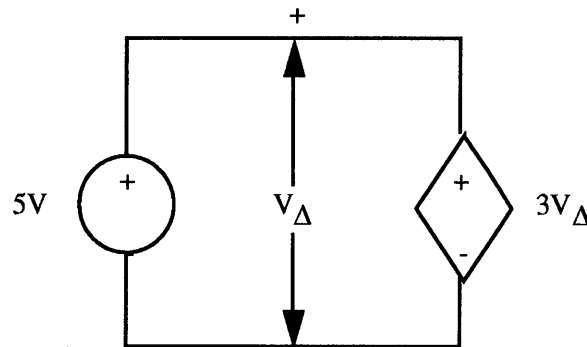


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Server Adobe PostScript version: 48.3
Server software version: V2.0
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Server name: crowford
Server job number: 1
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Client network node: util
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Submitted at: Thu Mar 18 09:52:43 1993 50 199320:41 1993
Printed at: Thu Mar 18 09:52:43 1993

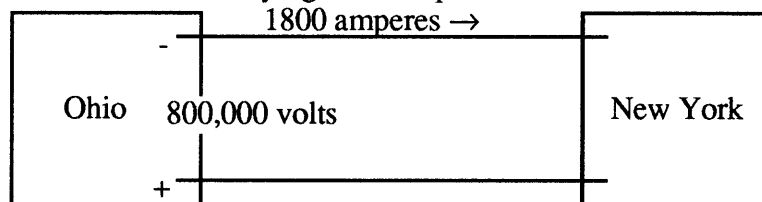
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Quiz_1.ps.732464623

1. Is the following circuit a valid circuit? Explain why or why not using the definitions of ideal voltage and current sources.



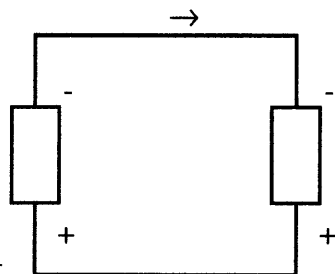
Answer: The circuit is invalid because the voltage source causes V_{Δ} to be 5V. This in turn causes the dependent (controlled) voltage source to force V_{Δ} to $3V_{\Delta}=15$ volts. V_{Δ} cannot be both +5 and +15 and the circuit is invalid because there is no common solution of the ideal voltage source v-i curves.
Misinterpreted source as a current source -10 points.

2. A high-voltage direct current transmission line between Ohio and New York is operating at 800,000 volts and carrying 1800 amperes as shown.



- (a) What is the power at the New York end?

Answer: -1.44×10^9 watts.

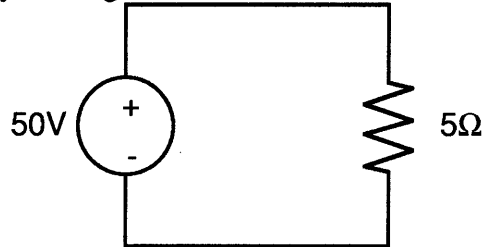


Redrawing the circuit we see that New York is opposite to the passive sign convention. The power is then $p = -vi = -(8 \times 10^5 \text{ volts})(1.8 \times 10^3 \text{ amps}) = -1.44 \times 10^9$ watts. Since this is a negative quantity New York is generating power.
-3 points for sign loss.

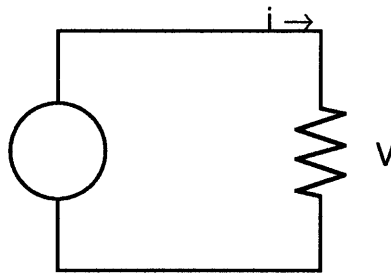
(b) Is Ohio absorbing or generating power? Explain your answer.

Ohio is absorbing power since it obeys the passive sign convention and $p=vi>0$ for Ohio.

3. (a) Calculate the values of v and i across the resistor. Be sure to label their directions and polarity on your diagram.



Answer: $v=50$ volts. $i=v/R=50/5=10$ amperes. Their polarity is as shown below. You lost -5 points for using the wrong form of Ohm's Law. -6 points for voltage polarity and current direction.



(b) What is the power dissipated in the resistor?
Answer: $P=vi=(50 \text{ volts})(10 \text{ amperes}) = 500 \text{ watts}$

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<i>Server Adobe PostScript version:</i>	48.3
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<i>Submitted at:</i>	Fri Mar 19 08:31:52 1993
<i>Printed at:</i>	Fri Mar 19 08:31:53 1993

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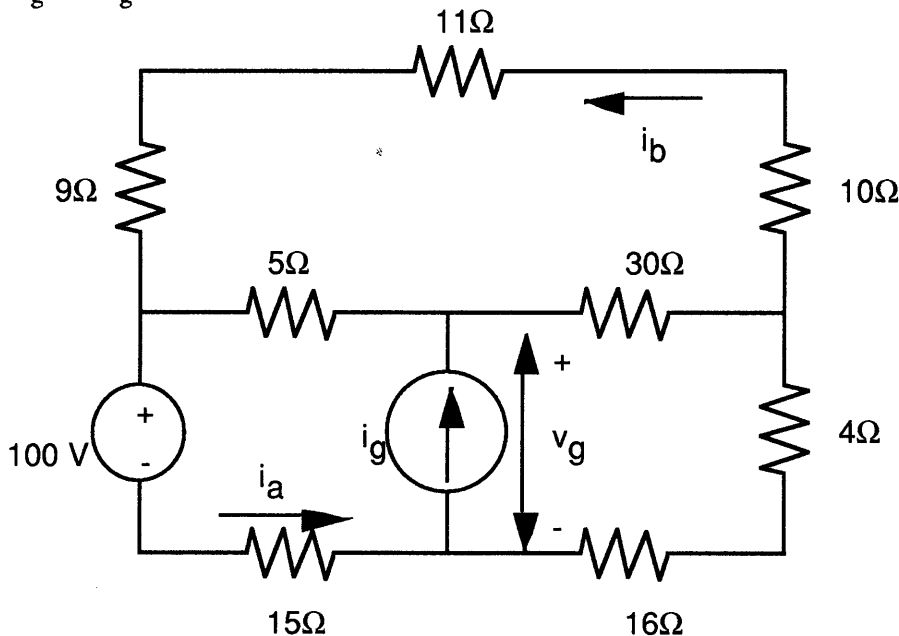
Quiz #2
EEAP 244

January 27, 1993

Topics on quiz:

- definitions of nodes, branches, loops, etc.
- ability to find currents, voltages and powers in simple resistive circuits
- validity of circuits due to current and voltage source constraints
- ability to determine currents, voltages and powers in multi-branch resistive circuits with independent and dependent sources
- circuit modeling and simple equivalent circuits
- simple KCL and KVL principles
- equivalent resistance of series and parallel connections and power delivered to resistance networks
- calculation of currents, voltages and powers developed in multi-branch circuits using KCL and KVL
- voltage and current divider circuits
- inductor voltages, currents and powers as a function of time
- capacitor voltages, currents and powers as a function of time

1. The currents i_a and i_b in the circuit shown below are 4A and -2A respectively. Determine i_g and v_g .



Answer: Solve the problem using loop mesh analysis. Use loops in the directions of i_a and i_b . Use a clockwise loop for i_c which includes the 30, 4 and 16Ω resistors as well as the current source. The resulting loop equations are:

$$\text{loop 1: } +i_b(9+10+11) + 5(i_b-i_a) + 30(i_b+i_c) = 0$$

$$\text{loop 2: } -v_g + 5(i_a-i_b) + 100 + 15i_a = 0$$

$$\text{loop 3: } +v_g + 30(i_c-i_b) + i_c(16+4) = 0$$

plus the constraint equation:

$$\text{eqn 4: } i_g = i_a + i_c$$

Since i_a and i_b are known I can use the loop 1 equation to find i_c

$$+(-2)(30) + 5(-2-4) + 30(-2+i_c) = 0$$

which gives $i_c=5$ amperes

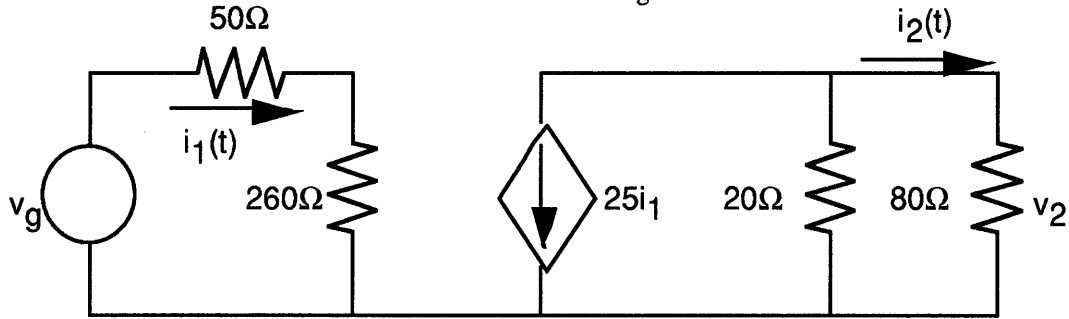
This, also tells me i_g since $i_g=i_a+i_c=+4+5=+9$ amperes

Now, using the equation for loop 2 I can solve for v_g :

$$-v_g + 5(4-(-2)) + 100 + 15(4) = 0$$

or, $v_g=+190$ volts.

2. Find i_2 and v_2 in the circuit shown below when $v_g = 5$ volts.



Answer:

Using Ohm's Law at the input loop:

$$i_1 = \frac{v_g}{50+260} = \frac{5}{310} = 16.13 \text{ mA}$$

Now, evaluating the current controlled current source

$$25i_1 = \frac{25(5)}{310} = 0.403 \text{ A}$$

The current i_2 is found from the current source current by using a current divider:

$$i_2 = -\frac{20}{20+80}(0.403 \text{ A}) = -80.6 \text{ mA}$$

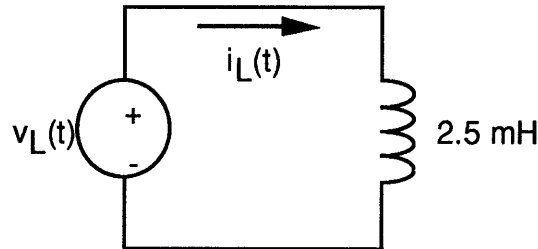
Use Ohm's Law to get the output voltage:

$$v_2 = iR = (-0.806\text{A})(80\Omega) = -6.45 \text{ volts}$$

3. The current through the 2.5-mH inductor in the circuit shown below is 1 ampere for $t \leq 0$. The voltage across the inductor for $t \geq 0$ is given by

$$v_L(t) = \begin{cases} 3e^{-4t} \text{ mV}, & 0^+ \leq t < 2 \text{ seconds} \\ -3e^{-4(t-2)} \text{ mV}, & 2^+ \leq t < \infty \end{cases}$$

Determine $i_L(t)$. Sketch your result.



Answer: Since $v_L(t) = L \frac{di_L(t)}{dt}$ we must find the current by integrating the voltage. For $0 \leq t < 2$ seconds we have

$$i_L(t) = \frac{1}{L} \int_{-\infty}^t v_L(x) dx = \frac{1}{L} \int_{-\infty}^0 v_L(x) dx + \frac{1}{L} \int_0^t v_L(x) dx$$

$$= 1 \text{ Ampere} + \frac{1}{2.5 \times 10^{-3}} \int_0^t 3 \times 10^{-3} e^{-4x} dx$$

$$= 1 \text{ A} + 1.2 \frac{e^{-4x}}{-4} \Big|_0^t = 1 - 0.3e^{-4t} + 0.3 \text{ A}$$

$$= 1.3 - 0.3e^{-4t} \text{ A for } 0 \leq t \leq 2 \text{ seconds}$$

Note that we used the fact that the current was 1 Ampere for $t \leq 0$

The analysis for $t > 2$ seconds is identical in form except for the initial condition which is $i_L(2) = 1.3 - 0.3e^{-4(2)} \text{ A} = 1.3 - 0.0003354 \approx 1.3$ Amperes

giving

$$i_L(t) = \frac{1}{L} \int_{-\infty}^t v_L(x) dx = \frac{1}{L} \int_{-\infty}^2 v_L(x) dx + \frac{1}{L} \int_2^t v_L(x) dx$$

$$= 1.3 \text{ Ampere} + \frac{1}{2.5 \times 10^{-3}} \int_2^t -3 \times 10^{-3} e^{-4(x-2)} dx$$

$$= 1.3 - 1.2 \frac{e^{-4(x-2)}}{-4} \Big|_2^t = 1.3 + 0.3e^{-4(t-2)} - 0.3 \text{ A}$$

$$= 1.0 + 0.3e^{-4(t-2)} \text{ A for } 2 \leq t \text{ seconds}$$

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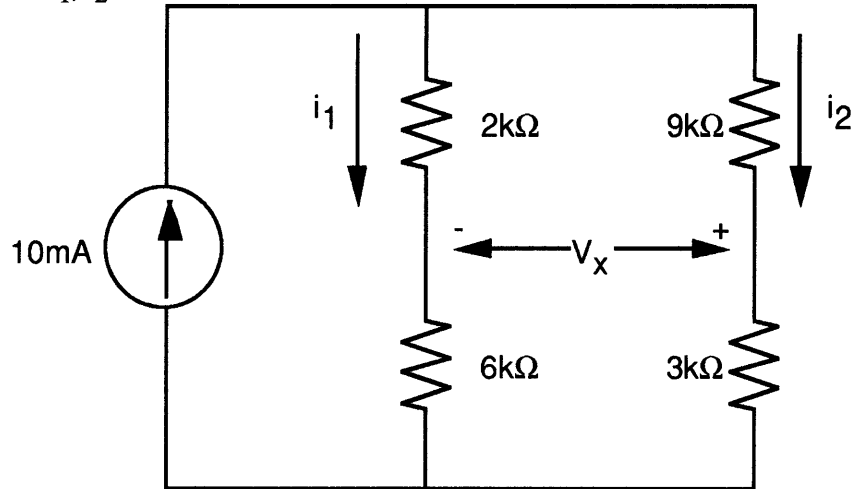
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<i>Submitted at:</i>	Fri Mar 19 08:33:59 1993
<i>Printed at:</i>	Fri Mar 19 08:34:00 1993

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Quiz #2 SOLUTIONS
EEAP 244

February 2, 1993

1. Determine i_1 , i_2 .

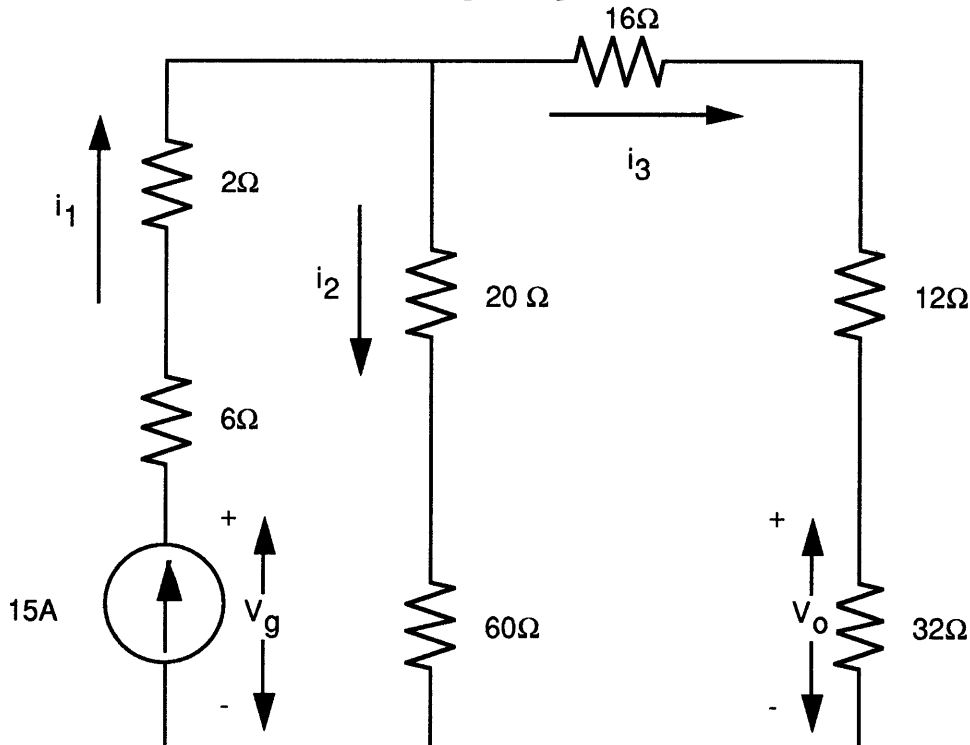


Answer: This problem can be done most efficiently using voltage and current dividers.

$$i_1 = \frac{R_{\text{branch 2}}}{R_{\text{branch 1}} + R_{\text{branch 2}}} i = \frac{(9\text{k} + 3\text{k})}{(2\text{k} + 6\text{k}) + (9\text{k} + 3\text{k})} 10\text{mA} = \frac{12\text{k}}{8\text{k} + 12\text{k}} 10\text{mA} = \frac{12\text{k}}{20\text{k}} 10\text{mA} = 6\text{mA}$$

Since KCL applies at the upper node, $i_2 = 10\text{mA} - i_1 = 10 - 6 = 4\text{mA}$

2. For the circuit shown below: (a) Determine i_1 , i_2 and i_3 ; (b) determine V_o and V_g .
NOTE: If you are pressed for time set up the equations but don't solve them.



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Answer:

(a)

You have to use one KCL equation (since there are two nodes), one source constraint, and one KVL equation.

Using KCL at node 1 with + in: $i_1 - i_2 - i_3 = 0$

The source provides: $i_1 = +15A$

KVL around the right hand loop (you can't go across the current source) gives:

$$-60i_2 - 20i_2 + 16i_3 + 12i_3 + 32i_3 = 0$$

These can be solved simultaneously:

$$15 - i_2 - i_3 = 0$$

$$-80i_2 + 60i_3 = 0$$

to give $i_1 = +15A$, $i_2 = 45/7 = 6.4 A$, $i_3 = 4/3(i_2) = 60/7 = 8.6 A$

(b) Solving for V_o is simple:

$$V_o = 32i_3 = 32\left(\frac{60}{7}\right) = \frac{1920}{7} \cong 274 \text{ volts}$$

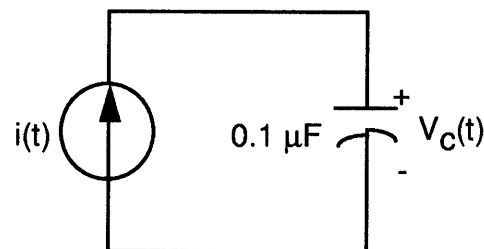
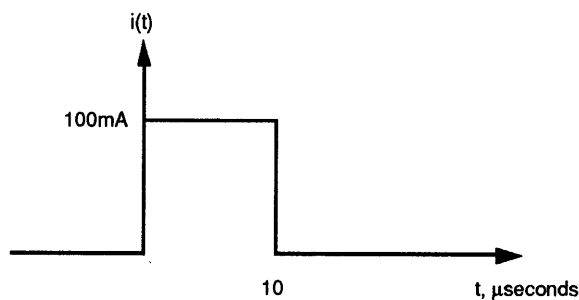
Solving for V_g is more complicated. You have to determine V at node 1 or do KVL around the loop containing the current source.

$$-V_g + 6i_1 + 2i_1 + 20i_2 + 60i_2 = 0$$

Using the above expressions for the currents we get:

$$V_g = +6(15) + 2(15) + 20(6.4) + 60(6.4) = 90 + 30 + 128 + 384 = 632 \text{ volts}$$

3. The current pulse shown at the left below is applied to the circuit shown at the right below. The capacitor is initially charged to $V_c(t) = -5$ volts for $t < 0$ seconds. Determine $V_c(t)$ as a function of time. Sketch your answer.



Answer:

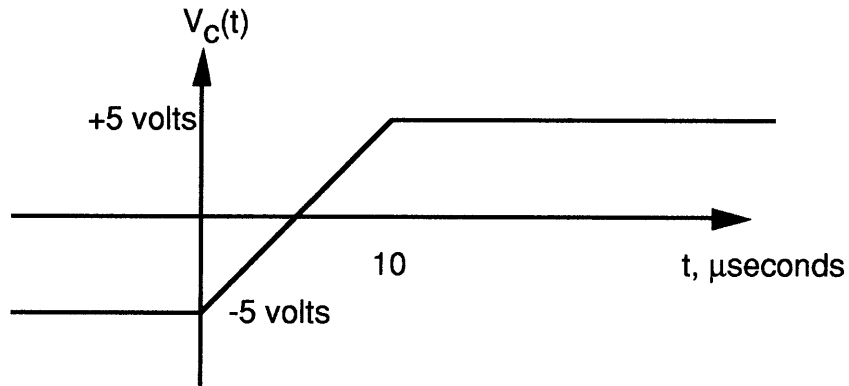
Solving for $0 \leq t \leq 10 \mu\text{seconds}$

$$\begin{aligned} V_c(t) &= \frac{1}{C} \int_{-\infty}^t i(x) dx = \frac{1}{0.1 \times 10^{-6}} \int_{-\infty}^t 100 \times 10^{-3} dx \\ &= \frac{100 \times 10^{-3}}{0.1 \times 10^{-6}} \Big|_0^t - 5 = 10^6 t - 5 \text{ for } 0 \leq t \leq 10 \mu \text{ seconds} \end{aligned}$$

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By inspection, $V_c(t) = -5$ for $t < 0$ and $V_c(t) = V_c(10\mu\text{seconds}) = +5$ volts for $t > 10$ $\mu\text{seconds}$



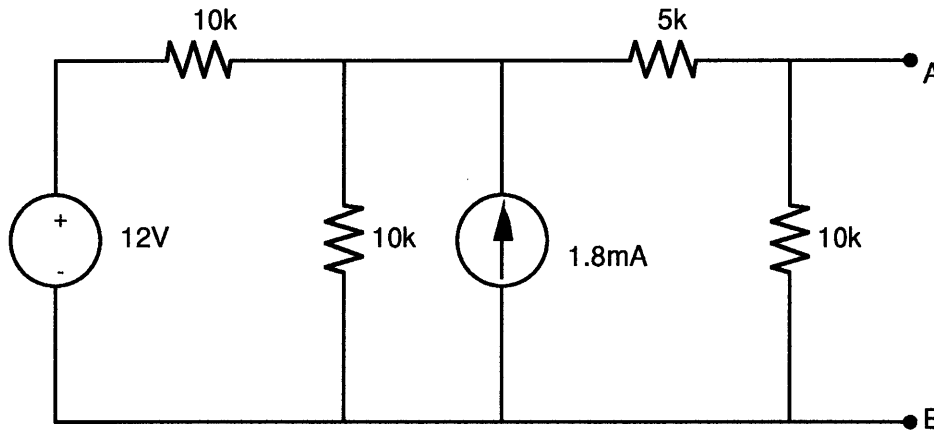
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<i>Submitted at:</i>	Fri Mar 19 08:35:35 1993
<i>Printed at:</i>	Fri Mar 19 08:35:36 1993

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quiz_3.ps.732547303

Quiz #3 SOLUTIONS
EEAP 244

February 8, 1993

1. Determine the open circuit voltage, the short circuit current, and the thevenin equivalent resistance for the circuit shown below.



Answer:

For the open circuit voltage this problem can be done by assigning node 1 to the junction of the 10k resistors and the current source and node 2 to the junction at the output, i.e. A.

At node 1, with + representing current into the node,

$$+\frac{12-V_1}{10} - \frac{V_1}{10} + 1.8 \text{ mA} + \frac{V_2-V_1}{5} = 0$$

At node 2, with + representing current into the node,

$$-\frac{V_2-V_1}{5} - \frac{V_2}{10} = 0$$

These equations can be readily solved:

$$12 - V_1 - V_1 + 18 + 2V_2 - 2V_1 = 0$$

$$-2V_2 + 2V_1 - V_2 = 0$$

Simplifying and re-arranging these equations:

$$-2V_1 + 2V_2 = -30 \text{ volts}$$

$$2V_1 - 3V_2 = 0$$

which has solution $V_2 = +7.5$ volts.

For the short-circuit current, the output short bypasses the 10k resistor between A and B.

Keeping the same node assignments KCL at node 1 can be written as:

$$+\frac{12-V_1}{10} - \frac{V_1}{10} + 1.8 \text{ mA} - \frac{V_1}{5} = 0$$

which is a single equation in only V_1 . Solving for V_1 gives $V_1 = +7.5$ volts. The short circuit current is then

$$i_{sc} = \frac{7.5 \text{ V}}{5 \text{ k}\Omega} = 1.5 \text{ mA}$$

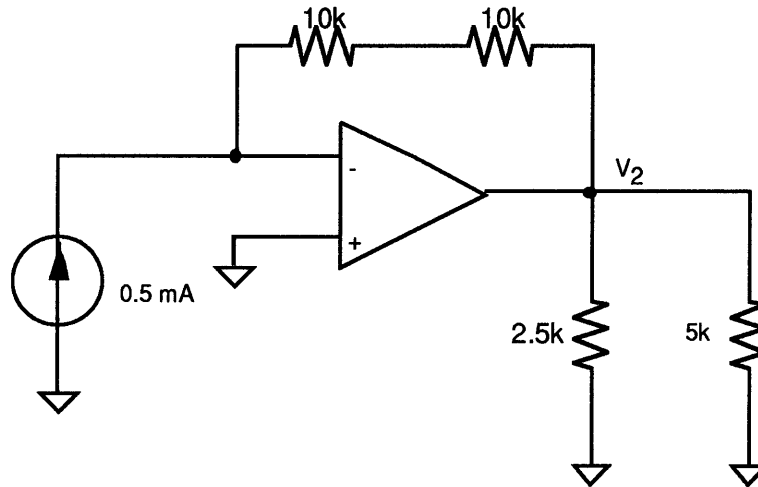
and the Thevenin resistance is

2. (a) For the circuit as shown below determine V_2 . NOTE: State any assumptions.

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- (b) If you place a short from V_2 to ground, what is the short circuit output current? Is there any current going through the op-amp output terminal?
- (c) What is the Thevenin equivalent circuit for the op-amp at the output terminal, i.e. at V_2 .



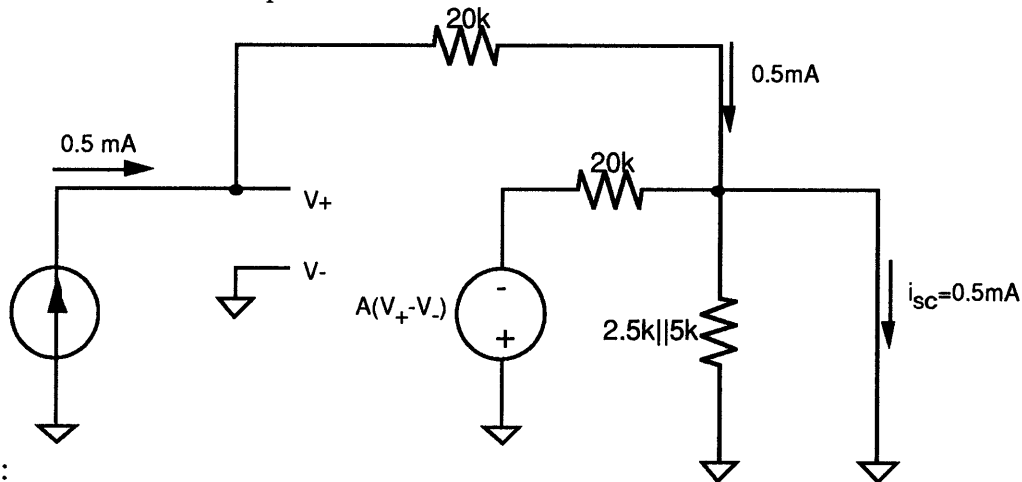
Answer:

(a) The important assumption is the virtual ground assumption (e.g., the virtual short) at the input. Using KCL at the - (i.e., inverting) input of the op-amp:

$$\frac{V_2 - 0}{20k} = 0.5 \text{ mA}$$

Solving for V_2 gives $V_2 = 10$ volts.

(b) By inspection, the short circuit current is the 0.5 mA coming from the input current source. Reasoning: the short bypasses the 2.5k and 5k load resistors. The only place any other current could come from is the op-amp itself. Since the voltage input is zero, the voltage source (in its equivalent circuit) is not producing any voltage and, thus, there is no additional current at the output. See the circuit



below:

As an aside, if you throw aside the ideal op-amp assumptions you will have some current from the op-amp. Problems 6.34 and 6.35 show this. For example, in 6.34 there is a 500k input resistance to the op-amp (between the V_+ and V_- terminals). This allows an input current into the op-amp input (you must use KCL at the V_- input to solve the problem), voltage across the input terminals (i.e., the virtual short assumption is not applicable if you have an input resistance), and you would have a current output from the

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op-amp if it were used in this problem. By the way, 6.34 was an assigned problem. Have a TA show you how to do it. If you want a similar problem to try do 6.35.

(c) No current goes through the op-amp because of the short. R_T is then given by $10\text{volts}/0.5\text{ma}=20\text{k}\Omega$

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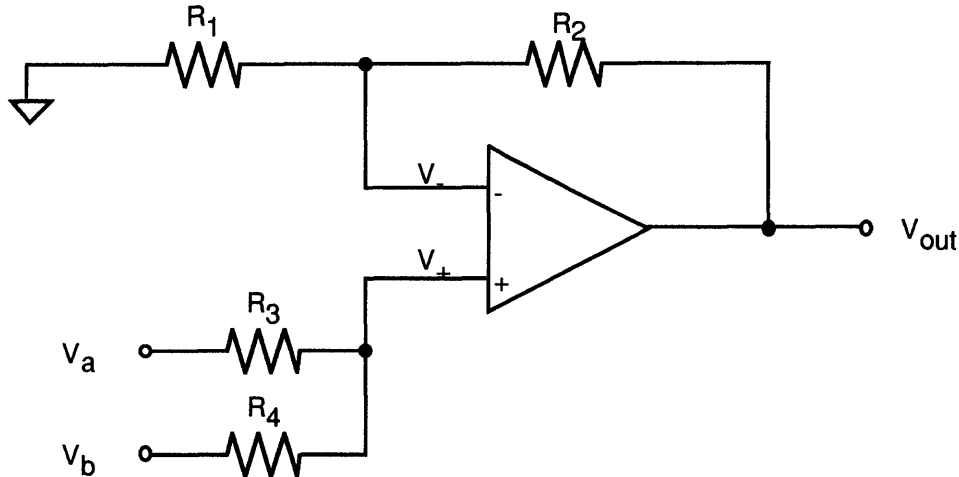
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<i>Submitted at:</i>	Fri Mar 19 08:37:07 1993
<i>Printed at:</i>	Fri Mar 19 08:37:07 1993

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quiz_4.ps.732547293

Quiz #4 SOLUTIONS
EEAP 244

February 15, 1993

1. You are given that $R_1=5k\Omega$, $R_2=10k\Omega$, $R_3=47k\Omega$, and $R_4=33k\Omega$ for the op-amp circuit shown below. Assuming that the op-amp is ideal, what is v_{out} if $v_a=5$ volts and $v_b=3$ volts?



Solution:

The formulas for this op-amp are relatively straight forward. Using KCL (+ in) at the + input of the op-amp:

$$\frac{v_a - V_+}{R_3} + \frac{v_b - V_+}{R_4} = 0$$

$$\frac{5 - V_+}{47k} + \frac{3 - V_+}{33k} = 0$$

And, solving for V_+ ,

$$165 - 33v_+ + 141 - 47v_+ = 0$$

giving

$$v_+ = 3.825 \text{ volts}$$

Since $V_- = V_+$ (the virtual short assumption) we proceed to the voltage divider at the inverting input to get

$$v_- = v_{out} \frac{R_1}{R_1 + R_2}$$

and, substituting,

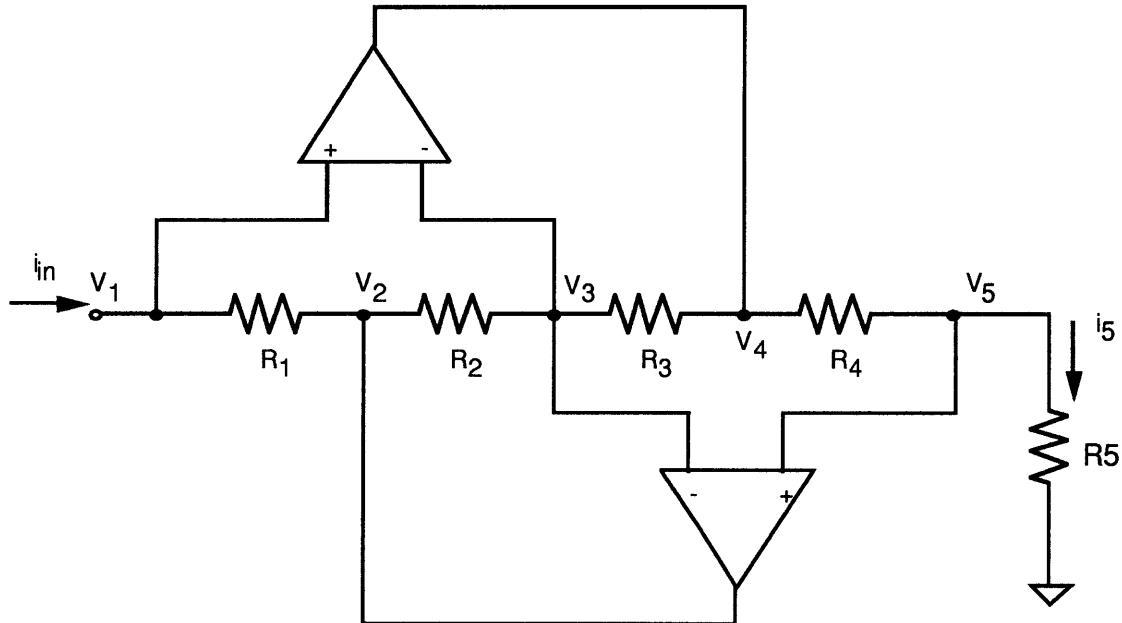
$$v_{out} = v_- \frac{R_1 + R_2}{R_1} = 3.825 \frac{5 + 10}{5} = 11.475 \text{ volts}$$

One of the biggest problems on this problem was PSC (Passive sign convention); the seconds biggest was units (dealing with $k\Omega$).

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2. This is a tough problem. You are given the following circuit which contains only ideal, identical op-amps and told $i_5=1\text{A}$. What is the value of i_{IN} , V_1 , and $R_T=V_1/i_{IN}$?



Answer: Since the op-amps are ideal they must satisfy the virtual short assumption, i.e. that $V_+=V_-$ for each op-amp. This means that $V_1=V_3=V_5=1(R_5)=R_5$ (assuming that $i_5=1\text{amp}$). I know V_1 and the only problem left is to determine the currents. The current through R_4 must also be 1A since no current can enter the op-amp. This means that $V_4=1(R_4)+V_5=R_4+R_5$. The current through R_3 is then given by KVL.

$$i_3 = \frac{V_3 - V_4}{R_3} = \frac{V_5 - V_4}{R_3} = \frac{R_5 - (R_4 + R_5)}{R_3} = \frac{R_5 - (R_4 + R_5)}{R_3} = \frac{-R_4}{R_3}$$

The voltage at V_3 is known. The voltage at V_2 is then given by

$$V_2 = i_2 R_2 + R_5 = \frac{-R_4}{R_3} R_2 + R_5 = \frac{-R_2 R_4 + R_3 R_5}{R_3}$$

The input current is then given by

$$i_{in} = \frac{1(R_5) - V_2}{R_1} = \frac{1(R_5) + \frac{R_2 R_4 - R_3 R_5}{R_3}}{R_1} = \frac{R_2 R_4}{R_1 R_3}$$

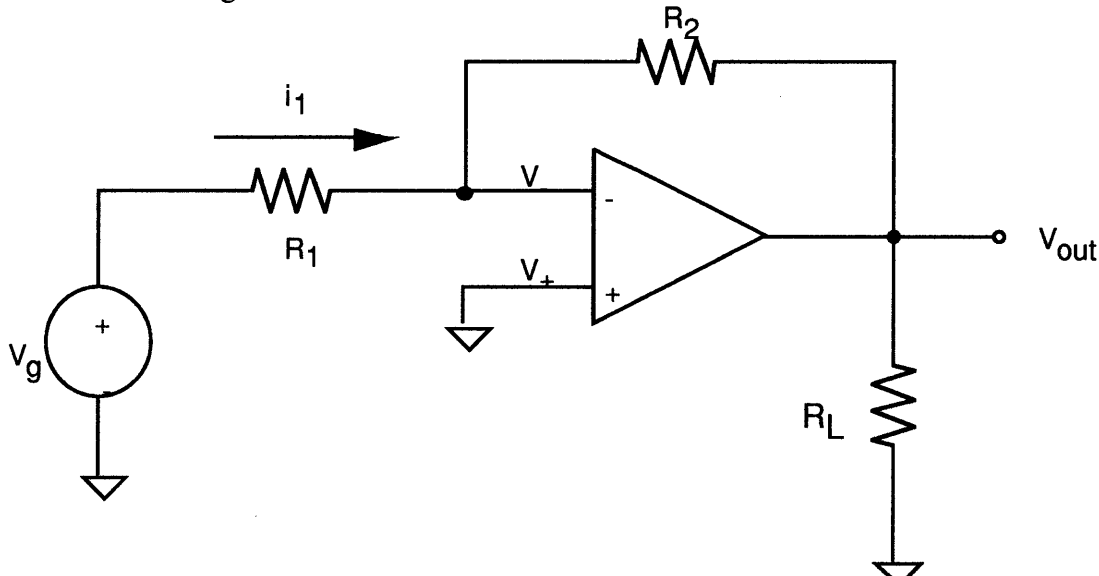
The input resistance is then

$$R_T = \frac{V_5}{i_{in}} = \frac{R_5}{\frac{R_2 R_4}{R_1 R_3}} = \frac{R_1 R_3 R_5}{R_2 R_4}$$

In grading, I basically looked for the virtual short assumption (e.g., $V_5=V_3=V_1$). I then looked for use of KCL at nodes 2 and 4. Finally, I checked to see if you included an op-amp output current in your calculations. Each of these was worth approximately 10 points.

3. This is a long problem. You are given the following circuit values for the circuit shown below.

$$\begin{aligned} R_1 &= 2\text{K}\Omega; \\ R_2 &= 180\text{k}\Omega; \\ R_L &= 1600\Omega \\ V_g &= 100\text{mV} \end{aligned}$$



- (a) Assuming an ideal op-amp, determine the values of v_{out} , i_1 and the Thevenin input resistance $R_T = V_g/i_1$.

The operational amplifier is no longer ideal. It has an open loop gain of 250,000, an input resistance of 500k Ω , and an output resistance of 5k Ω .

- (b) Draw the new circuit explicitly showing the non-ideal model for the op-amp.
 (c) Determine the voltages v_- and v_{out} .
 (d) What is the new value of the Thevenin resistance?

Answer:

(a) Since the amplifier is ideal you can use the virtual ground assumption for V_- . The input current is then given by

$$i_1 = \frac{V_g - V_-}{R_1} = \frac{0.1 - 0}{2000} = 50 \times 10^{-6} \text{ Amps}$$

This is also the current through R_2 and the output voltage V_o can be found to be

$$V_o = -i_1 R_2 = -(50 \times 10^{-6})(180 \times 10^3) = -9 \text{ volts}$$

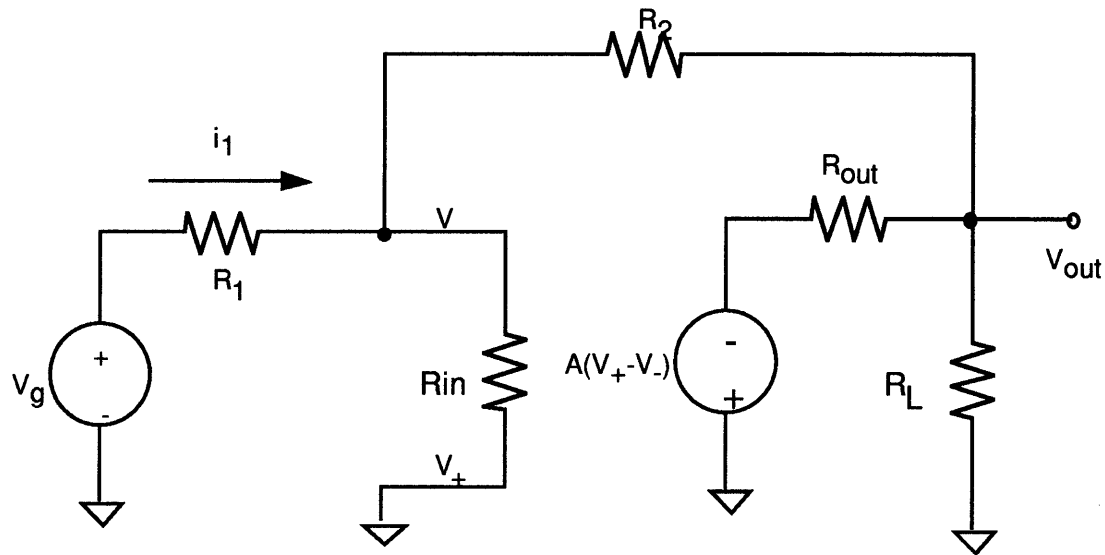
R_T is, quite simply,

$$R_T = \frac{V_g}{i_1} = \frac{0.1 \text{ volts}}{50 \mu\text{A}} = 2000 \Omega$$

I took points off for the sign of the output voltage. A number of people got the amount of current wrong or attempted to calculate Thevenin output resistance.

- (b) The complete non-ideal equivalent circuit is:

below:



I looked for the sign of the op-amp voltage source and took points off for it.

(c) The non-ideal op-amp can only be solved by setting up KCL at the input and output nodes of the op-amp as follows:

$$\frac{V_g - V_-}{R_1} - \frac{V_-}{R_{in}} + \frac{V_o - V_-}{R_2} = 0$$

$$-\frac{V_o - V_-}{R_2} - \frac{V_o + AV_-}{R_{out}} - \frac{V_o}{R_L} = 0$$

Many people did not get these equations right. The biggest mistake was PSC (passive sign convention). The second biggest mistake was combining voltages and currents.

Now, after substitution,

$$\frac{V_g - V_-}{2k} - \frac{V_-}{500k} + \frac{V_o - V_-}{180k} = 0$$

$$-\frac{V_o - V_-}{180k} - \frac{V_o + AV_-}{5k} - \frac{V_o}{1600} = 0$$

and re-arranging,

$$\left(-\frac{1}{2k} - \frac{1}{500k} - \frac{1}{180k}\right)V_- + \left(\frac{1}{180k}\right)V_o = -\frac{V_g}{2k}$$

$$\left(\frac{1}{180k} - \frac{A}{5k}\right)V_- + \left(\frac{1}{180k} - \frac{1}{5k} - \frac{1}{1.6k}\right)V_o = 0$$

These equations are solved to give $V_- = -0.001473$ volt and $V_o = -8.987$

(d) The input impedance is given quite simply as

$$R_T = \frac{V_g}{i_g} = \frac{V_g}{\frac{V_g - V_-}{R_1}} = \frac{0.1}{\frac{0.1 - 0.0014}{2k}} = 2003\Omega$$

Name: _____

SSN: _____

A lot of people calculated the Thevenin output resistance. The input was asked for in the problem and a circuit such as the one in this problem can have an output AND an input Thevenin resistance.

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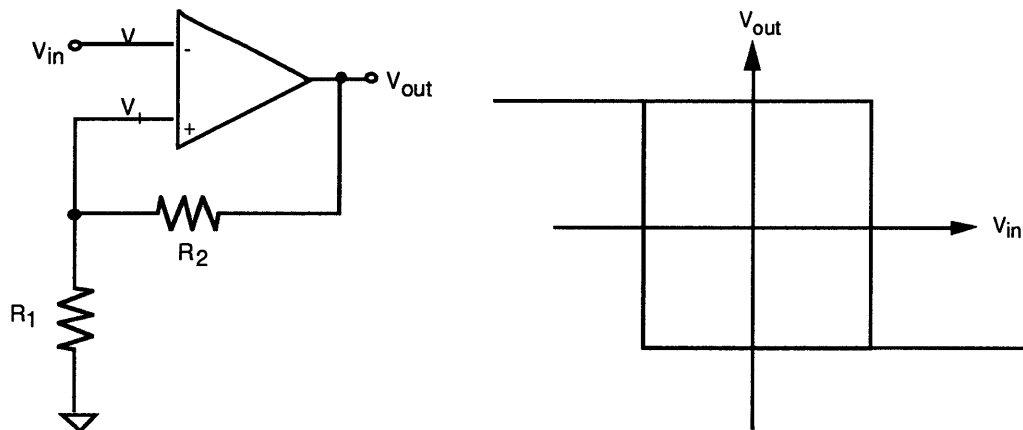
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<i>Client network node:</i>	util
<i>Client name:</i>	flm
<i>Client job name:</i>	quiz_5.ps.732547286
<i>Submitted at:</i>	Fri Mar 19 08:39:23 1993
<i>Printed at:</i>	Fri Mar 19 08:39:24 1993

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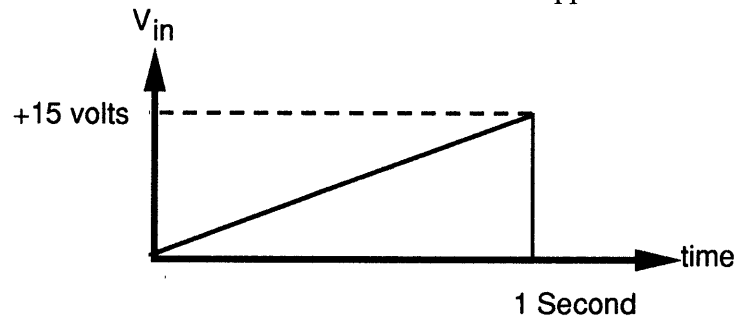
Quiz #5 SOLUTIONS
EEAP 244

February 22, 1993

1. Answer the following questions about the Schmitt trigger circuit shown below.



- (a) If $V_{in}=0$ volts and $V_{out}=+15$ volts describe what happens to V_{out} if $V_{in}(t)$ is:



Answer: The output voltage will remain constant at +15 as V_{in} increases to the trip voltage (determined by R_1 and R_2). When V_{in} reaches this trip voltage, the V_- input of the op-amp will dominate the V_+ input and the output will change to -15 volts and will remain there.

- (b) Using the input/output diagram at the above right, explain hysteresis as it applies to a Schmitt trigger. How can you change it in the given circuit.

Answer: Hysteresis is the difference between the upper and lower trip points. Basically, the circuit will trip at different voltages depending upon the current output voltage and the direction of the voltage, i.e. is it increasing or decreasing. The trip voltage is determined by the voltage divider relationship

- (c) How could you modify the above circuit so that the circuit operation is no longer centered about zero.

Name: _____

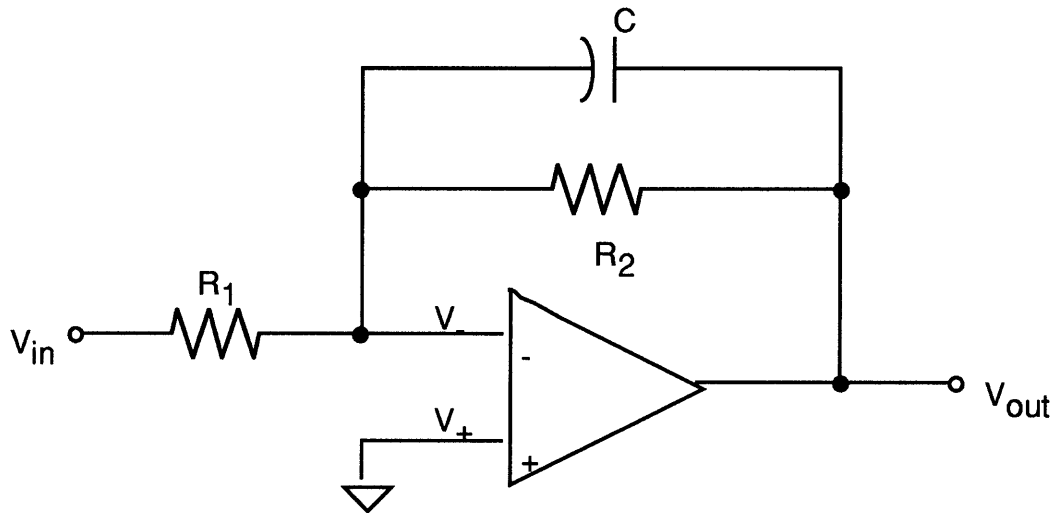
SSN: _____

Answer: By connecting a voltage source through a resistance to the non-inverting input, or, alternatively, by connecting a battery to the grounded end of R1.

Name: _____

SSN: _____

2. The figure below is a circuit for an inverting amplifier designed to be used as a filter. The input is the complex signal $V_{in}(t) = \text{Re}\{V_i e^{j\omega t}\}$.



You are to:

(a) determine the response, $V_{out}(t) = \text{Re}\{V_o e^{j\omega t}\}$.

Answer:

Use KCL at the inverting node to get $\frac{V_{in} - 0}{R_1} + \frac{V_{out} - 0}{Z} = 0$ where

$$Z = R_2 \parallel \frac{1}{j\omega C} = \frac{R_2 \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{j\omega C R_2 + 1} = \frac{R_2}{1 + j\omega R_2 C}$$

Solving for V_{out} gives

$V_{out} = -\frac{Z}{R_1} V_{in}$ which is the same formula we have derived before except that when we worked with resistive circuits we had $V_{out} = -\frac{R_2}{R_1} V_{in}$.

Using our expression for Z in our expression for V_{out} we get

$$\begin{aligned} V_{out} &= -\frac{\frac{R_2}{1 + j\omega R_2 C}}{R_1} V_{in} = \frac{-\frac{R_2}{R_1}}{1 + j\omega R_2 C} V_{in} = \frac{-\frac{R_2}{R_1}}{\sqrt{1 + (\omega R_2 C)^2} e^{j\theta}} V_{in} \\ &= \frac{-\frac{R_2}{R_1}}{\sqrt{1 + (\omega R_2 C)^2}} e^{-j\theta} V_{in} = \frac{-\frac{R_2}{R_1}}{\sqrt{1 + (\omega R_2 C)^2}} e^{-j\theta} V_i e^{j\omega t} \\ &= \frac{-\frac{R_2}{R_1}}{\sqrt{1 + (\omega R_2 C)^2}} e^{-j\theta} V_i e^{j\omega t} = \frac{-\frac{R_2}{R_1}}{\sqrt{1 + (\omega R_2 C)^2}} V_i e^{j(\omega t - \theta)} \end{aligned}$$

Taking the real part, the solution is

Name: _____

SSN: _____

$$V_{\text{out}}(t) = \frac{-\frac{R_2}{R_1}}{\sqrt{1 + (\omega R_2 C)^2}} V_i \cos(\omega t - \theta)$$

(a) determine the magnitude of the response V_o .

Answer: Already done in the above answer.

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<i>Print engine version:</i>	17
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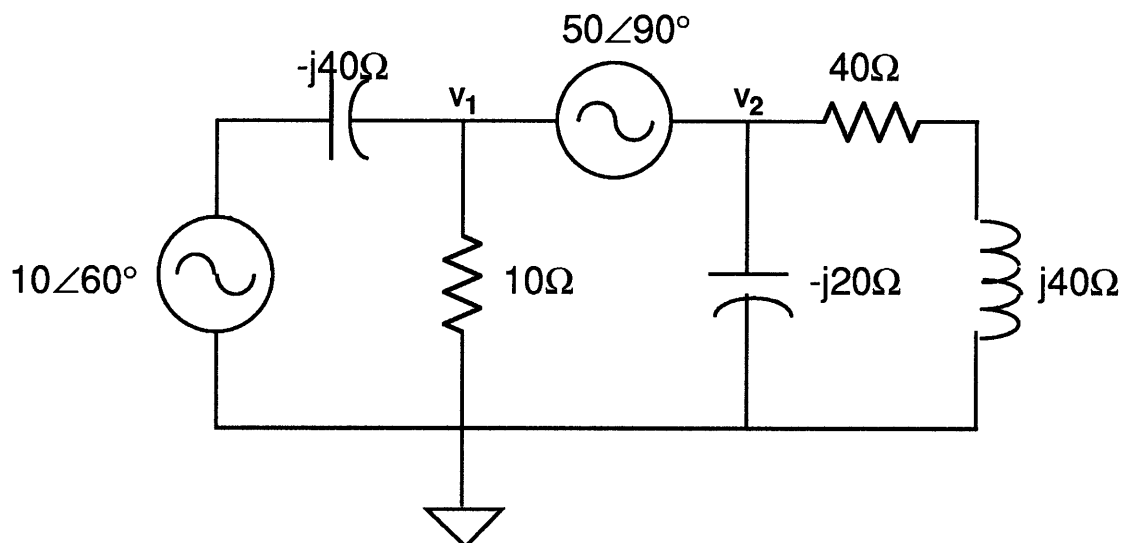
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Quiz #6 SOLUTIONS
EEAP 244

March 1, 1993

1. Solve for node voltages V_1 and V_2 in the circuit shown below. Define any circuit variables you use on the circuit drawing.

WARNING: The math in this problem can easily get out of hand. Set up the equations, reduce the problem to two equations in two unknowns, and leave the answer in that form.



HINT: The $50\angle 90^\circ$ voltage source defines a constraint between V_1 and V_2 . Define a new circuit variable as the current through the $50\angle 90^\circ$ voltage source and solve the resulting set of equations.

Answer:

Using KCL at V_1 :

$$-\frac{10\angle 60^\circ - V_1}{-j40} + \frac{V_1}{10} + I_s = 0$$

Using KCL at V_2 :

$$-I_s + \frac{V_2}{-j20} + \frac{V_2}{40 + j40} = 0$$

Now, adding the constraint,

$$V_1 = 50\angle 90^\circ + V_2$$

This defines three equations in three unknowns which can be solved.

The resulting KCL equations are then:

$$\left(\frac{1}{-j40} + \frac{1}{10}\right)V_1 + I_s = \frac{10\angle 60^\circ}{j40} = 0.25\angle -30^\circ$$

$$\left(\frac{1}{-j20} + \frac{1}{40 + j40}\right)V_2 - I_s = 0$$

Name: _____

SSN: _____

You can eliminate I_s by adding the two equations to get:

$$(0.1+j0.025)V_1 - (0.0125+j0.0375)V_2 = 0.25\angle-30^\circ$$

which, when combined with the constraint equation

$$V_1 - V_2 = 50\angle90^\circ$$

gives two equations in two unknowns. Your answer should have been

$$(0.1+j0.025)V_1 - (0.0125+j0.0375)V_2 = 0.25\angle-30^\circ$$

$$V_1 - V_2 = 50\angle90^\circ$$

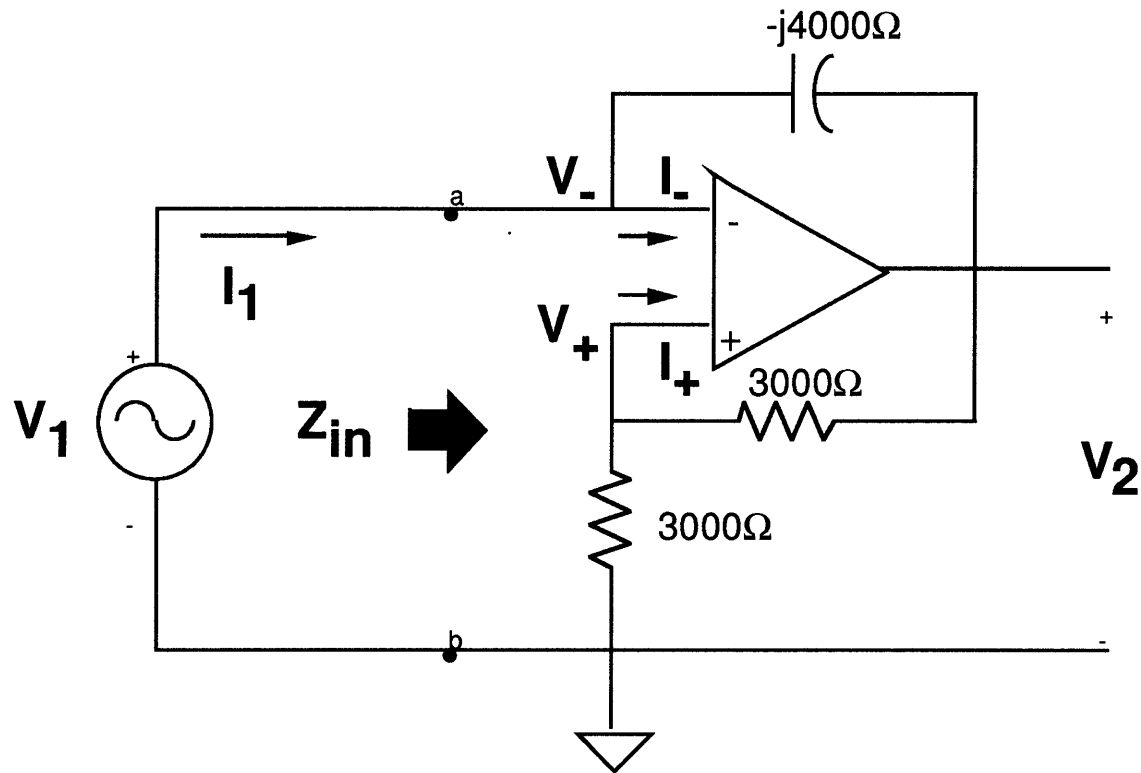
This system can be solved (I did it on my HP-48) to give

$$V_1=25.1\angle-11.6^\circ \text{ volts and } V_2=60.3\angle-65.9^\circ \text{ volts}$$

Name: _____

SSN: _____

2. Your overall task in this problem is to determine the input impedance Z_{in} seen at terminals a-b. You are given that $V_1 = 0.1 \angle 70^\circ$.



(a) The op-amp is ideal. What is the voltage V_- at the (-) input of the op-amp? What is the current I_- entering the (-) input of the op-amp? Why?

Answer: Because the op-amp is ideal I_- is zero. Furthermore, since the voltage source is directly connected to the inverting (-) input, $V_- = 0.1 \angle 70^\circ$ volts.

(b) What is the voltage V_+ at the (+) input of the op-amp? What is the current I_+ entering the (+) input of the op-amp?

Answer: Because the op-amp is ideal you use the virtual short assumption to realize that $V_+ = V_- = 0.1 \angle 70^\circ$. Furthermore, since both inputs are ideal $I_+ = 0$.

(c) What is the output voltage V_2 ?

Name: _____

SSN: _____

Answer: This now requires that you use KCL at the non-inverting input of the op-amp just as you did in the Schmitt trigger. The current through the 3000Ω resistor to ground is then

$$I = \frac{V_+}{3000\Omega} = \frac{0.1\angle 70^\circ}{3000\Omega} = 33.6\angle 70^\circ \mu\text{amps}$$

The output voltage V_2 is then

$$V_2 = (33.6 \times 10^{-6} \angle 70^\circ) 3000\Omega + 0.1\angle 70^\circ = 0.2\angle 70^\circ \text{ volts}$$

Knowing V_+ and V_2 you can calculate the current through the feedback resistor as

$$I_1 = \frac{V_2 - V_-}{-j4000} = \frac{0.2\angle 70^\circ - 0.1\angle 70^\circ}{-j4000} = 25 \times 10^{-6} \angle 160^\circ \text{ amps}$$

(d) What is the input impedance defined as $Z_{in} = V_1/I_1$?

Answer:

The input impedance is simply

$$Z_{in} = \frac{V_1}{I_1} = \frac{0.1\angle 70^\circ}{25 \times 10^{-6} \angle 160^\circ} = 2000 \angle -90^\circ \text{ ohms}$$

Name: _____

SSN: _____

Quiz #7 SOLUTIONS
EEAP 244

March 25, 1993

1. (40 points) Determine $f(t)$ for the following expressions:

$$(a) \quad F(s) = \frac{12}{s^2 + 8s + 12}$$

Answer:

$$F(s) = \frac{12}{s^2 + 8s + 12} = \frac{12}{(s+2)(s+6)} = \frac{a}{s+2} + \frac{b}{s+6}$$

Multiplying out and equating coefficients of s :

$$a(s+6) + b(s+2) = s(a+b) + (6a+2b) = 12$$

This gives two equations in two unknowns:

$$a + b = 0$$

$$3a + b = 6$$

which can be solved to give $a=3$ and $b=-3$.

$$F(s) = \frac{3}{s+2} - \frac{3}{s+6}$$

Inverse transforming,

$$f(t) = 3e^{-2t}u(t) - 3e^{-6t}u(t)$$

$$(b) \quad F(s) = \frac{300}{s(s+10)^2}$$

Answer: This was assigned problem 15.24(a).

$$F(s) = \frac{300}{s(s+10)^2} = \frac{a}{s} + \frac{b}{(s+10)^2} + \frac{c}{s+10}$$

Multiplying out and equating coefficients of s :

$$a(s+10)^2 + bs + c(s+10) = s^2(a+c) + s(20a+b+10c) + (100a) = 300$$

By inspection, $a=3$. It then follows that $c=-3$ and $b=-30$

The partial fraction expansion is then:

$$F(s) = \frac{3}{s} - \frac{30}{(s+10)^2} - \frac{3}{s+10}$$

The inverse transform of this expression is then:

$$f(t) = 3u(t) - 30te^{-10t}u(t) - 3e^{-10t}u(t)$$

You lost 10 points if you did not have the $\frac{c}{s+10}$ term present in your expression.

Name: _____

SSN: _____

2. (30 points) A student has arrived at the following integro-differential equation in analyzing a RLC circuit. Determine $V(s)$ for this equation. Assume $v(0^-)=0$.

$$\frac{dv(t)}{dt} + 6v(t) + 9 \int_0^t v(z)dz = 24(t-2)u(t-2)$$

Answer:

Transforming term by term

$$sV(s) - v(0^-) + 6V(s) + 9\frac{V(s)}{s} = 24e^{-2s}\frac{1}{s}$$

Since the initial condition was zero this reduces to

$$sV(s) + 6V(s) + 9\frac{V(s)}{s} = 24e^{-2s}\frac{1}{s}$$

Solving for $V(s)$

$$V(s) \left[s + 6 + \frac{9}{s} \right] = 24\frac{e^{-2s}}{s}$$

$$V(s) \left[\frac{s^2 + 6s + 9}{s} \right] = 24\frac{e^{-2s}}{s}$$

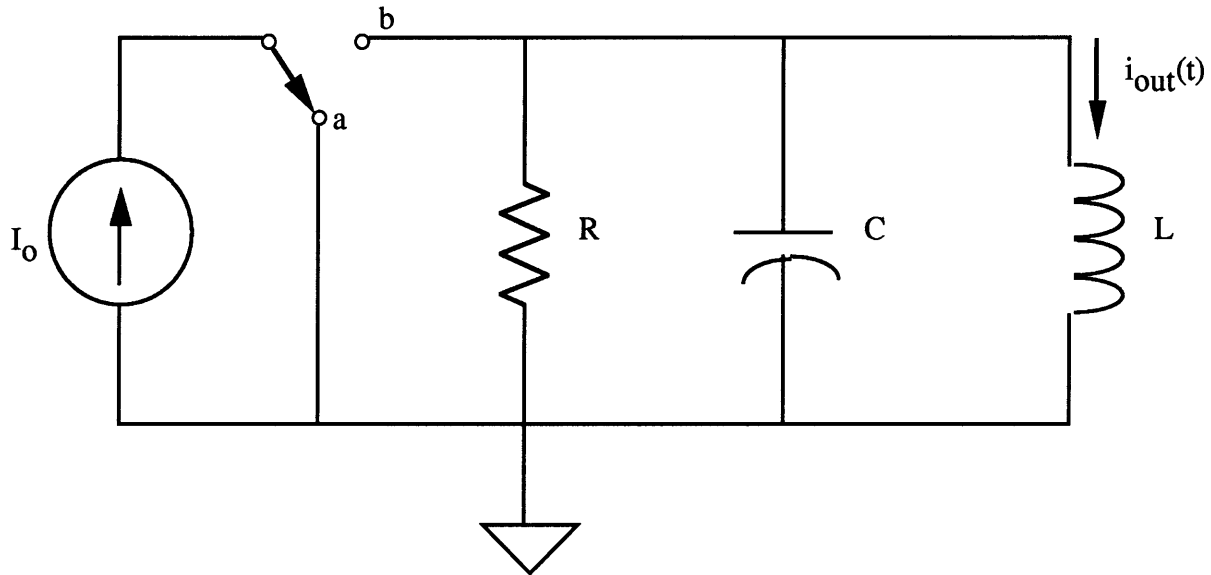
$$V(s) = 24\frac{e^{-2s}}{s^2 + 6s + 9}$$

You lost 15 points for not properly Laplace transforming the right side of the equation. The same number of points was allotted for the left hand side and everything else.

Name: _____

SSN: _____

3. (30 points) Determine an expression for $I_{out}(s)$ if the switch moves from a to b at $t=0$. I_o is a constant. Assume all initial conditions are zero.



Answer: This was assigned problem 15.21

You can solve for the output current by any of a number of techniques. I will use KCL at the node connected to B. The source looks like a current source with a unit step. The short at a is necessary since an ideal current source ALWAYS puts out the same current irregardless of the load.

$$\frac{I_o}{s} = \frac{V_b}{R} + \frac{V_b}{\frac{1}{sC}} + \frac{V_b}{sL} = V_b \left(\frac{1}{R} + sC + \frac{1}{sL} \right)$$

$$V_b = \frac{\frac{I_o}{s}}{\left(\frac{1}{R} + sC + \frac{1}{sL} \right)}$$

$$I_{out}(s) = \frac{V_b}{sL} = \frac{\frac{1}{sL} \frac{I_o}{s}}{\frac{1}{R} + sC + \frac{1}{sL}} = I_o \frac{\frac{1}{LC}}{s \left(s^2 + \frac{s}{RC} + \frac{1}{LC} \right)}$$

-5 points for not properly representing the source with a unit step

-8 points for not relating the voltage at node B and i_{out} , i.e. Ohm's Law

An Abbreviated List of Laplace Transform Pairs

f(t) (t>0-)	TYPE	F(s)
d(t)	(impulse)	1
u(t)	(step)	$\frac{1}{s}$
t	(ramp)	$\frac{1}{s^2}$
e ^{-at}	(exponential)	$\frac{1}{s+a}$
sin(ωt)	(sine)	$\frac{\omega}{s^2 + \omega^2}$
cos(ωt)	(cosine)	$\frac{s}{s^2 + \omega^2}$
te ^{-at}	(damped ramp)	$\frac{1}{(s+a)^2}$
e ^{-at} sin(ωt)	(damped sine)	$\frac{\omega}{(s+a)^2 + \omega^2}$
e ^{-at} cos(ωt)	(damped cosine)	$\frac{s+a}{(s+a)^2 + \omega^2}$

An Abbreviated List of Operational Transforms

f(t)	F(s)
Kf(t)	KF(s)
f ₁ (t) + f ₂ (t) - f ₃ (t) + ...	F ₁ (s) + F ₂ (s) - F ₃ (s) + ...
$\frac{df(t)}{dt}$	sF(s) - f(0 ⁻)
$\frac{d^2f(t)}{dt^2}$	s ² F(s) - sf(0 ⁻) - $\frac{df(0^-)}{dt}$
$\frac{d^n f(t)}{dt^n}$	s ⁿ F(s) - s ⁿ⁻¹ f(0 ⁻) - s ⁿ⁻² $\frac{df(0^-)}{dt}$ - s ⁿ⁻³ $\frac{d^2f(0^-)}{dt^2}$ - ... - $\frac{d^{n-1}f(0^-)}{dt^{n-1}}$
$\int_0^t f(x)dx$	$\frac{F(s)}{s}$
f(t-a)u(t-a), a>0	e ^{-as} F(s)
e ^{-at} f(t)	F(s+a)
f(at), a>0	$\frac{1}{a}f\left(\frac{s}{a}\right)$
tf(t)	$-\frac{dF(s)}{ds}$
t ⁿ f(t)	$(-1)^n \frac{d^n F(s)}{ds^n}$
$\frac{f(t)}{t}$	$\int_s^\infty F(u)du$

Name: _____

SSN: _____

Quiz #7 SOLUTIONS
EEAP 244

March 25, 1993

1. Determine $f(t)$ for the following expressions:

(a) $F(s) = \frac{12}{s^2 + 8s + 12}$

Answer:

$$F(s) = \frac{12}{s^2 + 8s + 12} = \frac{12}{(s+2)(s+6)} = \frac{a}{s+2} + \frac{b}{s+6}$$

Multiplying out and equating coefficients of s :

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This gives two equations in two unknowns:

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Name: _____

SSN: _____

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$$\frac{dv(t)}{dt} + 6v(t) + 9 \int_0^t v(z)dz = 24(t-2)u(t-2)$$

Answer:

Transforming term by term

$$sV(s) - v(0^-) + 6V(s) + 9\frac{V(s)}{s} = 24e^{-2s}\frac{1}{s^2}$$

Since the initial condition was zero this reduces to

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Solving for $V(s)$

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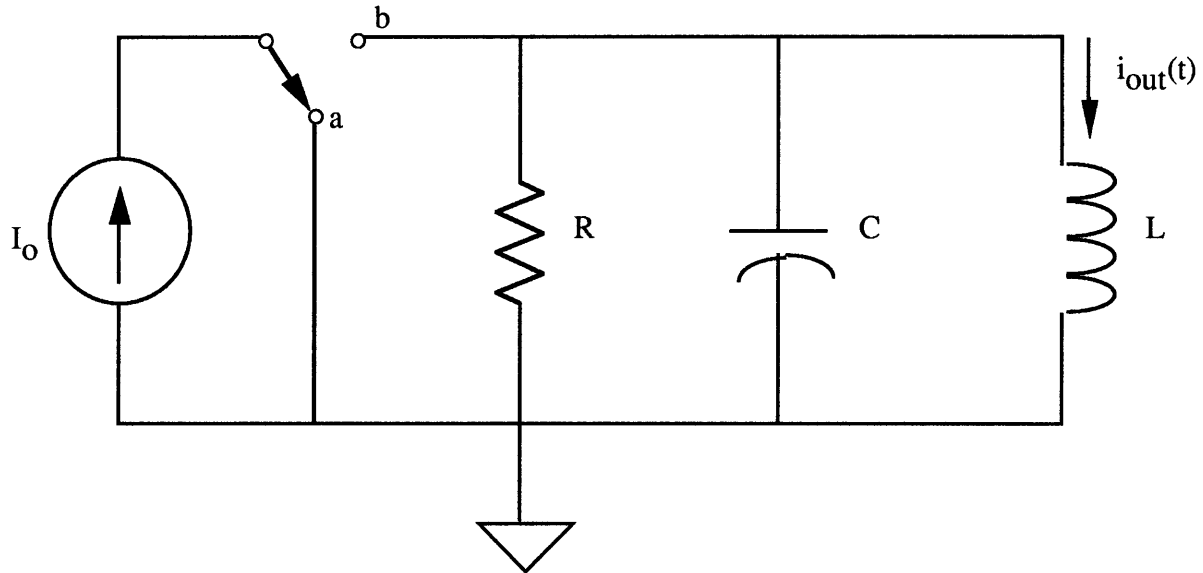
$$V(s) = 24\frac{e^{-2s}}{s(s^2 + 6s + 9)}$$

You lost 15 points for not properly Laplace transforming the right side of the equation. The same number of points was allotted for the left hand side and everything else.

Name: _____

SSN: _____

3. (30 points) Determine an expression for $I_{out}(s)$ if the switch moves from a to b at $t=0$. I_o is a constant. Assume all initial conditions are zero.



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You can solve for the output current by any of a number of techniques. I will use KCL at the node connected to B. The source looks like a current source with a unit step. The short at a is necessary since an ideal current source ALWAYS puts out the same current irregardless of the load.

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$$V_b = \frac{\frac{I_o}{s}}{\left(\frac{1}{R} + sC + \frac{1}{sL} \right)}$$

$$I_{out}(s) = \frac{V_b}{sL} = \frac{\frac{1}{sL} \frac{I_o}{s}}{\frac{1}{R} + sC + \frac{1}{sL}} = I_o \frac{\frac{1}{LC}}{s \left(s^2 + \frac{s}{RC} + \frac{1}{LC} \right)}$$

-5 points for not properly representing the source with a unit step

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te ^{-at}	(damped ramp)	$\frac{1}{(s+a)^2}$
e ^{-at} sin(ωt)	(damped sine)	$\frac{\omega}{(s+a)^2 + \omega^2}$
e ^{-at} cos(ωt)	(damped cosine)	$\frac{s+a}{(s+a)^2 + \omega^2}$

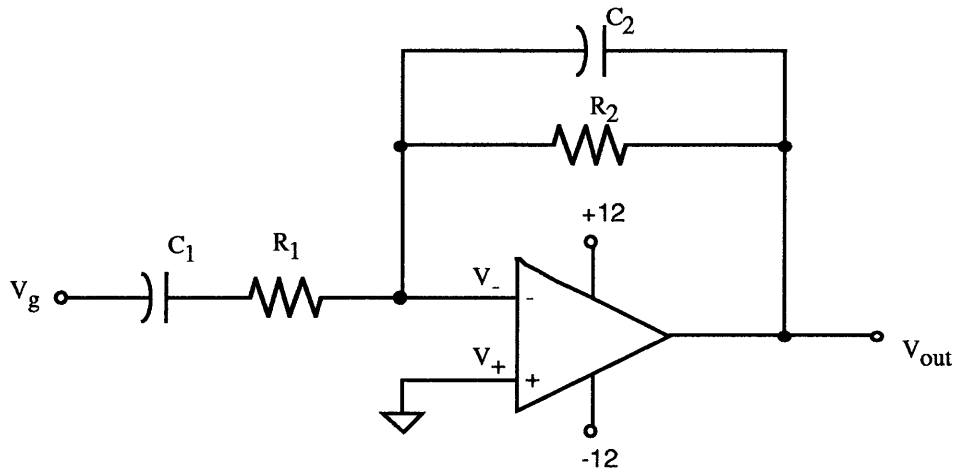
An Abbreviated List of Operational Transforms

f(t)	F(s)
Kf(t)	KF(s)
f ₁ (t) + f ₂ (t) - f ₃ (t) + ...	F ₁ (s) + F ₂ (s) - F ₃ (s) + ...
$\frac{df(t)}{dt}$	sF(s) - f(0 ⁻)
$\frac{d^2f(t)}{dt^2}$	s ² F(s) - sf(0 ⁻) - $\frac{df(0^-)}{dt}$
$\frac{d^nf(t)}{dt^n}$	s ⁿ F(s) - s ⁿ⁻¹ f(0 ⁻) - s ⁿ⁻² $\frac{df(0^-)}{dt}$ - s ⁿ⁻³ $\frac{d^2f(0^-)}{dt^2}$ - ... - $\frac{d^{n-1}f(0^-)}{dt^{n-1}}$
$\int_0^t f(x)dx$	$\frac{F(s)}{s}$
f(t-a)u(t-a), a>0	e ^{-as} F(s)
e ^{-at} f(t)	F(s+a)
f(at), a>0	$\frac{1}{a}f\left(\frac{s}{a}\right)$
tf(t)	$-\frac{dF(s)}{ds}$
t ⁿ f(t)	$(-1)^n \frac{d^n F(s)}{ds^n}$
$\frac{f(t)}{t}$	$\int_s^\infty F(u)du$

Quiz #8
EEAP 244

March 29, 1993

1. (60 points) The operational amplifier shown in the circuit below is ideal. There is no energy stored in the circuit at the time it is energized, i.e. the initial conditions for all capacitors is zero.



- (a) What is the transfer function of the amplifier in the s-domain. You can leave your answer in terms of R_1 , R_2 , C_1 and C_2 .

$$v_o = -\frac{Z_f}{Z_i} v_g$$

$$Z_f = R_2 \parallel \frac{1}{sC_2} = \frac{R_2 \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{1 + sR_2C_2}$$

$$v_o = -\frac{Z_f}{Z_i} v_g = -\frac{\frac{R_2}{1 + sR_2C_2}}{\frac{1 + sR_1C_1}{sC_1}} v_g = -\frac{sR_2C_1}{(1 + sR_2C_2)(1 + sR_1C_1)} v_g$$

- (b) the Laplace transform of the input $v_g(t) = tu(t)$ volts,

Name: _____

SSN: _____

- (c) Suppose the transfer function of the amplifier is $\frac{V_o(s)}{V_g(s)} = -\frac{200}{s}$, what is $v_o(t)$ for the input $v_g(t) = tu(t)$ volts? HINT: You may need to use the list of operational transforms provided.

- (d) Suppose $v_o(t) = [-15 + 30e^{-1000t} - 15e^{-2000t}]u(t)$ volts. Will the operational amplifier saturate and, if so, when will it saturate?

$$-15 + 30e^{-1000t} - 15e^{-2000t} = -12$$

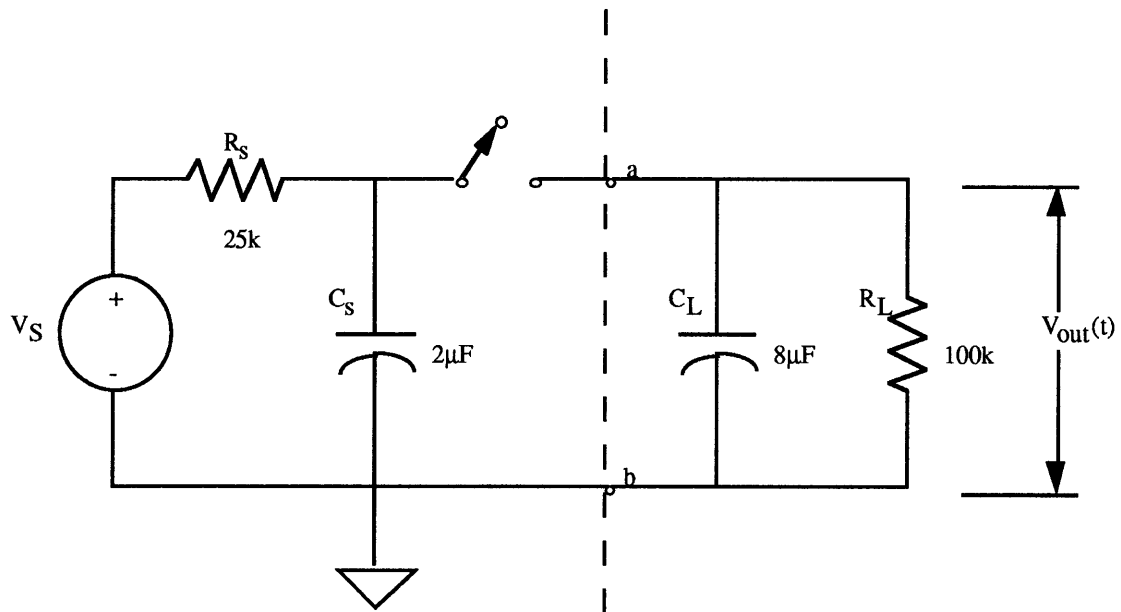
$$x = e^{-1000t} \quad 15x^2 - 30x + 3 = 0$$

$$t = -\frac{\ln(1 - \sqrt{4/5})}{1000} \approx 2.25 \text{ mS}$$

Name: _____

SSN: _____

2. (40 points) Determine the s-domain Thevenin equivalent of the circuit to the left of the terminals a,b.



$$V_{\text{thevenin}} = \frac{50}{s} \frac{\frac{5 \times 10^5}{s}}{25 \cdot 10^3 + \frac{5 \times 10^5}{s}} = \frac{50}{s} \frac{500}{25s + 500} = \frac{50}{s} \frac{20}{s + 20} = \frac{50}{s} \frac{20}{s + 20} = \frac{1000}{s(s + 20)}$$

$$Z_{\text{thevenin}} = \frac{(25 \times 10^3) \frac{5 \times 10^5}{s}}{25 \cdot 10^3 + \frac{5 \times 10^5}{s}} = (25 \times 10^3) \frac{20}{s + 20} = \frac{5 \times 10^5}{s + 20} \text{ ohms}$$

An Abbreviated List of Laplace Transform Pairs

f(t) (t>0⁻)	TYPE	F(s)
d(t)	(impulse)	1
u(t)	(step)	$\frac{1}{s}$
t	(ramp)	$\frac{1}{s^2}$
e ^{-at}	(exponential)	$\frac{1}{s+a}$
sin(ωt)	(sine)	$\frac{\omega}{s^2 + \omega^2}$
cos(ωt)	(cosine)	$\frac{s}{s^2 + \omega^2}$
te ^{-at}	(damped ramp)	$\frac{1}{(s+a)^2}$
e ^{-at} sin(ωt)	(damped sine)	$\frac{\omega}{(s+a)^2 + \omega^2}$
e ^{-at} cos(ωt)	(damped cosine)	$\frac{s+a}{(s+a)^2 + \omega^2}$

An Abbreviated List of Operational Transforms

f(t)	F(s)
Kf(t)	KF(s)
f ₁ (t) + f ₂ (t) - f ₃ (t) + ...	F ₁ (s) + F ₂ (s) - F ₃ (s) + ...
$\frac{df(t)}{dt}$	sF(s) - f(0 ⁻)
$\frac{d^2f(t)}{dt^2}$	s ² F(s) - sf(0 ⁻) - $\frac{df(0^-)}{dt}$
$\frac{d^nf(t)}{dt^n}$	s ⁿ F(s) - s ⁿ⁻¹ f(0 ⁻) - s ⁿ⁻² $\frac{df(0^-)}{dt}$ - s ⁿ⁻³ $\frac{d^2f(0^-)}{dt^2}$ - ... - $\frac{d^{n-1}f(0^-)}{dt^{n-1}}$
$\int_0^t f(x)dx$	$\frac{F(s)}{s}$
f(t-a)u(t-a), a>0	e ^{-as} F(s)
e ^{-at} f(t)	F(s+a)
f(at), a>0	$\frac{1}{a}f\left(\frac{s}{a}\right)$
tf(t)	$-\frac{dF(s)}{ds}$
t ⁿ f(t)	$(-1)^n \frac{d^n F(s)}{ds^n}$
$\frac{f(t)}{t}$	$\int_s^\infty F(u)du$

Name: _____

SSN: _____

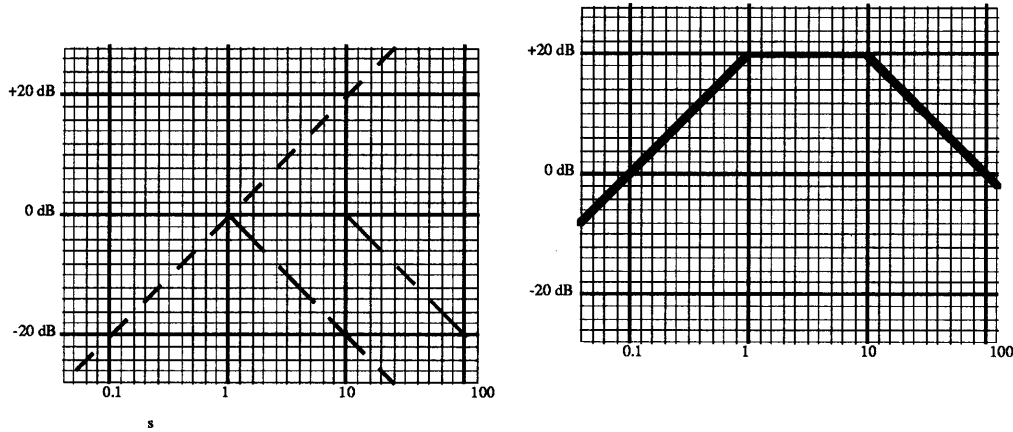
Quiz #9 SOLUTIONS
EEAP 244

April 5, 1993

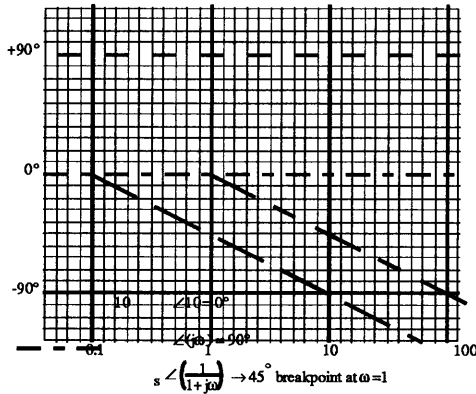
1. 60 points. Plot $|H(j\omega)|$ and $\angle H(j\omega)$, given that

$$H(s) = \frac{10s}{(1+s)(1+\frac{s}{10})}$$

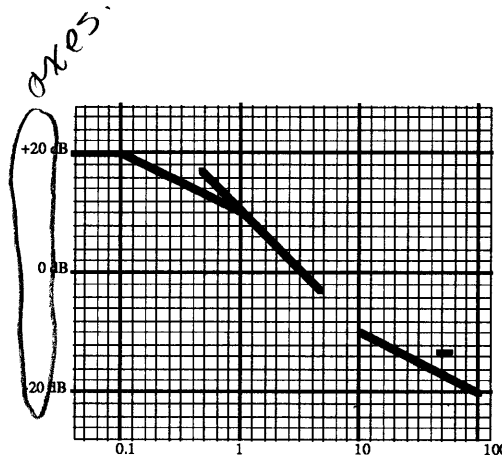
SOLUTIONS:



$\text{---} \frac{1}{1+s}$
 $\text{---} \frac{1}{1+\frac{s}{10}}$
 $\text{---} \text{---}$



$\text{---} \frac{1}{1+s} \angle \left(\frac{1}{1+\frac{\omega}{10}}\right) \rightarrow 45^\circ$ breakpoint at $\omega=10$
 $\text{---} \frac{1}{1+\frac{s}{10}}$



COMMENTS:

- You lost 5 points for not labeling your graph, i.e. horizontal or vertical units.
- You lost 5 points for not properly computing the overall phase response.
- You lost 5 points for missing terms in each expression.

Name: _____

SSN: _____

You lost 5 points for forgetting the $\angle 10\omega$ term.

You lost 10 points for using square phase response, i.e. constant phase. READ YOUR TEXTBOOK about this. The response is $\pm 45^\circ$ each side of the breakpoint.

You lost 20 points if you had some equations appropriate to the response but you did not plot them. The question explicitly told you to plot the responses.

For interested people, here is the PSpice simulation of the circuit response:

Quiz 9 problem 1

```
Vin 1 0 ac 1
```

```
Rin 1 0 1
```

```
E1 2 0 laplace {v(1)}={10*s/((1+s)*(1+s/10))}
```

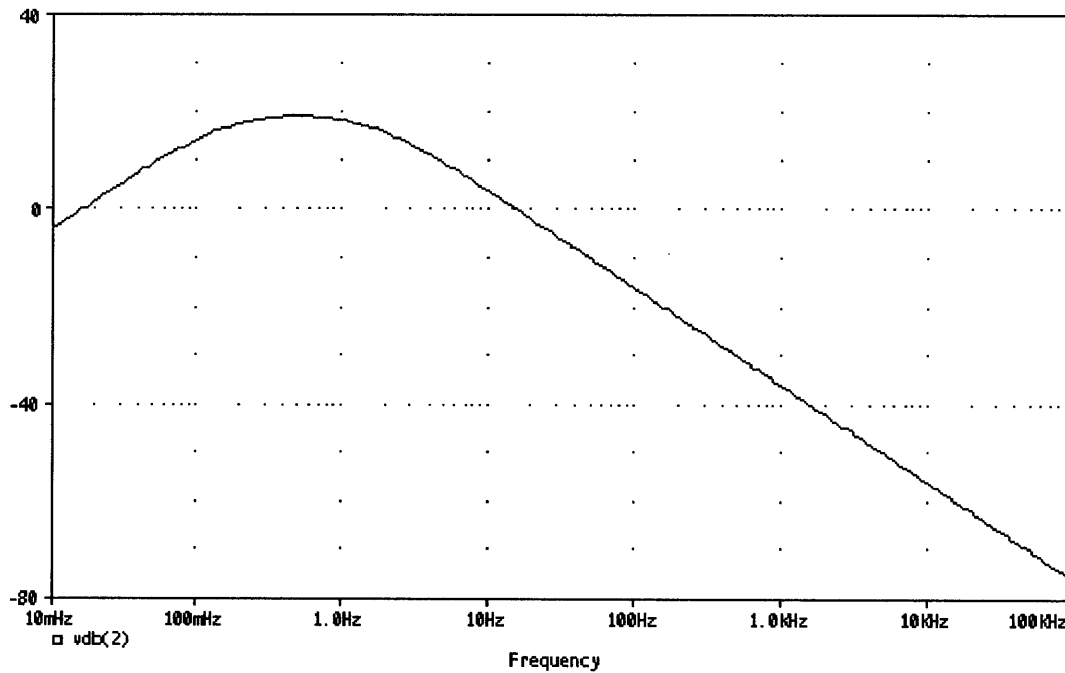
```
R1 2 0 1
```

```
.ac dec 40 .01 100K
```

```
.probe
```

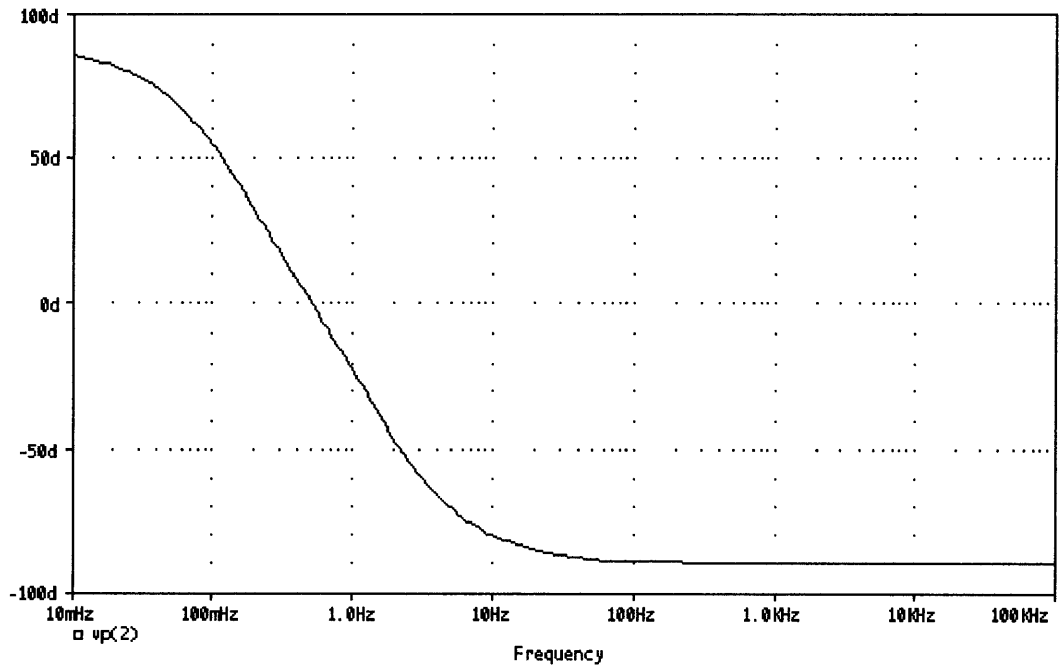
```
.end
```

Plot of solutions



Name: _____

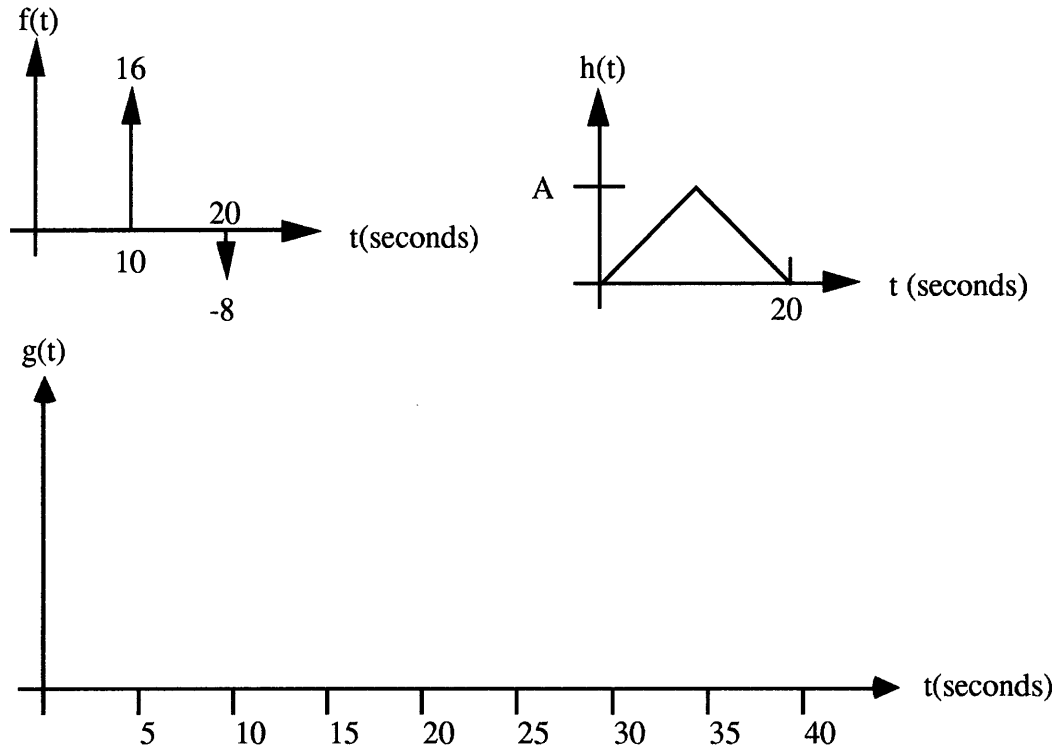
SSN: _____



Name: _____

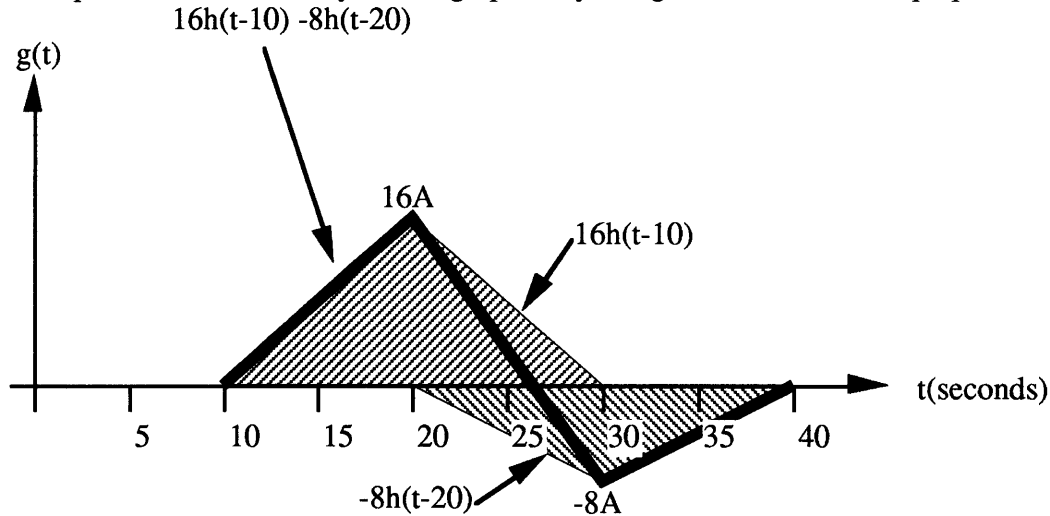
SSN: _____

2. The signal $f(t)$ shown below is the input to a linear system whose impulse response is given by $h(t)$. Sketch the output $g(t)$ of the filter for $0 \leq t \leq 40$ seconds. HINT: You can do your answer mathematically or graphically, but the answer must be sketched below.



Solution:

This problem is most easily solved graphically using convolution and superposition.



COMMENTS: You gained 10 points for representing the output as a convolution integral; you got 5 points for using the impulses correctly; you lost 10 points for not using the time-shifting property of impulses; you lost 20 points if there was no mention of or any comprehension of convolution either graphically or mathematically. The most common error (-10 points) was not using a delta function and/or not time shifting the the response functions.

An Abbreviated List of Laplace Transform Pairs

f(t) (t>0⁻)	TYPE	F(s)
d(t)	(impulse)	1
u(t)	(step)	$\frac{1}{s}$
t	(ramp)	$\frac{1}{s^2}$
e ^{-at}	(exponential)	$\frac{1}{s+a}$
sin(ωt)	(sine)	$\frac{\omega}{s^2 + \omega^2}$
cos(ωt)	(cosine)	$\frac{s}{s^2 + \omega^2}$
te ^{-at}	(damped ramp)	$\frac{1}{(s+a)^2}$
e ^{-at} sin(ωt)	(damped sine)	$\frac{\omega}{(s+a)^2 + \omega^2}$
e ^{-at} cos(ωt)	(damped cosine)	$\frac{s+a}{(s+a)^2 + \omega^2}$

An Abbreviated List of Operational Transforms

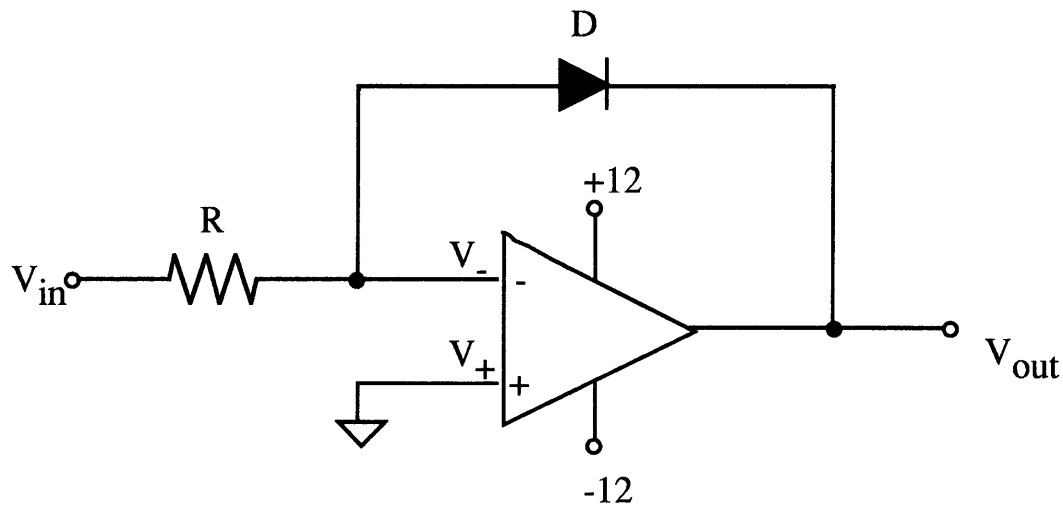
f(t)	F(s)
Kf(t)	KF(s)
f ₁ (t) + f ₂ (t) - f ₃ (t) + ...	F ₁ (s) + F ₂ (s) - F ₃ (s) + ...
$\frac{df(t)}{dt}$	sF(s) - f(0 ⁻)
$\frac{d^2f(t)}{dt^2}$	s ² F(s) - sf(0 ⁻) - $\frac{df(0^-)}{dt}$
$\frac{d^nf(t)}{dt^n}$	s ⁿ F(s) - s ⁿ⁻¹ f(0 ⁻) - s ⁿ⁻² $\frac{df(0^-)}{dt}$ - s ⁿ⁻³ $\frac{d^2f(0^-)}{dt^2}$ - ... - $\frac{d^{n-1}f(0^-)}{dt^{n-1}}$
$\int_0^t f(x)dx$	$\frac{F(s)}{s}$
f(t-a)u(t-a), a>0	e ^{-as} F(s)
e ^{-at} f(t)	F(s+a)
f(at), a>0	$\frac{1}{a}f\left(\frac{s}{a}\right)$
tf(t)	$-\frac{dF(s)}{ds}$
t ⁿ f(t)	$(-1)^n \frac{d^n F(s)}{ds^n}$
$\frac{f(t)}{t}$	$\int_s^\infty F(u)du$

INS TCP/IP PrintServer 20

Print engine name: PrintServer 20
Print engine version: 17
Printer firmware version: 32
Server Adobe PostScript version: 48.3
Server software version: V2.0
Server network node: crawford
Server name: crawford
Server job number: 86
Client software version: WRL-1.0
Client network node: util
Client name: flm
Client job name: quiz_10.ps.735168728
Submitted at: Sun Apr 18 17:57:00 19939933X
Printed at: Sun Apr 18 17:57:00 1993

flm@util
quiz_10.ps.735168728

1. (50 points) Problem 8.1-6 of your textbook. The operational amplifier shown in the circuit below is ideal. The diode is at room temperature. Determine V_{out} as a function of V_{in} . Use $R=1k\Omega$ and I_0 (for the diode) = $1\mu A$ in your answer.



ANSWER:

The basic equation for the resistor is Ohm's Law

$$i_1 = \frac{V_{in}}{R}$$

The diode is governed by the diode equation at room temperature, i.e.

$$i_2 = I_0(e^{-40V_{out}} - 1)$$

Applying KCL at the inverting input of the op-amp

$$i_1 - i_2 = 0$$

Or, after substitution,

$$\frac{V_{in}}{R} - I_0(e^{-40V_{out}} - 1) = 0$$

Rearranging

$$(e^{-40V_{out}} - 1) = \frac{V_{in}}{I_0 R}$$

and, assuming the 1 can be neglected on the left hand side, we get

$$e^{-40V_{out}} \approx \frac{V_{in}}{I_0 R}$$

After taking the natural log of both sides this becomes

$$-40V_{out} \approx -\frac{1}{40} \ln \left(\frac{V_{in}}{I_0 R} \right)$$

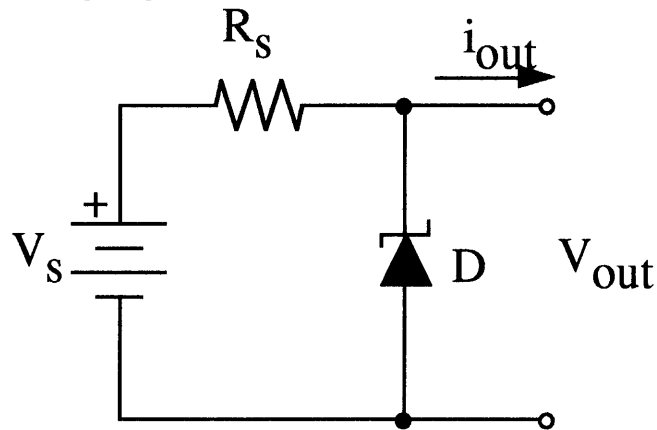
The final answer is

$$V_{\text{out}} \approx -\frac{1}{40} \ln \left(\frac{V_{\text{in}}}{(1\mu\text{A})(1\text{k}\Omega)} \right) = -\frac{1}{40} \ln (1000V_{\text{in}})$$

$$V_{\text{out}} \approx -0.025 \ln (1000V_{\text{in}}) = -0.025 \ln (1000) + 0.025 \ln (V_{\text{in}}) = -0.17269 - 0.025 \ln (V_{\text{in}})$$

COMMENTS: You lost 5 points for not recognizing that the diode was reverse biased.
You lost 5-10 points for reversing the input and output of the circuit.

2. (50 points) Problem 8.2-11 of your textbook. Answer the following questions about the zener diode voltage regulator circuit shown below.

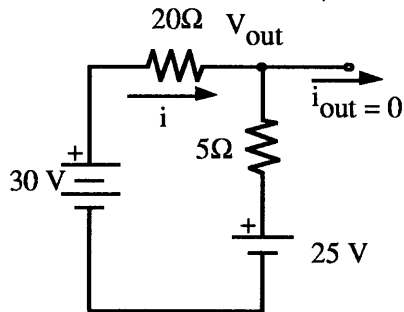


Use the circuit parameters: $V_S=30$ volts, $R_S=20$ ohms, $V_Z=25$ volts and $R_Z=5$ ohms for all parts of this problem.

- (a) Determine V_{out} for $i_{out}=0$ amperes.

ANSWER:

The circuit for the circuit, assuming conduction of the zener diode, is:



The current in the circuit is determined by Ohm's Law

$$i = \frac{30 - 25}{20 + 5} = \frac{5}{25} = 0.2 \text{ Amps}$$

The output voltage is then

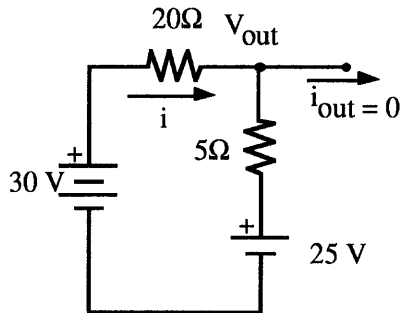
$$V_{out} = 30 - (0.2A)(20\Omega) = 30 - 4 = 26 \text{ volts}$$

The output voltage is greater than V_Z so the diode is conducting and our assumption is correct.

COMMENT: You lost 10 points for the wrong voltage.

- (b) Determine V_{out} for $i_{out}=0.5$ amperes.

ANSWER:



Since there is a non-zero i_{out} use KCL at the output terminal:

$$+ \frac{30 - V_{out}}{20 \Omega} - \frac{V_{out} - 25}{5 \Omega} - 0.5 = 0$$

Eliminating fractions,

$$30 - V_{out} - 4V_{out} + 100 - 10 = 0$$

which yields

$$120 - 5V_{out} = 0$$

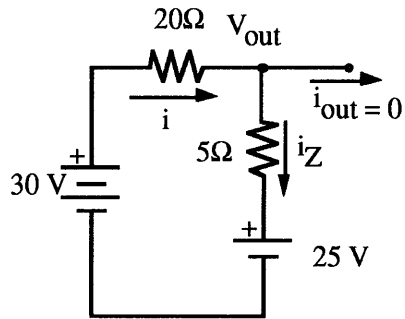
or $V_{out} = 24$ volts . Obviously the diode is not conducting and our assumption was wrong. Using Ohm's Law and ignoring the non-conducting diode

$$V_{out} = 30 - (20\Omega)(0.5 \text{ Amps}) = 30 - 10 = 20 \text{ volts}$$

COMMENT: You lost 15 points if you did not notice that the zener was OFF, said nothing about it being OFF, and did not compute the output voltage for it being OFF. You only lost 5-10 points if you noticed it was OFF and/or calculated the voltage for it being OFF.

- (c) At what value of i_{out} does the zener diode stop conducting?

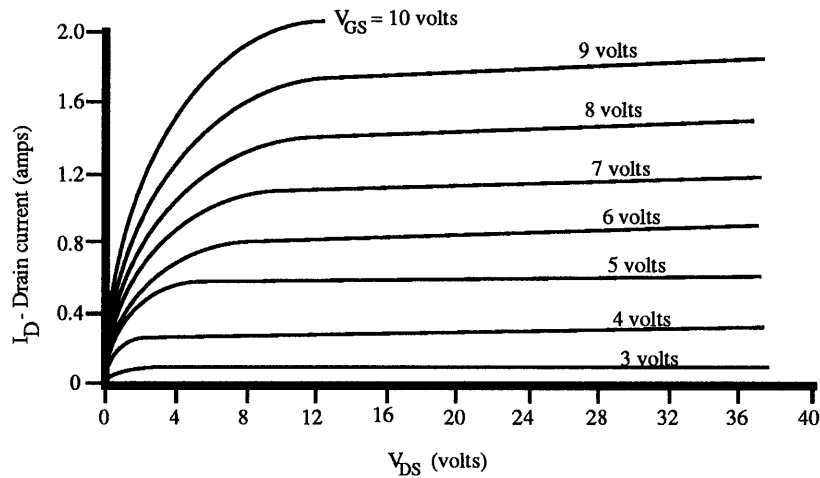
ANSWER:



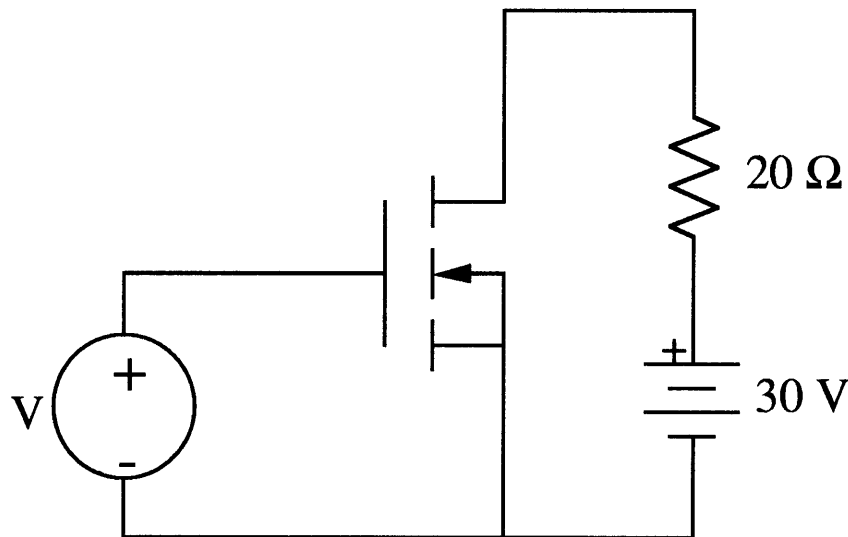
We know that the current through the zener diode must be zero when the diode just stops/starts conducting. Therefore, use $i_Z=0$ and, consequently, $V_{out}=25$ volts. For this situation

$$i_{out,max} = \frac{30 - 25 \text{ volts}}{20 \Omega} = \frac{5}{20} = 0.25 \text{ ampere}$$

1. (50 points) The following questions refer to the measured characteristics (shown below) of a 2N5447 MOSFET. The data was taken with a Tektronix 575 curve tracer.

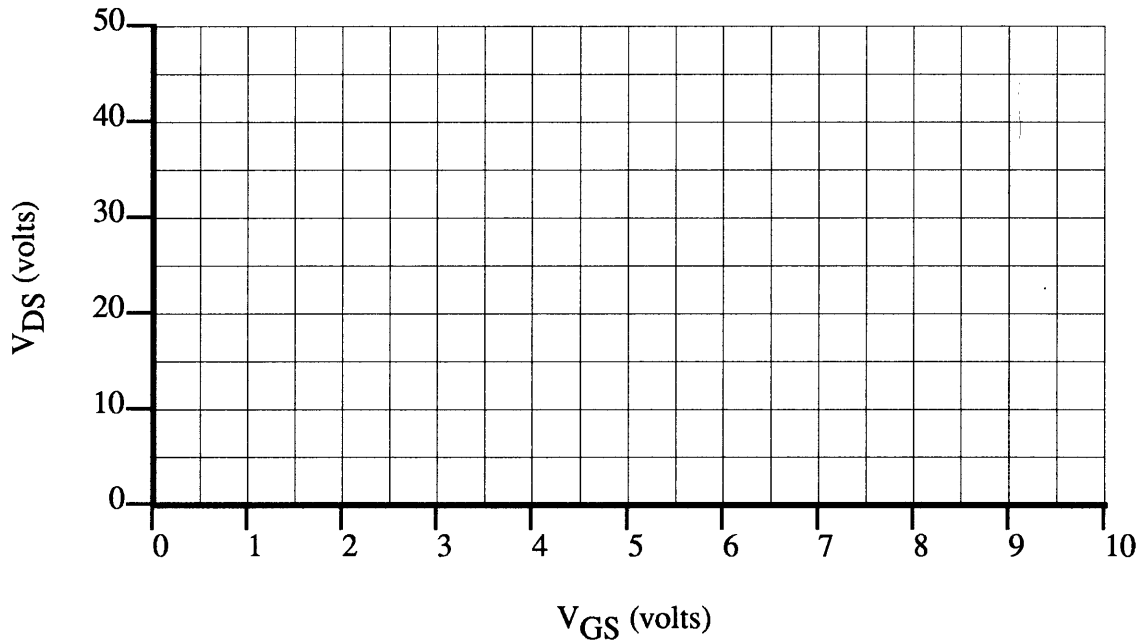
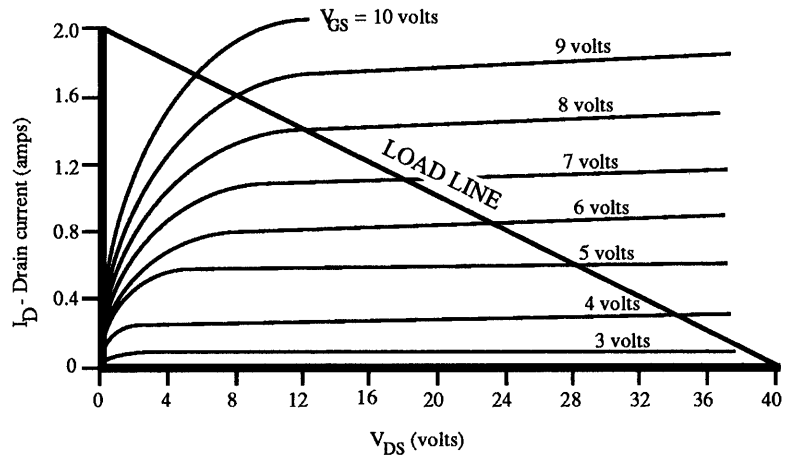


- (a) Estimate the threshold voltage (V_T) of the MOSFET from the data.
ANSWER: By inspection the threshold voltage must be smaller than 3 volts. Good answers would be 2-3 volts.
- (b) The MOSFET is connected in the circuit shown below. For $V_{GS}=6$ volts, determine the operating point of the MOSFET using a load line analysis.

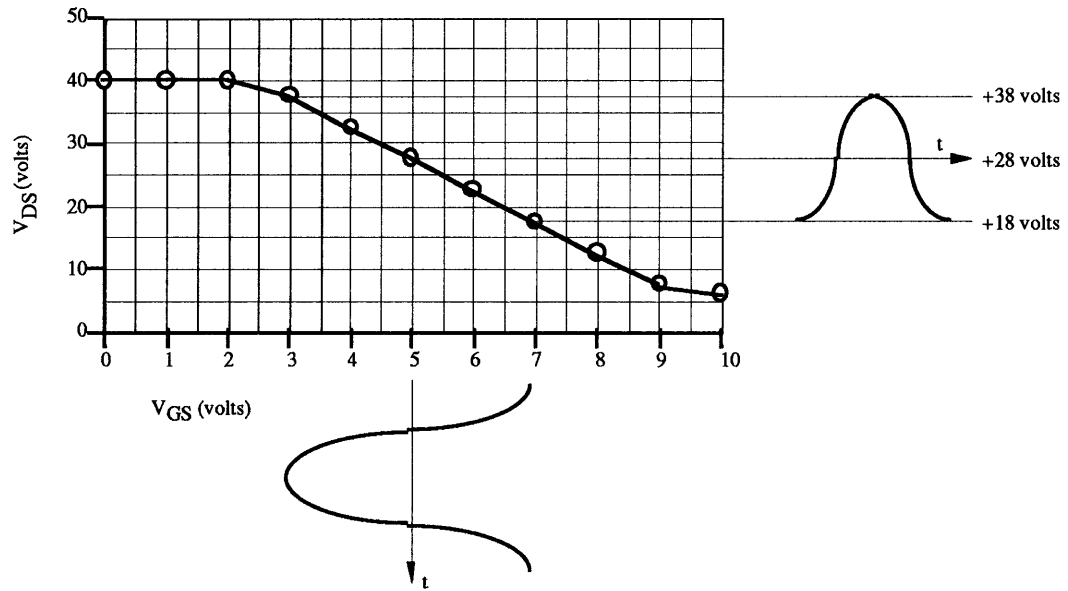


ANSWER: The x-intercept of the load line is 30 volts. The y-intercept of the load line is given by $i_D = \frac{30 \text{ volts}}{20 \Omega} = 1.5 \text{ amps}$. Drawing this line and estimating its intersection with the curve for $V_{GS}=6$ volts gives (12 volts, 0.8 amps) for the operating point.

- (c) Assuming the operation of the MOSFET circuit is described by the characteristic data/load line shown below, plot V_{DS} as a function of V_{GS} for V_{GS} ranging from 0 to 10 volts. If $V_{GS} = 6 \text{ volts} + 2\cos\omega t$, determine the change in V_{DS} as a function of V_{GS} . Do you think this MOSFET would make a good amplifier?

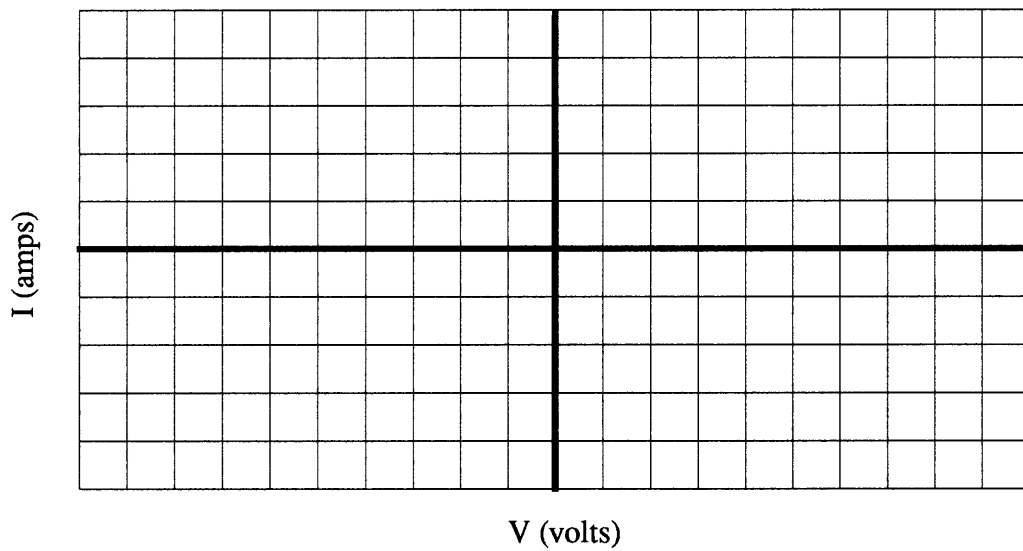
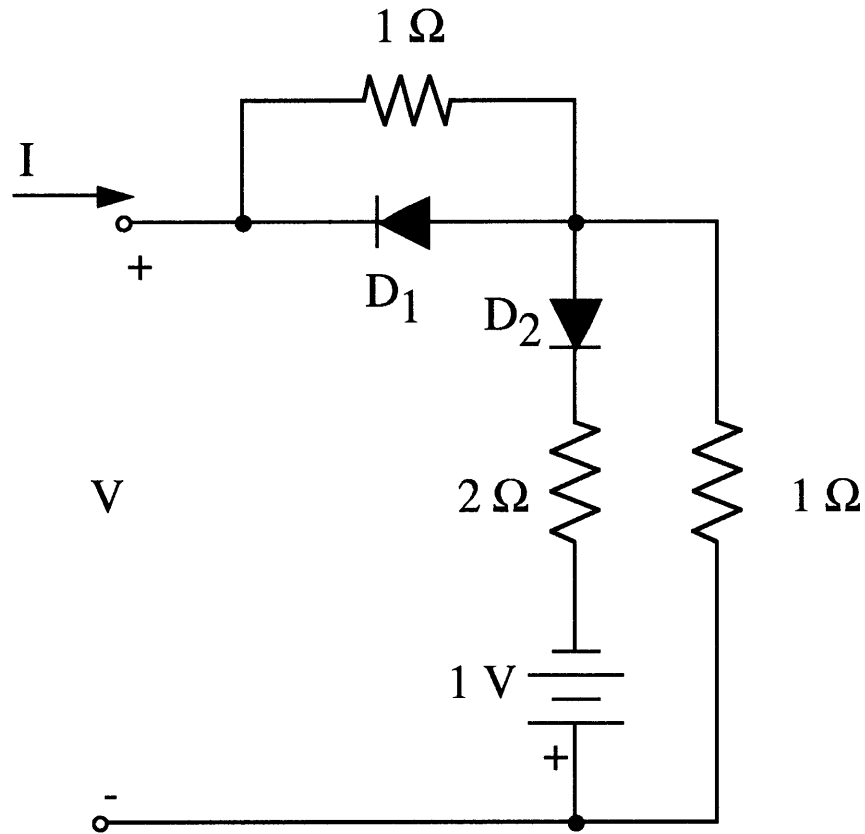


ANSWER:



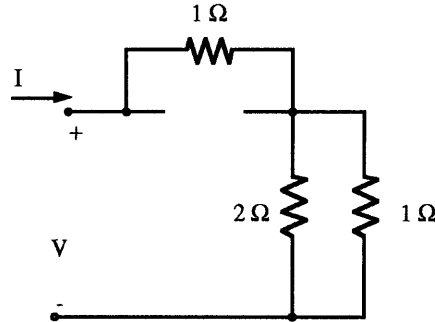
This circuit would make an excellent amplifier. It has a nice, long transfer curve and the input signal is approximately in the middle of the region resulting in undistorted output with a gain of approximately 5.

2. (50 points). For the circuit shown below, determine the two breakpoints and sketch I vs. V.



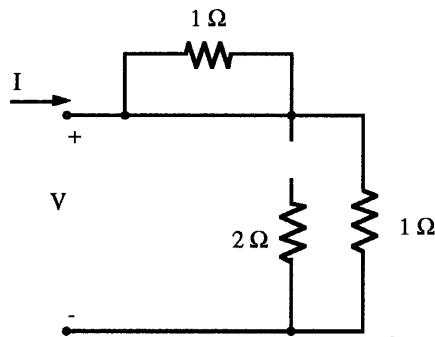
ANSWER:
There are four cases.

1. As $V \rightarrow +\infty$ D1 must be OFF and D2 is ON. The circuit then looks like



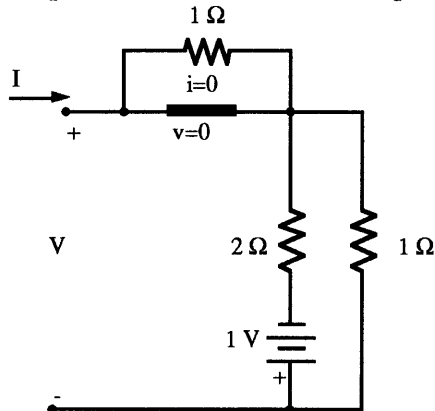
The battery is neglected because it is negligible compared to ∞ . The equivalent resistance of the circuit is then $1 + 2 \parallel 1 = 1 + 2/3 = 5/3\ \Omega$. The slope of the I-V curve is then $3/5$.

2. As $V \rightarrow -\infty$ D1 must be ON and D2 is OFF. The circuit then looks like



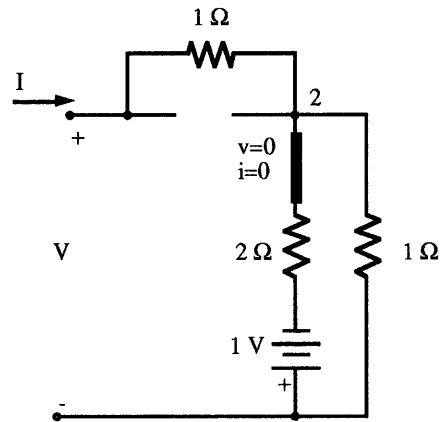
The equivalent resistance of the circuit is then simply the $1\ \Omega$ resistance and the slope of the I-V curve is 1.

3. Assume D1 is at a breakpoint and D2 is ON. The equivalent circuit is

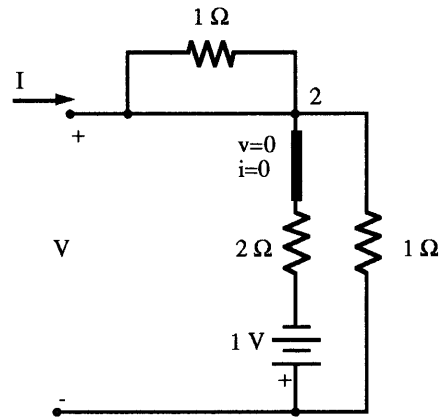


Since the voltage across D1 is zero, the voltage across the $1\ \Omega$ resistor is also zero and the current through it is also zero. This means immediately that $I=0$. The voltage at the junction of the diodes is determined by the $1\ \text{V}$ battery. The current flows from + to - meaning it flows through the $1\ \Omega$ $2\ \Omega$ battery loop in a counterclockwise direction. The magnitude of this current is $i = 1\ \text{V} / 3\ \Omega = 1/3\ \text{A}$. The voltage at the node is then the battery voltage PLUS the voltage drop across the $2\ \Omega$ resistor, i.e. $-1 + (2\ \Omega)(1/3\ \text{A}) = -1/3\ \text{V}$. Since there is no voltage drop across D1, the voltage at the input must be $-1/3\ \text{V}$. The first breakpoint is at $(-1/3\ \text{V}, 0\ \text{A})$.

4. Now assume D2 is at the breakpoint and let's assume D1 is OFF. The circuit is as shown below

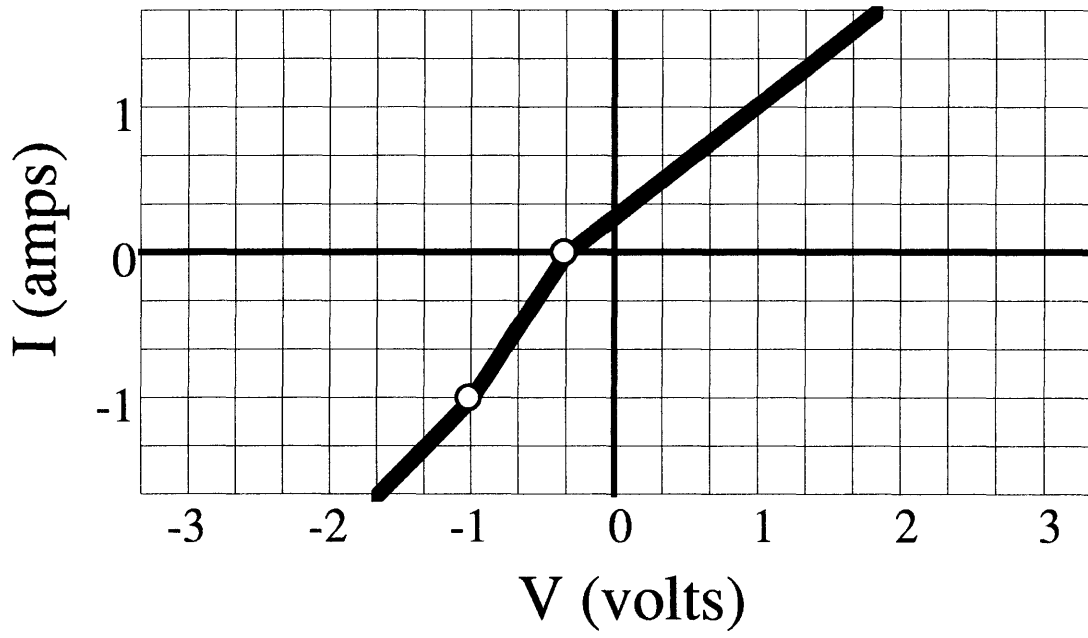


Since D2 has no current through it and no voltage across it, the voltage at node 2 must be -1 volts. This means that there is 1 amp of current flowing into node 2 from the 1Ω resistor to the right. Since there is no current through D2 this current must also be flowing through the 1Ω resistor across D1. This means that there is a positive voltage across D1 and D1 must be ON, not OFF as we assumed. Let's try again assuming that D1 is ON and D2 is at the breakpoint. This new circuit is

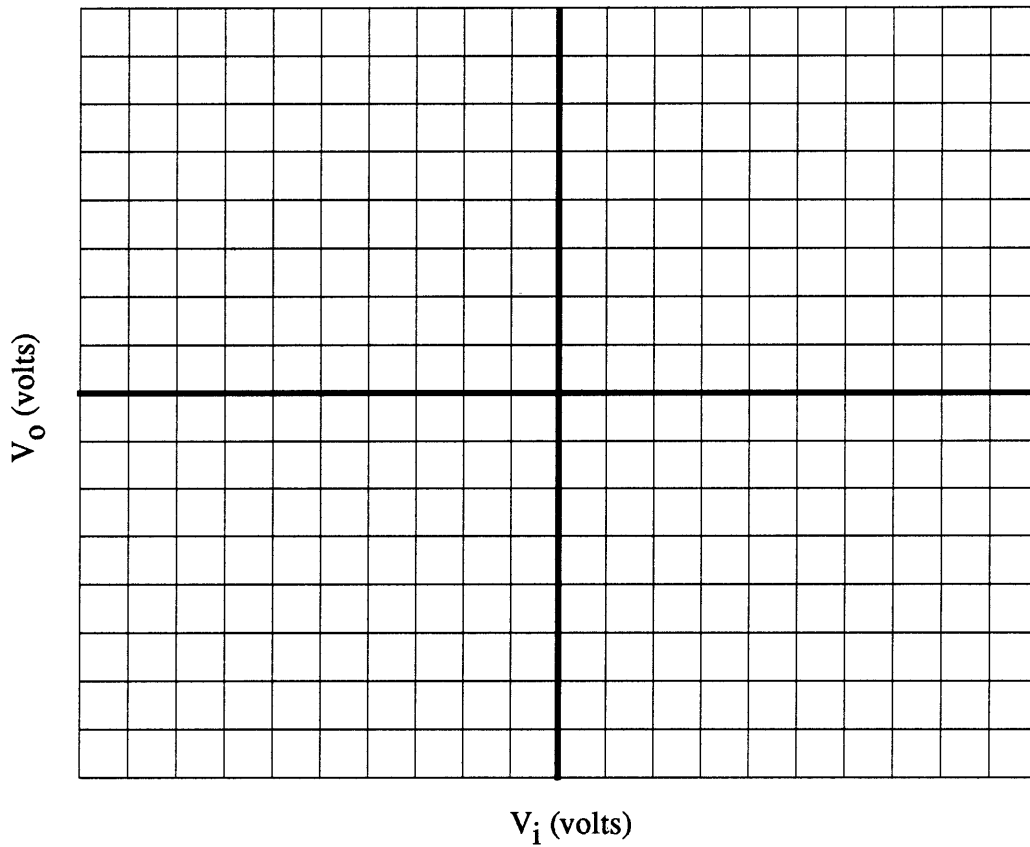
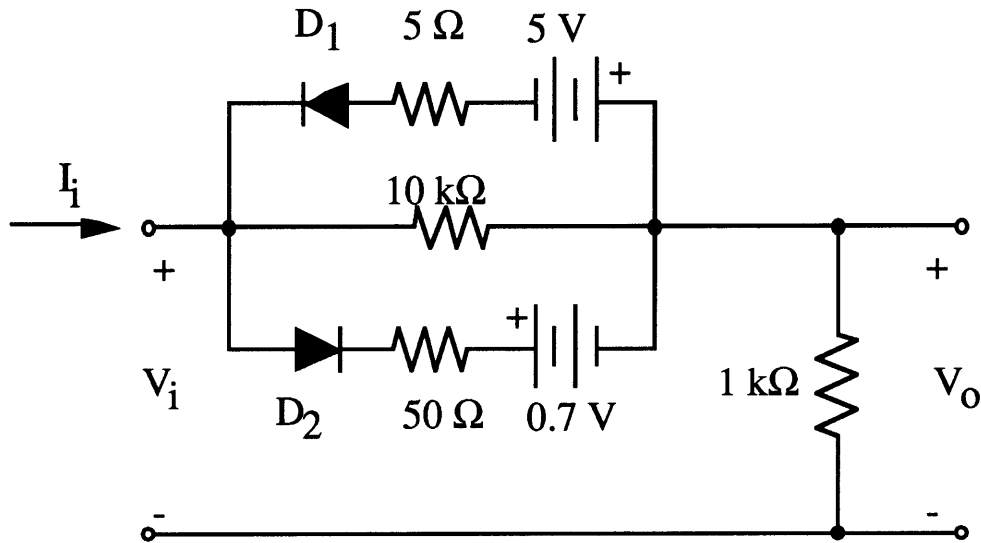


The analysis is exactly the same as before except that we can now see that for this situation $V=-1$ volt and $I=-1$ amp. Note that the current is negative, not positive.

The resulting V-I characteristic is shown below.



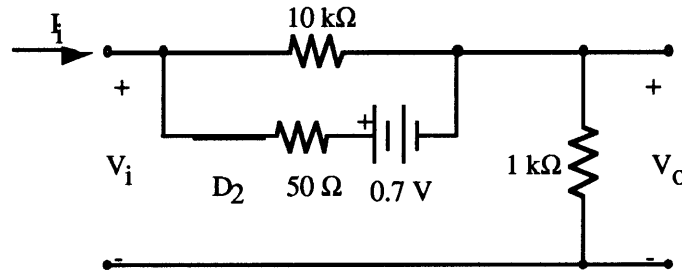
1. (50 points). For the circuit shown below, sketch the voltage transfer function V_o vs. V_i . Identify how many breakpoints and their values the circuit has.



ANSWER:

There are four cases.

1. As $V \rightarrow +\infty$ D1 must be OFF and D2 is ON. The circuit then looks like



I will solve the circuit exactly using KCL at the output node. Note that since V_i is positive the current I_i is also positive, i.e. in the direction shown.

$$\frac{V_i - V_o}{10\text{k}\Omega} + \frac{V_i - (V_o + 0.7)}{50\Omega} - \frac{V_o}{1\text{k}\Omega} = 0$$

Rationalizing and re-writing the equation:

$$V_i - V_o + 200(V_i - V_o - 0.7) - 10V_o = 0$$

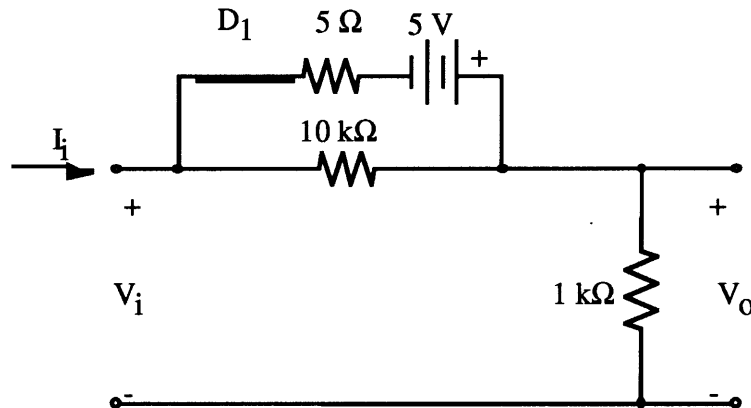
which gives

$$201V_i - 211V_o - 140 = 0$$

$$V_o = \frac{201}{211}V_i - \frac{140}{211} = 0.953V_i + 0.663$$

This is the equation of a line with a slope nearly one.

2. As $V \rightarrow -\infty$ D1 must be ON and D2 is OFF. The circuit then looks like



I will solve the circuit exactly using KCL at the output node. Note that since V_i is negative the current is also negative, i.e. opposite to the direction shown. Using + to indicate a current into the node:

$$\frac{V_o - V_i}{10\text{k}\Omega} + \frac{(V_o - 5) - V_i}{5\Omega} + \frac{V_o}{1\text{k}\Omega} = 0$$

Rationalizing and re-writing the equation:

$$V_o - V_i + 2000(V_o - 5 - V_i) + 10V_o = 0$$

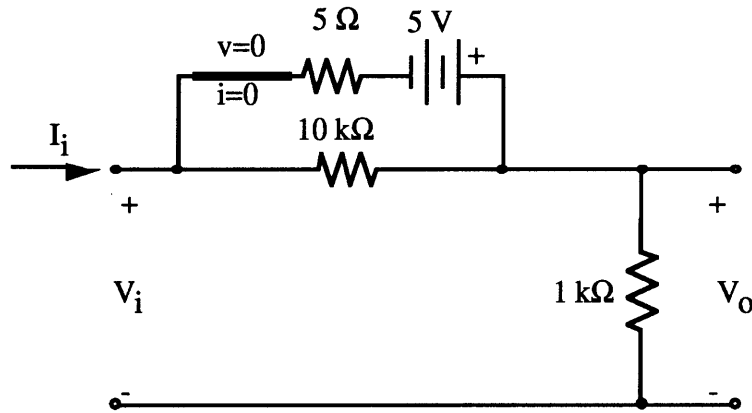
which gives

$$2011V_o - 2001V_i - 10000 = 0$$

$$V_o = \frac{2001}{2011}V_i - \frac{10000}{2011} = 0.995V_i - 4.973$$

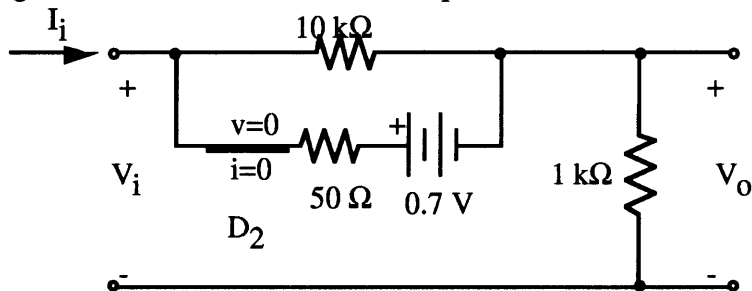
This is also the equation of a line with a slope nearly one.

3. Assume D1 is at a breakpoint. By inspection I will assume that D2 is OFF since the battery voltage across it is a reverse bias. Let's assume it is off and analyze the equivalent circuit:



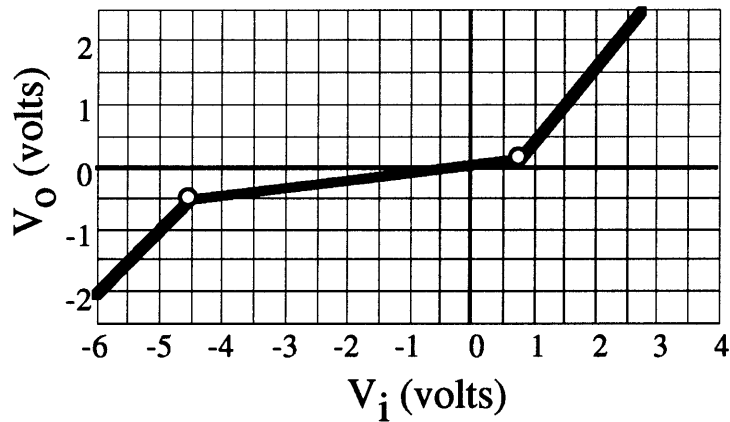
Since the current thru D1 is zero, the voltage across the 5Ω resistor is also zero and the voltage across the $10\text{k}\Omega$ resistor is 5 volts. This means immediately that $I_i = -5\text{V}/10\text{k} = -0.5\text{ mA}$. Note that the current is in the negative direction because of the battery direction. The corresponding voltage across the $1\text{k}\Omega$ output resistor is then $V_o = (-0.5\text{mA})(1\text{k}\Omega) = -0.5$ volts. The battery voltage relates the input and output voltages, i.e. $V_o = V_i + 5$. The input voltage is then $V_i = V_o - 5 = -0.5 - 5 = -5.5$ volts. The breakpoint occurs at $(-5.5, -0.5)$ volts.

4. Now assume D2 is at the breakpoint and let's assume D1 is OFF. This is reasonable as the battery voltages would reverse bias D1. The equivalent circuit is:



The analysis is exactly the same as for D1. Since the current thru D2 is zero, the voltage across the 50Ω resistor is also zero and the voltage across the $10\text{k}\Omega$ resistor is 0.7 volts. This means immediately that $I_i = 0.7\text{V}/10\text{k} = +0.07\text{ mA}$. Note that the current is now in the positive direction because of the battery direction. The corresponding voltage across the $1\text{k}\Omega$ output resistor is then $V_o = (0.07\text{mA})(1\text{k}\Omega) = +0.07$ volts. The battery voltage relates the input and output voltages, i.e. $V_i = V_o + 0.7$. The input voltage is then $V_i = 0.07 + 0.7 = 0.77$ volts. The breakpoint then occurs at $(0.77, 0.07)$ volts.

The resulting transfer characteristic is shown below.



COMMENTS ON GRADING:

$V_i \rightarrow +\infty$	10 points
$V_i \rightarrow -\infty$	10 points
D1 breakpoint	13 points
D2 breakpoint	13 points
graph	4 points

The number one cause of mistakes in the breakpoint analysis was thinking that the batteries forced the output voltage to 0.7 or 5 volts. The batteries CANNOT do this. At a breakpoint the current through the diode/resistor/battery combination is zero. Since the current through the resistor is zero the voltage drop across it is zero. The voltage drop across the diode is zero because it is at a breakpoint. This means that the voltage across the entire diode/resistor/battery combination is the battery voltage which then appears across the $10k\Omega$ resistor, NOT at the output terminal.

2. (50 points, This is basically problem 9.2-1, an assigned problem, with the numbers changed.) The transistor shown in Figure 1 below has the static characteristics shown in Figure 2. $R_k=0.5k\Omega$, $R_D=1k\Omega$, and $V_{DD}=18$ volts.

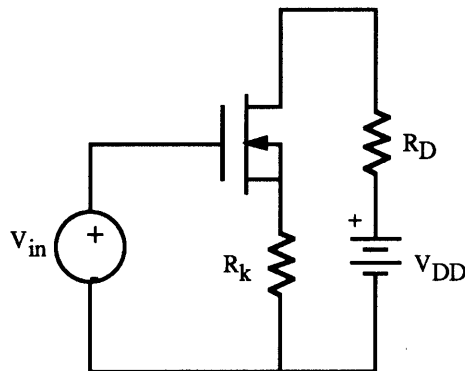


Figure 1 - Transistor Circuit

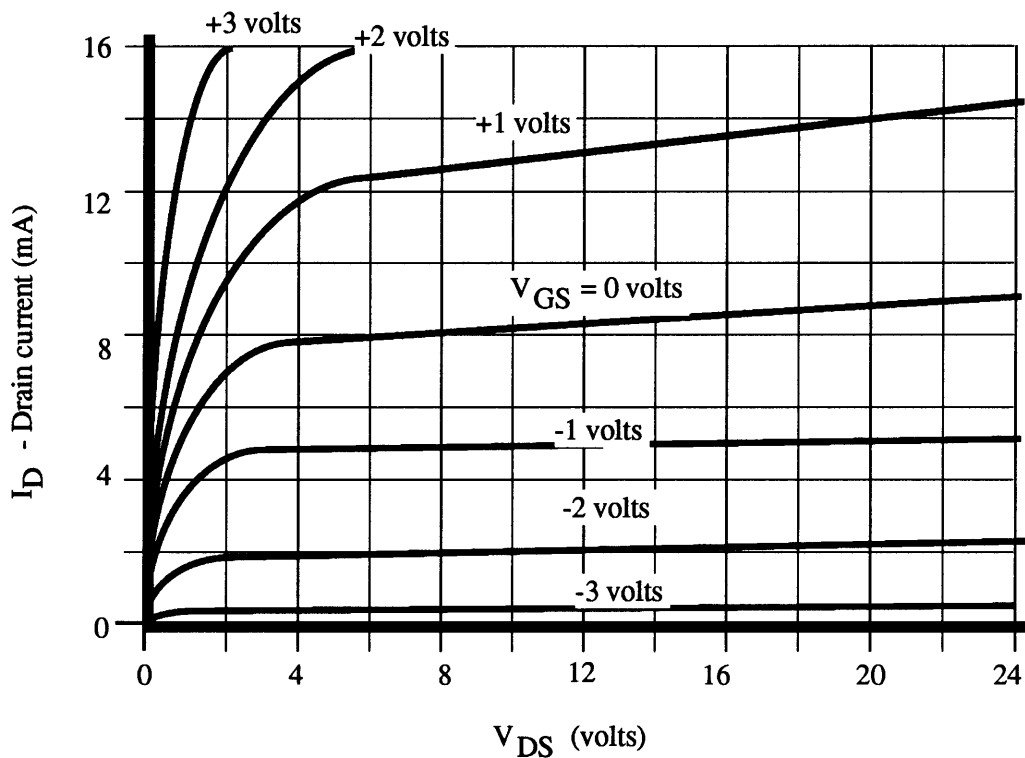
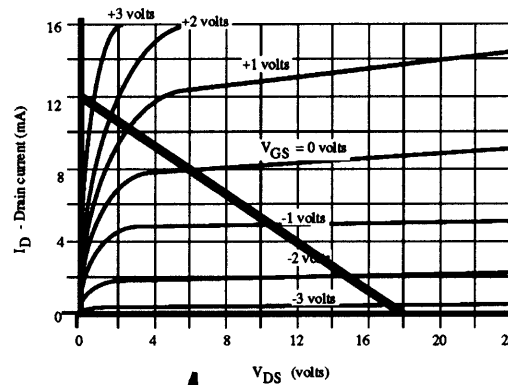


Figure 2 - Static Characteristics

- (a) What type of transistor is this?
ANSWER: The circuit symbol AND the static characteristics indicate that the transistor is a depletion mode n-channel MOSFET.
 (10 points)
- (b) Using a load line, estimate the value of v_{in} that yields $V_k=+3$ volts for the given circuit parameters. **NOTE:** V_k is the voltage at the source of the transistor.

ANSWER: The load line is given by its x-intercept [18 volts] and its y-intercept [18/(1+0.5k) = 12mA]. Since $R_k=0.5k\Omega$, we have $I_D=V_k/R_k=3$ volts/ $0.5k\Omega =$

6mA. Using the load line below we quickly find that this operating point [$I_D=6\text{mA}$] corresponds to a V_{GS} of approximately -0.6 volts. Thus, $V_{in} = V_{GS} + v_k = -0.6 + 3 = +2.4$ volts.



(20 points. You lost 8-10 points for no graphical analysis depending upon what you did. You lost 5 points for each end of the load line if it was wrong.)

- (c) The transistor in the circuit of Figure 1 is now described by the transistor parameters $V_P=5$ volts and $I_{DSS}=2.5$ mA. THIS IS NO LONGER THE TRANSISTOR DESCRIBED IN FIGURE 2. What is V_{GS} if $V_{in}=0$?

ANSWER: ,

The gate-source voltage is described by
 $V_{GS} = V_G - V_{GS} = 0 - I_D R_k$

Solving for I_D and equating this to the transistor characteristic curve

$$I_D = -\frac{V_{GS}}{R_k} = I_{DSS} \left(1 + \frac{V_{GS}}{V_P} \right)^2$$

Rearranging and expanding the function:

$$-\frac{V_{GS}}{I_{DSS} R_k} = 1 + 2 \frac{V_{GS}}{V_P} + \frac{V_{GS}^2}{V_P^2}$$

Substituting values

$$-\frac{V_{GS}}{(2.5 \text{ volt})(0.5 \text{ k}\Omega)} = 1 + 2 \frac{V_{GS}}{5 \text{ volts}} + \frac{V_{GS}^2}{25 \text{ volts}^2}$$

Rearranging

$$V_{GS}^2 + 30 V_{GS} + 25 = 0$$

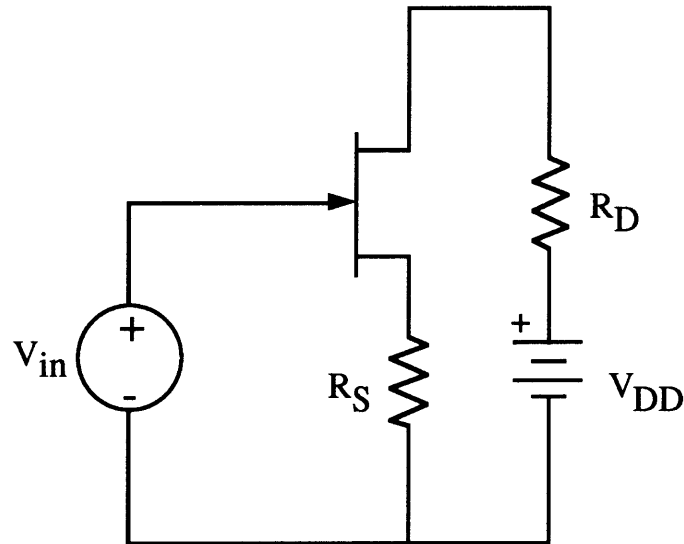
Solving,

$$V_{GS} = \frac{-30 \pm \sqrt{(30)^2 - 4(25)}}{2} = -15 \pm 14.1 = -29.1, -0.9 \text{ volts}$$

The answer is -0.9 volts since the other value would be beyond pinchoff.

(20 points. The most common error was not using the term $I_D = \frac{V_{GS}}{R_K}$ for which you lost 10 points)

1. (50 points) Consider the JFET transistor in the circuit shown. The circuit parameters are $V_{DD}=20$ volts, $R_D=3k\Omega$ and $R_S=2k\Omega$.



- (a) (35 points) If $V_{in}=0$ volts determine the operating point (i.e. V_{DS} , V_{GS} and I_D) for the JFET. The transistor parameters are $I_{DSS}=18mA$ and $V_P=6$ volts.

ANSWER:

This problem is very similar to problem 9.2-4 in your textbook. From the transistor characteristics

$$I_D = \frac{I_{DSS}}{V_P^2}(V_{GS} + V_P)^2$$

For the self-biased amplifier with $V_G=V_{IN}=0$:

$$V_S = I_D R_S, \text{ or}$$

$$I_D = + \frac{V_S}{R_S} = - \frac{V_{GS}}{R_S}$$

Combining these equations

$$- \frac{V_{GS}}{R_S} = \frac{I_{DSS}}{V_P^2}(V_{GS} + V_P)^2$$

Using numerical values the equation becomes:

$$- \frac{V_{GS}}{2} = \frac{18}{(6)^2}(V_{GS} + 6)^2$$

which can be re-arranged into the standard form

$$0.5V_{GS}^2 + 6.5V_{GS} + 18 = 0$$

with the solutions

$$V_{GS} = -4, -9 \text{ volts}$$

The -9 volt solution can be ignored since it is beyond V_P and no transistor operation is possible for such values. Using $V_{GS}=-4$ volts

$$I_D = \frac{18}{(6)^2}(-4 + 6)^2 = 2\text{mA}$$

With these values V_{DS} can be computed using KVL for the transistor drain-source circuit:

$$V_{DS} = V_{DD} - I_D R_D - I_D R_S = 20 - (2\text{mA})(3\text{k}) - (2\text{mA})(2\text{k}) = 20 - 6 - 4 = 10 \text{ volts}$$

The complete operating point is then:

$$\begin{aligned} V_{GS} &= -4 \text{ volts,} \\ I_D &= 2\text{mA, and} \\ V_{DS} &= 10 \text{ volts.} \end{aligned}$$

A detailed numerical analysis using an Excel spreadsheet is:

Problem 9.2-4 Analysis of a self-biased JFET amplifier

Circuit parameters:

Power supply voltage	VDD	20 volts
Load resistance	RD	3 kΩ
Source resistance	RS	2 kΩ

JFET parameters:

IDSS	18 mA
VP	6 volts

A. Determine Q-point of amplifier

VGS	a	0.5
	b	6.5
	c	18
	VGS1	-4 volts
	VGS2	-9 volts
	$0 \geq VGS \geq$	-6 volts

Pick operational value VGS= -4 volts <<<CHECK

2. Calculate drain current ID ID= 2 mA

3. Calculate VDS using KVL VDS= 10 volts

Condition for active operation:

or $V_{DS} > V_{GS} + V_P$ 2 volts
 $V_{DD} > V_{GS} + V_P + V_{DS}(R_D + R_S)$ 12 volts

GRADING SUMMARY:

- 15 starting with VDS to determine VGS
- 15 using a RD-RS voltage divider to find VDS
- 10 forgetting the term $I_D = -V_{GS}/R_S$ in computing VGS
- 30 only showing some relevant formulas; no calculations
- 35 no answer whatsoever
- 10 no coupling between the calculations for ID and VDS
- 10 saying $V_{GS} = V_{DS}$ since $V_{IN} = 0$

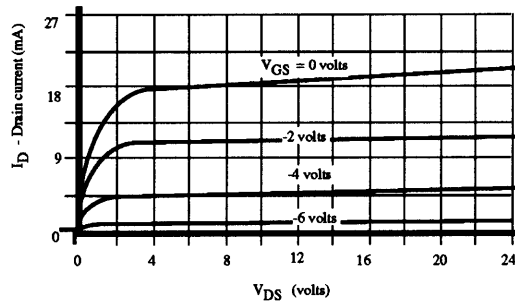
- (b) (15 points) Using the static and active transfer characteristics shown below for the transistor used in part (a), explain the basis for the following “conditions for active operation” used by the textbook, i.e. why does the book say

$$v_{DS} > v_{GS} + V_P \quad (1)$$

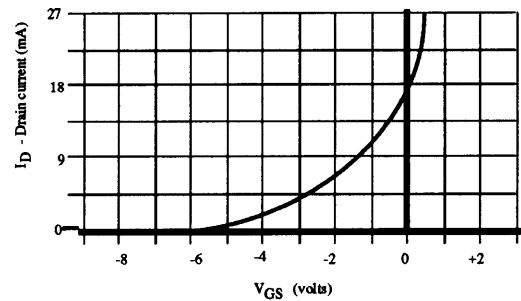
and

$$v_{GS} > -V_P \quad (2)$$

are required for active operation of a JFET amplifier?



(a) static characteristics



(b) active transfer characteristics

ANSWER:

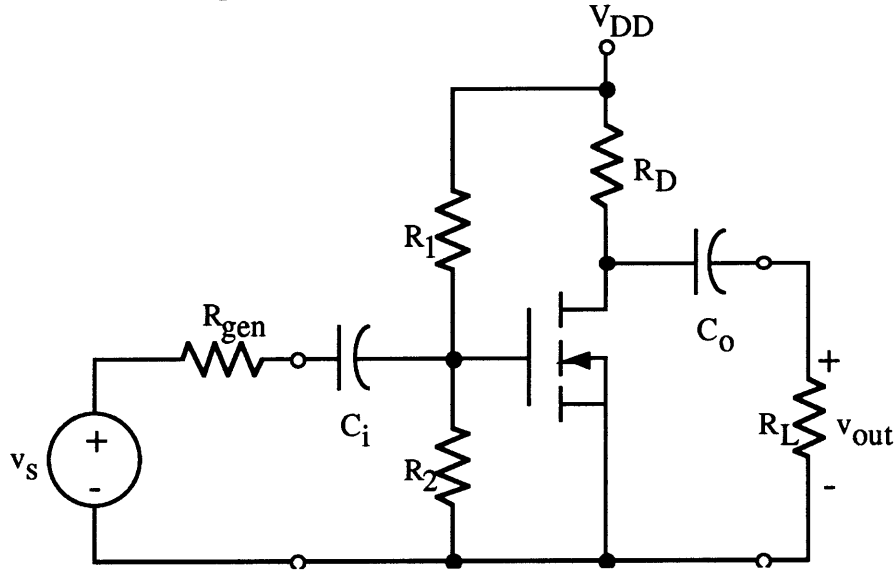
The first equation, (1), simply indicates that the transistor should be operating in the active region of the transistor static characteristics which begins at $v_{GS} + V_P$. In the figure (a), this is the point at which the characteristic curve stops following a curve (actually a parabola) and becomes a straight line. Amplifiers should operate in this “straight line” region of the static characteristic curve.

The second equation, (2), indicates that the transistor should be operating in the region where $0 > V_{GS} > V_P$. In (a) this means that the transistor must operate in the region below the $V_{GS} = 0$ curve and above the $V_{GS} = -6$ volts curve. Anything outside of this region is not appropriate for amplifier operation. The curve (b) simply amplifies this fact.

GRADING SUMMARY:

This part of the problem was worth 15 points. For Eqn.(1) and Figure (a) you needed to mention operating in the linear region of the curve. Also, acceptable was saying that you did NOT want to operate in the ohmic part of the curve. Depending upon how your answer was worded you lost 2-7 points for not mentioning these points. For Eqn.(2) and Fig.(b) you needed to mention that the transistor needed to be active.

2. For the MOSFET amplifier circuit shown below:



- (a) (20 points) Using the values $R_{gen}=10\text{k}\Omega$, $V_{DD}=18\text{V}$, $R_1=3.3\text{M}\Omega$, $R_2=1.2\text{M}\Omega$, $R_D=2\text{k}\Omega$, and $R_L=5\text{k}\Omega$ determine the DC operating point of the transistor, i.e. determine I_D , V_{DS} , and V_{GS} . The MOSFET is characterized by $I_{DSS}=6\text{mA}$ and $V_T=2.5$ volts.

ANSWER:

This problem is very similar to problem 11.1-3 with some of the numbers changed. The detailed analysis procedure is as follows:

There is no voltage at the source as it is connected directly to ground. V_{GS} is determined only by V_G which is set by the R_1 - R_2 voltage divider.

$$V_{GS} = \frac{R_2}{R_1 + R_2} V_{DD} = \frac{1.2\text{M}\Omega}{1.2\text{M}\Omega + 3.3\text{M}\Omega} (18\text{V}) = (0.27)(18\text{V}) = 4.8\text{V}$$

Since the transistor is an enhancement mode MOSFET we will switch to the convention of using k to describe the transistor where

$$k = \frac{I_{DSS}}{V_T^2} = \frac{6\text{mA}}{(2.5\text{V})^2} = 0.96 \frac{\text{mA}}{\text{V}^2}$$

With the transistor parameter k AND V_{GS} we can calculate the drain current as:

$$I_D = k (V_{GS} - V_T)^2 = 0.96 \frac{\text{mA}}{\text{V}^2} (4.8\text{V} - 2.5)^2 = 5.08\text{mA}$$

Once the drain current is known we can apply KVL to the loop from ground through the transistor, through R_D , and through the power supply to ground to get:

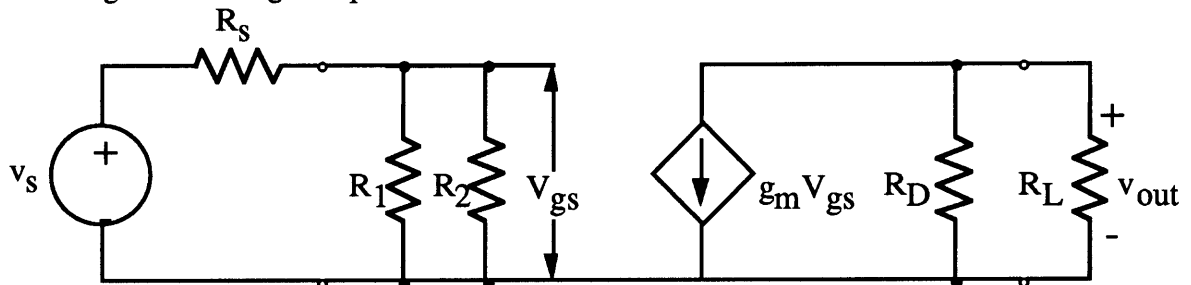
$$V_{DS} = V_{DD} - R_D I_D = 7.84\text{V}$$

COMMENTS: V_{GS} , I_D and V_{DS} were worth approximately 5 points apiece plus some points for the equations to get them.

(b) Draw the small signal equivalent circuit of the amplifier.

ANSWER:

Drawing the small signal equivalent circuit:



GRADING COMMENTS: The most common error was not replacing the power supply connection to ground. That lost 3-5 points depending upon how bad the error was. You also forgot to include the load resistance R_L in your circuits. That cost 3-8 points depending upon the rest of the circuit.

(c) (15 points) Using this small signal model determine A_v and R_{in} for this amplifier.

ANSWER:

For the ac analysis (i.e., voltage gain) we need to compute the transistor small signal parameter g_m :

$$g_m = 2k(V_{GS} - V_T) = 2 \left(0.96 \frac{\text{mA}}{\text{V}^2} \right) (4.8\text{V} - 2.5\text{V}) = 4.416 \text{ mS}$$

This can be used to determine the small signal gain. The ac input impedance of the amplifier is determined primarily by the bias resistors:

$$R_i = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{(3.3\text{M}\Omega)(1.2\text{M}\Omega)}{3.3\text{M}\Omega + 1.2\text{M}\Omega} = 880\text{k}\Omega \gg R_s$$

Since $R_1 \parallel R_2 \gg R_s$, basically the entire input voltage appears as V_{gs} and $V_{gs} = V_s$. The output voltage is then $-g_m V_{gs} (R_D \parallel R_L)$. The ratio of the output voltage to the input voltage becomes $-g_m V_s (R_D \parallel R_L)$ divided by V_s giving

$$A_v = -g_m (R_D \parallel R_L) = -g_m \frac{R_D R_L}{R_D + R_L} = -(4.416 \text{ mS}) \frac{(2\text{k}\Omega)(5\text{k}\Omega)}{2\text{k}\Omega + 5\text{k}\Omega} = -6.31$$

The problem was solved using an Excel spreadsheet:

Problem 9.2-3

Analysis of an enhancement MOSFET amplifier

Circuit parameters:

Power supply voltage	VDD	18 volts
Generator resistance	Rgen	10 kΩ
Drain resistance	RD	2 kΩ
Load resistance	RL	5 kΩ
Bias resistors	R1 (upper)	3300 kΩ
	R2 (lower)	1200 kΩ

Enhancement MOSFET parameters:

IDSS	6 mA
VT	2.5 volts

(a)

1. Calculate transistor param $k = 0.96 \text{ mA/volt}^2$

2. Calculate VGS $VGS = 4.8 \text{ volts}$

3. Calculate drain current $ID = 5.0784 \text{ mA}$

4. Calculate VDS $VDS = 7.8432 \text{ volts}$

(c)

5. Calculate gm $gm = 4.416 \text{ mS}$

6. Calculate Rin $Rin = 880 \text{ k}\Omega$

7. Calculate voltage gain $Av = -6.308571429$

GRADING COMMENTS: R_i and A_v were worth 8 points each with one extra point for the problem setup.