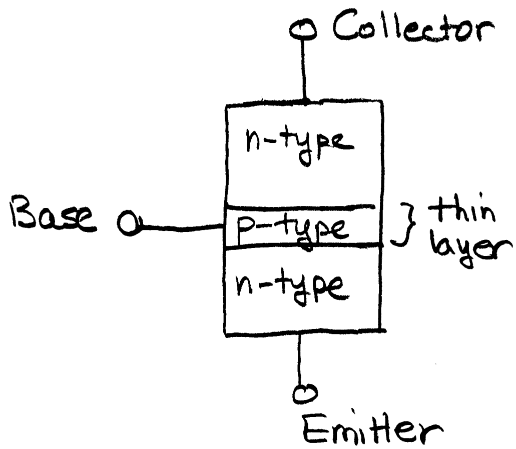
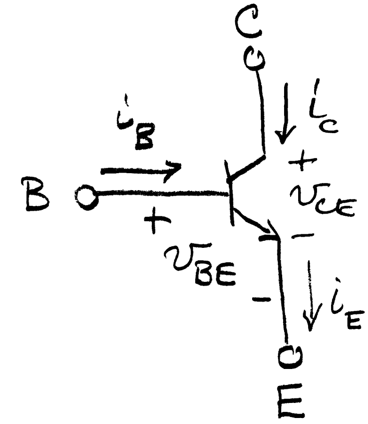
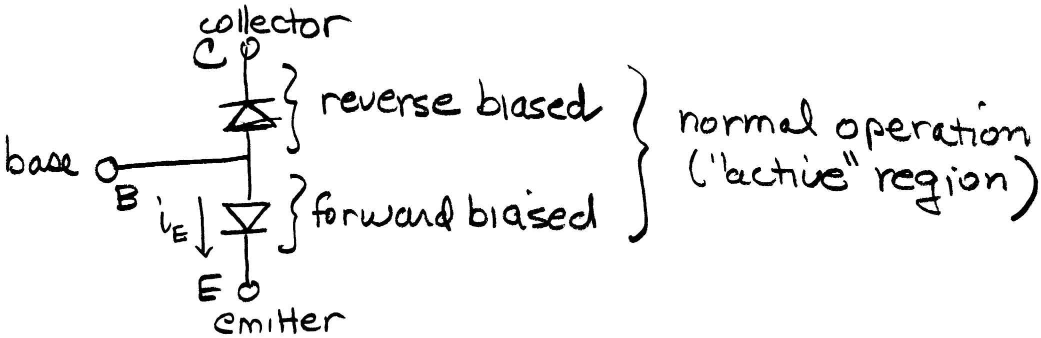
4.1 Basic Operation of the npn Bipolar Junction Transistor







How do we bias to make work?

(a) We need $v_{BE} \approx 0.6$ volts to forward bias bottom diode.

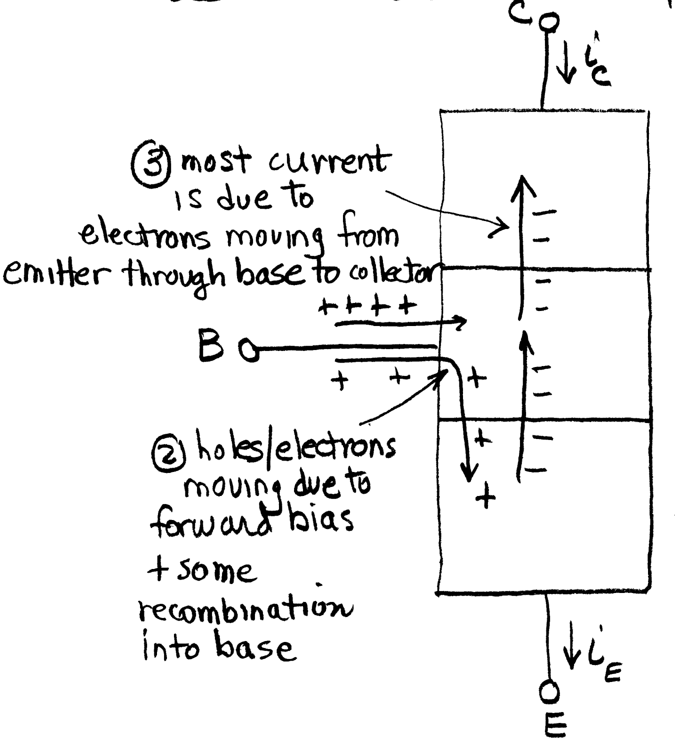
(b) If $v_{CE} > v_{BE}$ the upper diode is reverse biased.

The operation of the transistor fundamendally depends on the detailed, non-linear operation of the base-emitter junction

Shockley equation: $$i_E = I_{ES}\left(e^{\frac{v_{BE}}{V_T}} - 1\right)$$

$V_T \cong 26\text{mV}$ @ room temperature

How does a transistor work physically?

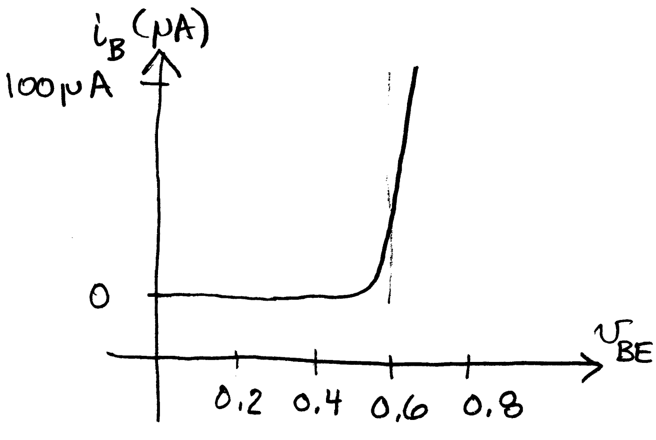


Note: ③ requires a "thin" base

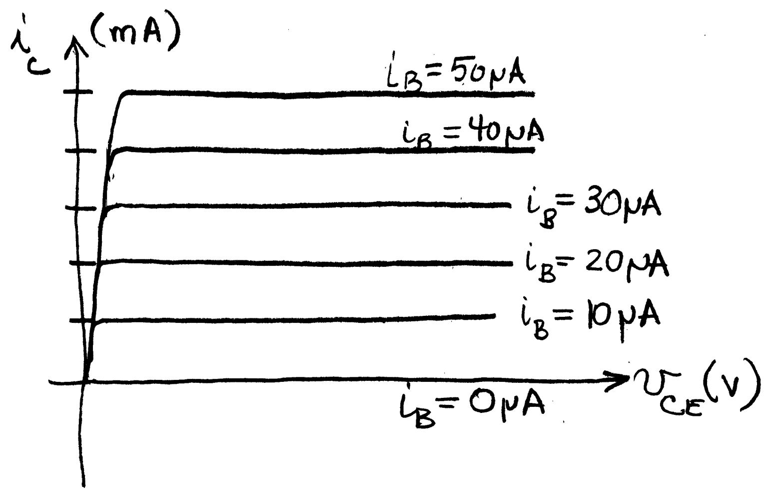
① emitter heavily doped to have lots of free electrons

④ most current across junction supplied by i_c rather than i_B
$\Rightarrow$ this leads to amplification

Measured characteristics



typical base emitter characteristics



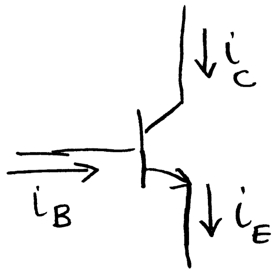
VERY SMALL CHANGE IN I_B CAUSES BIG CHANGE IN I_C

AMPLIFICATION

$$\beta \equiv \frac{i_c}{i_B}$$

typically 10–1000

Device equations



often refer all currents to emitter

by KCL $\quad i_E = i_C + i_B$

collector current is smaller than emitter current $\quad \alpha \equiv \dfrac{i_C}{i_E}$ (typically 0.9 – 0.999)

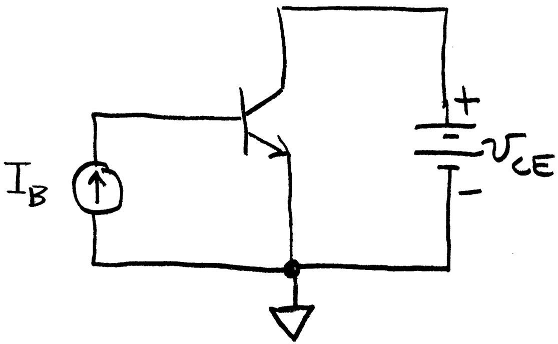
Then $\quad i_B = i_E - i_C = i_E - \alpha i_E = (1-\alpha)\, i_E$

where $\quad i_E = I_{ES}\left[e^{\frac{v_{BE}}{V_T}} - 1\right]$

Recall $\quad \beta = \dfrac{i_C}{i_B} = \dfrac{\alpha\, i_E}{(1-\alpha)\, i_E} = \dfrac{\alpha}{\alpha - 1}$

The equation we always use is $\quad i_C = \beta i_B$ ☆

Use PSpice to display characteristic curves



Use Qbreak n
click on transistor (turns red)

Under edit menu select edit model / edit instance model

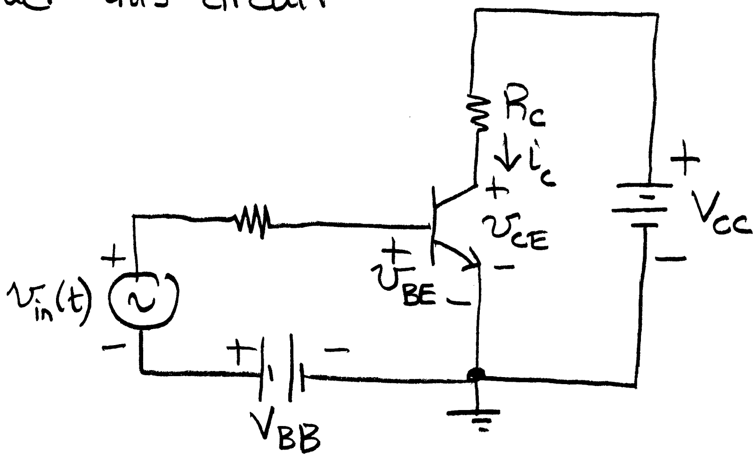
type in BF=100 (FORWARD BETA)

IS = 1E-14 (SCALE CURRENT)

VAF = 50 (EARLY VOLTAGE)

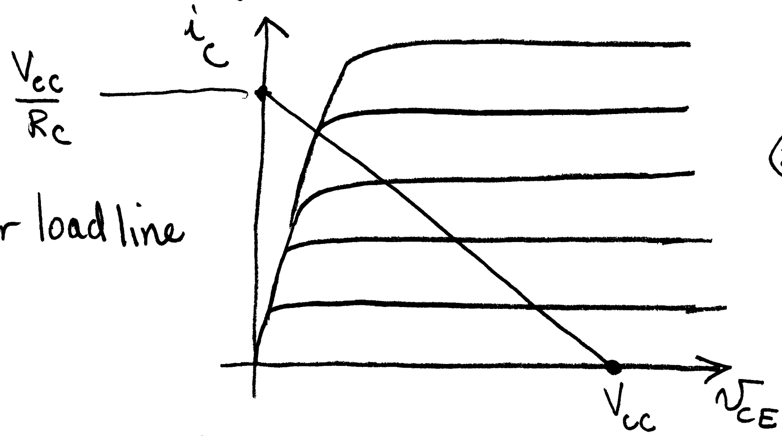
LOAD Line analysis

Consider this circuit



This is obviously a non-linear device which we can subject to load line analysis:

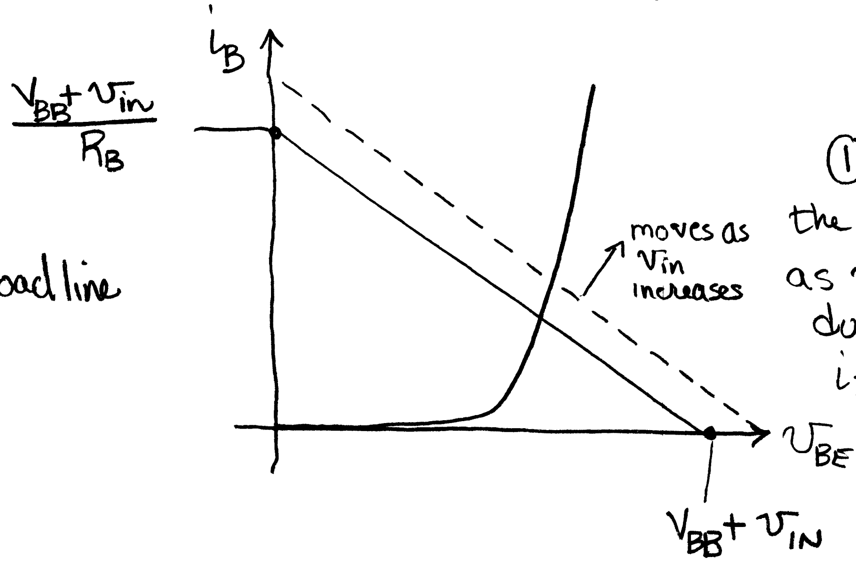
collector load line



(2) superimpose load line on $i_C - v_{CE}$ characteristics

(3) As v_{IN} changes, i_B changes and we move to a different operating point.

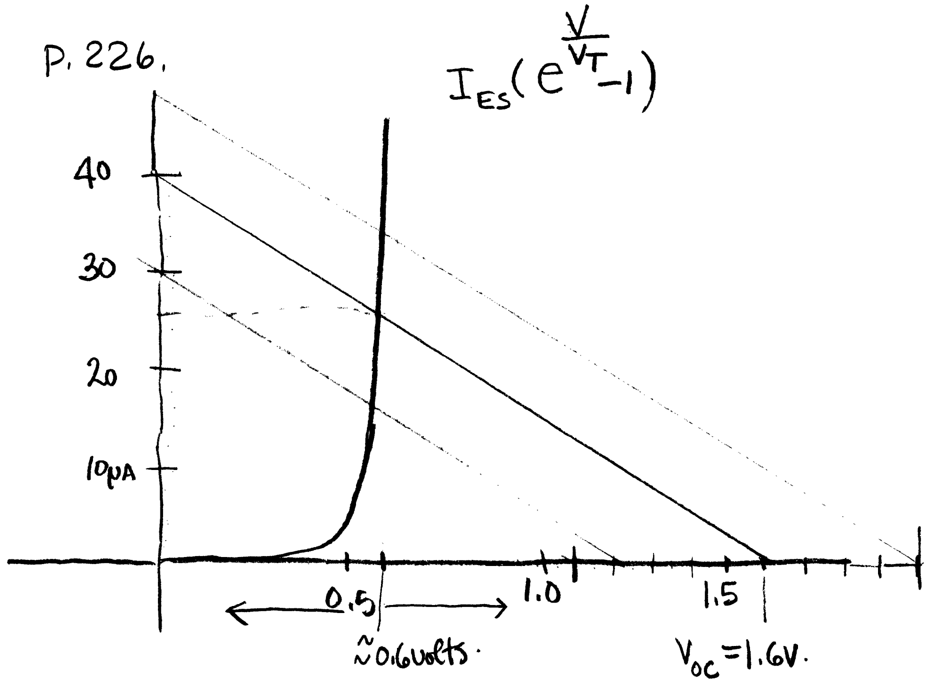
base load line

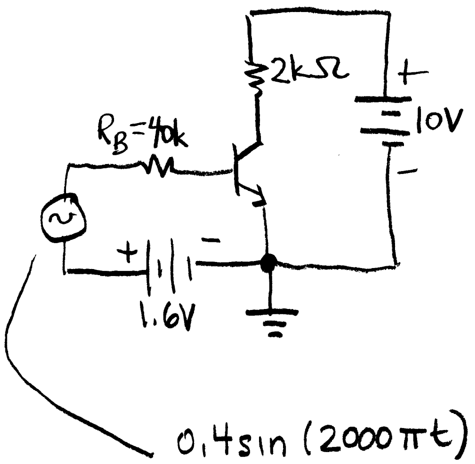


(1) As v_{IN} increases the load line moves up, as v_{IN} decreases it goes down. This changes i_B.

Do detailed analysis p. 226.

What makes the base analysis easier is the vertical diode characteristics





① Draw base load line

$V_{oc}=1.6V.$

$$I_{sc}=\frac{1.6\text{ volts}}{40\text{ k}}=40\mu A$$

② bias Q-point from intersection

③ Now add in signal voltage to perturb Q-point. This gives two new load lines.

④ find new Q-points for peaks of signal.

The variation is <u>VERY</u> linear

As V_s varies from $1.6-.4=1.2$

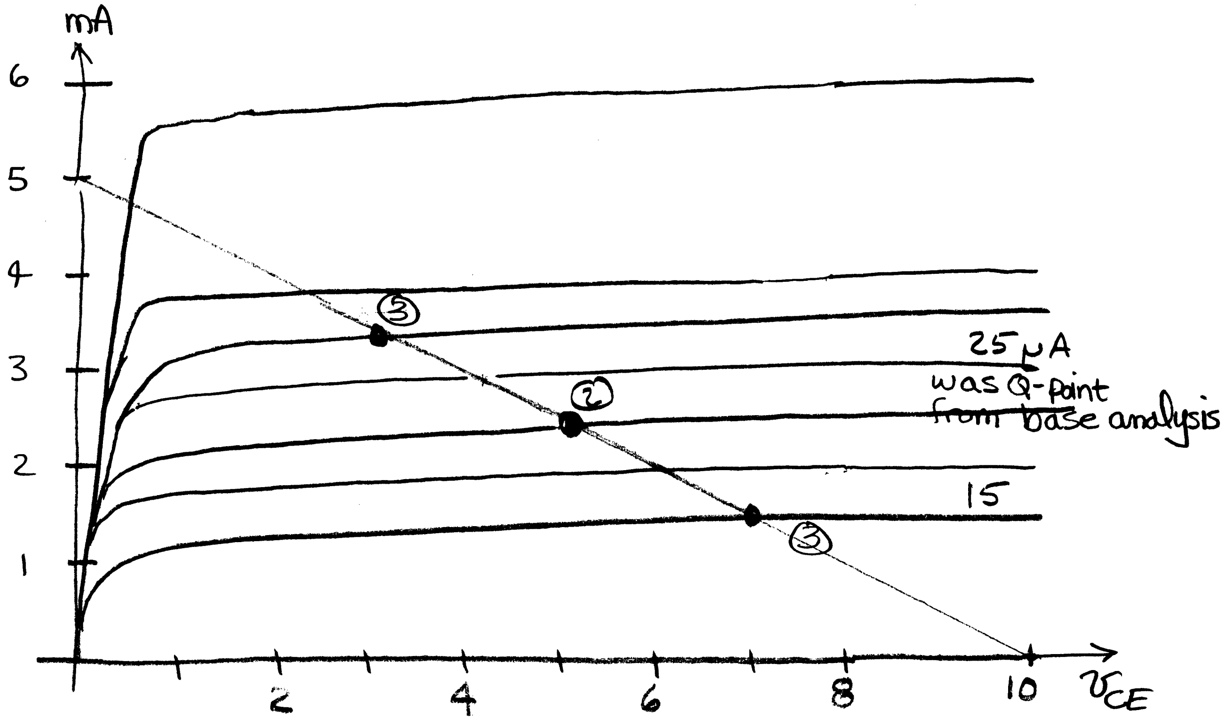
$1.6+.4=2.0$

Note V_{BE} almost does NOT change, ≈ constant at 0.6volts

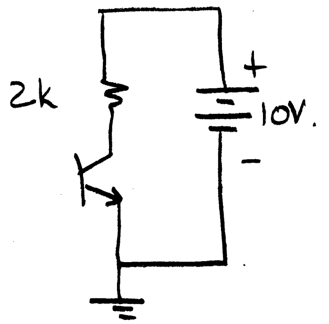
predict $I_{B1}=\dfrac{1.6-0.6+.4}{40k}=\dfrac{1.4}{40k}=35\mu A$

$I_{B2}=\dfrac{1.6-0.6-0.4}{40k}=\dfrac{0.6}{40k}=15\mu A$

Now do detailed load circuit analysis



① draw collector load line on characteristic curves for specific transistor



$V_{oc} = 10 \text{ volts}.$

$I_{sc} = \frac{10V}{2k} = 5mA.$

② identify base curve for DC bias current in this case $I_{BQ} = 25\mu A$.

This intersection is Q-point for collector circuit.

③ Now look at new Q-points corresponding to maximum and minimum of input signal

$I_{B+peak} = 35\mu A$

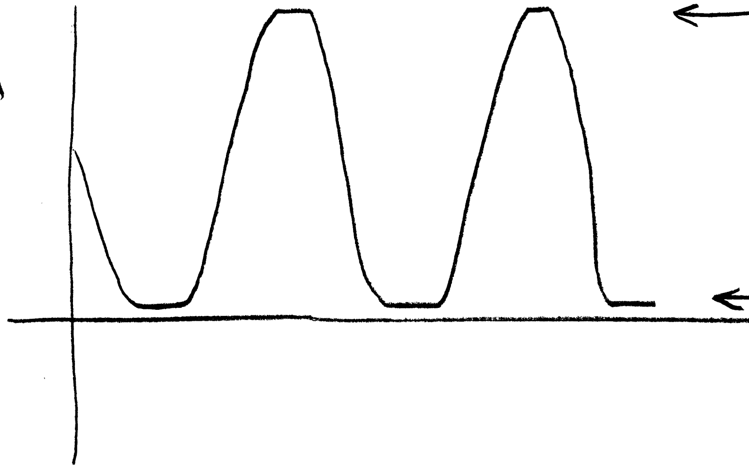
$I_{B-peak} = 15\mu A$

④ v_{CE} varies from 7 volts to 3 volts.

$$A_V = \frac{7-3 \text{ volts}}{+0.4+0.4} = \frac{4}{.8} = 5.$$

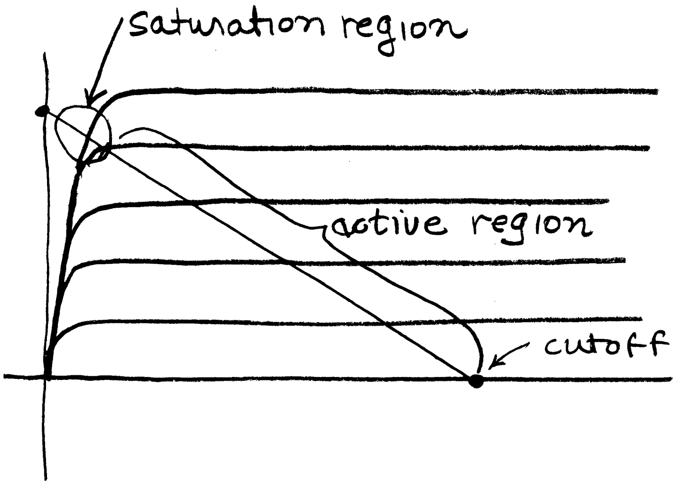
Distortion – amplifier is non-linear
due to curvature and nonuniform spacing
of characteristics

exaggerated
example

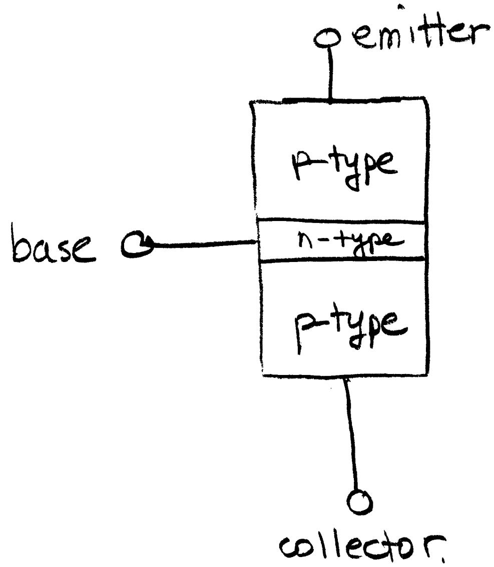


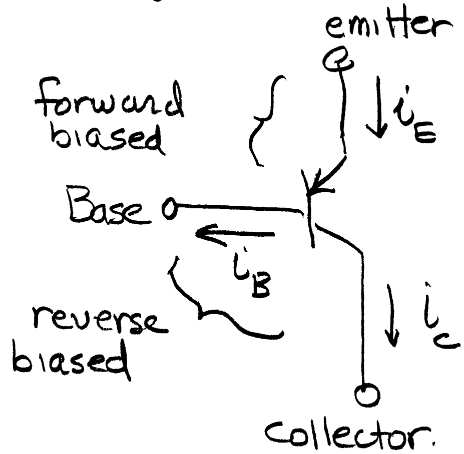
← clipping due to
exceeding power
supply voltage
(cutoff)

← i_B is large enough
to drive transistor
into highly non-linear
region
(saturation)



pnp transistor — the reverse of the npn transistor





note difference in sign of collector current

equations:

$$i_c = \alpha i_E$$

$$i_B = (1-\alpha) i_E$$

$$i_E = i_c + i_B$$

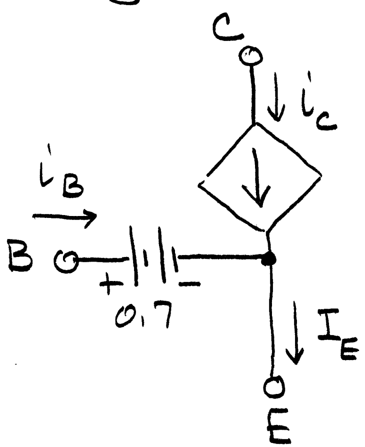
$$i_c = \beta i_B.$$

in active region:

$$i_E = I_{ES}\left[e^{-\frac{v_{BE}}{V_T}} - 1\right]$$

$$i_B = (1-\alpha) I_{ES}\left[e^{-\frac{v_{BE}}{V_T}} - 1\right]$$

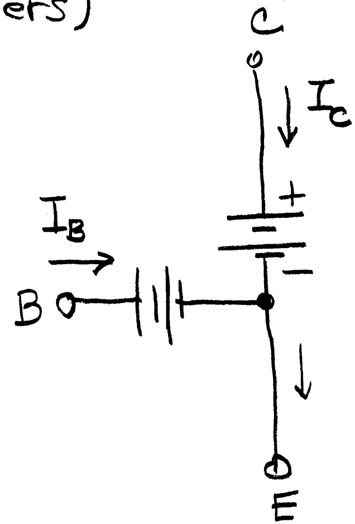
Large signal models
(usually for DC amplifiers)



active region

$I_B > 0$

$v_{CE} > 0.2V$.



saturation region

$I_B > 0$

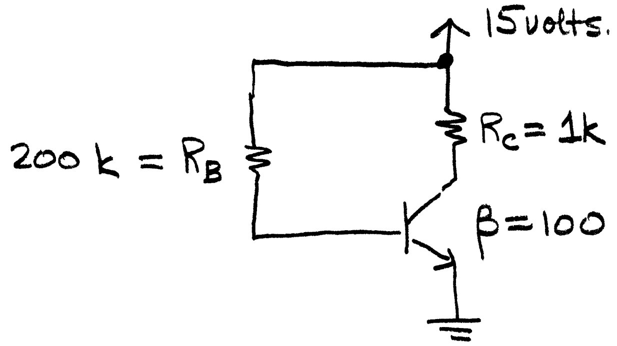
$\beta I_B > I_c > 0$

cutoff region

$v_{BE} < 0.5V$

$v_{BC} < 0.5V$

Three models.

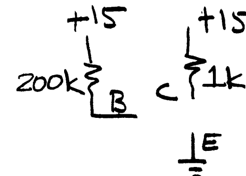


This is called a fixed bias point circuit. If β changes the operating point will change.

solve for I_c & V_{CE}

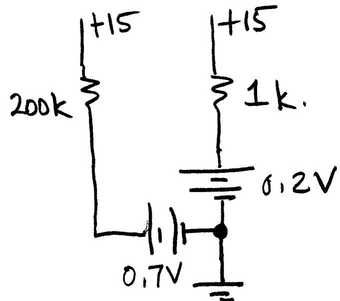
① is transistor active, cutoff, or saturated ?

assume cutoff then $v_{BE} = 15$ volts and transistor must be ON.



assume saturation

use this model.



$$I_c = \frac{V_{cc} - 0.2}{1k} = \frac{15 - 0.2}{1k} = 14.8\,mA$$

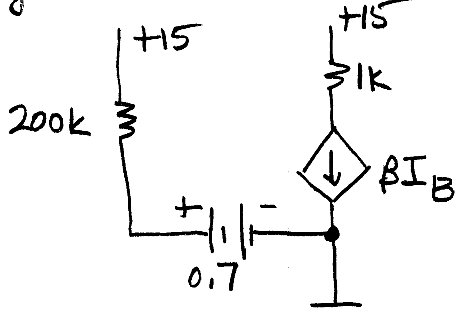
$$I_B = \frac{V_{cc} - 0.7}{200k} = 71.5\,\mu A.$$

$I_B > 0$ so ok

however $\beta I_B = (100)(71.5\,\mu A) = 7.15\,mA < 14.8\,mA$

$\Rightarrow$ transistor is active

In active region



$$I_B = \frac{15 - 0.7}{200k} = 71.5\,\mu A$$

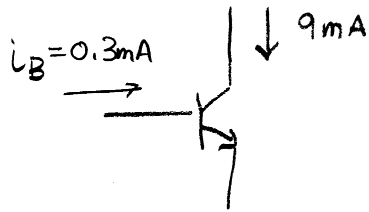
$$I_c = \beta I_B = 100(71.5\,\mu A)$$
$$= 7.15\,mA$$

$$V_{CE} = V_{CC} - I_c R_c$$
$$= 15 - (7.15\,mA)(1k)$$
$$V_{CE} = 7.85 \text{ volts.}$$

PSpice analysis gave $I_c = 7.18\,mA$ and $V_{CE} = 7.82$ volts.

Examples:

4.3 An npn transistor is operating with the base-emitter junction forward biased and the base-collector junction reverse biased. If $i_C = 9mA$ for $i_B = 0.3\ mA$, find i_E, α and β.



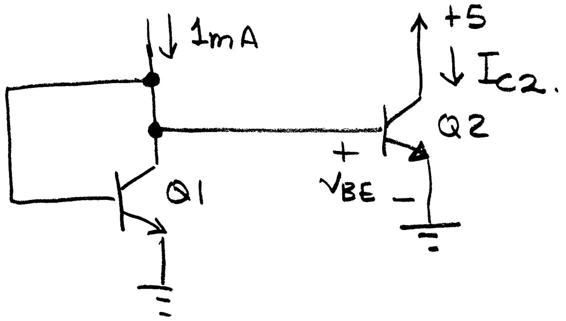
$$i_E = i_C + i_B = 9mA + 0.3mA = 9.3mA$$

$$\alpha = \frac{i_C}{i_E} = \frac{9mA}{9.3mA} = .9677$$

$$\beta = \frac{i_C}{i_B} = \frac{9mA}{0.3mA} = 30.$$

As a check $\beta = \frac{\alpha}{1-\alpha} = 30.$

4.11 Consider the circuit shown in Figure P4.11. The transistors Q_1 and Q2 are identical, both having $I_{ES} = 10fA = 10^{-14}A$. and $\beta = 100$. Find V_{BE} and I_{C2}. Assume a temperature of 300K.
Hint: both transistors are operating in the active region. Because the transistors are identical and have identical values of V_{BE}, their collector currents are equal.



Shockley equation $\quad i_E = I_{ES}\left[e^{\frac{v_{BE}}{v_T}} - 1\right]$

$$v_T = \frac{kT}{q} \cong 26mV \text{ @ } 300°K.$$

The key observation is that since the transistor bases are connected to the same point their V_{BE}'s are identical.

$$i_{B,Q1} + \beta i_{B,Q1} + i_{B,Q2} = 1mA$$

$$i_B + \beta i_B + i_B = 1mA$$

$$102\ i_B = 1mA$$

$$i_B = 9.8\mu A.$$

$$i_{C2} = \beta i_{B,Q2} = (100)(9.8\mu A) = 0.98\mu A.$$

@Q1 $\quad i_E = I_{ES}\left[e^{\frac{v_{BE}}{v_T}} - 1\right]$

$$(\beta+1)\,i_B = I_{ES}\left[e^{v_{BE}/v_T} - 1\right]$$

$$(101)\left(\frac{1mA}{102}\right) = 10^{-14}\left[e^{v_{BE}/v_T} - 1\right]$$

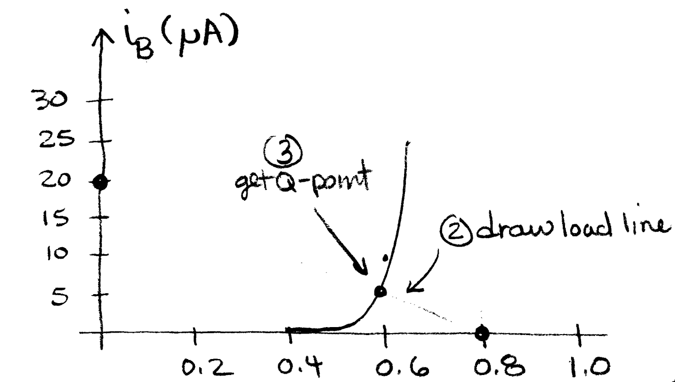
$$9.9\times10^{-4} = 10^{-14}\left[e^{v_{BE}/v_T} - 1\right]$$

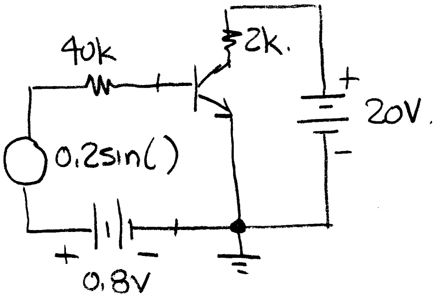
$$9.9\times10^{10} = \left[e^{v_{BE}/v_T} - 1\right]$$

$\therefore\ e^{v_{BE}/v_T} \cong 9.902\times10^{10}$ $\qquad \frac{v_{BE}}{v_T} = 25.318$ $\qquad v_{BE} \cong 658mV.$

Load line analysis of a common emitter amplifier

4.20. Consider the circuit of Figure 4.10. Assume $V_{CC} = 20V$, $V_{BB} = 0.8V$, $R_B = 40k\Omega$ and $R_C = 2k\Omega$. The input signal is a 0.2 volt peak, 1 kHz sinusoid given by $v_{in}(t) = 0.2 \sin(2000\pi t)$. The common-emitter characteristics for the transistor are shown in Figure P4.20. Find the maximum, minimum, and Q-point values for v_{CE}. What is the approximate voltage gain for this circuit.





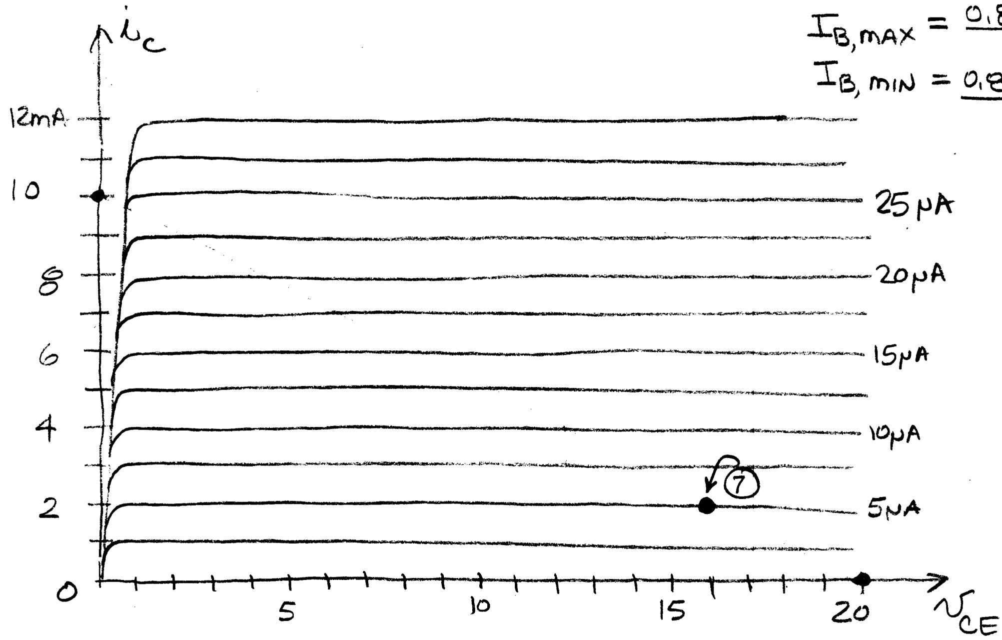
①
$$i_{sc} = \frac{0.8}{40k} = 0.02\ mA.$$

④ Alternatively, calculate

$$I_B = \frac{0.8 - 0.6\ V}{40k} = 5\mu A$$

$$I_{B,MAX} = \frac{0.8 - 0.6 + .2}{40k} = 10\mu A$$

$$I_{B,MIN} = \frac{0.8 - 0.6 - .2}{40k} = 0\mu A.$$



⑥ determine collector load line

$$I_{sc} = \frac{20V}{2k} = 10mA$$

$$V_{oc} = 20V$$

⑦ Determine Q-point for $I_B = 5\mu A$.

$$V_{CE,Q} = 16V.$$

$$I_{CE,Q} = 2mA$$

⑧ $\left.\begin{array}{l} v_{CE,MAX} \\ v_{CE,MIN} \end{array}\right\}$ Assuming symmetry and no distortion we get $v_{CE,Q} \pm 4V$

or $v_{CE,MAX} = 20$

$v_{CE,MIN} = 16 - 4 = 12V.$

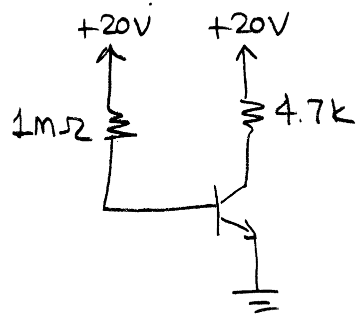
What does i_B do. $i_{B,MAX} = 10\mu A$ takes us to about 12V

likewise $i_{B,MIN} =$ [redacted] μA takes us to about 20V

⑨ $A_V \cong \frac{8\ volts}{.4\ volts} = 20$ ∎.

Large signal analysis of BJT circuits

4.33 Use the large signal models for the transistor illustrated to find I_C and V_{CE} for the circuits illustrated in Figure P4.33. Assume that $\beta = 100$. Repeat for $\beta = 300$ and compare the results.

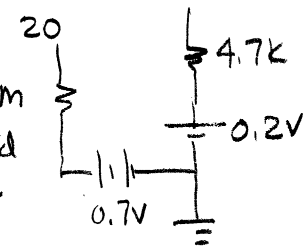


Is it cutoff, active, or saturated?

① cutoff has $v_{BE} = 20V \gg 0.7V$ so NOT this mode

② active has

saturated model



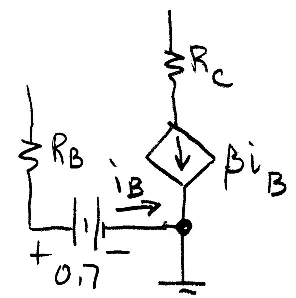
$$i_B = \frac{20 - 0.7}{1m} = 19.3\,\mu A > 0$$

$$i_C = \beta i_B = (100)(19.3\,\mu A)$$

$$i_C = 1.93\,mA.$$

$$i_C = \frac{20 - 0.2}{4.7k} = 4.2\,mA$$

active model



Since $\beta i_B < i_c$ we are in active region

$i_c = 1.93\,mA$ so $v_{CE} = 20 - (1.93)(4.7k) = 10.9$ volts.

@ $\beta = 300$ $\quad 300\, i_B = i_c = 5.79\,mA.$

$$v_{CE} = 20 - (5.79)(4.7) = -7.2 \text{ volts}$$

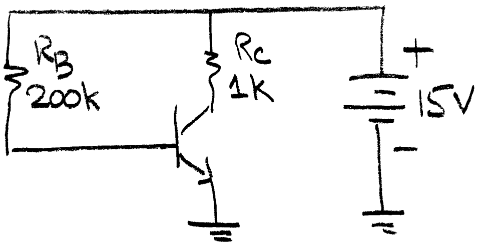
$\Rightarrow$ transistor is saturated.

BIAS CIRCUIT DESIGN

fixed – base circuit bias very dependent on β

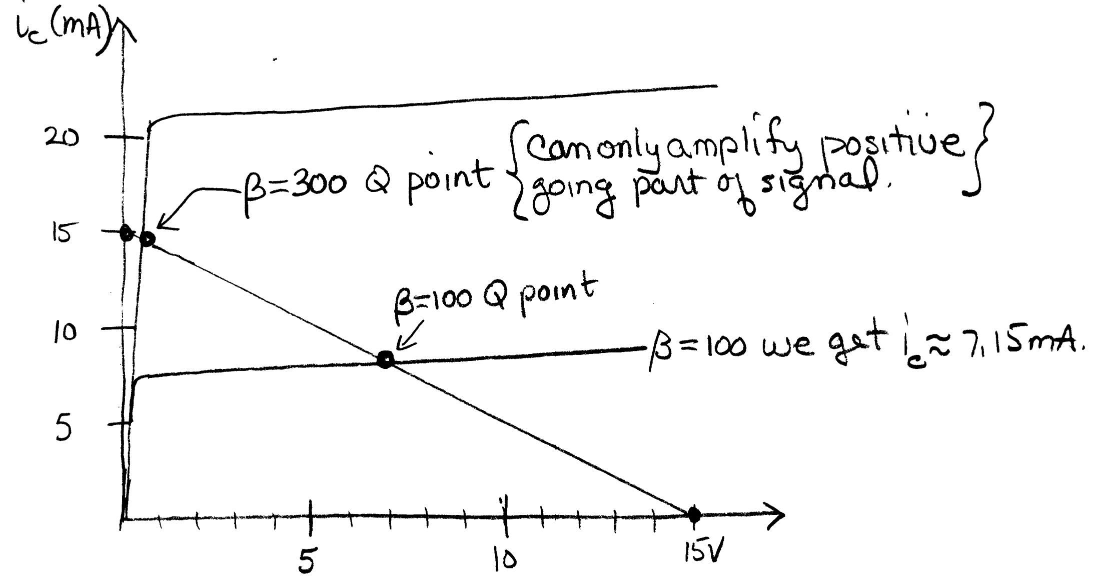
Consider a bias circuit that provides $I_B = 71.5\mu A$ to an npn transistor. The β for that transistor varies from 100 to 300.

The base circuit does NOT change, BUT the load circuit does since the transistor changes.



$$I_B = \frac{V_{cc} - 0.7}{R_B} = \frac{15 - 0.7}{200K} = 71.5\mu A.$$

This is independent of β. However, β influences the collector circuit.



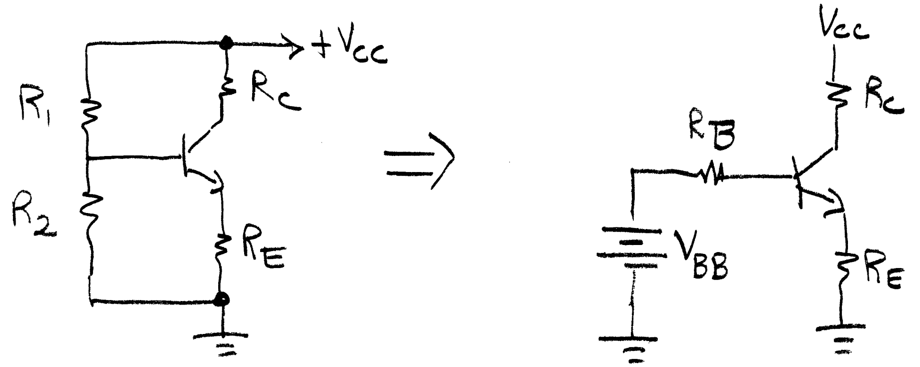
load line $V_{oc} = 15V$

$$I_{sc} = \frac{15V}{1k} = 15mA.$$

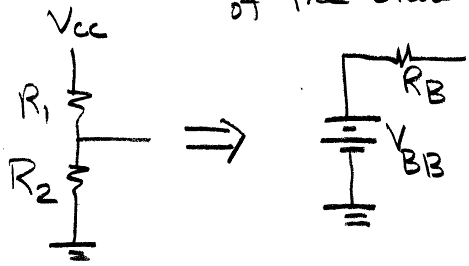
for $\beta = 100$ $i_c = \beta i_B = 7.15mA$

$\beta = 300$ $i_c = \beta i_B = 21.45mA.$

If you are designing a discrete BJT bias design you would use a 4-resistor bias circuit.



where R_B, V_{BB} are the Thevenin equivalent of the bias circuit



$$R_B = \frac{R_1 R_2}{R_1 + R_2}$$

$$V_{BB} = \frac{R_2}{R_1 + R_2} V_{cc}$$

Now do a base circuit analysis for the Thevenized bias circuit.

$$-V_{BB} + I_B R_B + v_{BE} + I_E R_E = 0.$$

Now use $v_{BE} \cong 0.7$ volts.

$$I_E = (\beta+1) I_B$$

Then $$I_B = \frac{V_{BB} - v_{BE}}{R_B + (\beta+1) R_E}$$

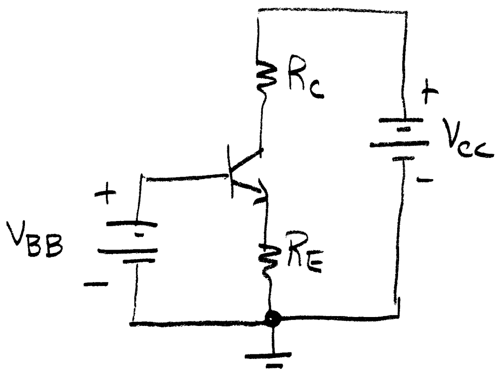
Now consider $$I_c = \beta I_B = \frac{\beta (V_{BB} - v_{BE})}{R_B + (\beta+1) R_E}$$

You can see that if $(\beta+1) R_E \gg R_B$ Then $I_c \cong \frac{\beta}{\beta+1}\left(\frac{V_{BB} - v_{BE}}{R_E}\right)$ which is effectively independent of β.

Ways to bias a transistor that are independent of β

1. emitter resistor
   a. 4-resistor bias network.
2. current source biasing.

Example 4.6



$V_{cc} = 15$
$V_{BB} = 5V$
$R_c = 2k\Omega$
$R_E = 2k\Omega$
$\beta = 100.$

We will assume $V_{BE} = 0.7$ volts.

For the base $\quad V_{BB} = 0.7 + I_E R_E$ or $I_E = \dfrac{V_{BB} - 0.7}{R_E} = \dfrac{5 - 0.7}{2k} = 2.15mA$

Note that I_E independent of β in this equation.

since $I_c = \beta I_B$ and $I_c + I_B = I_E$

we have $\beta I_B + I_B = (\beta+1) I_B = I_E$

$$\therefore \quad I_B = \frac{I_E}{\beta+1} = \frac{2.15\,mA}{100+1} = 21.3\mu A.$$

$$I_c = \beta I_B = (100)(21.3\mu A) = 2.13mA.$$

$$\begin{aligned} V_{CE} &= V_{cc} - I_c R_c - I_E R_E \\ &= 15 - (2.13)(2) - (2.15)(2) \\ &= 15 - 4.26 - 4.3 = 6.44. \end{aligned}$$

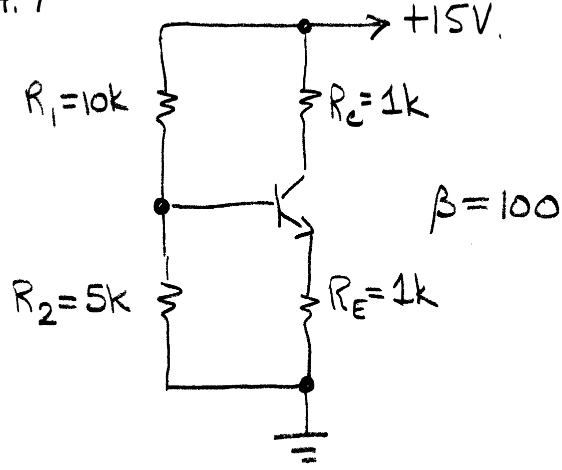
Now, this proves nothing until we compute the same thing for $\beta = 300$. Then

$$I_B = \frac{I_E}{\beta+1} = \frac{2.15mA}{300+1} = 7.14\mu A$$

$$I_c = \beta I_B = (300)(7.14\mu A) = 2.14mA$$

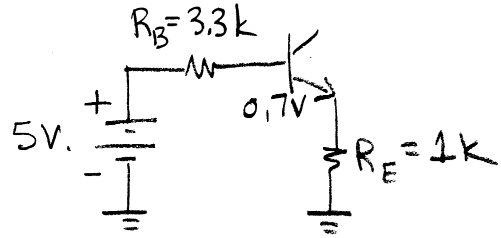
$$V_{CE} = V_{cc} - I_c R_c - I_E R_E = 15 - (2.14)(2) - (2.15)(2) = 6.42.$$

Example 4.7



Thevenize the input: $V_{BB} = \frac{5k}{5k + 10k} \cdot 15V = \frac{5}{15} \cdot 15 = 5$ volts.

$$R_B = \frac{R_1 R_2}{R_1 + R_2} = \frac{(10)(5)}{10+5} = \frac{50}{15} k = 3.3k$$



$$-5V + I_B(3.3k) + 0.7 + I_E(1k) = 0.$$

where $I_E = (\beta+1) I_B$

$$+5 - 0.7 = I_B(3.3k) + (\beta+1)(1k) I_B$$

$$I_B = \frac{5 - 0.7}{3.3k + (101)(1k)} = \frac{4.3}{104.3k}$$

$$\underline{I_B = 41.2 \mu A}$$

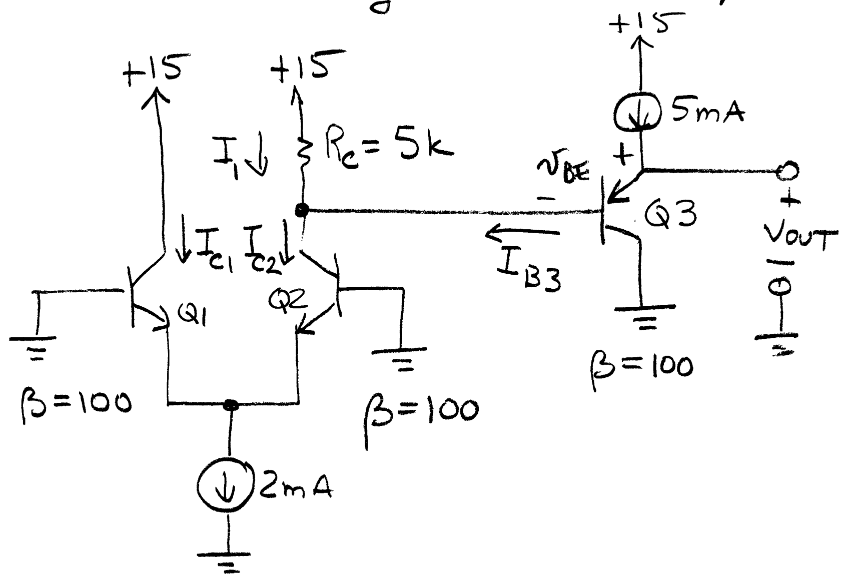
If $\beta = 300$ $\quad I_B' = \frac{4.3}{3.3k + (301)(1k)} = \frac{4.3}{304.3k} = 14.1 \mu A$

for $\beta = 100$ $\quad I_C = \beta I_B = (100)(41.2\mu A) = 4.12 mA$

$\beta = 300$ $\quad I_C' = \beta I_B = (300)(14.1 \mu A) = 4.23 mA$

small change!

Current source biasing (used in IC's)



Since Q_1, Q_2 are identical and their v_{BE}'s are identical

$$I_{E1} = I_{E2}.$$

From circuit $I_{E1} + I_{E2} = 2mA. \Rightarrow I_{E1} = I_{E2} = 1mA.$

$$I_{B1} = I_{B2} = \frac{I_{E1}}{\beta+1} = \frac{1mA}{100+1} = 9.9\mu A.$$

$$I_{C1} = I_{C2} = \beta I_{B1} = (100)(9.9\mu A) = 0.99mA.$$

Since I_E for $Q3 = 5mA$ we have

$$I_{B3} = \frac{I_{E3}}{\beta+1} = \frac{5mA}{100+1} = 49.5\mu A,$$

$$I_{C3} = \beta I_{B3} = (100)(49.5\mu A) = 4.95mA$$

Do KCL at collector of Q2

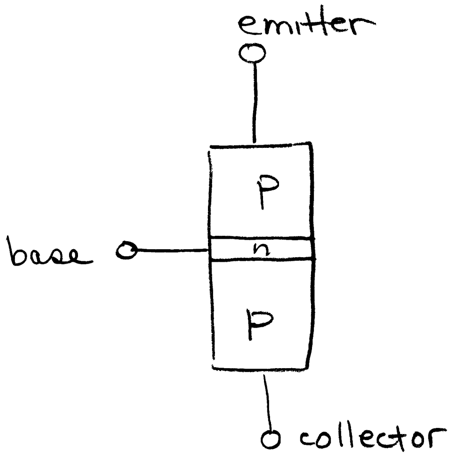
$$+I_1 - I_{C2} + I_{B3} = 0$$

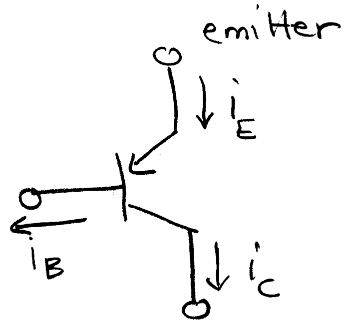
$$\therefore I_1 = I_{C2} - I_{B3} = 0.99mA - 49.5\mu A = 0.941\,mA.$$

Compute $V_o = 0.7 - I_1 R_c + 15$

$$= 0.7 - (0.941)(5) + 15 = 10.3 \text{ volts}.$$

pnp transistors

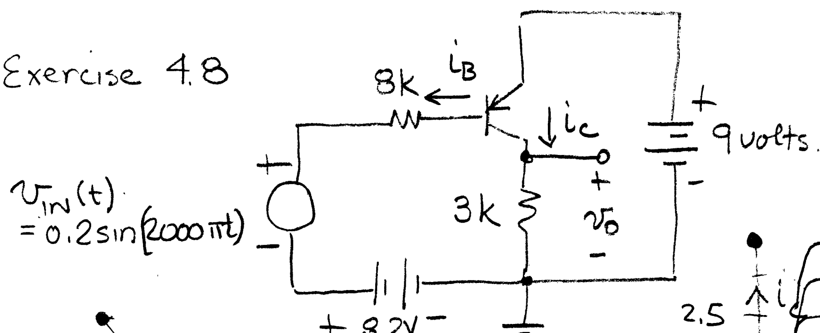




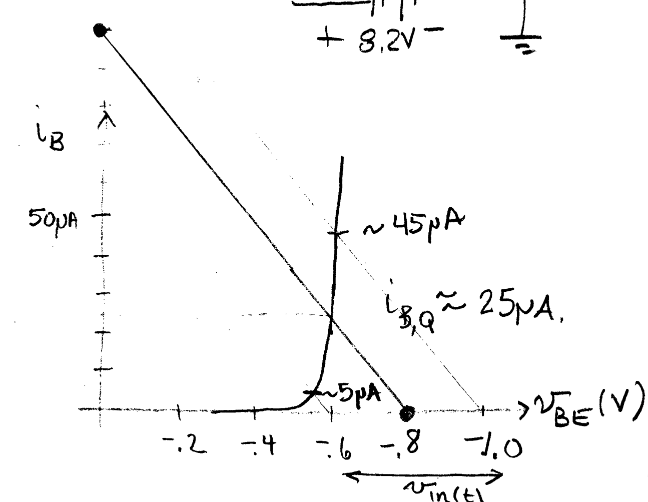
$$i_E = i_C + i_B$$
$$i_C = \beta i_B$$
$$i_E = I_{ES}\left(e^{-\frac{v_{BE}}{v_T}} - 1\right)$$

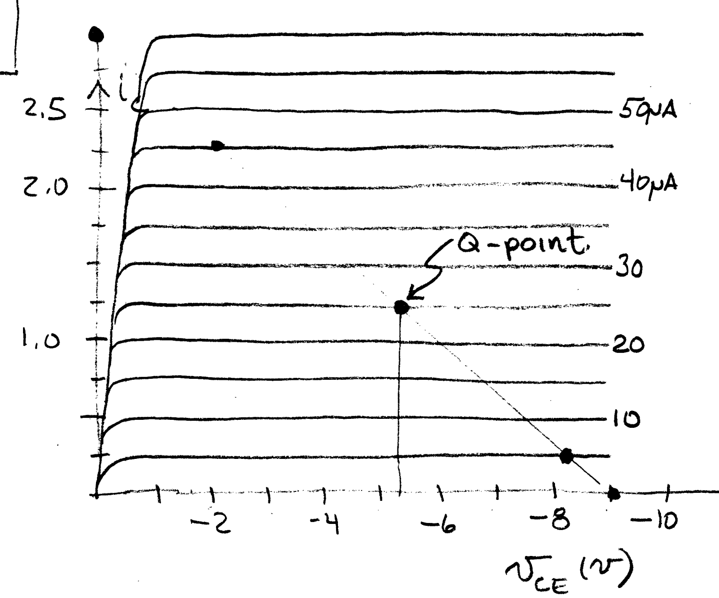
the equations for the pnp transistor are exactly the same as for the npn transistor.

Exercise 4.8



do analysis same as for npn but watch polarities





load line $V_{oc} = -0.8$ volts.

$$i_{sc} = \frac{0.8\text{ volts}}{8k} = 0.1\text{ mA} = 100\,\mu A.$$

$\Rightarrow i_{B,Q} \cong 25\,\mu A$.

load line: $V_{oc} = -9$ volts

$$i_{sc} = \frac{9\text{ volts}}{3K} = 3\text{ mA}.$$

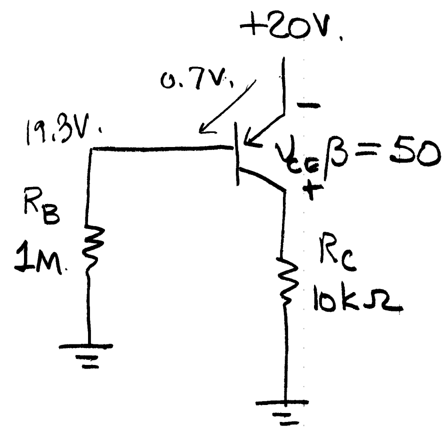
$\Rightarrow V_{CE,Q} \cong 5.3$ volts

$i_{C,Q} \cong 1.25$ mA.

$v_{CE,MAX} \cong -8.2$V,

$v_{CE,MIN} \cong -2$ volts

Another
pnp example.



(a) transistor is not cutoff since $V_{BE} > 0.5$ volts.

(b) active analysis:

$$I_B = \frac{20 - 0.7}{1M} = 19.3\,\mu A.$$

$$I_C = \beta I_B = (50)(19.3\,\mu A) = 965\,\mu A$$

at saturation $\quad I_{C,sat} = \dfrac{20 - 0.2}{10k} = \dfrac{19.8}{10}\,mA = 1.98\,mA$

$\Rightarrow$ transistor is active and

$$V_{CE} = 20 - I_C R_C = 20 - (.965\,mA)(10k)$$

$$= 20 - 9.65 = 10.35 \text{ volts}$$

Note polarity actually -10.35 volts.

4.6 Small Signal Equivalent Circuits

Little bit of theory.

$$i_B = (1-\alpha) I_{ES}\left[e^{\frac{v_{BE}}{V_T}} - 1\right]$$

write in terms of small signal variations about bias point

$$I_{B,Q} + i_b(t) = (1-\alpha) I_{ES}\left[e^{\frac{V_{BE,Q} + v_{be}(t)}{V_T}} - 1\right]$$

$$I_{B,Q} + i_b(t) \cong \underbrace{(1-\alpha) I_{ES}\, e^{\frac{V_{BE,Q}}{V_T}}}_{\text{but this is } I_{B,Q}} e^{\frac{v_{be}(t)}{V_T}}$$

$$I_{B,Q} + i_b(t) \cong I_{B,Q}\, e^{\frac{v_{be}(t)}{V_T}}$$

Now consider small-signals (on the order of millivolts)

$$e^x \cong 1 + x$$

using this approximation

$$I_{B,Q} + i_b(t) \cong I_{B,Q}\left[1 + \frac{v_{be}(t)}{V_T}\right]$$

$$\cancel{I_{B,Q}} + i_b(t) \cong \cancel{I_{B,Q}} + \frac{I_{B,Q}}{V_T} v_{be}(t)$$

$$i_b(t) = \frac{v_{be}(t)}{r_\pi} \quad \text{where } r_\pi \equiv \frac{V_T}{I_{B,Q}}$$

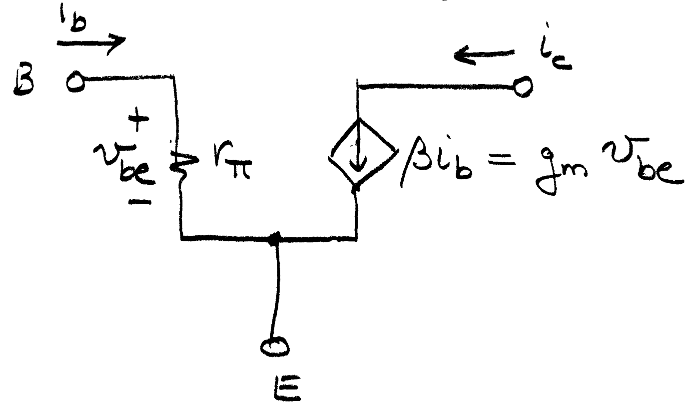
$$\text{or } r_\pi = \frac{\beta V_T}{I_{C,Q}}$$

$$I_{C,Q} + i_c(t) = \beta I_{B,Q} + \beta i_b(t)$$

so the small-signal components are related by

$$i_c(t) = \beta i_b(t).$$

This gives the <u>small-signal</u> equivalent circuit



4.42 (partial) An npn silicon transistor at room temperature has $\beta = 100$. Find the corresponding values of g_m and r_π if $I_{C,Q} = 1mA$. Assume that the device is operating in the active region.

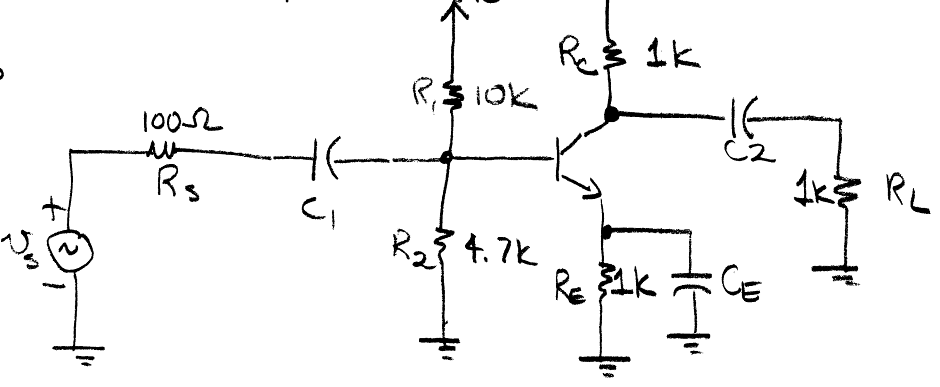
$$r_\pi = \frac{\beta V_T}{I_{C,Q}} = \frac{(100)(26mV)}{1mA} = 2600\Omega.$$

$$\beta = g_m r_\pi$$

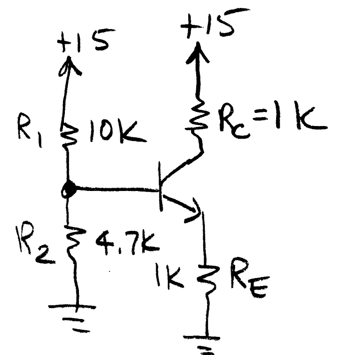
$$\text{or} \quad g_m = \frac{\beta}{r_\pi} = \frac{100}{2600}$$

Common-emitter amplifier

4.45



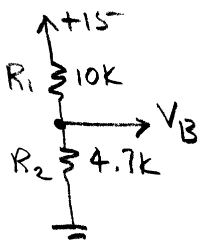
① DC circuit:



$\beta = 100$

$V_{CE,Q} = 0,7$

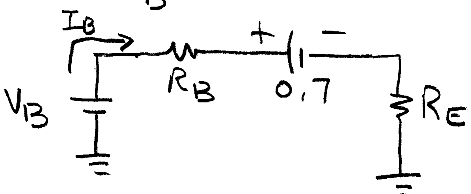
② do Thevenin of the bias circuit



$$V_B = \frac{4.7k}{10k + 4.7k} = 4.796 \text{ Volts}$$

$$R_B = R_1 \| R_2 = \frac{(10k)(4.7k)}{10k + 4.7k} = 3,197k.$$

③ Determine I_B



Doing KVL

$$-V_B + I_B R_B + 0.7 + (\beta+1) I_B R_E$$

or $$I_B = \frac{V_B - 0.7}{R_B + (\beta+1) R_E}$$

$$= \frac{4.796 - 0.7}{3,197k + (100+1)(1k)}$$

$$I_B = 39.3\,\mu A$$

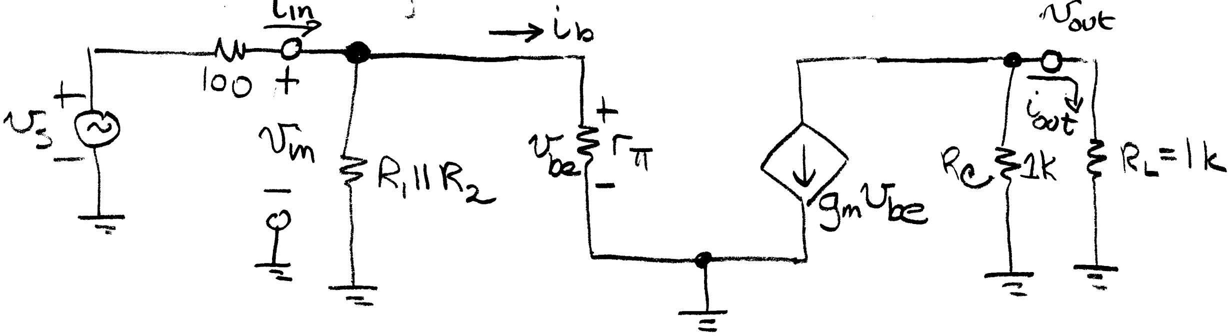
④ Determine Q-point

$$I_{c,Q} = \beta I_{B,Q} = (100)(39.3\mu A) = 3.93mA$$

$$V_{CE,Q} = V_{CC} - I_{c,Q} R_c - (\beta+1) I_{B,Q} R_E$$

$$= 15 - (3.93)(1) - (101)(.0393)(1) = 7.10V.$$

⑤ Now draw small signal circuit



⑥ Compute circuit values

$$R_1 \| R_2 = \frac{(10k)(4.7k)}{10k + 4.7k} = 3.198\,k$$

$$r_\pi = \frac{\beta V_T}{I_{C,Q}} = \frac{(100)(26mV)}{3.93\,mA} = 661.6\,\Omega$$

$$g_m = \frac{\beta}{r_\pi} = \frac{100}{661.6\Omega} = 0.151\,℧$$

⑦ Calculate v_{be}

In this case $v_{be} = v_{in} = \dfrac{(R_1 \| R_2) \| r_\pi}{(R_1 \| R_2) \| r_\pi + 100} v_s$.

$$(R_1 \| R_2) \| r_\pi = \frac{(3198)(661.6)}{3198 + 661.6} = 548.2$$

$$v_{be} = \frac{548.2}{548.2 + 100} v_s = 0.846\, v_s$$

⑧ Calculate v_{out}

Note: $v_{be} = v_{in}$

$$v_{out} = (-g_m v_{be})(R_C \| R_L)$$
$$= (-0.151)(\;v_{in}\;)\left(\frac{(1k)(1k)}{1k + 1k}\right)$$
$$= -75.5\, v_{in}$$

⑨ calculate A_v

$$A_v = \frac{v_{out}}{v_{in}} \cong -75.5$$

Note you could have computed this using βi_b as well ∎

(10) Compute no load gain

$$v_o = -g_m v_{be} R_C = -(0.151)(v_{in})(1000)$$

$$= -151\, v_{in}$$

$$A_{vo} = \frac{v_o}{v_{in}} = -151$$

(11) Calculate input impedance

$$Z_{in} = \frac{v_{in}}{i_{in}}$$ ← Note that $v_{in} = 0.846\, v_s$

we know v_{in} and must calculate i_{in}.

$$i_{in} = \frac{v_s}{100 + (R_1 \| R_2) \| r_\pi} = \frac{v_s}{100 + \frac{(3198)(661.6)}{3198 + 661.6}}$$

$$= \frac{v_s}{648.2\,\Omega}$$

$$\therefore \quad Z_{in} = \frac{0.846\, v_s}{\frac{v_s}{648.2\,\Omega}} = (0.846)(648.2)$$

since all done in terms of v_s this is OK.

$$= 548.4\,\Omega$$

(12) Calculate current gain

$$A_i = \frac{i_{out}}{i_{IN}} = \frac{v_{out}/R_L}{v_{in}/Z_{in}} = \frac{v_{out}}{v_{in}} \frac{Z_{in}}{R_L}$$

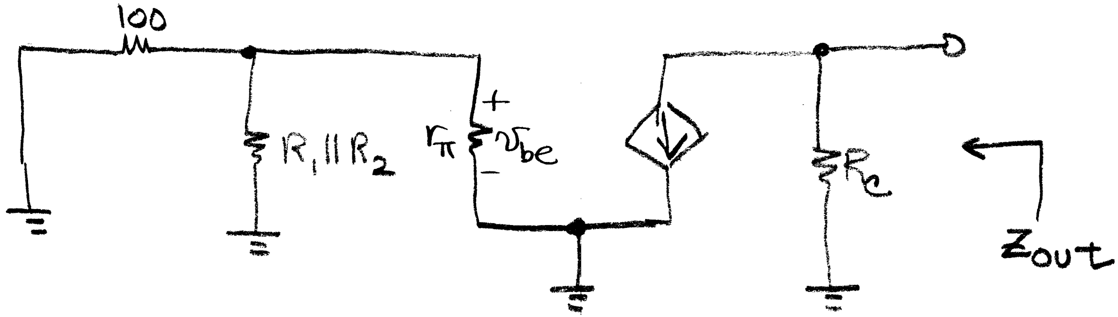
Note: R_L rather than R_L' because of where we defined the output

$$A_i = A_v \frac{Z_{in}}{R_L} = (-75.5)\,\frac{548.4\,\Omega}{1000\,\Omega} = -41.4$$

(13) Power gain

$$A_p = A_v A_i = (-75.5)(-41.4) = 3126$$

(14) Output impedance.



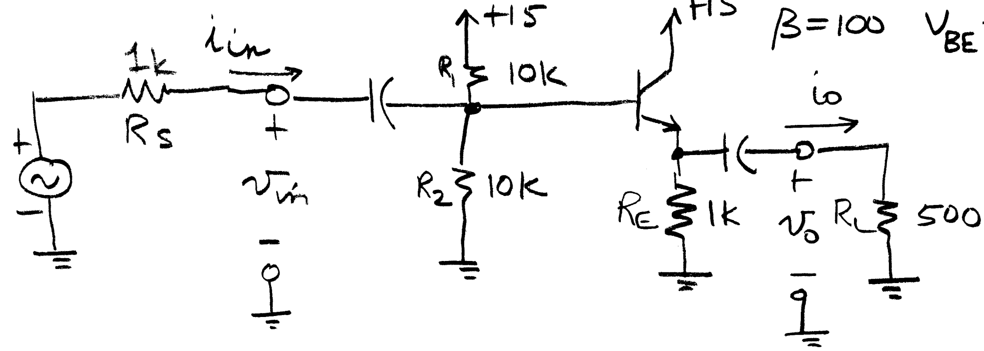
Z_{out} defined to be when $v_s = 0$.

If $v_s = 0$ then $v_{be} = 0$ and $g_m v_{be} = 0$

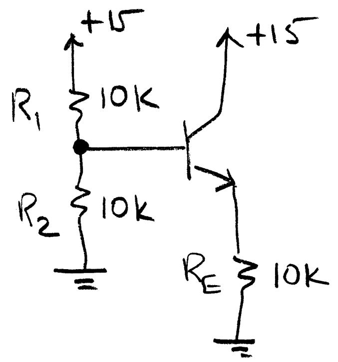
$\therefore \quad Z_{out} = R_c = 1k\Omega$

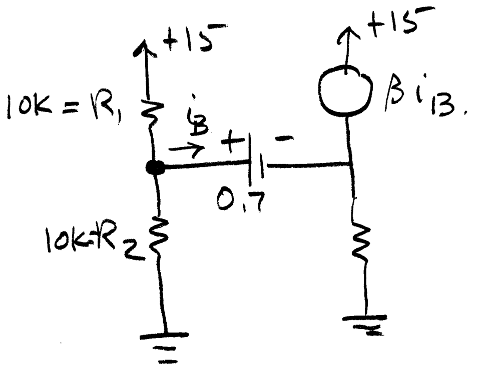
4.51 Consider the emitter follower amplifier. Draw the DC circuit and find $I_{C,Q}$. Find the value of r_π. Then calculate midband values for A_v, A_{vo}, A_i, G and Z_o.

$\beta = 100 \quad V_{BE} = 0.7\,V$.

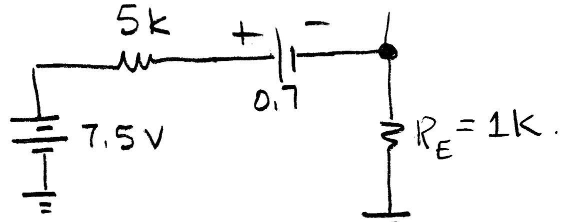


DC circuit





Thevenize input



doing KCL $\quad -7.5 + i_B(5k) + 0.7 + (\beta i_B)(1k) = 0$

$$i_B\,(5k + \beta(1k)) = 7.5 - 0.7$$

$$i_B = \frac{7.5 - 0.7}{5k + 100(1k)} = \frac{6.8}{105k} = 0.065\,mA$$

$$i_B = 65\,\mu A.$$

$i_{C,Q} = \beta i_B = 6.476\,mA$,

$i_{E,Q} = (\beta+1)\,i_B = (100+1)(65\,\mu A) = 6.54\,mA$.

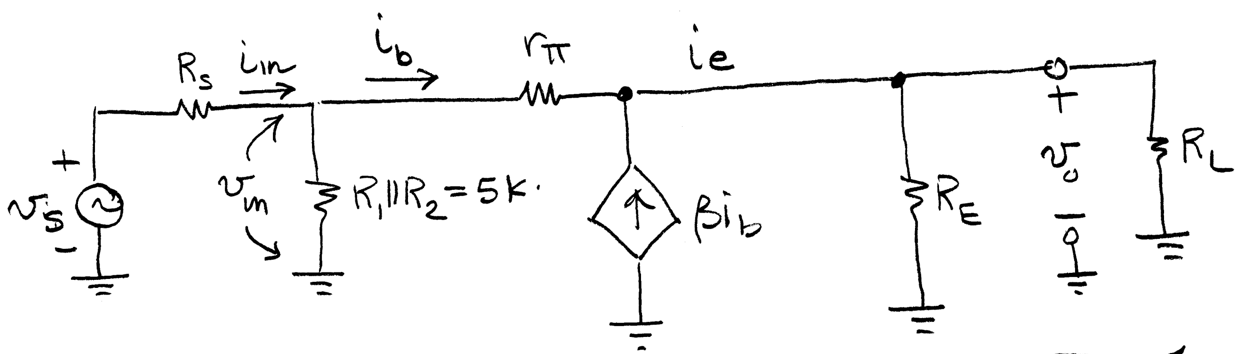
$V_E = i_E R_E = (6.54\,mA)(1k) = 6.54\,V$,

$V_{CE,Q} = 15 - 6.54\,V = 8.45\,V$.

Now calculate small signal parameters.

$$r_\pi = \frac{\beta V_T}{I_{c,Q}} = \frac{(100)(26mV)}{6.476\ mA} = 401.5\,\Omega$$

Now draw small signal circuit



$R_L' = R_E \| R_L$

$= \frac{(1000)(500)}{1000+500} = 333\,\Omega$

voltage gain:

write $v_o = i_e R_L' = (\beta+1) i_b R_L'$

then do KVL

$$-v_{in} + i_b r_\pi + (1+\beta) i_b R_L' = 0.$$

eliminate i_b

$$v_{in} = i_b \left[r_\pi + (\beta+1) R_L' \right]$$
$$v_o = i_b \left[(\beta+1) R_L' \right].$$

$$\therefore A_v = \frac{v_o}{v_{in}} = \frac{(\beta+1) R_L'}{r_\pi + (\beta+1) R_L'} = \frac{(101)\,333}{401.5 + (101)(333)}$$
$$= \frac{33667}{401.5 + 33667} = 0.988 \quad \text{(typical)}.$$

$$A_{vo} = \frac{v_o}{v_{in}} = \frac{i_b(\beta+1)R_E}{i_b[r_\pi + (\beta+1)R_E]}$$

$$A_{vo} = \frac{(\beta+1)R_L}{r_\pi + (\beta+1)R_L} = \frac{(101)(1000)}{401.5 + (101)(1000)} = 0.996$$

define the input impedance of the transistor Z_{it}

$$Z_{it} = \frac{v_{in}}{i_b} = \frac{i_b[r_\pi + (\beta+1)R_L']}{i_b}$$

$$Z_{it} = r_\pi + (\beta+1)R_L' = 401.5 + (101)333 = 34034.$$

$$Z_i = R_B \,\|\, Z_{it} = (5k)\,\|\,(34034) = \frac{(5)(34{,}034)}{5+34{,}034} = 4.36k$$

calculate the current gain

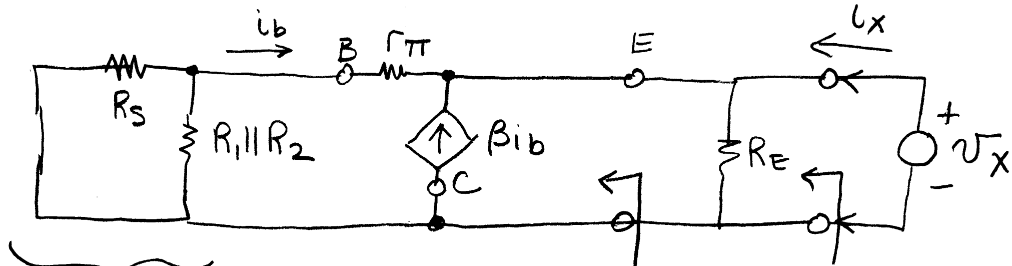
$$A_i = \frac{i_{out}}{i_{in}} = \frac{v_{out}/R_L}{v_{in}/Z_i} = \frac{v_{out}}{v_{in}} \cdot \frac{Z_i}{R_L} = A_v \frac{Z_i}{R_L}$$

$$A_i \quad (0.988)\,\frac{4360}{500} = 8.61$$

$$G = A_v A_i = (.988)(8.61) = 8.51$$

Finally calculate the output impedance

$$Z_o \equiv \frac{v_x}{i_x} \quad \text{where } v_s = 0.$$



$$R_s' = \frac{(1000)(5000)}{1000+5000} = 833.3\Omega \qquad Z_{ot} \qquad Z_o$$

Computing the output impedance is pretty complicated.

Write KVL for the outer loop as

$$i_b R_s' + i_b r_\pi + v_x = 0 \qquad (1)$$

Write KCL for the transistor's emitter

$$i_b + \beta i_b - \frac{v_x}{R_E} + i_x = 0 \qquad (2)$$

This gives us two equations which we can use to eliminate i_b

From (1) $\quad i_b(R_s' + r_\pi) = -v_x \quad$ or $\quad i_b = \dfrac{-v_x}{R_s' + r_\pi}$

From (2) $\quad i_b + \beta i_b - \dfrac{v_x}{R_E} + i_x = 0 \quad$ or $\quad i_b = \dfrac{v_x}{(\beta+1)R_E} - \dfrac{i_x}{\beta+1}$

Equating these results we get

$$\frac{-v_x}{R_s' + r_\pi} = \frac{v_x}{(\beta+1)R_E} - \frac{i_x}{\beta+1}$$

$$i_x = \frac{v_x}{R_E} + \frac{v_x}{(R_s' + r_\pi)/\beta+1} = \left[\frac{1}{R_E} + \frac{1}{\frac{R_s' + r_\pi}{\beta+1}}\right] v_x$$

$$\therefore \quad Z_o = \frac{v_x}{i_x} = \frac{1}{\frac{1}{R_E} + \frac{1}{\frac{R_s' + r_\pi}{\beta+1}}} = R_E \,\|\, \left(\frac{R_s' + r_\pi}{\beta+1}\right)$$

For this amplifier

$$Z_o = R_E \,\|\, \left(\frac{R_s' + r_\pi}{\beta+1}\right) = (1000) \,\|\, \left(\frac{833.3 + 401.5}{101}\right)$$

$$Z_o = 1000 \,\|\, 12.22 = \frac{(1000)(12.22)}{1000 + 12.22} = 12.1\,\Omega.$$