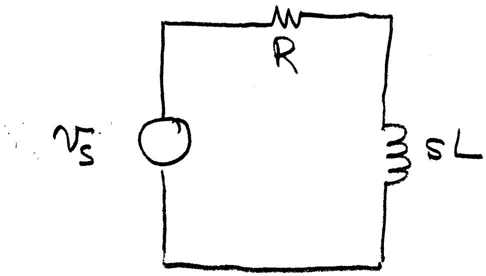
Normally we think of σ as a complex frequency $s = \sigma + j\omega$
Let's consider responses to exponential functions.



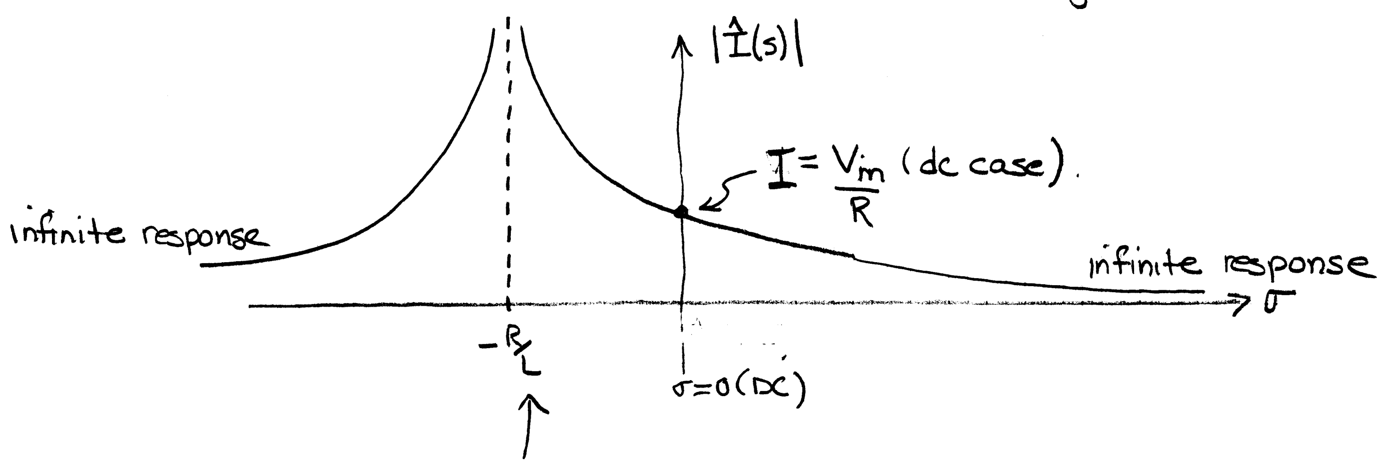
remember $s = \sigma + j\omega$

Consider a source of form $v_s = V_m e^{\sigma t}$

$$\hat{I} = \frac{V_m}{R + \sigma L} = \frac{V_m}{L} \frac{1}{\sigma + R/L}$$

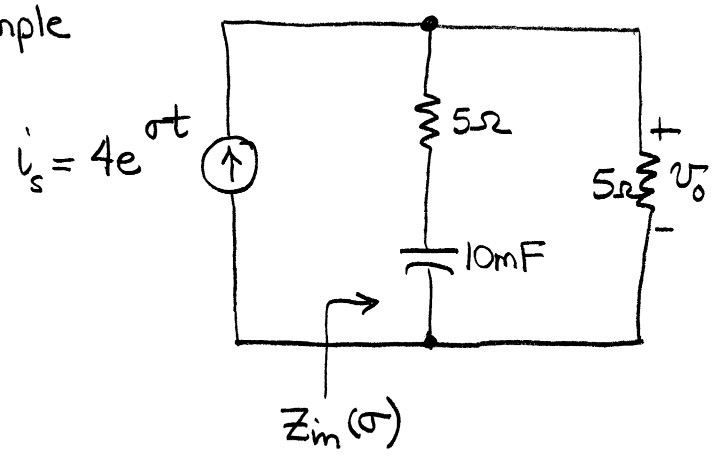
This can be inverse transformed to give $i(t) = \frac{V_m}{L} e^{-\frac{R}{L}t}$

What does $|\hat{I}(s)|$ look like as a function of σ?



at a pole response $\rightarrow \infty$
here $|I(s)| \rightarrow \infty$ as $s \rightarrow -R/L$

Example



Let's calculate input "impedance" to an exponential.

$$Z_{in}(s) = 5 \parallel \left(5 + \frac{1}{sC}\right) = 5 \parallel \left(5 + \frac{1}{\sigma(.01)}\right) \quad \text{since } s = \sigma + j\omega$$

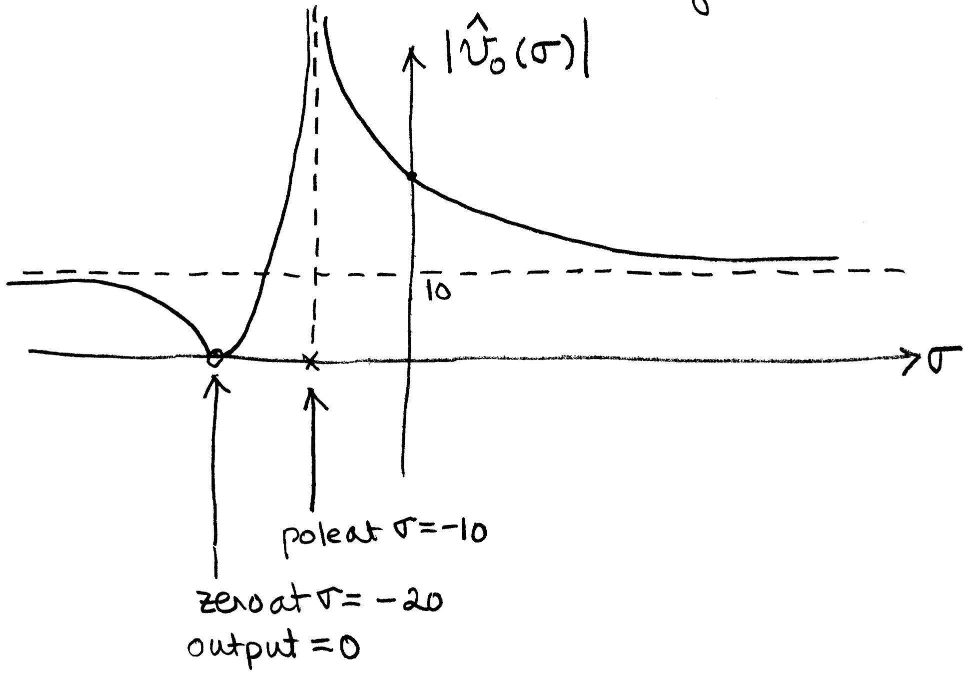
$$Z_{in}(\sigma) = \frac{5\,(5 + .01\sigma)}{5 + 5 + .01\sigma} = 2.5\,\frac{\sigma + 20}{\sigma + 10}$$

What does response look like in σ domain?

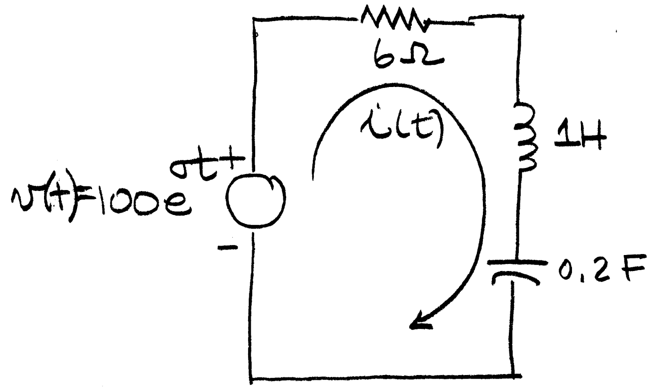
$$\hat{v}_o(s) = I_s Z_{in} = (4)\;2.5\,\frac{\sigma + 20}{\sigma + 10} = 10\,\frac{\sigma + 20}{\sigma + 10}$$

$e^{\sigma t}$ is assumed – just like e^{st}

Now consider this as a function of σ.



Example



Solving in the s-domain

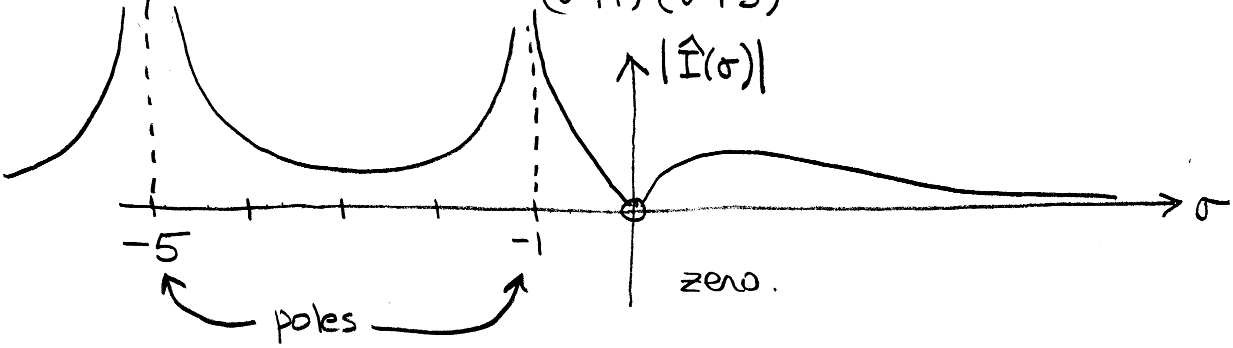
$$I(s) = \frac{V(s)}{R + sL + \frac{1}{sC}} = \frac{100}{6 + 1\cdot s + \frac{1}{s(.2)}}$$

$$= 100\,\frac{1}{6 + s + \frac{5}{s}} = 100\,\frac{s}{6s + s^2 + 5}$$

$$I(s) = 100\,\frac{s}{(s+1)(s+5)}$$

Now, since $s = \sigma + j\phi$ in this case

$$I(\sigma) = 100\,\frac{\sigma}{(\sigma+1)(\sigma+5)}$$



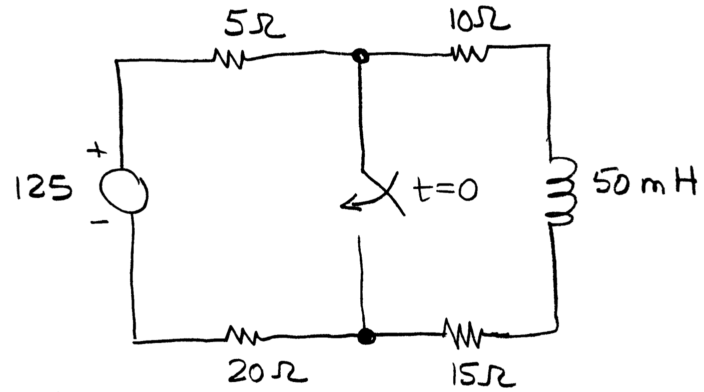
We can calculate forced response for any σ

Suppose $\sigma = -3$. Then $\hat{I} = 100\,\frac{-3}{(-3+1)(-3+5)} = 100\,\frac{-3}{(-2)(2)}$

$$= +75$$

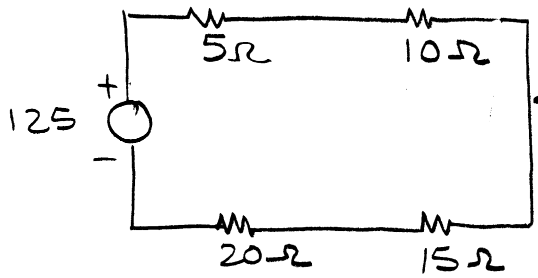
$$i(t) = 75e^{-3t}$$

Example problem



must determine $i_L(0^-)$

before the swith closes the circuit is this

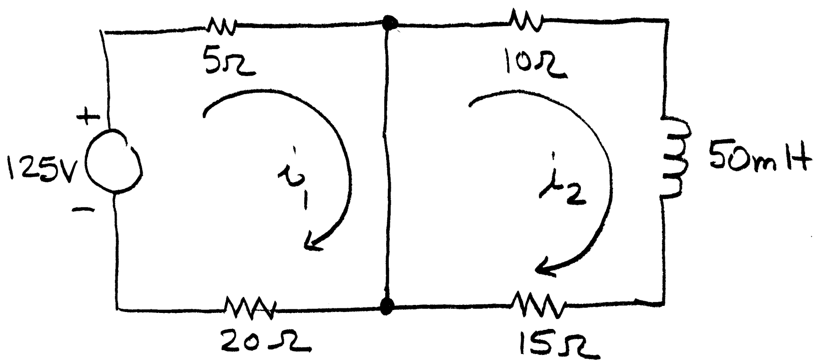


in dc analysis an inductor becomes a short since $v_L = L\frac{di}{dt} \to 0$

$$i_L(0^-) = \frac{125\,V}{5\Omega + 10\Omega + 15\Omega + 20\Omega} = \frac{125}{50} = 2.5\,A$$

Now the switch closes to give this circuit

for $t > 0$



doing the left-hand loop

$$-125V + 5i_1 + 20i_1 = 0$$

$$i_1 = 5 \text{ Amps.}$$

There is no voltage contribution thru the short so we see no i_2.

Now do the second loop. As before there is no voltage contribution from the short.

$$+10 i_2 + (50\times10^{-3}) \frac{di_2}{dt} + 15 i_2 = 0$$

$$25 i_2 + (50\times10^{-3}) \frac{di_2}{dt} = 0$$

Laplace transforming

$$25\, I_2(s) + (50\times10^{-3})\left[sI_2(s) - i_2(0^-)\right] = 0$$

$$25\, I_2(s) + (50\times10^{-3})\left[s\, I_2(s) - 2.5\right] = 0.$$

$$I_2(s)\left[25 + .05s\right] - 0.125 = 0$$

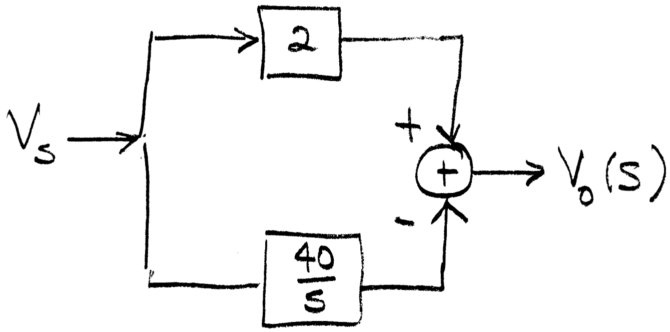
$$I_2(s) = \frac{(0.125)}{(.05s + 25)}\,\frac{20}{20} = \frac{2.5}{s+500}$$

$$i_2(t) = \mathcal{L}^{-1}\left[\frac{2.5}{s+500}\right] = 2.5e^{-500t}u(t).$$

This has the necessary 2.5 Amps at $t=0^+$ and decays to zero as we expect.

9.53 Block diagram of

(b) $V_o(s) = 2V_s(s) - \frac{40}{s} V_s(s)$



9.50 Use the initial & final value theorems to find the initial and final values of the waveform corresponding to the transforms in 9.26.

9-26 (a) $F(s) = \frac{(s+50)^2}{(s+10)(s+100)}$

Is not a proper rational function so

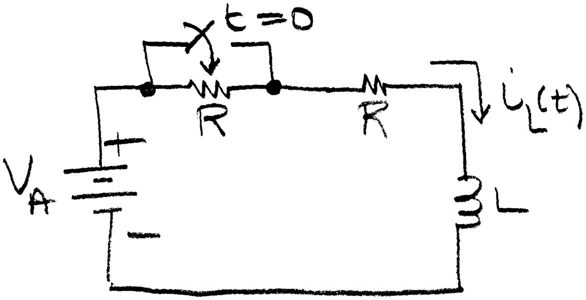
cannot compute $f(0) = \lim_{s \to \infty} sF(s)$

both poles in LHP so we can compute

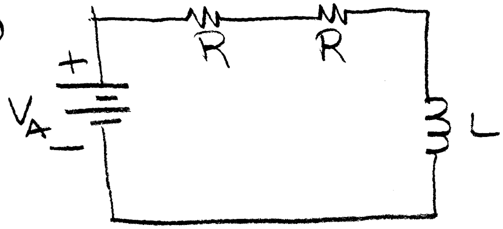
$f(\infty) = \lim_{s \to 0} sF(s)$

$= \lim_{s \to 0} \frac{s(s+50)^2}{(s+10)(s+100)} = 0$

10–11

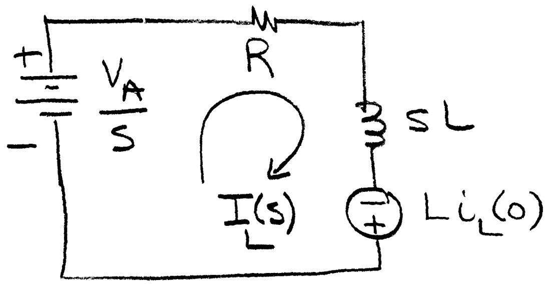


t<0



$$i_L(0^-) = \frac{V_A}{2R}$$

t>0

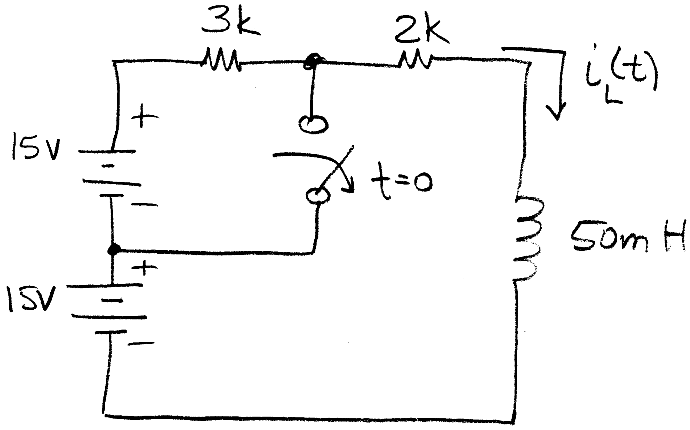


$$-\frac{V_A}{s} + I_L R + sLI_L - Li_L(0) = 0$$

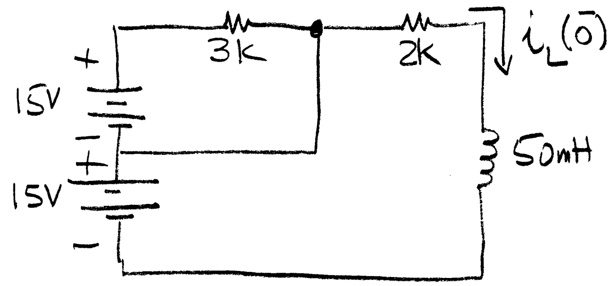
$$I_L = \frac{\frac{V_A}{s} + L\,i_L(0)}{R + sL} = \frac{\left(\frac{V_A}{s} + L\,\frac{V_A}{2R}\right)\frac{1}{L}}{(R + sL)\,\frac{1}{L}}$$

$$I_L = \frac{\frac{1}{s}\left(V_A/L + \frac{V_A}{2R}s\right)}{\frac{R}{L} + s} = \frac{V_A\left(\frac{1}{L} + \frac{s}{2R}\right)}{s\left(s + \frac{R}{L}\right)}$$

10–15.

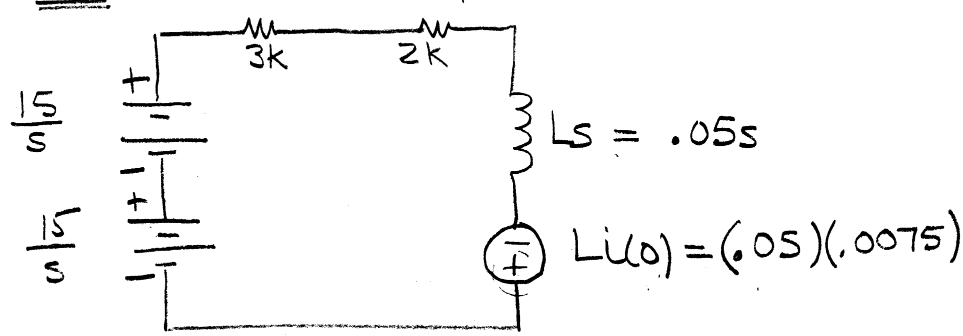


for $t<0$ switch is closed. In this configuration



$$i_L(0^-) = \frac{15V}{2k} = 7.5mA$$

for $\underline{\underline{t>0}}$ the switch is open



Note: (1) switch explicitly included on voltage sources
(2) initial condition explicitly showed

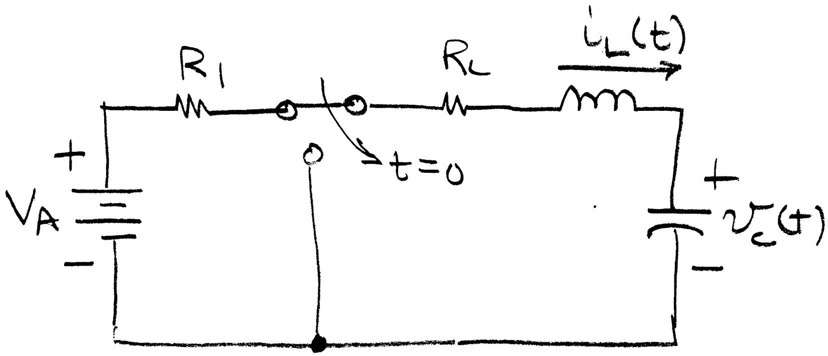
$$I_L(s) = \frac{\frac{15}{s} + \frac{15}{s} + (.05)(.0075)}{5000 + .05s}$$

$$= \frac{\left(\frac{30}{s} + .000375\right)}{(5000 + .05s)} \cdot \frac{\frac{1}{.05}}{\frac{1}{.05}} = \frac{\frac{600}{s} + .0075}{100000 + s}$$

$$I_L(s) = \frac{600 + .0075s}{s(s+100000)} = \frac{.006}{s} + \frac{.0015}{s + 100{,}000}$$ [using cover up algorithm].

$$i_L(t) = .006\,u(t) + .0015\,e^{-100{,}000t}\,u(t)$$

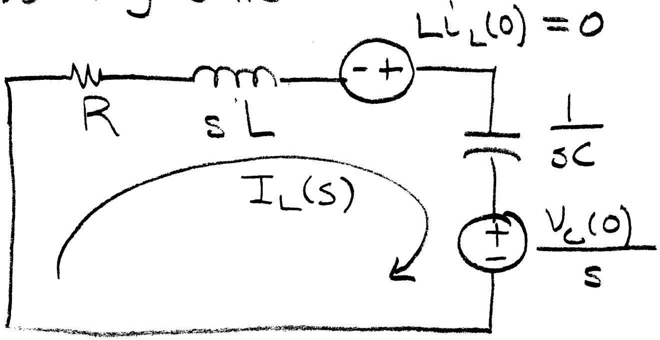
10-17



(a) $t<0$ the initial conditions are $i_L(0^-)=0$, $V_c(0^-)=V_A$

for $t>0$ in symbolic form

no switch since switch shorted to ground



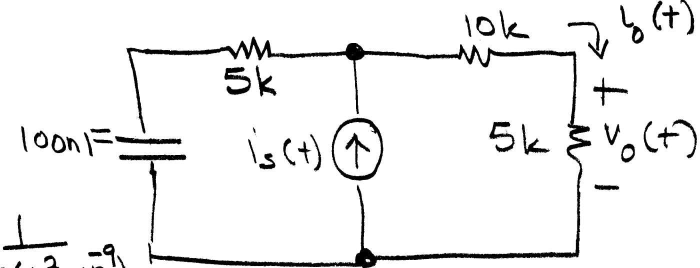
← $\frac{1}{s}$ explicitly in initial condition

using KVL

$$I = \frac{-\frac{v_c(0)}{s}}{R + sL + \frac{1}{sC}}$$ since other source is zero

$$= \frac{\left(-\frac{V_A}{s}\right)sC}{\left(R+sL+\frac{1}{sC}\right)sC} = \frac{-V_A C}{s^2LC+sRC+1}$$

10.23 There is no energy stored in the capacitor in Figure 10-23 at $t=0$. Transform the circuit into the s-domain and use current division to find $v_o(t)$ when $i(t) = 0.1\,e^{-100t}u(t)$. Identify the forced and natural poles in $V_o(s)$.



$$\frac{1}{sC} = \frac{1}{s(10^2 \times 10^{-9})} = \frac{10^7}{s}$$

$$I_o(s) = \frac{5k + \frac{1}{sC}}{\left(5k + \frac{1}{sC}\right) + (15k)} I_s(s)$$

$$= \left[\frac{5000 + \frac{10^7}{s}}{5000 + \frac{10^7}{s} + 15000}\right]\left[\frac{0.1}{s+100}\right]$$

$$= \frac{5\cancel{000}s + 10\,000\,\cancel{000}}{20\cancel{000}s + 10\,000\,\cancel{000}}\left(\frac{0.1}{s+100}\right)$$

$$= \frac{5s + 10000}{20s + 10000}\left(\frac{0.1}{s+100}\right) = \frac{s + 2000}{(4s + 2000)(s+100)}(0.1)$$

$$= \frac{0.25(s+2000)(0.1)}{(s+500)(s+100)} = \frac{0.025(s+2000)}{(s+500)(s+100)}$$

$$V_o(s) = 5000\,I_o(s) = \frac{125(s+2000)}{(s+500)(s+100)} = \left[\frac{a}{s+500} + \frac{b}{s+100}\right]125$$

$$as + 100a + bs + 500b = s + 2000$$

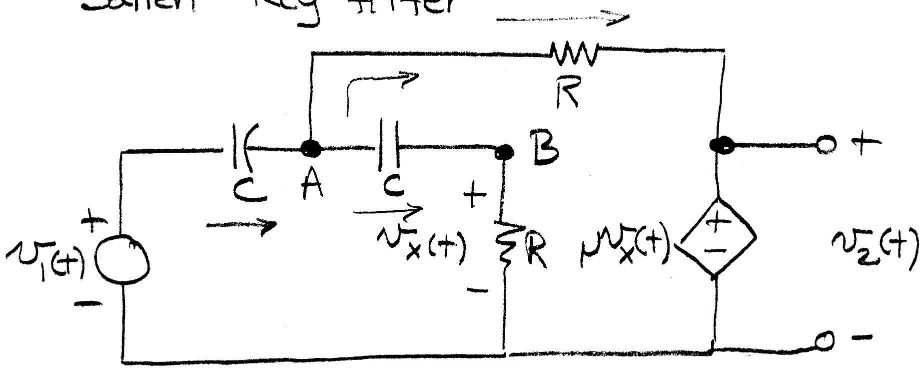
$$a + b = 1$$
$$a + 5b = 20$$

$$-4b = -19$$

$$b = \frac{19}{4} \qquad a = 1 - b = 1 - \frac{19}{4} = \frac{4-19}{4} = -\frac{15}{4}$$

$$V_o(s) = \underbrace{\frac{125\left(-\frac{15}{4}\right)}{s+500}}_{\text{natural.}} + \underbrace{\frac{125\left(\frac{19}{4}\right)}{s+100}}_{\text{forced}} \quad \text{or } V_o(t) = -468.8e^{-500t} + 593.8e^{-100t}$$

10-39 Sallen - Key filter



(a) Transform the circuit into the s-domain and find the circuit determinant

Use two nodes A & B.

KCL@ A $\quad \sum_{+in} i = 0 \qquad \dfrac{v_1 - v_A}{\frac{1}{sC}} - \dfrac{v_A - v_B}{\frac{1}{sC}} - \dfrac{v_A - \mu v_B}{R} = 0$

KCL@ B $\quad \sum_{+in} i = 0 \qquad + \dfrac{v_A - v_B}{\frac{1}{sC}} - \dfrac{v_B - 0}{R} = 0$

$$\left[-sC - sC - \tfrac{1}{R}\right] v_A + \left[sC + \mu/R\right] v_B + sCv_1 = 0$$

$$+ sC v_A + \left[-sC - \tfrac{1}{R}\right] v_B = 0$$

Rearranging

$$\left[\tfrac{1}{R} + 2sC\right] v_A - \left[\tfrac{\mu}{R} + sC\right] v_B = sCv_1$$

$$\left[sC\right] v_A + \left[sC + \tfrac{1}{R}\right] v_B = 0$$

circuit matrix

$$\begin{bmatrix} \frac{1}{R} + 2sC & -\left(\frac{\mu}{R} + sC\right) \\ -sC & \frac{1}{R} + sC \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} sCV_1 \\ 0 \end{bmatrix}$$

The determinant is

$$\Delta(s) = \det \begin{bmatrix} \frac{1}{R}+2sC & -\left(\frac{\mu}{R}+sC\right) \\ -sC & \frac{1}{R}+sC \end{bmatrix}$$

$$\Delta(s) = \frac{(RC)^2 s^2 + (3-\mu)RCs + 1}{R^2}$$

v_A & v_B are of the form $\frac{\Delta}{\Delta(s)}$

(b) Select R, C & μ so the circuit has poles at $s = \pm j5000$

The zeros of $\Delta(s)$ are the poles of solutions for v_A and v_B.

For $\Delta(s)$ to be a zero let's require $\mu = 3$

Then $(RC)^2 s^2 + 1 = 0$

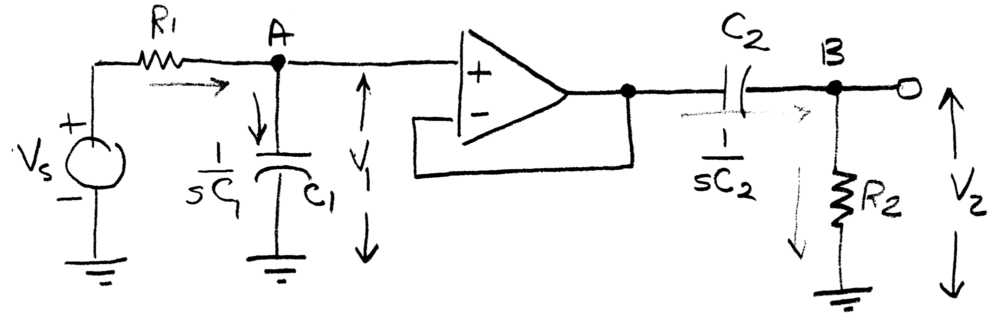
$$s^2 = \frac{1}{(RC)^2} \quad \text{or} \quad s = \frac{1}{RC} = 5000$$

$\therefore$ pick $C = 20\,nF$

Then $R = \dfrac{1}{5000\,C} = \dfrac{1}{(5\times10^3)(20\times10^{-9})}$

$$= \frac{1}{10^{-4}} = 10^4$$

10-48



(a) Do KCL @ node A $\quad \dfrac{V_s - V_1}{R_1} - \dfrac{V_1 - 0}{\frac{1}{sC_1}} = 0 \qquad \dfrac{V_s}{R_1} = \dfrac{V_1}{R_1} + sC_1 V_1$

Do KCL @ node B $\quad \dfrac{V_1 - V_2}{\frac{1}{sC_2}} - \dfrac{V_2 - 0}{R_2} = 0 \qquad sC_2 V_1 = sC_2 V_2 + \dfrac{V_2}{R_2}$

Solving $\quad V_s = V_1 (1 + R_1 C_1 s) \quad \therefore \quad V_1 = \dfrac{V_s}{1 + R_1 C_1 s}$

$$sR_2C_2 V_1 = sR_2C_2V_2 + V_2 \qquad V_2 = \frac{R_2C_2 s}{1 + R_2C_2 s} V_1$$

$$V_2(s) = \frac{R_2C_2 s}{1 + R_2C_2 s} \, \frac{1}{1 + R_1C_1 s} \, V_s$$

poles at $\quad s = -\dfrac{1}{R_1C_1} = \dfrac{-1}{(10^4)(50 \times 10^{-9})} \qquad s = -\dfrac{1}{R_2C_2} = \dfrac{-1}{(10^4)(100 \times 10^{-9})}$

$s = -2000 \qquad\qquad s = -1000$

(b) If $V_s = 10\,u(t)$

$$V_1(s) = \frac{V_s}{1 + \frac{s}{2000}} = \underbrace{\frac{2000}{s + 2000}}_{\text{natural pole}} \cdot \frac{1}{s}$$

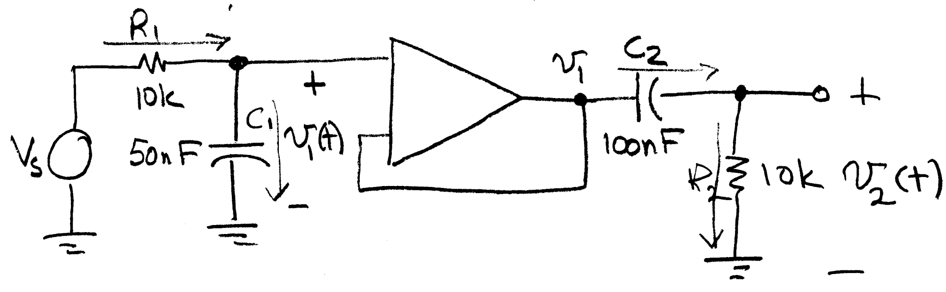
forced pole

(c)

$$V_2(s) = \frac{\frac{s}{1000}}{1 + \frac{s}{1000}} \cdot \frac{2000}{s+2000} \cdot \frac{1}{s} = \underbrace{\frac{\cancel{s}}{s+1000} \cdot \frac{2000}{s+2000}}_{\text{two natural poles}} \cdot \frac{1}{\cancel{s}}$$

forced pole canceled

10.48



(a) What are the natural poles of this circuit?

At input
$$+\frac{V_s - V_1}{R_1} - \frac{V_1 - 0}{\frac{1}{sC_1}} = 0$$

at output
$$+\frac{V_1 - V_2}{\frac{1}{sC_2}} - \frac{V_2 - 0}{R_2} = 0$$

@ input
$$\frac{V_s}{R_1} = \frac{V_1}{R_1} + sC_1 V_1 = \frac{V_1(1 + sR_1C_1)}{R_1}$$
$$\therefore \quad V_1 = \frac{V_s}{1 + sR_1C_1}$$

@ output
$$sC_2 V_1 - sC_2 V_2 - \frac{V_2}{R_2} = 0$$
$$sR_2C_2V_1 - sR_2C_2V_2 - V_2 = 0$$
$$sR_2C_2V_1 = V_2(1 + sR_2C_2)$$
$$V_2 = \frac{R_2C_2 s}{R_2C_2 s + 1} V_1$$

$$V_o(s) = \left[\frac{R_2C_2 s}{R_2C_2 s + 1}\right]\left[\frac{1}{R_1C_1 s + 1}\right] V_s$$

poles at $s = -\frac{1}{R_2C_2} = -\frac{1}{(10^4)(10^2 \times 10^{-9})} = -\frac{1}{10^{-3}} = -1000$

$s = -\frac{1}{R_1C_1} = -\frac{1}{(10^4)(5 \times 10 \times 10^{-9})} = -\frac{1}{5} \times 10^4 = -.2 \times 10^4$
$= -2000$

$R_1C_1 = 5 \times 10^{-4}$ $\qquad$ $R_2C_2 = 10^{-3}$