From Goodman

Chapter 2 - Analysis of two dimensional linear systems

Definition of two-dimensional Fourier transform

$$G = \Im[g] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y) e^{-j2\pi\left(xf_x + yf_y\right)} dxdy \qquad (2\text{-}1)$$

$$g = \Im^{-1}[G] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} G\left(f_x, f_y\right) e^{+j2\pi\left(xf_x + yf_y\right)} df_x df_y \qquad (2\text{-}2)$$

Definitions of functions

$$rect(x) = \begin{cases} 1, |x| \le \frac{1}{2} \\ 0, otherwise \end{cases}$$

$$\sin c(x) = \frac{\sin(\pi x)}{\pi x}$$

$$\text{sgn}(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$$

$$\Lambda(x) = \begin{cases} 1 - |x|, |x| \le 1 \\ 0. otherwise \end{cases}$$

$$comb(x) = \sum_{n=-\infty}^{+\infty} \delta(x - n)$$

$$circ(r) = \begin{cases} 1, r \le 1 \\ 0, otherwise \end{cases}$$

Some transform pairs

$e^{-\pi\left(x^2+y^2\right)}$	$e^{-\pi\left(f_x{}^2+f_y{}^2\right)}$
$rect(x)rect(y)$	$\sin c\left(f_x\right)\sin c\left(f_y\right)$
$\Lambda(x)\Lambda(y)$	$\sin c^2\left(f_x\right)\sin c^2\left(f_y\right)$
$\delta(x,y)$	1
$e^{-j\pi(x+y)}$	$\delta\left(f_x - \frac{1}{2}, f_y - \frac{1}{2}\right)$
$\text{sgn}(x)\text{sgn}(y)$	$\frac{1}{j\pi f_x}\frac{1}{j\pi f_y}$
$comb(x)comb(y)$	$comb\left(f_x\right)comb\left(f_y\right)$
$circ(r)$ where $r = \sqrt{x^2 + y^2}$	$\frac{J_1(2\pi\rho)}{\rho}$ where $\rho = \sqrt{f_x{}^2 + f_y{}^2}$

Superposition integral:

$$g_2(x_2, y_2) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g_1(\xi, \eta) h(x_2, y_2; \xi, \eta) d\xi d\eta \tag{2-21}$$

Chapter 3 - Scalar Diffraction Theory

Monochromatic wave $\hat{U}(P) = U(P)e^{-j\phi(P)}$

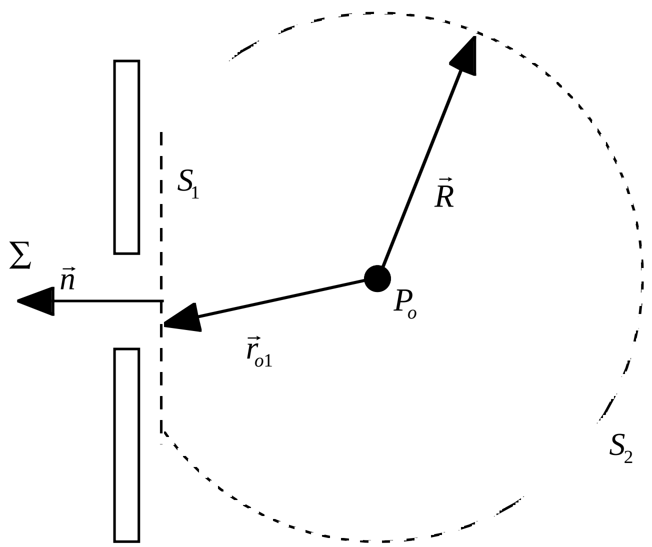
Wave equation $\left(\nabla^2 + k^2\right)\hat{U} = 0$ where $k = \frac{2\pi}{\lambda} = 2\pi\frac{\nu}{c}$

Hemholtz-Kirchoff integral theorem

$$\hat{U}(P_o) = \frac{1}{4\pi}\iint_S \left\{\frac{\partial\hat{U}}{\partial n}\frac{e^{jkr_{o1}}}{r_{o1}} - \hat{U}\frac{\partial}{\partial n}\left(\frac{e^{jkr_{o1}}}{r}\right)\right\}dS$$

This expresses the field at point P_o in terms of the boundary conditions of the wave on any closed surface surrounding P_o assuming a spherical wave expanding about P_o.

Kirchoff diffraction by a plane screen



where $G = \frac{e^{jkr_{o1}}}{r_{o1}}$

Evaluated on S_2: $\frac{\partial G}{\partial n} = \left(jk - \frac{1}{R}\right)\frac{e^{jkR}}{R} \cong jkG$ if $R >> 1$

As $R \to \infty$ the integral should vanish.

$$U(P_o) = \frac{1}{4\pi}\iint_{S_1+S_2}\left\{\frac{\partial\hat{U}}{\partial n}\hat{G} - \hat{U}\frac{\partial\hat{G}}{\partial n}\right\}dS$$

The Sommerfeld radiation condition says the integral over S_2 vanishes.

$$U(P_o) = \frac{1}{4\pi}\iint_{S_1}\left\{\frac{\partial \hat{U}}{\partial n}\hat{G} - \hat{U}\frac{\partial \hat{G}}{\partial n}\right\}dS$$

Kirchoff boundary conditions on diffraction:

I. across Σ $\hat{U}$ and $\frac{\partial \hat{U}}{\partial n}$ are exactly the same as if no screen were present

II. over S_1 lying in the shadow of the screen $\hat{U}$ and $\frac{\partial \hat{U}}{\partial n}$ are zero.

$$U(P_o) = \frac{1}{4\pi}\iint_{\Sigma}\left\{\frac{\partial \hat{U}}{\partial n}\hat{G} - \hat{U}\frac{\partial \hat{G}}{\partial n}\right\}dS$$

Fresnel-Kirchoff diffraction formula:

I. if $r_{o1} >> \lambda$ then $\frac{\partial \hat{G}}{\partial n} \cong jk\vec{n} \bullet \vec{r}_{o1}\frac{e^{jkr_{o1}}}{r_{o1}}$

II. spherical wave illumination $\hat{U}(P_1) = A\frac{e^{jkr_{21}}}{r_{21}}$ at aperture

$$U(P_o) = \frac{A}{j\lambda}\iint_{\Sigma}\left\{\frac{e^{jk(r_{21}+r_{o1})}}{r_{o1}r_{21}}\frac{\vec{n} \bullet \vec{r}_{o1} - \vec{n} \bullet \vec{r}_{21}}{2}\right\}dS$$

The trouble with the Kirchoff boundary conditions is that they mathematically cause $U(P_o)$ to identically vanish over all space.

Chapter 4 - Fresnel and Fraunhofer diffraction

(Diffraction of monochromatic light by a finite aperture Σ in an infinite opaque screen.)

Huyghens-Fresnel principle

$$\hat{U}(x_o, y_o) = \iint_\Sigma h(x_o, y_o; x_1, y_1)\hat{U}(x_1, y_1)dx_1 dy_1$$

where $h(x_o, y_o; x_1, y_1) = \frac{1}{j\lambda}\frac{e^{jkr_{o1}}}{r_{o1}}\cos(\vec{n} \bullet \vec{r}_{o1})$

Simplifying assumptions:

(1) z>> characteristic size of Σ

(2) confine interest near axis, i.e., paraxial assumption gives $\cos(\vec{n} \bullet \vec{r}_{o1}) \cong 1$ and

$$h(x_o, y_o; x_1, y_1) \cong \frac{1}{j\lambda z} e^{jkr_{o1}}$$

Fresnel approximations:

If $r_{o1} = \sqrt{z^2 + (x_o - x_1)^2 + (y_o - y_1)^2} = z\sqrt{1 + \left(\frac{x_o - x_1}{z}\right)^2 + \left(\frac{y_o - y_1}{z}\right)^2}$

The Fresnel approximation is that $r_{o1} \cong z\left(1 + \frac{1}{2}\left(\frac{x_o - x_1}{z}\right)^2 + \frac{1}{2}\left(\frac{y_o - y_1}{z}\right)^2\right)$

This gives $h(x_o, y_o; x_1, y_1) \cong \frac{1}{j\lambda z} e^{jkz} e^{j\frac{k}{2z}\left[(x_o - x_1)^2 + (y_o - y_1)^2\right]}$ which is an approximation of the spherical wave by a quadratic surface.

Interpretation of the Fresnel approximations:

I. $\hat{U}(x_o, y_o) = \left[h(x_o, y_o; x_1, y_1) * U(x_1, y_1)\right]$

II. $\hat{U}(x_o, y_o) = e^{j\frac{k}{2z}\left[x_o^2 + y_o^2\right]} \frac{e^{jkz}}{\lambda z} \Im\left[U(x_1, y_1) * e^{j\frac{k}{2z}\left[x_1^2 + y_1^2\right]}\right]\Bigg|_{f_x = \frac{x_o}{\lambda z},\, f_y = \frac{y_o}{\lambda z}}$

$$H(f_x, f_y) = e^{jkz} e^{-j\pi\lambda z\left(f_x^2 + f_y^2\right)}$$

Fraunhofer approximation

If $z >> \left.\frac{k(x_1^2 + y_1^2)}{2}\right|_{\max}$

Then $\hat{U}(x_o, y_o) \cong \Im\left[U(x_1, y_1)\right]\Big|_{f_x = \frac{x_o}{\lambda z},\, f_y = \frac{y_o}{\lambda z}}$

Chapter 5 - Fourier Transforming and Imaging Properties of Lenses

A thin lens may be modeled as a transmissivity of the form
$t_\ell(x,y) = e^{jk\Delta_o} e^{jk(n-1)\Delta(x,y)}$
where Δ_o is the maximum thickness of the lens and $\Delta(x,y)$ is the thickness of the lens.

If x and y are small in comparison to the radii of the surfaces of the lens (paraxial assumption) then

$$t_\ell(x,y) \cong e^{jkn\Delta_o} e^{-j\frac{k}{2f}\left(x^2+y^2\right)}$$

i.e., the spherical surfaces of the lens have been approximated by parabolic surfaces. The sign of " f " determines whether the lens produces a convergent or divergent spherical wave.

Pupil function - accounts for the finite size of the lens.

Fourier transforming - transmittance in front of the lens at a distance d from the lenbs

$$F_o\left(f_x, f_y\right) = \Im\left\{\Delta t_o\right\}$$

Incident on lens is $F_\ell\left(f_x, f_y\right) = \Im\left\{U_\ell\right\}$

General propagation using Fresnel approximations:

$$F_\ell\left(f_x, f_y\right) = F_o\left(f_x, f_y\right) e^{-j\pi\lambda d_o\left(f_x^2+f_y^2\right)}$$

General response of the lens is given by

$$U_f\left(x_f, y_f\right) = \frac{e^{j\frac{k}{2f}\left(x_f^2+y_f^2\right)}}{j\lambda f} F_\ell\left(\frac{x_f}{\lambda f}, \frac{y_f}{\lambda f}\right)$$

where $f_x = \frac{x_f}{\lambda f}$ and $f_y = \frac{y_f}{\lambda f}$.

This gives $U_f\left(x_f, y_f\right) = \dfrac{e^{j\frac{k}{2f}\left(1-\frac{d_o}{f}\right)\left(x_f^2+y_f^2\right)}}{j\lambda f} F_o\left(\dfrac{x_f}{\lambda f}, \dfrac{y_f}{\lambda f}\right)$ which becomes an exact Fourier transform if $d_o = f$.

Image formation

Response of a lens of focal length f to a point source at d_o in front of the lens. The response is in the plane d_i behind the lens

$$h(x_i, y_i; x_o, y_o) \cong \frac{1}{\lambda^2 d_o d_i} e^{j\frac{k}{2d_i}(x_i^2+y_i^2)} e^{j\frac{k}{2d_o}(x_o^2+y_o^2)} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P(x,y) e^{j\frac{k}{2}\left(\frac{1}{d_o}+\frac{1}{d_i}-\frac{1}{f}\right)(x^2+y^2)} e^{-jk\left(\frac{x_o}{d_o}+\frac{x_i}{d_i}\right)} e^{-jk\left(\frac{y_o}{d_o}+\frac{y_i}{d_i}\right)} dxdy$$

After approximations

$$h(x_i, y_i; x_o, y_o) \cong \frac{1}{\lambda^2 d_o d_i} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P(x,y) e^{j\frac{k}{2}\left(\frac{1}{d_o}+\frac{1}{d_i}-\frac{1}{f}\right)(x^2+y^2)} e^{-jk\left[\left(\frac{x_o}{d_o}+\frac{x_i}{d_i}\right)+\left(\frac{y_o}{d_o}+\frac{y_i}{d_i}\right)\right]} dxdy$$

The lens law states that $\frac{1}{d_o}+\frac{1}{d_i}-\frac{1}{f}=0$, i.e., at the point where the image reforms

$$h(x_i, y_i; x_o, y_o) \cong \frac{1}{\lambda^2 d_o d_i} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P(x,y) e^{-jk\left[\left(\frac{x_o}{d_o}+\frac{x_i}{d_i}\right)+\left(\frac{y_o}{d_o}+\frac{y_i}{d_i}\right)\right]} dxdy$$

We can define the magnification $M = \frac{d_o}{d_i}$ to get

$$h(x_i, y_i; x_o, y_o) \cong \frac{1}{\lambda^2 d_o d_i} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P(x,y) e^{-j\frac{2\pi}{\lambda d_i}\left[(x_i+Mx_o)x+(y_i+My_o)y\right]} dxdy$$

which is the Fraunhofer diffraction of the lens aperture centered at $x_i = -Mx_o$ and $y_i = -My_o$.

Chapter 6 -Frequency Analysis of Optical Imaging Systems

An imaging system is diffraction limited if it converts an incoming spherical wave into a new, perfectly spherical wave.

Diffraction limited coherent system

$$U_i(x_i,y_i)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\tilde{h}(x_i-\tilde{x}_o,y_i-\tilde{y}_o)U_g(\tilde{x}_o,\tilde{y}_o)d\tilde{x}_o d\tilde{y}_o=\tilde{h}*U_g$$

where $U_g(\tilde{x}_o,\tilde{y}_o)$ is the geometric optics image and

$$\tilde{h}(x_i,y_i)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}P(\lambda d_i\tilde{x},\lambda d_i\tilde{y})e^{-j2\pi(x_i\tilde{x}+y_i\tilde{y})}d\tilde{x}d\tilde{y}$$

$$G_g(f_x,f_y)=\Im\left[U_g(\tilde{x}_o,\tilde{y}_o)\right]$$

$$G_i(f_x,f_y)=\Im\left[U_i(x_i,y_i)\right]$$

$$H(f_x,f_y)=\Im\left[\tilde{h}(x_i,y_i)\right]=P(\lambda d_i f_x,\lambda d_i f_y)$$

$$G_i=HG_g$$

diffraction limited incoherent system

$$I_i(x_i,y_i)=\kappa\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left|\tilde{h}(x_i-\tilde{x}_o,y_i-\tilde{y}_o)\right|^2 I_g(\tilde{x}_o,\tilde{y}_o)d\tilde{x}_o d\tilde{y}_o$$

$$\mathcal{G}_g(f_x,f_y)=\frac{\Im\left[I_g(\tilde{x}_o,\tilde{y}_o)\right]}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}I_g(\tilde{x}_o,\tilde{y}_o)d\tilde{x}_o d\tilde{y}_o}$$

$$\mathcal{G}_i(f_x,f_y)=\frac{\Im\left[I_i(x_i,y_i)\right]}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}I_g(x_i,y_i)dx_i dy_i}$$

$$\mathcal{H}(f_x,f_y)=\frac{\Im\left[\left|\tilde{h}\right|^2\right]}{\Im\left[\left|\tilde{h}\right|^2\right]\Big|_{f_x=0,\,f_y=0}}$$

$\mathcal{H}$ is called the OTF (optical transfer function)
$|\mathcal{H}|$ is called the MTF (modulation transfer function)

$$\mathcal{H}(f_x, f_y) = \frac{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P\left(\xi + \frac{\lambda d_i f_x}{2}, \eta + \frac{\lambda d_i f_y}{2}\right) P\left(\xi - \frac{\lambda d_i f_x}{2}, \eta - \frac{\lambda d_i f_y}{2}\right) d\xi d\eta}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P^2(\xi,\eta) d\xi d\eta}$$

The numerator represents the area of overlap of two displaced pupil functions, one centered at $\left(+\frac{\lambda d_i f_x}{2}, +\frac{\lambda d_i f_y}{2}\right)$ and the second centered at $\left(-\frac{\lambda d_i f_x}{2}, -\frac{\lambda d_i f_y}{2}\right)$. The denominator simply normalizes the area of overlap by the total area of the pupil.

The OTF of a diffraction limited system extends to a frequency that is twice the cutoff of the coherent transfer function. Coherent cutoff refers to components of image amplitude, while incoherent cutoff refers to frequency components of image intensity.

The frequency spectrum of the image intensity

incoherent: $\Im[I_i] = (\mathcal{H} \otimes \mathcal{H})(\mathcal{G}_g \otimes \mathcal{G}_g)$
coherent: $\Im[I_i] = \mathcal{H}\mathcal{G}_g \otimes \mathcal{H}\mathcal{G}_g$
where $\otimes$ denotes the autocorrelation function

Chapter 7 - Spatial Filtering and Optical Information Processing

Photographic film

$\exp osure(E) = \mathcal{I}_T$

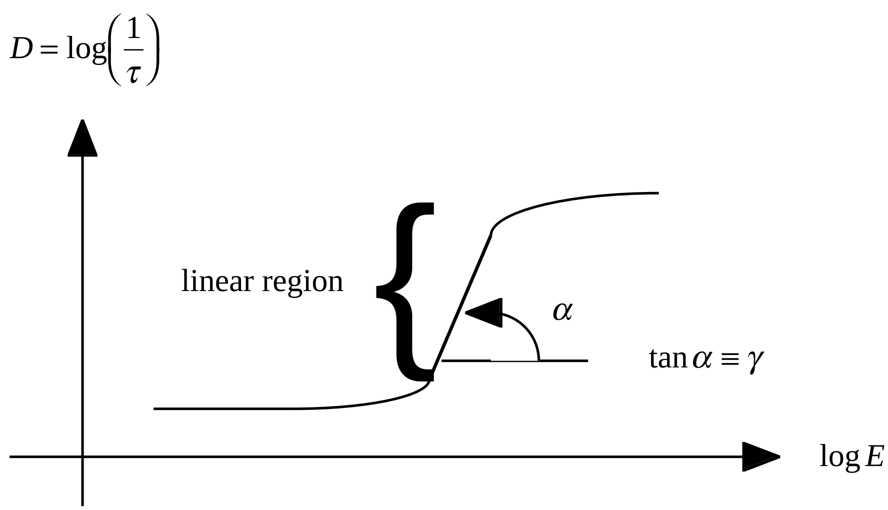
where $\mathcal{I}$ is the incident intensity and T is the exposure time.

$$\tau(x,y) \equiv local\_average\left[\frac{I[transmitted\_at\_(x,y)]}{I[incident\_at\_(x,y)]}\right]$$

$$D = \log\left(\frac{1}{\tau}\right)$$

The intensity transmittance of the developed transparancy is

H&D curve:



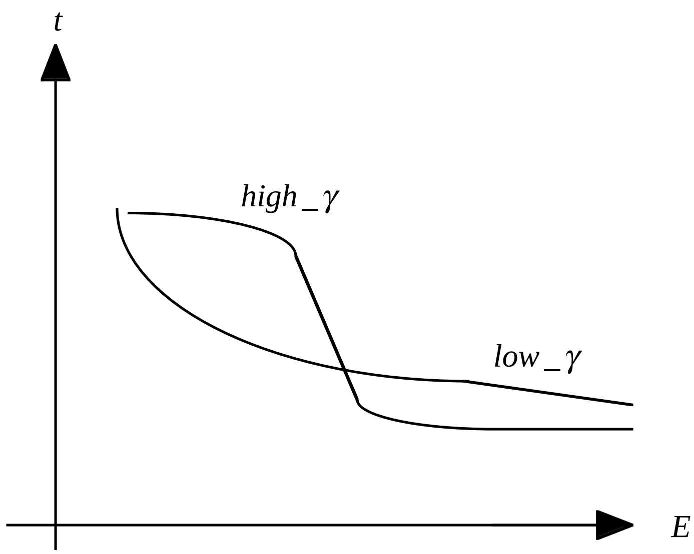
Film in an incoherent optical system $\tau_m = \kappa_m \mathcal{I}^{-\gamma_n}$

Film in a coherent optical system

The amplitude transmittance is $t_n = k_n \mathcal{I}^{-\frac{\gamma_n}{2}}$

$$t_p = k_p \mathcal{I}^{\frac{\gamma_p}{2}}$$

Notice that if $\gamma_p = 2$ the film acts as a square law mapping of intensity


t
high_γ
low_γ
E

Chapter 8 - Holography

Reference wave $\hat{A}(x,y) = A(x,y)e^{-j\psi(x,y)}$

Signal with which $\hat{A}(x,y)$ interferes is $\hat{a}(x,y) = a(x,y)e^{-j\phi(x,y)}$
$\mathcal{I}(x,y) = (\hat{A} + \hat{a})(\hat{A}^* + \hat{a}^*) = A^2 + a^2 + 2Aa\cos(\psi(x,y) - \phi(x,y))$

Recording medium:
1. linear mapping of intensity into amplitude transmittance, i.e., $\gamma_p = 2$
2. MTF of emulsion extends high enough to record the incident spatial structure

III. assume has constant intensity across the recording surface

$t_f(x,y) = t_b + \beta'\left(\hat{a}^2 + \hat{A}^*\hat{a} + \hat{A}\hat{a}^*\right)$

illuminate by a reference beam $\hat{B}$

light transmitted by a transparancy is $B(x,y)t_f(x,y)$

if B is A then $\Delta t_f(x,y) = t_b A + \beta'\,\hat{a}^2\hat{A} + \beta'\,\hat{A}^2 a + \beta'\,AAa^*$ where the last two terms are the ones of interest

Lecture 9

I. mode control
   A. transverse mode control using an iris
      Ref. Guesic et al, "Coherent Optical Sources"
      Ref. Kruger et al, "Laser Modes: Some basic Concepts"
      A. Relative size of modes
      B. Divergence angle and diffraction loss due to finite mirror size
   B. Longitudinal mode control
      B. Fabry-Perot etalon - Yariv Section 4.1

II. Diffraction gratings
   (1) Double slit interference - JMA, p. 163-166, 168-171
   (2) The grating equation
      1. diffraction orders
      2. angular dispersion
      3. resolving power
   (3) Types of gratings
      1. Blazed gratings
      2. Echelon

Lecture 10

Fiber Optics: Re: Research Toward Optical Fiber Transmission Systems
Yariv - Guided Wave optics, Section 13 - Guided wave modes

I. Basic concepts - differeing index of refraction to confine the rays
   A. Total internal reflection - JWA p.18-19
   B. Numerical aperture - JWA p. 74-76

II. Types of fiber
   A. Rectangular index profile
   C. parabolic index profile
      Marcatelli - "What kind of optical fiber for long distance research"

III. Fields and modes
   A. Propagation constant - Di Domenico $\beta = n_1(b\Delta + 1)$
   B. Modes as a function of guide diameter
   C. Common single and multi-mode fiber types (Miller et al)
   D. Dispersion
      1. Modal

         $\Delta t \approx \left(\frac{n}{c}\Delta\right)L$ for rectangular

         $\Delta t \approx \left(\frac{n}{2c}\Delta^2\right)L$ for parabolic

      2. Chromatic

         $$\Delta t \approx \frac{1}{c}\frac{\Delta\lambda}{\lambda}\lambda^2\frac{d^2n}{d\lambda^2}$$

IV. Losses due to
   A. bends

B. cladding losses
C. material absorption
D. Rayleigh scattering
E.

Lecture #11 Photometry

I. Photometry
  A. Radiometric quantities
  B. Photometric quantities
  C. Relationships between systems
II. Photodetectors
  A. Figures of Merit - Limperis
    1. Responsivity $R = \frac{V_{S,RMS}}{P_{S,RMS}}$
    2. Noise equivalent power
    3. Detectivity - $D, D^{\star}, D^{\star\star}$

Begin lecture #12 here
  B. Noise, Yariv Chapter 10
    1. Example of NEP - 10.1
    2. Noise - Basic definitions and theorems
      a. resolution (integration time)
      b. power spectral density
    3. spectral density of a train of randomly occurring events
      a. Carson's theorem - $S(\omega) = \frac{\overline{N}|F(\omega)|^2}{\pi}$
    4. Shot noise
    5. Johnson noise - statistical derivation only
    6. Spontaneous emission noise (discussion only)

lecture #13
I. The detection of optical radiation (Yariv, chapter 11)
  A. Optically induced transition rates - coherent "beating"
  B. The photomultiplier
    1. avalanche mechanism
    2. NEP - video detection (point out high value of R_L if shot noise limited and that high gain minimizes Johnson noise contribution)
    3. NEP - heterodyne detection
Lecture #14
  C. photoconductive detectors
    1. basic circuit operation
    2. solid state
      a. energy bands (conduction and valence bands in conductors, semiconductors and insulators)

b. intrinsic semiconductors
c. extrinsinic semiconductors ("doping")
3. generation-recombination noise
4. heterodyne detection

lecture#15 here

D. photodiodes
1. the basic p-n junction
2. photodiode
a. electron-hole production
b. p-i-n photodiode
c. frequency response
d. S/N for video detection
3. avalanche photodiode to improve S/N
a. mechanism of operation
b. sensitivity
c. S/N for video detection

Lecture #16

Source: Motorola Application Notes AN-440, AN-508, AN-561

I. Phototransistor

A. photodiode base-emitter juntion
B. equivalent circuit model
1. photocurrent source
2. frequency response
3. linearity of response
4. switching times

II. How to use phototransistors with incandescent light

A. sensitivity as a function of color temperature (for incandescent light)
1. compute source intensity
2. compute irradiance
3. computer expected photocurrent

B. use of lens to increase sensitivity