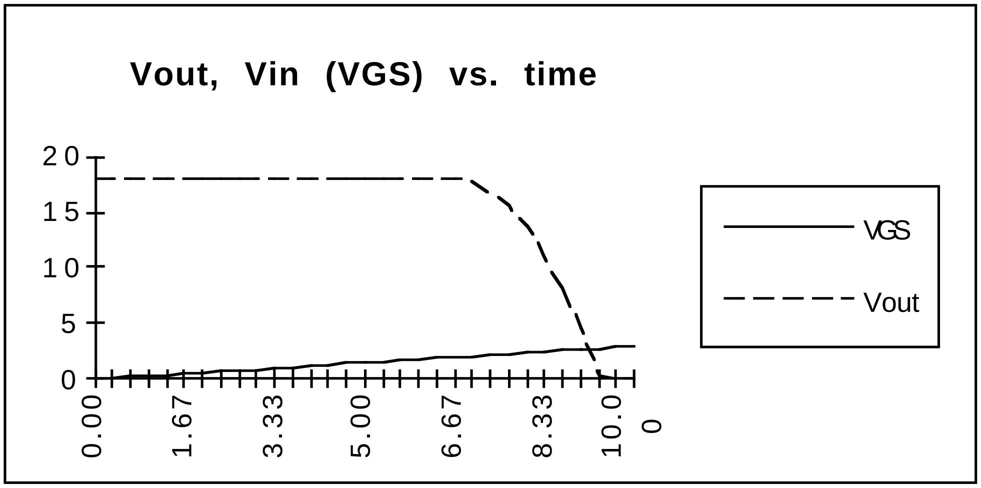

As you can see this is not a great amplifier over this range of input voltage, but might be a good logic switch.

GRADING
(a) V_{DS}, V_{GS} and V_G were worth 2 points each
(b) V_{DS}, V_{GS} and V_G were worth 2 points each
(c) The overall graph was worth 6 points.
Several students plotted Vout as rising to a maximum and then dropping off to zero. I am assuming you were confused by the square in the expression for I_D. However, the MOSFET does not behave that way – remember we typically get two values of I_D but one of them is not realizable because it corresponds to V_{GS} less than V_{TO}. In this case the transistor cannot be on for $V_{GS}<V_{TO}=2$ volts so the transistor must be off, i.e, $V_{OUT}=18$ volts until you reach V_{TO}. You lost –3 for having an output for $V_G<V_{TO}$.
(d) The calculation of the voltage gain was worth 3 points. The definition is simply the slope of the $\Delta V_{OUT}/\Delta V_G$ curve. You lost 2 points for calculating $\Delta V_{OUT}/\Delta V_G$, and you lost 1 point for not correctly getting the sign right — the slope at $V_G=2.4$ volts is negative.
(e) This graph was worth 3 points. Since this graph was based upon your results from (c) I did not take points off twice. If you got a V_{OUT} which seemed to correspond to the input using your graph from (c) I gave you full credit. The most common error was simply plotting V_G as a function of time. This is the simple ramp as shown above and you received no credit for this.

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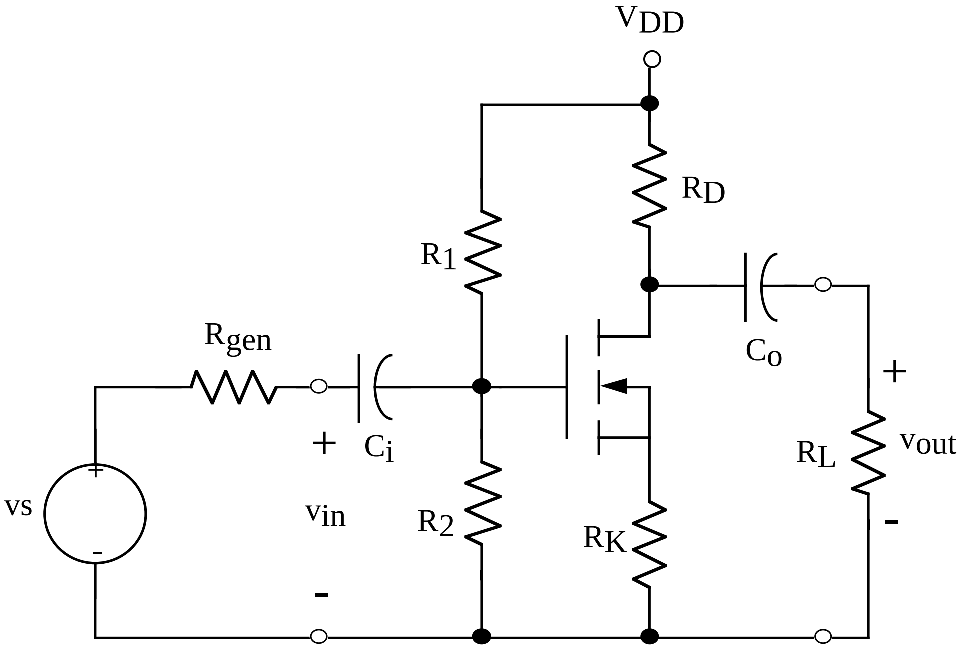
3. MOSFETs (AC AMPLIFIER)

The amplifier circuit shown below uses an enhancement mode MOSFET with k=2.7 mA/V^2 and V_T=2.5 volts operating at I_{DQ}=1.2 mA. The circuit parameters are R_{gen}=600Ω, R_K=1kΩ, R_L=10kΩ, R_1=1.285MΩ, and R_2=237kΩ. You may neglect r_d for the MOSFET.

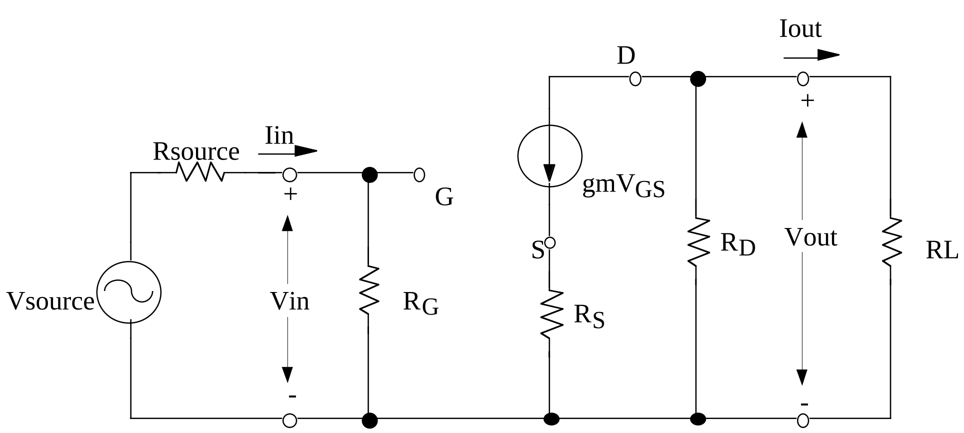


(a) Draw a small signal equivalent circuit for this amplifier indicating the values of all small signal parameters. Note that R_K is NOT bypassed.

(b) Derive an expression for the small signal gain $A_v = \dfrac{V_{out}}{V_{in}}$ of this amplifier.

ANSWER:

(a) The small signal model is



I combined R_1 and R_2 into the single bias resistor R_G since both go to ground. All the circuit paramerers are known except for R_G and g_m from the previous circuit diagram. The bias

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resistor is given by $R_G = \frac{R_1 R_2}{R_1 + R_2} = \frac{(1285k\Omega)(237k\Omega)}{(1285k\Omega)+(237k\Omega)} = 200k\Omega$. g_m can be computed as

$$g_m = 2\sqrt{KI_{DQ}} = 2\sqrt{\left(2.7\frac{ma}{V^2}\right)(1.2mA)} = 0.0036S$$

To determine the small voltage gain we first need to determine V_{GS}. We can do KVL around the loop defined by V_{in}, V_{GS} and R_S, i.e., $-V_{in} + V_{GS} + (g_m V_{GS})R_K = 0$. This gives $V_{GS} = \frac{V_{in}}{1+g_m R_K}$. The output voltage is simply given as $V_{out} = -(g_m V_{GS})(R_D \parallel R_L)$. These two results can be combined to give $V_{out} = -(g_m)\left(\frac{V_{in}}{1+g_m R_K}\right)(R_D \parallel R_L)$ or

$$\frac{V_{out}}{V_{in}} = -\frac{g_m}{1+g_m R_K}(R_D \parallel R_L).$$

GRADING:
(a) The small signal equivalent circuit was worth 10 points. The most common error was putting RK in parallel with RD and RL for which you typically lost 4 points. Other students improperly connected RK to the gate. Another common error was improperly calculating gm. Many of you did not properly compute the square root for which you lost 1 point.
(b) The second part of the problem was worth 15 points but was very hard to grade. Determining that V_{GS} was not equal to V_{in} was worth 8 points. You lost varying amounts of point depending upon what you did. Determining V_{OUT} in terms of V_{GS}, R_D and R_L was worth 7 points. The most common error was simply setting V_{GS}=V_{in}. Again you lost varying amounts of points depending upon how you set up and solved your equations.

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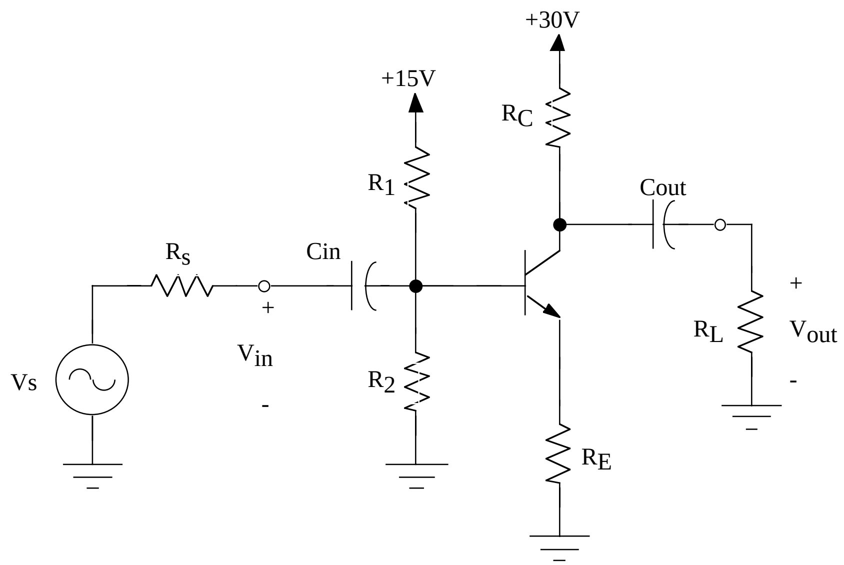
4. SMALL SIGNAL BJT ANALYSIS



Consider the above BJT amplifier where R_S=500Ω, R_1=10kΩ, R_2=10kΩ, R_E=2000Ω, R_L=10kΩ and Rc=5kΩ. The transistor is characterized by β=100. The amplifier is biased such that $I_{C,Q}$=3.3mA. You may assume that C_{IN} and C_{OUT} have a low impedance at mid-frequency and that r_o is so large it can be neglected.

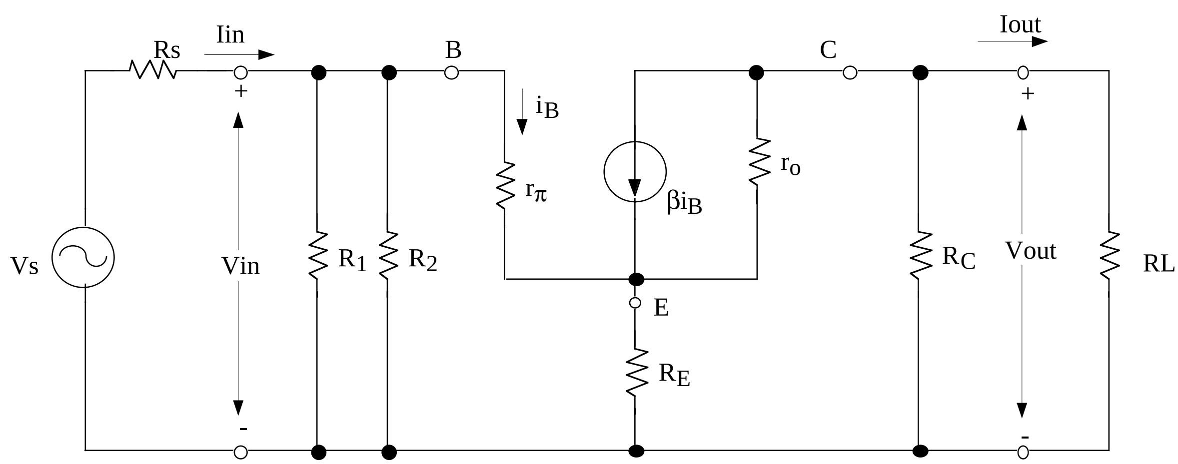
(a) Draw the small-signal equivalent circuit for this BJT small signal amplifier at mid-frequency. Indicate the values of all small signal parameters in your circuit.

(b) Calculate R_{in} for this amplifier.

(c) What is the voltage gain A_v for this amplifier.

ANSWER:

(a)



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All small signal circuit parameters except r_π are known. r_o can be neglected. r_π can be calculated as $r_\pi = \frac{\beta V_T}{I_{C,Q}} = \frac{(100)(26mV)}{(3.3mA)} = 788\Omega$

(b)

Many of you assumed that the bias resistor equivalent resistance $R_B = R_1 \| R_2 = \frac{(R_1)(R_2)}{R_1 + R_2} = \frac{(10k\Omega)(10k\Omega)}{10k\Omega + 10k\Omega} = 5k\Omega$ was so large that it could be neglected in this problem. That was not so since it is only 5000 ohms.

<u>Here is the solution if you ignored the bias resistors.</u>

We use the definition of input resistance $R_{in} = \frac{V_{in}}{I_{in}}$. The input voltage can be calculated as $V_{in} = i_B r_\pi + (\beta+1) i_B R_E = i_B\left(r_\pi + (\beta+1)R_E\right)$ and, when substituted into the expression for R_{in}, gives $R_{in} = \frac{V_{in}}{I_{in}} = \frac{i_B\left(r_\pi + (\beta+1)R_E\right)}{I_{in}} = r_\pi + (\beta+1)R_E$ or, numerically, $R_{in} = 202788\Omega$

<u>Here is the solution which properly included the bias resistors.</u>

We continue to use the definition of input resistance $R_{in} = \frac{V_{in}}{I_{in}}$. The input current is now calculated as $I_{in} = \frac{V_{in}}{R_1 \| R_2} + i_B$. The corresponding input voltage can be calculated as $V_{in} = i_B r_\pi + (\beta+1) i_B R_E = i_B\left(r_\pi + (\beta+1)R_E\right)$. Solving for i_B gives $i_B = \frac{V_{in}}{r_\pi + (\beta+1)R_E}$ which can be substituted into the expression for V_{in} to give $I_{in} = \frac{V_{in}}{R_1 \| R_2} + \frac{V_{in}}{r_\pi + (\beta+1)R_E}$. Using this expression in that for R_{in} gives $R_{in} = \frac{V_{in}}{I_{in}} = \frac{V_{in}}{\frac{V_{in}}{R_1 \| R_2} + \frac{V_{in}}{r_\pi + (\beta+1)R_E}} = \frac{1}{\frac{1}{R_1 \| R_2} + \frac{1}{r_\pi + (\beta+1)R_E}}$ which can be recognized as $R_{in} = (R_1 \| R_2) \| \left(r_\pi + (\beta+1)R_E\right)$.

Numerically this gives $R_{in} = (5k\Omega) \| \left(788 + (100+1)2000\right) = (5k\Omega) \| (202788\Omega) = 4880\Omega$ compared to the above result of $R_{in} = 202788\Omega$

(c) Note that i_B is independent of R_{in} and is given by $i_B = \frac{V_{in}}{r_\pi + (\beta+1)R_E}$. The collector current is then $i_C = \beta i_B = \beta \frac{V_{in}}{r_\pi + (\beta+1)R_E} = \frac{\beta}{r_\pi + (\beta+1)R_E} V_{in}$. The output voltage is then

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$V_{out} = -i_C(R_C \parallel R_L) = -\frac{\beta}{r_\pi + (\beta+1)R_E} V_{in}(R_C \parallel R_L)$. Solving for the voltage gain gives

$$\frac{V_{out}}{V_{in}} = -\frac{\beta(R_C \parallel R_L)}{r_\pi + (\beta+1)R_E}.$$

Numerically,

$$\frac{V_{out}}{V_{in}} = -\frac{\beta(R_C \parallel R_L)}{r_\pi + (\beta+1)R_E} = -\frac{100(5000\Omega \parallel 10000\Omega)}{788\Omega + (100+1)2000\Omega} = -1.64$$

GRADING:
Again this was a very difficult problem to grade.
(a) You received 9 points for a properly drawn small signal equivalent circuit. You lost 3 points each for g_m and r_π if you did not include them. You lost 1 point for not calculating each one, or improperly calculating the,. NOTE you could have used $g_m V_\pi$ instead of βi_b in your small signal equivalent circuit.

(b) The input resistance was worth 6 points. You lost 1 point if you failed to include the bias resistance $R_1 \parallel R_2$ in your calculations. You typically lost 1 point for R_E without the $(\beta+1)$ term. A number of students said that the input resistance was $R_1 \parallel R_2$ without any reference to the $r_\pi + (\beta+1)R_E$ term. Since this statement is true I was unable to take off any points even though you might have never considered the $r_\pi + (\beta+1)R_E$ resistance.

(c) The voltage gain was worth 10 points. There was an unbypassed emitter resistance which complicated the problem. Most people were able to write an expression of the form $V_{OUT} = -g_m V_{be} R_L'$ for the output voltage but you were unable to determine what V_{be} would be to compute the gain. You lost 3-6 points for that depending upon your detailed calculations.

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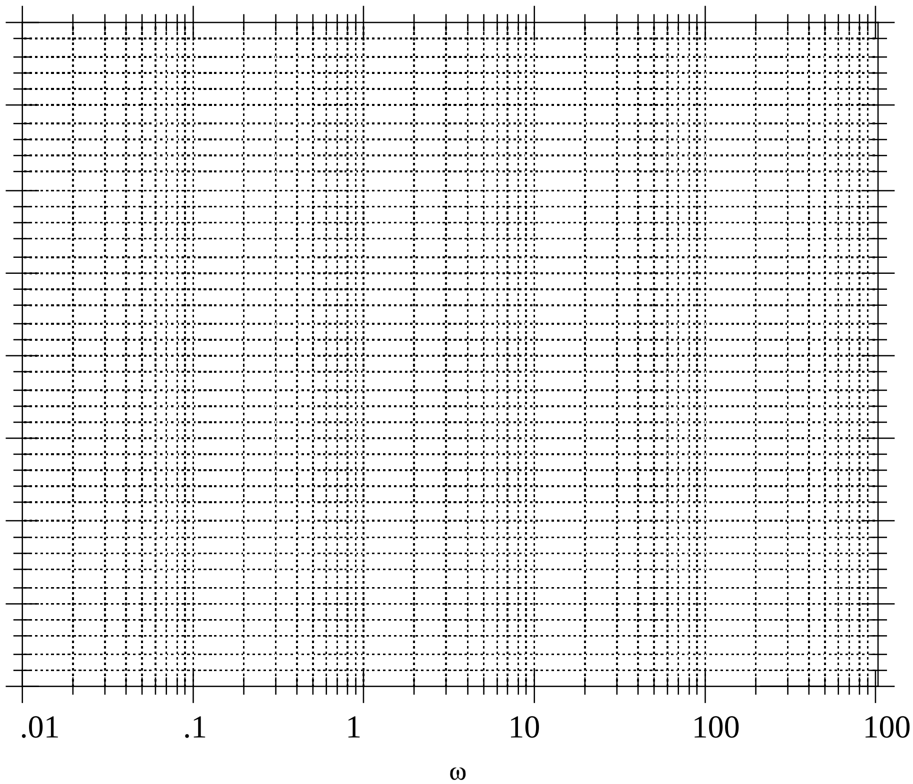
5. FREQUENCY RESPONSE & BODE PLOTS

Plot |H(jω)| given that

$$H(s) = \frac{10s}{(1+s)(1+\frac{s}{10})}$$



SOLUTION:

This problem is very similar to many of the homework problems. The only expression that gave students trouble was the zero at $\omega = 0$. Evaluating the transfer function at $s = j\omega$ and taking the logarithm gives

$$20\log|H(j\omega)| = +20\log|10| + 20\log|j\omega| - 20\log|1+j\omega| - 20\log\left|1+\frac{j\omega}{10}\right|$$

Since the first term is a constant we have

$$20\log|H(j\omega)| = +20 + 20\log|j\omega| - 20\log|1+j\omega| - 20\log\left|1+\frac{j\omega}{10}\right|$$

These terms can be plotted as shown below.

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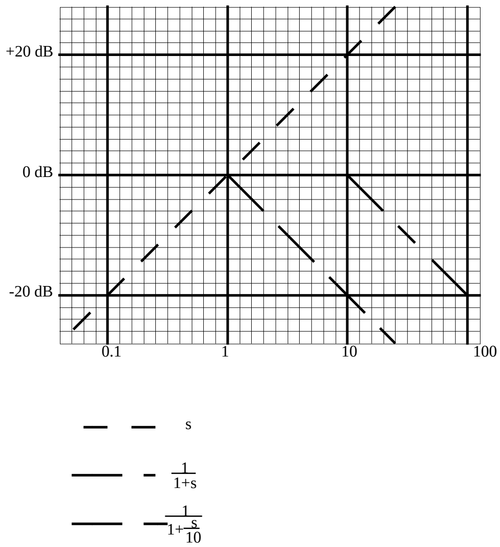

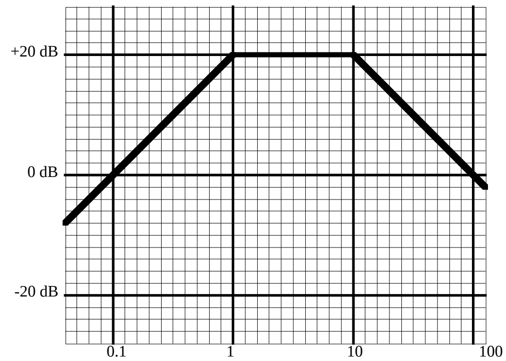

COMMENTS:

You lost 20 points if you simply did a partial fraction expansion with no plots.

You lost 3 points if you had the peak of the passband at 60dB. This resulted from not understanding how to plot the log|1+jw| term.

Two very major errors also resulted from not understanding how to plot the terms. Two common errors resulted in a high pass characteristic (-15 points) and a low pass characteristic (-10 points).

Other errors resulted in:

-5 log(20ω) was plotted as a constant

-10 didn't properly add terms together for plot

-10 did filter characteristic by plugging in values of ω. Typically did not plot vertical in decibels.

For interested people, here is a old style PSpice simulation of the circuit response:

```
Final exam problem 5
Vin 1 0 ac 1
Rin 1 0 1
E1 2 0 laplace {v(1)}={10*s/((1+s)*(1+s/10))}
R1 2 0 1
.ac dec 40 .01 100K
.probe
.end
```

Plot of solution

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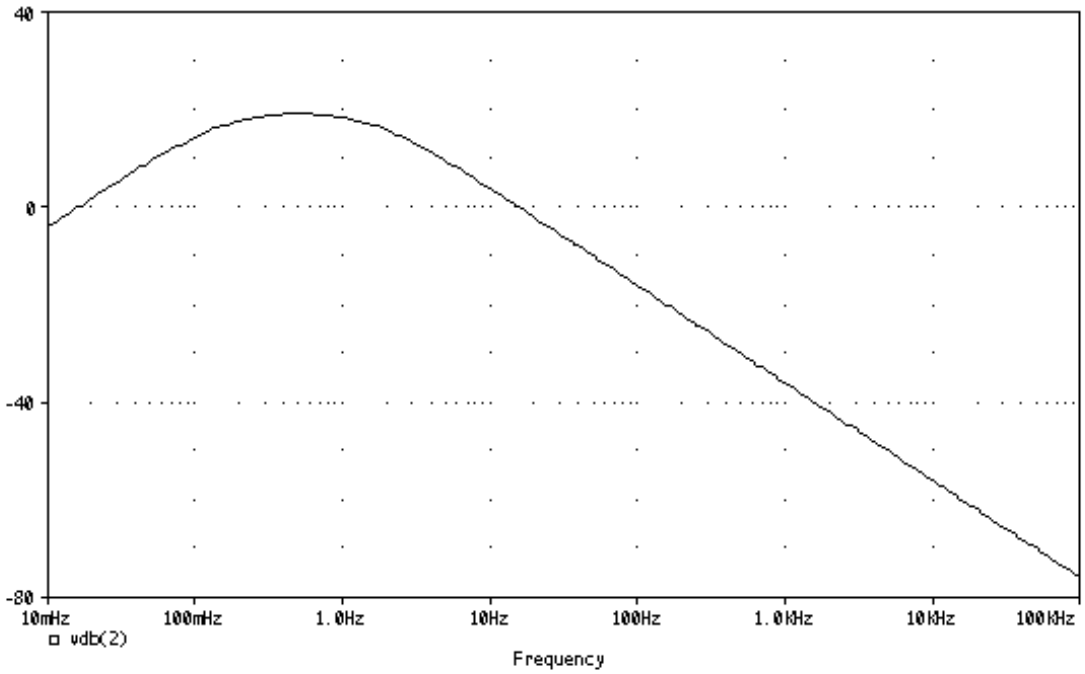

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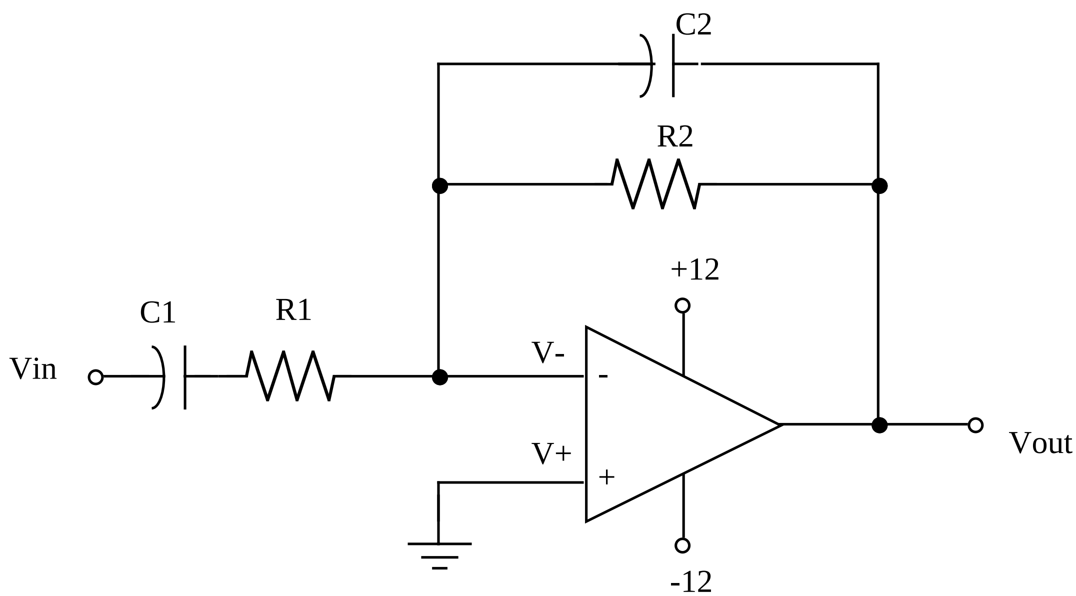
6. s-DOMAIN CIRCUIT ANALYSIS

The operational amplifier shown in the circuit below is ideal. There is no energy stored in the circuit at the time it is energized, i.e. the initial conditions for all capacitors is zero.



(a) What is the transfer function of the amplifier in the s-domain. You can leave your answer in terms of R_1, R_2, C_1 and C_2.

ANSWER:

You can start at any point including basic principles. If I call the impedance of $R_2 \| C_2$ Z_f and the impedance of R_1 in series with C_1 Z_i we can solve the problem using KCL at the input of the op-amp. Using KCL at the input we have

$$+\frac{V_{in}-0}{Z_i}+\frac{V_{out}-0}{Z_f}=0$$

where + indicates current into the node. Solving for vo:

$$V_{out}=-\frac{Z_f}{Z_i}V_{in}$$

Now solving for the impedances:

$$Z_f=R_2 \,\|\, \frac{1}{sC_2}=\frac{(R_2)\frac{1}{sC_2}}{R_2+\frac{1}{sC_2}}=\frac{R_2}{1+sR_2C_2}$$

and

$$Z_i=R_1+\frac{1}{sC_1}$$

The overall transfer function in the s-domain becomes

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$$v_o = -\frac{Z_f}{Z_i} v_g = -\frac{\dfrac{R_2}{1 + sR_2C_2}}{\dfrac{1 + sR_1C_1}{sC_1}} v_g = -\frac{sR_2C_1}{(1 + sR_2C_2)(1 + sR_1C_1)} v_g$$

$$T(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{Z_f}{Z_i} = -\frac{\dfrac{R_2}{1+sR_2C_2}}{R_1 + \dfrac{1}{sC_1}} = -\frac{\dfrac{sR_2C_1}{1+sR_2C_2}}{1+sR_1C_1} = -\frac{sR_2C_1}{(1+sR_2C_2)(1+sR_1C_1)}$$

(b) What is the Laplace transform of the input $v_g(t)$=tu(t) volts?

ANSWER:
Since the input $v_g(t)$=tu(t), the Laplace transform of the input signal is:
$$tu(t) \leftrightarrow \frac{1}{s^2}$$

(c) Suppose the transfer function of the amplifier turns out to be $\frac{V_o(s)}{V_g(s)} = -\frac{200}{s}$, what is $v_o(t)$ for the input $v_g(t)$=tu(t) volts? HINT: You may need to use the list of operational transforms provided.

ANSWER:
By definition of the transfer function, the output voltage is given by
$$V_o(s) = \frac{V_o(s)}{V_g(s)} V_g(s) = -\frac{200}{s} \times \frac{1}{s^2} = -\frac{200}{s^3}$$
The inverse transform is then given using the attached tables as:
$$u(t) \leftrightarrow \frac{1}{s}$$
Using the result for tu(t) and the derivative of 1/s we have
$$tf(t) = tu(t) \leftrightarrow -\frac{dF(s)}{ds} = -\frac{d}{ds}\left(\frac{1}{s}\right) = -\left(-\frac{1}{s^2}\right) = \frac{1}{s^2}$$
Continuing one more time we have the desired result
$$tf(t) = t\left[t^2u(t)\right] \leftrightarrow -\frac{dF(s)}{ds} = -\frac{d}{ds}\left(\frac{1}{s^2}\right) = -\left(-\frac{2}{s^3}\right) = \frac{2}{s^3}$$

Using this to invert $-\frac{200}{s^3}$, we get $v_o(t) = -100t^2u(t)$.

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(d) Determine the magnitude of the response V_{out} if the input is the signal $V_{in}(t)=2\cos(1800t+45^\circ)$. Assume the transfer function is $T(s)=\frac{10000s}{(s+1000)(s+1800)}$

Answer:
Simply evaluate the transfer function at $s=j\omega=j1800$ to get the magnitude and phase.

$$T(s=j1800)=\frac{100(j1800)}{(j1800+1000)(j1800+1800)}$$

Using my complex number calculator I get $T(s=j1800)=3.3-j.943=3.43\angle-15.95^\circ$

The output is given by $V_{out}(t)=\left|T(s=j1800)\right|2\cos\left(1800t+45^\circ+\angle T(s=j1800)\right)$ or
$V_{out}(t)=(3.43)2\cos(1800t+45^\circ-15.95^\circ)=6.86\cos(1800t+45^\circ-15.95^\circ)$

GRADING:
(a) 7 points for this part of the question. The most common error was in the algebraic manipulation of the terms
(b) 6 points for this part
(c) 6 points for this part. Many people did not get this part.
(d) 6 points for this part. Many people did not do the complex number math correctly.

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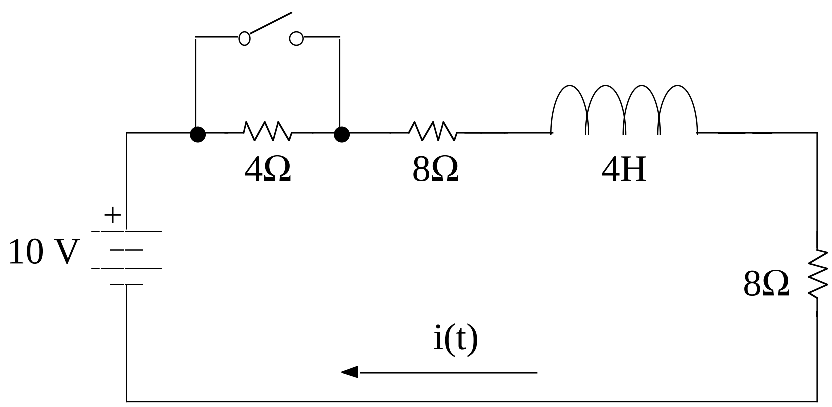
7. s-DOMAIN CIRCUIT ANALYSIS

The switch in the circuit below has been open for a long time and is closed at t=0.

(a) Transform the circuit into the s-domain and write the s-domain expression for the current I(s).

(b) Determine the current i(t).

(c) Identify the forced and natural components of the response to the switch closing.

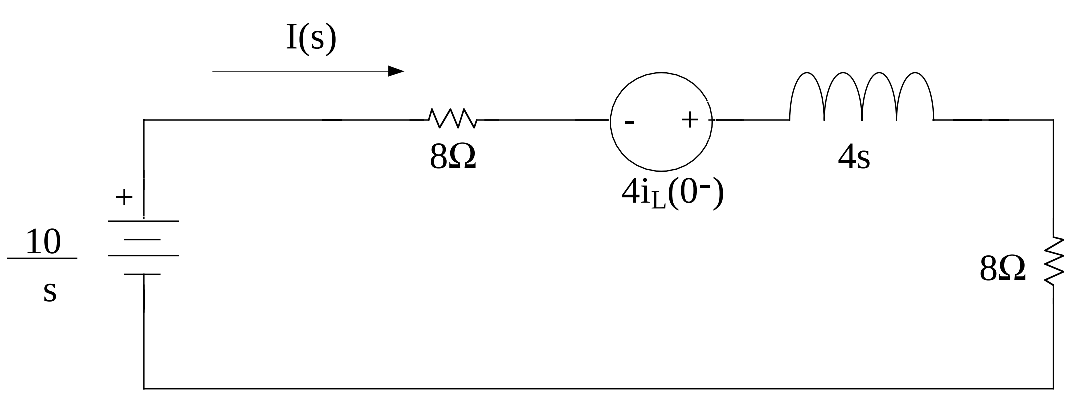


ANSWER:

There is an initial current $i_L(0^-)$ through the 4 henry inductor of

$i_L(0^-) = \frac{10 volts}{4\Omega + 8\Omega + 8\Omega} = \frac{10 volts}{20\Omega} = \frac{1}{2}$.

We redraw the circuit in the s-domain for t>0 to get



Note that the initial condition is now represented as a voltage source of magnitude $Li_L(0^-) = (4)\left(\frac{1}{2}\right) = 2$. We next use KVL to get $-\frac{10}{s} + 8I(s) - 2 + 4sI(s) + 8I(s) = 0$. Solving

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for the current gives $I(s)=\frac{10+2s}{s(16+4s)}=\frac{1}{2}\frac{s+5}{s(s+4)}$. We next use a partial fraction expansion to separate the terms. The coefficients are given by the cover-up algorithm as $\left.(s+4)\frac{1}{2}\frac{s+5}{s(s+4)}\right|_{s=-4}=\frac{1}{2}\frac{-4+5}{-4}=-\frac{1}{8}$ and $\left.(s)\frac{1}{2}\frac{s+5}{s(s+4)}\right|_{s=0}=\frac{1}{2}\frac{5}{4}=\frac{5}{8}$. The result is

$$I(s)=\frac{1}{2}\frac{s+5}{s(s+4)}=\frac{5/8}{s}+\frac{-1/8}{s+4}$$

(b) $i(t)=\frac{5}{8}u(t)-\frac{1}{8}e^{-4t}u(t)$

Note that this satisfies the initial conditions that $i(0)=\frac{5}{8}-\frac{1}{8}=\frac{1}{2}$ and $i(\infty)=\frac{5}{8}$

(c) The forced response is the term with the same form as the source (a $\frac{1}{s}$) and is given by $-\frac{1}{8}u(t)$. The natural response is given by $\frac{5}{8}e^{-4t}u(t)$. A simple definition is to say that the forced response is associated with the input (in this case the switch corresponding to the pole at s=0) and the natural response is associated with the circuit which has a pole at s=-4 (independent of the input).

GRADING:

(a) 10 points. A very common error was not having the $\frac{1}{s}$ associated with the source. This was worth 5 points. You lost 3 points for not correctly including the initial condition, including its value. Many students also did not remove the 4Ω resistor after the switch shorted it out. This was worth 2 points.

Many students lost terms in s while determining I(s) or made other math errors. These were typically worth 1-2 points.

(b) 10 points. You were supposed to find i(t) in this part. If you correctly inverse transformed your result from (a) you typically got full credit. However, if what you came up with was not related to your expression for I(s) you lost up to all 10 points.

The most common error was in losing an s term in the numerator. This was worth 2 points.

(c) 5 points. The most common error was to say that the constant, such as 5/8, was the forced response and the unit step was the natural response, or vice versa. This cost you all 5 points.

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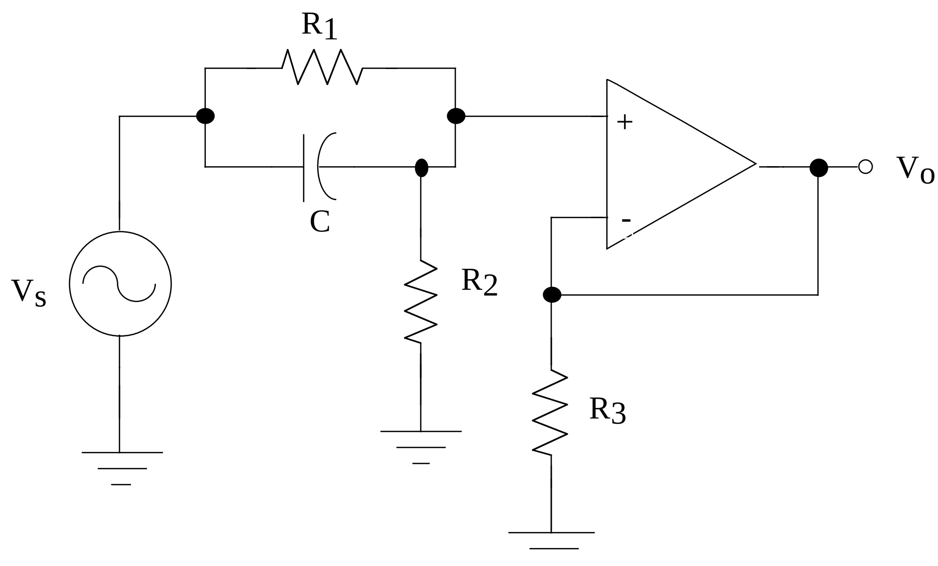
8. s-DOMAIN CIRCUIT ANALYSIS

Consider the OP-AMP circuit shown below.

(a) Determine the system transfer function.

(b) Sketch the gain of the circuit as a function of ω. Assume that R_1=1000Ω, R_2=200Ω, R_3=1000Ω and C=1μF.

(c) What type of filter is this: low-pass, high-pass, etc.?



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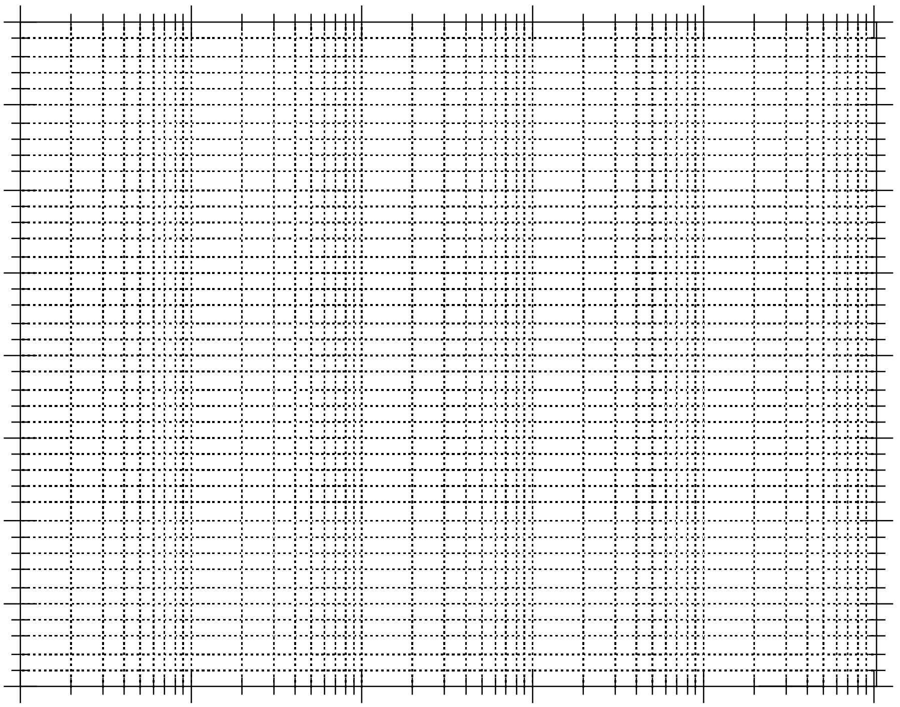

ANSWER:

(a) Apply KCL at the inverting input of the op-amp. Assuming that current into the node is positive we can write

$$\frac{V_s - V_o}{R_1} + \frac{V_s - V_o}{\frac{1}{sC}} - \frac{V_o}{R_2} = 0$$

Solving for the transfer function gives

$$\left(\frac{1}{R_1} + sC\right)V_s = \left(\frac{1}{R_1} + \frac{1}{R_2} + sC\right)V_o$$

$$\frac{V_o}{V_s} = \frac{\frac{1}{R_1} + sC}{\frac{1}{R_1} + \frac{1}{R_2} + sC} = \frac{s + \frac{1}{R_1 C}}{s + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{C}} = \frac{s + \frac{1}{R_1 C}}{s + \frac{1}{(R_1 \| R_2)C}}$$

This transfer function has a single pole and a single zero.

(b) For the given values, the transfer function evaluates to

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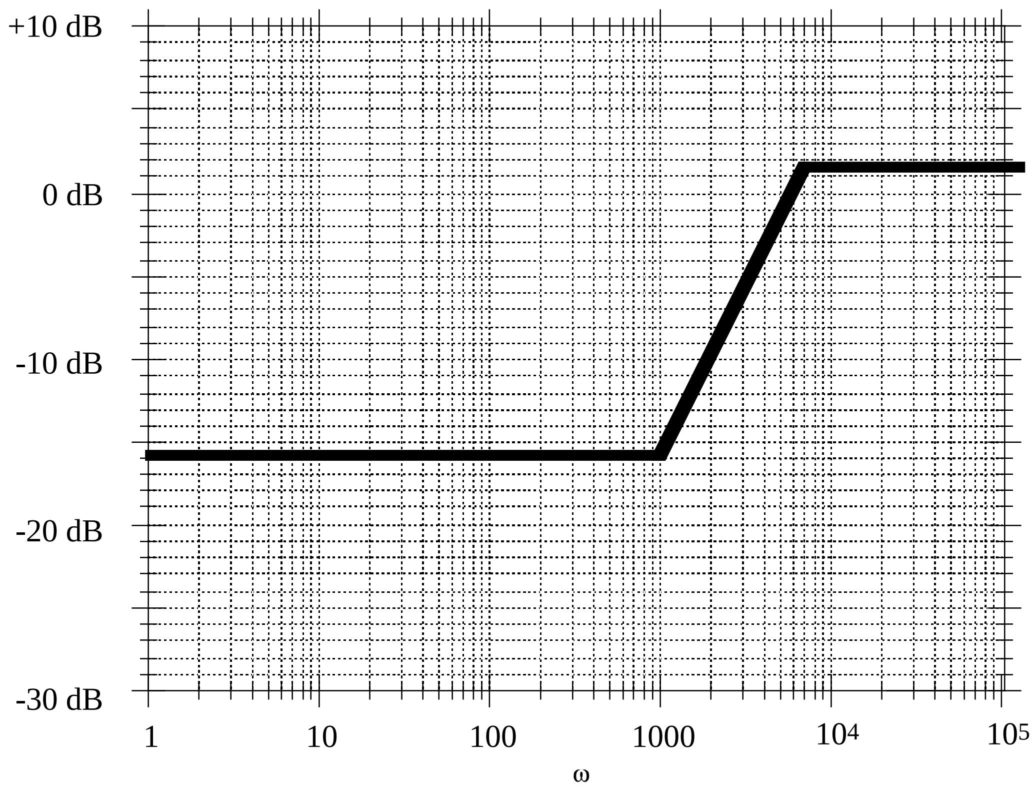
$$\frac{V_o}{V_s}=\frac{s+\dfrac{1}{(1000)(1\times10^{-6})}}{s+\dfrac{1}{\left(\dfrac{(1000)(200)}{1000+200}\right)(1\times10^{-6})}}=\frac{s+1000}{s+6000}=0.167\frac{1+\dfrac{s}{1000}}{1+\dfrac{s}{6000}}$$

$$20\log\left|\frac{V_o}{V_s}\right|=20\log(0.167)+20\log\left|1+\frac{s}{1000}\right|-20\log\left|1+\frac{s}{6000}\right|$$

The first term is approximately –15.6 dB. See the plot below.

(c) See the plot below. This is a high-pass filter.



COMMENTS ON GRADING:

(a) This part was worth 10 points. Aside from being in the s-domain this op-amp problem was similar to many you have already done this semester. The most common problem was in computing the parallel impedance of R1 and C. This was typically worth 3 points. The second most common problem was in computing the input at the + terminal of the op-amp. Depending upon the severity of your errors this typically was worth 3-8 points.

(b) This part was also worth 10 points. If your sketched frequency response corresponded to your transfer function (even if the transfer function was wrong) you received credit. I took points off for incorrect break points, improperly adding together contributions from poles and

zeros, incorrect decibel values. Several students had frequency responses which had nothing in common with their transfer functions — this usually cost you all 10 points.
(c) This part was worth five points. Since this answer was based upon your sketch from (b) I gave full credit if you correctly interpreted your frequency response.

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An Abbreviated List of Laplace Transform Pairs

f(t) ($t>0^-$)	**TYPE**	**F(s)**
$\delta(t)$	(impulse)	1
$u(t)$	(step)	$\frac{1}{s}$
t	(ramp)	$\frac{1}{s^2}$
e^{-at}	(exponential)	$\frac{1}{s+a}$
$\sin(\omega t)$	(sine)	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	(cosine)	$\frac{s}{s^2+\omega^2}$
te^{-at}	(damped ramp)	$\frac{1}{(s+a)^2}$
$e^{-at}\sin(\omega t)$	(damped sine)	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	(damped cosine)	$\frac{s+a}{(s+a)^2+\omega^2}$

An Abbreviated List of Operational Transforms

f(t)	**F(s)**
$Kf(t)$	$KF(s)$
$f_1(t)+f_2(t)-f_3(t)+\ldots$	$F_1(s)+F_2(s)-F_3(s)+\ldots$
$\frac{df(t)}{dt}$	$sF(s)-f(0^-)$
$\frac{d^2f(t)}{dt^2}$	$s^2F(s)-sf(0^-)-\frac{df(0^-)}{dt}$
$\frac{d^nf(t)}{dt^n}$	$s^nF(s)-s^{n-1}f(0^-)-s^{n-2}\frac{df(0^-)}{dt}-s^{n-3}\frac{d^2f(0^-)}{dt^2}-\ldots-\frac{d^{n-1}f(0^-)}{dt^{n-1}}$
$\int_0^t f(x)dx$	$\frac{F(s)}{s}$
$f(t-a)u(t-a)$, $a>0$	$e^{-as}F(s)$
$e^{-at}f(t)$	$F(s+a)$
$f(at)$, $a>0$	$\frac{1}{a}f\left(\frac{s}{a}\right)$
$tf(t)$	$-\frac{dF(s)}{ds}$
$t^nf(t)$	$(-1)^n\frac{d^nF(s)}{ds^n}$
$\frac{f(t)}{t}$	$\int_s^\infty F(u)du$

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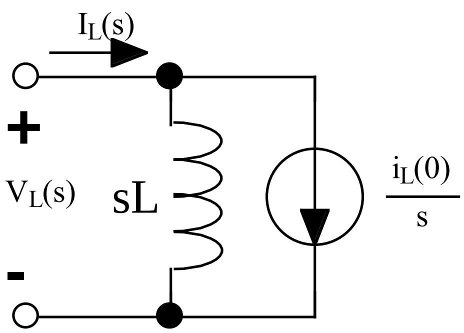
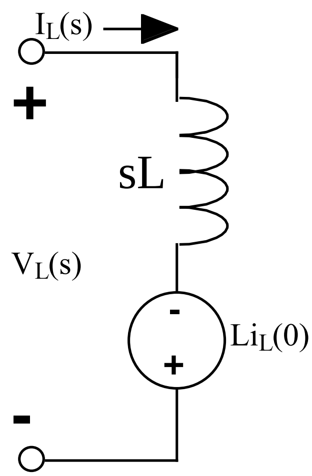
Summary of initial conditions for Laplace transforms:

Inductor:

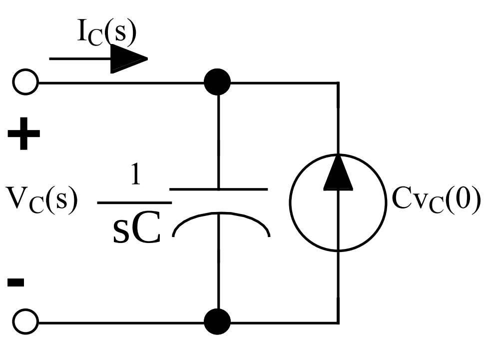


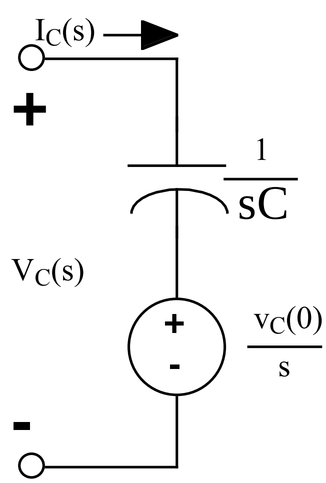


$$I_L(s) = \frac{1}{Ls}V_L(s) + \frac{i_L(0)}{s}$$

$$V_L(s) = LsI_L(s) - Li_L(0)$$

Capacitor:





$$I_C(s) = CsV_C(s) - Cv_C(0)$$

$$V_C(s) = \frac{1}{Cs}I_C(s) + \frac{v_C(0)}{s}$$

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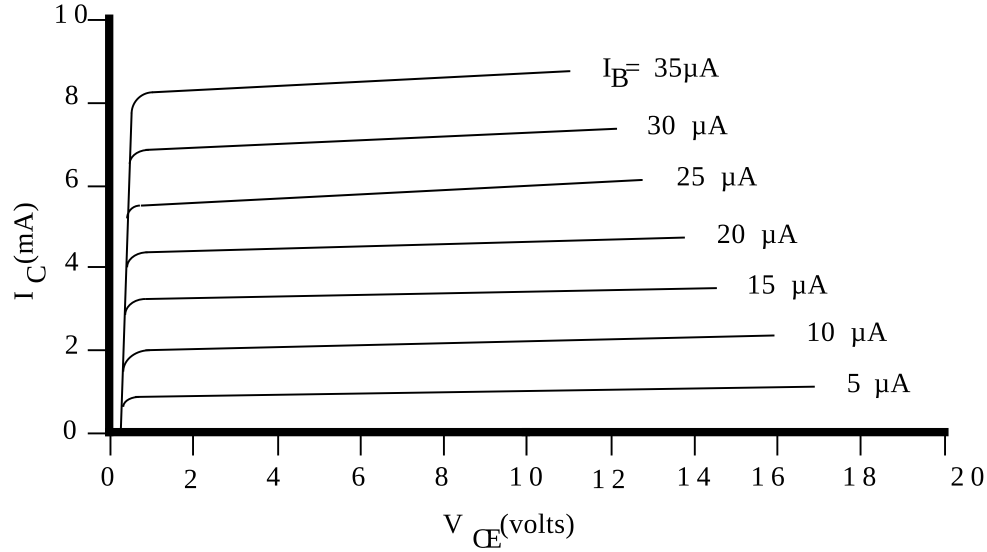
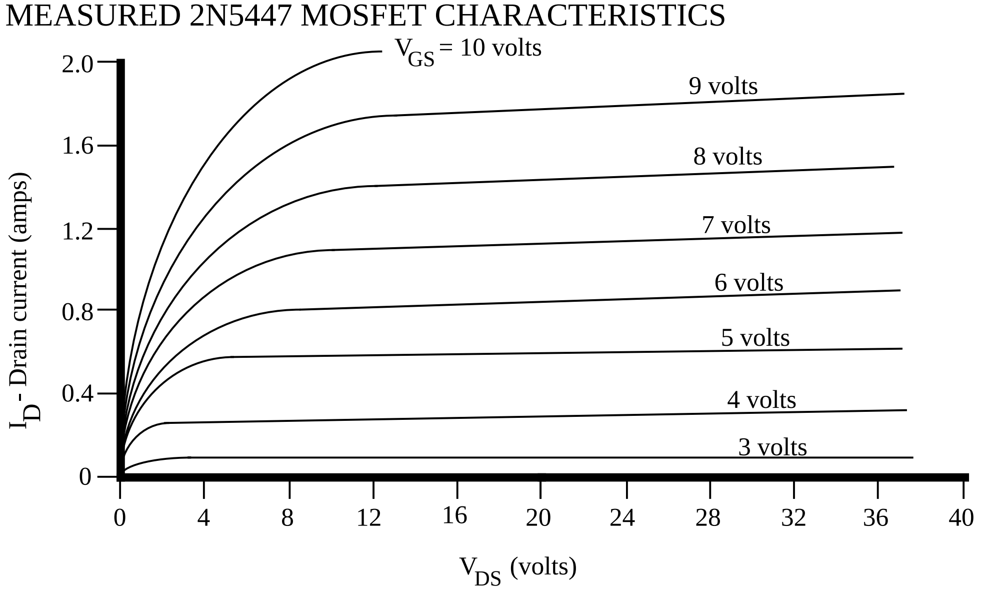
TRANSISTORS

MEASURED 2N2222 BJT CHARACTERISTICS



MEASURED 2N5447 MOSFET CHARACTERISTICS



MOSFETs

$$I_D = K\left(V_{GS} - V_{TO}\right)^2$$

$$g_m = 2K\left(V_{GS} - V_{TO}\right) = 2\sqrt{KI_{DQ}}$$

BJTs

V_T=26mV @ 300˚ C

$$r_\pi = \frac{\beta V_T}{I_{C,Q}}$$

$$g_m = \frac{\beta}{r_\pi} = \frac{I_{C,Q}}{V_T}$$