Print engine name:	PrintServer 20
Print engine version:	17
Printer firmware version:	32
Server Adobe PostScript version:	48.3
Server software version:	V2.0
Server network node:	crawford
Server name:	crawford
Server job number:	1
Client software version:	WRL-1.0
Client network node:	util
Client name:	flm
Client job name:	Circuits_acdc.ps.732464587
Submitted at:	Thu Mar 18 09:39:19 199320:41 1993
Printed at:	Thu Mar 18 09:39:19 1993

flm@util
Circuits_acdc.ps.732464587

ELECTRICAL CIRCUITS
DC AND AC

INDEX

The section numbers refer to chapters and sections in Engineering in Training Review Manual, 8th Edition by Michael Lindeburg.

Current may be defined by a derivative, i.e. the rate at which charge is moved:

$$i = \frac{dq}{dt}$$

or in the form of an integral as the total charge moved:

$$q = \int_0^t i(t)\, dt$$

where i is in units of coulombs/second , or amperes.

47.1 Voltage

Voltage is a measure of the work required to move a unit charge through an electric field (usually inside an electrical circuit element such as a resistor). It requires one joule of energy to move one coloumb of charge through a potential difference of one volt.

$$w = \int v\, dq$$

where w is in joules, v is in volts and dq is in coulombs.

47.2 Resisitivity and Resistance

The resistance of ordinary wire can be calculated provided one knows the resistivity of the wire as

$$R = \frac{\rho l}{A}$$

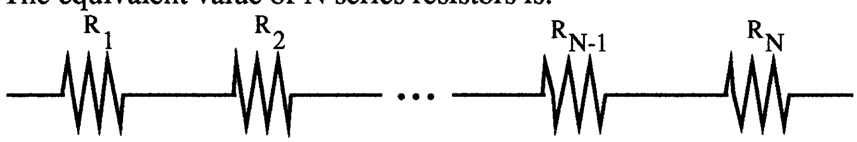
where l is the length of the wire and A is its cross-sectional area. The resistivity ρ is a property of the material and a function of temperature as given by

$$\rho = \rho_0(1 + \alpha\Delta t)$$

where $\rho_0 = 1.7241\times10^{-6}\ \Omega\text{-cm}^2/\text{cm}$ at 20°C, α=0.00382/°C for hard-drawn copper of the type most commonly used for electrical wiring, and ΔT is the temperature difference between the temperature of the desired resistance and that at which ρ_0 is specified, in this case $\Delta T = T_{specified} - 20°C$.

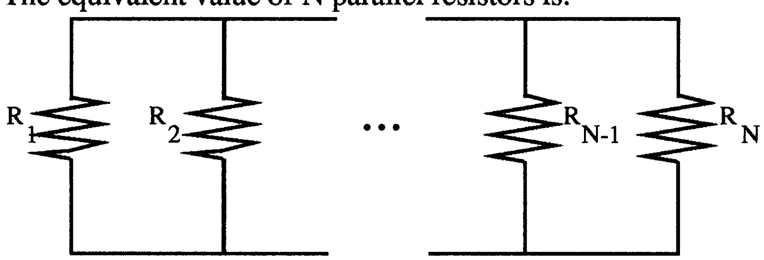
Do problem 47-1.
Problem 47-3 will be done as an example.

The equivalent value of N series resistors is:



$$R_{eq} = \sum_{j=1}^{N} R_j$$

The equivalent value of N parallel resistors is:

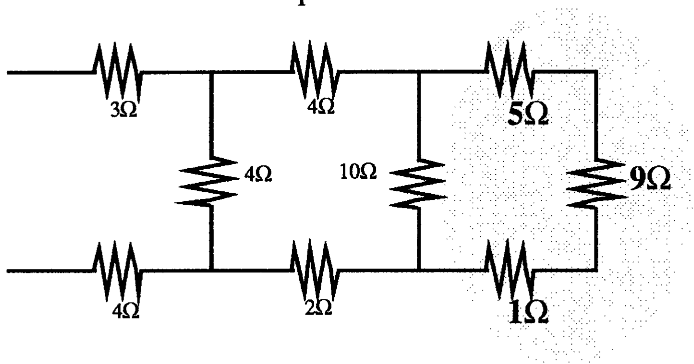


$$R_{eq} = \frac{1}{\sum_{j=1}^{N} \frac{1}{R_j}}$$

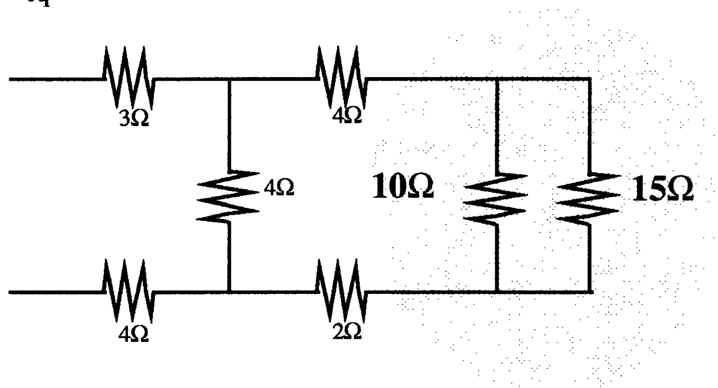
For the important case of two resistors in parallel:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

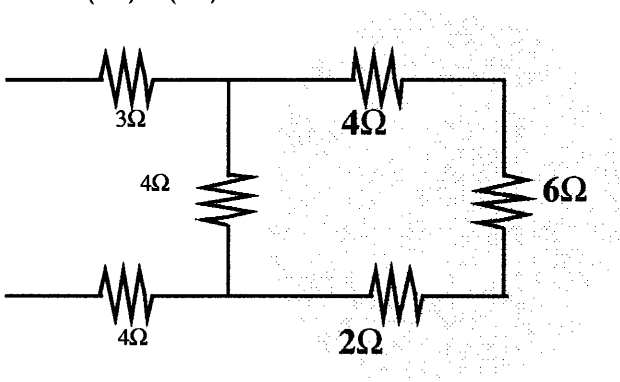
Equivalent resistance for a complex network:



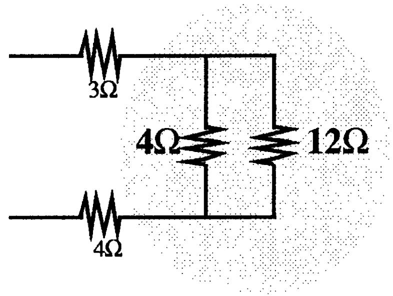
Looks like three resistors in series:
$R_{eq} = 5\Omega + 1\Omega + 9\Omega = 15\Omega$



Looks like two resistors in parallel:
$$R_{eq} = \frac{(10)(15)}{(10)+(15)} = 6\ \Omega$$

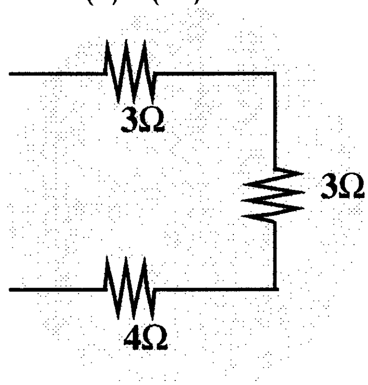


Looks like three resistors in series
$R_{eq} = 4 + 6 + 2 = 12\Omega$



Looks like two resistors in parallel:

$$R_{eq} = \frac{(4)(12)}{(4)+(12)} = 3\ \Omega$$

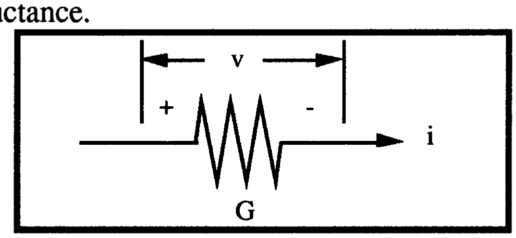


Looks like three resistors in series:

R_{eq}= 3 + 3 + 4 = 10Ω

47.3 Conductivity and conductance

Ohm's Law can also be written in terms of conductance which is simply 1/R, i.e. v=i/G where G is in mhos, the unit of conductance.

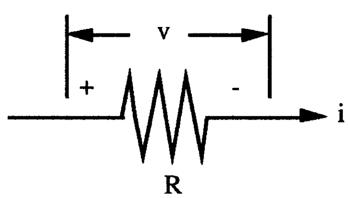


47.4 Ohm's Law

Electrical resistance

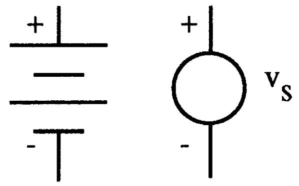
v=iR

where R is in units of ohms.

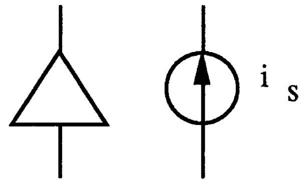


47.5 Energy sources

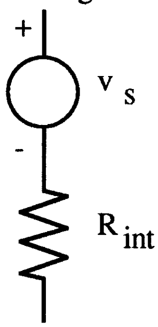
independent ideal voltage source:



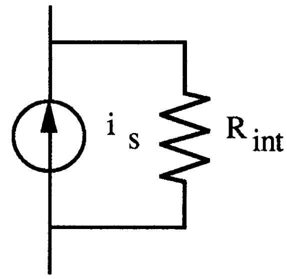
independent ideal current source:



independent real voltage source:



independent real current source:



Perfect voltage and current sources have the following characteristics:

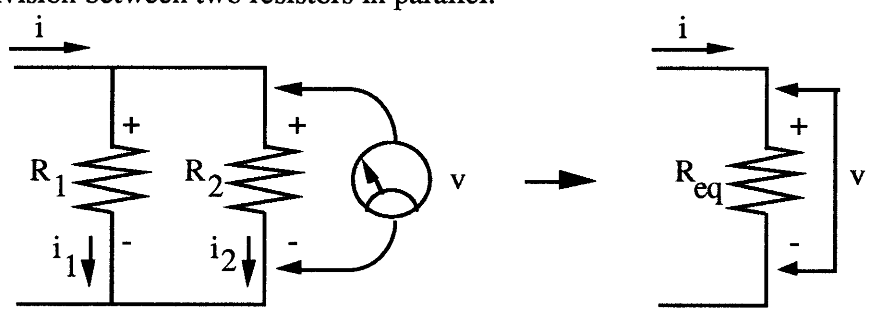
A perfect (ideal) voltage source has R_{int}=0.
A perfect (ideal) current source has R_{int}=∞.

47.6 Voltage Sources in Series and Parallel

Voltage sources that are in series (even if there are intervening resistances) can be algebraically combined into a single equivalent resistance.

47.10 Voltage and Current dividers

Current division between two resistors in parallel:



Since the resistors are in parallel they MUST have the same voltage across them

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$$

$$v = iR_{eq} = \frac{R_1R_2}{R_1 + R_2}i$$

$$i_1 = \frac{v}{R_1} = \frac{R_2}{R_1 + R_2}i$$

$$i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2}i$$

This is known as a current divider.

Voltage division between two resistors in series (see the figure below).
As the resistors are in series they MUST have the same current thru them.

$$R_{eq} = R_1 + R_2$$

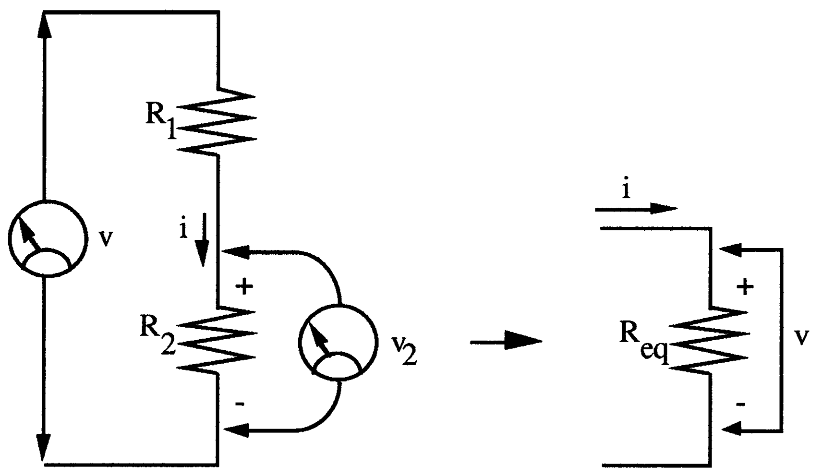
Using Ohm's Law

$$i = \frac{v}{R_{eq}} = \frac{v}{R_1 + R_2}$$

Knowing the current through each resistor we can apply Ohm's Law again to get

$$v_2 = iR_2 = \frac{v}{R_1 + R_2} R_2 = \frac{R_2}{R_1 + R_2} v$$

This result is known as a voltage divider.



Do problem 47-12.
Also look at the units in Problem 47-13 which will be assigned in section 47.15.

47.11 Power

Power may also be defined, using our previous relationships, as

$p=vi$.

The unit of power is joules/second, or watts. 746 watts = 1 horsepower is a very common conversion.

Do problem 47-7.

47.12 Decibels

Decibels are units used to express power ratios

$$db = 10 \log_{10}\left(\frac{P_2}{P_1}\right)$$

or voltage and current ratios

$$db = 20 \log_{10}\left(\frac{V_2}{V_1}\right)$$

$$db = 20 \log_{10}\left(\frac{I_2}{I_1}\right)$$

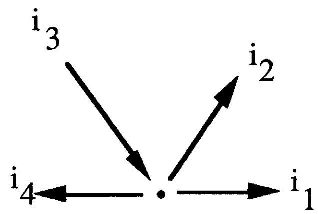
Do problem 47-14. Problem 47-15 will be done as an example.

47.14 Kirchoff's Laws

Kirchoff's current Law: the algebraic sum of all currents entering or leaving a node is zero. Mathematically,

$$\sum_{j=1}^{N} i_j = 0$$

For a simple example: $i_3-i_1-i_2-i_4=0$ where we used the negative sign to indicate current leaving the node. IMPORTANT: It does not matter whether you use the positive or negative sign to indicate current leaving the node AS LONG AS YOU ARE CONSISTENT.

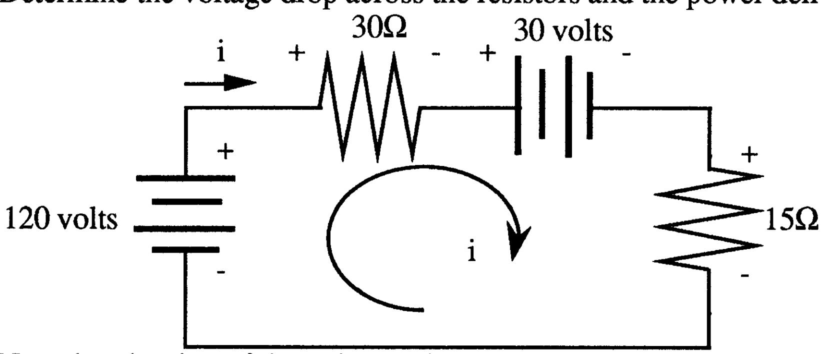


Kirchoff's voltage law states that the algebraic sum of the voltages around any closed path in a circuit is zero, i.e.

$$\sum_{j=1}^{N} v_j = 0$$

47.15 Simple Series Circuit

Determine the voltage drop across the resistors and the power delivered by the batteries.



Note that the sign of the voltages drops agrees with our previously defined convention. Summing the voltages around the circuit in a clockwise direction:

$$-120 + 30i + 30 + 15i = 0$$
$$45i = 90$$
$$i = 2 \text{ amps}$$

The voltage drop across the 30Ω resistor is:

$$v_{30\Omega} = iR = (2)(30) = 60 \text{ volts}$$
$$v_{15\Omega} = iR = (2)(15) = 30 \text{ volts}$$

The power supplied by the batteries is:

$$P_{120V} = vi = (120)(2) = 240 \text{ watts}$$
$$P_{30V} = vi = (30)(2) = 60 \text{ watts}$$

The power dissipated by the resistors is:

$$P_{30\Omega} = i^2R = (2)^2(30) = 120 \text{ watts}$$
$$P_{15\Omega} = i^2R = (2)^2(15) = 60 \text{ watts}$$

where we used the following formula to calculate the power:

$$P = vi = i^2R = \frac{v^2}{R}$$

Note that the batteries use a lot of power just canceling each other out.

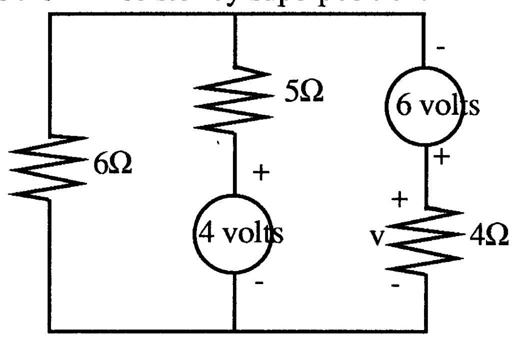
Do problem 47-13.

47.21 Superposition

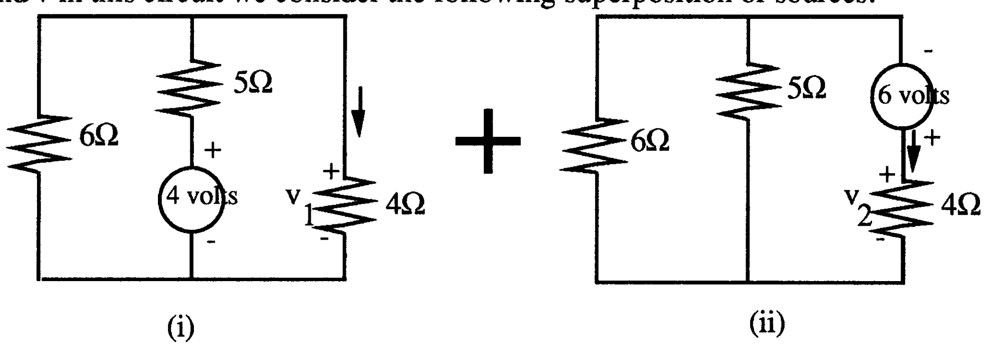
Superposition can be used to determine node voltages but is usually more complex than the loop current method discussed in the next section. To solve a problem by superposition we consider only one voltage or current source at a time (all the other voltage sources are replaced by shorts, all the other current sources are replaced by open circuits) and sum up the resulting voltages or currents. Superposition works because electrical sources are linear.

Example:

Find the voltage v across the 4Ω resistor by superposition.



To find v in this circuit we consider the following superposition of sources:



In circuit (i), the total resistance seen by the source is 5 + 4||6 = 5 + 2.4 = 7.4Ω. The total current is then i=4volts/7.4Ω=0.54 ampere. This current goes through the 5Ω resistor and, then, splits with 0.54(6/(4+6)) = 0.324 amperes going through the 4Ω resistor. The voltage drop v_1 is then v_1= iR = (0.324 amperes)(4Ω)= 1.3 volts.

In circuit (ii), the total resistance seen by the source is 4 + 5||6 = 4 + 2.73 = 6.73Ω. The total current is i=6volts/6.73Ω=0.892 ampere. This current goes through the 4Ω resistor directly below the source. The voltage drop v_2 across the 4Ω resistor is then v_2= iR = (0.892 amperes)(4Ω)= 3.57 volts.
Since the currents going through the 4Ω resistor are in the same direction they add giving, for the original circuit, $v = v_1 + v_2$ = 1.3 + 3.57 = 4.87 volts.

Do problem 47-19.

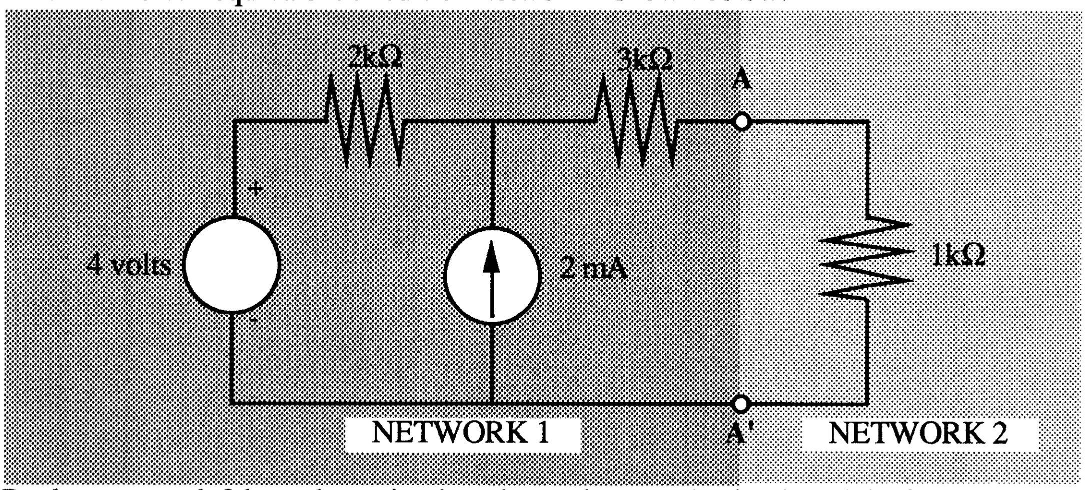
47.22 Norton's Theorem

Norton's Theorem (formal definition)

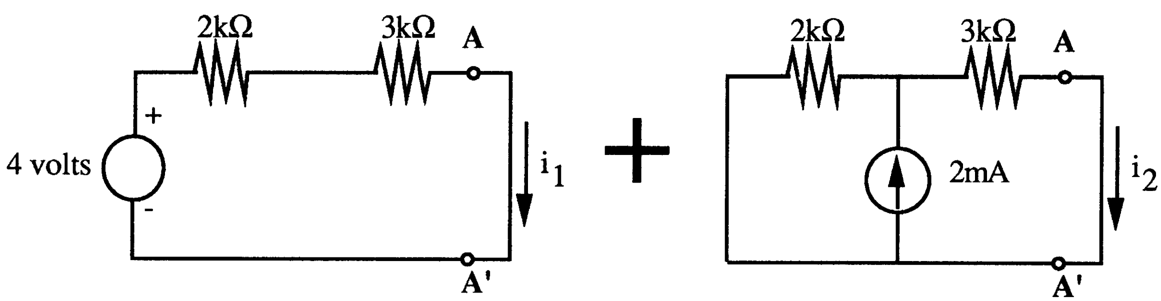
Given any linear circuit, rearrange it in the form of two networks 1 and 2 that are connected together by two zero resistance conductors. Define a current i_{sc} as the short-circuit current which would appear at the terminals A and A' of network 1 if network 2 were replaced by a short circuit. Then, all the currents and voltages in network 2 will remain unchanged if network 1 is killed (all independent voltage sources and current sources in network 1 are replaced by short circuits and open circuits, respectively) and an independent current source i_{sc} is connected, with proper polarity, in parallel with the equivalent resistance of the dead (inactive) network 1.

Example:

Find the Norton equivalent circuit of Network 1 shown below.



Replace network 2 by a short-circuit and superimpose the 4 volt and the 2 ma sources to find i_{sc}.



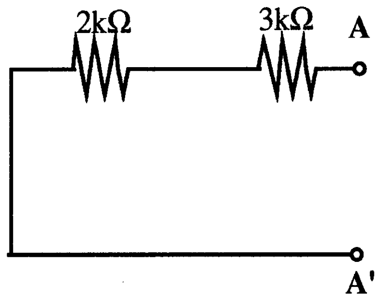
The current $i_1 = 4 \text{ volts}/5k\Omega = 0.8mA$. The current i_2 is found using the current divider relationship.

$$i_2 = 2mA\left(\frac{2k\Omega}{2k\Omega + 3k\Omega}\right) = 2mA\left(\frac{2}{5}\right) = 0.8 \text{ mA}$$

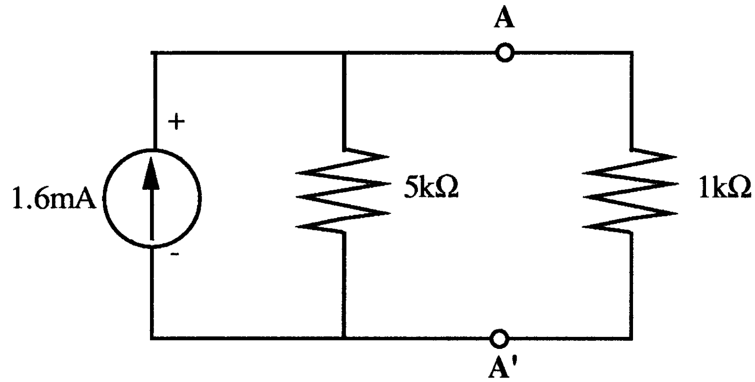
Both currents are in the same direction so $i_{sc} = i_1 + i_2 = 0.8mA + 0.8mA = 1.6mA$.

Shorting out the voltage sources and opening the current sources yields:

Shorting out the voltage sources and opening the current sources yields:



The terminal resistance $R_T = 2k\Omega + 3k\Omega = 5k\Omega$
The final Norton equivalent circuit is then:

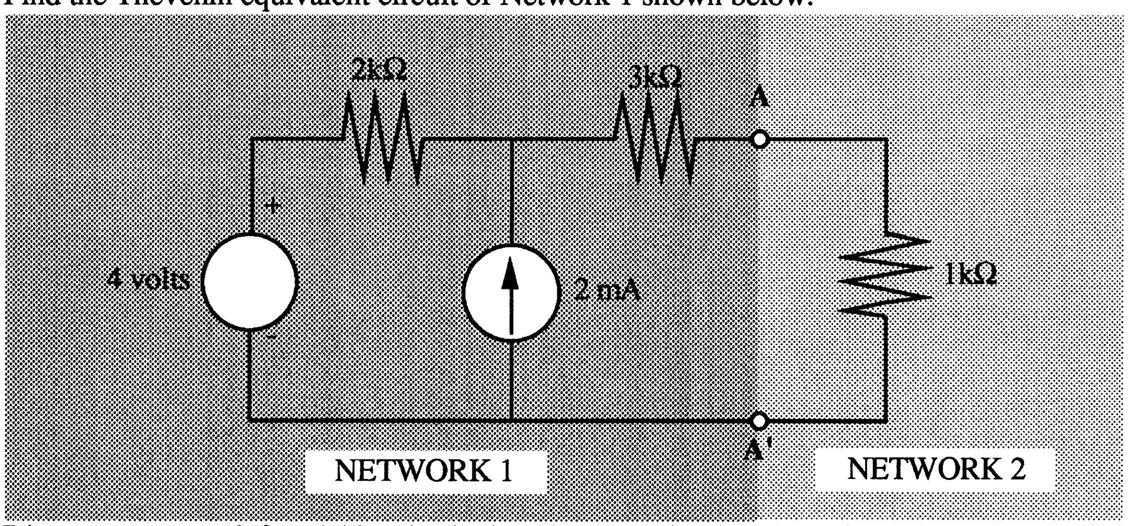


which is certainly easier to analyze than the original circuit with multiple sources.

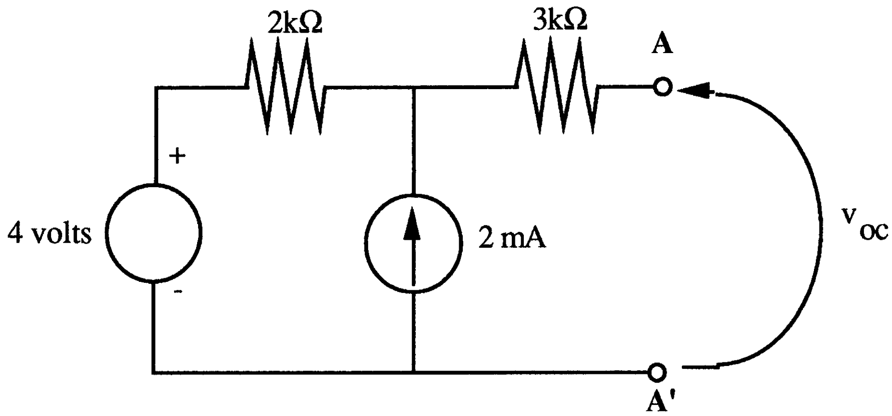
47.23 Thevenin's Theorem

Thevenin's Theorem (formal definition):
Given any linear circuit, rearrange it in the form of two networks 1 and 2 that are connected together by two zero resistance conductors at A and A'. Define a voltage v_{oc} as the open-circuit voltage which would appear between the terminals A and A' if network 2 were disconnected so that no current is drawn from network 1. Then, all the currents and voltages in network 2 will remain unchanged if network 1 is killed (i.e., all independent voltage sources and current sources in network 1 are replaced by short circuits and open circuits, respectively) and an independent voltage source v_{oc} is connected, with proper polarity, in series with the equivalent resistance of the dead (inactive) network 1.

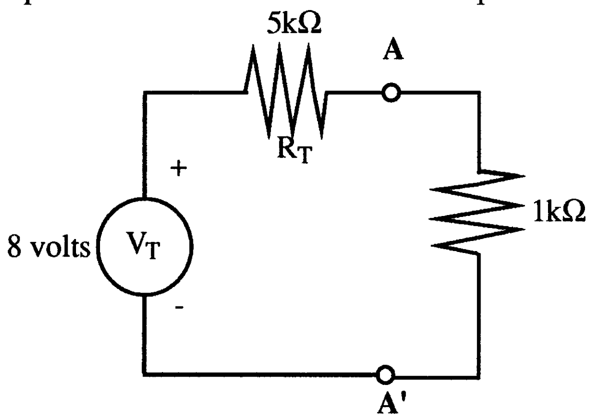
Example:
Find the Thevenin equivalent circuit of Network 1 shown below.



Disconnect network 2 at AA' and calculate the open circuit voltage from network 1.

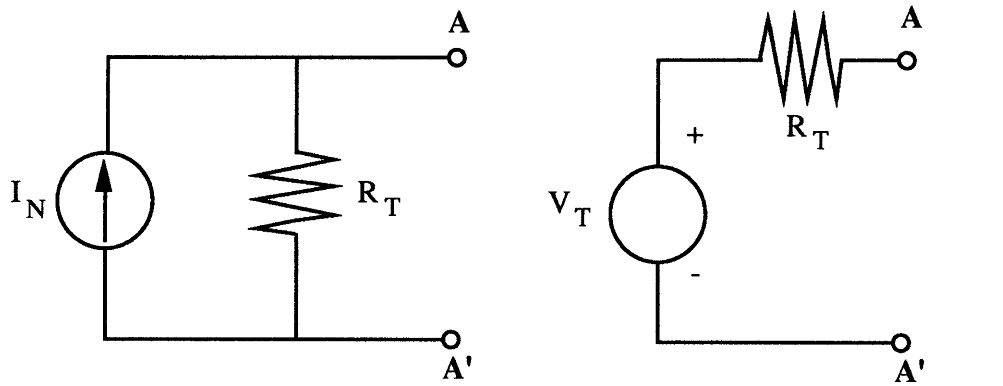


No current flows through the 3kΩ resistor. Use superposition to find v_{oc}. From the voltage source only, $v_1 = 4$ volts. From the current source only $v_2 = (2k\Omega)(2mA) = 4$ volts. The voltages are of the same polarity so $v_{oc} = 4 + 4 = 8$ volts. The equivalent resistance is found by replacing the 4 volt source by a short and the 2mA current source by an open and computing the resultant Thevenin resistance $R_T = 2k\Omega + 3\ k\Omega = 5k\Omega$. The Thevenin equivalent circuit to connect at AA' in place of network 1 is then:



Equivalence of Norton and Thevenin Equivalent Circuits

If you know the Norton equivalent circuit, the Thevenin equivalent circuit is directly computable from Ohm's Law. This observation also works the other way.



Norton equivalent circuit Thevenin equivalent circuit

Note that R_T is the same in both circuits and $V_T = I_N R_T$.

Perfect voltage and current sources have the following characteristics:

A perfect (ideal) voltage source has $R_T=0$.

A perfect (ideal) current source has $R_T=\infty$.

For a perfect (ideal) source,

if $V_T=0$ then the source is replaced by a short, $R_T=0$.

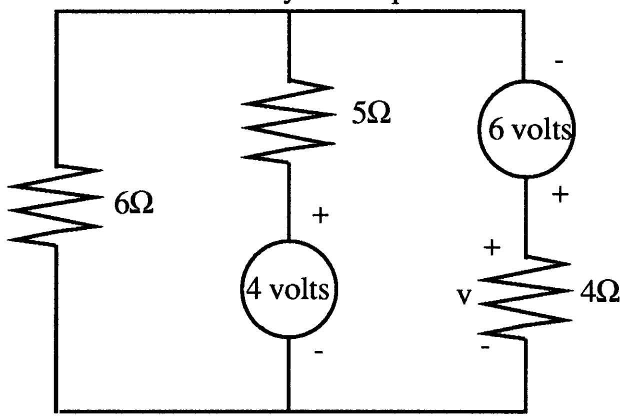
if $I_N=0$ then the source is replaced by an open, $R_T=\infty$.

Do problem 47-18. HINT: Directly apply the Thevenin equivalent circuit shown above.

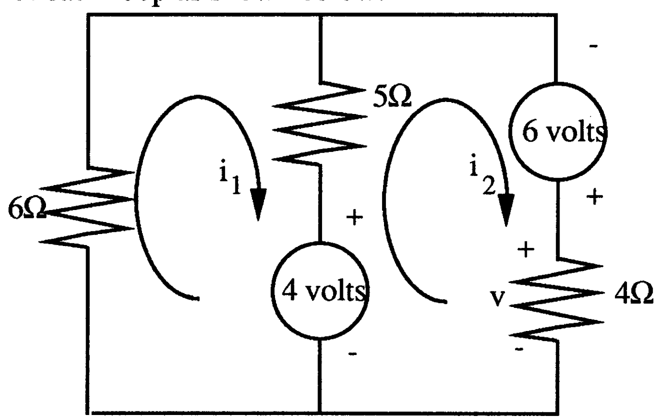
47.24 Loop Current Method

Example:

Find the voltage v across the 4Ω resistor by the loop current method.



To find v in this circuit we assume current directions for the chosen loops and write Kirchoff's voltage law for each loop as shown below:



Writing the loop equations:

loop1: $6i_1 + 5i_1 - 5i_2 + 4\text{volts} = 0$

loop2: $-4\text{volts} + 5i_2 - 5i_1 - 6\text{volts} + 4i_2 = 0$

Simplifying,

loop1: $11i_1 - 5i_2 = -4\text{volts}$

loop2: $-5i_1 + 9i_2 = +10\text{volts}$

which can be solved to give

$i_2 = +1.216$ amperes and $i_1 = 0.19$ amperes.

The voltage v across the 4Ω resistor is then

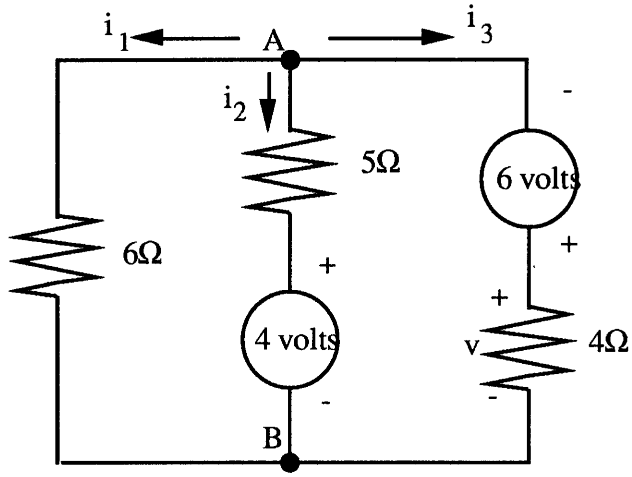
$v = i_2(4\Omega) = (1.216)(4) = 4.864$ volts

Do problem 47-21.

47.25 Node Voltage Method

Example:

Find the voltage v across the 4Ω resistor by the node voltage method.



To find v in this circuit we assume that the node B is ground (0 volts). Typically, the ground is associated with the negative side of a voltage source or other voltage reference. In this case, ground is the negative side of the voltage drop v. The only other node in the circuit is A and we will define the voltage at A to be V_A. Using the node voltage method we apply Kirchoff's current law to node A.

At A: $i_1 + i_2 + i_3 = 0$

where

$V_A = i_1(6\Omega)$, or $\quad i_1 = \frac{V_A}{6}$

$V_A = i_2(5\Omega) + 4$ volts, or $\quad i_2 = \frac{V_A - 4}{5}$

$V_A = i_3(4\Omega) - 6$ volts, or $\quad i_3 = \frac{V_A + 6}{4}$

Substituting these results into Kirchoff's current law and solving for V_A:

$$\frac{V_A}{6} + \frac{V_A - 4}{5} + \frac{V_A + 6}{4} = 0$$

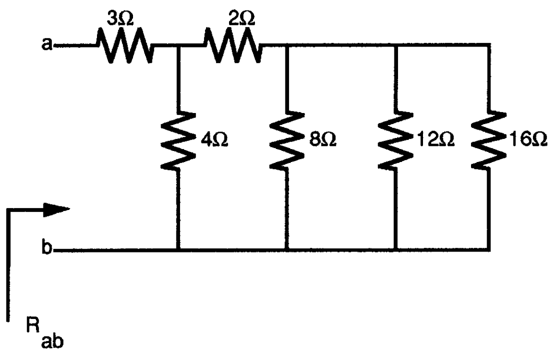
$V_A = -1.135$ volts.

Then, $i_3 = (-1.135+6)/4 = 1.216$ amperes and $v = i_3(4\Omega) = (1.216 \text{ amperes})(4\ \Omega) = 4.864$ volts.

Notice that all three examples give the same answer. You may judge whether one is particularly easier or faster than the others.

CIRCUITS 1

The equivalent resistance R_{ab} is closest to
(a) 2 ohms
(b) 4 ohms
(c) 6 ohms
(d) 8 ohms
(e) 10 ohms



Solution:
R_{eq} for the 8Ω, 12Ω and 16Ω resistors in parallel is

$$R_{eq} = \frac{1}{\frac{1}{8}+\frac{1}{12}+\frac{1}{16}} = 3.68\Omega$$

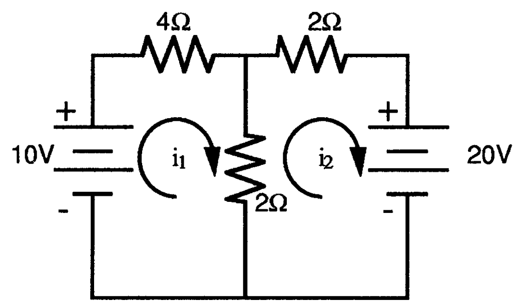
and

$$R_{ab} = 3 + 4||(2+3.68) = 3 + \frac{4\times(2+3.68)}{4+(2+3.68)} = 3 + 2.35 = 5.35\Omega$$

The correct answer is (c).

CIRCUITS 4

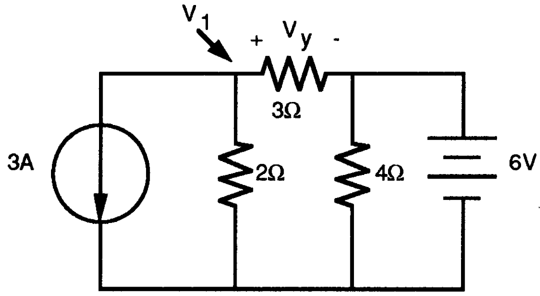
The power supplied by the 10 volt source is
(a) 12 watts
(b) 0 watts
(c) -12 watts
(d) 16 watts
(e) -16 watts



Solution:
Call the clockwise loop currents i_1 and i_2 as shown in the drawing above. Use KCL to obtain two equations in two unknowns
$6i_1 - 2i_2 = 10$
$-2i_1 + 4i_2 = -20$
Multiplying the first equation by two gives
$12i_1 - 4i_2 = 20$
and adding the last two equations we get the solution that $i_1=0$.
$P_{10 \text{ volt source}} = i_1(10 \text{ volts}) = 0$
The correct answer is (b).

CIRCUITS 5

The voltage V_y is closest to
(a) 0 volts
(b) 3.6 volts
(c) -1.2 volts
(d) 7.2 volts
(e) -7.2 volts



Solution:
Call the voltage drop (from top to bottom) across the 2Ω resistor V_1. Using KCL to sum the currents at the node pointed to by V_1 in the above drawing gives the following expression

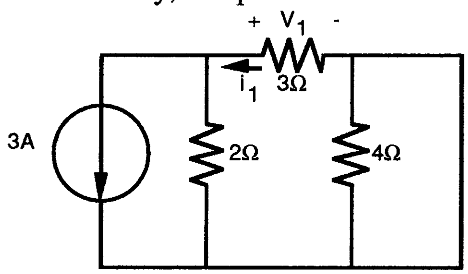
$$\frac{V_1}{2} + \frac{V_1 - 6}{3} = -3$$

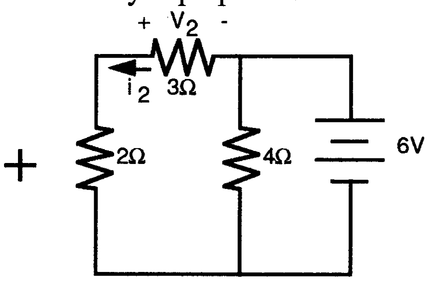
Note that we used the + sign for currents coming out of the node. Solving for V_y gives V:

$$V_y = V_1 - 6 = -7.2 \text{ volts}$$

or $V_1 = -1.2$ volts. The correct answer is (e).

Alternatively, this problem could have been solved by superposition





Since the 4Ω resistor is shorted out by the 6 volt source in the circuit on the left we can solve for the loop current as

$$i_1 = \frac{2}{2+3}(3A) = 1.2 \text{ A}$$

The voltage across the 3Ω resistor is then

$$V_1 = -\left(\frac{6}{5}A\right)(3\Omega) = -\frac{18}{5} \text{ A}$$

Examing the right hand circuit we can solve for the current i_2 as

$$i_2 = \frac{6\text{ V}}{5\Omega} = \frac{6}{5} \text{ A}$$

The voltage across the 3Ω resistor is then given by

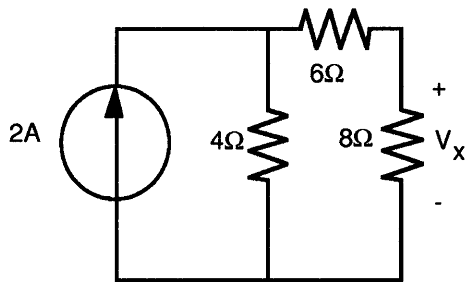
$$V_2 = -\left(\frac{6}{5}\text{ A}\right) 3\Omega = -\frac{18}{5} \text{ volts}$$

Vy is then the sum of the voltages across the 3Ω resistor, i.e.

$$V_y = V_1 + V_2 = -\frac{18}{5} - \frac{18}{5} = -\frac{36}{5} = -7.2 \text{ volts}$$

CIRCUITS 2

The voltage V_x is closest to
(a) 16 volts
(b) 8 volts
(c) 3.55 volts
(d) 6.42 volts
(e) 4.65 volts



Solution:
Using current division the current through the 8Ω resistor is

$$i_{8\Omega} = 2 \times \frac{4}{4 + (6+8)} = \frac{8}{18} \text{ Amps}$$

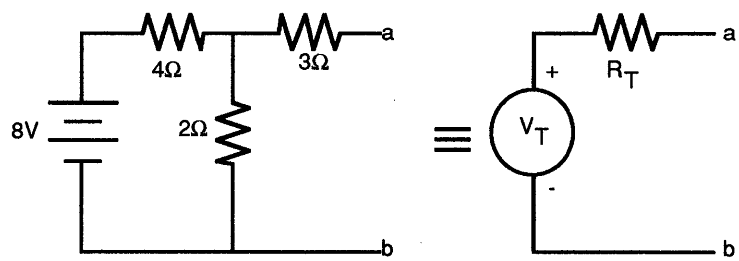
The voltage across the 8Ω resistor is then given by Ohm's Law as

$$V_x = i_{8\Omega} \times 8\Omega = \frac{8}{18} \times 8 = \frac{64}{18} = 3.55 \text{ Volts}$$

The correct answer is (c).

CIRCUITS 10

The Thevenin equivalent at terminals a-b is closest to
(a) V_T=5.33 volts, R_T=5Ω
(b) V_T=1.84 volts, R_T=4.33Ω
(c) V_T=1.84 volts, R_T=5Ω
(d) V_T=5.33 volts, R_T=4.33Ω
(e) V_T=2.67 volts, R_T=4.33Ω



Solution:
V_T is the open-circuit voltage from terminals a-b. The open circuit voltage at a-b is given by the voltage divider relationship

$$V_T = 8 \times \frac{2}{2+4} = 2.67 \text{ Volts}$$

Note that the 3Ω resistor does enter this relationship since no current flows through it if terminals a-b are open. R_T is the equivalent resistance with all the sources replaced by their equivalent impedances. For a voltage source this is zero, a short, placing the 4Ω and 2Ω resistors in parallel and their resultant in series with the 3Ω resistor to give

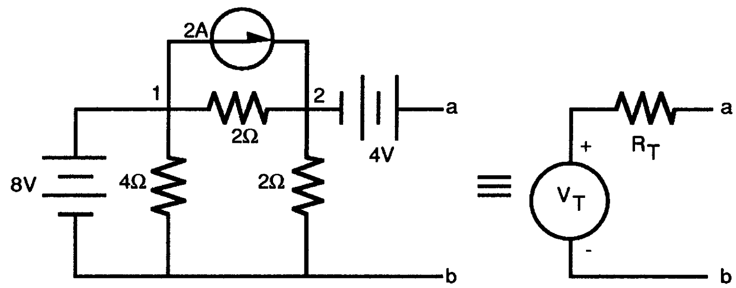
$$R_T = 3 + \frac{2\times 4}{2 + 4} = 3 + \frac{8}{6} = 4.33\ \Omega$$

The correct answer is (e).

CIRCUITS 11

The Thevenin equivalent at terminals a-b is closest to

(a) V_T=10 volts, R_T=1.5 Ω
(b) V_T=12 volts, R_T=1.0 Ω
(c) V_T=12 volts, R_T=1.5 Ω
(d) V_T=10 volts, R_T=1.0 Ω
(e) V_T=0 volts, R_T=1.0 Ω



Solution:
Since this is a more complex circuit than the previous example we must find V_T indirectly by first finding the voltage V_2 at node 2. Using KCL at node 2 gives the relationship for V_2 as

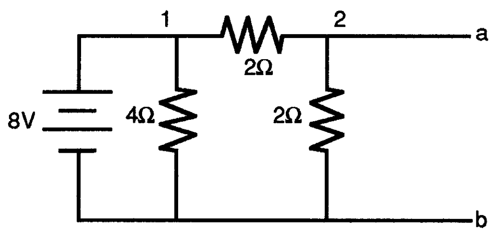
$$\frac{V_2}{2} + \frac{V_2 - 8}{2} - 2 = 0$$

where currents leaving the node are positive. Note that V_2-8 is the voltage across the resistor between nodes 1 and 2 since the voltage at node 1 must be 8 volts. Solving for V_2, V_2=6 volts. Using KVL across the 2Ω output resistor and the 4V voltage source we can get the voltage V_{ab} as $V_{ab}=V_2 + 4 = 6 + 4 = 10$ Volts.
R_T is relatively easy to determine. Replacing all of the sources by their equivalent impedances, the voltage sources are replaced by shorts and the current source is replaced by an open. The 4Ω resistor is shorted out by the 8V voltage source leaving only two 2Ω resistors in parallel. R_T is

$$R_T = 2||2 = \frac{2 \times 2}{2+2} = 1\Omega$$

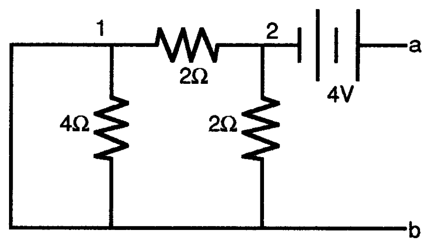
The correct answer is (d).
V_T could also be determined by superposition; however, there will be three components to V_T due to the fact that three sources are present. These contributions are shown below:

From the 8 volt source



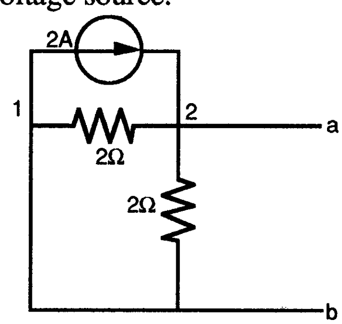
V_{ab}=+4 volts

From the 4 volt source

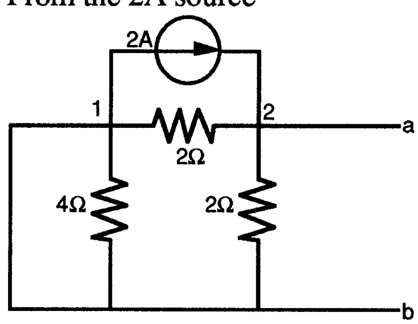


V_{ab}= +4 volts

Redrawing the original circuit to explicitly show the 2Ω resistors in parallel as a result of the shorting out of the 4Ω resistor by the 8V voltage source.



From the 2A source



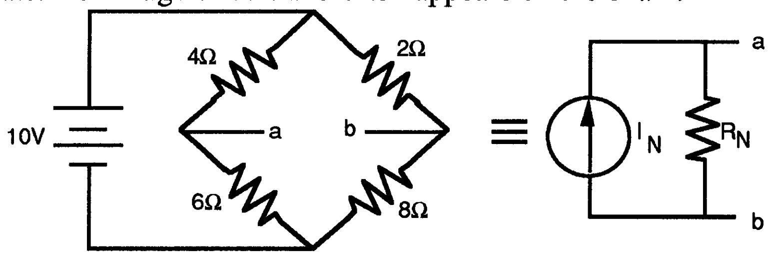
$V_{ab}=2A \times 1\Omega = +2$ volts

The resulting V_{ab} is then the sum of all the voltage contributions, i.e.
V_{ab}=4+4+2 = 10 volts as before.

CIRCUITS 12 This is theWheatstone Bridge circuit and often appears on the exam.

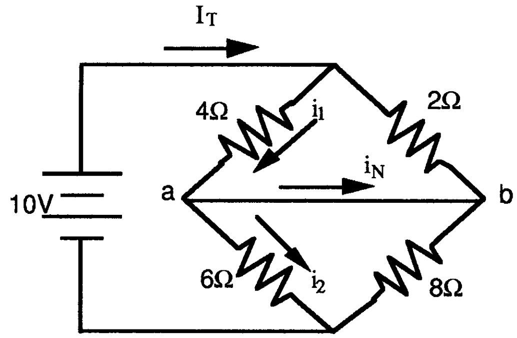
The Norton equivalent at terminals a-b is closest to

(a) I_N=0.2 amps, R_N=4 Ω
(b) I_N=0.2 amps, R_N=5 Ω
(c) I_N=-0.5 amps, R_N=4 Ω
(d) I_N=0.5 amps, R_N=5 Ω
(e) I_N=-0.5 amps, R_N=2 Ω



Solution:

The Norton current I_N is defined as the current between terminals a and b when terminals a and b are shorted together. The resulting circuit looks like a series combination of 4Ω||2Ω and 6Ω||8Ω.



The total current I_T supplied by the 10 volt source is then

$$I_T = \frac{10 \text{ volts}}{\frac{4\times2}{4+2}+\frac{8\times6}{8+6}} = \frac{10 \text{ volts}}{\frac{8}{6}\Omega+\frac{48}{14}\Omega} = 2.1\text{Amps}$$

Using the current divider relationship the current i_1 through the 4Ω resistor and the current i_2 through the 6Ω resistor can be calculated as

$$i_1 = 2.1 \text{ Amps} \times \frac{2}{4+2} = 0.7 \text{ Amps}$$

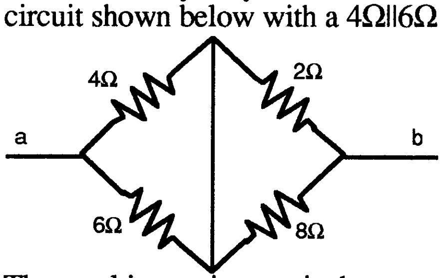
and

$$i_2 = 2.1 \text{ Amps} \times \frac{8}{6+8} = 1.2 \text{ Amps}$$

The Norton current I_N can then be found by applying KCL to node a

$i_N = i_1 - i_2 =$ 0.7 Amps - 1.2 Amps = -0.5 Amps

R_N is relatively easy to determine. Replacing the voltage source by a short we are now facing the circuit shown below with a 4Ω||6Ω combination in series with the 2Ω||8Ω combination.



The resulting resistance is then

$$R_N = \frac{4\times6}{4+6}+\frac{8\times2}{8+2} = 4.0\Omega$$

The correct answer is (c).

I_N could alsobe determined by finding the Thevenin equivalent and converting it to a Norton equivalent. The 4Ω and 6Ω resistors form a voltage divider at terminal a. The voltage at terminal a is then

$V_a = \frac{6}{4+6}(10 \text{ volts}) = 6 \text{ volts}$

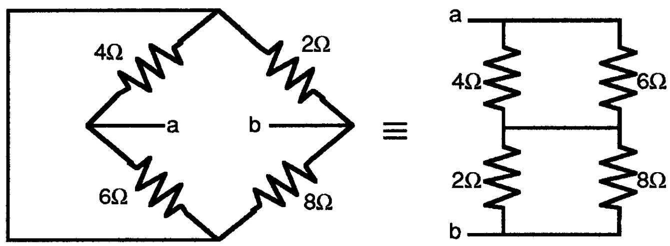
The voltage at terminal b can be found in a similar manner

$V_b = \frac{8}{2+8}(10 \text{ volts}) = 8 \text{ volts}$

The Thevenin voltage is the voltage difference between a and b.

$V_T = V_a - V_b = 6 - 8 = -2 \text{ volts}$

The Thevenin resistance R_T is found by shorting the voltage source and computing the resistance between terminals a and b as shown below

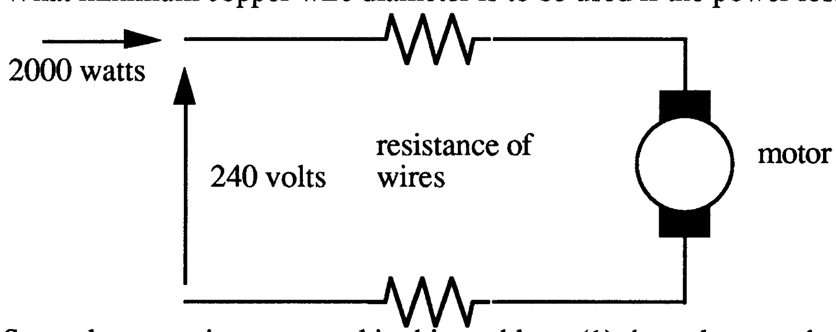


$$R_T = \frac{4\times6}{4+6} + \frac{2\times8}{2+8} = 2.4 + 1.6 = 4.0\Omega$$

Note that R_T=R_N and that V_T and I_N satisfy Ohm's Law

$$I_N = \frac{V_T}{R_T} = \frac{-2 \text{ volts}}{4\Omega} = -0.5 \text{ Amps}$$

Problem 47-3 A 240 volt motor requiring 2000 watts is located 1 km from a power source. What minimum copper wire diameter is to be used if the power loss is to be kept less than 5%?



Several assumptions are used in this problem: (1) the voltage at the line input is 240 volts, not at the motor; (2) that the 2000 watts are required by the motor, not at the input.

The power loss due to the resistors is 0.05×2000=100 watts. The total power consumed by the circuit is then

$P = VI = (240 \text{ volts})\, I = 2100 \text{ watts}$

Solving for I gives I=8.75 amps. We can now use this current to find the wire resistance.

$P = I^2R = (8.75)^2R = 100 \text{ watts}$

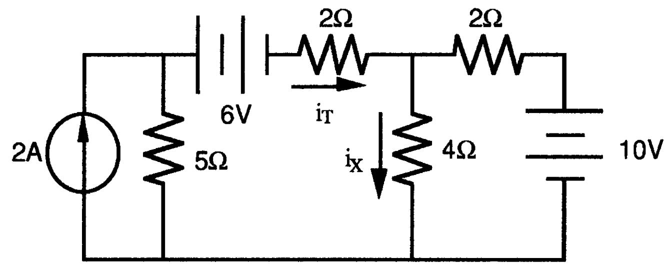
or R=1.306Ω. Remember that this is the total resistance of the wire. Calculating the wire resistance using the total wire distance of 2km we can solve for the diameter of the wire.

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi\left(\frac{D}{2}\right)^2} = \frac{\left(0.6788\times10^{-6}\,\frac{\Omega\text{-in}^2}{\text{in}}\right)\left(2\text{km}\times3281\frac{\text{ft}}{\text{km}}\times12\frac{\text{in}}{\text{ft}}\right)}{\pi\left(\frac{D}{2}\right)^2} = 1.306\Omega$$

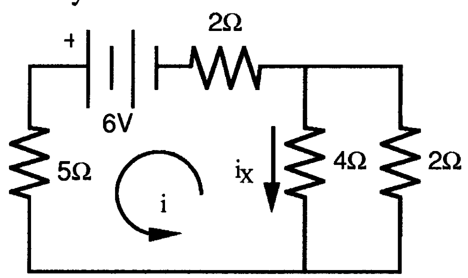
or D=0.228 inches

CIRCUITS 13

When solving for i_X using superposition, the contribution due to the 6V source is
(a) -0.24 amps
(b) 0.24 amps
(c) 0 amps
(d) 0.72 amps
(e) -0.72 amps



Solution:
To find the contribution from the 6V source, we replace the 2 amp source by an open circuit and the 10 volt source by a short. Redrawing the circuit to show the 6V source circuit a little more clearly



The total current i due to the 6V source is given as

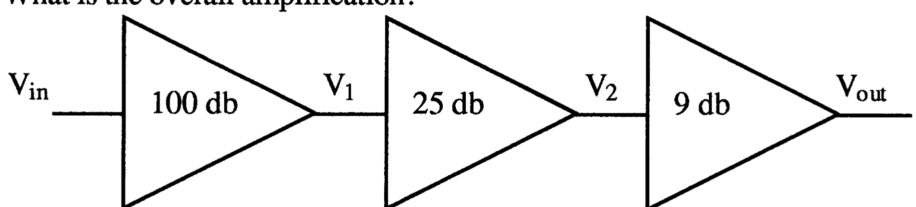
$$i = \frac{6\text{ volts}}{5 + 4||2 + 2} = \frac{6\text{ volts}}{5 + \frac{4\times2}{4+2} + 2} = \frac{18}{25} = 0.72\text{ amps}$$

The fraction of this current flowing through the 4Ω resistor is given by a current divider

$$i_X = -\frac{2}{2+4}(0.72\text{ amps}) = -0.24\text{ amps}$$

Note the use of the minus sign since I defined i to be in the opposite direction to i_X.
The correct answer is (a).

Problem 47-15 Three cascaded amplifier stages have amplifications of 100 db, 25db and 9db. What is the overall amplification?



The gain relationships can be written as

$$100\text{ db} = 20\log\left(\frac{V_1}{V_{in}}\right) = 20\log V_1 - 20\log V_{in}$$

$$25\text{ db} = 20\log\left(\frac{V_2}{V_1}\right) = 20\log V_2 - 20\log V_1$$

$$9\text{ db} = 20\log\left(\frac{V_{out}}{V_2}\right) = 20\log V_{out} - 20\log V_2$$

and can be combined by adding the right and left hand sides of the above expressions

$$134\text{ db} = 20\log\left(\frac{V_{out}}{V_{in}}\right) = 20\log V_{out} - 20\log V_{in}$$

47.28 Transient analysis

Initial Conditions

Before we can solve transient problems involving inductors and capacitors we must understand the initial conditions that apply to the differential equations:

Because no circuit can supply infinite power:
(1) the current through an inductor cannot change instantaneously
(2) the voltage across a capacitor cannot change instantaneously

These rules are the boundary conditions which apply to inductors and capacitors and, for purposes of writing time dependent eqpressions for voltage and current, can be written as:

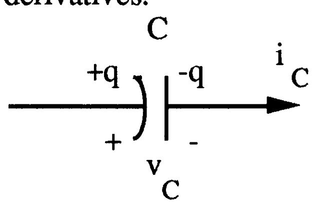
For an inductor
$i(0-)=i(0+)$ where it is assumed that a switch has been opened or closed in the network at $t=0$.

For a capacitor
$v(0-)=v(0+)$ where it is assumed that a switch has been opened or closed in the network at $t=0$.

Basically, this means that for inductors the current is continuous, whereas for capacitors the voltage is continuous.

Preview (This material is in 48.11 and 48.12):

The voltage current relationship for a capacitor cannot be written without using integrals or derivatives.

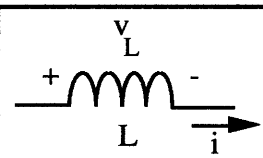


Using the above definitions we can write "Ohm's Law" for a capacitor as

$$i_C = \frac{dq}{dt} = C\frac{dv}{dt}$$

or, in integral form, as

$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i_C(t)\,dt$$



For the inductor, "Ohm's Law" is

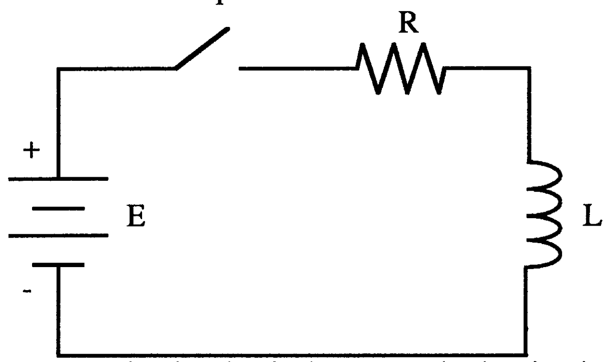
$$v_L = L\frac{di}{dt}$$

or, in integral form,

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v_L(t)\,dt$$

A transient is what occurs whenever you open or close a switch in an electrical network. The voltages and currents must quickly re-adjust themselves to the new network. The presence of capacitors and inductors in the network means that that the voltage and current readjustments occur according to the differential equations describing these circuit components.

Consider the simple series circuit shown below:



The question is what is the current in the circuit as a function of time for t≥0, the switch closes at t=0. This requires the use of differential equations. To describe this circuit begin with Kirchoff's voltage law

$$\sum V_n$$

$$E - iR - L\frac{di}{dt} = 0$$

This is a differential equation in i

$$L\frac{di}{dt} + iR = E$$

The homogeneous (or transient) solution comes from solving the equation with the right hand side of the equation set to zero.

$$L\frac{di}{dt} + iR = 0$$

The solution to this equation is an exponential of the form

$$i(t) = Ae^{kt}$$

where A and k are constants to be determined. Substituting this solution into the above differential equation we can find k

$$LAke^{kt} + Ae^{kt}R = 0$$

Dividing through by Ae^{kt}

$$Lk+R=0$$

or

$$k=-R/L$$

The constant A must come from the steady-state solution of the original differential equation. In the steady-state all derivatives are zero since nothing is changing. Therefore, the differential equation for this solution reduces to

$$iR=E$$

or

$$i = \frac{E}{R}$$

The resulting total solution is the sum of the steady-state and transient solutions

$$i(t)=i_{steady\text{-}state} + i_{transient}$$

or

$$i(t) = \frac{E}{R} + Ae^{-\frac{R}{L}t}$$

To find A we must apply the boundary conditions on i, i.e. that
$i(0-)=i(0^+)$. Let $i(0^-)=i_0$. At $t=0^+$

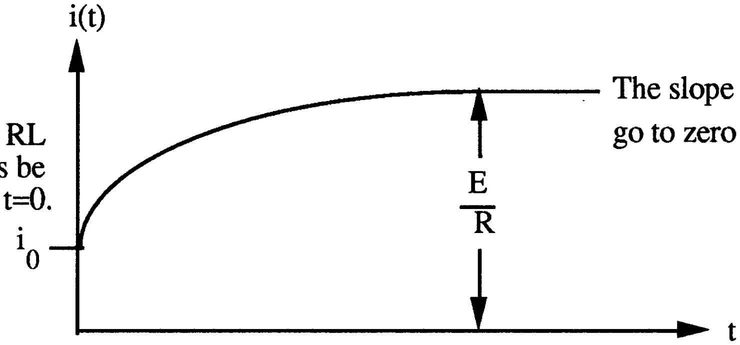
$$i_0 \approx \frac{E}{R} + A$$

$$A = i_0 - \frac{E}{R}$$

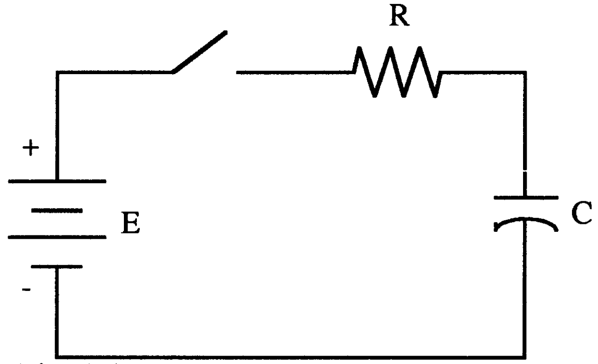
Therefore, the final time dependent expression for i(t) is

$$i(t) = \frac{E}{R} + \left(i_0 - \frac{E}{R}\right)e^{-\frac{R}{L}t}$$

The factor L/R is called the time constant of the circuit since it determines how rapidly the current changes in the circuit.



For a similar series RC circuit, the objective is to predict the voltage across the capacitor as a function of time given that the capacitor has a voltage v_O across it at time t=0.



Using KVL

$$E - iR - \frac{1}{C}\int_{t_0}^{t} i(t')\ dt' = 0$$

We cannot use i as the variable for capacitors - we must use voltage.

$$v_c(t) = \frac{1}{C}\int_{t_0}^{t} i(t')\,dt'$$

which is the integral form of

$$i = C\frac{dv_c}{dt}$$

The differential equation is then

$$E - R\left(C\frac{dv_c}{dt}\right) - v_c = 0$$

or, rewriting the equation in a more standard form,

$$RC\frac{dv_c}{dt} + v_c = E$$

As before this equation has a homogeneous and a steady-state solution.
For the homogeneous solution:

$$RC\frac{dv_c}{dt} + v_c = 0$$

Letting $v_c(t)=Ae^{kt}$

$$RCAke^{kt} + Ae^{kt} = 0$$

or, solving for k,

$$k = \frac{-1}{RC}$$

The steady-state solution comes from setting all time derivatives to zero to give

$$v_c = E$$

The total solution is then

$$v_C(t) = v_{steady\text{-}state} + v_{transient}$$

$$v_c(t) = E + Ae^{-\frac{t}{RC}}$$

At t=0, $v_C(t)=v_O$ so

$$v_c(0) = v_0 = E + A$$

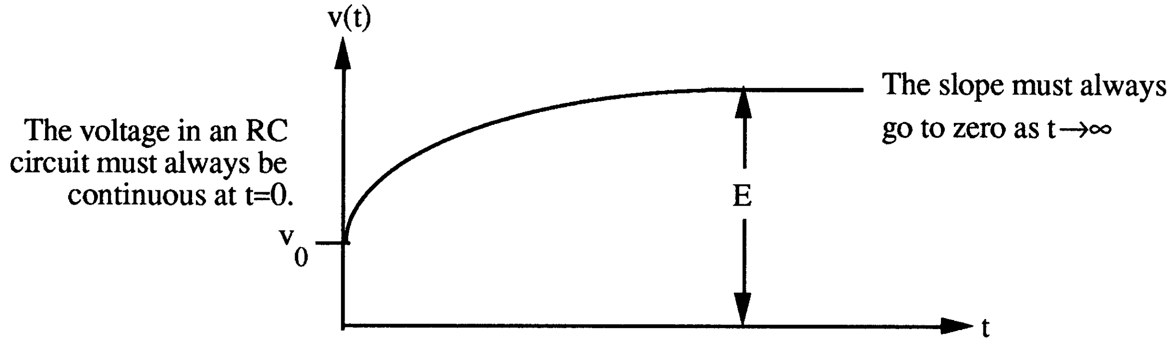
and

$$A = v_O - E$$

The final solution is then

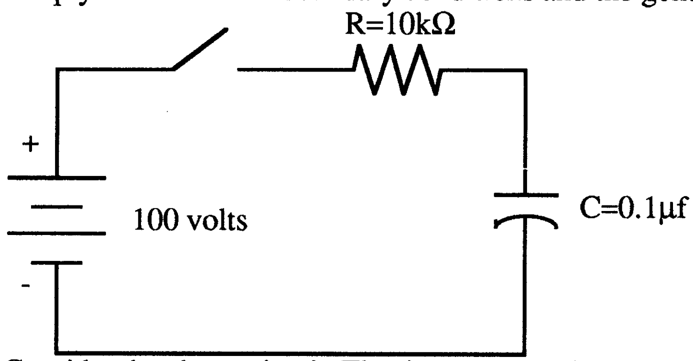
$$v_c(t) = (v_0 - E)e^{-\frac{t}{RC}} + E$$

which is shown graphically below



Example:

Most of the time you do not need any fancy math as shown above to solve the problem. Simply remember the boundary conditions and the general form of the solutions.



Consider the above circuit. The time constant is simply computed as

$$\tau = RC = (10\times10^{-3})(0.1\times10^{-6}) = 10^{-3} \text{ seconds}$$

Assume $v_O=0$ since it was not given. The problem is to determine v across the capacitor at $t=5\times10^{-4}$ seconds. The general form of the solution was shown graphically above. Expressing this general solution mathematically

$$v(t) = E + (v_O - E)e^{-\frac{t}{RC}}$$

which for the given constants becomes

$$v(t) = 100 + (0-100)e^{-\frac{t}{10^{-3}}}$$

and, at $t=5\times10^{-4}$ seconds, is

$$v(5\times10^{-4}) = 100\left(1 - e^{-\frac{5\times10^{-4}}{10^{-3}}}\right) = 100\left(1 - e^{-\frac{1}{2}}\right)$$

$$v(5\times10^{-4}) \approx 39.35 \text{ volts}$$

Summary of important relationships for solving transient problems.

Component	Initial condition	t=0+	t=∞	τ	Ohm's Law
Capacitor	V(0-)=V(0+)	short	open	RC	$I = C\frac{dV}{dt}$ or $V = \frac{1}{C}\int I\,dt$
Inductor	I(0-)=I(0+)	open	short	L/R	$V = L\frac{dI}{dt}$

The differential equation

$$A\frac{dI}{dt} + BI = C$$

has two solutions. One solution is the d.c. (also called steady state solution) which occurs when all the derivatives go to zero. This solution is simply I=C/B. The other solution is the transient solution (also called the homogeneous solution) and requires that C=0, i.e. solve

$$A\frac{dI}{dt} + BI = 0$$

This solution is always of the form $I(t) = Ae^{mt}$ and can be solved for by simply substituting this expression for I(t) into

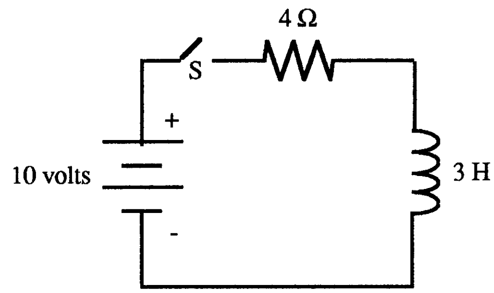
$$A\frac{dI}{dt} + BI = 0$$

Do problems 47-28, 47-29 (initial conditions)

CIRCUITS 31

The switch S closes at T=0. The complete response for i(t) for t>0 is

(a) $2.5 + Ae^{-0.75t}$
(b) $2.5 - 2.5e^{-0.75t}$
(c) $2.5 + 2.5e^{-1.33t}$
(d) 0
(e) $2.5 - 2.5e^{-1.33t}$



We use KVL to write the differential equation for the circuit using the correct expression for the impedance of the inductor.

$$-10 + 4i + 3\frac{di}{dt} = 0$$

Re-writing this in conventional form with the sources on the right hand side of the equation

$$3\frac{di}{dt} + 4i = 10$$

The dc (or homogeneous) solution is obtained by setting the derivatives equal to zero or, in this case

$$4i = 10$$

giving i=2.5 amps.

The transient solution is always an exponential in form. Substituting $i(t)=Ae^{kt}$ into the differential equation and setting the source (the right hand side of the equation) equal to zero we obtain

$$3\frac{di}{dt} + 4i = 0$$

$$3kAe^{kt} + 4Ae^{kt} = 0$$

$$3k+4 = 0$$

$$k=-4/3$$

The total solution is then $i(t) = i_{transient} + i_{homogeneous} = Ae^{-1.33t} + 2.5$

The coefficient A is solved for by using the boundary condition that $i(0^+)=i(0^-)=0$.

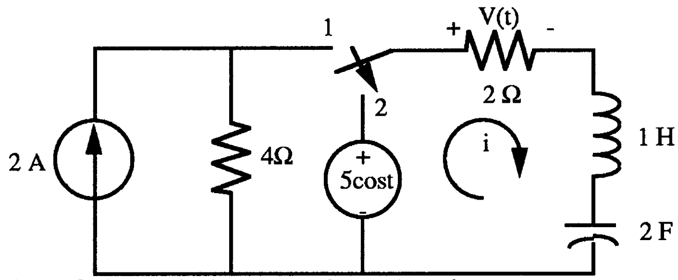
This requires that $i(0^+) = A + 2.5 = 0$, or that A=-2.5.

Then $i(t) = 2.5 - 2.5e^{-1.33t}$ and the correct answer is (e).

Circuits 32

The form of the transient part of V(t) for t>0 is

(a) $(A_1+A_2)e^{-0.5t}$
(b) $A_1\cos(0.5t) + A_2\sin(0.5t)$
(c) $A_1e^{-t}\cos(0.5t) + A_2e^{-t}\sin(0.5t)$
(d) $A_1e^{-1.71t} + A_2e^{-0.29t}$
(e) 0



At t=0 the switch moves from 1 to 2.

Solution:
For t>0 the equation of the circuit is

$$\frac{di}{dt} + 2i + \frac{1}{2}\int_0^t i(\alpha)d\alpha + V_C(0^+) = 5\cos(t)$$

where $V_C(0^+)$, the initial voltage on the capacitor, is zero.
Differentiating the above equation to remove the integral

$$\frac{d^2i}{dt^2} + 2\frac{di}{dt} + \frac{1}{2}i(t) = -5\sin(t)$$

The left hand side of this equation describes the transient response. For the transient

$$\frac{d^2i}{dt^2} + 2\frac{di}{dt} + \frac{1}{2}i(t) = 0$$

If we let $i(t)=Ae^{mt}$ we get

$$m^2Ae^{mt} + 2mAe^{mt} + \frac{1}{2}Ae^{mt} = 0$$

which reduces to the characteristic equation

$$m^2 + 2m + 0.5 = 0$$

This equation can be solved using the quadratic formula

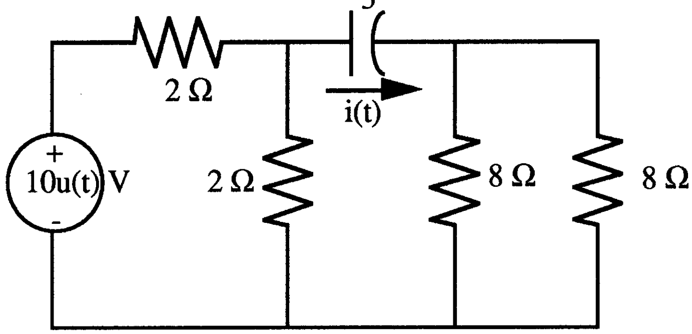
$$m = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(0.5)}}{2} = -1.71 \text{ and } -0.29$$

The only answer with these exponents is (d).

Circuits 36

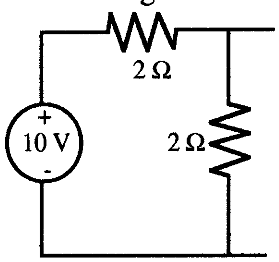
i(t) is

(a) $e^{-0.6t}$
(b) $e^{-1.67t}$
(c) $-e^{-1.67t}$
(d) $-e^{-0.6t}$
(e) $2-e^{-1.67t}$



Solution:
There are many ways to solve this problem but, perhaps, the easiest way is to Thevenize the left hand side of the circuit (the voltage source and the two 2Ω resistors) and replace the right hand side of the circuit (the two 8Ω resistors in parallel) by its equivalent resistance.

Thevenizing the left hand side of the circuit



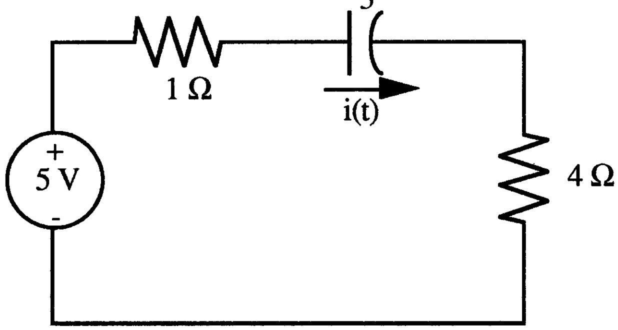
$$V_T = \frac{2}{2+2} 10 = 5 \text{ volts}$$

and

$$R_T = \frac{2\times 2}{2+2} = 1\ \Omega$$

NOTE: This is a good technique to use to get rid of current sources in problems.

The 8 Ω resistors in parallel can be replaced by a 4 Ω resistor. Redrawing the original circuit and replaceing the left hand side by its Thevenin equivalent and replacing the two 8Ω resistors by a single 4Ω resistance, we get the following circuit



Since $V_C(0^+)=V_C(0^-)=0$

$$i_C(0^+) = \frac{5 \text{ volts}}{1\Omega + 4\Omega} = 1 \text{ amp}$$

The time constant for the circuit can be directly computed as

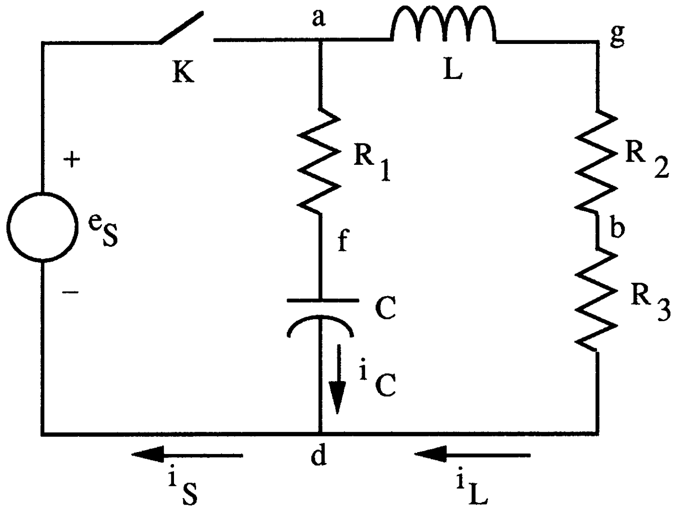
$$\tau = RC = (5\ \Omega)(\tfrac{1}{3}\text{ F}) = 1.67 \text{ seconds}$$

The solutions should be of the form

$$Ae^{-\frac{t}{1.67}} = Ae^{0.6t}$$

The correct answer is (a).

Example afternoon problem (this is actually tougher than most afternoon problems):



You are given that $e_S(t)=E+E_1\sin(500t)+E_2\sin(1000t)$, L=10 millihenries, C=200 microfarads, R_1=10 ohms, R_2=5.0 ohms and R_3=5.0 ohms in the above circuit.

For questions 11–14 assume that switch K is closed at t=0 and answer the questions for the instant immediately after the switch is closed, i.e. for time $t=0^+$.

11. If E=30V, E_1=40V and E_2=20V, the current i_C is most nearly
(A) 0.0 amperes
(B) 1.5 amperes
(C) 2.8 amperes
(D) 3.0 amperes
(E) 6.0 amperes

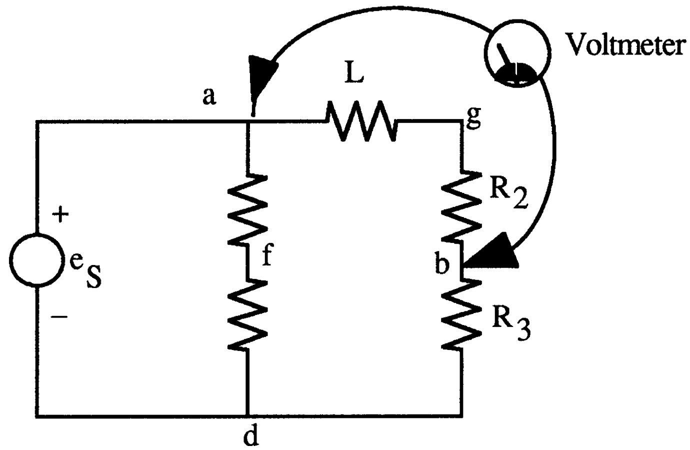
This problem is most easily solved by recalling the initial conditions for capacitors which require that V(0-) = V(0+). The initial voltage on the capacitor is 0 volts so the voltage across the capacitor immediately after the switch is closed must also be 0 volts. The applied voltage $e_S(t=0+) \approx 30$ since $\sin(0+) \approx 0$. At t=0+ e_S appears enitrely across R_1 and the resulting current (which is equal to i_C since R_1 and C are in series) must be given by

$$i_C = \frac{e_S}{R_1} = \frac{30\text{ volts}}{10\Omega} = 3.0\text{ Amperes}$$

The correct answer is (D).

12. If E=30V, E_1=40V and E_2=20V, the magnitude of the voltage between points a and b is most nearly
(A) 5.0 volts
(B) 7.5 volts
(C) 15 volts
(D) 22 volts
(E) 30 volts

The voltage being referred to is across the series combination of the inductor and R_2 as shown in the diagram below.



As in question #11 the voltage $e_S(0^+) \approx 30$ volts, $i_L(0^+)=i_L(0^-)$ since the current through inductors is continuous, and $V_C(0^+)=V_C(0^-)$ since the voltage across capacitors is continuous. Since there is no current flow through L at $t=0^+$ the inductor represents an open circuit. The potential at a is +30 volts; the potential at b is zero since it is connected to ground through R_3 and no current is flowing through R_3. The potential v_{ab} is then 30 volts. The correct answer is (E)

13. If E, E_1 and E_2 are magnitudes such that i_C at $t=0^+$ is 2.0 amperes, the rate of change of the voltage between points f and d is most nearly
(A) 0.0 volts/second
(B) 20 volts/second
(C) 5×10^2 volts/second
(D) 5×10^3 volts/second
(E) 1.0×10^4 volts/second
This question is a lot simpler than it sounds and is a direct application of of the definition of capcitance. By definition,

$$i_C = C\frac{dv}{dt}$$

which includes a direct expression of the rate of change of voltage. Evaluating this expression for $t=0^+$,

$$i_C(0^+) = C\frac{dv}{dt}\bigg|_{0^+}$$

which can be solved for the time rate of change of voltage at $t=0^+$

$$\frac{dv}{dt}\bigg|_{t=0^+} = \frac{i_C(0^+)}{C} = \frac{2 \text{ Amperes}}{200\times10^{-6} \text{ Farads}} = 10^4 \frac{\text{volts}}{\text{second}}$$

The closest answer is (E).

14. If E, E_1 and E_2 are magnitudes such that the voltages between points a and g is 40 volts at $t=0^+$, the rate of change of i_L is most nearly
(A) 0.0 amps/second
(B) 4×10^{-2} amps/second
(C) 4×10^3 amps/second
(D) 2×10^4 amps/second
(E) 4×10^4 amps/second

The solution of this problem is almost identical to that of problem #13 with the exception that the voltage expression must be that for an inductor, i.e.

$$V_L = L\frac{di}{dt}$$

Evaluating this expression at t=0+

$$\left.\frac{di}{dt}\right|_{t=0^+} = \frac{V_L(0^+)}{L} = \frac{40 \text{ volts}}{10 \times 10^{-3} \text{ Henrys}} = 4000\,\frac{\text{amps}}{\text{second}}$$

The answer is (C).

15. If E=30V, E_1=40V and E_2=20V, the magnitude of the current i_C is most nearly
(A) 0.0 amperes
(B) 1.5 amperes
(C) 2.8 amperes
(D) 3.0 amperes
(E) 6.0 amperes

$e_S(t)=E+E_1\sin(500t)+E_2\sin(1000t)$

$i_C(0^-)=0$ and $i_L(0^-)=0$ since there are no voltage sources for t<0. Using our rules for boundary conditions on inductors and capacitors $i_L(0^+)=i_L(0^-)$ but $i_C(0^+)\neq i_C(0^-)$. At t=0+ $e_S(0^+)=30+40\sin(500\times0^+)+20\sin(1000\times0^+) \approx 30$ volts since the sine of a small number is approximately zero. Since $V_C(0^+)=V_C(0^-)$ (remember that the voltage across a capacitor is continuous) the current i_C through R_1 at t=0+ must then be given by i_C = 30volts/10ohms = 3 amperes. The answer is (D).