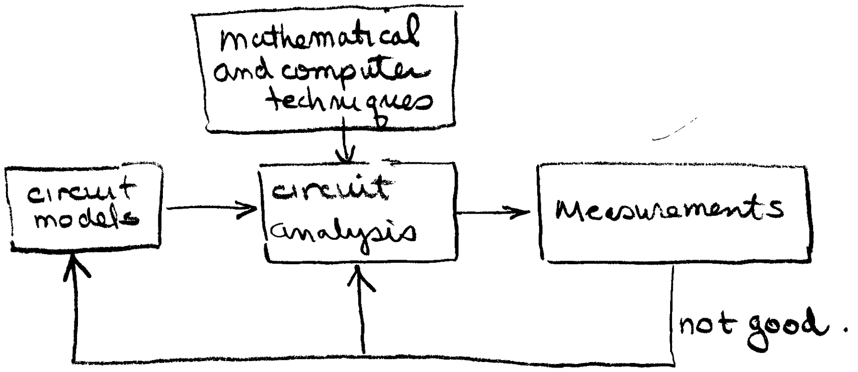
EEAP 244

1.3



circuits — mathematical models for the behavior of electrical systems

components — elements that make up the circuit

ideal circuit elements. (Thurs & Fri lectures)

practicing engineers try to use mature circuit models. (typically 20-100 years old).

1.4 voltage & current

energy is expended when positive & negative charges are separated

these are vectors.

$v = \frac{dw}{dq}$ energy (joules) / charge (coulombs)

voltage (volts)

energy per unit charge created by the charge separation

$i = \frac{dq}{dt}$ — charge (coulombs) / time (seconds).

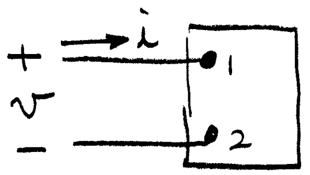
current (amperes)

$e = 1.6022 \times 10^{-19}$ coulomb.

components can be modeled strictly in terms of the voltage & current at the terminals

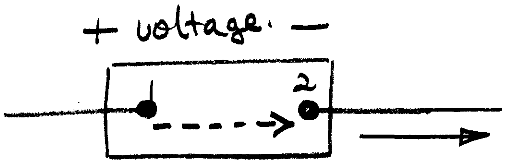
- usually don't need to know internal behavior
- use physics to develop circuit models.

1.5 Basic circuit element



voltage polarity
current direction.

attributes
1. has only two terminals
2. described mathematically in terms of voltage and/or current
3. cannot be subdivided into other elements



current flowing through the device.

current flow in this direction

If voltage > 0 there is a voltage drop across device

if voltage < 0 there is a voltage rise across device.

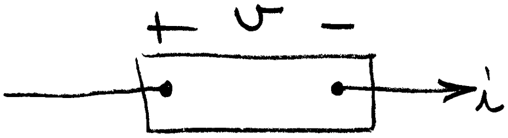
if $i > 0$ current flowing from 1 to 2

if $i < 0$ current flowing from 2 to 1.

not a function of components.

assignment of polarities is entirely arbitrary but you can never change convention in a problem

passive sign convention



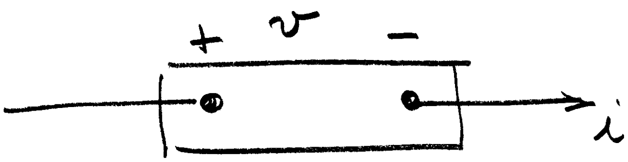
ideal − does not exist as a realizable circuit component

basic − cannot be further reduced or subdivided into other elements.

can better model "real" devices as connections of "ideal" devices.

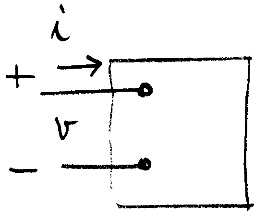
1.6 Power and energy.

$$\underbrace{p}_{\text{power (watts)}} = \frac{\overset{\text{energy (joules)}}{dw}}{\underbrace{dt}_{\text{time (sec).}}} = \underbrace{\frac{dw}{dq}}_{\text{voltage}}\;\underbrace{\frac{dq}{dt}}_{\text{current}}$$

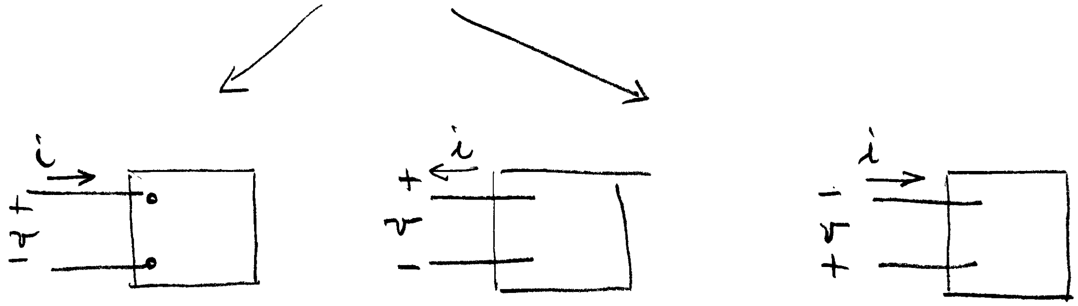


$p = vi > 0$ power is being delivered to circuit

$p = vi < 0$ power is being extracted from circuit



power being delivered.

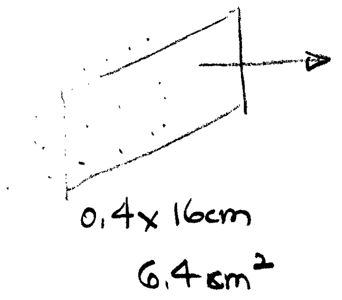


$p = vi$
$= (+10)(+4)$
$= +40.$

$i = +4$
$v = -10$
$p = -vi = -(-10)(4)$
$= +40$

Example problems:

1.7 A current of 1600 A exists in a rectangular (0.4 × 16 cm) copper bus bar. The current is due to free electrons moving through the bus bar at an average velocity of v meters/sec. If the concentration of free electrons is 10^{29} electrons/m^3 and if they are uniformly dispersed throughout the bus bar, then what is the average velocity of an electron.



$$q/m^3 = 1.6022 \times 10^{-19} \frac{\text{coul}}{\text{electron}} \cdot \frac{10^{29}\ \text{electron}}{m^2}$$

$$q/m^3 = 1.6022 \times 10^{10} \frac{\text{coul}}{m^3}$$

$$1.6022 \times 10^{10} \frac{\text{coul}}{m^3} \times \underbrace{(6.4 \times 10^{-4} m^2)}_{\text{assuming constant cross section}} = 10.25 \times 10^{6} \frac{\text{coul}}{m}.$$

$$i = 10.25 \times 10^{6} \frac{\text{coul}}{m} \bar{v}$$

$$1600 = 10.25 \times 10^{6} \bar{v}.$$

$$\bar{v} = 156 \times 10^{-6} \frac{m}{\text{sec}}.$$

1.12 A 12-volt battery supplies 100mA to a radio. How much energy does the battery supply in 4 hours.

$$p = vi = \frac{dw}{dt}$$

$$\underbrace{(12)(10^{-1})}_{\text{units are watts}} = \frac{\Delta W}{14400 \text{ sec}}$$

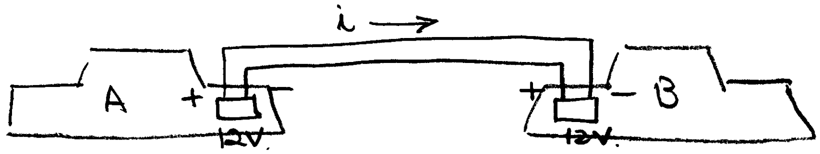
$$4 \text{ hrs} \times \frac{60 \text{ min}}{\text{hr}} \times \frac{60 \text{ sec}}{\text{min}}.$$

$$\Delta W = 17280 \text{ joules}.$$

1.20 When a car has a dead battery it can often be started by connecting the battery from another car across its terminals. The positive terminals are connected together as are the negative terminals. The connection is illustrated in Fig P1.20. Assume the current i in Fig P1.20 is measured and found to be $-40A$.

(a) Which car has the dead battery?

(b) If this connection is maintained for 1.5 minutes, how much energy is transferred to the dead battery?



(a) $i = -40 A$ so current is flowing from B to A.

A is dead.

$$vi = \frac{dw}{dt}.$$

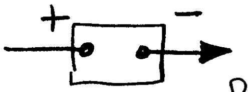
$$\text{or } w = \int_0^{1.5 \text{min}} vi \, dt = \int_0^{90} (12)(40) \, dt = 43,200 \text{ joules}$$

1.26 The numerical values of the voltages and currents in the interconnection seen in Fig. P1.26 are:

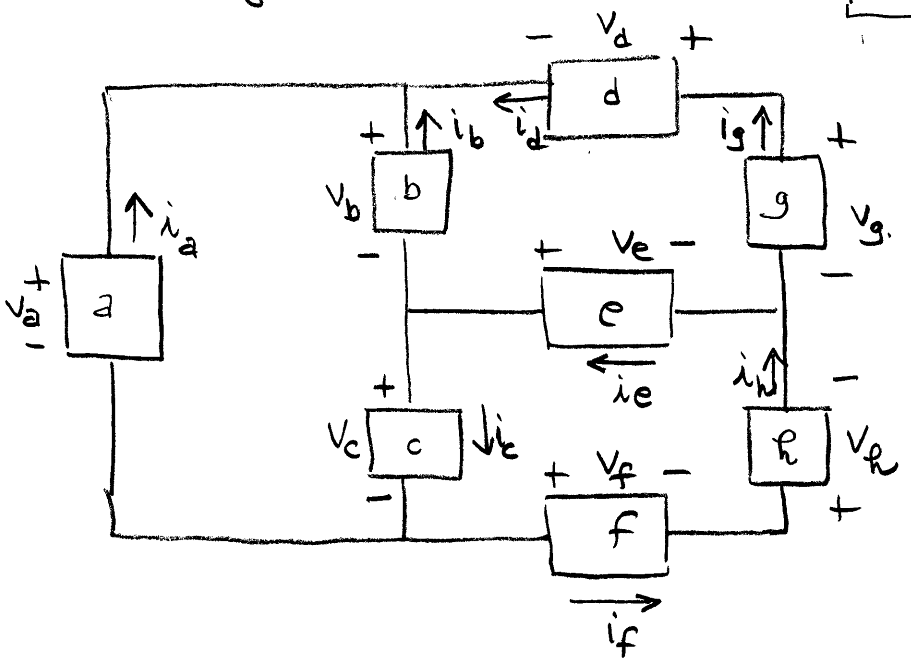
$v_a = 990V$,	$i_a = -22.5A$.	$-vi$	$-(990)(-22.5) = +22275$
$v_b = 600V$,	$i_b = -30A$	$-vi$	$-(600)(-30) = +18000$
$v_c = 300V$,	$i_c = 60A$	$+vi$	$+(300)(60) = +18000$
$v_d = 105V$	$i_d = 52.5A$	$+vi$	$+(105)(52.5) = +5512.5$
$v_e = -120V$	$i_e = 30A$	$-vi$	$-(-120)(30) = +3600$
$v_f = 165V$	$i_f = 82.5A$	$+vi$	$+(165)(82.5) = +13612.5$
$v_g = 585V$	$i_g = 52.5A$.	$-vi$	$-(585)(52.5) = -30712.5$
$v_h = -585V$	$i_h = 82.5A$	$+vi$	$-(585)(82.5) = -48262.5$

Does the interconnection satisfy the power check?

① Check for sign conventions!



$p = vi > 0$.
passive sign convention.



summing $+81000 \longleftarrow P_{absorbed}$

$-78975 \longleftarrow P_{delivered}$

$P_{abs} \neq P_{del}$.

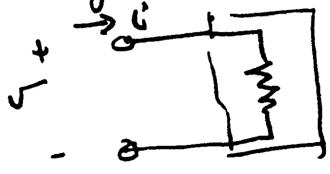
EEAP 243 - 1/20/93

The concepts of voltage and current are based on the physics of charges in electric fields. Voltage is associated with work on charges or work that can be produced by charge motion.

Take the ~~bod~~ human body

If we are subjected to a few hundred volts, under ~~most~~ some circumstances this may produce a non-lethal shock because it is the flow of current through the body that causes the damage. and if the work necessary to cause the lethal flow exceeds this voltage - for reasons of skin resistance to flow or the path through the body to the source involves insulating material, such as ~~rubber~~ dry-rubber soled shoes, we may be made aware of our foolishness by a minor shock. The work required to push a lethal flow of charges exceeds the work available!

Clearly, in analogy to water flow in a pipe, the narrower the pipe, the more work will be required to push the same flow of water - we require more voltage to push a given number of charges through a resistive material per second. This resistor impedes current flow. The relationship between work (voltage) and charge flow (current) is called Ohm's law.

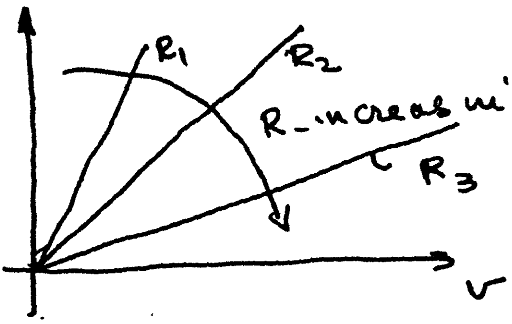


$v/i = R$ note defined direction of current flow and voltage signs!

This is ~~[illegible]~~ convention.

Notice the linear relationship between the two variables v, i.

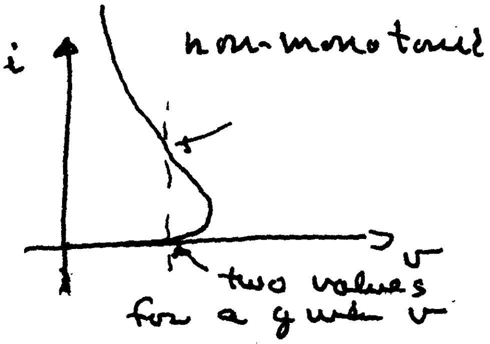
1/20/93



$R_3 > R_2 > R_1$

This nice linear behavior between voltage and current isn't a necessity for a device. We could have

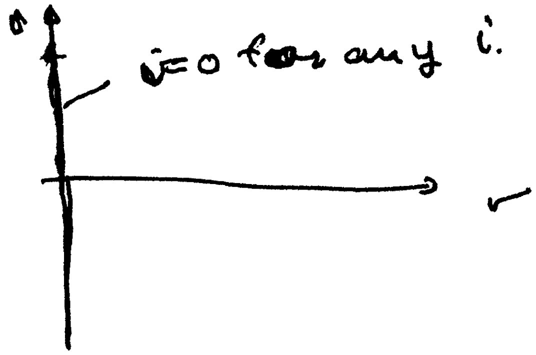
or this



These are i, v characteristics for real physical devices -BUT NOT A RESISTOR. which is <u>defined</u> by ohm's law, $v = iR$.

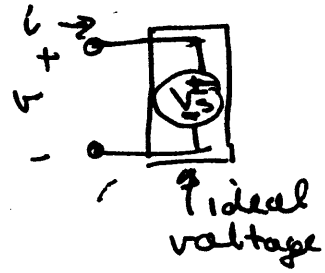
An open circuit is an infinite resistance

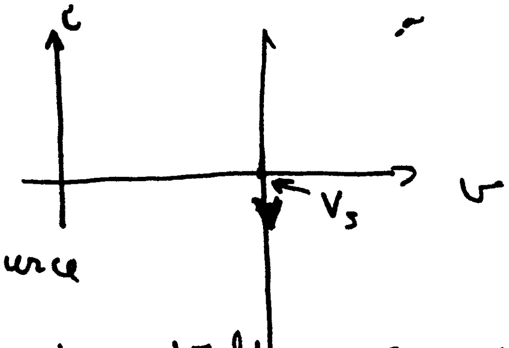
A short circuit is a zero-valued resistor

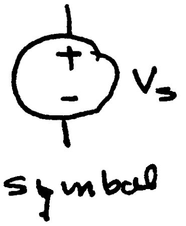


1/20/93

- we have been talking about voltages and currents. We also know that batteries produce voltages - i.e. they are <u>voltage sources</u> They, too, have terminal characteristics

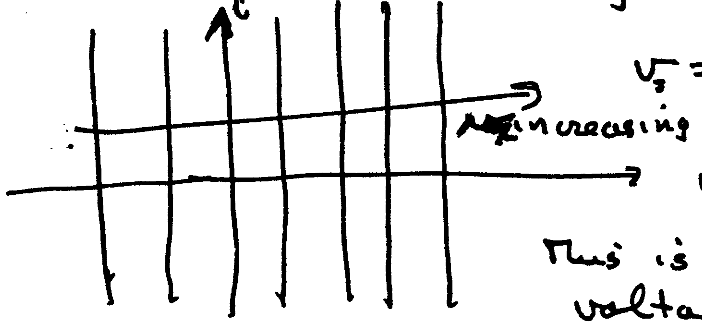




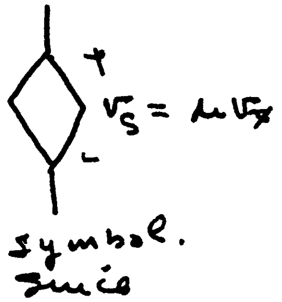


In some sense the voltage source is a displaced (in V) short circuit.

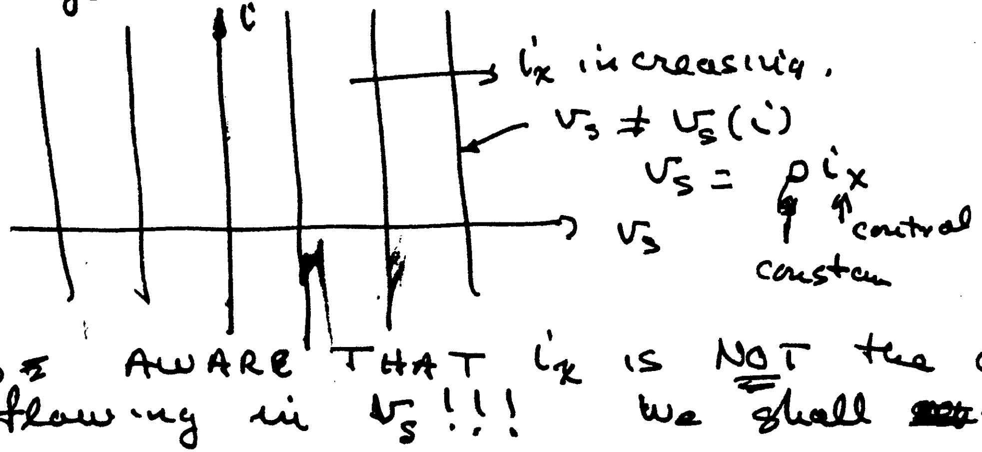
You can imagine that a voltage source is made to vary, perhaps as easily as turning a knob mechanically. This could be done electronically (as we shall see later)



$v_s = \overset{\text{constant}}{\mu}\ \underset{\text{variable}}{v_x}$



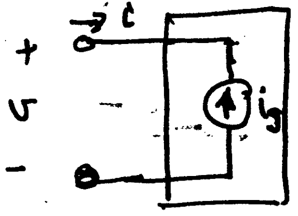
This is a voltage controlled voltage source. Since currents are also possible means of controlling voltage sources → current controlled voltage sourc

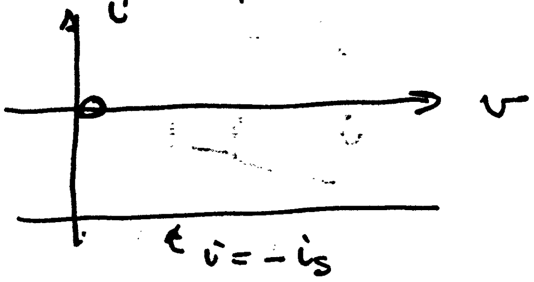


$v_s = \underset{\text{constant}}{\rho}\ \underset{\text{control}}{i_x}$

BE AWARE THAT i_x IS <u>NOT</u> the current flowing in v_s!!! We shall ~~ex~~ examine these two strange, but interesting boasts later in the course

Just as we have sources of voltage, we can have sources of current. This means that whatever the resistance placed in its path, the same current will flow.





This appears to be a displaced (along current axis) infinite resistor. Note: Because of the definitions (i, v, i_s) we wind up with $i=-$ Since they are in opposite directions - physically the current flows out of the + terminal in spite of our definition of positive current flow in. This is characteristic of sources - (see how current definition went into the voltage source while we should expect current to flow out of the (+) terminal.

We also have the possibility of controlled current sources. (we need to exhaust all cases)

Current control $i_s = \beta i_x$ controlling current (where is it ?)

Voltage control $i_s = \alpha v_x$ controlling voltage

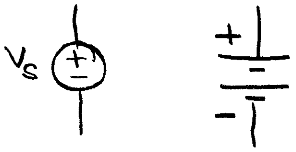
← proportionality constant.

Chapter 2 Circuit Elements

basic circuit elements

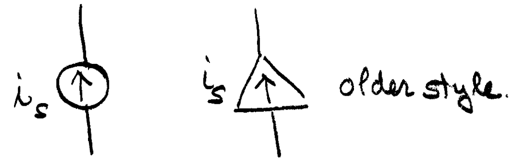
- voltage sources
- current sources
- resistors
- inductors — See section 7.1
- capacitors — See section 7.2
- switches (not in text) see problem 2.25
- diodes (not in text) See Carlson p. 250 - 251
- op-amps (not basic) See section 6.1

ideal independent voltage source.



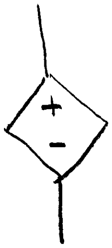
maintains voltage across its terminals regardless of current

ideal independent current source.



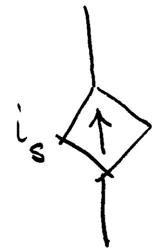
maintains the current through its terminals regardless of voltage.

ideal dependent (controlled) voltage source.



$v_s = \mu v_x$ or $v_s = \rho i_x$

can be controlled by a current <u>or</u> a voltage ideal dependent (controlled) current source

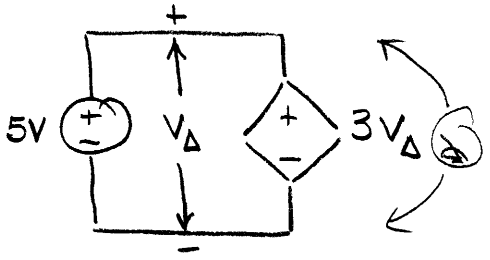


$i_s = \alpha v_x$ $i_s = \beta i_x$

can be controlled by a voltage or a current

Example 2.2.

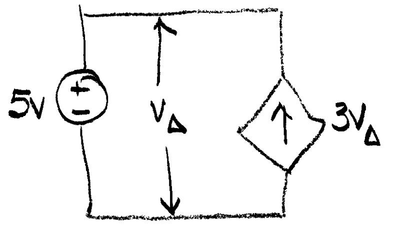
(a)



invalid

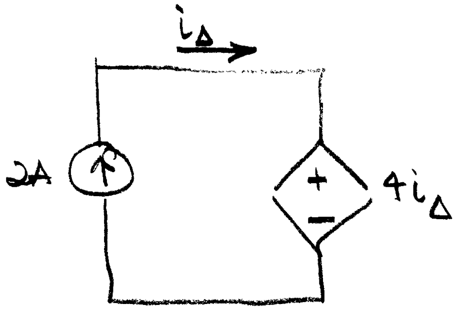
this voltage can't be both 5V and 15V.

(b)



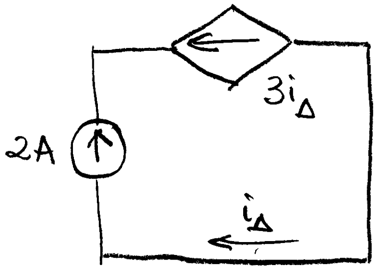
OK.

(c)



OK.

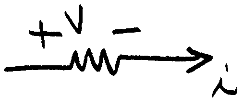
(d)



invalid.

2.2 Electrical resistance (Ohm's Law)

circuit symbol resistance in Ohms (Ω)



$v = iR$ Ohm's Law

voltage drop.

or $i = \dfrac{v}{R}$

properties of resistance

1. linear, time-invariant (constant)
   independent of voltage, current or time
2. bilateral
   if polarity of v reverses, current reverses
3. lumped
   independent of spatial dimensions

A positive resistor always absorbs power from the circuit since $P > 0$

$$P = vi > 0$$

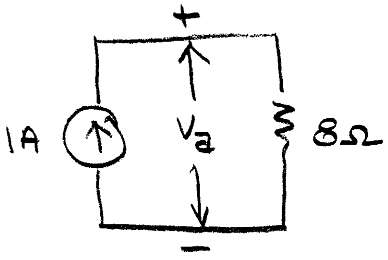
power

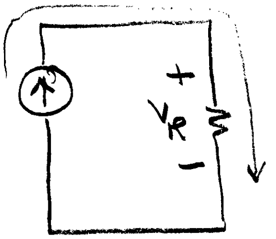
$$= (iR)i = i^2R$$

$$= v\left(\frac{v}{R}\right) = \frac{v^2}{R}$$

conductance $G = \dfrac{1}{R}$ Siemens

Example 2.3 (a)



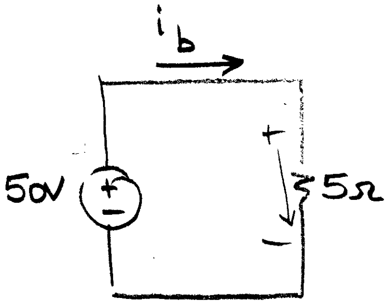


signs due to current

$$V_a = V_R = iR = (1)(8) = 8V.$$

$$P_{8\Omega} = vi = (8V)(1A) = 8W.$$

(b)



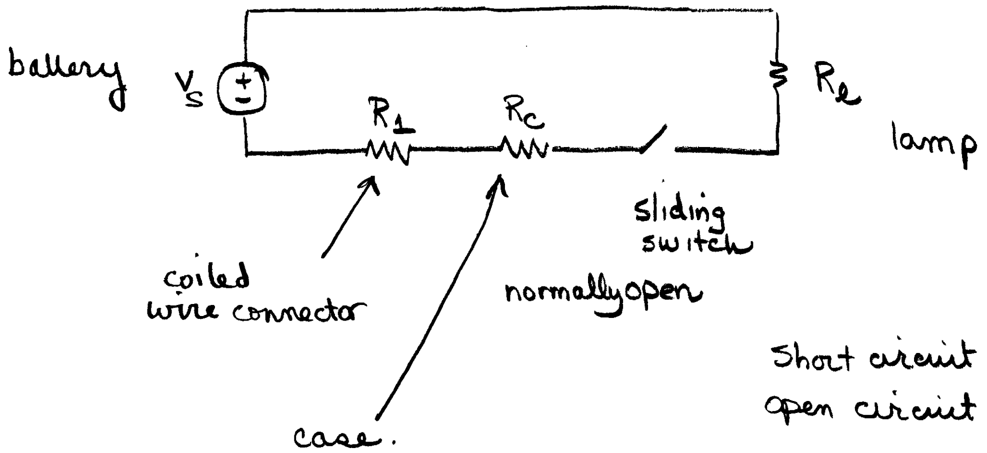
$$V_{5\Omega} = i_b R.$$

$$50 = i_b 5$$

$$i_b = 10A.$$

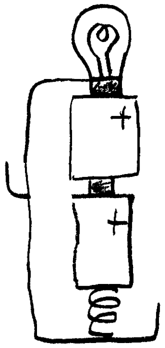
$$P_{5\Omega} = vi = (50V)(10A) = 500W.$$

2.3 Simple Circuits

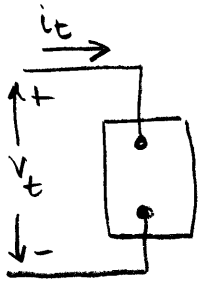


Short circuit $R = 0$
open circuit $R = \infty$

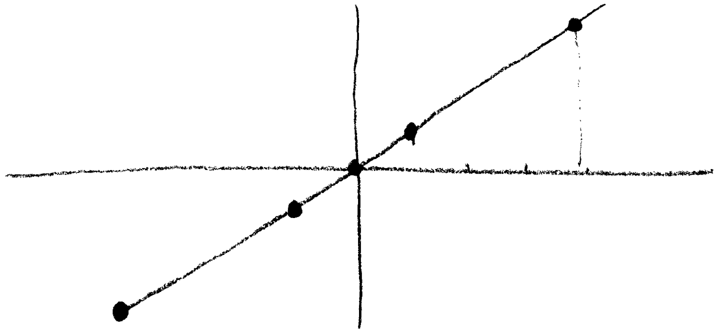
physically



Example 2.5.



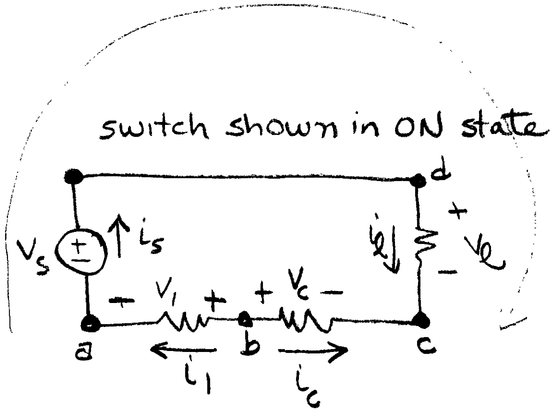
v_t	i_t
−40	−10
−20	−5
0	0
20	5
40	10



Note $v = k i$
where k must be a constant resistance.

2.4 Kirchoff's Law's

battery example



switch shown in ON state

V_s known

$$\underbrace{i_s,\ i_\ell,\ V_\ell,\ V_1,\ i_1,\ V_c,\ i_c \text{ are unknown}}_{\text{want to solve for these}}$$

$$\left.\begin{aligned} V_1 &= i_1 R_1 \\ V_c &= i_c R_c \\ V_\ell &= i_\ell R_\ell \end{aligned}\right\} \text{3 equations in 6 unknowns}$$

R's are known.

Kirchoff's voltage laws & current law have additional
(KVL) (KCL)
constraints that let us solve for the remaining unknowns.

node ≜ point at which two or more circuit elements join
a, b, c, d are nodes in above diagram

KCL: The <u>algebraic</u> sum of the currents at any node is zero.

You can choose + to be into the node or − to be into the node but you <u>MUST</u> be consistent at each node.

applying to battery circuit: positive sign leaving

$$\left.\begin{aligned} a:&\quad i_s - i_1 = 0 \\ b:&\quad i_1 + i_c = 0 \\ c:&\quad -i_c - i_\ell = 0 \\ d:&\quad -i_s + i_\ell = 0 \end{aligned}\right\}$$

4 more equations
only 3 of which are independent
need one more equation

KVL: The algebraic sum of all the voltages around any closed path in a circuit is zero

must choose a direction <u>clockwise</u> in this case

$$-V_s + V_\ell - V_c + V_1 = 0$$

7 equations in 7 unknowns is a mess to solve.

observations

1. use Ohm's Law to eliminate variables
   reduces number of equations
2. you only need one current if only
   two components connect at a node

EEAP 243
1/22/93

I. Lab reports
1. Statement of lab goals (short)
2. Response to questions posed
3. Graphics where appropriate - data analyses.
4. Special observations -
    - comments on lab reasonableness
    - suggestions.

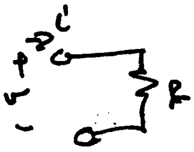
II. Lockers - name on labels
- not responsible for losses
- locks to be removed at end of semester

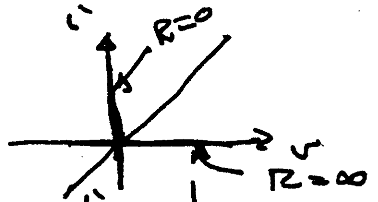
III Horowitz & Hill is NOT required.
4(?) copies on 2 hour reserve at Sears.

IV Judd Gardner hours M-Th 1430-1700.

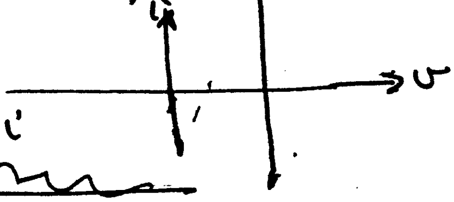
CIRCUIT ELEMENTS

Resistor $v = iR$
Ohm's law

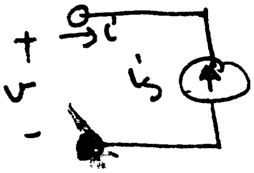


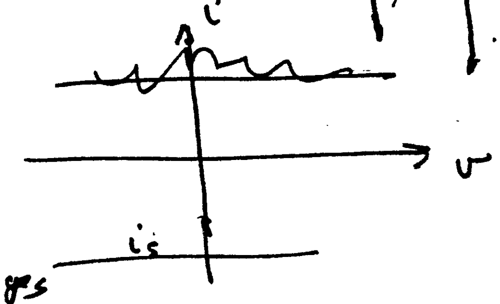


Voltage source
also VCVS, CCVS

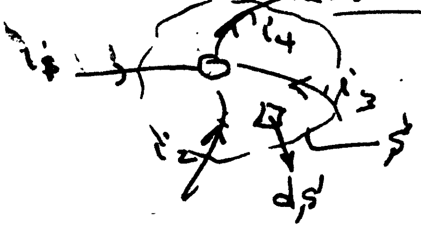


Current Source





$i = -i_s$
explain by motion of charges



$$\oint_S J \cdot ds = 0 = -i_1 - i_2 + i_4 - i_3 = 0$$

outward

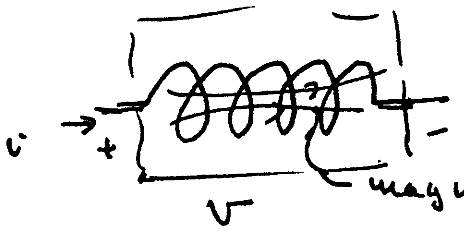
KCL

1/22/93

④

NEW ELEMENTS - Chapter 7 pps 227 - 240 ~~of text~~.

① The inductor



The terminal relationship is based upon Maxwell's equation

$$\oint \underline{E}\cdot d\underline{l} = -\frac{d}{dt}\oint_S \underline{B}\cdot d\underline{s}$$

$B \sim i$ ~~where~~

$\underline{B} = \underline{k}_L \, i$ geometric vector

$$-v = -\frac{di}{dt}\boxed{\oint_S \underline{k}_L \cdot d\underline{s}}$$

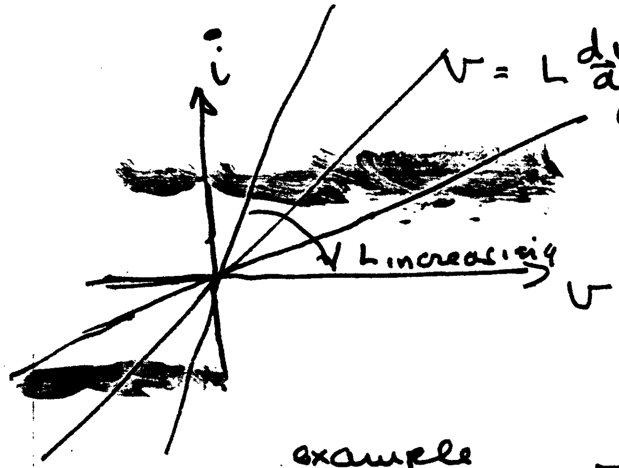
→ geometry, not current, nor time varying

↳ this is $\underline{\underline{\text{inductance}}}$, L (henries)

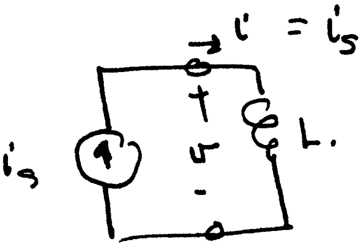
$v = L\frac{di}{dt} + i\frac{dL}{dt} \rightarrow 0$ for this course.

we cannot plot (i, v) except for specific cases.

we can plot i, v



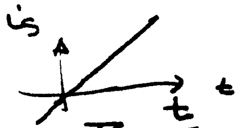
example



constant

$i_s = I_s \neq I_s(t)$

$v = 0$



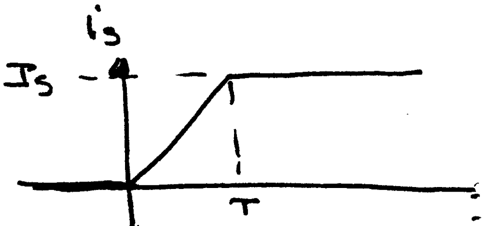
$i_s = I_s \frac{t}{T}$

$$\frac{di}{dt} = \frac{di_s}{d} = \frac{d(I_s/T)t}{dt} = \frac{I_s}{T}$$

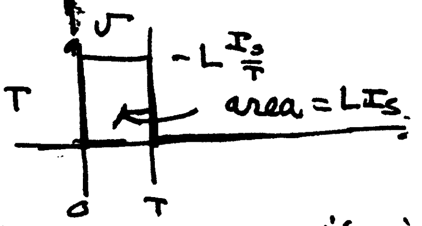
hence $v = \frac{L I_s}{T}$

11/22/93

$$i_s = \begin{cases} 0 & t < 0 \\ I_s \, t/T & 0 \le t \le T \\ I_s & T < t \end{cases}$$

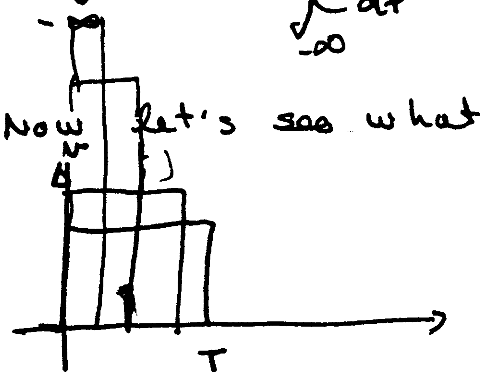


$$v = L\frac{di}{dt} = L\frac{di_s}{dt} = \begin{cases} 0 & t<0 \\ L\frac{I_s}{T} & 0 \le t \le T \\ 0 & 0 < t \end{cases}$$



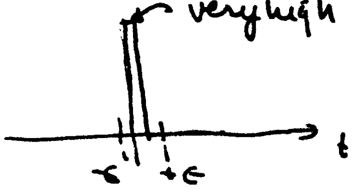
$$\int_{-\infty}^{t} v\,dt = \int_{-\infty}^{t} \left(L\frac{di}{dt}\right) dt = \int_{-\infty}^{t} L\,di = L\,i(t)$$ assumes $i(-\infty) = 0$

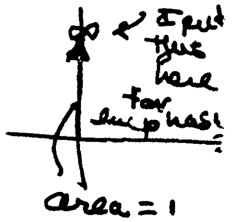
Now let's see what happens as T gets smaller.



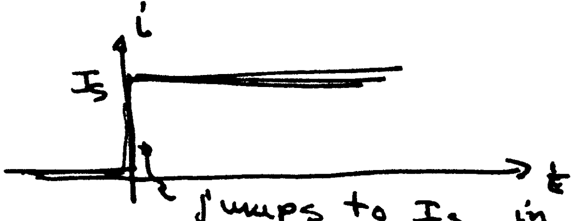
even as $T \to 0$ the area must be LI_s, so the height must go to ∞ that is, infinite volts for an infinitesmal time but the area is constant (LI_s). The function which has zero width and ~~finite~~ unit area (infinite height) is called an impulse $\delta(t)$

$$\int_{t=-\epsilon}^{t=+\epsilon} \delta(t)\,dt = 1$$ as $\epsilon \to 0$

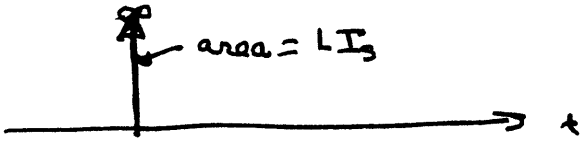


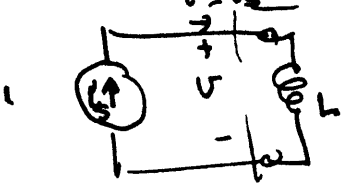


So if



jumps to I_s in zero time ($T \to 0$) at t=0



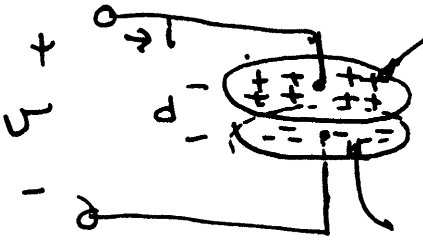


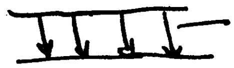
let $i_s = I_s \cos\omega t$ for all time

$$v = L\frac{di_s}{dt} = -\omega L I_s \sin\omega t$$

while its current varies as a cosine its voltage varies as a sine

1/22/93 The capacitor.





~~maxwell's continuity equation~~

$$\cancel{\oint_S J \cdot dS + \frac{d}{dt}\int \epsilon E \cdot d}$$

Total Charges on a surface are proportional to the voltage.

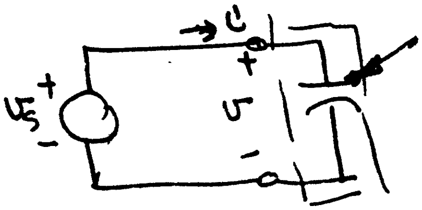
$q = C v$ the constant of proportionality is the capacitance. But we know $i = \frac{dq}{dt}$

So $i = \frac{d(CV)}{dt} = c\frac{dv}{dt} + v\frac{dc}{dt}$ by chain rule.

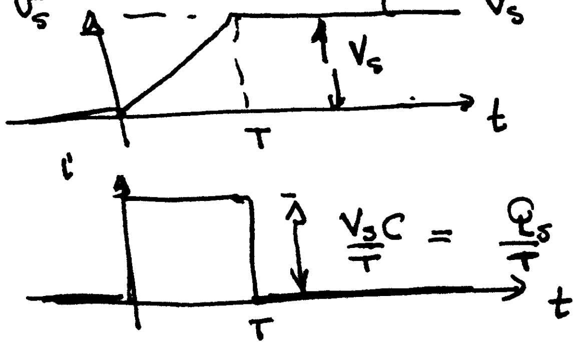
if the capacitance is not time varying (fixed geometry) then

ended here on 1/22 $i = C\frac{dv}{dt}$ which works for this course

units are farads



$$v = v_s(t) = \begin{cases} 0 & t < 0 \\ V_s\, t/T & 0 \le t \le T \\ V_s & t > T \end{cases}$$

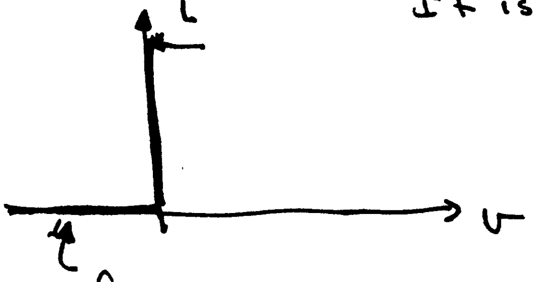


remember dimensions i = charge/time $\left[\frac{\text{coulombs}}{\text{sec}}\right]$

1/22/93 Another new element - a diode (ideal for now

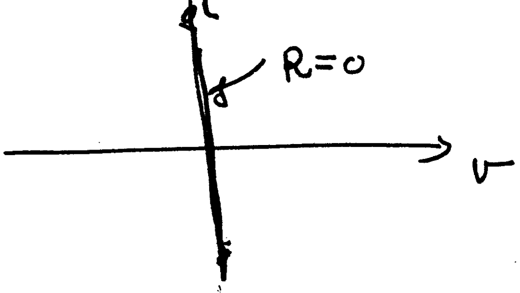
This is our first non-linear element.

It is defined by the v-i characterist



$i = 0$ for $v < 0$;

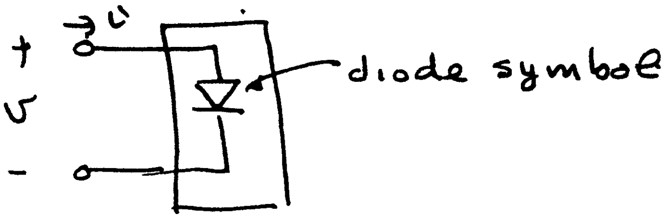
i can be anything for $v = 0$, note that it won't allow v to be non-zero. what does this remind us of ?



so for any attempt to get $v > 0$ it becomes a short circuit! in a sense, the diode is a voltage controlled resistor

$R = \infty$ $v < 0$

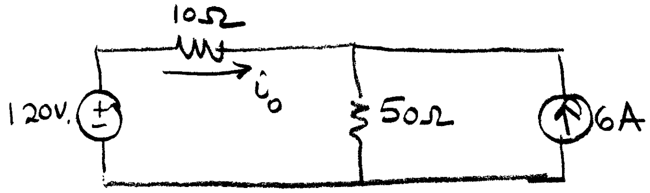
$R = 0$ $v = 0$. Again, it won't let v be non-zero positive like a switch



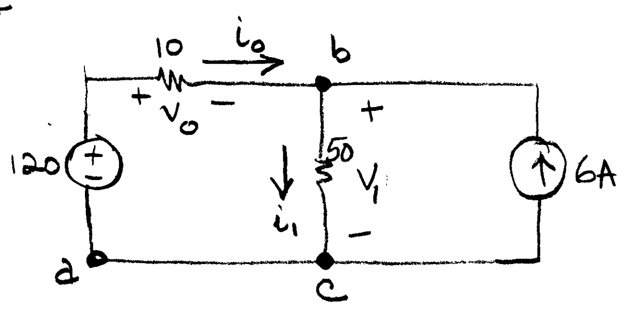
Example 2.8

(a) Use Kirchoff's laws and Ohm's Law to find i_o in the circuit shown in Fig. 2.20.

(b) Test the solution by verifying that the total power generated equals the total power dissipated



redraw



in PSpice you number the reference node (usually ground) as zero. All others are positive numbers

How many unknowns: i_o, i_1

V_o & V_1 are known thru Ohm's Law

need two equations in two unknowns

KCL at either c or b: at b using + in

$\sum i = 0$ $\qquad i_o - i_1 + 6 = 0$

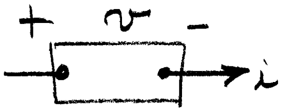
need second equation

KVL around a c b ↻

$\sum v = 0$ $\qquad -120 + V_o + V_1 = 0$

$-120 + i_o 10 + 50 i_1 = 0$

$$\left.\begin{aligned} i_o - i_1 &= -6 \\ 10 i_o + 50 i_1 &= +120 \end{aligned}\right\}$$ 2 equations in two unknowns



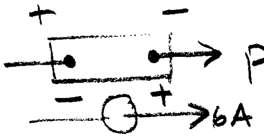
passive convention

$i_o = -3A$ $\qquad i_1 = 3A.$

$P_{120V} = -vi = -(120V)(-3) = +360W.$ power absorbed.

↑ opposite to correct passive convention

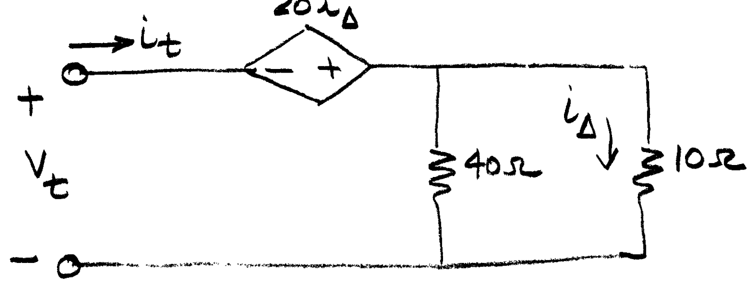
$v_1 = 50 i_1 = 150V.$

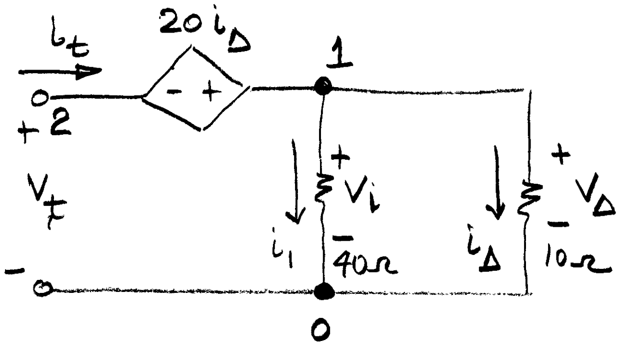


$P_{6A} = -(v)(i) = -(150)(6) = -900W$ calculate others the same way.

Drill 2.6

In section 2.2 we mentioned that a negative resistance can appear in the circuit model of a device. With this thought in mind, verify that the circuit inside the dashed box can be modeled with a negative resistance.





at node 1:

\+ going in use KCL $+i_t - i_1 - i_\Delta = 0$ (1)

using KVL around (clockwise) outside loop.

$$-V_t - 20 i_\Delta + V_\Delta = 0. \quad (2)$$

Solve (2) for V_t and using Ohm's Law $V_t = -20 i_\Delta + V_\Delta = -20 i_\Delta + 10 i_\Delta = -10 i_\Delta$

now, using (1) $i_t = i_1 + i_\Delta$ ③

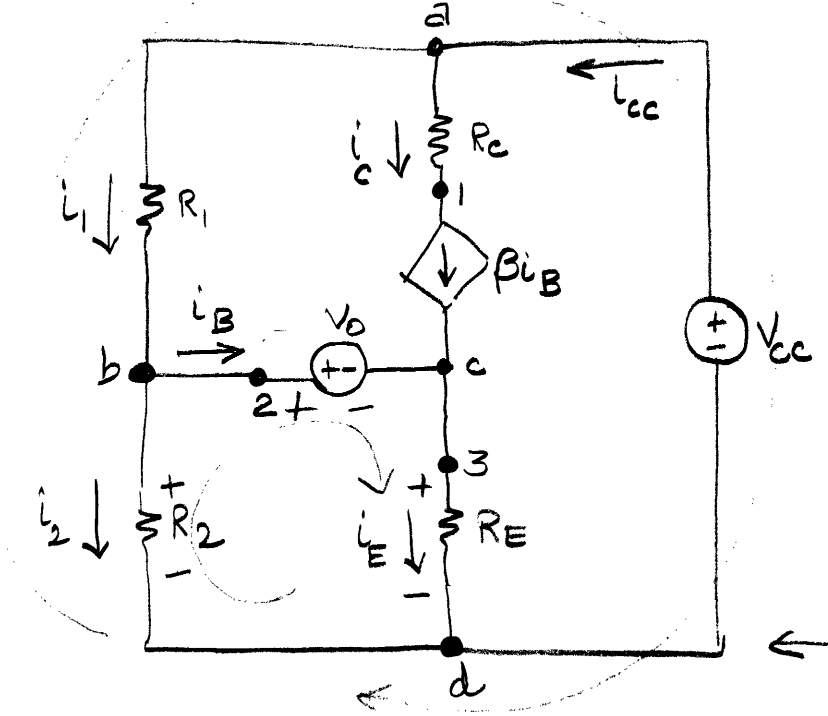
$V_1 = V_\Delta$ ← this point is not made clearly enough in textbook V_1 and V_Δ in parallel

using Ohm's Law $i_1 40 = i_\Delta 10$

Now use this for i_1 in ③ $i_t = i_1 + i_\Delta = i_\Delta \left(\frac{10}{40}\right) + i_\Delta = 1.25 i_\Delta$

at terminals $R_t = \frac{V_t}{i_t} = \frac{-10 i_\Delta}{1.25 i_\Delta} = -8\Omega$

2.5 Analysis of a circuit containing a dependent source



These are consequences of Kirchoff's Laws.

① currents thru things in series are the same

② voltages across things in parallel are the same.

this would be node Ø in PSpice

all resistors V_{cc}, V_O and β are known.

use **currents** as your unknowns : 6 unknowns

KCL @ a,b,c use out as positive

a: $+i_1 + i_c - i_{cc} = 0$ (1)

b: $-i_1 + i_2 + i_B = 0$ (2)

c: $-i_B - i_c + i_E = 0$ (3)

3 equations in 6 unknowns.

controlled sources always produce constraints

$$i_c = \beta i_B \quad (4)$$

Now use KVL to get two more equations

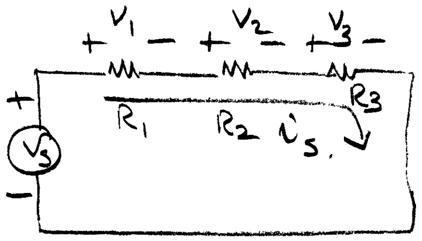
DO NOT DO KVL across a current source !

bcdb: $-i_2 R_2 + V_O + i_E R_E = 0$

badb: $-i_2 R_2 - i_1 R_1 + V_{cc} = 0$

Solve: $$i_B = \frac{\dfrac{V_{cc} R_2}{R_1 + R_2} - V_O}{\dfrac{R_1 R_2}{R_1 + R_2} + (1+\beta) R_E}$$

3.1 Resistors in Series

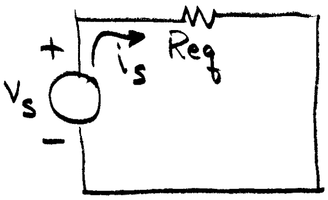


know currents will be the same

voltage is known by Ohm's Law.

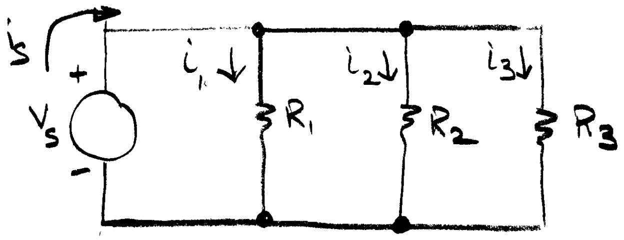
KVL $\quad -V_s + i_s R_1 + i_s R_2 + i_s R_3 = 0.$

$$V_s = i_s(R_1 + R_2 + R_3) = i_s R_{eq}$$



$$R_{eq} = \sum_{i=1}^{k} R_i$$

3.2 Resistors in parallel



know voltages across R_1, R_2, R_3 will be the same

from ohm's law

$$V_s = V_1 = V_2 = V_3$$

$$V_s = i_1 R_1 = i_2 R_2 = i_3 R_3$$

$$i_1 = \frac{V_s}{R_1} \quad i_2 = \frac{V_s}{R_2} \quad i_3 = \frac{V_s}{R_3}$$

using KCL (+ out) $\quad -i_s + i_1 + i_2 + i_3 = 0$

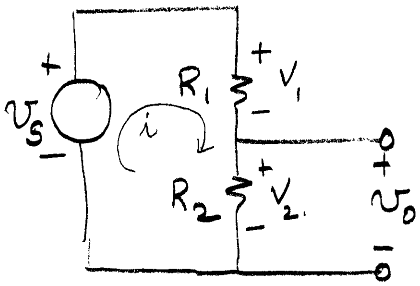
$$i_s = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3} = V_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{1}{R_{eq}} = \frac{i_s}{V_s} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Very important special case, 2 resistors in parallel

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}.$$

3.3 Voltage Divider Circuit



assume clockwise current i

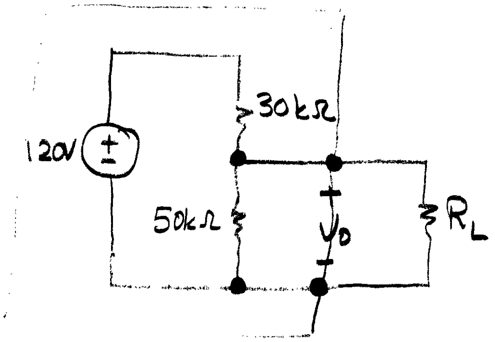
$$-v_S + V_1 + V_2 = 0$$

$$-v_S + iR_1 + iR_2 = 0$$

$$i = \frac{v_S}{R_1 + R_2}$$

$$v_0 = V_2 = iR_2 = \frac{R_2}{R_1+R_2} v_S \quad \blacksquare$$

Drill Exercise 3.2

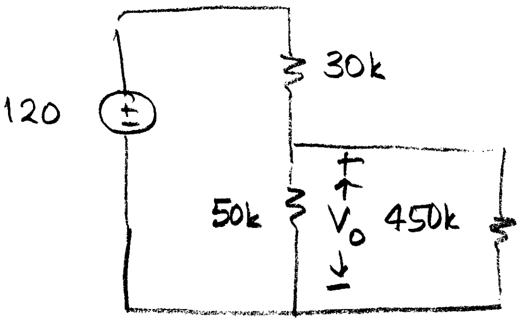


(a) Find the no-load value of v_o in the circuit shown.

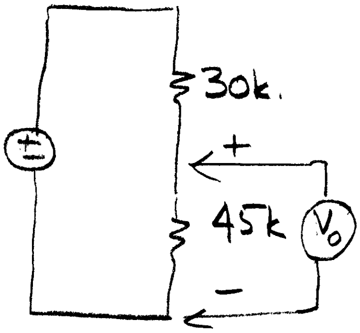
By inspection it's a voltage divider

$$v_o = \frac{50\,k}{30k+50k}\,120 = \frac{5}{8}(120) = 75V.$$

(b) Find v_o when R_L is 450 kΩ.



⇒



$$R = \frac{50 \cdot 450}{50+450} = 45$$

Now use voltage divider

$$V_o = \frac{45k}{45k+30k}(120) = 72\,\text{Volts} \quad \text{lower.}$$

Chapter 4 **Reading** 4.1, 4.2 (intro only), 4.5 (4.6, 4.7?)
4.8 4.9, 4.10, 4.11
Thevenin & Norton
4.13 superposition

4.1 terminology

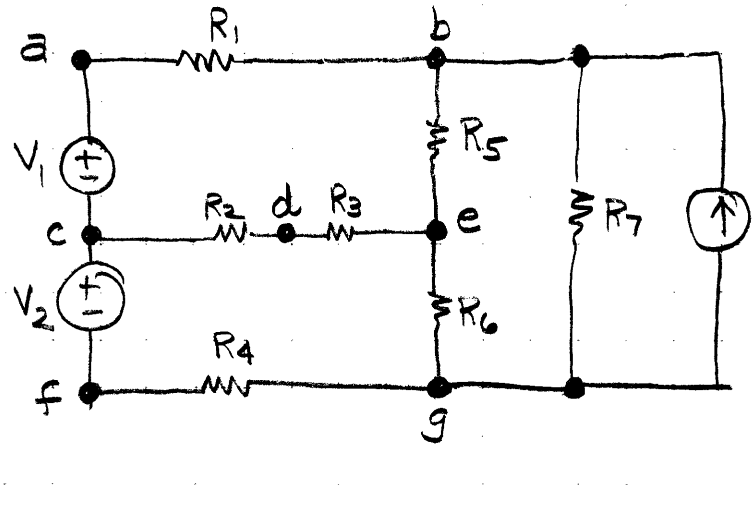
nodes – point where 2 or more circuit elements join

path – formed when a set of adjoining basic circuit elements is traced, in order, without passing throuch a connecting node more than once.

branch – path that connects two nodes

mesh – special type of loop that doesn't contain any other loops within it.

closed path (loop) – start at a selected node, trace a set of connected basic circuit elements and return to the original starting node without passing through any intermediate node more than once.



seven nodes: a, b, c, d, e, f, g.
ten branches: v_1, R_1, R_2, R_3, v_2, R_4, R_5, R_6, R_7, I
four meshes: $v_1 - R_1 - R_5 - R_3 - R_2$, $v_2 - R_2 - R_3 - R_6 - R_4$
$R_5 - R_6 - R_7$, $R_7 - I$
also several loops that are not meshes $v_1 - R_1 - R_5 - R_6 - R_4 - v_2$, etc.

essential nodes → connect three or more elements
i.e. c and b

essential branches → connect essential nodes without passing through essential nodes.
i.e. $v_1 - R_1$

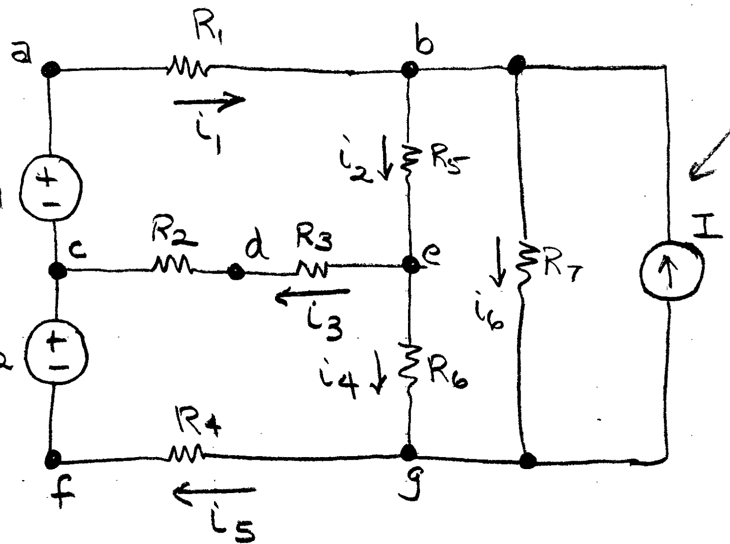
\# of nodes = n
\# of branches = b

simultaneous equations
KCL to $n-1$ junctions to get $n-1$ equations

KVL to $b-(n-1)$ **meshes** to get $b-(n-1)$ independent eqns.

illustration of the systematic approach.

Fig. 4.4.



4.7 more about current sources!

four essential nodes
b, c, e, g

six essential branches
$V_1 - R_1$
$R_2 - R_3$
$V_2 - R_4$
R_5
R_6
R_7 (I)

6 unknowns — correspond to branches.

apply KCL at nodes b, c, e 3 essential nodes.

+ out @b $-i_1 + i_2 + i_6 - I = 0$
+ out @c $i_1 - i_3 - i_5 = 0$ can only do n-1
+ out @e $i_3 + i_4 - i_2 = 0$.

now derive loop equations using KVL need 3 can't use R_7-I.

abec $R_1 i_1 + R_5 I_2 + i_3 (R_2 + R_3) - v_1 = 0$
cegf $- i_3 (R_2 + R_3) + i_4 R_6 + R_4 i_5 - V_2 = 0$
bg. $- i_2 R_5 + i_6 R_7 - i_4 R_6 = 0$.

re-arrange in form of matrix

$$\begin{array}{cccccccc}
-i_1 & i_2 & - & - & - & i_6 & = & I \\
i_1 & & -i_3 & & -i_5 & & = & 0 \\
& -i_2 & +i_3 & i_4 & & & = & 0 \\
R_1 i_1 & R_5 i_2 & (R_2+R_3) i_3 & & & & = & v_1 \\
& & -(R_2+R_3) i_3 & R_6 i_4 & R_4 i_5 & & = & v_2 \\
& R_5 i_2 & & -R_6 i_4 & & R_7 i_6 & = & 0.
\end{array}$$

6 equations in 6 unknowns.

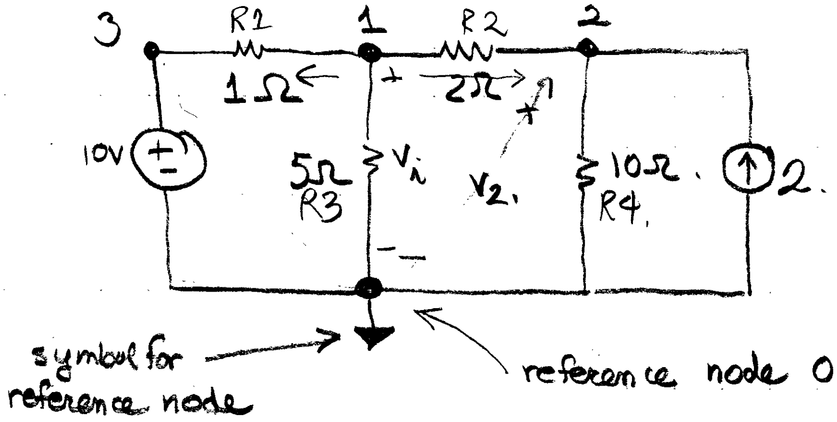
new methods allow us to use fewer equations:
specifically $n-1$ equations by using new variables

node-voltage $n-1$ equations

mesh-current $b-(n-1)$ equations.

How computer analysis works

4.2 Node-voltage method. (refers to KCL at each node).



1. draw a planar circuit with no branches crossing
2. label reference node as zero
3. label all other nodes

   all nod voltages are with respect to this reference node.

4. apply KCL at every essential node.

at node 1
+ out

$V_1 < V_2$.

$$\frac{V_1 - 10}{1} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = 0$$

at node 2
+ out

$$\frac{V_2 - V_1}{2} + \frac{V_2}{10} - 2 = 0$$

$$10V_1 - 100 + 2V_1 + 5V_1 - 5V_2 = 0.$$

$$5V_2 - 5V_1 + V_2 - 20 = 0.$$

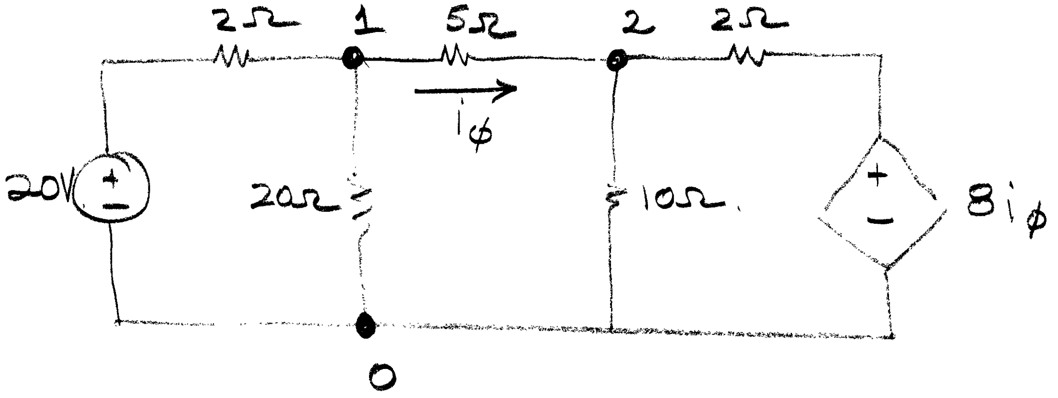
$$17V_1 - 5V_2 = 100$$
$$-5V_1 + 6V_2 = 20$$

according to book: $V_1 = \frac{100}{11} = 9.09V$

$$V_2 = \frac{120}{11} = 10.91V.$$

Node voltage and different sources:

how to add dependent sources to node-voltage analysis



at 1:
+ out

$$\frac{v_1-20}{2}+\frac{v_1}{20}+\underbrace{\frac{v_1-v_2}{5}}_{i_\phi}=0.$$

at 2
+ out.

$$\frac{v_2-v_1}{5}+\frac{v_2}{10}+\frac{v_2-8i_\phi}{2}=0.$$

opposite

2 eqns in 3 unknowns v_1, v_2, i_ϕ

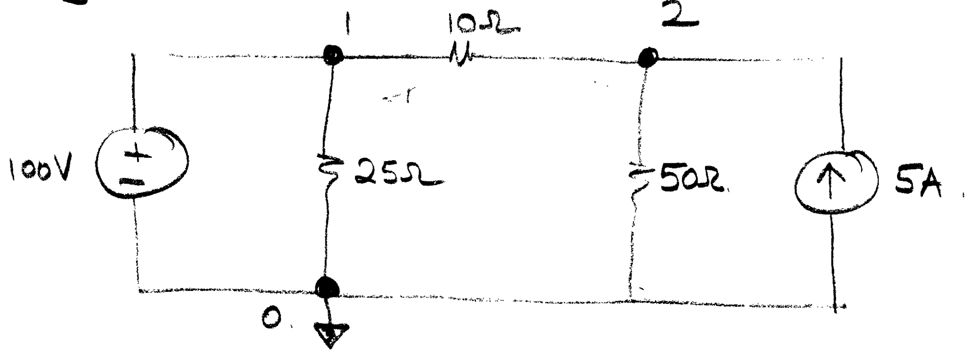
need source equation to finish

$$i_\phi=\frac{v_1-v_2}{5}$$

add a constraint equation.

special problems: voltage sources..

only a voltage source in branch between 0 and 1.



problem: what is current from voltage source.
solution: write as difference of voltages
look at node 2: + out

$$\frac{v_2 - v_1}{10} + \frac{v_2}{50} - 5 = 0.$$

assume $v_2 > v_1$
then use
passive current convention which is + $\leftarrow$

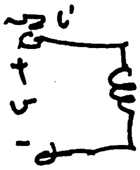
then looking at circuit shows $v_1 = 100V$ so no problem.

$\Rightarrow$ voltage sources reduce # of equations

LAB ANNOUNCEMENTS

1. KITS SHOULD BE COMPLETED & TESTED BY FEB 3,
2. LAB 1 DU 2/5
3. DO NOT DIG INTO THE EQUIPMENT - NOR MOVE IT!

INDUCTOR

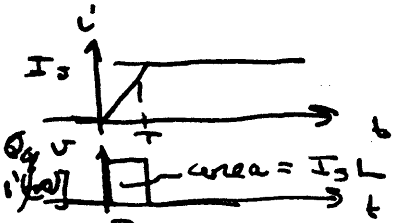


$v = L\frac{di}{dt}$

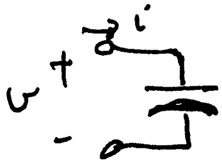
$\int_{-\infty}^{t} v\,dt = \int_{-\infty}^{t} L\,di$

Area = $L[i(t) - \cancel{i(-\infty)}]$

so $t > T$ = LI_s



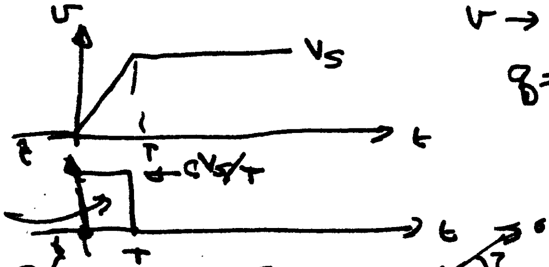
CAPACITOR



$i = C\frac{dv}{dt}$ (note analogy

$v \rightarrow i$, $i \rightarrow v$, $L \rightarrow$

$q = CV \rightarrow \underbrace{Li}_{\text{flux linkage}}$

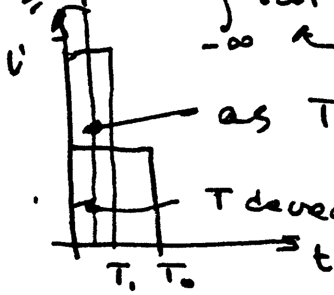


area = $\int_{-\infty}^{t} i\,dt$ [

$C\int_{-\infty}^{t} dv = C[v(t) - \cancel{v(-\infty)}]$

For $t \geq T$ $\int_{-\infty}^{t} i\,dt = CV_s$ a constant

← what is this? Q

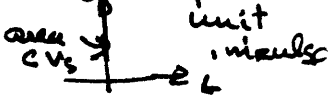


as $T \rightarrow 0$ area is still CV

T decreasing

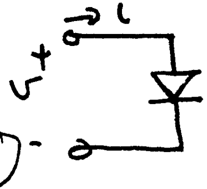
$i \rightarrow CV_s\,\delta(t)$

unit impulse

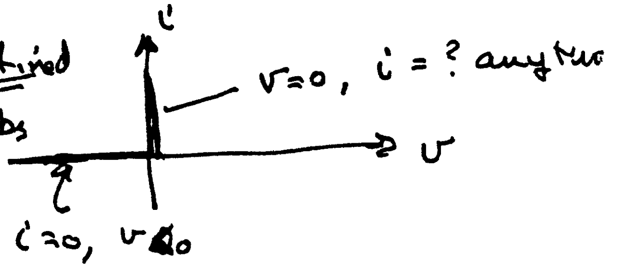


New Element

IDEAL DIODE



defined by



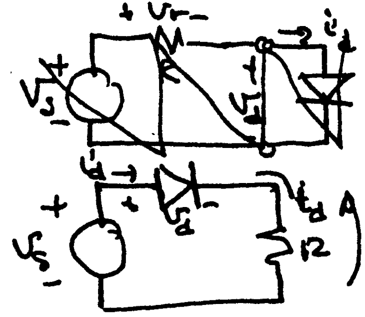
NON-LINEAR ELEMENT

~~It is like~~ a voltage controlled switch.

NOTE arrow points in direction of possible current flow

1/27/93

let's examine this circuit



$v_R = i_d R$

KVL ⇒ $v_s - i_d R - v_d = 0$

Σ voltage rises = 0

$v_d = v_s - i_d R$

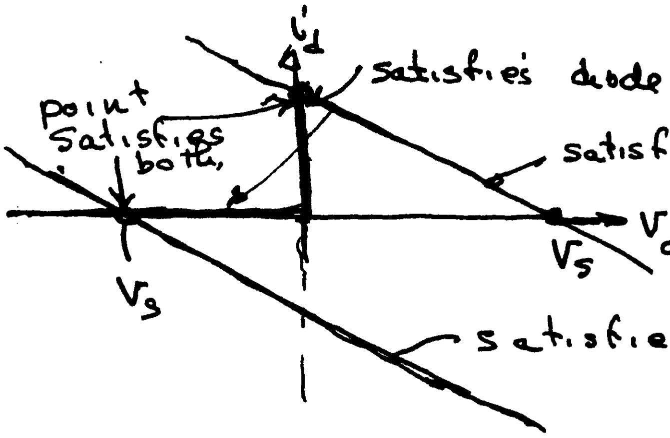
if $v_s > 0$ then current can flo

$0 = v_s - i_d R$, $v_R = v_s$

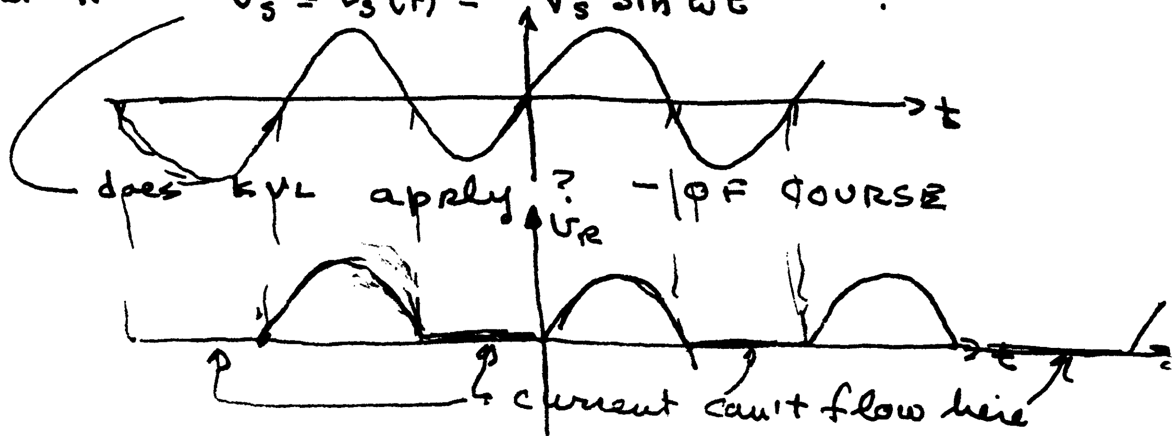
if $v_s < 0$ no current can flow

$v_d = v_s$ ($v_s < 0$)

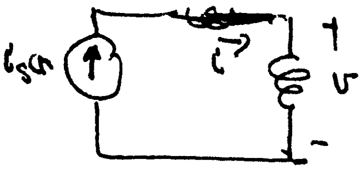
and $v_R = 0$



what if $v_s = v_s(t) = V_s \sin \omega t$?

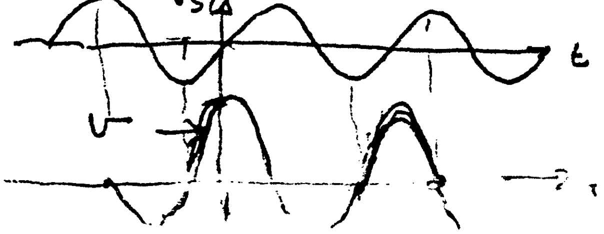


Now that we've introduced a time-varying source let's examine the case for a current source $i_s(t) = I_s \sin \omega t$ and an inductor, L.



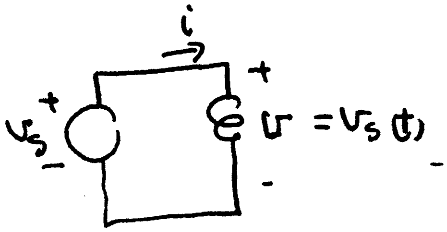
$$v = L\frac{di}{dt} = L\frac{di_s}{dt}$$

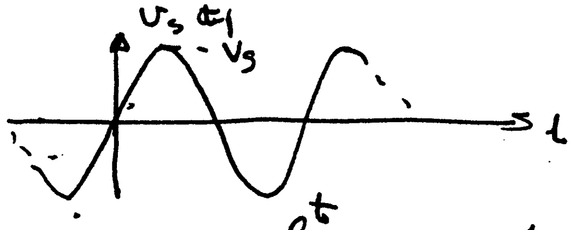
$$v = \omega L I_s \cos \omega t$$



1/27/93

Also





$v = L\frac{di}{dt}$, $\int_{-\infty}^{t} v\,dt = L\,i(t)$

$$\frac{1}{L}V_s\int^{t}\sin\omega t\,dt = i(t)$$

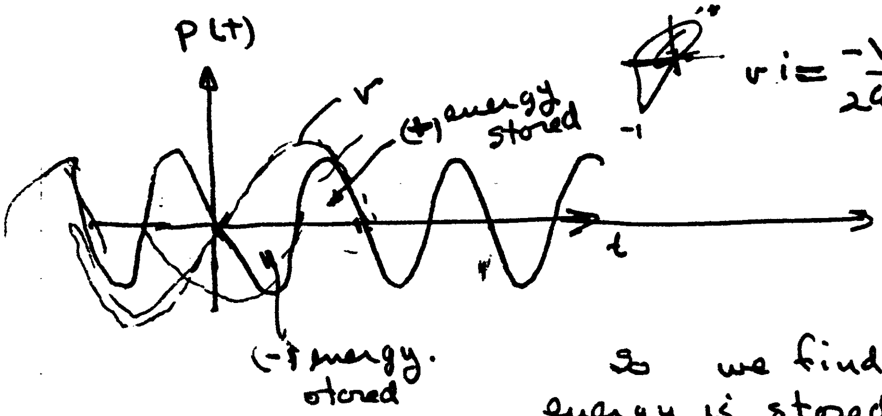
$$-\frac{V_s}{\omega L}\cos\omega t = i(t)$$

check it $L\frac{d}{dt}\left(-\frac{V_s}{\omega L}\cos\omega t\right) = V_s\sin\omega t$ OK

Power into inductor $= p(t) = v\,i = V_s\sin\omega t \times \left(-\frac{V_s}{\omega L}\cos\omega t\right)$

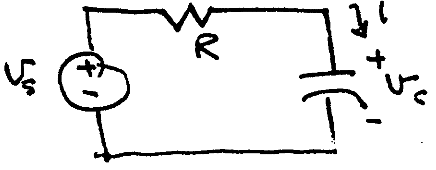
$$\sin A\cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$$

$$v\,i = \frac{-V_s^2}{2\omega L}\left[\sin 2\omega t\right]$$



So we find that alternatively energy is stored then returned to the source

Source power input = ⊖ $V_s i$ (sign convention)



KVL $V_s - iR - V_c = 0$
.. and $i = C\frac{dV_c}{dt}$ so we arrive at a single equation for V_c in terms of V_s, C, and R

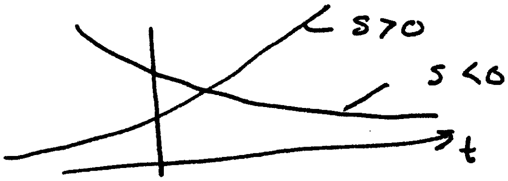
$V_s - RC\frac{dV_c}{dt} - V_c = 0$ ← V_c is the unknown

$RC\frac{dV_c}{dt} + V_c = V_s$ ← source

called a linear differential equation.
with constant coefficients

1/27/93

How to solve? Let's use a source that is $V_S(t) = V_s e^{st}$



note that if we guess that $v_c = \overset{\text{unknown constant (not function of t}}{V_c} e^{st}$ as well then substitute

$$RCsV_c e^{st} + V_c e^{st} = V_s e^{st}$$

Notice e^{st} factor throughout.

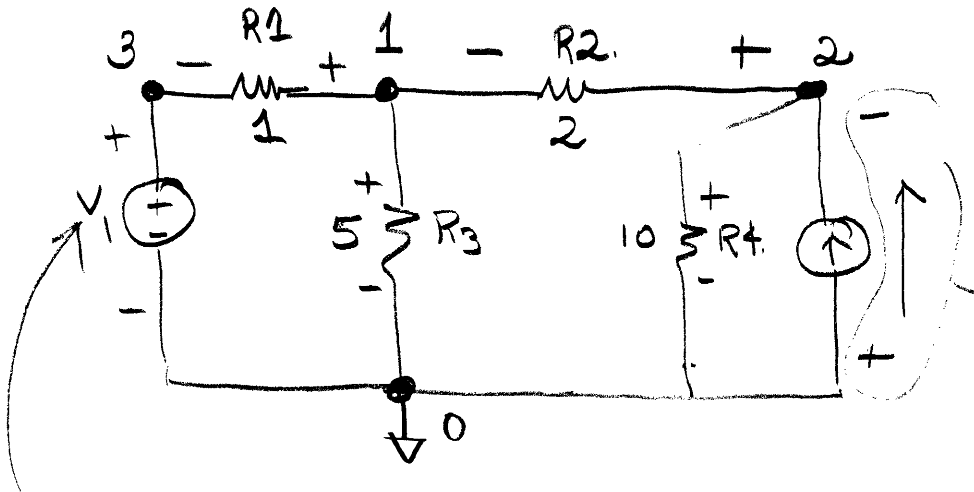
An exponential is returned when a derivative (or integral) is taken and this is very important $\frac{d}{dt}(e^{st}) = s\, e^{st}$ same time dependence.

$$\underbrace{[(RCsV_c + V_c) - V_s]}_{\text{this must be zero}} e^{st} = 0 \quad \text{if this is non-zero then}$$

$$V_c = \left[\frac{V_s}{1+RCs}\right]$$ creates a good solution - check by substitution

$$RCs\left[\frac{V_s}{1+RCs}\right]e^{st} + \frac{V_s}{1+RCs}e^{st} = \underset{\text{OK.}}{V_s e^{st}} \quad !!$$

```
Problem 4.2
V1 3 0 DC 10
I1 0 2 DC 2
R1 1 3 1
R2 2 1 2
R3 1 0 5
R4 2 0 10
.PROBE
.END
```



whenever Pspice prints a current it measures it <u>through</u> a device, i.e. passive sign convention

V1 ordinary convention

calculated

V(1) = +9.0909

V(2) = +10.9090

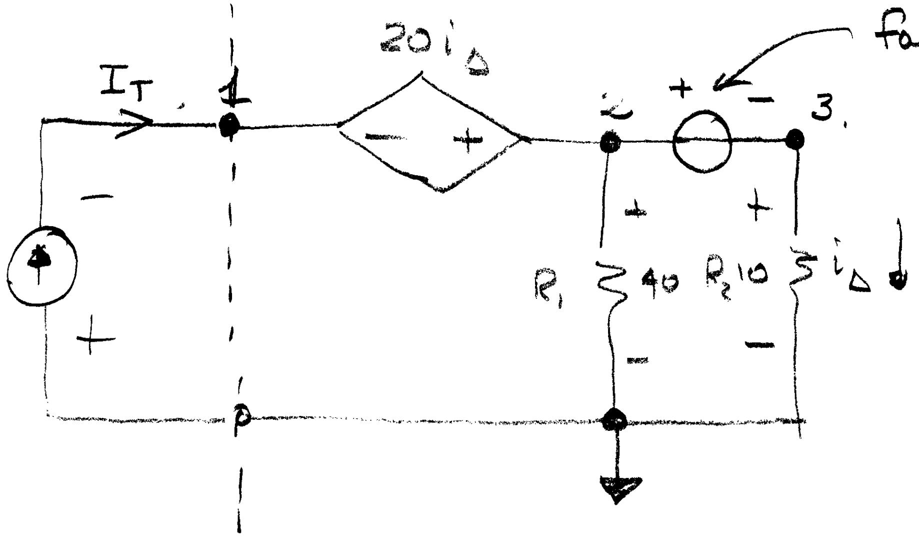
V(3) = +10.0000

V1 current = −9.091E−01.

```
**** 01/27/93 17:54:08 *********** Evaluation PSpice (Jan 1992) **************

 Problem 4.2


 ****     CIRCUIT DESCRIPTION


******************************************************************************



V1 3 0 DC 10
I1 0 2 DC 2
R1 1 3 1
R2 2 1 2
R3 1 0 5
R4 2 0 10
.PROBE
.END

**** 01/27/93 17:54:08 *********** Evaluation PSpice (Jan 1992) **************

 Problem 4.2


 ****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C


******************************************************************************



 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE


(    1)    9.0909  (    2)   10.9090  (    3)   10.0000




    VOLTAGE SOURCE CURRENTS
    NAME         CURRENT

    V1           -9.091E-01

    TOTAL POWER DISSIPATION   9.09E+00  WATTS


         JOB CONCLUDED

         TOTAL JOB TIME            .72
```

$R = \frac{V}{I}$.

Problem: want to know It.



fake zero value voltage source V_{delta} ie. a volt meter.

P SPice Program.

```
* Equivalent Resistance.
I1 0 1   DC   1A.
H1   2   0   Vdelta   20
Vdelta  2   3   DC   0
R1      2   0   40
R2      3   0   10
.END.
```

fake → see p.10.

called transresistance

this is a current controlled voltage source.

```
**** 01/27/93 18:19:58 *********** Evaluation PSpice (Jan 1992) **************

 * Equivalent Resistance calculation


 ****     CIRCUIT DESCRIPTION


******************************************************************************



I1     0     1     DC     1A
H1     2     1     Vdelta  20
Vdelta  2     3     DC     0
R1     2     0     40
R2     3     0     10
.END
```

ADD .DC I1 0.1 20 0.1

start stop increment

to sweep values.

```
**** 01/27/93 18:19:58 *********** Evaluation PSpice (Jan 1992) **************

 * Equivalent Resistance calculation


 ****     SMALL SIGNAL BIAS SOLUTION     TEMPERATURE =   27.000 DEG C


******************************************************************************



 NODE   VOLTAGE    NODE   VOLTAGE    NODE   VOLTAGE    NODE   VOLTAGE


(   1)  -8.0000 (   2)   8.0000 (   3)   8.0000




    VOLTAGE SOURCE CURRENTS
    NAME         CURRENT

    Vdelta       8.000E-01

    TOTAL POWER DISSIPATION   0.00E+00  WATTS


          JOB CONCLUDED

          TOTAL JOB TIME            .83
```

Note I1 was 1A

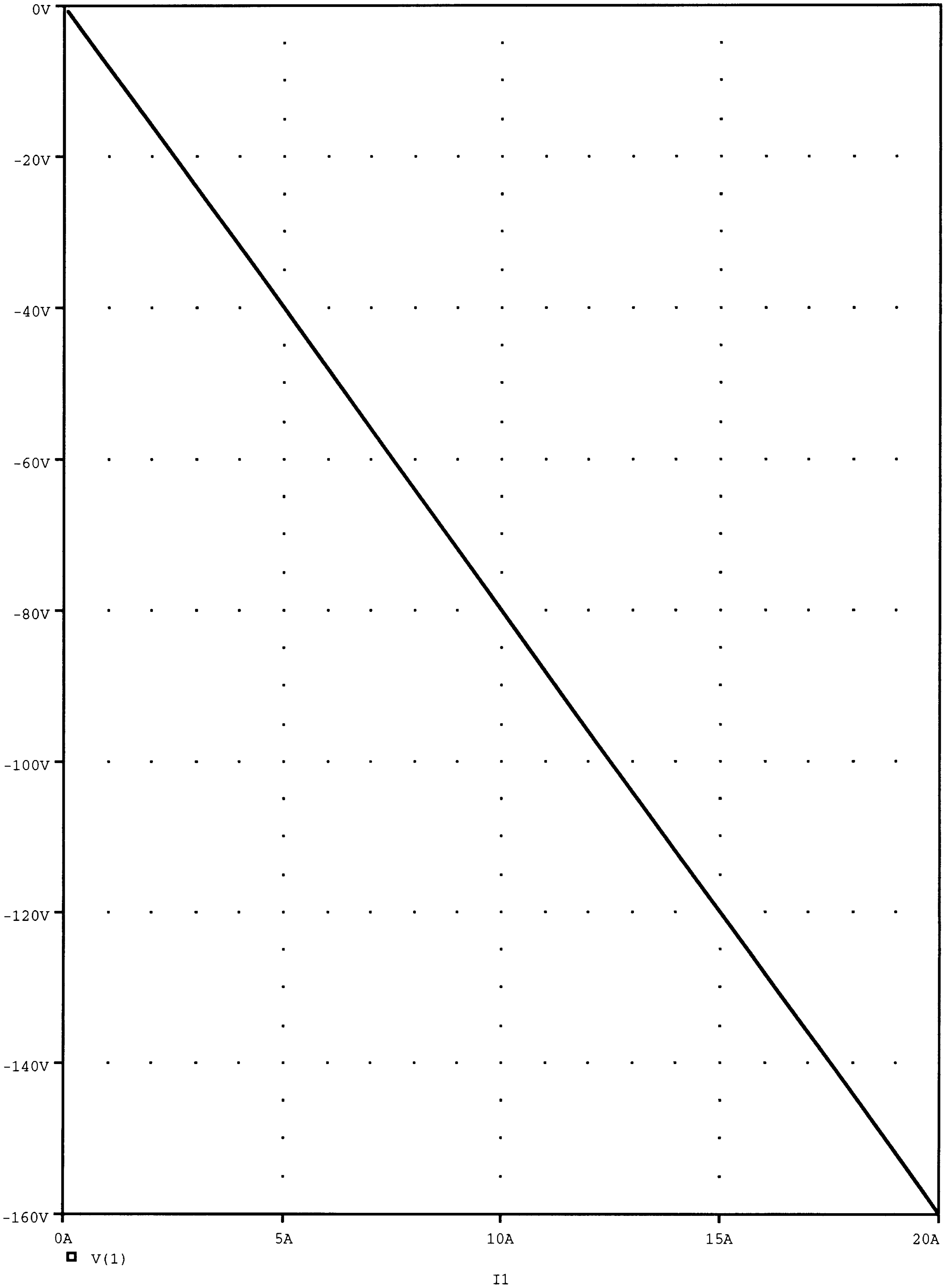
$V(1) = -8$

$$R_{in} = \frac{V(1)}{I(1)} = -8$$

```
**** 01/27/93 18:27:07 *********** Evaluation PSpice (Jan 1992) **************

 * Equivalent Resistance calculation


 ****     CIRCUIT DESCRIPTION


******************************************************************************



I1      0      1      DC      1A
H1      2      1      Vdelta  20
Vdelta  2      3      DC      0
R1      2      0      40
R2      3      0      10
.DC     I1     0.1    20      0.1
.PROBE
.END

        JOB CONCLUDED

        TOTAL JOB TIME          5.37
```

← now with DC sweep.

← for probe.



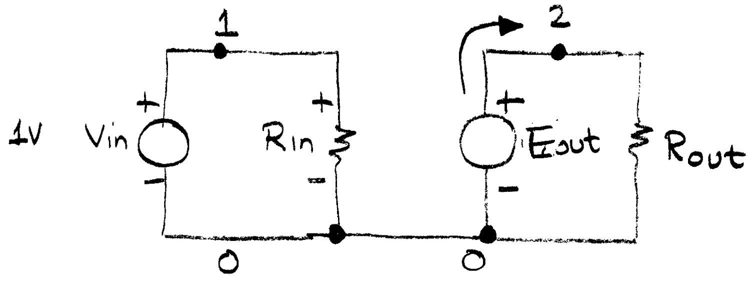
"* Equivalent Resistance calculation" 01/27/93 18:27:07 27.0°
Evaluation Probe 5.1 © 1992 MicroSim Corp.

Ref. Tuinenga p. 37

```
* simple gain circuit
Vin   1   0   DC   1 volt
Rin   1   0   1 ohm
Eout  2   0   1   0   5.0
Rout  2   0   1 ohm
.PROBE
.TF  V(2)  Vin
.END
```

gain

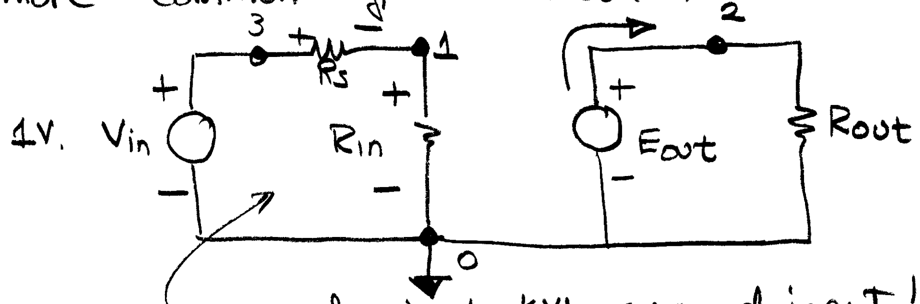
current sources use a reverse notation



Exx voltage controlled voltage source

E	VCVS
F	CCCS
G	VCCS
H	CCVS

more common type of circuit:

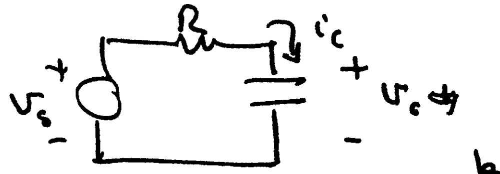


in analysis do KVL around input loop.

change Vin 3 0 DC 1 volt.
add Rs 3 1 600 ohms.

1/28/93 $\qquad$ unknowns i_c, v_c but $i_c = C\frac{dv_c}{dt}$

we were examining the circuit



KVL $\Rightarrow$ $v_s - i_c R - v_c = 0$

but we know that $i_c = C\frac{dv_c}{dt}$ for the capacitor thus we wind up with

$$RC\frac{dv_c}{dt} + \overset{\text{response}}{v_c} = \overset{\text{source}}{v_s(t)}$$

Suppose that $v_s = \begin{cases} 0 & t \leq 0 \\ \alpha_1 + \beta_1 t + \gamma_1 t^2 & t \geq 0 \end{cases}$

Then we assume a solution of the same form as the source

$$v_c(t) = \begin{cases} 0 & \\ \alpha_2 + \beta_2 t + \gamma_2 t^2 & t \geq 0 \end{cases}$$

and substitute

$$RC\left[\beta_2 + 2\gamma_2 t\right] + \alpha_2 + \beta_2 t + \gamma_2 t^2 = \alpha_1 + \beta_1 t + \gamma_1 t^2$$

and equate coefficients of like powers of t

hence
$$RC\beta_2 + \alpha_2 = \alpha_1$$
$$2RC\gamma_2 + \beta_2 = \beta_1$$
$$\gamma_2 = \gamma_1$$

then
$$\beta_2 = \beta_1 - 2RC\gamma_1$$
$$RC\left[\beta_1 - 2RC\gamma_1\right] + \alpha_2 = \alpha_1$$

or
$$\alpha_2 = \alpha_1 - RC\left[\beta_1 - 2RC\gamma_1\right]$$

But this was an easy example since it involved a "power-series" for the time functions — NOT VERY REALISTIC.

As we shall see later on, we can very often express ~~signals~~ complex, but important, signals as exponentials. One thing we know is that

$$\mathrm{Cos}\,\omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2} .$$

Let's use the funny signal $\bar{V}_s e^{j\omega t}$ as a source — we shall see shortly that this is a very useful way of writing sinusoidal signals. Then from our previous example — no way can we match the source and response — unless the response varies as $\bar{V}_c e^{j\omega t}$. So let's assume this as a response

$$RCj\omega \bar{V}_c e^{j\omega t} + \bar{V}_c e^{j\omega t} = \bar{V}_s e^{j\omega t}$$

now for every time this equation can be true if

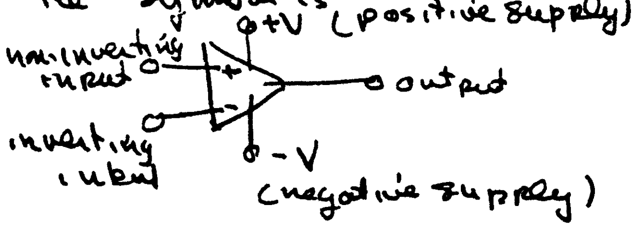
$$\bar{V}_c \left[RCj\omega + 1\right] = \bar{V}_s$$

$$\text{or} \quad \bar{V}_c = \frac{\bar{V}_s}{\left[1 + RCj\omega\right]}$$

This satisfies KVL (substitute and see). we shall return to this another time.

Chapter 6 Another new element - THE OPERATIONAL AMPLIFIER (OP-AMP for short). This device is ~~not~~ formed ~~at all a single~~ not by a single simple element but ~~thousands~~ hundreds of elements within a capsule — a chip. The first solid-state one was introduced in 1968. - μA 741 and is still manufactured

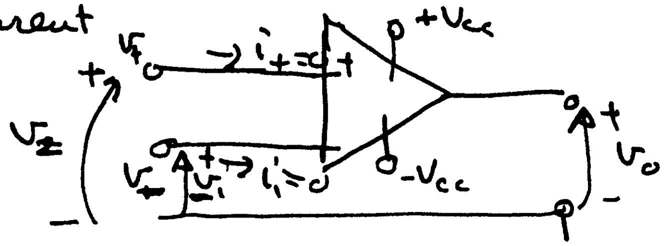
The symbol is



1/29/93

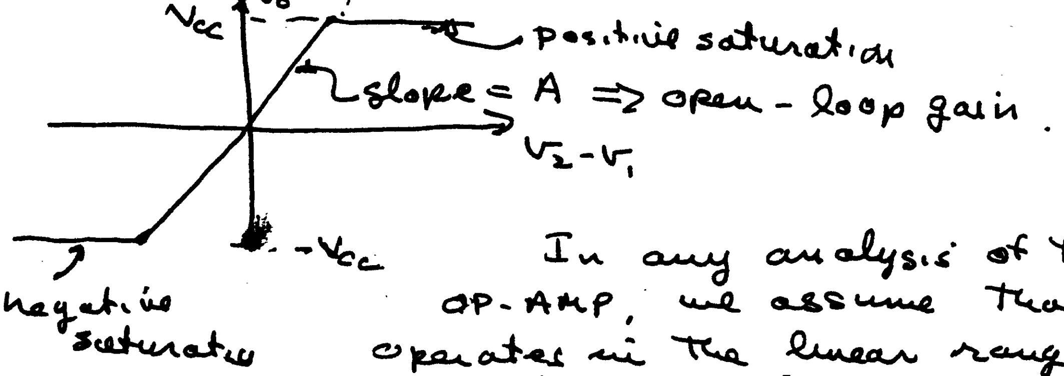
Just as with R, v, i, etc where we specified the devices by their terminal relationships, we do the same with the OP-AMP.

The ideal OP-AMP inputs do not conduct current



They amplify their inputs so that $V_o = A(V_2 - V_1)$

as long as $-V_{cc} \le V_o \le V_{cc}$ so we can describe the OP-AMP by the "transfer-characteristic".

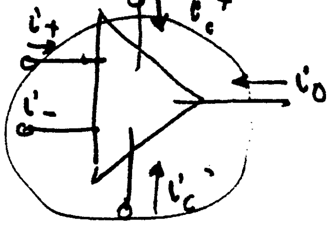


In any analysis of the OP-AMP, we assume that it operates in the linear range and the test to see if $|V_o| < V_{cc}$.

A typical value for the open loop gain is $\approx 10^5$ so if the output is just at saturation

$$|v_2 - v_1| = V_{cc}/10^5 = \frac{15}{10^5} = 150\ \mu\text{volts}.$$

very small. So we almost always can assume $v_2 \approx v_1$ to keep the OP-AMP in its linear region.



$$\cancelto{0}{i_+} + \cancelto{0}{i_-} + i_c^+ + i_c^- + i_o = 0$$

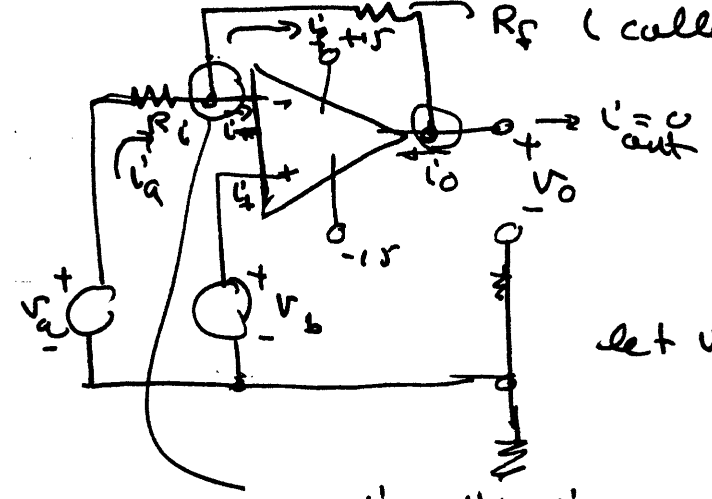
so $i_o = -(i_c^+ + i_c^-)$

the supplies − supply the current for the output.

1/29/93

Examples 6.1 - The inverting amplifier

why don't we use the theoretic gain of an OP-AMP - It is flaky - it may be 10^5 for one chip & vary by as much as 20% from this with another. We can't rely on uniformity - and there are other reasons which will be discussed later.



let $v_b = 0$

KCL. $i_a - i_- - i_f = 0$, $i_f - i_o - \overset{0}{\cancel{i_{out}}} = 0$

but $i_- = 0$ for ideal op-amp $\underline{i_f = i_o}$

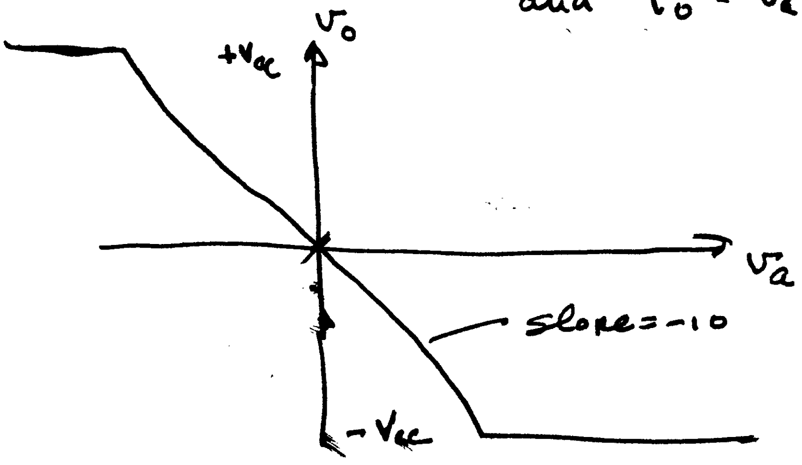
so $i_a = i_f = i_o$

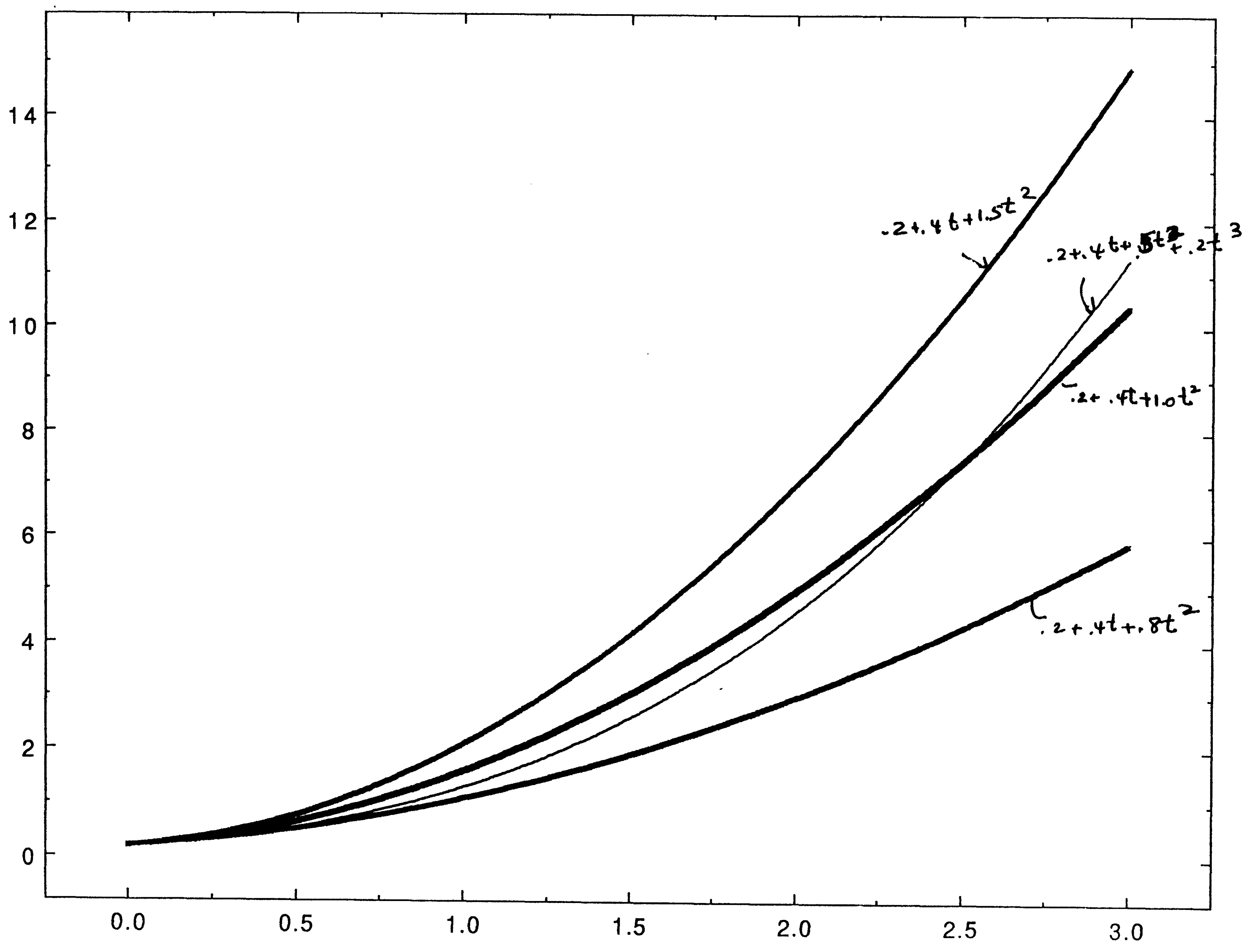
then since $v_a \approx v_b = 0$ $i_a = v_a/R_i = i_o$

$$v_o = -i_f R_f = -v_a \times R_f/R_i$$

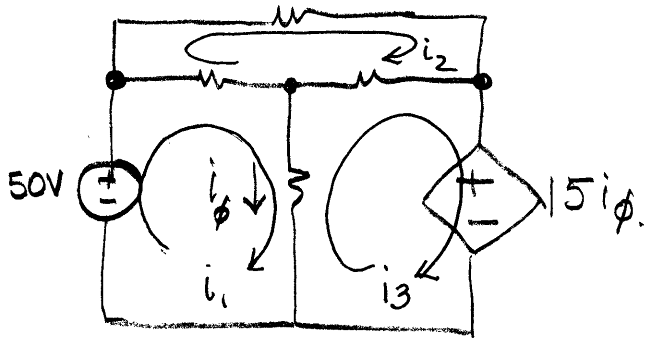
let $R_f/R_i = 10$ then $v_o = -10 v_a$

and $i_o = v_a/R_i$




.2+.4t+1.5t^2
.2+.4t+.5t^2+.2t^3
.2+.4t+1.0t^2
.2+.4t+.8t^2
0
2
4
6
8
10
12
14
0.0
0.5
1.0
1.5
2.0
2.5
3.0

4.6 dependent sources.



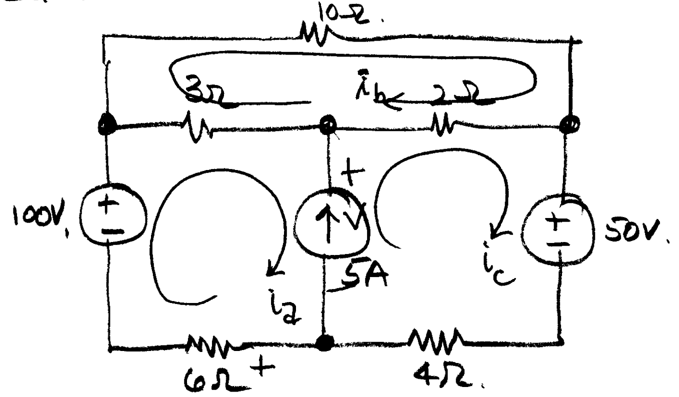
three mesh currents! $i_1, i_2\ i_3 \Rightarrow$ 3 equations in 4 unknowns

supplemental equation due to i_ϕ

$$i_\phi = i_1 - i_3$$

substitute this into first three and get 3 eqns in 3 unknowns.

4.7 special cases: current sources



5 essential branches with unknown currents.

4 essential nodes.

need $5-(4-1)$ mesh equations to solve circuit

$5-3=2$

problem is that voltage v across 5A source is unknown

Solution: write both eqns in terms of v and add

a: $-100 + 3(i_a - i_b) + v + 6i_a = 0$

b: $-v + 2(i_c - i_b) + 50 + 4i_c = 0$

a+b: $50 = 9i_a - 5i_b + 6i_c$

now use constraint: $i_c - i_a = 5A.$

comparison of methods:

both are systematic, both reduce # of equations

may not need complete solutions.

prefer: node voltage for computer analysis

mesh current for manual analysis

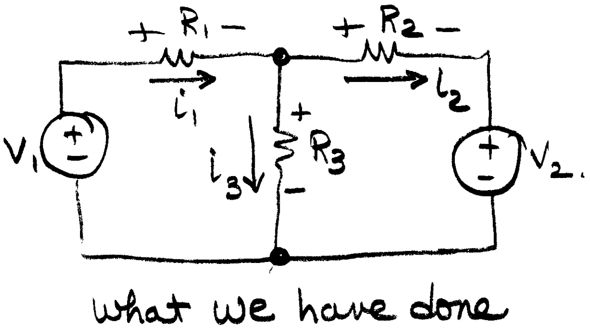
4.5 mesh currents.

mesh – cannot have any other loops inside it

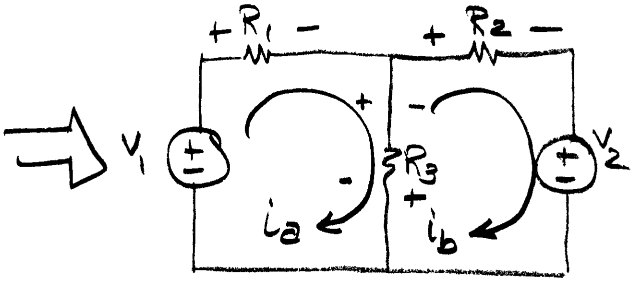
$b - (n-1)$

# of branches.

# of nodes.



what we have done



mesh currents i_a, i_b.

use KCL $i_1 - i_2 - i_3 = 0$ (1)

use KVL left hand loop: $-v_1 + i_1 R_1 + i_3 R_3 = 0$ (2)

use KVL right hand loop $-i_3 R_3 + i_2 R_2 + v_2 = 0$. (3)

reduce to two eqns by solving for i_3 in (1) and eliminate from others.

i.e. $i_3 = i_1 - i_2$.

$$-v_1 + i_1 R_1 + (i_1 - i_2) R_3 = 0.$$

$$-(i_1 - i_2) R_3 + i_2 R_2 + v_2 = 0$$

$$i_1 (R_1 + R_3) - i_2 R_3 = v_1$$

$$-i_1 R_3 + i_2 (R_2 + R_3) = -v_2.$$

two eqns in 2 currents.

automatically eliminates KCL equations.

use i_a and i_b.

KVL i_a:

$$-V_1 + R_1 i_a + R_3 (i_a - i_b) = 0.$$

KVL i_b:

$$+ R_3 (i_b - i_a) + i_b R_2 + V_2 = 0.$$

now rearrange:

$$i_a (R_1 + R_3) - i_b R_3 = v_1$$

$$-i_a R_3 + (R_3 + R_2) i_b = -V_2.$$

now identify

$i_1 = i_a$

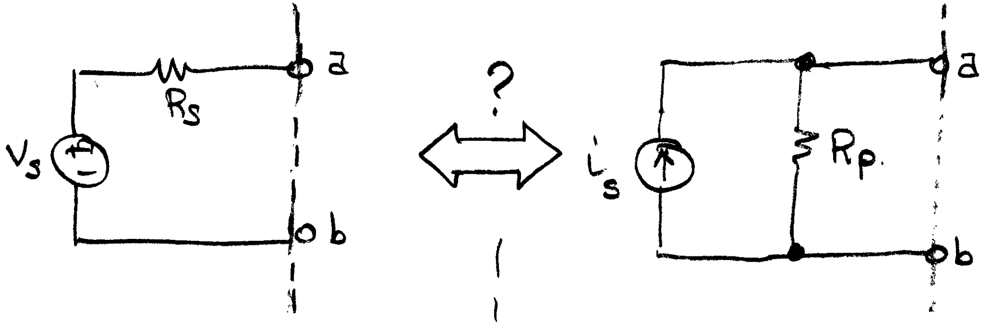
$i_2 = i_b$

$i_3 = i_1 - i_2 = i_a - i_b$

difference between mesh currents

do by inspection in future!

4.9 source transformations

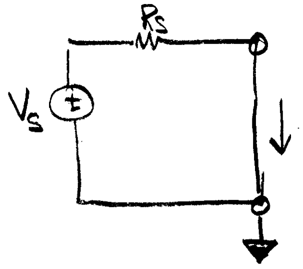


under what conditions are these equivalent at their terminals

Statement if they are equivalent they must be equivalent for all values of R connected across a, b.

Natural to use extreme values: 0, ∞

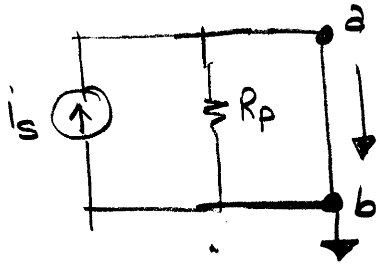
for $R_L = 0$



$V_a = 0$

$i_{ab} = \frac{V_s}{R_s}$

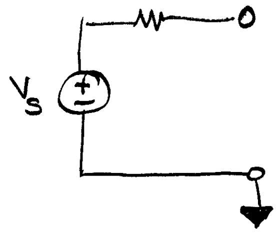
=



$i_{ab} = i_s$

$V_a = 0.$

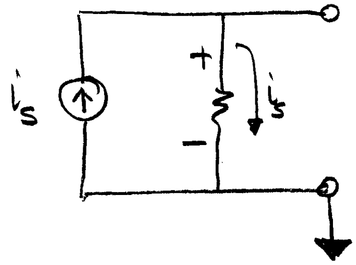
for $R_L = \infty$



$V_a = V_s$

$i_{ab} = 0$

=



$V_a = +i_s R_p$

$i_{ab} = 0,$

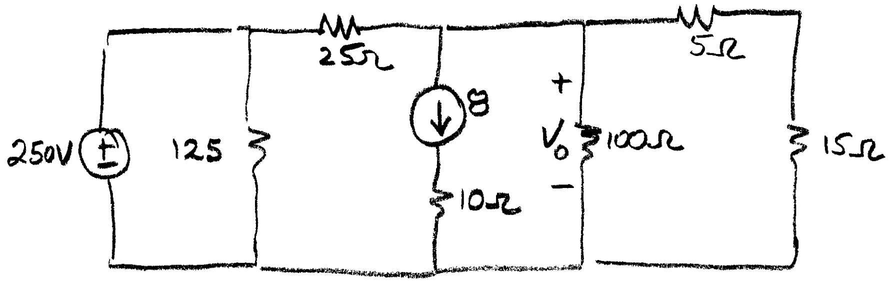
by inspection $\frac{V_s}{R_s} = i_s$ or $R_s = \frac{V_s}{i_s}$

$V_s = i_s R_p$ or $R_p = \frac{V_s}{i_s}$

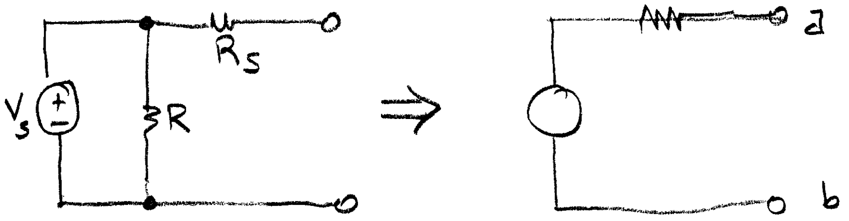
∴ if $R_s = R_p$ these circuits are equivalent!

Example 4.8

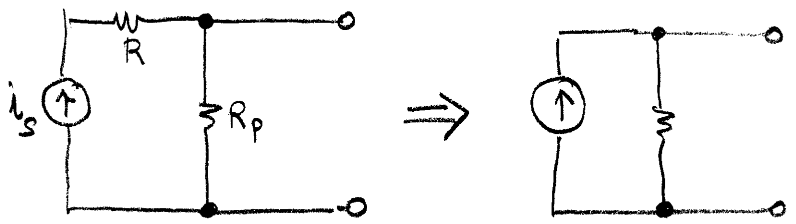
(a) Use source transformations to find v_o.



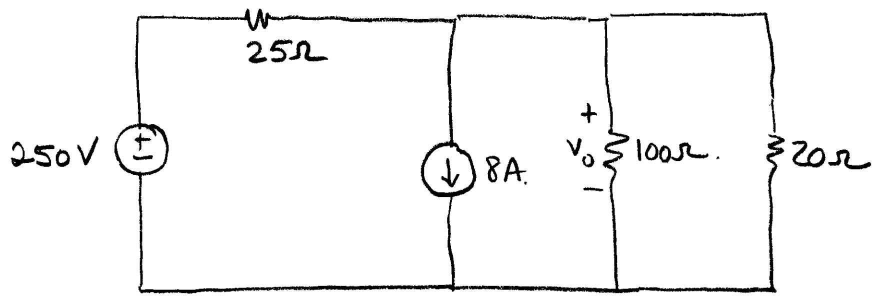
IMPORTANT RULE: If there is a resistance R in parallel with a voltage source or in series with a current source, the resistance R has no effect on the equivalent circuit.

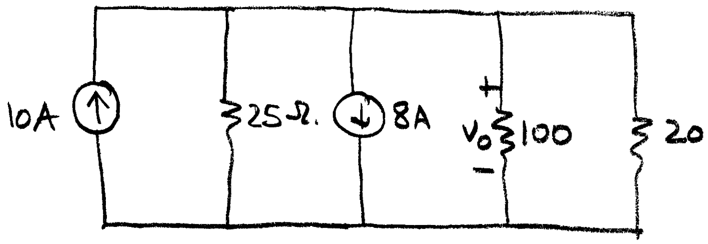


no change in output.

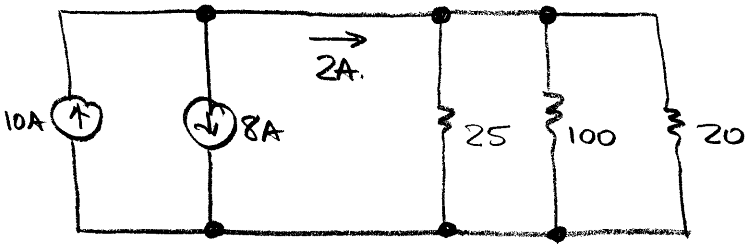


with no change.



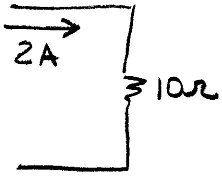


$$i = \frac{250V}{25\Omega} = 10\ A.$$



$$\frac{1}{R} = \frac{1}{25} + \frac{1}{100} + \frac{1}{20} = .04 + .01 + .05 = 0.10$$

$$R = 10\Omega.$$



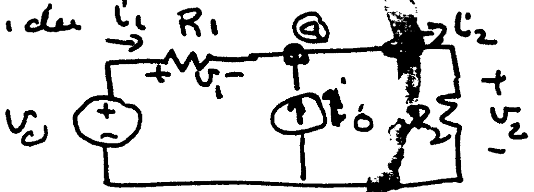
$$V = (2A)(10\Omega) = 20V.$$

2/3/93

SUPERPOSITION

The Thevinin or Norton equivalent circuits are often obtained using the concept of superposition when multiple sources are present in the circuit

Consider



KVL of outer loop $V_0 = v_1 + V_2$

KCL at ⓐ $i_0 + i_1 = i_2$

hence $V_0 = i_1R_1 + i_2R_2$ we can then solve for i_1 and i_2

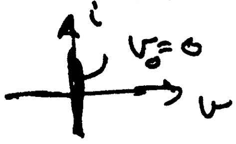
$$i_1 = \frac{V_0}{R_1+R_2} - i_0\frac{R_2}{R_1+R_2}$$

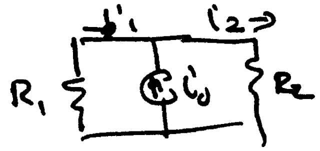
and $$i_2 = \frac{V_0}{R_1+R_2} + i_0\frac{R_1}{R_1+R_2}$$

The fact is that we see the effects of the two sources as a sum. - each term is exactly proportional to the source.

Suppose V_0 were zero (a short circuit!)

then

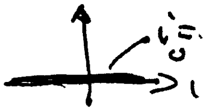


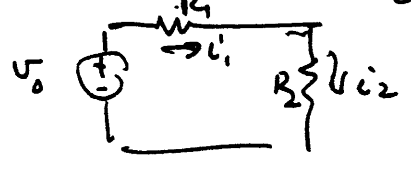


$i_1\Big|_{V_0=0} = -i_0\frac{R_2}{R_1+R_2}$ (current dividers)

$i_2\Big|_{V_0=0} = i_0\frac{R_1}{R_1+R_2}$

now let's set $i_0 = 0$ (an open circuit)



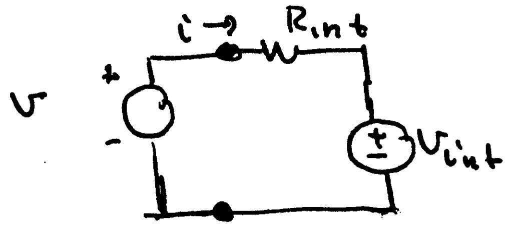


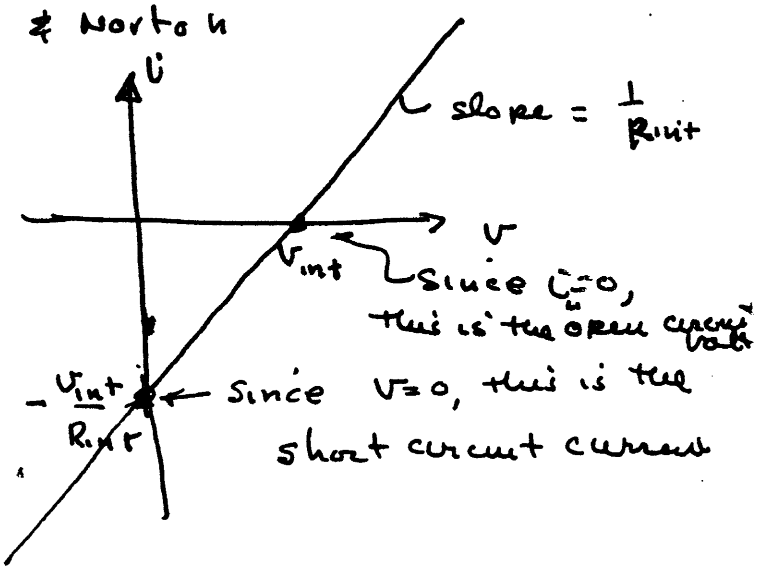
$i_1 = i_2 = \frac{V_0}{R_1+R_2}$

their sum is exactly what we first obtained

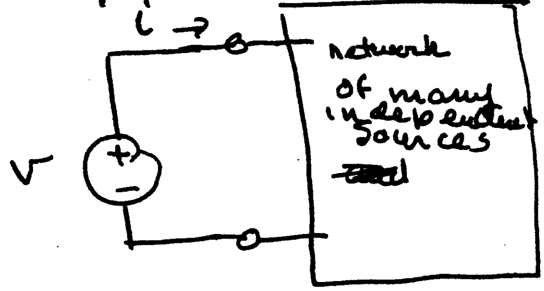
2/5/97

What about Thevenin & Norton





Now consider a complicated network driven by v.



$$i = Gv - \underbrace{[a_1 v_1 + a_2 v_2 + \cdots + a_N v_N + b_1 i_1 + \cdots b_M i_M}$$

these are the superposition effect of N - voltage sources (indep

and M - current sources (indep

We can find these constants as follows:

to find G, set all internal sources to zero and find i for a given v. (or set $v=1$ then $i = G$)

to find a_1 set v, and $v_n = 0$ $n \neq 1$, $i_m = 0$ all m

b_1 set v, and $v_n = 0$ all n, $i_m = 0$ $m \neq 1$

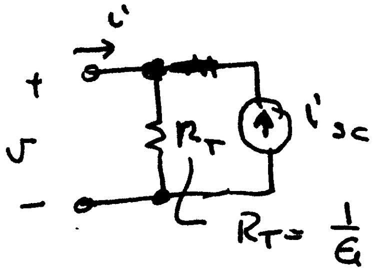
Note that if $v = 0$ with all sources on we have the short circuit current

when $i = 0$; v = the open circuit voltage

2/5/93

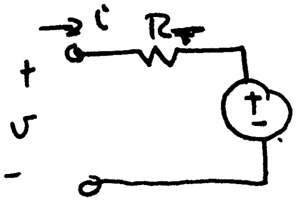
Thus $i = Gv - i_{sc}$

$\longrightarrow$ circuit $\longrightarrow$ for this.



this is the Norton Equivalent Circuit.

when $v = v_{oc}$ $i = 0$ so



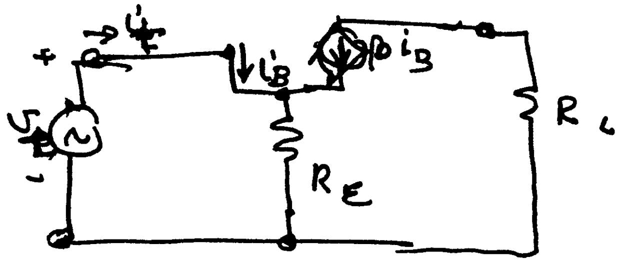
$V_T = i_{sc} * R_T$

we can find $R_T = \frac{v_{oc}}{i_{sc}}$

2/5/93

Now lets consider superposition with controlled sources

example

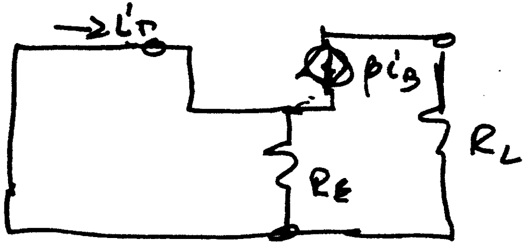


find the Thevenin equivalent of the input

Can we do the following:

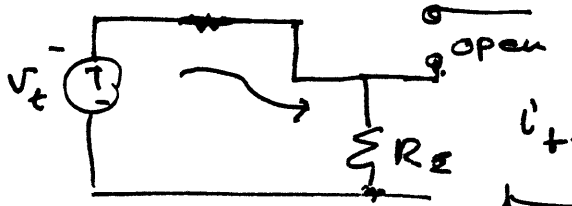
① We apply v_T and set the current source $\beta i_B = 0$ then find the current i_T due to v_t alone

② Then set $v_t = 0$ ~~and find~~ which then makes the circuit



~~But~~ this results in $i_t = -\beta i_B$.

③ Now suppress βi_B and use v_t alone



$i_t = \frac{v_t}{R_e}$

~~hence~~

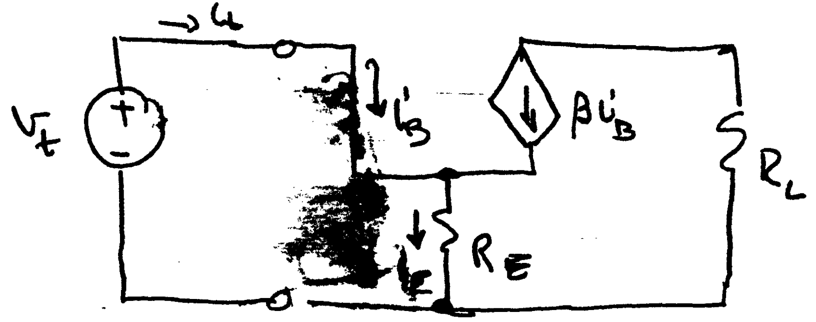
④ hence $i_t = \frac{v_t}{R_E} - \beta i_B$

NO WAY why? because the current source βi_B is a DEPENDENT source - it must exist if βi_B exists.

2/5/93

⑤

The correct solution

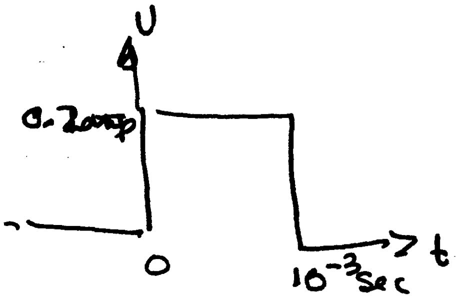


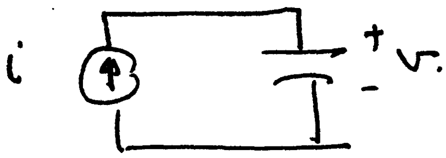
$i_t = i_B$, $\quad i_E = i_B + \beta i_B = (1+\beta)\, i_B$

$V_E = R_E(1+\beta)\, i_B = v_t$ by KVL

hence $i_t = \dfrac{v_t}{R_E(1+\beta)}$ ∴ $R_{th} = v_t / i_t$

$= (1+\beta) R_E$





$$i = C\frac{dv}{dt}$$

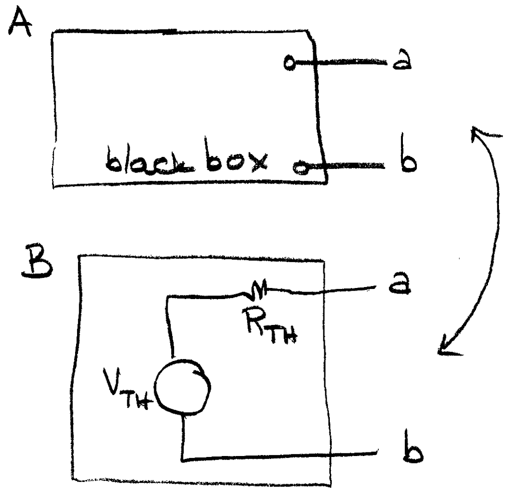
$$\int_{t=t_1}^{t=t_2} dv = \int_{t_1}^{t_2} \frac{1}{C}\, i\, dt$$

$$\int_{t=-\infty}^{t} dv = v(t) - v(-\infty) = \frac{1}{C}\int_{-\infty}^{t} i(t)\, dt$$

$$= \frac{1}{C}\begin{cases} 0 & t< \\ 0.2t & 0 \le t \le \\ 2\times[illegible] & t \ge 1 \end{cases}$$

hence $v(t) = v(-\infty) + \frac{1}{C}\Bigg\{ \qquad \Bigg\}$

4.10 Thevenin and Norton equivalent circuits



we want these to be equivalent

this can be done for any circuit of linear elements.

Note 1: If A & B are equivalent.

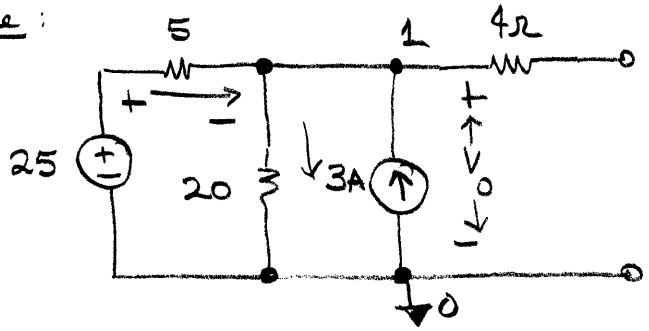
For $R_{ab} = \infty$

measure $V_{ab} = V_{TH}$

For $R_{ab} = 0$

measure $i_{ab} = \frac{V_{TH}}{R_{TH}}$

Example:



① calculate v_{oc}

no load

use KCL @ node 1 + in.

$$+\frac{25-v_0}{5} - \frac{v_0}{20} + 3 = 0 \Rightarrow v_{oc} = 32V.$$

② calculate i_{sc}

$$+\frac{25-v_0}{5} - \frac{v_0}{20} + 3 - \frac{v_0}{4} = 0 \Rightarrow v_0 = 16V.$$

$$i_{sc} = \frac{16}{4} = 4A$$

③ $R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{32}{4} = 8\Omega$

Norton equivalent: source transformation

$$i_N = \frac{V_{TH}}{R_{TH}}$$

and we know $R_{TH} = R_N$

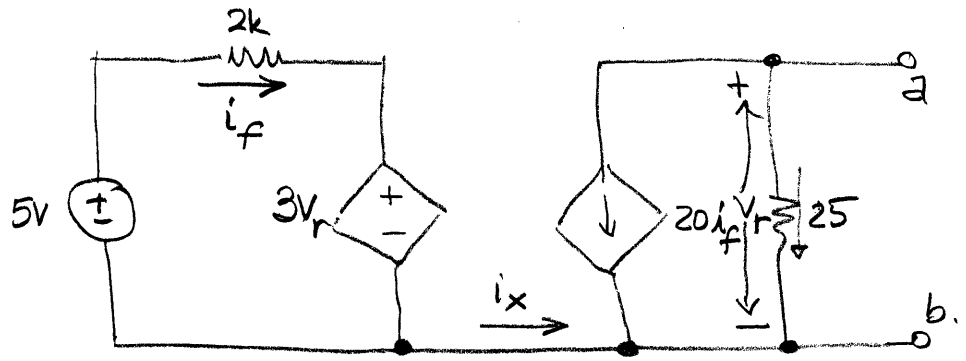
Finding R_{TH} with
in Dependent sources — don't use above method

replace voltage sources with shorts
current sources with opens.

dependent sources

1. deactivate all independent sources
2. apply a test voltage (or current) to outputs a, b

Good example



V_{TH}

① recognize $i_x = 0$

② $V_{oc} = V_{ab} = +i25 = (-20i_f)25 = -500\, i_f$

③ $i_f = \dfrac{5 - 3V_r}{2k}$

solving $V_r = -5V = V_{TH}$

i_{sc}

by inspection $i_{sc} = i_{ab} = -20 i_f$

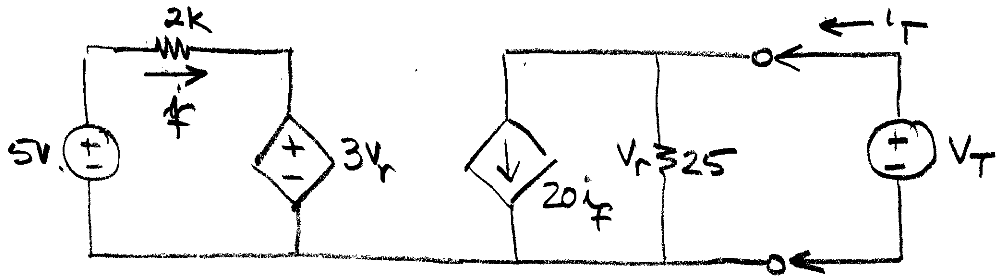
$V_r = 0$ if ab is a short $\Rightarrow 3V_r = 0$ in input

$$i_f = \frac{5-3(0)}{2k} = 2.5\ mA$$

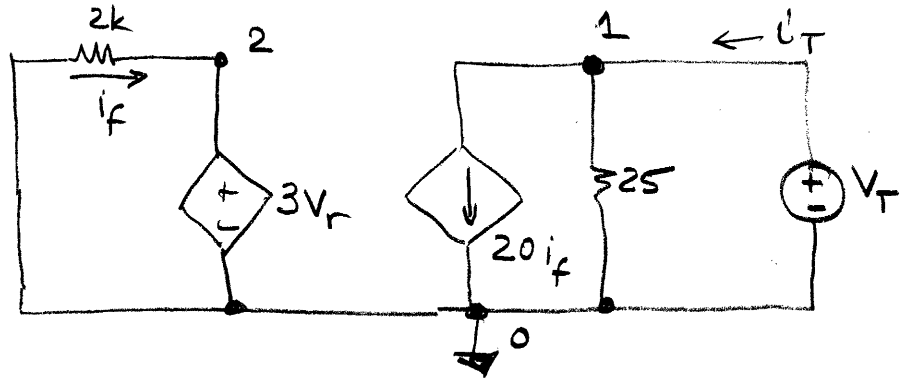
$$i_{sc} = -20(2.5mA) = -50\ mA$$

$$R_{TH} = \frac{V_{TH}}{i_{sc}} = \frac{-5}{-50mA} = 100\Omega$$

alternate method:



all independent sources ⟷ opens or shorts



at node 1 + out $$-i_T + \frac{V_T}{25} + 20\, i_f = 0.$$

at input use Ohm's Law $$i_f = -\frac{3V_t}{2}\ \text{mA}$$

substituting. $$-i_T + \frac{V_T}{25} + 20\left(-\frac{3V_T}{2k}\right) = 0.$$ units

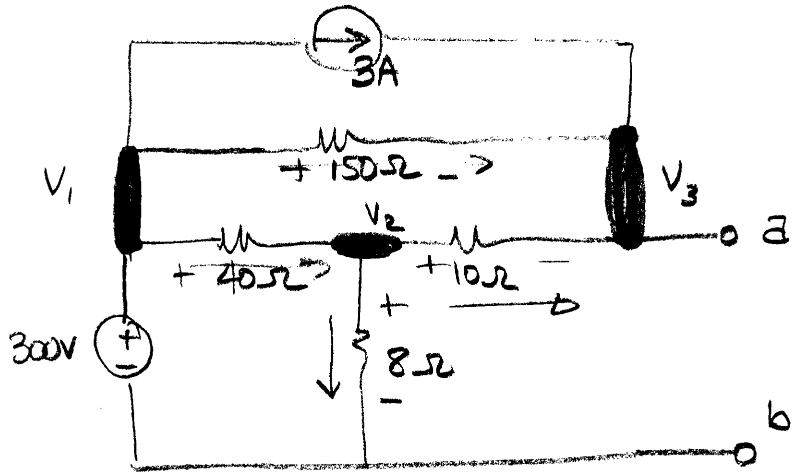
$$i_T = \frac{1}{25}V_T - \frac{60}{2000}V_T$$

$$\frac{i_T}{V_T} = \frac{1}{25} - \frac{60}{2000} = \frac{1}{100}$$

$$R_{TH} = \frac{V_T}{i_T} = 100\Omega.$$

must use
alternative
method if only
resistors and
dependent sources.

4.56.



by inspection $V_1 = +300V.$

KCL@3 + in $\quad 3A + \frac{300 - V_3}{150} + \frac{V_2 - V_3}{10} \cdot \frac{15}{15} = 0.$

KCL@2 + in $\quad \frac{300 - V_2}{40} - \frac{V_2}{8} \cdot \frac{5}{5} \; \frac{V_2 - V_3}{10} \cdot \frac{4}{4} = 0.$

should be. $V_3 = 120V$

$450 + 300 - V_3 + 15V_2 - 15V_3 = 0$

$300 - V_2 - 5V_2 - 4V_2 + 4V_3 = 0$

$15V_2 - 16V_3 = -750$

$-10V_2 + 4V_3 = -300$

$15V_2 - 16V_3 = -750$

$+5V_2 + 2V_3 = -150$

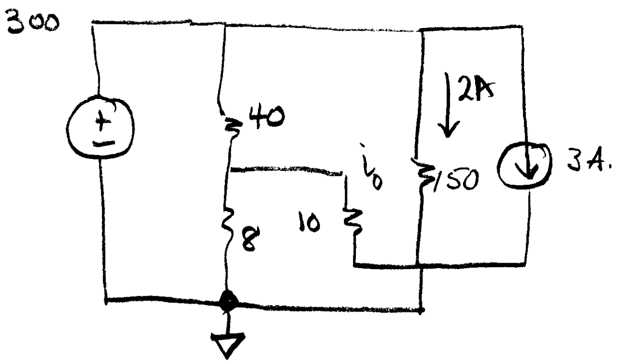
$15V_2 - 16V_3 = -750$

$-15V_2 + 6V_3 = -450$

$-10V_3 = -1200$

$V_3 = +120. V. \quad = V_{TH}.$

Now short output.



$\frac{300V}{150\Omega} = 2A$

$8 \| 10 = \frac{8 \cdot 10}{8+10} = \frac{80}{18} = \frac{40}{9}.$

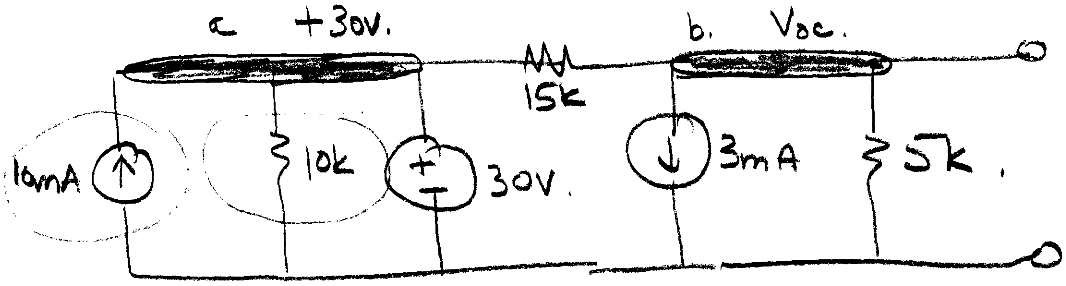
$i = \frac{\varepsilon}{R} = \frac{300}{40 + \frac{40}{9}} = \frac{300}{\frac{360+40}{9}} = \frac{2700}{400} = \frac{27}{4}$

$i_0 = \frac{\overset{2}{\cancel{8}}}{8+10} \cdot \frac{27}{\cancel{4}} = \frac{2 \cdot 3}{2} = 3A$

$i_{TOT} = 3 + 2 + 3 = 8A.$

Go back and clean up examples

4.58



No Source transformations

KCL at b
+ out

$$\frac{V_{oc}-30}{15} + \left(3mA\right)^{15}_{15} + \left(\frac{V_{oc}}{5k}\right)^{3}_{3} = 0.$$

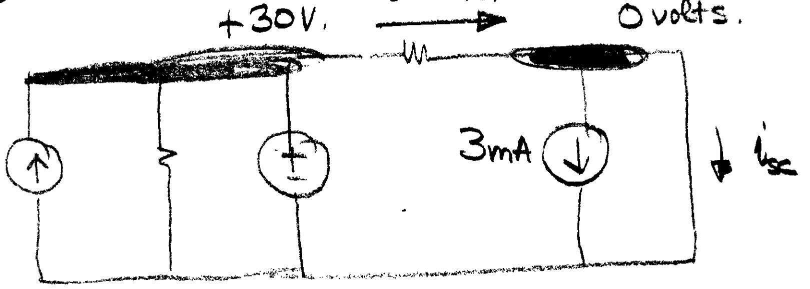
$$V_{oc} - 30 + 45 + 3V_{oc} = 0.$$

$$4V_{oc} = -15.$$

$$V_{oc} = -\frac{15}{4}$$

Now do i_{sc}

$$i = \frac{30V}{15k} = 2mA.$$



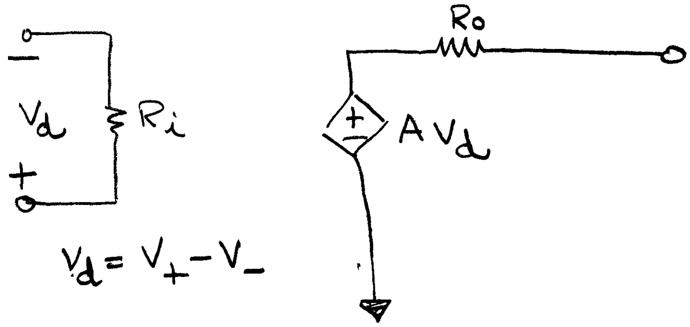
$$i_{sc} = -1mA$$

$$R_T = \frac{V_{oc}}{i_{sc}} = \frac{-45/4}{-1\,mA} =$$

$$V_T = iR$$
$$= 3.75V.$$

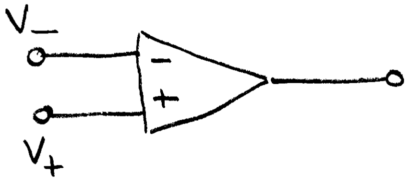
6.7 equivalent circuits for the operational amplifier

ideal op-amp

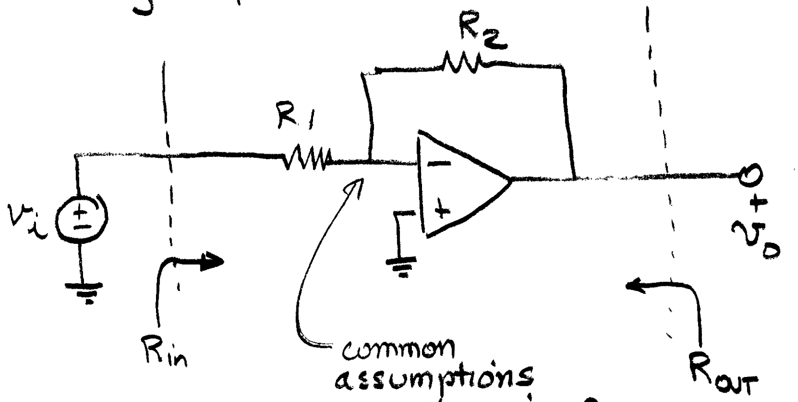


$V_d = V_+ - V_-$

symbol



linear inverting amplifier



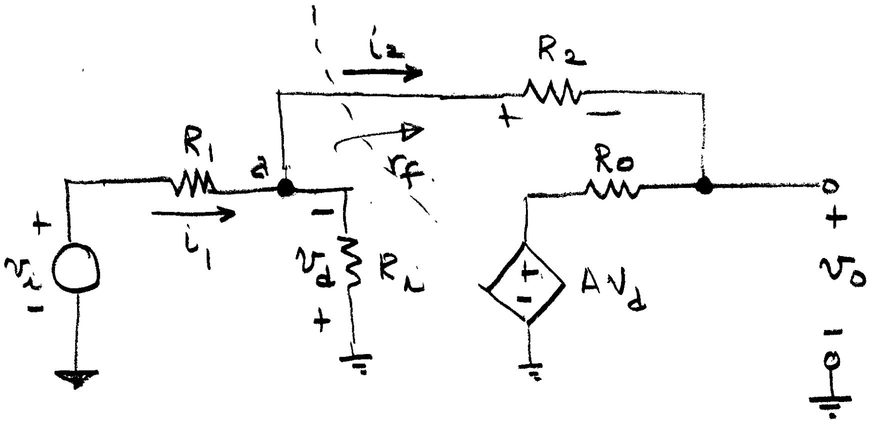
common assumptions
$V_1 = 0,\ i_1 = 0.$

$A_v \cong -\frac{R_2}{R_1}$

$R_{IN} \cong R_1$

$R_o \cong 0.$

equivalent circuit



Simple analysis

for an ideal op-amp $\boxed{R_i \to \infty}$, $V_d \to 0$

$$i_1 = \frac{v_i - (-V_d)}{R_1} = \frac{v_i + V_d}{R_1}$$

$$i_2 = \frac{-V_d - v_o}{R_2}$$

using KCL at a $\quad -i_1 + i_2 = 0$ or $i_1 = i_2$.

$$\frac{v_i + \cancel{V_d}^{\to 0}}{R_1} = \frac{-\overset{\to 0}{V_d} - v_o}{R_2}$$

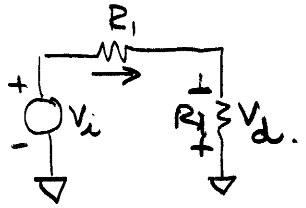
$$\text{or} \quad \frac{R_2}{R_1} = -\frac{v_o}{v_i}$$

$$\Rightarrow A_v = -\frac{R_2}{R_1} \quad \text{for an Ideal op-amp}$$

This is a classical result for an inverting amplifier
Typically pick R_1, R_2 so $|A_v| < 50$ and $R_2 < 100\,k\Omega$.

Input impedance of the inverting amplifier

$$r_i = \frac{v_i}{i_1}$$



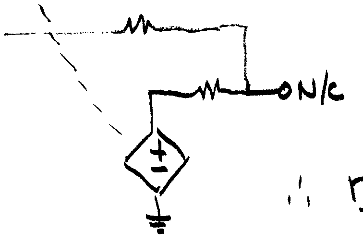
KVL around input $\quad -v_i + i_1 R_1 - \cancel{V_d}^{\to 0} = 0$.

(for an ideal op-amp) $\quad \Rightarrow \quad r_i = \frac{v_i}{i_1} = \frac{i_1 R_1}{i_1} = R_1$

for a non-ideal op amp $\quad$ no output connected

KVL around feed back loop

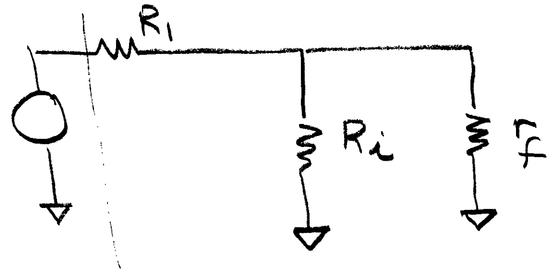
$$+V_d + i_2 R_2 + i_2 R_o + A V_d = 0$$



$$V_d(1+A) = -i_2(R_2 + R_o)$$

$$\therefore r_f = -\boxed{\frac{V_d}{i_2}} = \frac{R_2 + R_o}{1 + A_d}$$

input circuit



$$\Rightarrow \quad R_{IN} = R_1 + R_i \| r_f = R_1 + \frac{R_i r_f}{R_i + r_f}$$

$$= R_1 + \frac{R_i\left(\frac{R_2+R_o}{1+A_d}\right)}{R_i + \frac{R_2+R_o}{1+A_d}} = R_1 + \frac{\frac{R_i(R_2+R_o)}{1+A_d}}{\frac{R_i(1+A_d)+R_2+R_o}{1+A_d}}$$

$$R_{IN} = R_1 + \frac{R_i(R_2+R_o)}{R_i(1+A_d)+(R_2+R_o)}$$

in practice $R_2 = 10k$, $R_o = 100\Omega$, $R_i = 100k\Omega$, $A_d = 10^5$

then $R_{IN} = R_1 + \dfrac{(10^5)(10^4+10^2)}{10^5(1+10^5)+(10^4+10^2)}$

$\frac{110\ 100}{10000110100}$

$$= R_1 + 1.1 \times 10^{-5}$$

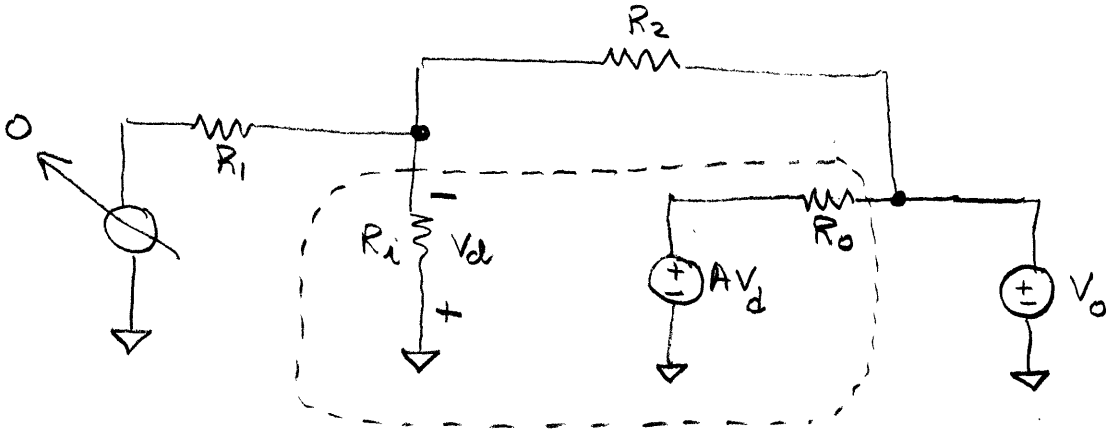
Output impedance of the inverting amplifier

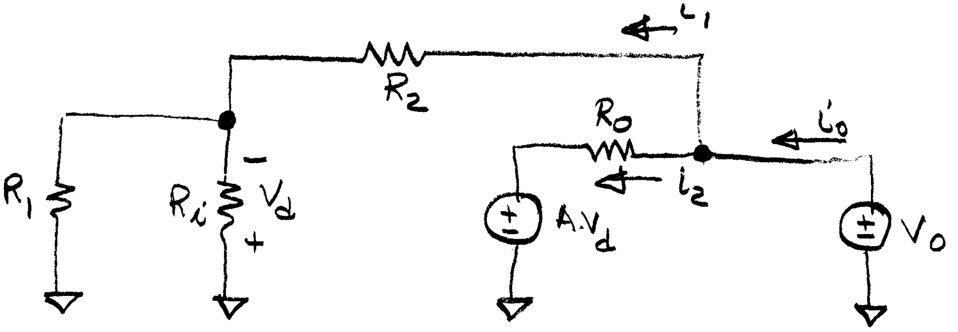
procedure is very important

set $v_i = 0$

insert a test voltage source at output

measure current drawn from test source





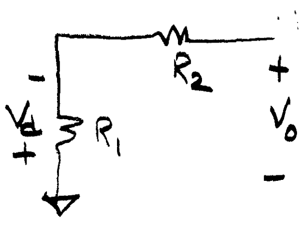
$$i_0 = i_1 + i_2$$

where $i_1 = \dfrac{V_0}{R_2 + R_1 \| R_i} \cong \dfrac{V_0}{R_2 + R_1}$

$$i_2 = \frac{V_0 - AV_d}{R_0}$$

$$i_0 = i_1 + i_2 = \frac{V_0}{R_1 + R_2} + \frac{V_0 - AV_d}{R_0} \qquad (1)$$

Again for $R_i >> R_1$ very large.



$$-V_d = \frac{R_1}{R_1 + R_2} V_0. \qquad (2)$$

e

$$\frac{1}{r_o} = \frac{i_o}{V_o} = \frac{1}{V_o}\frac{V_o}{R_1+R_2} + \frac{1}{V_o}\frac{V_o - AV_d}{R_o}$$

$$= \frac{1}{R_1+R_2} + \frac{1}{R_o} - \frac{A}{R_o}\frac{V_d}{V_o}$$

$$= \frac{1}{R_1+R_2} + \frac{1}{R_o} - \frac{A}{R_o}\left(\frac{-R_1}{R_1+R_2}\right)\cancel{\frac{V_o}{V_o}}$$

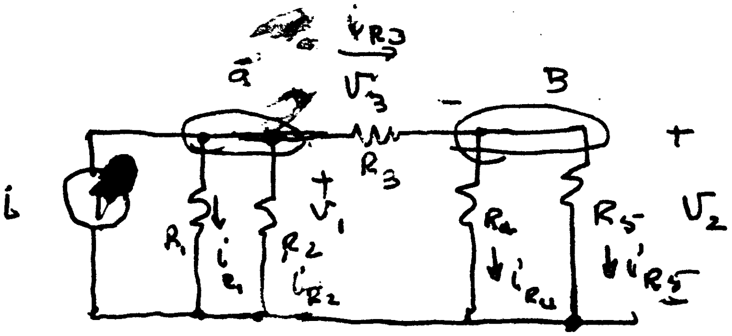
$$= \frac{1}{R_1+R_2} + \frac{1 + A\left(\frac{R_1}{R_1+R_2}\right)}{R_o}$$

for an ideal op-amp $A \to \infty$ $\frac{1}{r_o} \to \infty$

so $r_o \to 0$

P 379 - 390   2/10/93.

②

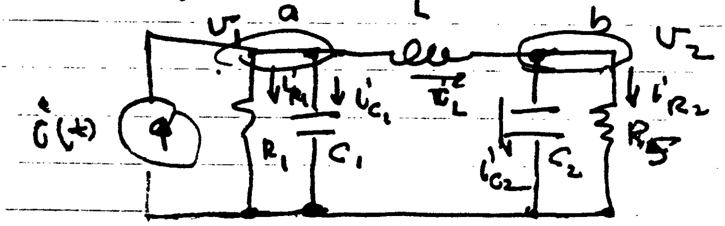


$$i = i_{R_1} + i_{R_2} + i_{R_3} = \frac{v_1}{R_1} + \frac{v_1}{R_2} + \frac{v_1 - v_2}{R_3}$$

$$i_{R_3} = i_{R_4} + i_{R_5} \Rightarrow \frac{v_1 - v_2}{R_3} = \frac{v_2}{R_4} + \frac{v_2}{R_5}$$

now solve for v_1 and v_2.

Of ~~course~~ we could have simplified this circuit quite abit as we know $R_1 \| R_2$ and $R_4 \| R_5$. But now let's look at a circuit with identical topology (layout) but different elements



$$v_L = L\frac{di_L}{dt}$$

and so $i_L = \frac{1}{L}\int^t v_L\,dt$

$$v_L = v_1 - v_2$$

~~again~~ at node a

at ⓐ $i = i_{R_1} + i_{C_1} + i_L = \frac{v_1}{R_1} + C_1\frac{dv_1}{dt} + \frac{1}{L}\int^t (v_1 - v_2)dt$

at ⓑ $i_L = \frac{1}{L}\int^t (v_1 - v_2)dt = i_{C_2} + i_{R_2} = C_2\frac{dv_2}{dt} + \frac{v_2}{R_5}$

This is called an integro-differential equation. We can wish to find a nice simple solution like for the resistor manifestation of the topology. In fact there is a simple approach for ~~a very~~ special kind of a time function, [illegible].

2/10/93

①ⓐ

in matrix form for resistor circuits

$$\begin{bmatrix} \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i \\ 0 \end{bmatrix}$$

· matrix form for L, R, C. with $i = I e^{j\omega t}$

$$\begin{bmatrix} \left(\frac{1}{R_1}+\left(\frac{1}{\frac{1}{j\omega C_1}}\right)+\frac{1}{j\omega L}\right) & -\frac{1}{j\omega L} \\ -\frac{1}{j\omega L} & \frac{1}{R_5}+\left(\frac{1}{\frac{1}{j\omega C_2}}\right)+\frac{1}{j\omega L} \end{bmatrix} \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

Let $\frac{1}{R_1} = G_1$, $\frac{1}{R_2} = G_2$, $\frac{1}{R_3} = G_3$, $\frac{1}{R_4} = G_4$, $\frac{1}{R_5} = ($

$$\begin{bmatrix} (G_1+G_2+G_3) & -G_3 \\ -G_3 & (G_4+G_5+G_3) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \left(G_1 + j\omega C_1 + \frac{1}{j\omega L}\right) & -\left(\frac{1}{j\omega L}\right) \\ -\frac{1}{j\omega L} & \left(G_5 + j\omega C_2 + \frac{1}{j\omega L}\right) \end{bmatrix} \begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

we find $\hat{V}_1$ & $\hat{V}_2$ but the complete solution is

$$v_1 = \hat{V}_1 e^{j\omega t}$$
$$v_2 = \hat{V}_2 e^{j\omega t}$$

2/10/93

②

$I = [\text{amps}]$

Let's use a source $i(t) = I e^{j\omega t} =$ [illegible] $[\cos\omega t + j\sin\omega$ as we once did before. Again we assume that the voltage must also vary with time as $e^{j\omega t}$ with a constant multiplier to be determined.

So $v_1(t) = \hat{V}_1 e^{j\omega t}$ $\quad$ $\hat{V}_{1,2} \neq \hat{V}_{1,2}(t)$ ~~[volts]~~

and $v_2(t) = \hat{V}_2 e^{j\omega t}$ $\quad$ ~~as time would~~

Now we substitute these into our ~~node~~ equations

$$I e^{j\omega t} = \frac{\hat{V}_1 e^{j\omega t}}{R_1} + j\omega C_1 \hat{V}_1 e^{j\omega t} + \frac{1}{j\omega L}\left[\hat{V}_1 e^{j\omega t} - \hat{V}_2 e^{j\omega t}\right]$$

$$\frac{1}{j\omega L}\left[\hat{V}_1 e^{j\omega t} - \hat{V}_2 e^{j\omega t}\right] = j\omega C_2 \hat{V}_2 e^{j\omega t} + \frac{\hat{V}_2}{R_5} e^{j\omega t}$$

Notice that everywhere time enters as $e^{j\omega t}$ and we can cancel this factor throughout.

$$I = \frac{\hat{V}_1}{R_1} + \frac{\hat{V}_1}{(1/j\omega C_1)} + \frac{\hat{V}_1 - \hat{V}_2}{j\omega L}$$

$$\frac{\hat{V}_1 - \hat{V}_2}{j\omega L} = \frac{\hat{V}_2}{(1/j\omega C_2)} + \frac{\hat{V}_2}{R_5}$$

Now compare the two sets of equations

see that $\left(\frac{1}{j\omega C_1}\right)$ plays the role of R_2

$j\omega L$ " " " " R_3

$\left(\frac{1}{j\omega C_2}\right)$ " " " " R_4

$\frac{[\text{volts}]}{[\text{amps}]}$ was called ohms for a resistor

but $\frac{[\text{volts}]}{[\text{amps}]} =$ $j\omega L$ is reactance

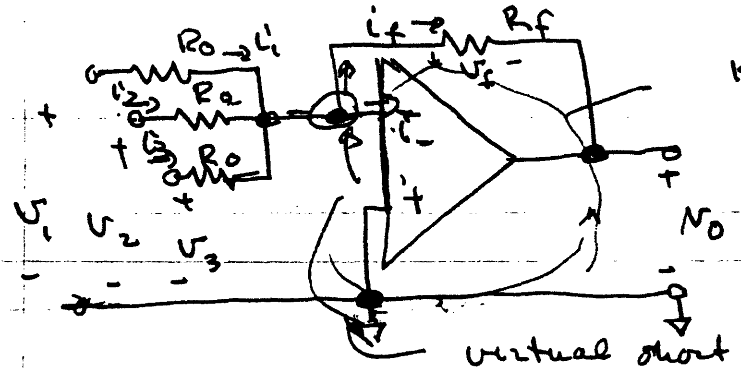
or $= (1/j\omega C)$ is reactance.

to return once more

2/10/93

③

More on OP-AMPS CIRCUITS



KVL ⇒ $v_o + v_f = 0$

$v_o = -v_f = i_f R_f$

$$i_1 + i_2 + i_3 = \overset{0 \text{ for ideal op amp}}{\cancel{i_-}} + i_f$$

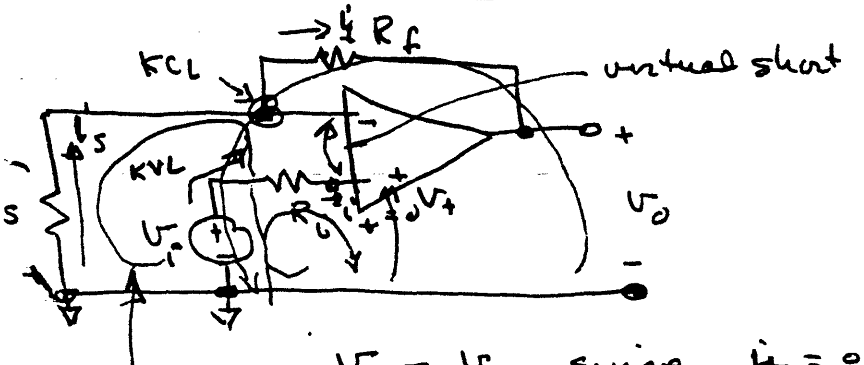
$$\frac{v_1}{R_o} + \frac{v_2}{R_o} + \frac{v_3}{R_o} = i_f \quad \text{hence}$$

$$-(v_1 + v_2 + v_3)\frac{R_f}{R_o} = v_o$$

~~[illegible]~~ or the system acts as an analog adder. In fact we could add as many inputs as we desire. We can also have

$v_o = -10 v_1$ ~~+~~ v_2 or any linear combination we choose.

Another amplifier circuit



$v_+ = v_i$ since $i_+ = 0$

therefor $v_- = v_i$

KVL $-R_s i_s - v_- = 0$

so $i_s = -\frac{v_-}{R_s} = -\frac{v_i}{R_s}$

again $i_f = i_s = -\frac{v_i}{R_s}$, ~~[illegible]~~

2/10/93

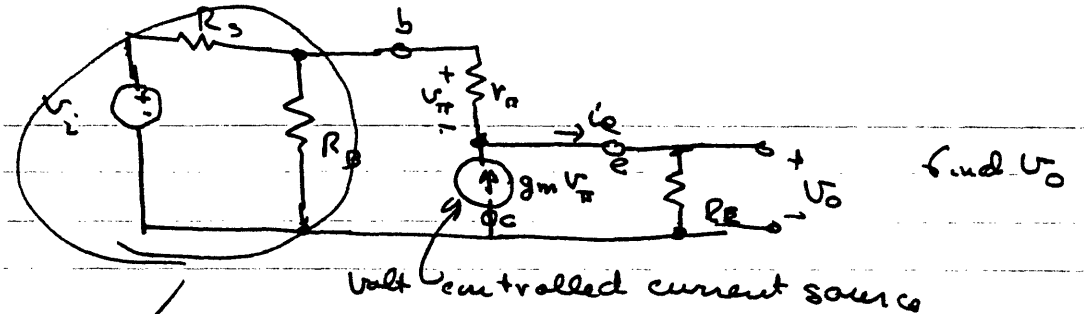
④ Now take KVL through v_o, R_f v_-

$$v_o + i'_f R_f \; v_- = 0$$

$$v_o - \frac{R_f}{R_S} v_i - v_i = 0 \quad \text{or} \quad v_o = v_i\left(1 + \frac{R_f}{R_S}\right)$$

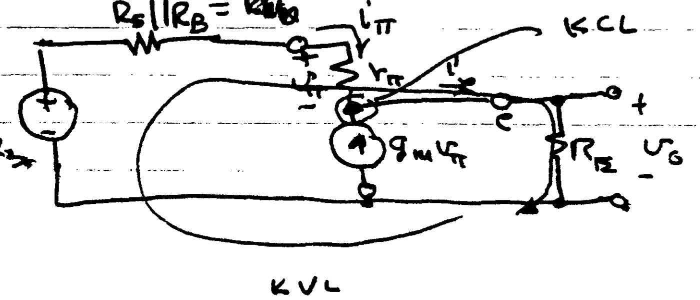
<u>NOTE</u> that this is <u>non</u>-inverting!

<u>Example</u> (called an emitter follower (using a transistor



convert to thevenin equivalent

$$v_{TH} = v_i \cdot \frac{R_B}{R_S + R_B}$$



$$V_{TH} - i_\pi R_{TH} - v_\pi - i_e R_E = 0$$

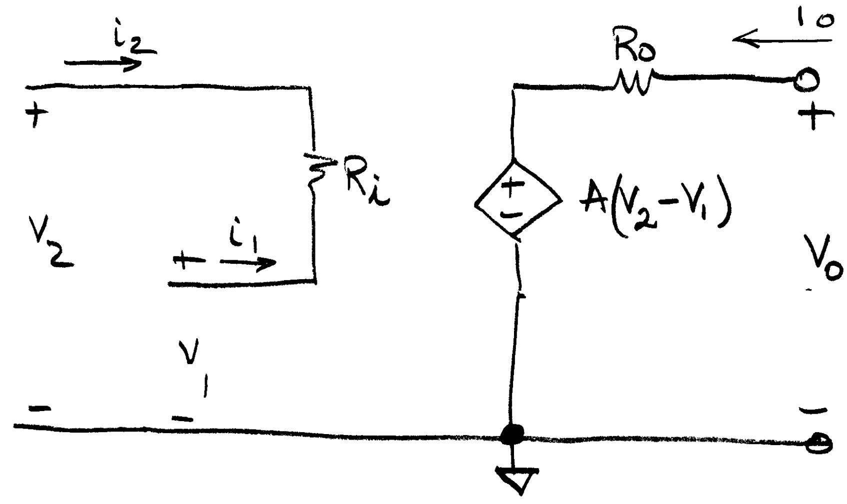
$$i_e = g_m v_\pi + i_\pi = (g_m r_\pi + 1) i_\pi$$

$$V_{TH} - i_\pi \left[R_{TH} + r_\pi + R_E(g_m r_\pi + 1)\right] = 0$$

$$i_\pi = \frac{V_{TH}}{R_{TH} + r_\pi + (1 + g_m r_\pi) R_E}$$

$$v_o = i_e R_E = \frac{R_E(g_m r_\pi + 1) V_{TH}}{R_{TH} + r_\pi + (g_m r_\pi + 1) R_E}$$

6.7 Equivalent circuits for the operational amplifier



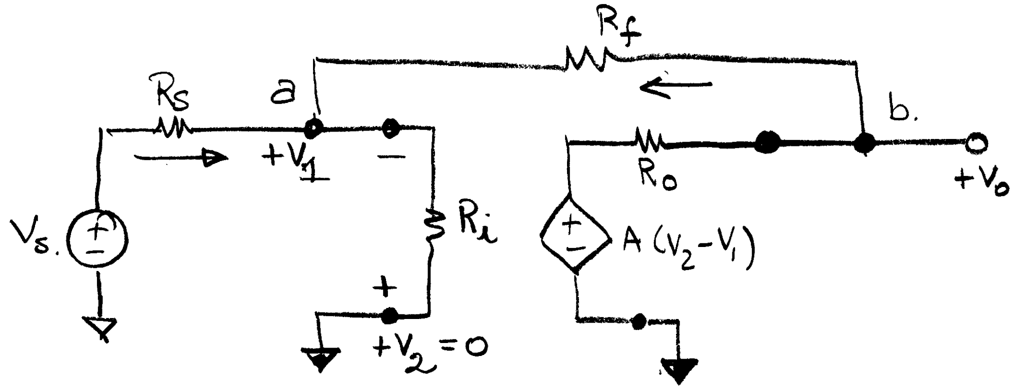
μA 740

$R_i = 2\times10^6$

$A = 10^5$

$R_o = 75$

input impedance of the inverting op-amp circuit using the more realistic model.



KCL @ a + in
$$\frac{V_s - V_1}{R_s} - \frac{V_1}{R_i} + \frac{V_o - V_1}{R_f} = 0.$$

KCL @ b + in
$$-\frac{V_o - V_1}{R_f} - \frac{V_o - A(\overset{\nearrow 0.}{\cancel{V_2}} - V_1)}{R_o} = 0$$

$$V_1\left(-\frac{1}{R_s} - \frac{1}{R_i} - \frac{1}{R_f}\right) + V_o\left(\frac{1}{R_f}\right) = \frac{1}{R_s}V_s$$

$$V_1\left(+\frac{1}{R_f} - \frac{A}{R_o}\right) + V_o\left(-\frac{1}{R_f} - \frac{1}{R_o}\right) = 0.$$

(b)

$$V_o = \frac{-A + \frac{R_o}{R_f}}{\frac{R_s}{R_f}\left(1 + A + \frac{R_o}{R_i}\right) + \left(\frac{R_s}{R_i} + 1\right) + \frac{R_o}{R_f}} v_s.$$

as $A \rightarrow \infty$ $$V_o \rightarrow \frac{-A}{\frac{R_s}{R_f} A} v_s = -\frac{R_f}{R_s} v_s.$$

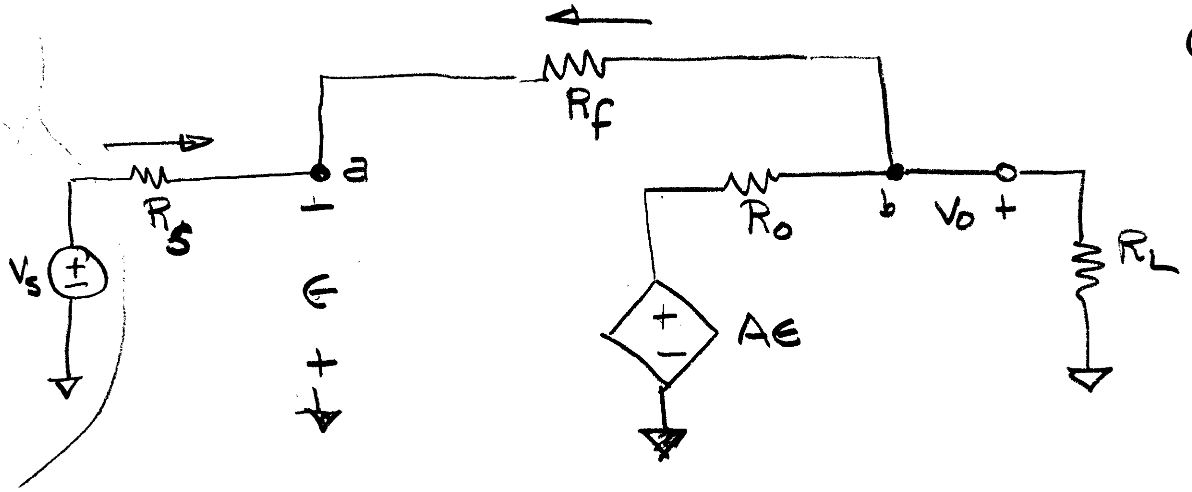
out of curiousity what is v_1 ?

$$v_1 = \frac{\begin{bmatrix} \frac{1}{R_s} V_s & \frac{1}{R_f} \\ 0 & -\frac{1}{R_f} - \frac{1}{R_o} \end{bmatrix}}{\begin{bmatrix} \left(-\frac{1}{R_s} - \frac{1}{R_i} - \frac{1}{R_f}\right) & \frac{1}{R_f} \\ \frac{1}{R_f} - \frac{A}{R_o} & -\frac{1}{R_f} - \frac{1}{R_o} \end{bmatrix}} = \frac{\frac{1}{R_s}\left(-\frac{1}{R_f} - \frac{1}{R_o}\right) V_s}{\left(-\frac{1}{R_s} - \frac{1}{R_i} - \frac{1}{R_f}\right)\left(-\frac{1}{R_f} - \frac{1}{R_o}\right) - \left(\frac{1}{R_f}\right)\left(\frac{1}{R_f} - \frac{A}{R_o}\right)}$$

$$V_1 = \frac{-\frac{1}{R_S R_S} - \frac{1}{R_S R_O}}{-\frac{1}{R_S R_f} - \frac{1}{R_i R_f} - \frac{1}{R_f^2} + \underline{\frac{1}{R_S R_o}} + \frac{1}{R_i R_O} + \underline{\frac{1}{R_o R_f}} - \frac{1}{R_f^2} + \underline{\frac{A}{R_o R_f}}} V_s.$$

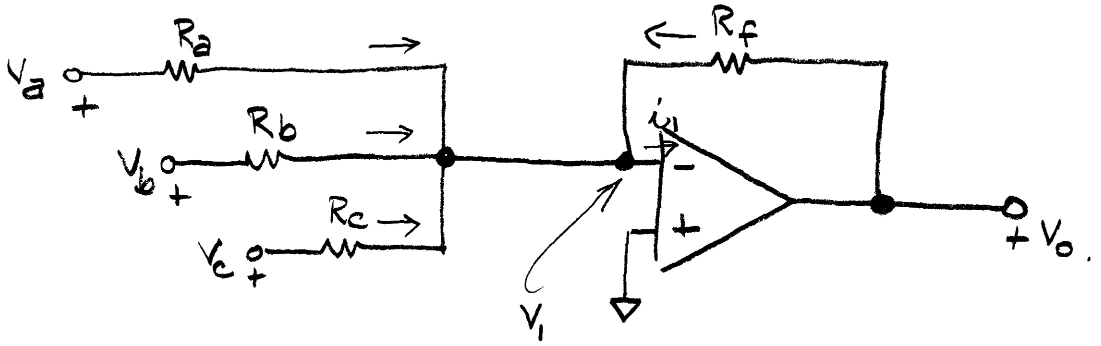
as $A \rightarrow \infty$ $V_1 \rightarrow 0.$

(c)



kcL@a + in $\quad + \underline{V_s + \epsilon}$

6.4. The summing amplifier



KCL at inverting input terminal since $V_1 \to 0$

$$\frac{\overset{\checkmark}{V_a} - V_1}{R_a} + \frac{\overset{\checkmark}{V_b} - V_1}{R_b} + \frac{\overset{\checkmark}{V_c} - V_1}{R_c} + \frac{V_o - V_1}{R_f} - \cancelto{0}{i_1} = 0$$

ideal op amp

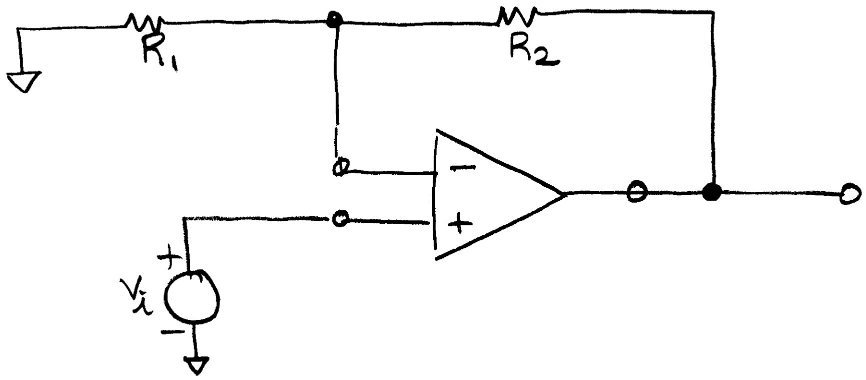
$$\therefore \quad \frac{V_a}{R_a} + \frac{V_b}{R_b} + \frac{V_c}{R_c} + \frac{V_o}{R_f} = 0.$$

$$V_o = -R_f\left(\frac{V_a}{R_a} + \frac{V_b}{R_b} + \frac{V_c}{R_c}\right).$$

suppose $R_a = R_b = R_c = R_s$.

$$V_o = -\frac{R_f}{R_s}\left(V_a + V_b + V_c\right).$$

Linear non-inverting amplifier



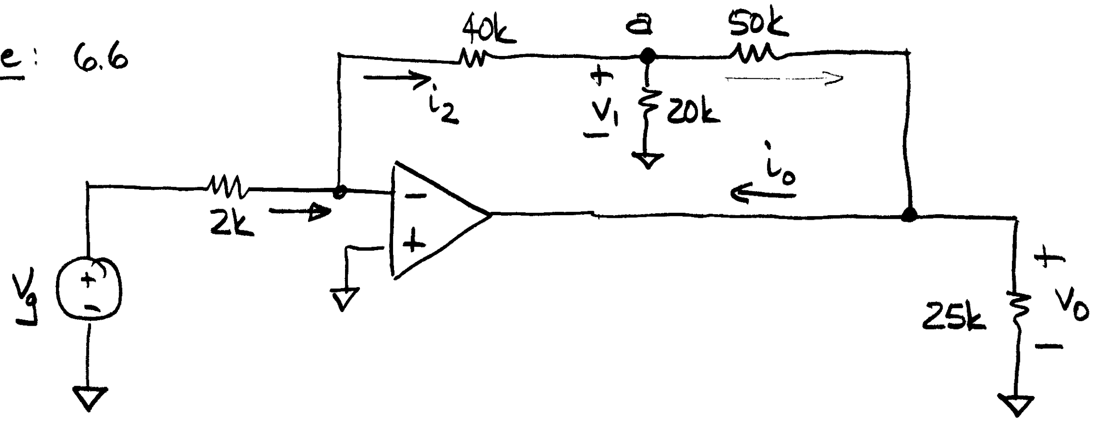
differences

$$A_v \cong 1 + \frac{R_2}{R_1}$$

$$R_{IN} \cong \frac{A\,R_i}{1 + \frac{R_2}{R_1}}$$

$$R_{OUT} \cong 0.$$

Example: 6.6



Assume ideal op-amp. $V_g = 0.15$ volt Calculate V_1, V_o, i_2.

use KCL at inverting input of op-amp. − in

$$-\frac{0.15-0}{2} + \frac{0-V_1}{40} = 0$$

$$-\frac{0.15}{2} - \frac{V_1}{40} = 0 \qquad V_1 = -3V$$

use KCL at node a. + out

$$\frac{V_1-0}{40} + \frac{V_1}{20} + \frac{V_1 - V_o}{50} = 0$$

$$\frac{V_1}{40} + \frac{V_1}{20} + \frac{V_1}{50} - \frac{V_o}{50} = 0.$$

$$V_1\left(\frac{1}{4} + \frac{2}{4} + \frac{1}{5}\right) = \frac{V_o}{5}$$

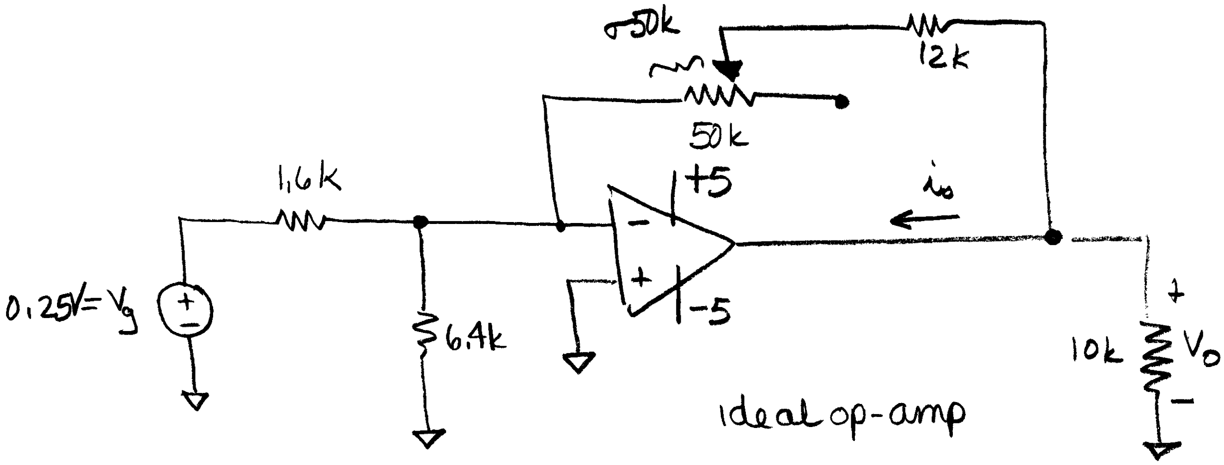
$$V_1\left(\frac{3}{4} + \frac{1}{5}\right) = \frac{V_o}{5}$$

$$V_o = 5V_1(.95) = \quad 5(-3)(.95) = -14.25$$

use Ohm's Law

$$i_2 = \frac{0-V_1}{40k} = \frac{-(-3)}{40k} = 0.075\,mA = 75\,\mu A.$$

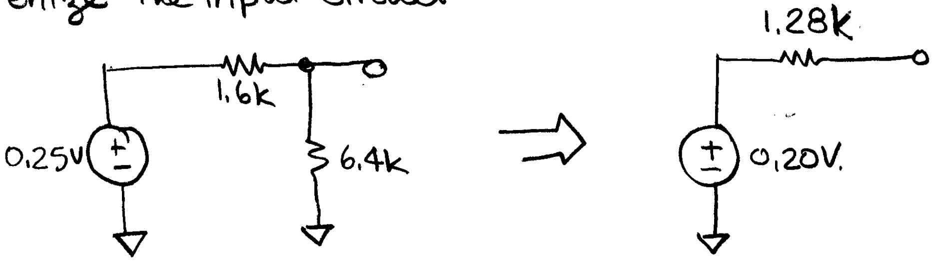
6.8



(a) Find the range of σ for which the op-amp does not saturate.

(b) Find i_o when σ = 0.272

Thevenize the input circuit

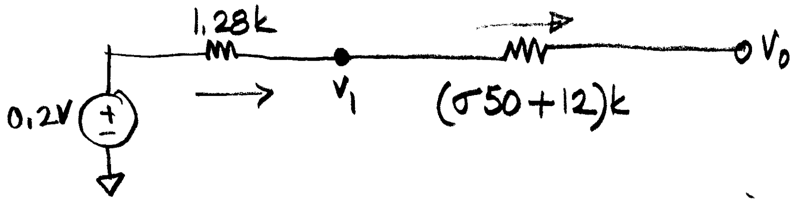


$$V_{oc} = \frac{6.4}{1.6+6.4}\,0.25 = \frac{6.4}{8.0}\,0.25 = 0.20\text{V.}$$

$$i_{sc} = \frac{0.25\text{V}}{1.6\text{k}} = 0.15625 \text{ mA}$$

$$R = \frac{V_{oc}}{i_{sc}} = 1.28\text{k}$$

Then

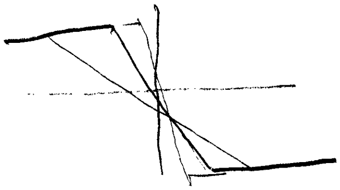


using KCL at the inverting input.

$$\frac{0.2-0}{1.28\text{k}} = \frac{0-V_0}{(\sigma 50+12)\text{k}}$$

$$V_0 = -\frac{(12+\sigma 50)}{1.28}\,0.2\text{ V.}$$

6.8 (cont.) the amplifier will saturate at ± 5V,



$$V_o = -5V = -\frac{(12+\sigma 50)}{1.28} 0.2$$

solving for σ $\sigma = 0.4$

$\therefore$ operational amplifier will not saturate for $0 \leq \sigma \leq 0.4$

When $\sigma = 0.272$

$$V_o = -\frac{(12+.272(50))}{1.28}(0.2) = -4V.$$

$\Rightarrow$ using KCL at output node.

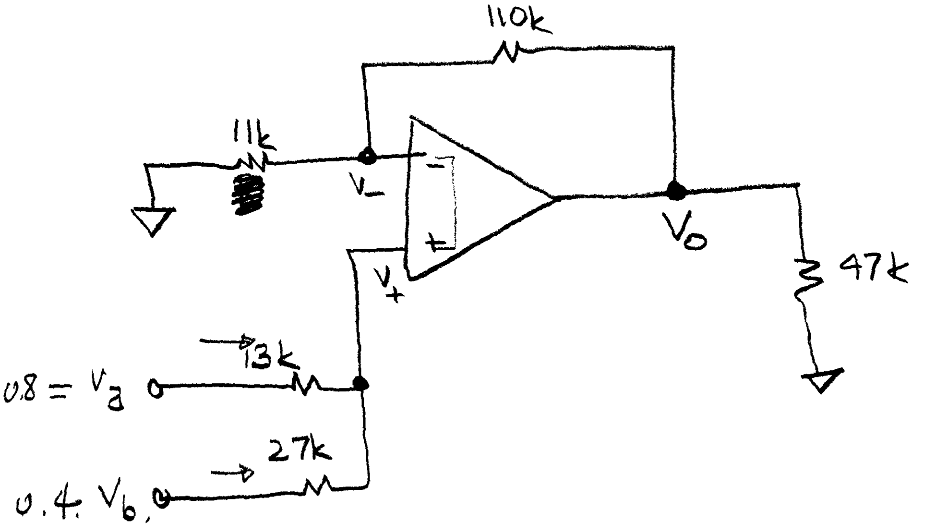
$$-\frac{0 - V_o}{(.272)50+12} + i_o + \frac{V_o}{10} = 0$$

$$\frac{V_o}{25.6} + i_o + \frac{V_o}{10} = 0.$$

$$i_o = -\frac{V_o}{10} - \frac{V_o}{25.6} = 0.556 \text{ mA}.$$

6.21 The op-amp is ideal $V_a = 0.8\text{ V}$,
$V_b = 0.4\text{V}$.

Calculate v_o, i_a, i_b



ideal so $V_+ = V_- = V_g$ source.
input current $= 0$

since $i_{IN} = 0$.
$$V_- = \frac{11k}{11k + 110k} V_o$$

but
$$V_+ = V_- = \frac{11k}{11k + 110k} V_o.$$

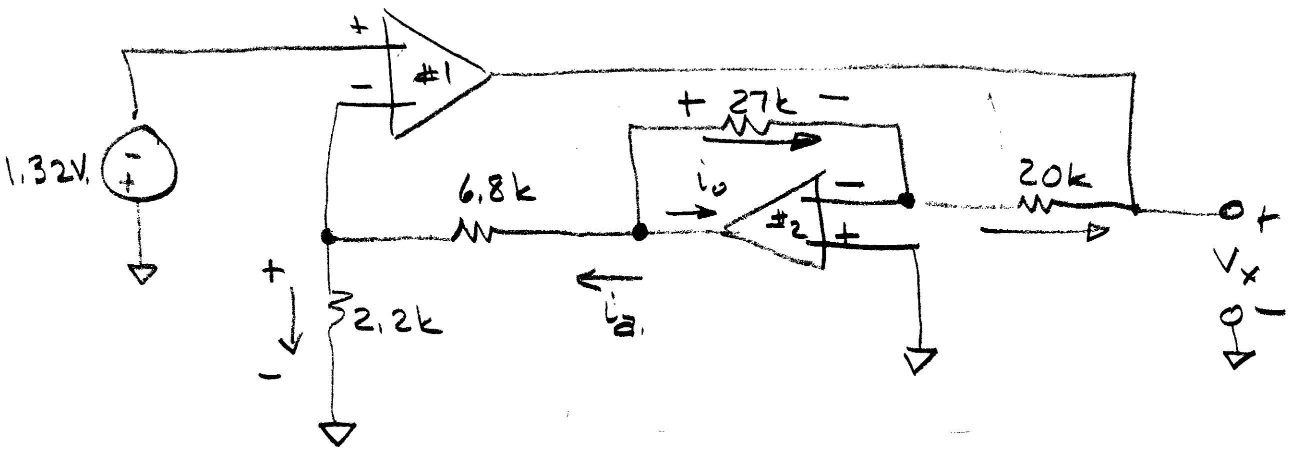
$$V_o = \frac{110k + 11k}{11k} V_+ \Rightarrow V_o = \frac{121}{11} V_+ = 11\ V_+$$

What is V_+?

Use KCL
$$\frac{V_a - V_+}{13k} + \frac{V_b - V_+}{27k} = 0.$$
$$27V_a - 27V_+ + 13V_b - 13V_+ = 0.$$
$$27V_a + 13V_b = 40V_+$$
$$V_+ = \frac{27}{40} V_a + \frac{13}{40} V_b$$
$V_+ = V_-$

$V_o = 11\ (V_+) = 7.37$ volts.

6.39 op amps are ideal.



reasoning V_+ #1 = − 1.32V.

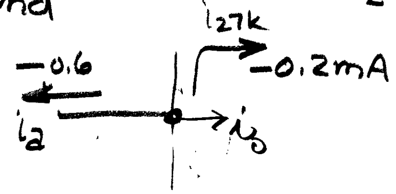
V_- #1 = − 1.32V.

⟹ current thru 2.2k resistor is $\frac{-1.32V}{2.2k} = -0.6mA$

by inspection $i_a = -0.6mA$.

V_o #2 = $-0.6mA\,(2.2k + 6.8k) = -5.4V$.

i_{27k} resistor using virtual ground $\frac{(-5.4V) - (0)}{27k} = -0.2mA$



$i_o + i_a + i_{27k} = 0$

$i_o = -i_a - i_{27k}$

$= -(-0.6) - (-0.2)$

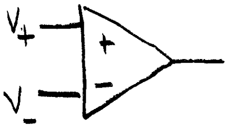
$= +0.8mA$.

What's V_x? $V_- = V_x + i_{27k}\,20k$.

$0 = V_x + (-0.2mA)(20k) = 4V$.

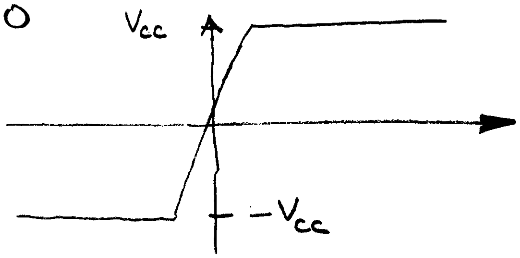
comparator – determine which of two signals is larger
– know when a signal exceeds a given threshold

simplest comparator – high-gain differential amplifier

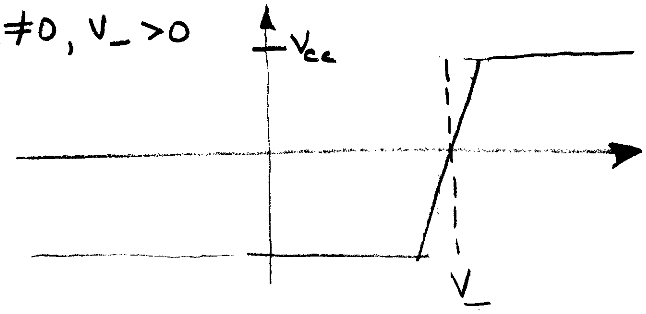


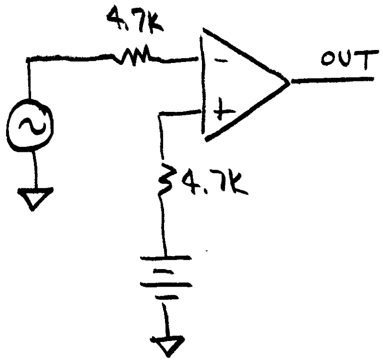
$V_{OUT} = A(V_+ - V_-)$

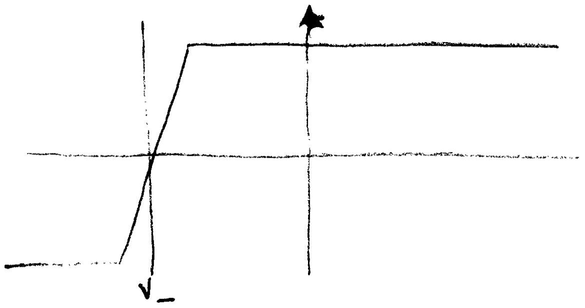
if $V_- = 0$



if $V_- \neq 0$, $V_- > 0$





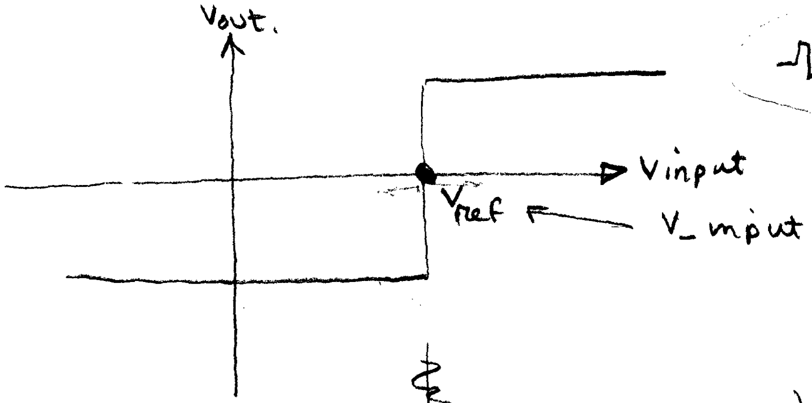


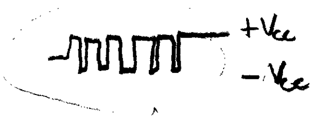
excellent for alarms

op-amps are usually not good comparators – use specialized IC chips such as LM311 with better output circuits

Never use negative feedback for a comparator – unstable.
need high gain for comparator
lower gain reduces comparator action

problems with noise

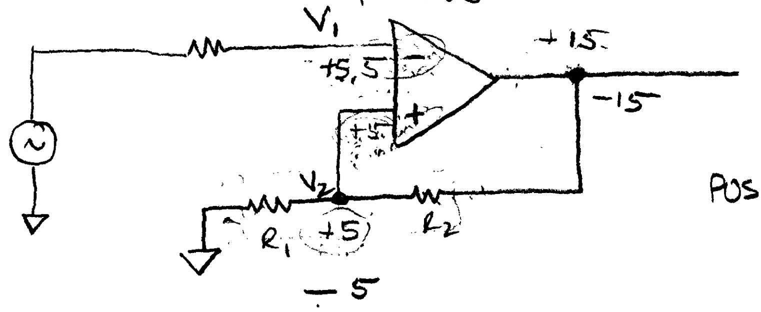




Comparator requires clean signal.

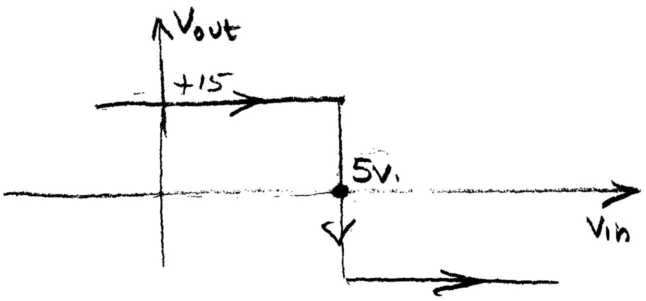
suppose you have 5V input with 0.1 volt noise

how to handle noise – use $\overset{\text{positive}}{\wedge}$ feedback to get hysteris.

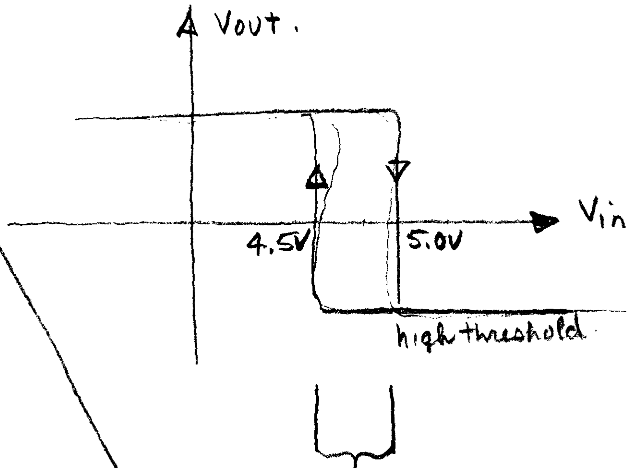


POSITIVE FEEDBACK!

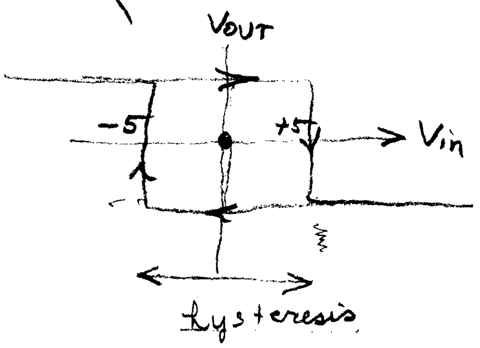
no feedback.



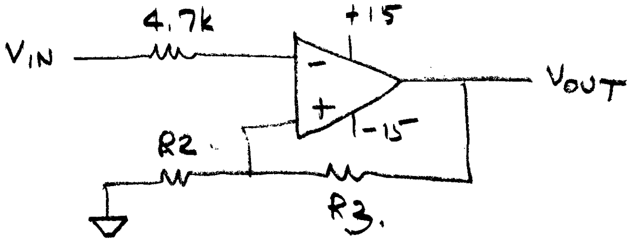
V_1 case 1 < 5 V.
case 2 = 5V.
case 3 > 5V.



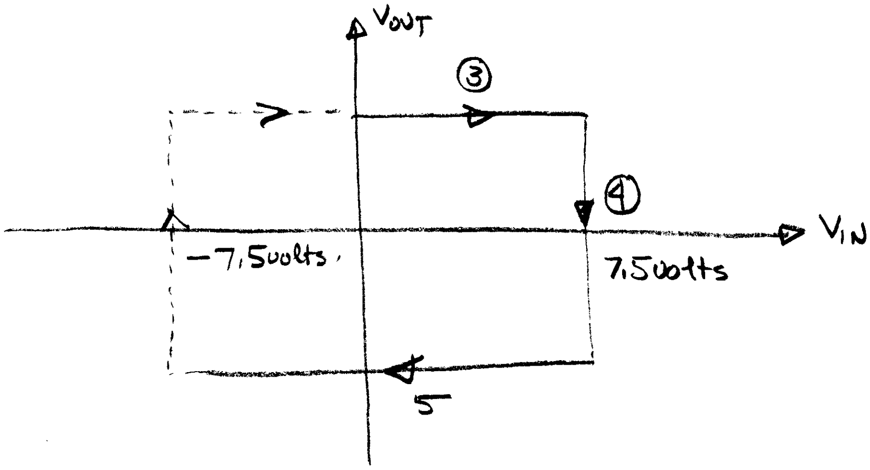
this is hysteresis



Schmitt trigger



1. If $V_{IN} = 0$, $V_{OUT} = +15$.
2. If $R_2 = R_3$, then $V_+ = 7.5$ volts forcing $V_{OUT} = 15$ volts
3. If the input voltage V_{IN} rises to +7.5 volts, no change in V_{OUT}
4. When input rises to 7.50001 volts, output switches to −15. and V_+ changes to −7.5 V,
5. V_{IN} drops to − 7.5 volts.



You can change switch points by changing ratio of R_2 and R3,

i.e. $V_{switch} = \dfrac{R_2}{R_2 + R_3}$

EEAP 243　　　　Lab Work:
Lab 8　　　　Questions Due:

NON-LINEAR OP-AMP CIRCUITS

READING ASSIGNMENT: Horowitz, pgs. 124-127.

Abstract:
This laboratory will examine the principles and operation of non-linear op-amp circuits. The Schmitt trigger exhibits hysteresis and is often used to "square up" analog signals. The comparator is often used to monitor the presence or absence of analog signals. Effectively, these circuits convert analog signals into 0's or 1's. The performance of these circuits in the presence of simulated "noise" will be examined.

As a pre-lab exercise, you should determine the switching points and output voltages for the comparator and Schmitt trigger circuits and the voltage and resistor values needed to modify the Schmitt trigger. Remember to connect $+V_{CC}$ to +15 V and $-V_{CC}$ to -15 V.

Part 1 - The Comparator and Noise

When an op-amp is used without feedback, it is said to be "open-loop". Under these conditions the gain of the amplifier is $A(IN^+ - IN^-)$ where A is on the order of 100,000. For $(IN^+ - IN^-) > +150\ \mu V$, the theoretical output is higher than $+V_{CC}$ and the output will saturate at +15 V. For $(IN^+ - IN^-) <$ $-150\ \mu V$, the theoretical output is lower than $-V_{CC}$ and the output will saturate at -15 V. The region of linear operation is very small and, for the most part, the amplifier's output will saturate at either +15 V or -15 V, depending on which of its inputs is greater. This type of circuit is known as a "comparator" because it compares the two input voltages to see which is greater and produces an output which is one of two states (+15 V or -15 V), depending on the result of the comparison. The circuits used in this part compare the input voltage at the inverting input to a "reference" voltage at the non-inverting input. Other configurations are possible; the reference may be on the inverting input and both the reference and input voltages may be any voltage between $+V_{CC}$ and $-V_{CC}$.

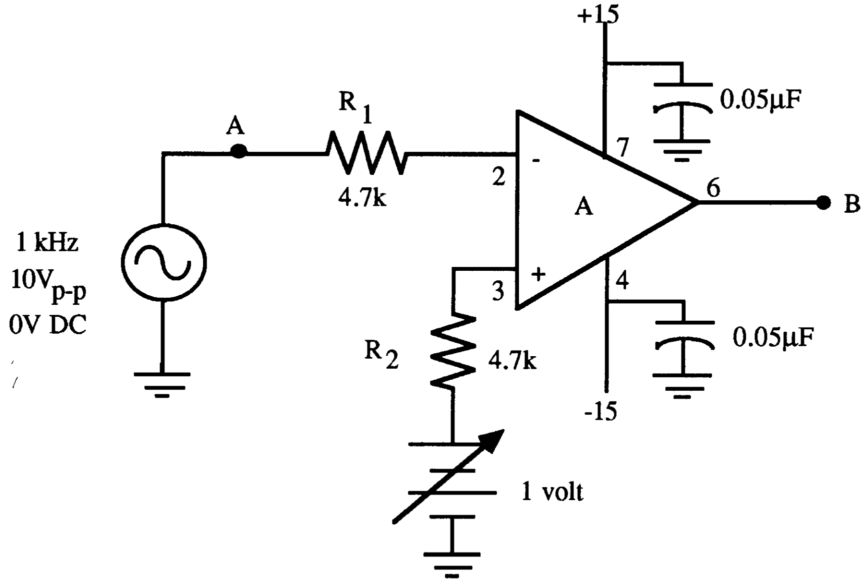


Figure 1 - Comparator circuit

(1) Build the comparator circuit of Fig.1.

(2) Display points A and B simultaneously on the oscilloscope and record each waveform in Table 8.1.

(3) Carefully note the phase difference between A and B and note the two switching points.

(4) Set the DC offset to +3 V. To do this, set the scope channel connected to point A to 2 V/cm. A +3 V DC offset will make the triangle wave jump up 1.5 cm when the mode is switched from AC to DC.

(5) Record the waveforms in Table 8.2 and make note of the phase difference between A and B.

(6) Repeat step 5 for a DC offset of -3 V.

A "noisy" signal can be thought of as the sum of a desired signal and other, unwanted signals. The result is a signal which generally follows the desired one, but which deviates from it. If a noisy signal is fed into a comparator, the noise will cause unwanted changes of state at the output as the desired signal crosses zero volts.

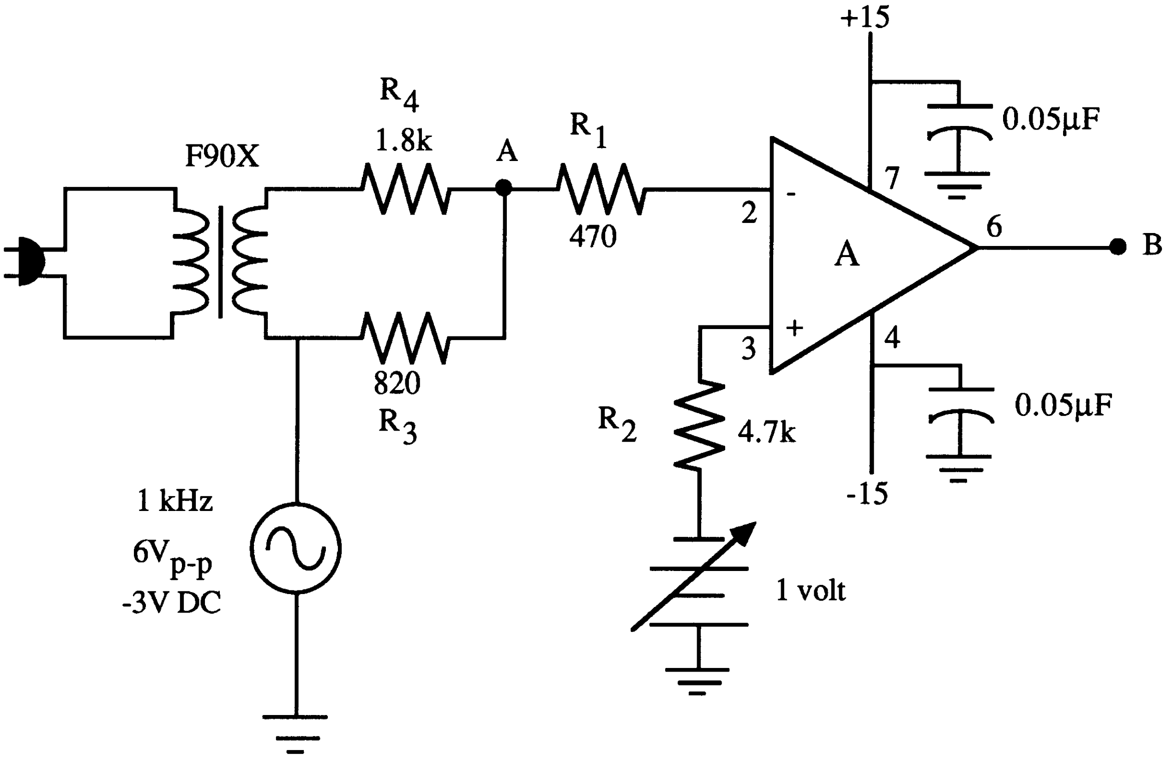


Figure 2 - Signal + "Noise" generator

(1) Disconnect the signal generator from your circuit and adjust its output to that shown in Fig.2.

(2) Build the circuit shown in Fig.2. Plug in the transformer. The transformer produces a sine wave at 60 Hz, which is added to the 1 KHz from the signal generator.

(3) If the composite signal at A is not around 14 V_{p-p} you have not connected the circuit properly.

(4) Display the waveforms at A and B and record them in Table 8.3 making special note of the phase difference between them. It will not be possible to get an absolutely stable trace on the oscilloscope.

(5) Make note of exactly what takes place when the waveform at B changes state.

Part 2 - The Schmitt Trigger and Noise

The amplifiers you built in Lab 6 and 7 used negative feedback - a resistor feeding the output signal back to the inverting input. By using positive feedback - a resistor feeding the output back to the non-inverting input - the two switching points can be moved further apart. Consider the circuit of Fig.3 and assume that the input is zero, that the output is +15 V, and that $R_2 = R_3$. R_2 and R_3 set the voltage at the non-inverting input to +7.5 V, $(IN^+-IN^-) > 0$, and the output is indeed forced to +15 V. If we raise the input to +7.50015 V, then $A(IN^+-IN^-) = -15$ V and the output switches to -15 V. This changes the voltage at the non-inverting input to -7.5 V which makes (IN^+-IN^-) very negative, maintaining the output at -15 V. The output will stay at -15 V until the input drops to 7.50015 V which will make $A(IN^+-IN^-) = +15$ V, setting the output to +15 V, setting the non-inverting input to +7.5 V. This makes (IN^+-IN^-) very positive, maintaining the output at +15 V. We can summarize the operation of the circuit by saying that, when the input goes above the upper switching point, the output will go negative and stay negative until the input goes below the lower switching point. This characteristic is called "hysteresis" and this type of circuit is called a "Schmitt trigger".

PLEASE CALL A TEACHING ASSISTANT TO CHECK YOUR DATA BEFORE CONTINUING.

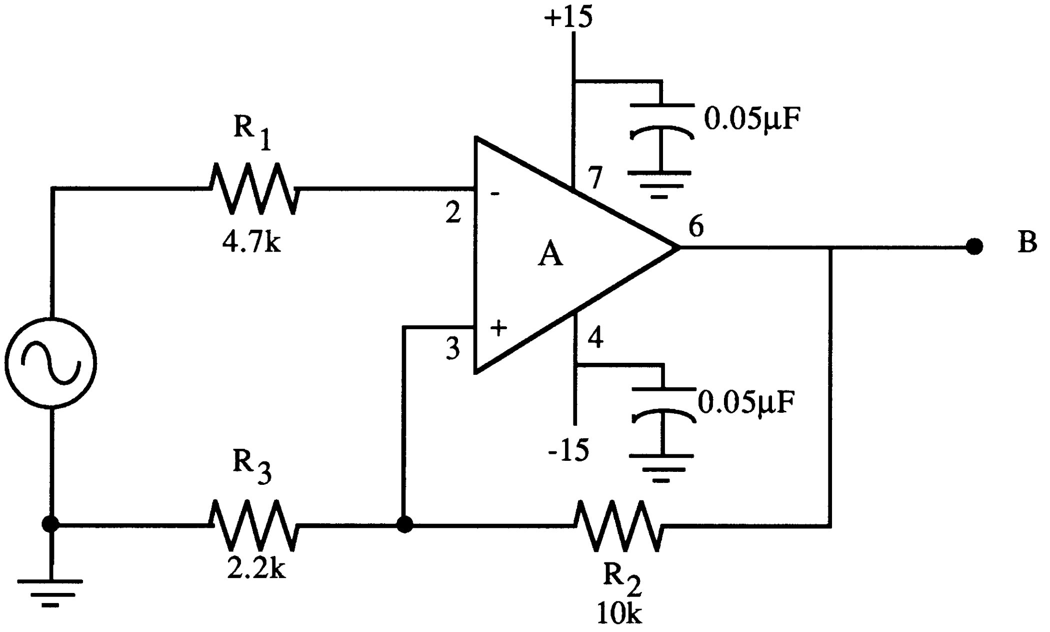


Figure 3 - Schmitt trigger

(1) Build the Schmitt trigger circuit of Fig. 3 with R_2 = 10K and R_3 = 2.2K.
(2) Display points A and B simultaneously on the oscilloscope and record each waveform in Table 8.4.
(3) Carefully note their phase difference and the two switching points.
(4) Slowly vary the DC offset between +3 V and -3 V and observe what happens at point B.

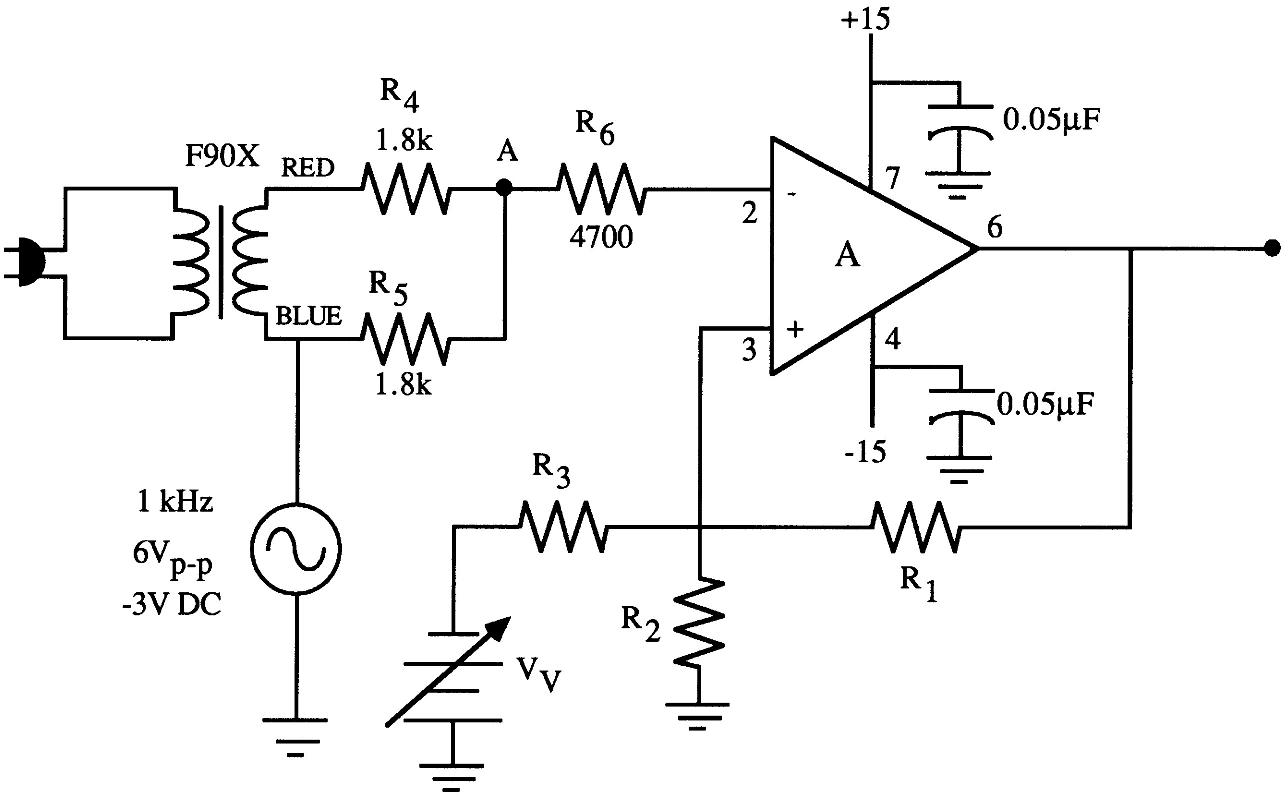


Figure 4

The Schmitt trigger can be used to eliminate signal noise.

(1) Disconnect the signal generator from your circuit and set it as shown in Fig. 4.
(2) Build the circuit shown in Fig. 4 and plug in the transformer. Select a variable supply voltage and standard values for R_1, R_2, and R_3 so that the switching points are +1 V and -7 V.
(3) If the composite signal at A is not around 14 $V_{p\text{-}p}$ you have not connected the circuit properly.
(4) Display the waveforms at A and B and record them in Table 8.5 making special note of their phase difference. It will not be possible to get an absolutely stable trace on the oscilloscope.
(5) Make note of exactly what takes place when the waveform at B changes state.

Questions:

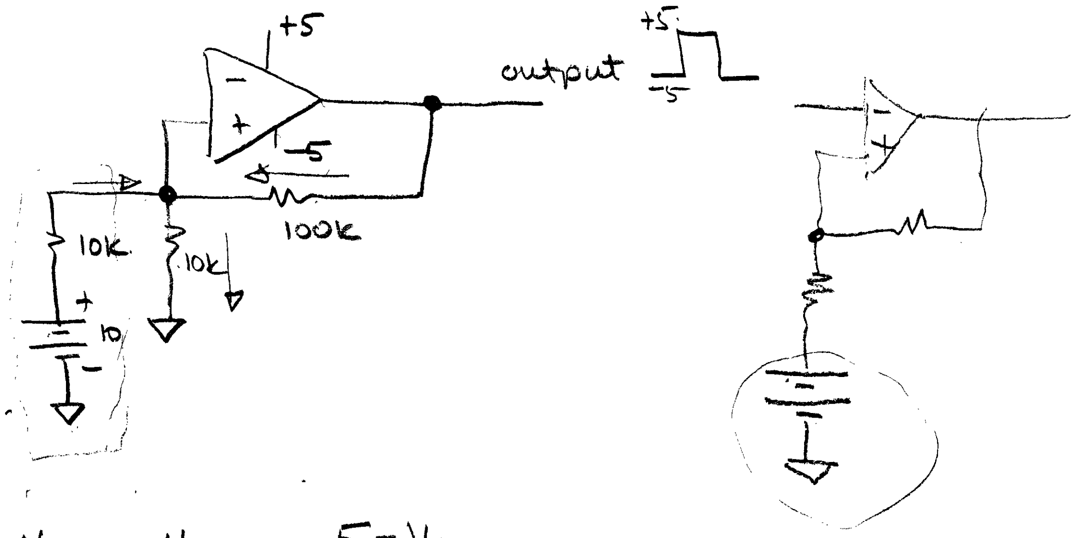
1. (a) What are the two switching points?
   (b) Describe what point B does as the triangle wave goes from its lowest to highest and back to its lowest voltage.
   (c) Explain why the relative phase acts as it does when you change the DC offset.

2. (a) Describe the input waveform.
   (b) Describe why point B changes state several times before stabilizing.
   (c) Why does this sort of comparator have problems with a "noisy" signal?

3. (a) Calculate the switching points without using your experimental data.
   (b) What switching points are indicated by the scope?
   (c) Why aren't they the same as your calculated values?
   (d) Explain why the signal at point B acts as it does when you change the DC offset.

4. (a) Show your calculations for R_1, R_2, R_3, and V_v.
   (b) How is the output different from that seen in Part 1?
   (c) Why is a Schmitt trigger better at handling signals which contain undesirable noise?
   (d) Are both R_2 and R_3 really needed?

sample calculation



+in $$+\frac{10-V_+}{10k} - \frac{V_+}{10k} + \frac{5-V_+}{100k} = 0.$$

$$100 - 10V_+ - 10V_+ + 5 - V_+ = 0.$$

$$105 - 21V_+ = 0.$$

$$V_+ = \frac{105}{21.}$$ for positive output.

$$\frac{10-V_+}{10k} - \frac{V_+}{10k} + \frac{-5-V_+}{100k} = 0.$$

$$100 - 10V_+ - 10V_+ - 5 - V_+ = 0$$

$$95 - 21V_+ = 0.$$

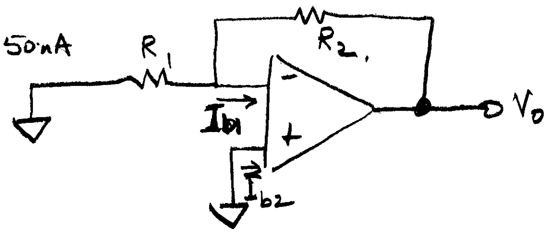
$$V_+ = \frac{95}{21}$$

Real op amps

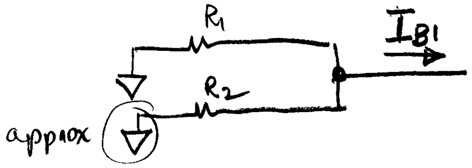
$\boxed{\text{symmetry}}$ if you ground V_- and V_+ you expect 0 output. This is NOT true with real op-amps.

Input offset current: difference in input currents when you make $V_O = 0$. typically 50pA–50nA

bias current in input transistors.

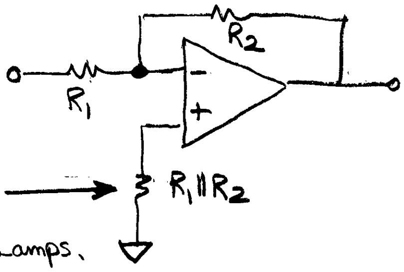


If V_o small, $V_d = V_+ - V_- \cong -I_{b1} R_1 \| R_2$ differential input voltage



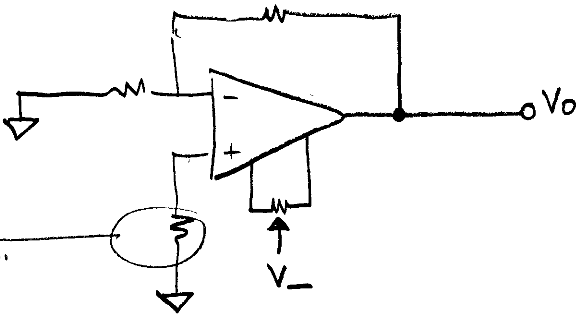
output offset voltage $V_o = A\, V_d$. $\leq 1\text{mV}$ in practice

practical solution connect a resistor $R_p = R_1 \| R_2$ from V_+ to ground.



compensating resistor for BJT input op-amps.

input offset voltage

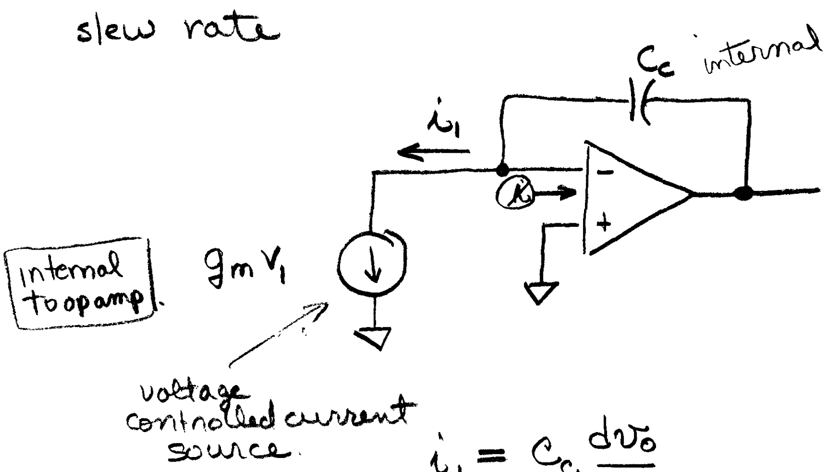


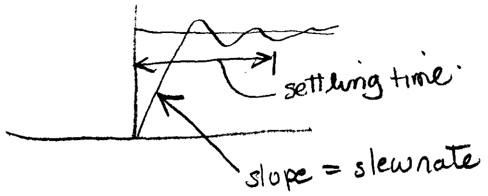
not needed with FET, etc.

voltage that must be applied to make $V_o = 0$ when $V_d = 0$.

unity gain bandwidth

slew rate





$$i_1 = C_c \frac{dv_o}{dt}$$

$$\text{S.R.} \triangleq \frac{dv_o}{dt}$$

$$\Rightarrow \quad \text{S.R.} = \frac{i_1}{C_c}$$

for the 741 $C_c = 30\text{pf}$. $i_1 = 20\mu A$.

$$SR = \frac{20 \times 10^{-6}}{30 \times 10^{-12}} \cong 0.67 \, V/\mu\text{sec}.$$

general purpose op-amp

741, 702, 709

Bi-FET FET's for input, BJT's elsewhere.

LF351, etc.

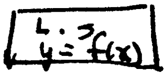
Special

AD380 high slew rate 500V/μsec.

TEXT PP 379 — 409 excluding 395-398 2/17/93

①

we were discussing linear systems at our last meeting



example [illegible]

$$x = a_0 \frac{dy}{dt} + a_2 \frac{d^2y}{dt^2} + \cdots + a_N \frac{d^Ny}{dt^N}$$

source response.

if x_1 is the source, we find the response is y_1
if Ax_1 is the source, we find the response is Ay_1

$$A(x_1 = a_1 \frac{dy_1}{dt} + a_2 \frac{d^2y_1}{dt^2} + \cdots + a_N \frac{d^Ny}{dt^N})$$

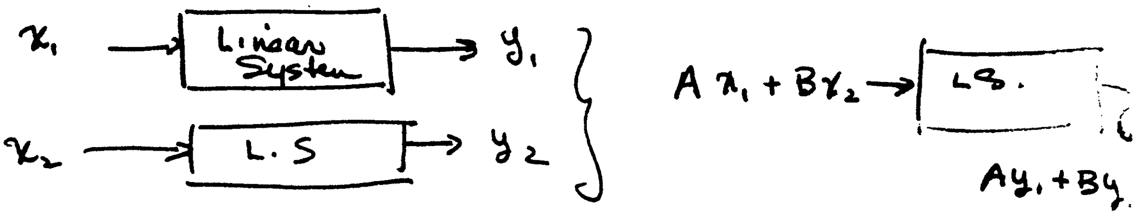
if x_2 is the source we then find the response is

then if Bx_2 is the source " " " " " is By
If $Ax_1 + Bx_2$ is the source

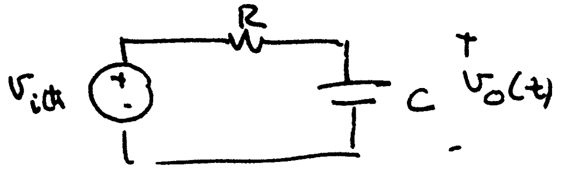
$$Ax_1 + Bx_2 = a_1\left[A\frac{dy_1}{dt} + B\frac{dy_2}{dt}\right] + a_2\left[A\frac{dy_1}{dt} + B\frac{dy_2}{dt}\right] + \cdots$$

hence it has the solution $Ay_1 + By_2$

Summary



Now how does that relate to the example we had last week?



, $v_i(t) = RC\frac{dv_o}{dt} + v_o$

we found that

if $v_i(t) = V_s \cos\omega t$,
then trying $v_o = A\cos\omega t + B\sin\omega t$
we found $V_o(t)$

$$v_o(t) = \frac{V_s}{1+(RC\omega)^2}\cos\omega t + \frac{V_s(RC\omega)}{1+(RC\omega)^2}\sin\omega t$$

2/17/93

If we used the imaginary source

$$v_i'(t) = V_s e^{j\omega t} \quad \text{to find } v_o'(t) = ?$$

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

we found

$$v_o'(t) = \left(\frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}\right) V_s e^{j\omega t} = \frac{V_s(1 - jR\omega C)\, e^{j\omega t}}{(R\omega C)^2 + 1}$$

~~$e^{j\omega t}$~~

$$v_o'(t) = \frac{V_s}{1+(RC\omega)^2}\cos\omega t + \frac{V_s R\omega C \sin\omega t}{1+(RC\omega)^2}$$

$$+ j\left[\frac{V_s}{1+(RC\omega)^2}\sin\omega t - \frac{V_s R\omega C}{1+(RC\omega)^2}\cos\omega t\right]$$

& observed that the real part of $V_o'(t) \Rightarrow \text{Re}\{v_o'(t)\}$ is exactly the correct solution of the $V\cos\omega t$ source.

$$\text{Note} \quad v_i'(t) = \underbrace{\overset{\text{real input}}{\boxed{V_s}}\cos\omega t}_{\text{input 1}} + \underbrace{\overset{\text{extra input}}{jV_s\sin\omega t}}_{\text{input 2}} \quad \text{to L.S. } \underbrace{\text{L.T.I}}_{\text{linear, time invari.}}$$

the j acts like a tag and produces an output which has j multiplying it.

Rules

① If we have a source $v_i(t) = \hat{V}_s\cos\omega t$

② change it to $v_i'(t) = \hat{V}_s e^{j\omega t}$

③ then solve LTI system

④ the real part of the solution is due to the ~~real~~ source $\cos\omega t$

New Rules

① if we have a source $v_i(t) = \hat{V}_s \sin\omega t$

② we change it to $v_i'(t) = \hat{V}_s e^{j\omega t} = V_s\cos\omega t + j\hat{V}_s \sin\omega$

③ then ~~solve~~ find LTI solution, v_o'

④ Take the imaginary part of the solution

General Rule

$$v_i(t) = \hat{V}_s\cos(\omega t + \phi) = \cancel{\hat{V}_s e^{j\omega t + j\phi}}$$

+j

③ 2/17/93

Then we can use $v_i'(t) = V_s e^{j(\omega t+\phi)} = \underbrace{V_s e^{j\phi}}_{\hat{V}_s} e^{j\omega t}$

as a source, again taking the real part of the result

Phasors

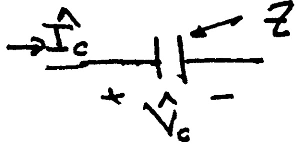
s.s.s.s with $e^{j\omega t}$ sources

The terminal relations

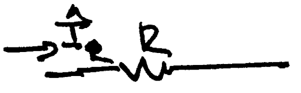
$$i_c = C\frac{dv_c}{dt} \Rightarrow \hat{I}_c e^{j\omega t} = j\omega C \hat{V}_c e^{j\omega t}$$

$\hat{I}_c$ & $\hat{V}_c$ are called phasors.

$$\hat{I}_c = j\omega C \hat{V}_c \quad \text{or} \quad \hat{V}_c = \left(\frac{1}{j\omega C}\right)\hat{I}_c$$

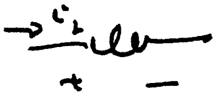


$Z = \frac{1}{j\omega C}$, $\quad \hat{V}_c = Z I_c$

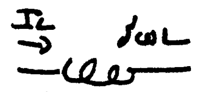


$\hat{V}_R = R\hat{I}_R$

analogous.



$v_L = L\frac{di_L}{dt}$ $\Rightarrow$

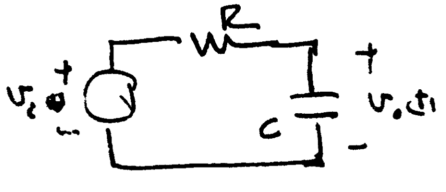


$\hat{V}_L = (j\omega L)\hat{I}_L$

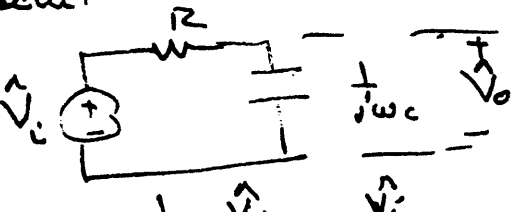
$\hat{i}_L = \hat{I}_L e^{j\omega t}$

$\hat{v}_L = \hat{V}_L e^{j\omega t}$

Now let's return to our circuit



$\Rightarrow$



$$\hat{V}_o = \frac{\frac{1}{j\omega C}\hat{V}_i}{\underbrace{R+\frac{1}{j\omega C}}_{\text{voltage divider!}}} = \frac{\hat{V}_i}{1+jR\omega C}$$

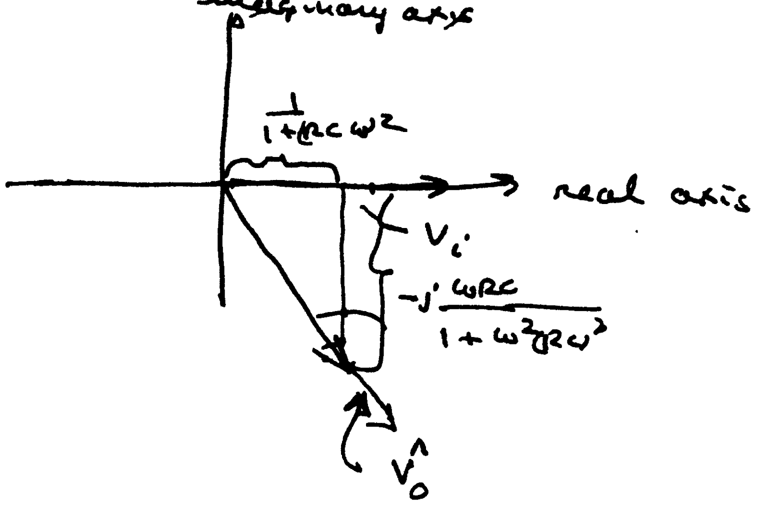
$$\hat{V}_o = \left(\frac{1-j\omega RC}{1+(R\omega C)^2}\right)\hat{V}_i$$

then $v_o'(t) = \hat{V}_o e^{j\omega t}$, $\quad v_o(t) = \text{Re}\{v_o'(t)\}$

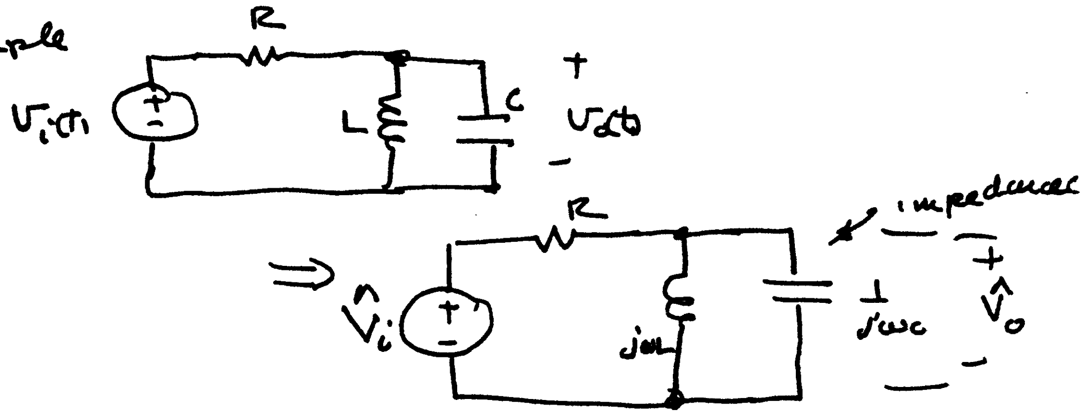
2/1/93

Phasors

Let $\hat{V}_i = V_i$ (real)



Example



use voltage divider

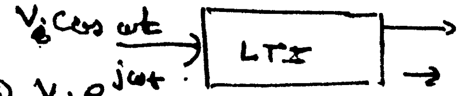
$$\hat{V}_o = \hat{V}_i \frac{(j\omega L \| \frac{1}{j\omega C})}{R + (j\omega L \| \frac{1}{j\omega C}} =$$

$$j\omega L \| \frac{1}{j\omega C} = \frac{(j\omega L)(\frac{1}{j\omega C})}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1-\omega^2 LC}$$

so $$V_o = \frac{V_i \left(\frac{j\omega L}{1-\omega^2 LC}\right)}{\left(R + \frac{j\omega L}{1-\omega^2 LC}\right)}$$

2/19/93 - EEAP 243

We have been studying the sinusoidal-steady-state ~~at~~ response of linear systems.



$V_i \cos \omega t \rightarrow$ LTI $\rightarrow V_{o1} \cos\omega t + V_{o2} \sin \omega t$

$\Rightarrow V_i e^{j\omega t} \rightarrow$ ~~$\hat{V}_o \cos\omega t$~~ $\hat{V}_o e^{j\omega t}$

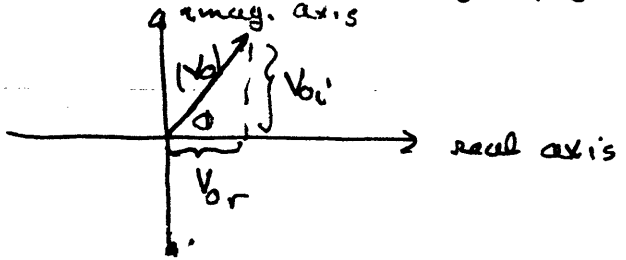
true signal $\mathrm{Re}\{V_i e^{j\omega t}\}$ $\rightarrow$ true response $= \mathrm{Re}\{\hat{V}_o e^{j\omega t}\}$

Two forms of complex numbers

- Rectangular $\hat{V}_o = V_{or} + j V_{oi}$

  $\mathrm{Re}\{\hat{V}_o\}$ V_{or} $\mathrm{Im}\{\hat{V}_o\} = V_{oi}$ (<u>not</u> to include j !!)

- Polar $\hat{V}_o = |V_o| e^{j\Phi} = |V_o| [\cos\Phi + j \sin \Phi]$



$V_{or} = V_o \cos\Phi$

$V_{oi} = V_o \sin \Phi$

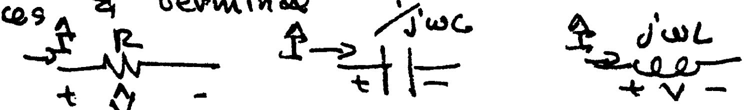
① $\mathrm{Re}\{\hat{V}_o e^{j\omega t}\} = \mathrm{Re}\{(V_{or} + j V_{oi})(\cos\omega t + j \sin\omega t)\}$

$= V_{or} \cos\omega t - V_{oi} \sin \omega t$

equivalent (equal)

② $\mathrm{Re}\{\hat{V}_o e^{j\omega t}\} = \mathrm{Re}\{V_o e^{j\Phi} e^{j\omega t}\} = V_o \cos(\omega t + \Phi)$

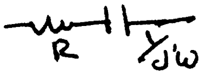
<u>Impedances</u> & terminal relations



$\hat{V} = \hat{I} R$ $\hat{V} = \hat{I}\left(\frac{1}{j\omega C}\right)$ $\hat{V} = (j\omega L) \hat{I}$

series

$Z = R + j\omega L$



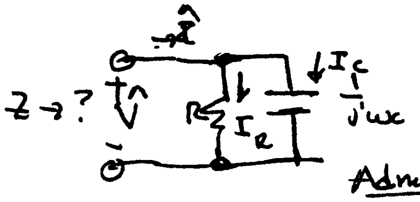
$R + \frac{1}{j\omega C} = R - \frac{j}{\omega C}$

$\mathrm{Re}\{Z\}$ called Resistance.

$\mathrm{Im}\{Z\}$ called Reactance $\begin{cases} + \text{ sign} & \text{called Inductive React} \\ - \text{ sign} & \text{called Capacitive "} \end{cases}$

Po

parallel elements

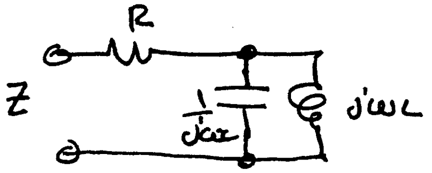


$$Z = \frac{\hat V}{\hat I} = \frac{\hat V}{\frac{\hat V}{R} + \frac{\hat V}{1/j\omega C}} = \frac{1}{\frac{1}{R} + j\omega C}$$

Admittance $$\hat Y \triangleq \frac{1}{Z} = \frac{\hat I}{\hat V} = \frac{V/R + j\omega C \hat V}{V} = \left(\frac{1}{R} + j\omega C\right)$$

Re$\{\hat Y\}$ called conductance, (here = $\frac{1}{R}$)

Im$\{Y\}$ called susceptance, (here = ωC)



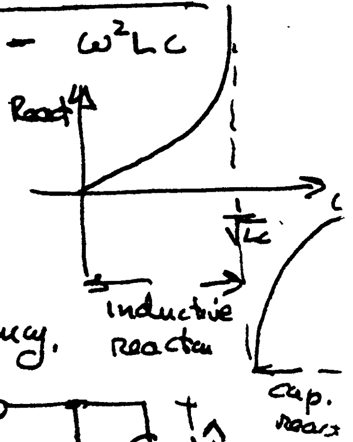
$$Z = R + \frac{1}{j\omega C} \| j\omega L$$

$$= R + \frac{L/C}{\frac{1}{j\omega C} + j\omega L}$$

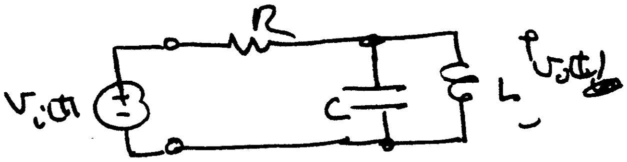
$$= R + \frac{j\omega L}{1 - \omega^2 LC}$$

Re$\{Z\}$ = R

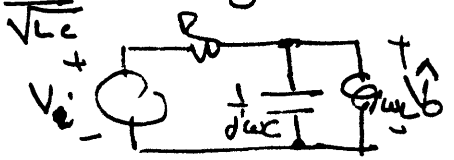
Im$\{Z\}$ = $\frac{\omega L}{1-\omega^2 LC}$



The frequency where Im$\{Z\} \to \infty$ is called the resonant frequency.
$2\pi f_{res} = \frac{1}{\sqrt{LC}}$



⇒

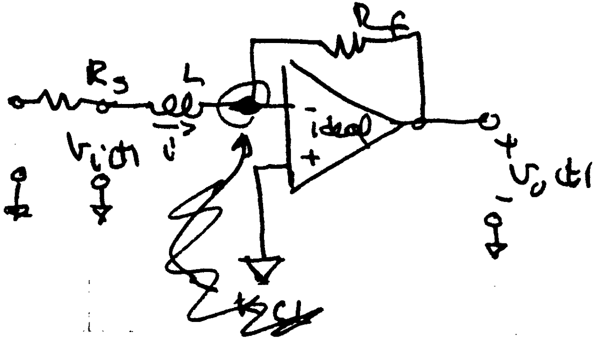


Phasor domain version

voltage divider $$\hat V_o = \frac{j\,\text{Im}(Z)}{\text{Re}\{Z\} + j\,\text{Im}\{Z\}} V_i$$

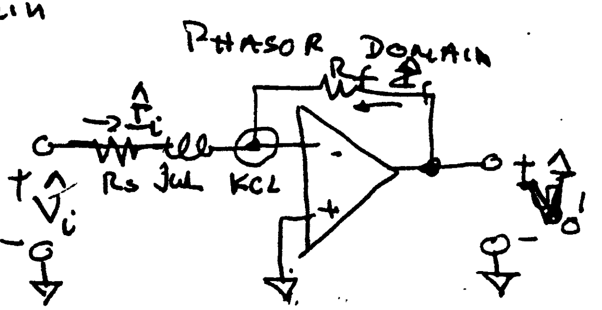
at $\omega = \omega_{res} = \frac{1}{\sqrt{LC}}$ $\hat V_o = V_i$

OP-AMP in Phasor domain



⇒

PHASOR DOMAIN



$$+\hat{I}_i + \hat{I}_f = 0$$

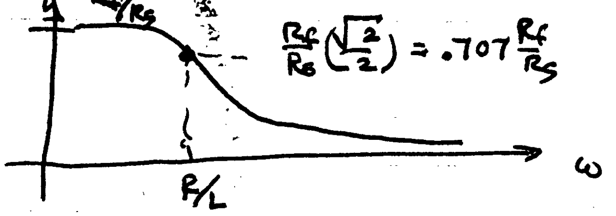
$$\hat{I}_f = -\hat{I}_i = \frac{-\hat{V}_i}{R_s + j\omega L}$$

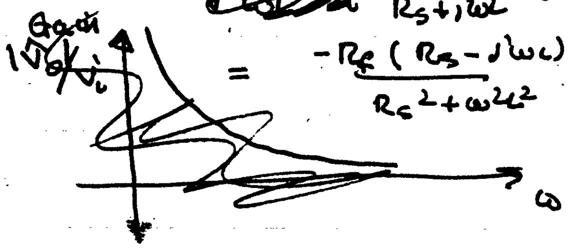
$$\hat{V}_o = + I_f R = -\frac{R_f}{R_s + j\omega L}\hat{V}_i$$

$$= \cancel{[illegible]} \; -\frac{R_f}{R_s + j\omega L} V_i$$

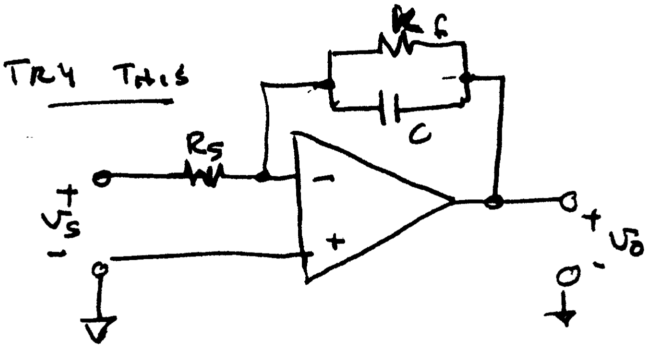
$$= \frac{-R_f(R_s - j\omega L)}{R_s^2 + \omega^2 L^2} V_i$$

$$\text{Gain} \triangleq \frac{|V_o|}{|V_i|} = \left[\frac{R_f}{R_s} \Big/ \sqrt{1 + \frac{\omega^2 L^2}{R_s^2}}\right]$$





TRY THIS



⇒ PHASOR DOMAIN VIEW OF CIRCUIT?

REVIEW

① Sinusoidal steady state

② $\cos\omega t \to e^{j\omega t}$

③ Redraw circuit in phasor domain

④ solve for "output phasor" in terms of "input phasor"

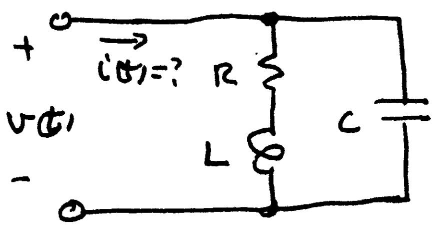
⑤ multiply output phasor by $e^{j\omega t}$

⑥ take real-part to get solution as a function of time

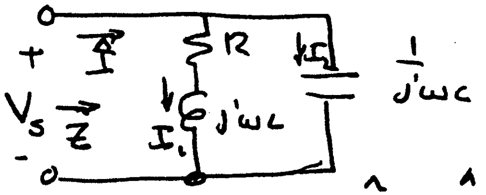
2/19/93

Another circuit

PHASOR DOMAIN



⇒



$v(t) = V_s \cos\omega t$

$\hat{I} = ? = \hat{I}_1 + \hat{I}_2$

$\hat{I}_1 = \frac{V_s}{R+j\omega L}$, $\hat{I}_2 = \hat{V}_s \times j\omega c$

$\hat{I} = V_s\left[\frac{1}{R+j\omega L} + j\omega c\right]$

$\hat{Z} = \frac{\hat{V}_s}{\hat{I}} = \frac{1}{\frac{1}{R+j\omega L}+j\omega C}$

$= \frac{R+j\omega L}{1+j\omega RC-\omega^2 LC}$

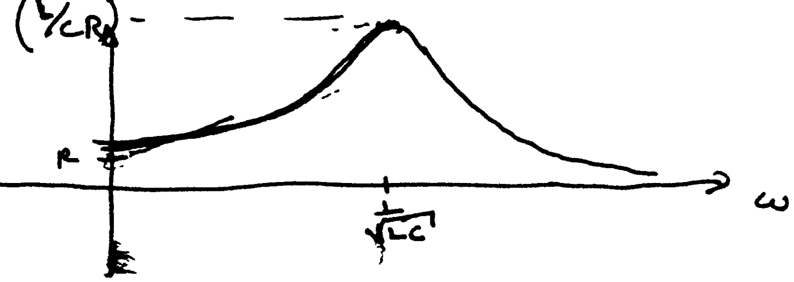
$$Z = \frac{(R+j\omega L)\left[(1-\omega^2 LC)-j\omega RC\right]}{(1-\omega^2 LC)^2+\omega^2 R^2 C^2} = \frac{R(1-\omega^2 LC)+R\omega^2 LC}{(1-\omega^2 LC)^2+\omega^2 R^2 C^2}$$

$Z = R + jX$

$$+ \frac{j\left[\omega L(1-\omega^2 LC)-\omega R^2 C\right]}{(1-\omega^2 LC)^2+\omega^2 R^2 C^2}$$

$$R = \text{Resistance} = \frac{R(1-\omega^2 LC)+R^2\omega^2 LC}{(1-\omega^2 LC)^2+\omega^2 R^2 C^2} = \frac{R}{(1-\omega^2 c)^2+\omega^2 RC^2}$$

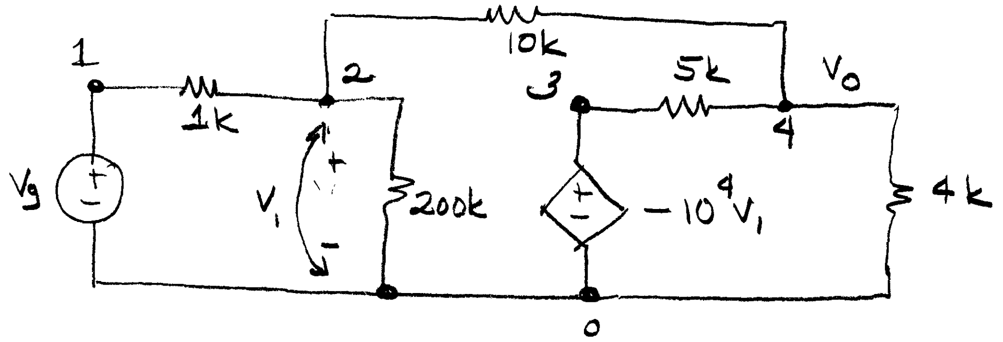
$$X = \text{Reactance} = \frac{\omega L(1-\omega^2 LC) - \omega^2 R^2 C}{(1-\omega^2 LC)^2+\omega^2 R^2 C^2}$$



for ω = 0 $j\omega L = 0$, $j\omega c = 0$

R (all that's left)

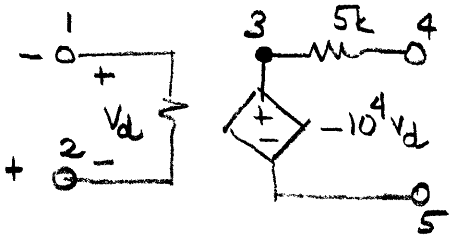
PSpice model of op amp.



```
Vg   1   0    DC   1
Rs   1   2    1000
Ri   2   0    200e3    ———> ∞
Rf   2   4    10e3
E1   3   0    2  0    -10e3 ———> -∞
Ro   3   4    5e3 ———> 0
R1   4   0    4e3
.END
```

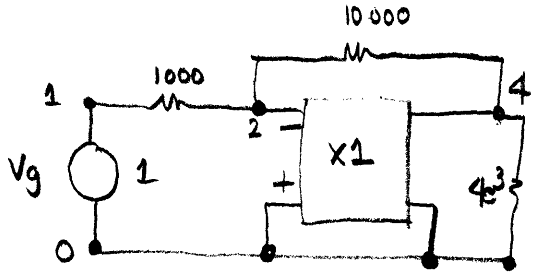
OP AMP SUBCIRCUIT

```
.SUBCKT   OPAMP   1  2  4  5
R1     2   1   200e3
E1     3   5   2   1   10e3
Ro     3   4   5e3
.ENDS  OPAMP
```



overall program

```
Vg   1   0   DC   1
RS   1   2   1000
Rf   2   4   10e3
R1   4   0   4e3
┌──────────────────────────────┐
 .SUBCKT  OPAMP  1  2  4  5
   {
 .ENDS  OPAMP
└──────────────────────────────┘
X1   2  0  4  0   OPAMP
.END.
```



using a µA741 model.

requires explicit power supplies

```
.subckt  uA741   1   2   3   4   5
                 |   |   |   |   └ output
                 |   |   |   └──── negative power supply
                 |   |   └──────── positive power supply
                 |   └──────────── inverting input
                 └──────────────── non-inverting input
```

```
X1    0   2   5   6   4
VCC1  5   0   DC  12
VEE1  0   6   DC  12

.LIB  EVAL.LIB               start
                             stop
.DC  VG   0,5  0,5  1  <──── increment.
.PROBE
.END
```

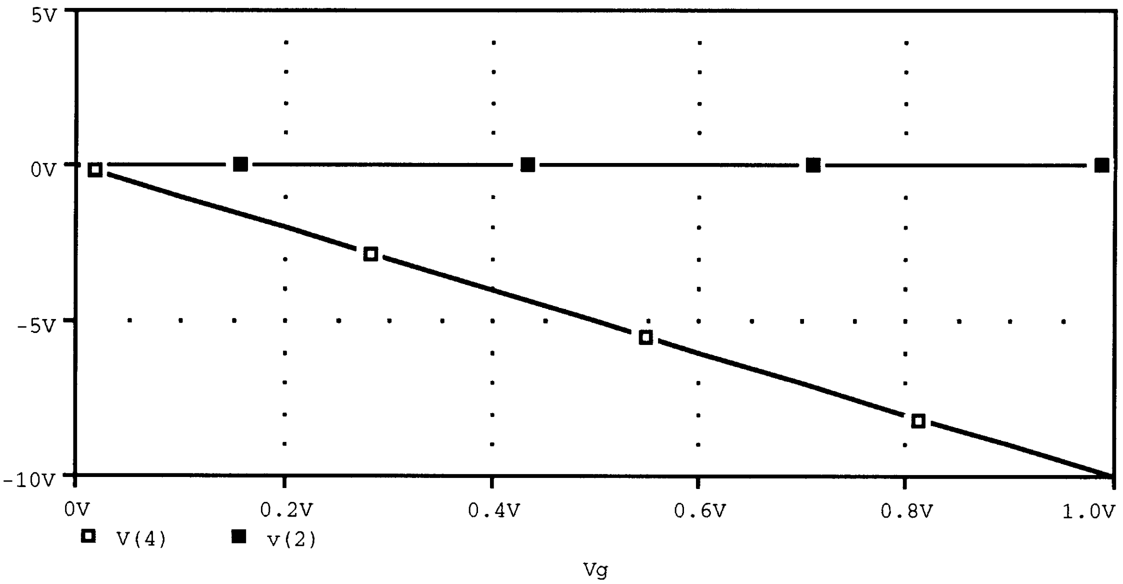
Step response.

```
                     T1 V1  T2    V2   T2       V3
V1   1   0   PWL( 0  0   1e-6  21  1000 e-6  21 )

.TRAN  20e-6  3000e-6
        #       #
```

```
**** 02/21/93 23:03:52 *********** Evaluation PSpice (Jan 1992) **************

 Simple op-amp analysis using voltage controlled voltage source


 ****     CIRCUIT DESCRIPTION


**************************************************************************
Vg    1    0    DC    1
Rs    1    2    1000
Ri    2    0    200e3
Rf    2    4    10e3
E1    3    0    2    0    -10e3
Ro    3    4    5e3
R1    4    0    4e3
.probe
.end

**** 02/21/93 23:03:52 *********** Evaluation PSpice (Jan 1992) **************

 Simple op-amp analysis using voltage controlled voltage source


 ****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C


**************************************************************************
******



 NODE  VOLTAGE    NODE  VOLTAGE    NODE  VOLTAGE    NODE  VOLTAGE


(   1)   1.0000  (   2)    .0027  (   3)  -27.4180  (   4)   -9.9697




   VOLTAGE SOURCE CURRENTS
   NAME        CURRENT

   Vg         -9.973E-04

   TOTAL POWER DISSIPATION   9.97E-04  WATTS



      JOB CONCLUDED

      TOTAL JOB TIME           1.22
```

```
**** 02/21/93 23:08:33 *********** Evaluation PSpice (Jan 1992) **************

 Simple op-amp analysis using voltage controlled voltage source


 ****     CIRCUIT DESCRIPTION


********************************************************************************
Vg    1     0     DC     1
Rs    1     2     1000
Rf    2     4     10e3
R1    4     0     4e3
.SUBCKT OPAMP  1     2     4     5
R1    2     1     200e3
E1    3     5     2     1     10e3
Ro    3     4     5e3
.ENDS  OPAMP
X1    2     0     4     0     OPAMP
.probe
.end

**** 02/21/93 23:08:33 *********** Evaluation PSpice (Jan 1992) **************

 Simple op-amp analysis using voltage controlled voltage source


 ****     SMALL SIGNAL BIAS SOLUTION      TEMPERATURE =   27.000 DEG C


********************************************************************************

 NODE  VOLTAGE    NODE  VOLTAGE    NODE  VOLTAGE    NODE  VOLTAGE


(    1)   1.0000  (    2)    .0027  (    4)   -9.9697  ( X1.3)  -27.4180




   VOLTAGE SOURCE CURRENTS
   NAME        CURRENT

   Vg        -9.973E-04

   TOTAL POWER DISSIPATION   9.97E-04  WATTS



      JOB CONCLUDED

      TOTAL JOB TIME             .90
```

```
**** 02/21/93 23:16:04 *********** Evaluation PSpice (Jan 1992) **************

 Simple op-amp analysis using 741 library model


 ****     CIRCUIT DESCRIPTION
******************************************************************************


Vg     1     0     DC     1
Rs     1     2     1000
Rf     2     4     10e3
R1     4     0     4e3
X1     0     2     5     6     4     uA741
VCC1   5     0     DC     12
VEE1   0     6     DC     12
.LIB   EVAL.LIB
.DC    Vg    0     1     0.1
.probe
.end

**** 02/21/93 23:16:04 *********** Evaluation PSpice (Jan 1992) **************

 Simple op-amp analysis using 741 library model


 ****     Diode MODEL PARAMETERS

******************************************************************************
           X1.dx
      IS  800.000000E-18
      RS   1


**** 02/21/93 23:16:04 *********** Evaluation PSpice (Jan 1992) **************

 Simple op-amp analysis using 741 library model


 ****     BJT MODEL PARAMETERS

******************************************************************************
           X1.qx
           NPN
      IS  800.000000E-18
      BF   93.75
      NF   1
      BR   1
      NR   1

      JOB CONCLUDED

      TOTAL JOB TIME           3.35
```

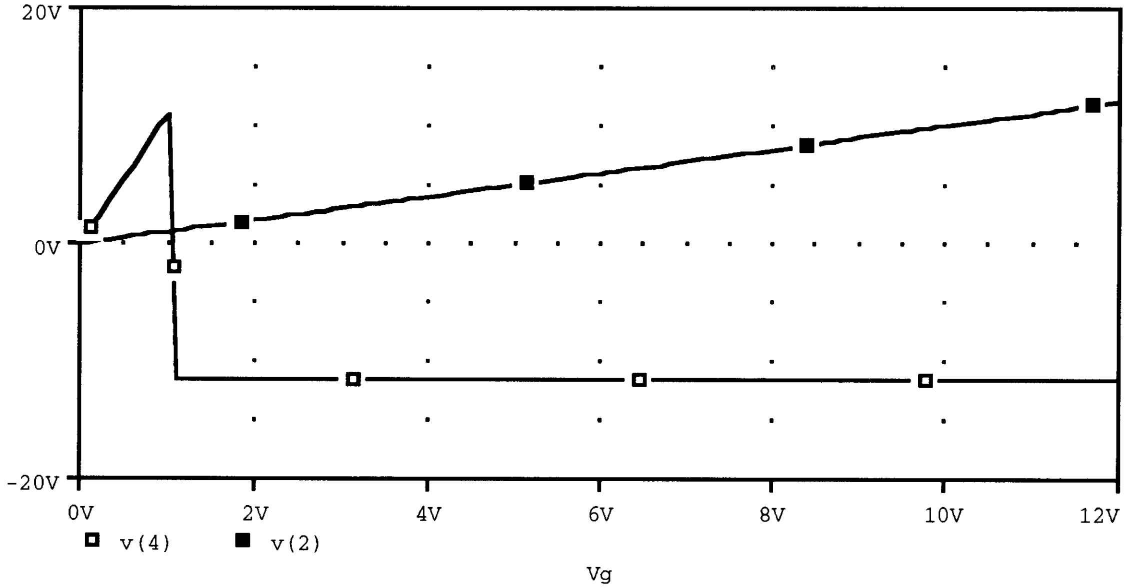


"Simple op-amp analysis using 741 library model" 02/21/93 23:16:04 27.0°

Evaluation Probe 5.1 © 1992 MicroSim Corp.

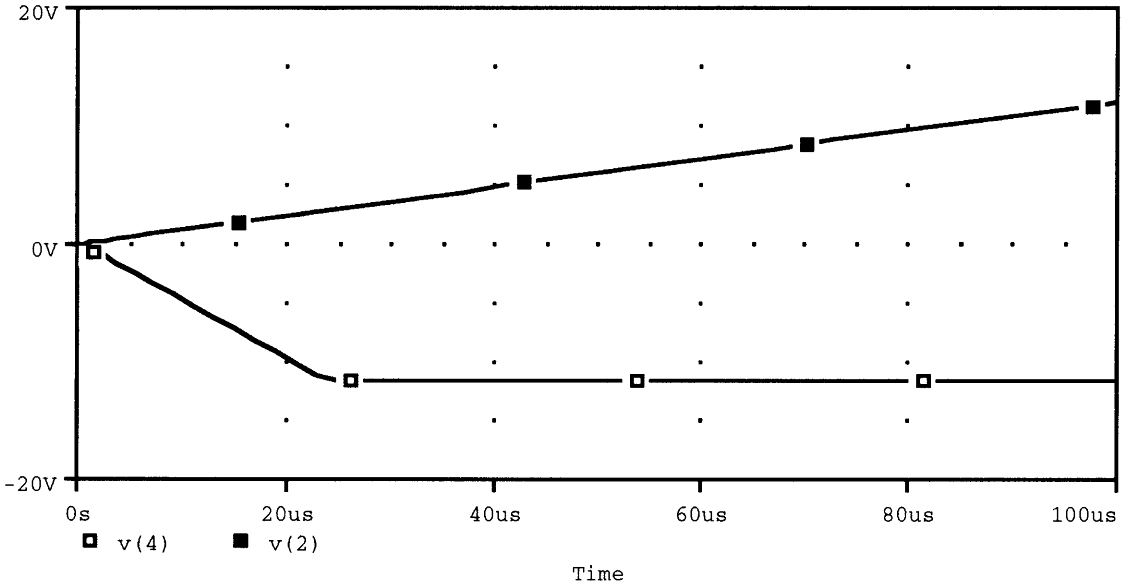
```
**** 02/21/93 23:23:05 *********** Evaluation PSpice (Jan 1992) **************

 Schmitt trigger using uA741 model

 ****    CIRCUIT DESCRIPTION
**********************************************************************
Vg    1     0     DC     1
Rs    1     2     1000
Ra    3     0     1e3
Rb    3     4     10e3
R1    4     0     4e3
X1    3     2     5     6     4     uA741
VCC1   5     0     DC     12
VEE1   0     6     DC     12
.LIB   EVAL.LIB
.DC    Vg    0     12    0.1
.probe
.end

**** 02/21/93 23:23:05 *********** Evaluation PSpice (Jan 1992) **************

 Schmitt trigger using uA741 model


 ****    Diode MODEL PARAMETERS
**********************************************************************
           X1.dx
      IS  800.000000E-18
      RS   1


**** 02/21/93 23:23:05 *********** Evaluation PSpice (Jan 1992) **************

 Schmitt trigger using uA741 model

 ****    BJT MODEL PARAMETERS
**********************************************************************
X1.qx
           NPN
      IS  800.000000E-18
      BF  93.75
      NF  1
      BR  1
      NR  1



      JOB CONCLUDED

      TOTAL JOB TIME          22.80
```

```
**** 02/21/93 23:33:53 *********** Evaluation PSpice (Jan 1992) **************

 Schmitt trigger using uA741 model


 ****     CIRCUIT DESCRIPTION
************************************************************************
Vg     1     0     PWL(0 0  100u 12)
Rs     1     2     1000
Ra     3     0     1e3
Rb     3     4     10e3
R1     4     0     4e3
X1     3     2     5     6     4     uA741
VCC1   5     0     DC    12
VEE1   0     6     DC    12
.LIB   EVAL.LIB
.TRAN  1u     100u
.probe
.end

**** 02/21/93 23:33:53 *********** Evaluation PSpice (Jan 1992) **************

 Schmitt trigger using uA741 model


 ****    Diode MODEL PARAMETERS
************************************************************************
X1.dx
       IS  800.000000E-18
       RS    1


**** 02/21/93 23:33:53 *********** Evaluation PSpice (Jan 1992) **************

 Schmitt trigger using uA741 model


 ****    BJT MODEL PARAMETERS
************************************************************************
X1.qx
          NPN
       IS  800.000000E-18
       BF   93.75
       NF    1
       BR    1
       NR    1


**** 02/21/93 23:33:53 *********** Evaluation PSpice (Jan 1992) **************

 Schmitt trigger using uA741 model
```



"Schmitt trigger using uA741 model" 02/21/93 23:23:05 27.0°
Evaluation Probe 5.1 © 1992 MicroSim Corp.

```
 ****    INITIAL TRANSIENT SOLUTION      TEMPERATURE =   27.000 DEG C

************************************************************************
******


 NODE   VOLTAGE    NODE   VOLTAGE    NODE   VOLTAGE    NODE   VOLTAGE

(   1)    0.0000  (    2)-79.76E-06  (    3)-99.49E-06  (    4)-297.1E-06

(   5)   12.0000  (    6)  -12.0000  ( X1.6) 8.773E-06  ( X1.7)    -.0308

( X1.8)    -.0308  ( X1.9)    0.0000  (X1.10)    -.6078  (X1.11)   11.9600

(X1.12)   11.9600  (X1.13)    -.5939  (X1.14)    -.5939  (X1.53)   11.0000

(X1.54)  -11.0000  (X1.90)    -.6104  (X1.91)   40.0000  (X1.92)  -40.0000

(X1.99)    0.0000



    VOLTAGE SOURCE CURRENTS
    NAME         CURRENT

    Vg          -7.976E-08
    VCC1         -1.337E-03
    VEE1        -1.337E-03
    X1.vb        8.773E-11
    X1.vc        1.100E-11
    X1.ve        1.100E-11
    X1.vlim      -6.104E-04
    X1.vlp      -4.061E-11
    X1.vln      -3.939E-11

    TOTAL POWER DISSIPATION   3.21E-02  WATTS


         JOB CONCLUDED

         TOTAL JOB TIME           10.53
```



"Schmitt trigger using uA741 model"   02/21/93 23:33:53   27.0°

Evaluation Probe 5.1 © 1992 MicroSim Corp.

$v(t) \leftrightarrow Be^{j\phi}$
where the $e^{j\omega t}$ is assumed.

The real (as opposed to complex) time-dependent waveform can be written as $v \cos(\omega t+\theta) = Re\{ve^{j\omega t+\theta}\}$ where *Re* denotes the real part of the rectangular form obtained using the Euler identity. A complex voltage $V(t)=V_m\cos(\omega t+\theta)$ can be written in phasor form as
$\hat{V} = |V|e^{j\theta} = V_m e^{j\theta}$

where the vertical bars denote the magnitude of V and $e^{j\omega t}$ is assumed.

Do problems 3-13, 3-14, 3-15 and 3-17

The advantages of phasor notation will be shown in the following section in which ac problems will be easily solved using Ohm's Law and complex currents, voltages and impedances expressed in phasor form.

14.8 AC circuits, phasors, impedance

The real voltage (or any other type of waveform) is the real part of the phasor voltage $v = Re\{\hat{v}e^{j\omega t}\}$, etc.

	instantaneous	phasor
R:	$v=iR$	$\hat{v}=R\hat{i}$
L:	$v=L\frac{di}{dt}$	$\hat{v}=j\omega L\hat{i}$
C:	$i=C\frac{dv}{dt}$	$\hat{i}=j\omega C\hat{v}$

where $\omega=2\pi f$ radians/second and f is in Hertz.

$$\hat{v} = R\hat{i} + j\omega L\hat{i} + \frac{\hat{i}}{j\omega C} = \left(R + j\omega L + \frac{1}{j\omega C}\right)\hat{i}$$

The input impedance is then given by

$$\hat{Z} = \frac{\hat{v}}{\hat{i}} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

The circuit is in series resonance when

$$\omega L - \frac{1}{\omega C} = 0$$

The frequency for which this occurs is the resonant frequency ω_0 and is given by

$$\omega_0 L - \frac{1}{\omega_0 C} = 0$$

Therefore,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

and

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

The input admittance is simply the reciprocal of the impedance

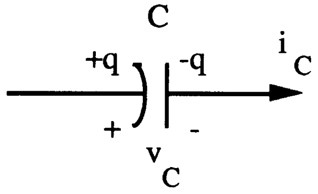
$$\hat{Y} = \frac{\hat{i}}{\hat{v}} = \frac{1}{\hat{Z}}$$

48.11 Capacitors

A capacitor can be simply defined by the relationship

$$q=Cv$$

where C is the capacitance in farads, q is the charge on the plates and v is the voltage between the plates; however, just like an inductor the voltage current relationship for a capacitor cannot be written without using integrals or derivatives.



Using the above definitions we can write the "Ohm's Law" for a capacitor as

$$i_C = \frac{dq}{dt} = C\frac{dv}{dt}$$

or, in integral form, as

$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i_C(t)\, dt$$

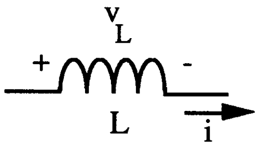
The energy stored in the capacitor can be computed as

$$W_C = \int_{t_0}^{t} v(t)i(t)\, dt = \int_{t_0}^{t} v\left(C\frac{dv}{dt}\right) dt = \int_{v(t_0)}^{v(t)} Cv\, dv$$

which is usually written in the more useful form

$$W_C = \frac{1}{2}C\left[v^2(t) - v^2(t_0)\right] \text{ joules}$$

48.12 Inductance



An inductor, unlike a simple resistor, requires a differential or integration relation to describe the relationship between voltage and current. For the inductor, this form of "Ohm's Law" is

$$v_L = L\frac{di}{dt}$$

or, in integral form,

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v_L(t)\, dt$$

The energy stored in the inductor is

$$W_L = \int_{t_0}^{t} p(t)\, dt = \int_{t_0}^{t} \left(L\frac{di}{dt}\right)(i\, dt) = \int_{i(t_0)}^{i(t)} Li\, di$$

which is more commonly written in the more useful form

$$W_L = \frac{1}{2}L\left[i^2(t) - i^2(t_0)\right] \text{ joules}$$

48.13 Combining inductances and capacitances

Inductors and capacitors obey rules very similar to resistors for combining. Remember that you CANNOT combine inductors AND capacitors. You can only combine like components.

For inductors in series:

$$L_{\text{equivalent}} = \sum_{n=1}^{N} L_n$$

For inductors in parallel:

$$L_{\text{equivalent}} = \frac{1}{\sum_{n=1}^{N} \frac{1}{L_n}}$$

For capacitors in series:

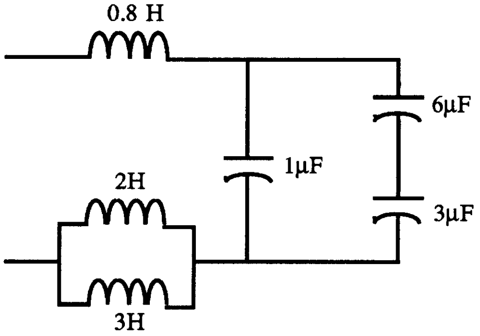
$$C_{\text{equivalent}} = \frac{1}{\sum_{n=1}^{N} \frac{1}{C_n}}$$

For capacitors in parallel:

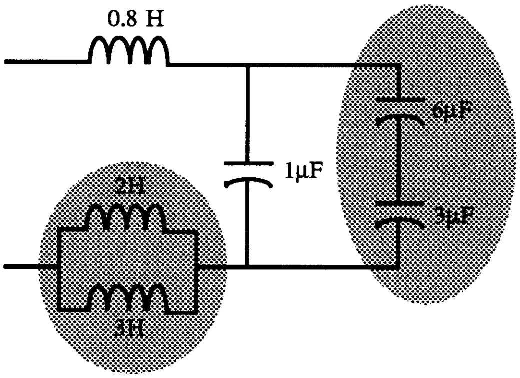
$$C_{\text{equivalent}} = \sum_{n=1}^{N} C_n$$

Example:

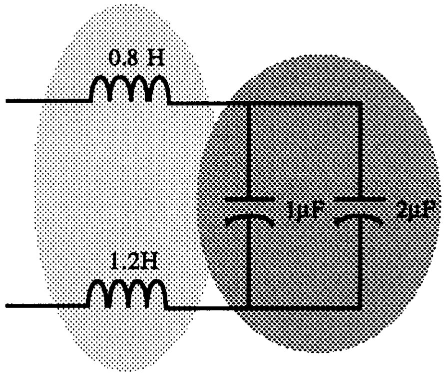
Simplify the complex network of capacitors and inductors shown below.



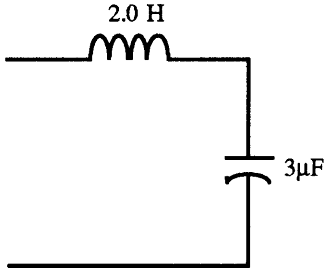
which can be reduced using the above expressions



Combining the two series capacitors and the two parallel inductances:



Where the parallel capacitances and series inductors can again be combined to give:

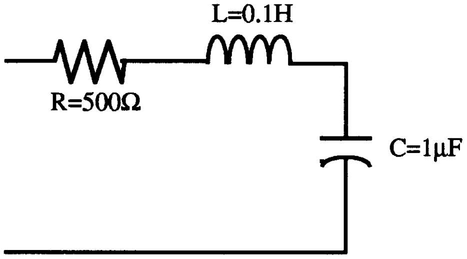


The series combination of the inductances in this last circuit may not be possible for certain applications.

48.17 Series Circuits

Example:

If the frequency is ω=1000 radians/second, determine the input impedance and the resonant frequency



$$\hat{Z} = 500 + j(10^3)(0.1) - j\frac{1}{(10^3)(10^{-6})} = 500 + j100 - j1000$$

$$\hat{Z} = 500 \ -j900 = 1029\angle -60.9°$$

$$|\hat{Z}| = 1029\Omega$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(10^{-1})(10^{-6})}} = 3162 \frac{\text{radians}}{\text{second}}$$

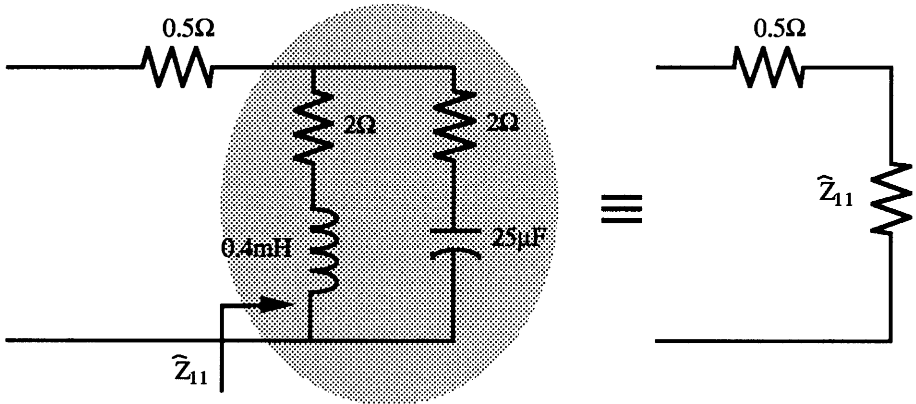
$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(10^{-1})(10^{-6})}} = 503 \text{ Hz}$$

Do problem 48-6 HINT: It is a series circuit.

48.18 Parallel Circuits

Example:

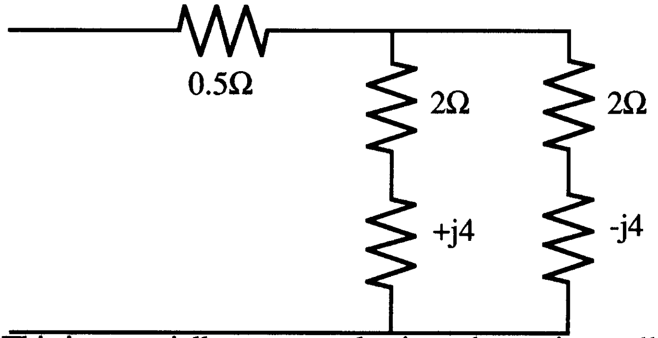
At a frequency of $\omega=10^4$ radians/second, determine the input impedance of the circuit shown below.



$$j\omega L = j(10^4)(0.4\times10^{-3}) = j4\ \Omega$$

$$\frac{1}{j\omega C} = \frac{1}{j(10^4)(25\times10^{-6})} = \frac{1}{j0.25} = -j4\ \Omega$$

The equivalent circuit is then



This is essentially two complex impedances in parallel with an equivalent parallel impedance given by

$$\hat{Z}_{11} = \frac{\hat{Z}_1\times\hat{Z}_2}{\hat{Z}_1+\hat{Z}_2} = \frac{(2+j4)(2-j4)}{(2+j4)+(2-j4)} = \frac{4+16}{4} = 5\ \Omega$$

The input impedance of this circuit is the parallel impedance added to the 0.5Ω series resistance, i.e.

$$\hat{Z}_{IN} = 0.5 + \hat{Z}_{11} = 0.5 + 5 = 5.5\ \Omega$$

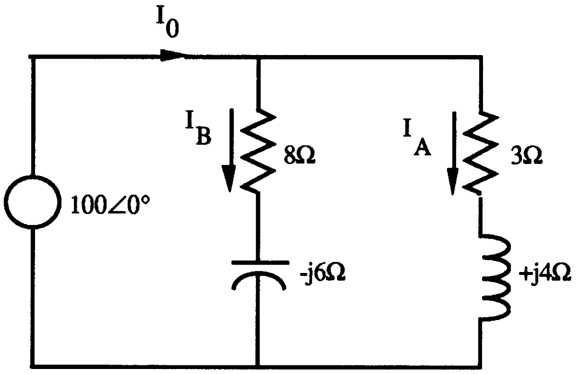
Since the input impedance is real, this circuit is at resonance. The input admittance is

$$\hat{Y}_{IN} = \frac{1}{\hat{Z}_{IN}} = \frac{1}{5.5} = 0.182 \text{ mhos}$$

Do problem 48-9

Example:

Find I_A and I_B as real currents in the circuit shown below where the source voltage is specified to be 100 volts at a phase angle of 0°



Using the Euler identity $Ve^{j(\omega t+\theta)} = V\cos(\omega t+\theta) + jV\sin(\omega t+\theta)$

$$\hat{I}_A = \frac{\hat{V}}{\hat{Z}} = \frac{100\angle 0°}{3+j4} = \frac{100\angle 0°}{5\angle 53.2°} = 20\angle{-53.2°}$$

$$\hat{I}_B = \frac{\hat{V}}{\hat{Z}} = \frac{100\angle 0°}{8-j6} = \frac{100\angle 0°}{10\angle{-36.8°}} = 10\angle{+36.8°}$$

$\hat{I}_0 = \hat{I}_A + \hat{I}_B = 20\angle{-53.2°} + 10\angle{+36.8°}$

These currents must be combined vectorially as shown in the diagram below:

$= 20(e^{-j53.2°})+10(e^{+j36.8°})$

$= 20(\cos 53.2°-j\sin 53.2°)+10(\cos 36.8°+j\sin 36.8°)$

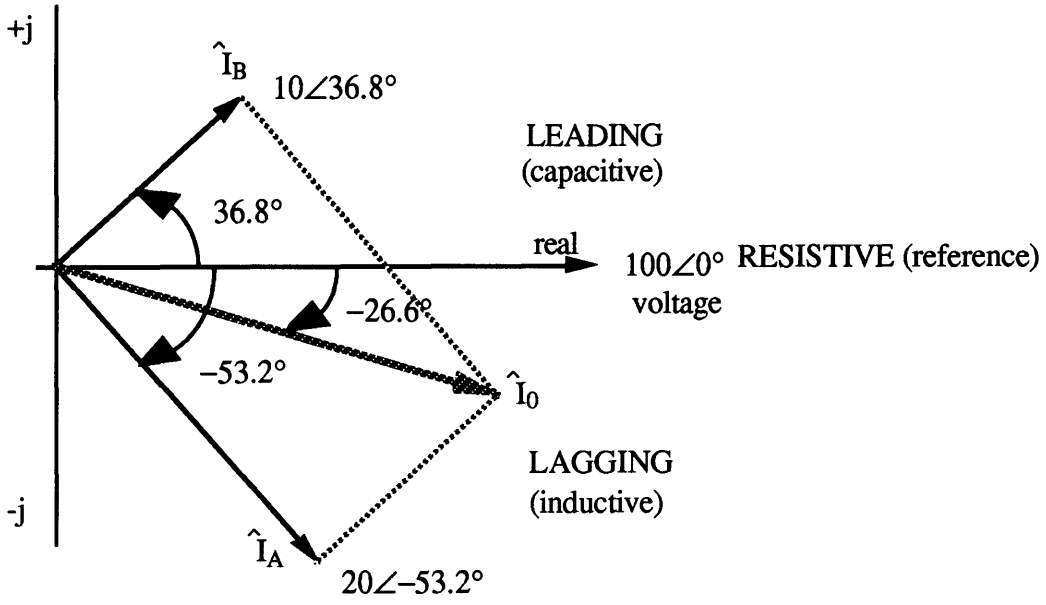
$= 20\cos 53.2°-j20\sin 53.2°+10\cos 36.8°+j10\sin 36.8°$

$= 11.98-j16.01+8+j5.99$

$= 19.98-j10.02$

$= 22.35\angle{-26.6°}$

Phasor diagram:



Recalling that $Re\{e^{j\omega t}\} = Re\{\cos\omega t + j\sin\omega t\} = \cos\omega t$, we can write the time dependent voltages in the circuit as

$v(t) = 100 \cos(\omega t+0°)$

$i_A(t) = 20 \cos(\omega t-53.2°)$

$i_B(t) = 10 \cos(\omega t+36.8°)$

$i_0(t) = 22.35 \cos(\omega t-26.6°)$

where the voltages and currents are now instantaneous.

CIRCUITS 14

Given: 3cos(10t) - 4sin(10t-45°) + X(t) = 0 X(t) is

(a) 6.4cos(10t)

(b) 1.0cos(10t-135°)

(c) 6.4cos(10t+154°)

(d) 1.0cos(10t+135°)

(e) 6.4cos(10t-154°)

Solve the problem using phasors. Before using the Euler identity to convert the expression into phasor form, we must use the identity sinθ=cos(θ-90°) to convert the expression 4sin(10t-45°) = 4cos(10t–45°–90°) = + 4cos(10t-135°). Substituting this into the given equation:

3cos(10t) - 4cos(10t-135°) + X(t) = 0

Converting to phasor form:

$3\angle 0° - 4\angle -135° + X = 0$

Solving for X and converting back to real:

$X = -3\angle 0° + 4\angle -135° = -3 + (-2.828 - j2.828) = -5.828 - j\,2.828 = 6.478\angle -154°$

$\mathrm{X} = \mathrm{Re}\{X\} = \mathrm{Re}\{6.478\angle -154°\} = 6.478\cos(10t-154°)$

The correct answer is (e).

CIRCUITS 21

$\bar{I}_X$ is

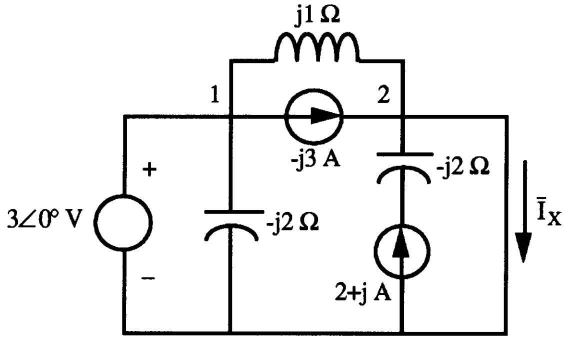
(a) 2.7 - j6.2 amps

(b) 2.7 + j6.2 amps

(c) 2 + j3 amps

(d) 2 - j3 amps

(e) 5.4∠-68°



Solution:

The trick of this problem is to notice the short from node 2 to the circuit common (the voltage reference). Because of this short the voltage at node 2 is zero and the voltage at node 1 (and across the j1 Ω inductor) is 3 volts.

Using KCL at node 2

$\bar{I}_X = (2+j) - j3 + \frac{3}{j} = 2 - j5 = 5.4\angle -68°$

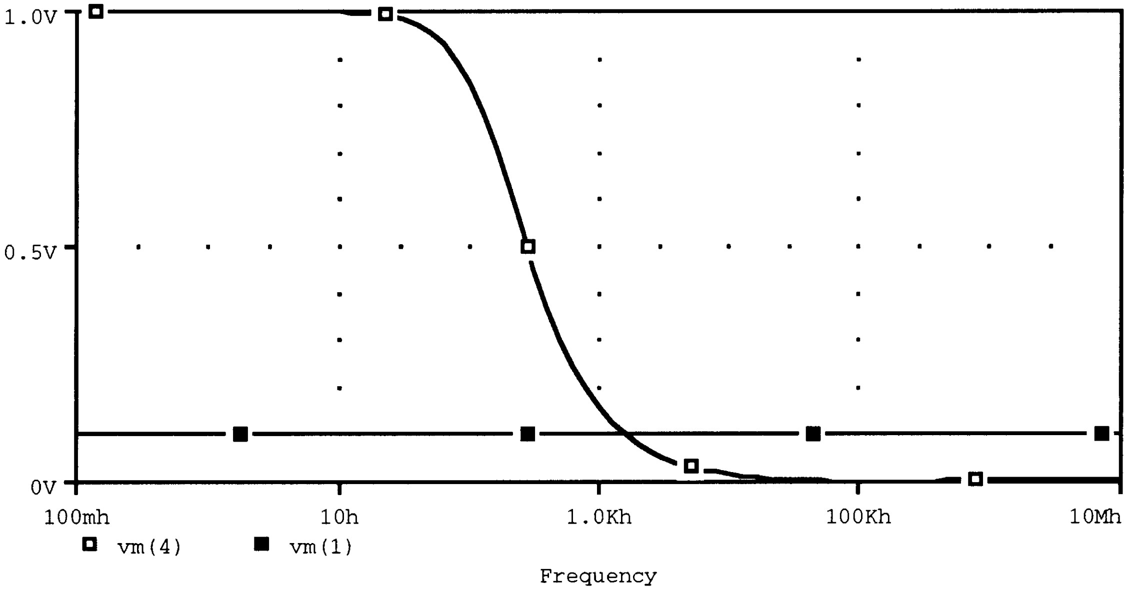
The correct answer is (e).

```
**** 02/22/93 22:50:00 *********** Evaluation PSpice (Jan 1992) **************

 Low pass op-amp filter analysis using 741 library model


 ****     CIRCUIT DESCRIPTION


********************************************************************************



Vg     1      0      AC      100m
Rs     1      2      1000
Rf     2      4      10e3
C      2      4      0.0001micro
R1     4      0      4e3
X1     0      2      5      6      4      uA741
VCC1   5      0      DC      12
VEE1   0      6      DC      12
.LIB   EVAL.LIB
.AC    DEC    10     0.1    10000000
.PLOT  AC     vm(1)  vm(4)
.probe
.end

**** 02/22/93 22:50:00 *********** Evaluation PSpice (Jan 1992) **************

 Low pass op-amp filter analysis using 741 library model


 ****     Diode MODEL PARAMETERS


********************************************************************************




          X1.dx
      IS  800.000000E-18
      RS    1


**** 02/22/93 22:50:00 *********** Evaluation PSpice (Jan 1992) **************

 Low pass op-amp filter analysis using 741 library model


 ****     BJT MODEL PARAMETERS


********************************************************************************
```

```
          X1.qx
          NPN
     IS  800.000000E-18
     BF   93.75
     NF    1
     BR    1
     NR    1


**** 02/22/93 22:50:00 *********** Evaluation PSpice (Jan 1992) **************

 Low pass op-amp filter analysis using 741 library model


 ****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C

******************************************************************************



 NODE   VOLTAGE    NODE   VOLTAGE    NODE   VOLTAGE    NODE   VOLTAGE


(    1)    0.0000  (    2) 18.22E-06  (    4) 998.1E-06  (    5)   12.0000

(    6)  -12.0000  ( X1.6)-19.34E-06  ( X1.7)     .0682  ( X1.8)     .0682

( X1.9)    0.0000  (X1.10)    -.6077  (X1.11)   11.9600  (X1.12)   11.9600

(X1.13)    -.5938  (X1.14)    -.5938  (X1.53)   11.0000  (X1.54)  -11.0000

(X1.90)    1.3431  (X1.91)   40.0000  (X1.92)  -40.0000  (X1.99)    0.0000




    VOLTAGE SOURCE CURRENTS
    NAME         CURRENT

    Vg           1.822E-08
    VCC1        -1.337E-03
    VEE1        -1.337E-03
    X1.vb       -1.934E-10
    X1.vc        1.100E-11
    X1.ve        1.100E-11
    X1.vlim      1.343E-03
    X1.vlp      -3.866E-11
    X1.vln      -4.135E-11

    TOTAL POWER DISSIPATION   3.21E-02  WATTS

**** 02/22/93 22:50:00 *********** Evaluation PSpice (Jan 1992) **************

 Low pass op-amp filter analysis using 741 library model
```

```
****     AC ANALYSIS                    TEMPERATURE =   27.000 DEG C

******************************************************************************


 LEGEND:

*: VM(1)
+: VM(4)

  FREQ        VM(1)


(*)----------   1.0000E-01  1.0000E+00  1.0000E+01  1.0000E+02  1.0000E+03
(+)----------   1.0000E-03  1.0000E-02  1.0000E-01  1.0000E+00  1.0000E+01
                  _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _
  1.000E-01  1.000E-01 .*         .         .         +         .
  1.259E-01  1.000E-01 .*         .         .         +         .
  1.585E-01  1.000E-01 .*         .         .         +         .
  1.995E-01  1.000E-01 .*         .         .         +         .
  2.512E-01  1.000E-01 .*         .         .         +         .
  3.162E-01  1.000E-01 .*         .         .         +         .
  3.981E-01  1.000E-01 .*         .         .         +         .
  5.012E-01  1.000E-01 .*         .         .         +         .
  6.310E-01  1.000E-01 .*         .         .         +         .
  7.943E-01  1.000E-01 .*         .         .         +         .
  1.000E+00  1.000E-01 .*         .         .         +         .
  1.259E+00  1.000E-01 .*         .         .         +         .
  1.585E+00  1.000E-01 .*         .         .         +         .
  1.995E+00  1.000E-01 .*         .         .         +         .
  2.512E+00  1.000E-01 .*         .         .         +         .
  3.162E+00  1.000E-01 .*         .         .         +         .
  3.981E+00  1.000E-01 .*         .         .         +         .
  5.012E+00  1.000E-01 .*         .         .         +         .
  6.310E+00  1.000E-01 .*         .         .         +         .
  7.943E+00  1.000E-01 .*         .         .         +         .
  1.000E+01  1.000E-01 .*         .         .         +         .
  1.259E+01  1.000E-01 .*         .         .         +         .
  1.585E+01  1.000E-01 .*         .         .         +         .
  1.995E+01  1.000E-01 .*         .         .         +         .
  2.512E+01  1.000E-01 .*         .         .         +         .
  3.162E+01  1.000E-01 .*         .         .         +         .
  3.981E+01  1.000E-01 .*         .         .         +         .
  5.012E+01  1.000E-01 .*         .         .         +         .
  6.310E+01  1.000E-01 .*         .         .         +         .
  7.943E+01  1.000E-01 .*         .         .         +         .
  1.000E+02  1.000E-01 .*         .         .         +         .
  1.259E+02  1.000E-01 .*         .         .        +.         .
  1.585E+02  1.000E-01 .*         .         .        +.         .
  1.995E+02  1.000E-01 .*         .         .       + .         .
  2.512E+02  1.000E-01 .*         .         .      +  .         .
  3.162E+02  1.000E-01 .*         .         .      +  .         .
  3.981E+02  1.000E-01 .*         .         .     +   .         .
  5.012E+02  1.000E-01 .*         .         .    +    .         .
  6.310E+02  1.000E-01 .*         .         .   +     .         .
  7.943E+02  1.000E-01 .*         .         .  +      .         .
```

```
1.000E+03 1.000E-01 .*         .         . +       .         .
1.259E+03 1.000E-01 .*         .         .+        .         .
1.585E+03 1.000E-01 .*         .         +         .         .
1.995E+03 1.000E-01 .*         .        +.         .         .
2.512E+03 1.000E-01 .*         .       + .         .         .
3.162E+03 1.000E-01 .*         .      +  .         .         .
3.981E+03 1.000E-01 .*         .     +   .         .         .
5.012E+03 1.000E-01 .*         .    +    .         .         .
6.310E+03 1.000E-01 .*         .   +     .         .         .
7.943E+03 1.000E-01 .*         .  +      .         .         .
1.000E+04 1.000E-01 .*         . +       .         .         .
1.259E+04 1.000E-01 .*         . +       .         .         .
1.585E+04 1.000E-01 .*         .+        .         .         .
1.995E+04 1.000E-01 .*        +.         .         .         .
2.512E+04 1.000E-01 .*       + .         .         .         .
3.162E+04 1.000E-01 .*      +  .         .         .         .
3.981E+04 1.000E-01 .*     +   .         .         .         .
5.012E+04 1.000E-01 .*    +    .         .         .         .
6.310E+04 1.000E-01 .*   +     .         .         .         .
7.943E+04 1.000E-01 .*  +      .         .         .         .
1.000E+05 1.000E-01 .*  +      .         .         .         .
1.259E+05 1.000E-01 .* +       .         .         .         .
1.585E+05 1.000E-01 .* +       .         .         .         .
1.995E+05 1.000E-01 .* +       .         .         .         .
2.512E+05 1.000E-01 .* +       .         .         .         .
3.162E+05 1.000E-01 .*  +      .         .         .         .
3.981E+05 1.000E-01 .*   +     .         .         .         .
5.012E+05 1.000E-01 .*    +    .         .         .         .
6.310E+05 1.000E-01 .*     +   .         .         .         .
7.943E+05 1.000E-01 .*     +   .         .         .         .
1.000E+06 1.000E-01 .*      +  .         .         .         .
1.259E+06 1.000E-01 .*       + .         .         .         .
1.585E+06 1.000E-01 .*       + .         .         .         .
1.995E+06 1.000E-01 .*       + .         .         .         .
2.512E+06 1.000E-01 .*      +  .         .         .         .
3.162E+06 1.000E-01 .*      +  .         .         .         .
3.981E+06 1.000E-01 .*      +  .         .         .         .
5.012E+06 1.000E-01 .*      +  .         .         .         .
6.310E+06 1.000E-01 .*     +   .         .         .         .
7.943E+06 1.000E-01 .*     +   .         .         .         .
1.000E+07 1.000E-01 .*     +   .         .         .         .
          - - - - - - - - - - - - - - - - - - - - - - - - - - -


     JOB CONCLUDED

     TOTAL JOB TIME            9.77
```

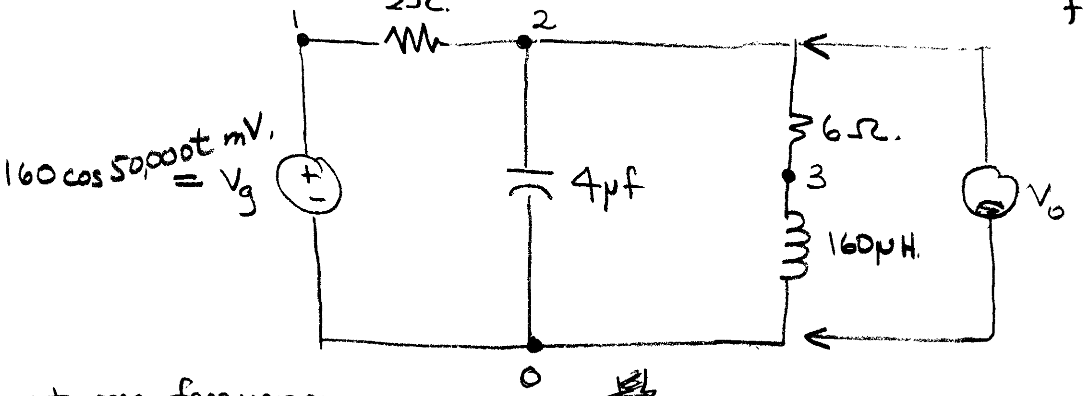


"Low pass op-amp filter analysis using 741 library model" 02/22/93 22:50:00 27.0°

Evaluation Probe 5.1 © 1992 MicroSim Corp.

modified 10.36.

$f = \frac{50,000}{2\pi} = 7957.$



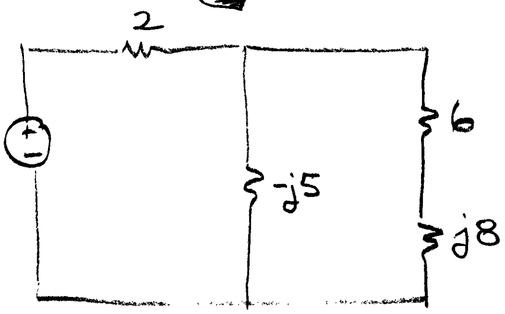
modified 10.36.

```
Vg 1 0  AC 160mV
R1 1 2  2
C1 2 0  4p
R2 2 3  6
L1 3 0  160U
.AC LOG 10 10 10000
.PROBE
.END.
```

at one frequency
(given).

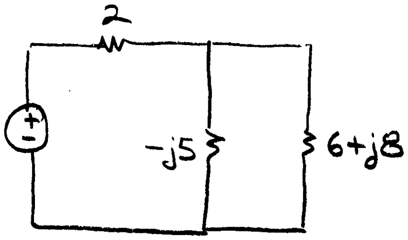
$160\angle 0^\circ$

$\omega = 50,000$



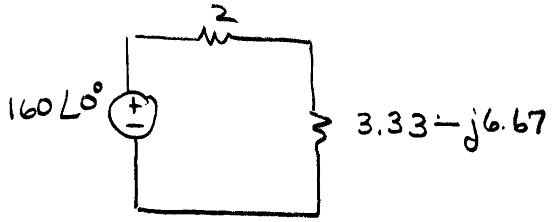
$$\frac{1}{j\omega C} = \frac{1}{j(50000)(4\times 10^{-6})} = -j5$$

$$j\omega L = j(50000)(160\times 10^{-6}) = j8$$



$$\frac{(-j5)(6+j8)}{-j5+6+j8} = \frac{40-j30}{6+j3}$$

$$= (3.33, -6.67)$$

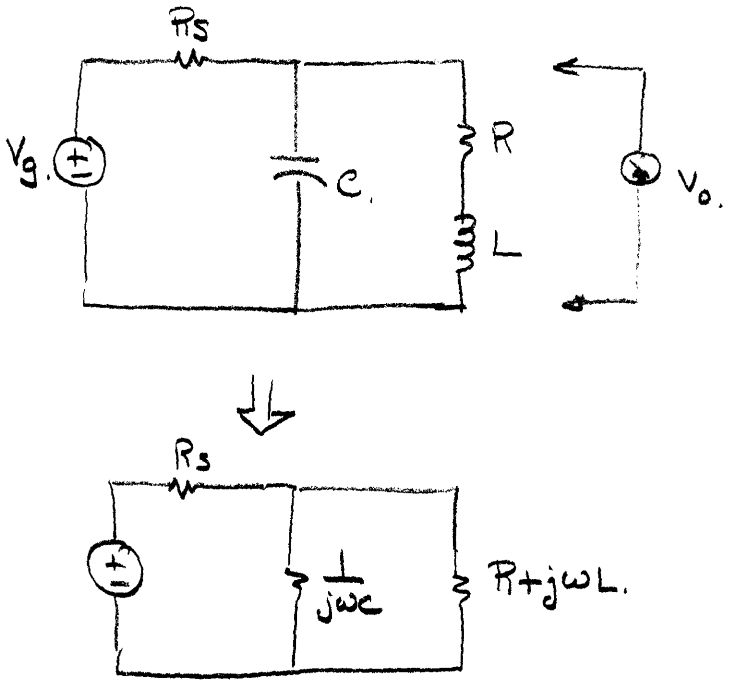


$$V_o = \frac{3.33-j6.67}{2+3.33-j6.67}(160\angle 0) = \frac{3.33-j6.67}{5.33-j6.67}(160\angle 0)$$

$$= (0.853, -0.182)(160\angle 0)$$

$$= (136.59, -29.19)\ \text{mV}$$

$$= 139.67\angle -12^\circ$$



$$\frac{1}{j\omega C} \| (R+j\omega L) = \frac{\frac{1}{j\omega C}(R+j\omega L)}{\frac{1}{j\omega C}+R+j\omega L} = \frac{R+j\omega L}{1+j\omega RC-\omega^2 LC}$$

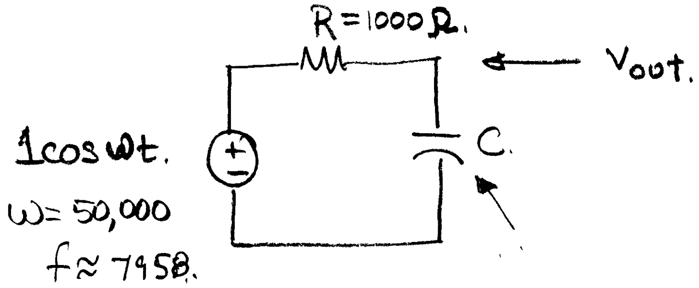
$$= \frac{R+j\omega L}{(1-\omega^2 LC)+j\omega RC}$$

$$= \frac{(R+j\omega L)\left[(1-\omega^2 LC)-j\omega RC\right]}{(1-\omega^2 LC)^2+\omega^2 R^2 C^2}.$$

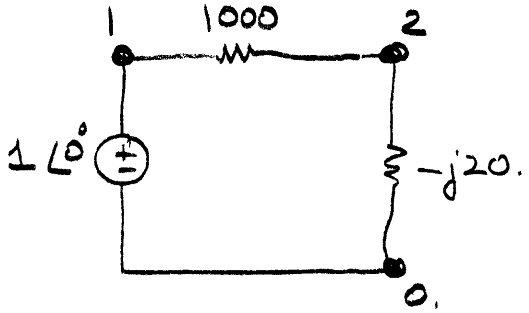
$$= \frac{R(1-\cancel{\omega^2 LC})+j\omega L(1-\omega^2 LC)-j\omega R^2 C+\cancel{\omega^2 LRC}}{(1-\omega^2 LC)^2+\omega^2 R^2 C^2}.$$

$$= \frac{R+j\omega L(1-\omega^2 LC)-j\omega R^2 C}{(1-\omega^2 LC)^2+\omega^2 R^2 C^2}$$

$$= \frac{R}{(1-\omega^2 LC)^2+\omega^2 R^2 C^2} + j\,\frac{\omega L(1-\omega^2 LC)-\omega R^2 C}{(1-\omega^2 LC)^2+\omega^2 R^2 C^2}.$$



⇒



RC = 1 msec.

$$C = \frac{1\times10^{-3}}{1000} = 1\mu f.$$

$$\frac{1}{j\omega c} = \frac{1}{j\,50{,}000\,(10^{-6})} = -j20$$

at one frequency

$$V_{OUT} = \frac{-j20}{1000-j20}\;1.$$

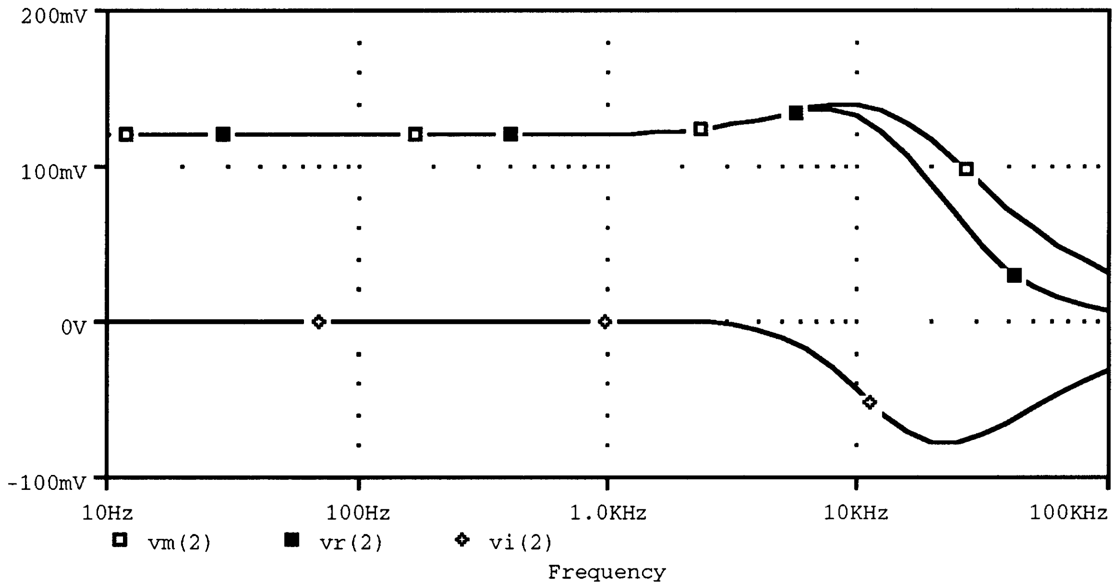
$$= (4\times10^{-4}, -2\times10^{-2})$$

$$\boxed{V_{OUT.} \cong .02 \angle -88.85^{0}}$$

Pspice Analysic

```
RC circuit
Vg  1  0  AC1 volt.   PWL( 0  0  1e-6  1).
R1  1  2    1000
C1  2  0    1p
⇒ .AC   DEC  10  1   1000,000
⇒ .TRAN  20e-6  2000e-6
   .PROBE
   .END.

⇒ .DC   Vg.  0   1  0.1
```

```
**** 02/24/93 22:03:50 *********** Evaluation PSpice (Jan 1993) **************

 modified problem 10.36  w=50,000 radians/second


 ****     CIRCUIT DESCRIPTION


******************************************************************************



Vg    1    0    AC    160mvolt
R1    1    2    2
C1    2    0    4u
R2    2    3    6
L1    3    0    160u
.AC   DEC   10    10    100000
.PROBE
.END

**** 02/24/93 22:03:50 *********** Evaluation PSpice (Jan 1993) **************

 modified problem 10.36  w=50,000 radians/second


 ****     SMALL SIGNAL BIAS SOLUTION      TEMPERATURE =   27.000 DEG C


******************************************************************************



 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE


(    1)    0.0000  (    2)    0.0000  (    3)    0.0000




    VOLTAGE SOURCE CURRENTS
    NAME         CURRENT

    Vg           0.000E+00

    TOTAL POWER DISSIPATION   0.00E+00  WATTS


        JOB CONCLUDED
```

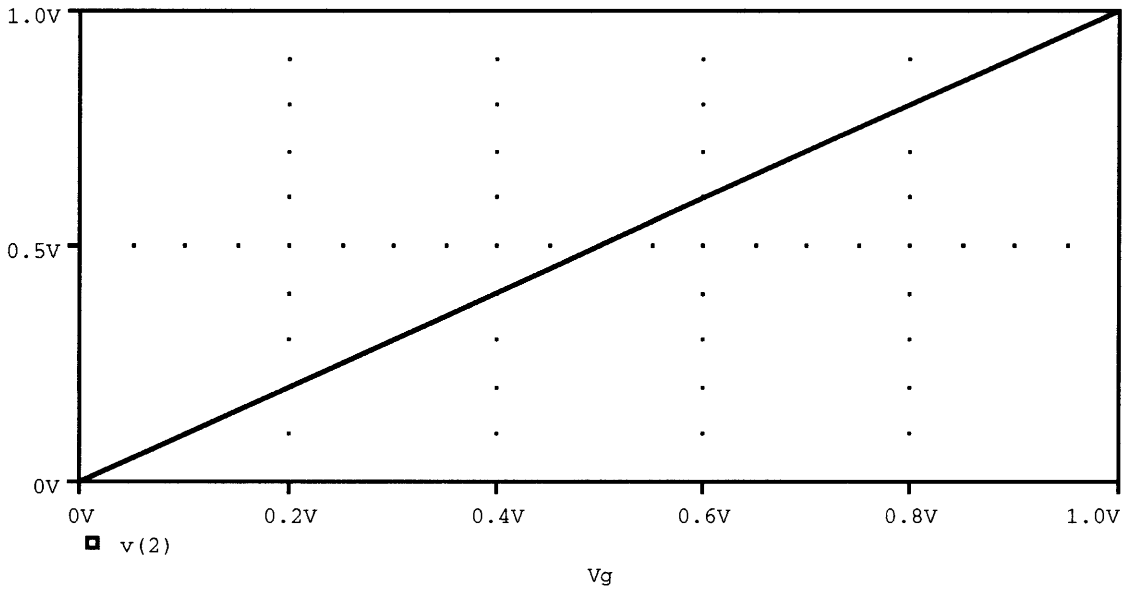


"modified problem 10.36 w=50,000 radians/second" 02/24/93 22:03:50 27.0°

Evaluation Probe 5.3 © 1993 MicroSim Corp.

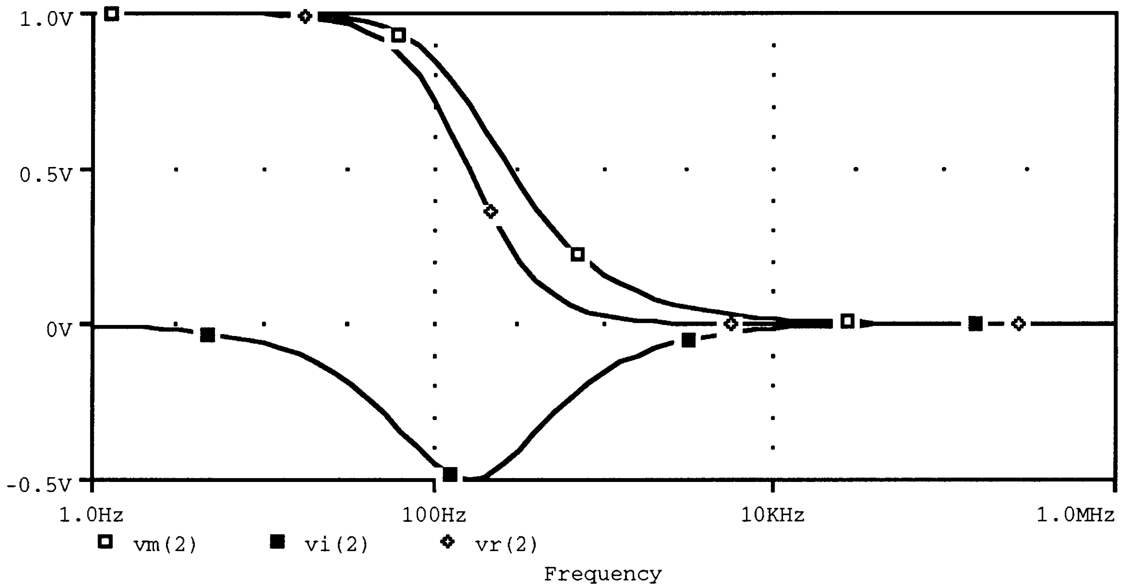
```
**** 02/24/93 22:48:45 *********** Evaluation PSpice (Jan 1993) **************

 Example SPICE analysis using RC circuit


 ****     CIRCUIT DESCRIPTION


******************************************************************************


Vg     1     0     AC     1volt  PWL(0  0     1e-6   1)
R1     1     2     1000
C1     2     0     1u
.AC    DEC    10    1     1000000
.TRAN  20e-6  10000e-6
.DC    Vg    0     1     0.1
.PROBE
.END

**** 02/24/93 22:48:45 *********** Evaluation PSpice (Jan 1993) **************

 Example SPICE analysis using RC circuit


 ****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C


******************************************************************************


 NODE  VOLTAGE    NODE  VOLTAGE    NODE  VOLTAGE    NODE  VOLTAGE


(   1)   0.0000  (   2)   0.0000




   VOLTAGE SOURCE CURRENTS
   NAME        CURRENT

   Vg          0.000E+00

   TOTAL POWER DISSIPATION   0.00E+00  WATTS

**** 02/24/93 22:48:45 *********** Evaluation PSpice (Jan 1993) **************

 Example SPICE analysis using RC circuit


 ****     INITIAL TRANSIENT SOLUTION       TEMPERATURE =   27.000 DEG C
```

```
******************************************************************************

 NODE  VOLTAGE   NODE  VOLTAGE   NODE  VOLTAGE   NODE  VOLTAGE

(    1)   0.0000  (    2)   0.0000



    VOLTAGE SOURCE CURRENTS
    NAME         CURRENT

    Vg           0.000E+00

    TOTAL POWER DISSIPATION   0.00E+00  WATTS


         JOB CONCLUDED

         TOTAL JOB TIME            5.63
**** 02/24/93 22:48:51 *********** Evaluation PSpice (Jan 1993) **************
 .PROBE

 ****     CIRCUIT DESCRIPTION

******************************************************************************

.END

WARNING -- There are no devices in this circuit, (this message will be printed
if there are blank lines after the last .END statement)
```



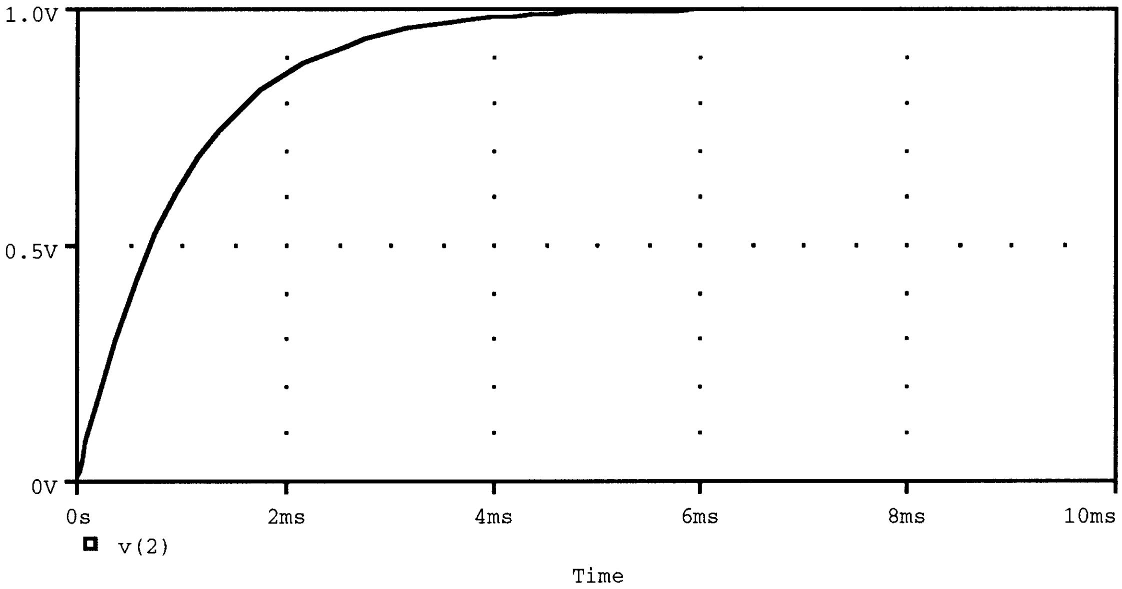
"Example SPICE analysis using RC circuit"   02/24/93 22:44:44   27.0°

Evaluation Probe 5.3 © 1993 MicroSim Corp.



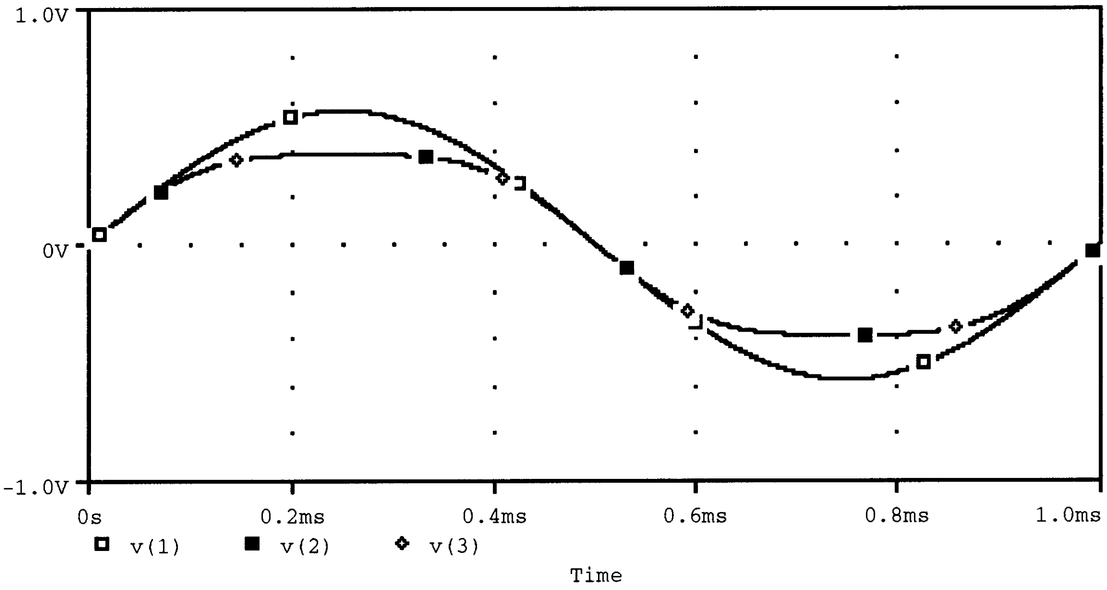
"Example SPICE analysis using RC circuit"   02/24/93 22:44:44   27.0°

Evaluation Probe 5.3 © 1993 MicroSim Corp.



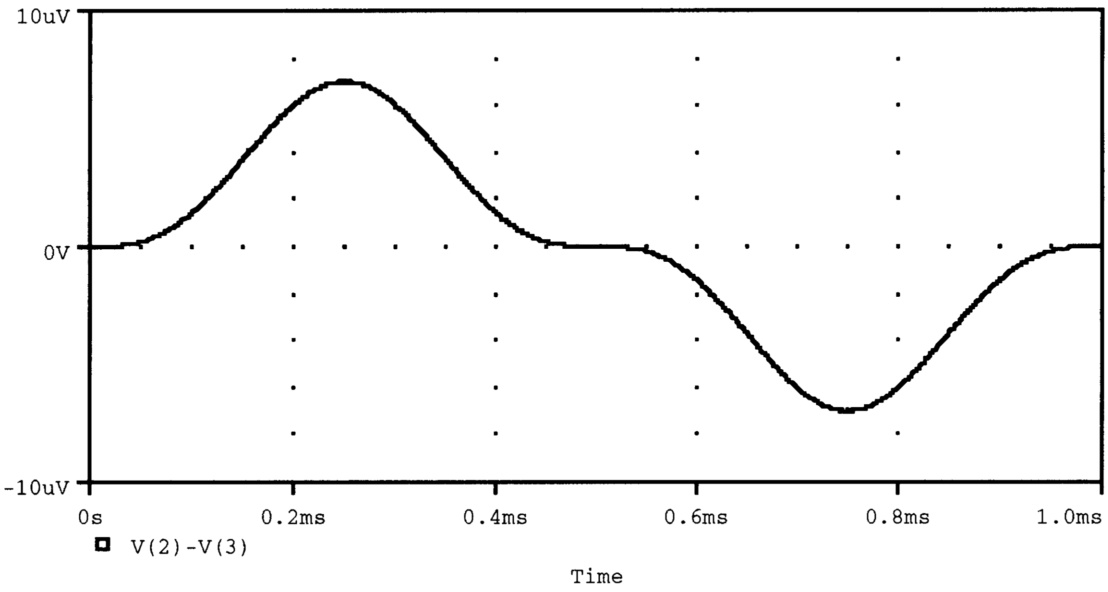
"Example SPICE analysis using RC circuit"   02/24/93 22:48:45   27.0°

Evaluation Probe 5.3 © 1993 MicroSim Corp.



"Example 3 showing superposition of waveforms" 02/24/93 22:55:30 27.0°

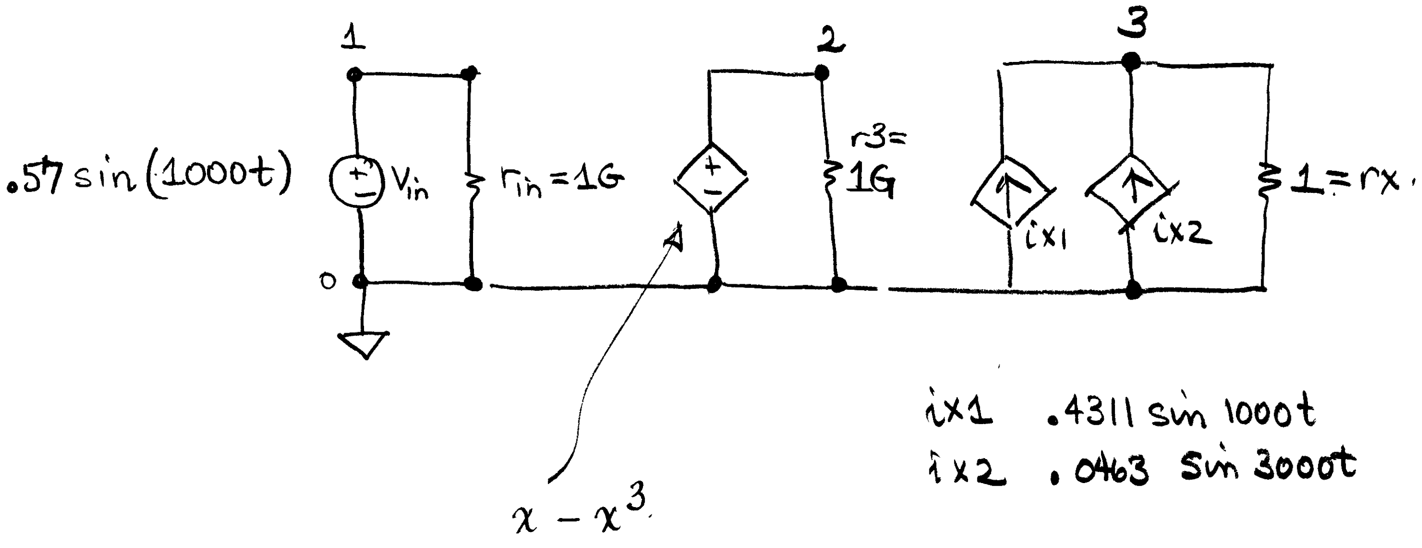
Evaluation Probe 5.3 © 1993 MicroSim Corp.



"Example 3 showing superposition of waveforms"   02/24/93 22:55:30   27.0°

Evaluation Probe 5.3 © 1993 MicroSim Corp.

```
Vin   1 0   sin (0  .57   1000)
rin   1 0   1G
e3    2 0   poly(1) (1,0)  0 1 0 -1
r3    2 0   1G
ix1   0 3   sin (0 .4311  1000)
ix3   0 3   sin (0 .0463  3000)
rx    3 0   1.
.tran 1μ  1m.
.probe
.end.
```



ix1 .4311 sin 1000t
ix2 .0463 sin 3000t

poly(1) (1,0) 0 1 0 -1

x^0 x^1 x^2 x^3

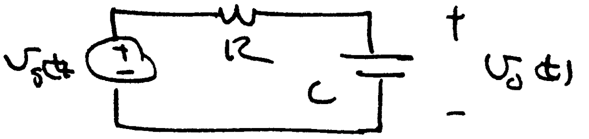
control input v(1) - v(0).

1 controlling input

```
**** 02/24/93 22:55:30 *********** Evaluation PSpice (Jan 1993) **************

 Example 3 showing superposition of waveforms


 ****     CIRCUIT DESCRIPTION


******************************************************************************
Vin    1     0     sin(0 0.57 1000)
Rin    1     0     1G
e3     2     0     poly(1) (1,0) 0 1 0 -1
r3     2     0     1G
ix1    0     3     sin(0 .4311 1000)
ix3    0     3     sin(0 .0463 3000)
rx     3     0     1
.TRAN  1u    1m
.PROBE
.END

**** 02/24/93 22:55:30 *********** Evaluation PSpice (Jan 1993) **************

 Example 3 showing superposition of waveforms


 ****     INITIAL TRANSIENT SOLUTION       TEMPERATURE =   27.000 DEG C


******************************************************************************



 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE


(    1)    0.0000  (    2)    0.0000  (    3)    0.0000




    VOLTAGE SOURCE CURRENTS
    NAME        CURRENT

    Vin         0.000E+00

    TOTAL POWER DISSIPATION   0.00E+00  WATTS



         JOB CONCLUDED

         TOTAL JOB TIME          38.08
```

2/26/93 EEAP 243

We have been considering the linear differential equation which satisfies Kirchoff's laws for an R-C circuit



$$v_s(t) = RC\frac{dv_o}{dt} + v_o \leftarrow \text{first-order diff eq}$$

We ~~examined~~ solved the case for $v_s(t) = V_0\,u(t)$ using two different methods

① Separation of variables
- works easily for first-order circuits

② solution by superposition

$$v_o(t) = v_o'(t) + v_{on}(t)$$

where the two sources are

① the constant V_0

② the zero source (actually an impulse response)

particular solution → $V_0 = RC\overset{=0}{\cancel{\frac{dv_o'}{dt}}} + v_o'$

homogeneous equation $0 = RC\frac{dv_{on}}{dt} + v_{on}$ } add these up

which's analogous to impulse driven.

$$V_0 = RC\frac{dv_o}{dt} + (v_o' + v_{on}) \quad = v_o$$

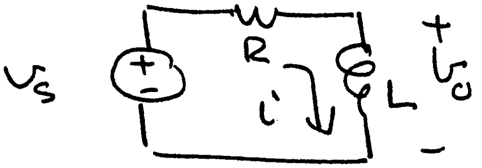
The solution was found to be

$$v_o(t) = \begin{cases} 0 & t<0 \\ V_0\left(1 - e^{-t/\boxed{RC}}\right) & t \geq 0 \end{cases}$$

called the time constant

2/26/93

Let's examine an RL circuit:



$i(t) = ?$

$v_o(t) = ?$ $L\frac{di}{dt}$

$v_s(t) = V_0\, u(t)$

$v_s(t) = Ri + L\frac{di}{dt}$ So we find i, then use $v_0 = L\frac{di}{dt}$

$\frac{v_s}{R} = i + \boxed{\frac{L}{R}}\frac{di}{dt} \Rightarrow v_s = v_0 + \boxed{RC}\frac{dv_0}{dt}$ → time constants.

Let's use our 2nd method

$i = i' + i_n$

PARTICULAR
SOLUTION

① what is the steady-state? since we have a constant source (V_0) we know $di/dt = 0$ for steady-state

hence $i' = V_0/R$

② $0 = Ri_h + L\frac{di_n}{dt}$ ← homogeneous equation

HOMOGENEOUS
→ TRANSIENT
— IMPUSE

try $i_h = Ie^{st}$

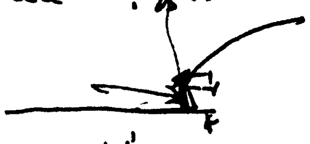
$0 = RIe^{st} + LsIe^{st}$

hence $s = -R/L$ and I is to be determined.

$i = i' + i_n = \frac{V_0}{R} + Ie^{-t/(L/R)}$ $t \geq 0$

at $t = 0^+$ $i(0) = \frac{V_0}{R} + I$ for $t = 0^-$, $i = 0$

hence



this jump means $\frac{di}{dt} \to \infty$ - which requires ∞ voltage & that's not what our source is

hence $i(0^+) = 0 = V_0/R + I$, $I = -V_0/R$

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + a_2 \frac{d^{n-2} y}{dt^{n-2}} + \cdots + a_n y = x(t)$$

homogeneous $x(t) \Rightarrow 0$. and it is known that there will be n-independent solutions, $y_n^{(n)}(t)$ and there is a particular solution $y_p(t)$

The total solution is

$$y(t) = C_1 y_n^{(1)}(t) + C_2 y_n^{(2)}(t) + \cdots + C_n y_n^{(n)}(t) + y_p(t)$$

The homogeneous (transient) solutions are always of the form $y = ce^{st}$. Thus.

$$\frac{d^m y}{dt^m} = c s^m e^{st}$$ .. substituting

gives us

$$c e^{st}(\underbrace{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots a_n}_{\text{this must be zero.}}) = 0$$

characteristic equation $\rightarrow$ $s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n = 0$

the values of s (there are n-roots) are called the natural frequencies of the system.

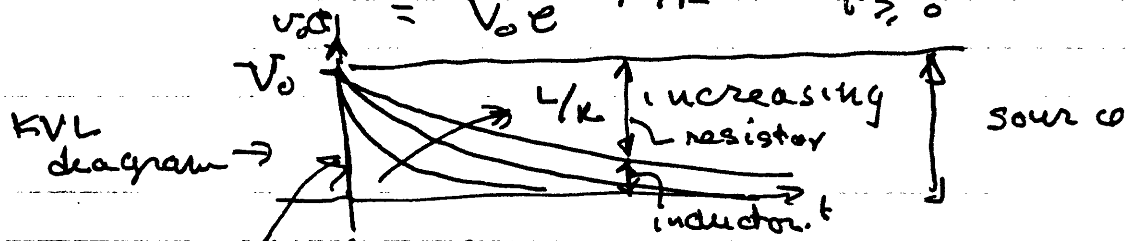
2/26/93

EEAP 243

Finally $i = \frac{V_0}{R}\left(1 - e^{-t/\left(\boxed{L/R}\right)}\right)$ → $t \geq 0$ called the time constant

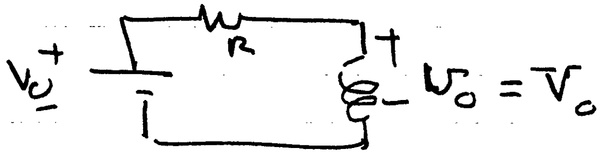
and $v_0 = L\frac{di}{dt} = L\left(\frac{R}{L}V_0\right)e^{-t/L/R}$

$$v_L(t) = V_0 e^{-t/L/R} \qquad t \geq 0$$



at t=0 all of the voltage is across the inductor so no current can flow ($v_R = 0$)

at $t = 0^+$



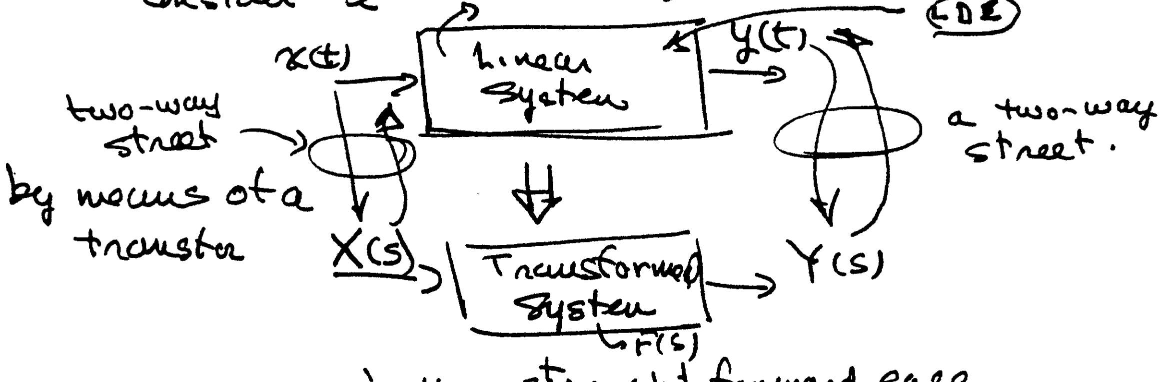
Now what can we do to deal with more complicated circuits

① continue to find the homogeneous + particular solutions

→ ② find other means

It turns out that there exists a very effective method for solving linear diff. eq with constant coefficients which utilizes the concept of transforms - in this case we call it the Laplace Transform.

Consider a ~~$x(t) = a_0 y + a_1 y + a_2\frac{dy}{dt} + \cdots + a_n\frac{dy^n}{dt^n}$~~ (LDE)



in the straightforward case we find $Y(s) = X(s)\,F(s)$

2/26/93

PP 601-625

Then we can find y(t) by transforming back — called the inverse transform. Here's how it works

$$X(s) = \int_0^\infty x(t) e^{-st}\, dt$$

also we write this symbolically as

$$X(s) = \mathcal{L}\{x(t)\}$$

where s is a complex constant in this integration

Let's take a specific case.

let's choose $x(t) = u(t) = \begin{cases} 0 & t<0 \\ 1 & t \geq 0 \end{cases}$

then $X(s) = \int_0^\infty u(t) e^{-st}\, dt = \int_0^\infty e^{-st}\, dt =$

$$= -\frac{1}{s}\left[e^{-st}\Big|_0^\infty \right]$$

so as long as the $\text{Re}\{s\} > 0$ the value at ∞ is zero.

Let $s = \sigma + j\omega$ $\quad$ ($\text{Re}\{s\} = \sigma$)

$\text{Im}\{s\} = \omega$

$$e^{-\sigma t} e^{-j\omega t}\Big|_0^\infty = -1 + \underset{\substack{\uparrow \\ \text{goes} \\ \text{to} \\ \text{zero} \\ \text{if } \sigma > 0}}{e^{-\sigma\infty}} \underset{\substack{\text{magnit} \\ \text{is uni}}}{e^{-j\omega\infty}}$$

So we get $X(s) = \frac{1}{s}$, for $x(t) = u(t)$

called a transform pair

$x(t)$	$X(s)$
$u(t)$	$1/s$
$\delta(t)$	1

2/26/93

If we find an $X(s) = \frac{1}{s}$ we know that we have a $y(t) = u(t)$. That is the two are uniquely related.

Let's use an impulse for $x(t)$

$$X(s) = \int_0^\infty \delta(t)\, e^{-st}\, dt = \int_0^\infty \delta(t)\, dt = 1$$

the impulse in the integrand causes it to be evaluated at the position of the impulse.

Next let's consider $y(t) = \frac{dx}{dt}$ to find $Y(s)$

$$Y(s) = \mathcal{L}\{y(t)\} = \int_0^\infty \left(\frac{dx}{dt}\right) e^{-st}\, dt$$

and integrate by parts let $u = e^{-st}$, $du = -se$

$dv = \frac{dx}{dt}\, dt$

$v = x$

$$Y(s) = \int_0^\infty \frac{dx}{dt} e^{-st}\, dt = x e^{-st}\Big|_0^\infty - \int_0^\infty x\underset{\uparrow\text{constant}}{(-s)} e^{-st}\, dt$$

$$= -x(0) + s\int_0^\infty x(t)\, e^{-st}\, dt$$

$$= s\,X(s) - x(0)$$

example $v(t) = L\frac{di}{dt}$

$$V(s) = \mathcal{L}\{v(t)\} = \mathcal{L}\left\{L\frac{di}{dt}\right\} = LS\,I(s) - L\,i($$

so $V(s) = L\,s\,I(s) - L\,i(0)$

if $i(0) = 0$ and $i(t) = u(t)$

$I(s) = \frac{1}{s}$

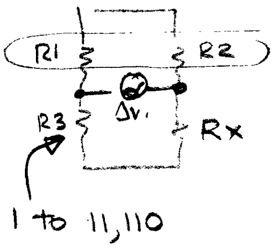
then $V(s) = Ls\left(\frac{1}{s}\right) = L$ ← constant

$v(t) = L\,\delta(t)$ ← transform pair.

notes about PSpice HW

Not too long!

You should.



$R2/R_1$ = .001, .01, ..., 1000

at extremes
4 & 5.

null by setting $R_x = \frac{R_2}{R_1} R_3$

R3 adjustable

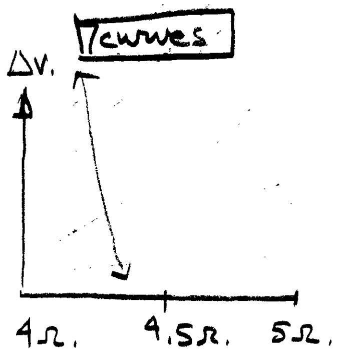
.001, .01, .1, 1, 10, 100, 1000

$R_x = 4$ $\frac{R2}{R1}$ = .001, ..., 1000

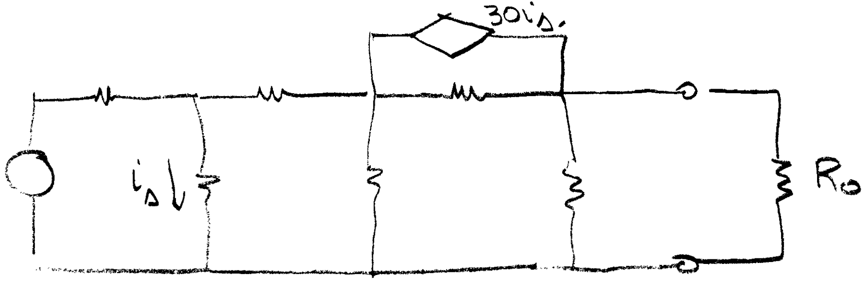
$R_x = 5$ = .001, ..., 1000

How linear is ΔV?

Plot



4.79



program.

Answer (a), (b), (c)

Laplace transform problems.

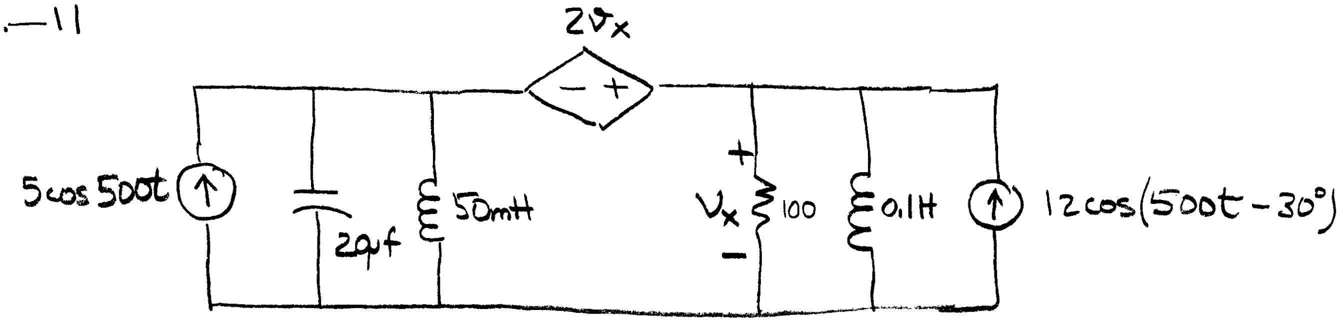
15.7, 15.13 — simple math

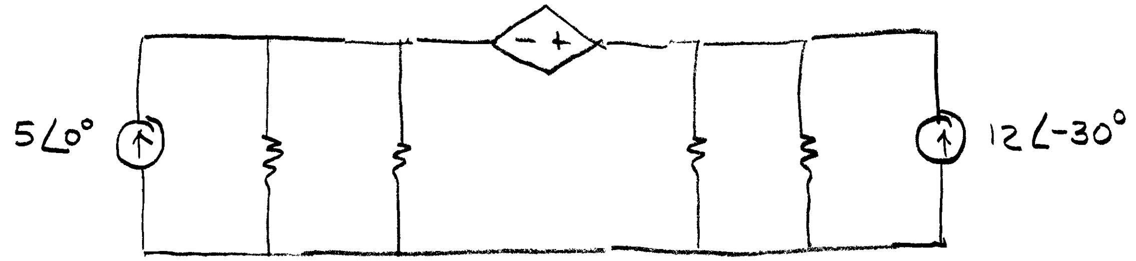
15.19, 15.20 } RLC circuit
15.21, 1522.

15.23, 15.24, 15.25. partial fractions

Phasor analysis.

P11.—11





$$j\omega L = j(500)(50\times10^{-3}) = 25\Omega$$

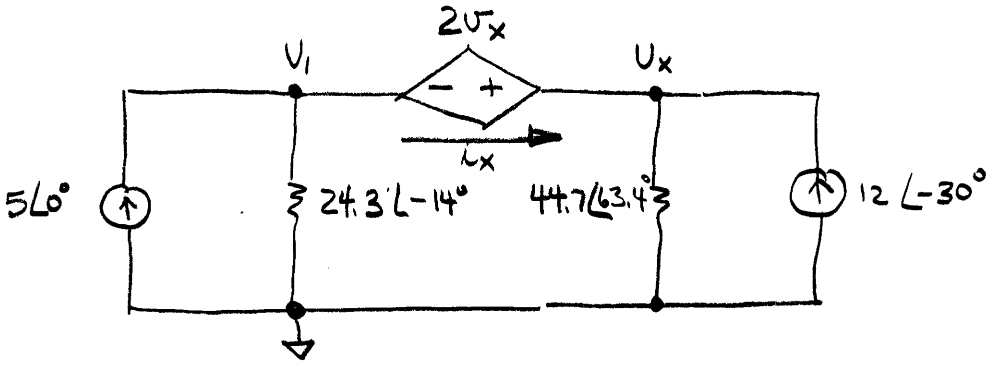
$$\frac{1}{j\omega C} = \frac{1}{j(500)(20\times10^{-6})} = -j100$$

$$\frac{(25)(-j100)}{25+(-j100)} = (23.5, -5.88)$$

$$24.3\angle -14°$$

$$j\omega L = j(500)(0.1) = j50$$

$$\frac{(100)(j50)}{100+j50} = 20+j40 = 44.7\angle 63.4°$$

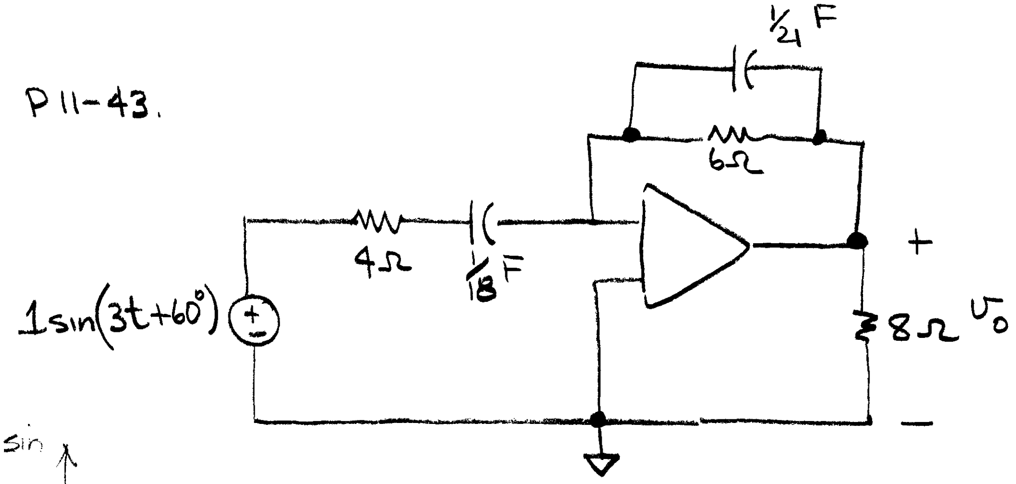


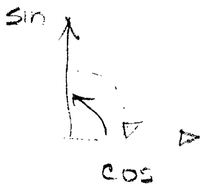
+out@1 $$-5\angle 0° + \frac{v_1}{24.3\angle -14°} + i_x = 0$$

+out@2 $$-i_x + \frac{v_x}{44.7\angle 63.4°} - 12\angle -30° = 0$$

$$v_1 + 2v_x = v_x$$

P11-43.



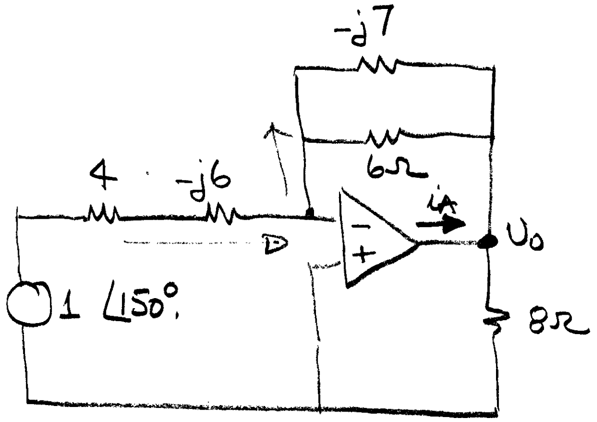


$\sin(x-90°) = \cos x$

$\sin(3t+150°-90°) = \cos(3t+150°)$

$\frac{1}{j\omega c} = \frac{1}{j(3)(\frac{1}{18})} = -j6$

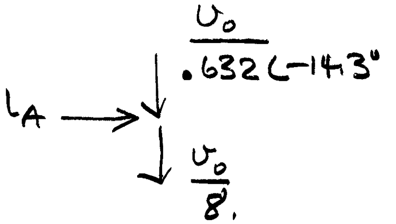
$\frac{1}{j\omega c} = \frac{1}{j3(\frac{1}{21})} = -j7$



$\frac{(-j7)(6)}{6-j7} = \frac{-j42}{6-j7}$

$= (3.46, -2.96)$

$\frac{1\angle 150° - 0}{4-j6} - \frac{0 - v_o}{(3.46, -2.96)} = 0.$



$\frac{1\angle 150°}{4-j6} + \frac{v_o}{(3.46, -2.96)} = 0.$

$v_o = -\frac{1\angle 150°}{4-j6}(3.46, -2.96) = (.612, -.156)$

$v_o = .632\angle -14.3°$

$\frac{v_o}{(3.46, -2.96)} + i_A = \frac{v_o}{8.}$

$i_A = \frac{v_o}{8} - \frac{v_o}{(3.46, -2.96)} = (-.0479, -.0809)$

$= .094\angle -121°$

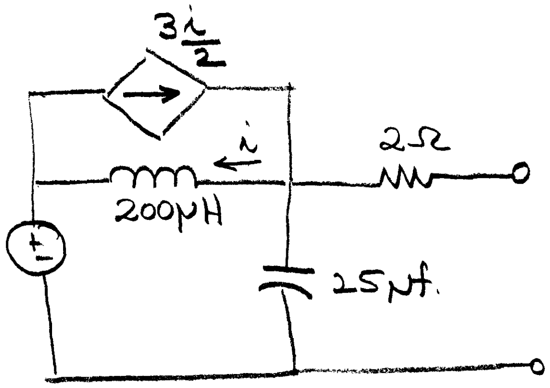
AP 11-1 CD - player output.

$\omega = 10{,}000$

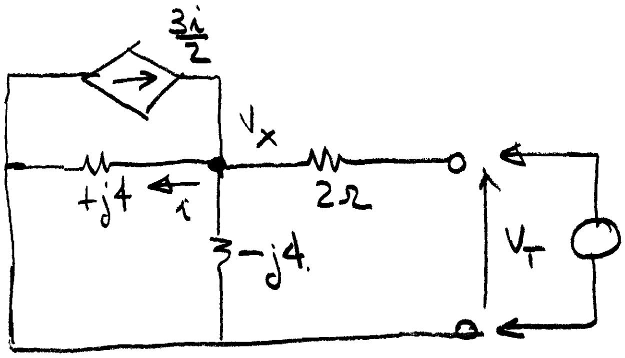
$v_s = 10\cos(\omega t + 53.1°)$

$$\frac{1}{j\omega C} = \frac{1}{j(10{,}000)(25\times10^{-6})} = -j4$$

$$j\omega L = j(200\times10^{-6})(10\,000) = +j4$$



R_T out



$$+\frac{3}{2}\left(\frac{V_x}{j4}\right) - \cancel{\frac{V_x}{j4}} - \cancel{\frac{V_x}{-j4}} + \frac{V_T - V_x}{2} = 0.$$

$$\left(\frac{3}{j8} - \frac{1}{2}\right)V_x + V_t = 0.$$

$$V_x = \frac{V_t}{\frac{1}{2} - \frac{3}{j8}}$$

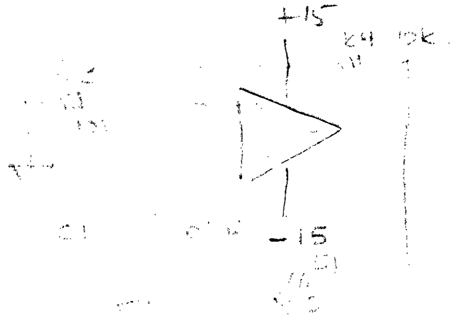
$$I_T = \frac{V_T - V_x}{2} = \frac{V_T - \dfrac{V_T}{\frac{1}{2} - \frac{3}{j8}}}{2.}$$

$$Z_T = \frac{V_T}{I_T} = \frac{V_T}{\frac{1}{2}\left(V_T - \dfrac{V_T}{\frac{1}{2} - \frac{3}{j8}}\right)} = \frac{2\left(\frac{1}{2} - \frac{3}{j8}\right)}{\frac{1}{2} - \frac{3}{j8} - 1}$$

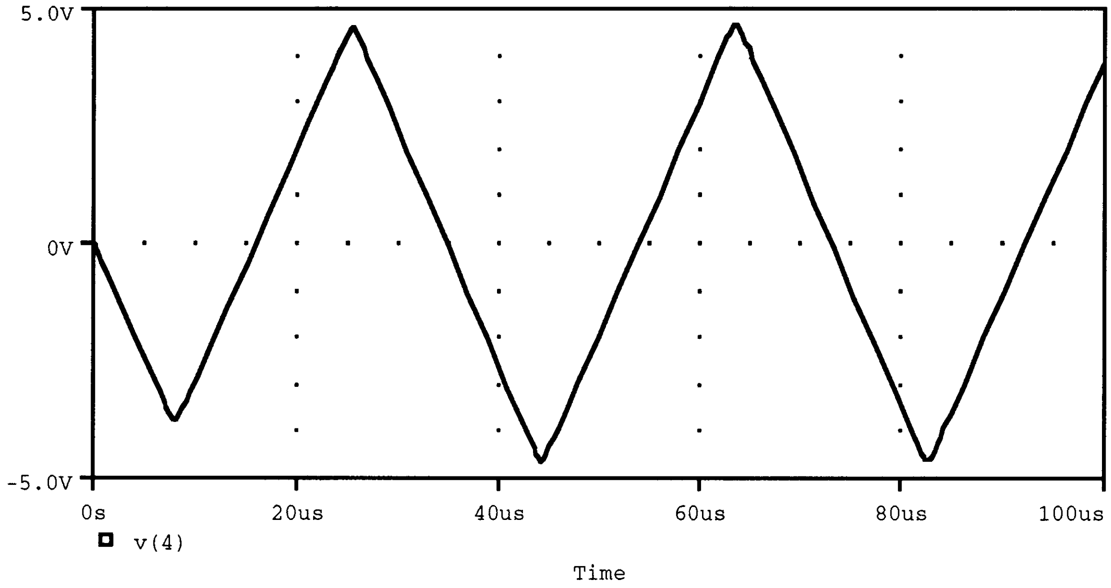
$$= \frac{2\left(\frac{1}{2} + j\frac{3}{8}\right)}{-\frac{1}{2} + j\frac{3}{8}} = \frac{2(4 + j3)}{(-4 + j3)} = (-.56, -1.92$$

$$= 2\angle 106°$$

```
**** 03/04/93 07:04:51 *********** Evaluation PSpice (Jan 1993) **************

 LAB 4 OSCILLATOR


****    CIRCUIT DESCRIPTION
******************************************************************************
C1    2    0    0.001u  IC=0.1
R4    4    3    10K
R2    2    4    5K
R3    3    0    10K
X1    3    2    5    6    4    uA741
VCC    5    0    DC    15
VEE    6    0    DC    -15
.LIB   EVAL.LIB
.TRAN  10u 100u      UIC
.PROBE
.END
```



```
**** 03/04/93 07:04:51 *********** Evaluation PSpice (Jan 1993) **************

 LAB 4 OSCILLATOR


 ****    Diode MODEL PARAMETERS


******************************************************************************
           X1.dx
      IS  800.000000E-18
      RS   1

**** 03/04/93 07:04:51 *********** Evaluation PSpice (Jan 1993) **************

 LAB 4 OSCILLATOR


 ****    BJT MODEL PARAMETERS


******************************************************************************
           X1.qx
           NPN
      IS  800.000000E-18
      BF  93.75
      NF   1
      BR   1
      NR   1


      JOB CONCLUDED

      TOTAL JOB TIME          23.93
```



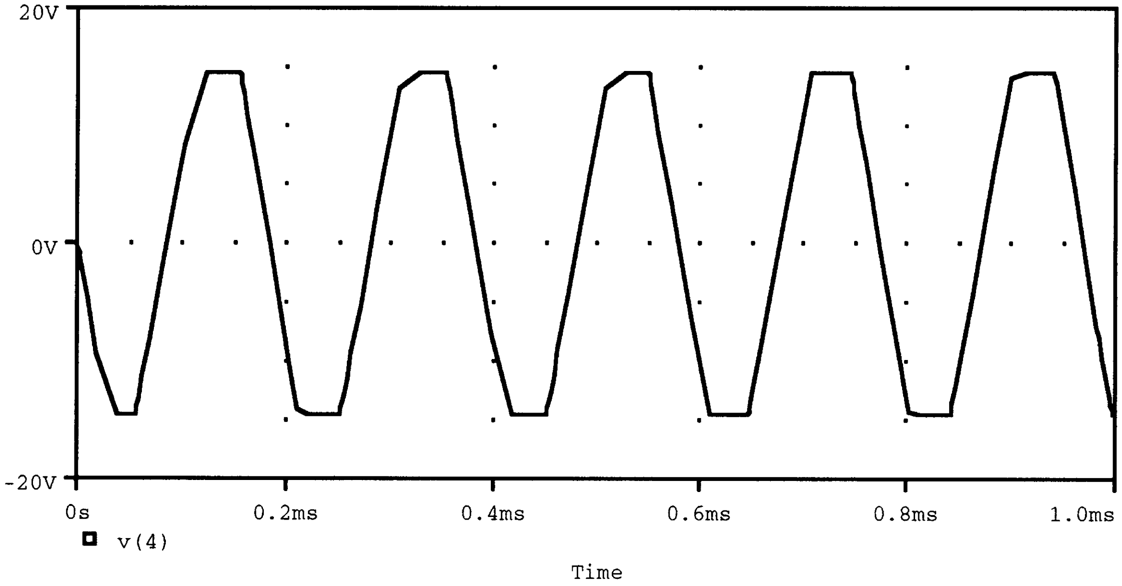
"LAB 4 OSCILLATOR"   03/04/93 07:04:51   27.0°

Evaluation Probe 5.3 © 1993 MicroSim Corp.

C=.001

741.

```
**** 03/04/93 07:10:52 *********** Evaluation PSpice (Jan 1993) **************

 LAB 4 OSCILLATOR


 ****     CIRCUIT DESCRIPTION
******************************************************************************
C1     2     0     0.01u  IC=0.1
R4     4     3     10K
R2     2     4     5K
R3     3     0     10K
X1     3     2     5     6     4      uA741
VCC    5     0     DC    15
VEE    6     0     DC    -15
.LIB   EVAL.LIB
.TRAN  10u 1000u     UIC
.PROBE
.END

**** 03/04/93 07:10:52 *********** Evaluation PSpice (Jan 1993) **************

 LAB 4 OSCILLATOR


 ****    Diode MODEL PARAMETERS


******************************************************************************
          X1.dx
      IS  800.000000E-18
      RS   1


**** 03/04/93 07:10:52 *********** Evaluation PSpice (Jan 1993) **************

 LAB 4 OSCILLATOR


 ****    BJT MODEL PARAMETERS


******************************************************************************
X1.qx
          NPN
      IS  800.000000E-18
      BF  93.75
      NF   1
      BR   1
      NR   1

      JOB CONCLUDED

      TOTAL JOB TIME          57.33
```



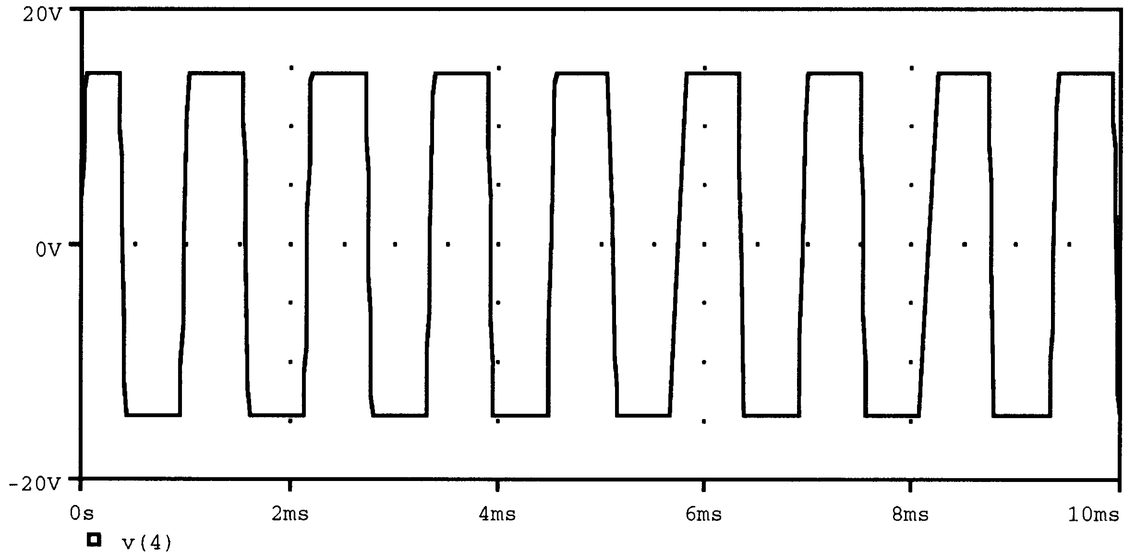
"LAB 4 OSCILLATOR" 03/04/93 07:10:52 27.0°

Evaluation Probe 5.3 © 1993 MicroSim Corp.

C = ,01

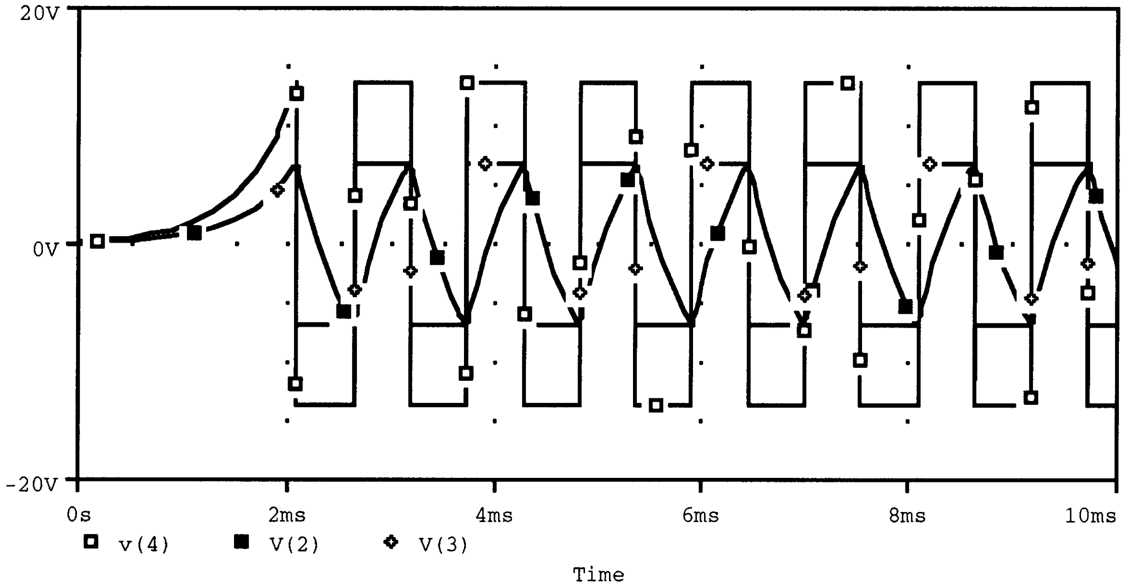
741

```
**** 03/04/93 07:14:51 *********** Evaluation PSpice (Jan 1993) **************

 LAB 4 OSCILLATOR


 ****     CIRCUIT DESCRIPTION
******************************************************************************
C1    2    0    0.1u   IC=0.1
R4    4    3    10K
R2    2    4    5K
R3    3    0    10K
X1    3    2    5    6    4    uA741
VCC    5    0    DC    15
VEE    6    0    DC    -15
.LIB   EVAL.LIB
.TRAN  10u 10000u    UIC
.PROBE
.END

**** 03/04/93 07:14:51 *********** Evaluation PSpice (Jan 1993) **************

 LAB 4 OSCILLATOR


 ****    Diode MODEL PARAMETERS


******************************************************************************
X1.dx
       IS  800.000000E-18
       RS    1


**** 03/04/93 07:14:51 *********** Evaluation PSpice (Jan 1993) **************

 LAB 4 OSCILLATOR


 ****    BJT MODEL PARAMETERS


******************************************************************************
X1.qx
          NPN
       IS  800.000000E-18
       BF  93.75
       NF   1
       BR   1
       NR   1

       JOB CONCLUDED

       TOTAL JOB TIME          101.72
```



"LAB 4 OSCILLATOR"   03/04/93 07:14:51   27.0°
Evaluation Probe 5.3 © 1993 MicroSim Corp.

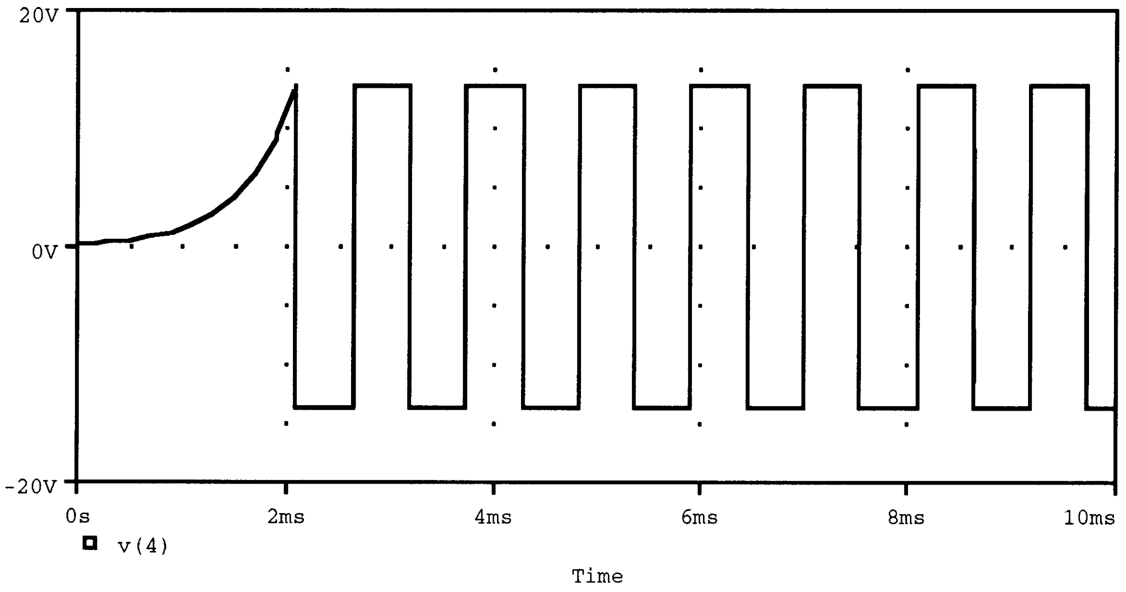
C = 0.1
741



"LAB 4 OSCILLATOR"   03/04/93 07:42:12   27.0°

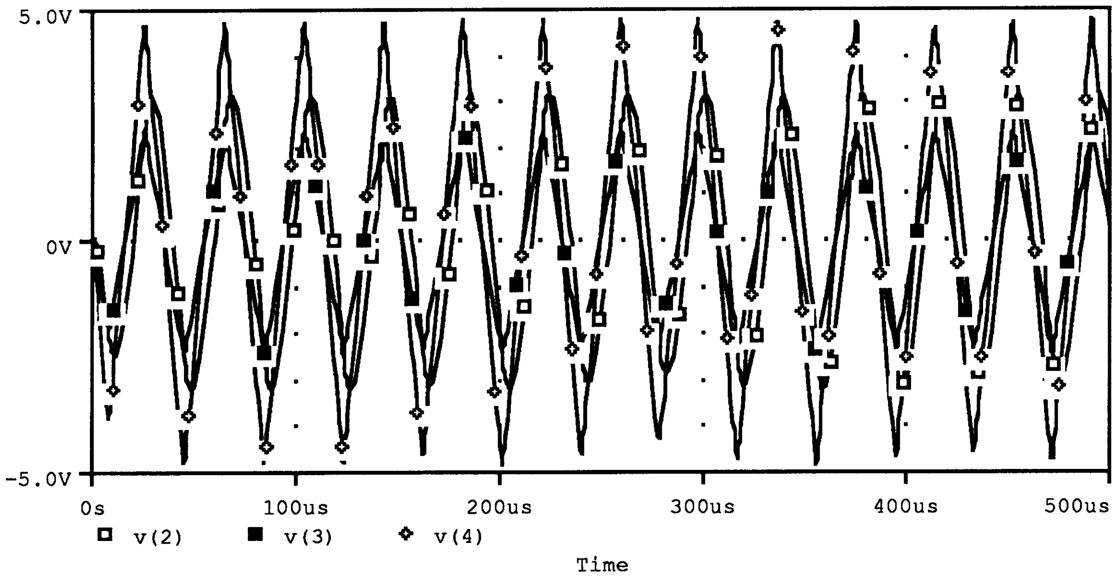
Evaluation Probe 5.3 © 1993 MicroSim Corp.

LM318.



"LAB 4 OSCILLATOR" 03/04/93 07:42:12 27.0°
Evaluation Probe 5.3 © 1993 MicroSim Corp.

LM318.



"LAB 4 OSCILLATOR"   03/03/93 17:09:18   27.0°
Evaluation Probe 5.3 © 1993 MicroSim Corp.

analysis

forcing function $\longrightarrow \boxed{\text{circuit}} \longrightarrow$ output

originally dc $V_0 e^0$

transient $V_0 e^{kt}$

sinusoids $V_0 e^{j\theta} e^{j\omega t}$

response

in general $V_0 e^{j\theta} e^{\underbrace{\overset{=s}{(\sigma+j\omega)}}_{\text{complex number}} t}$

dc $s = 0$

transient (exponential) $s = \sigma$

sinusoidal $s = j\omega$

exponential sinusoid $s = \sigma + j\omega$

mathematically

do the problem in the transform domain

$$v(t) = \frac{1}{2\pi j}\int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} \underbrace{e^{st} V(s)\, ds}_{\substack{\text{express } v(t) \text{ as the sum (integral)} \\ \text{of an infinite number of vanishingly small} \\ \text{terms with complex frequency } \sigma + j\omega}}$$

bilateral Laplace transform

$\boxed{\sigma \text{ governs convergence}}$

$$V(s) = \int_{-\infty}^{\infty} e^{-st} v(t)\, u(t)\, dt = \int_0^{\infty} e^{-st} v(t)\, dt.$$

one sided Laplace transform
Laplace transform.

use definition $V(s) = \int_0^\infty e^{-st} v(t)\,dt = \mathcal{L}[v(t)]$.

$$v(t) = \frac{1}{2\pi j}\int_{\sigma_0-j\infty}^{\sigma_0+j\infty} e^{st} V(s)\,ds = \mathcal{L}^{-1}[V(s)]$$

$$v(t) \Longleftrightarrow V(s)$$

one-to-one transform.

we will not do this everytime. (it's easier than it looks.

unit step function

$$\mathcal{L}[u(t)] = \int_0^\infty e^{-st} u(t)\,dt = \int_0^\infty e^{-st}\,dt = -\frac{1}{s} e^{-st}\Big|_0^\infty = \frac{1}{s}.$$

impulse

$$\underset{t_0 > 0^-}{\mathcal{L}[\delta(t-t_0)]} = \int_{0^-}^\infty e^{-st}\delta(t-t_0)\,dt = e^{-st_0}$$

$$\delta(t-t_0) \Longleftrightarrow e^{-st_0}$$

$$\delta(t) \Longleftrightarrow 1.$$

exponential

$$\mathcal{L}[e^{-\alpha t}u(t)] = \int_0^\infty e^{-\alpha t} e^{-st}\,dt = -\frac{1}{s+\alpha} e^{-(s+\alpha)t}\Big|_0^\infty = \frac{1}{s+\alpha}$$

$$e^{-\alpha t}u(t) \Longleftrightarrow \frac{1}{s+a}$$

initial conditions we know $v(t) \Longleftrightarrow V(s)$.

$$\mathcal{L}\left\{\frac{dv}{dt}\right\} = \int_0^\infty e^{-st}\frac{dv}{dt}dt. = \int_0^\infty e^{-st}dv.$$

$$= e^{-st}v\Big|_{0^-}^{\infty} - \int_{0^-}^{\infty} -s e^{-st} v(t)\,dt.$$

$$= e^{-st}v\Big|_{0^-}^{\infty} + s\int_0^\infty e^{-st}v(t)\,dt.$$

$$= 0 - v(0^-) + sV(s).$$

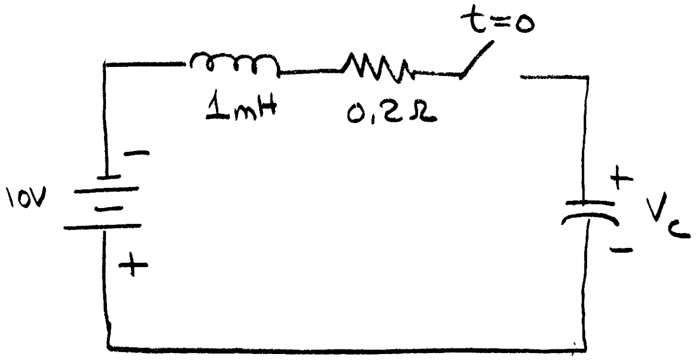
$$\mathcal{L}\left\{\frac{dv}{dt}\right\} = sV(s) - v(0^-).$$

#5

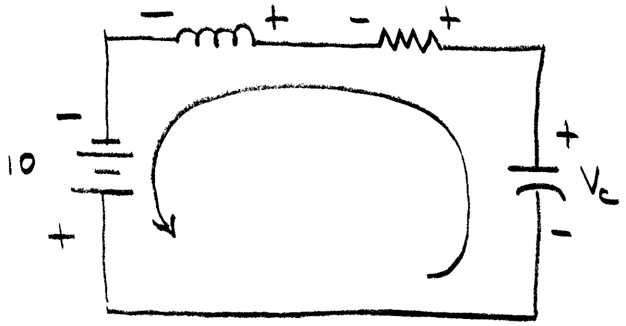
P.77

([illegible] 19)

Engineering Circuit
Analysis with
PSpice and Probe



for switch closed



we wll solve for $i(t)$.

use KVL $\quad +10 - L\frac{di}{dt} - iR + V_c = 0.$

where $V_c(t) = \frac{1}{C}\int_0^t i(x)\,dx$

How do we get Laplace transform of $V_c(t)$?

$$\mathcal{L}\left[\int_0^t i(x)dx\right] = \int_0^\infty e^{-st}\left[\int_0^t i(x)\,dx\right]dt$$

Now we integrate by parts. $\quad \int_0^\infty u\,dw = uw\Big|_0^\infty - \int_0^\infty w\,du$

$$u = \int_0^t i(x)\,dx \quad \Rightarrow \quad du = i(t)\,dt$$

$$dw = e^{-st}dt \qquad w = \frac{e^{-st}}{-s} = -\frac{1}{s}e^{-st}$$

$$\mathcal{L}\left[\int_0^t i(x)dx\right] = \left[\int_0^t i(x)dx\right]\left[-\frac{e^{-st}}{s}\right]\Bigg|_0^\infty + \int_0^\infty \frac{e^{-st}}{s}\,i(t)\,dt$$

$$\underbrace{\begin{matrix}\text{as } t\to\infty \quad e^{-st}\to 0 \\ \text{as } t\to 0 \quad \int^t i(x)dx \to 0\end{matrix}}_{0} + \frac{1}{s}\,I(s)$$

back to our problem :

$$10 - L\frac{di}{dt} - iR + \frac{1}{C}\int_0^t i(x)\,dx = 0.$$

$$10 - L\left[s\,I(s) - i(0^-)\right] - I(s)R + \frac{1}{C}\left[\frac{1}{s}I(s)\right] = 0.$$

before switch closes $i(0^-) = 0.$

$$10 - sL\,I(s) - R\,I(s) + \frac{1}{sC}I(s) = 0.$$

$$10 + I(s)\left(\frac{1}{sC} - R - sL\right) = 0.$$

$$I(s) = \frac{-10}{\frac{1}{sC} - R - sL.}$$

$$= \frac{-10sC}{1 - sRC - s^2LC}$$

$$I(s) = \frac{-10s(\cancel{10^3}\times\cancel{10^{-6}})}{10^3 - s(0.2)(\cancel{10^3}\times\cancel{10^{-6}}) - s^2(1\times10^{-3})(\cancel{10^3})\times\cancel{10^{-6}}}$$

don't know how to do this yet →

$$= \frac{-10s}{10^3 - 0.2s - 10^{-3}s^2} = \frac{-10,000\,s}{10^6 - 200s - s^2}$$

$$= \frac{10,000s}{s^2 + 200s - 10^6}$$

do quadratic solution

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$s = \frac{-200 \pm \sqrt{(200)^2 - 4(1)(10^6)}}{2} = -100 \pm \frac{1}{2}j1990$$

What is this ?

$$I(s) = \frac{+10{,}000\,s}{[s-(-100+j995)][s-(-100-j995)]}.$$

<u>What is this?</u>

$$= \frac{+10{,}000\,s}{(s+100-j995)(s+100+j995)}$$

$$= \frac{a}{s+100-j995} + \frac{b.}{s+100+j995}$$

$$a(s+100+j995) + b(s+100-j995) = -10000s.$$

$$as + bs = +10{,}000\,s.$$

$$a+b = +10{,}000$$

$$a(100+j995) + b(100-j995) = 0.$$

$$a = -b\,\frac{(100-j995)}{(100+j995)}$$

$$-b\,\frac{(100-j995)}{(100+j995)} + b = +10000.$$

$$b = \frac{+10{,}000}{-\frac{100-j995}{100+j995}+1} = \frac{+10{,}000}{\frac{-(100-j995)+(100+j995)}{100+j995}}$$

$$= \frac{+10{,}000\,(100+j995)}{\cancel{-100}+j995\cancel{+100}+j995}$$

$$= \frac{+10{,}000\,(100+j995)}{j1990.}$$

$$b = \frac{-10{,}000\,j\,(100+j995)}{j1990} = -j5.03\,(100+j995)$$

$$b = +5000 - j503$$

$$a + b = +10{,}000$$

$$a = -b + 10{,}000 = -5000 + j503 + 10{,}000 = +5000 + j503$$

$$I(s) = \frac{+5000 + j503}{s + 100 - j995} + \frac{+5000 - j503}{s + 100 + j995}.$$

recall $\mathcal{L}^{-1}\left[\frac{1}{s+a}\right] = e^{-at}.$

$$\mathcal{L}^{-1}\left[I(s)\right] = \left[(+5000 + j503)\,e^{-(100-j995)t} + (+5000 - j503)\,e^{-(100+j995)t}\right]u(t)$$

$$= e^{-100t}\left[(+5000 + j503)\,e^{-j995t} + (+5000 - j503)\,e^{+j995t}\right]$$

$$= e^{-100t}\left[+5000\left(e^{-j995t} + e^{+j995t}\right) + j503\left(+e^{-j995t} - e^{+j995t}\right)\right]$$

$e^{j\theta} = \cos\theta + j\sin\theta$

$e^{-j\theta} = \cos\theta - j\sin\theta$

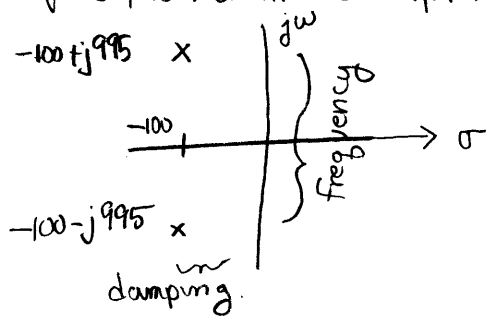
$e^{j\theta} - e^{-j\theta} = 2j\sin\theta$

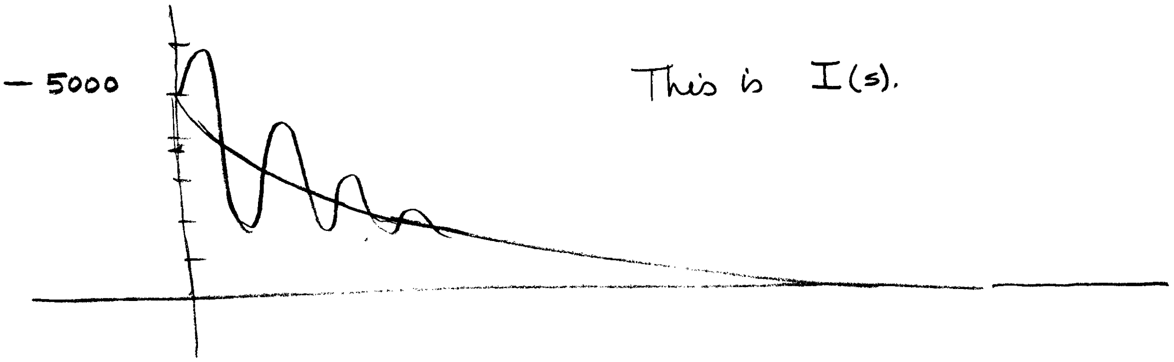
$e^{j\theta} + e^{-j\theta} = 2\cos\theta$

$$= e^{-100t}\left[+5000\;2\cos(995t) + j503\left(-2j\sin 995t\right)\right].$$

$$= e^{-100t}\left[+10000\cos(995t) + 1006\sin 995t.\right].$$

The poles may be plotted in the complex domain as





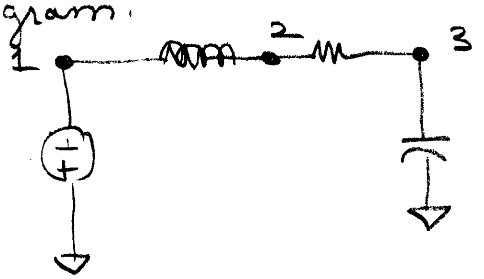
This is I(s).

Plan to do simulation of this.

$t = \frac{1}{100} = 10$ msec.

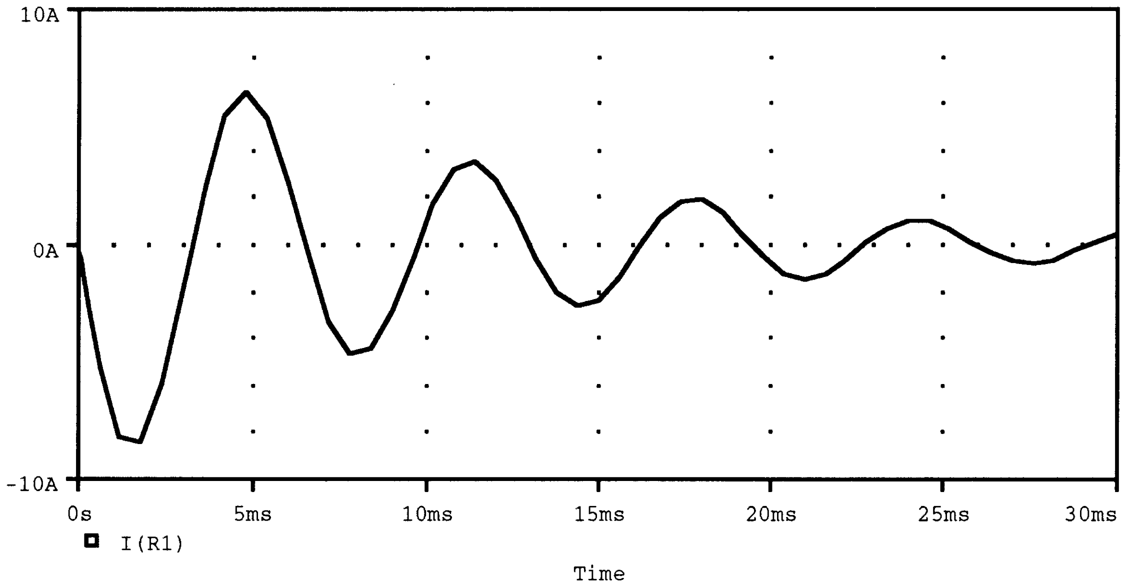
everything done in 30 milliseconds

PSpice program.



```
RLC circuit
Vs  0  1   PWL 0,0  1μ,1
L   1  2   1m
R   2  3   0.2
C   3  0   1000μ
.tran 30m 30m.
.probe
.end.
```

```
**** 03/04/93 07:56:36 *********** Evaluation PSpice (Jan 1993) **************

 CLASSROOM DEMO RLC CIRCUIT


 ****     CIRCUIT DESCRIPTION


**********************************************************************
Vs     0      1      PWL 0,0 1u,10
L1     1      2      1m
R1     2      3      0.2
C1     3      0      1000u
.TRAN   30m 30m
.PROBE
.END

**** 03/04/93 07:56:36 *********** Evaluation PSpice (Jan 1993) **************

 CLASSROOM DEMO RLC CIRCUIT


 ****     INITIAL TRANSIENT SOLUTION       TEMPERATURE =   27.000 DEG C


**********************************************************************
 NODE   VOLTAGE    NODE   VOLTAGE    NODE   VOLTAGE    NODE   VOLTAGE


(    1)    0.0000  (    2)    0.0000  (    3)    0.0000




   VOLTAGE SOURCE CURRENTS
   NAME        CURRENT

   Vs          0.000E+00

   TOTAL POWER DISSIPATION   0.00E+00  WATTS



       JOB CONCLUDED

       TOTAL JOB TIME           3.98
```



"CLASSROOM DEMO RLC CIRCUIT" 03/04/93 07:56:36 27.0°

Evaluation Probe 5.3 © 1993 MicroSim Corp.

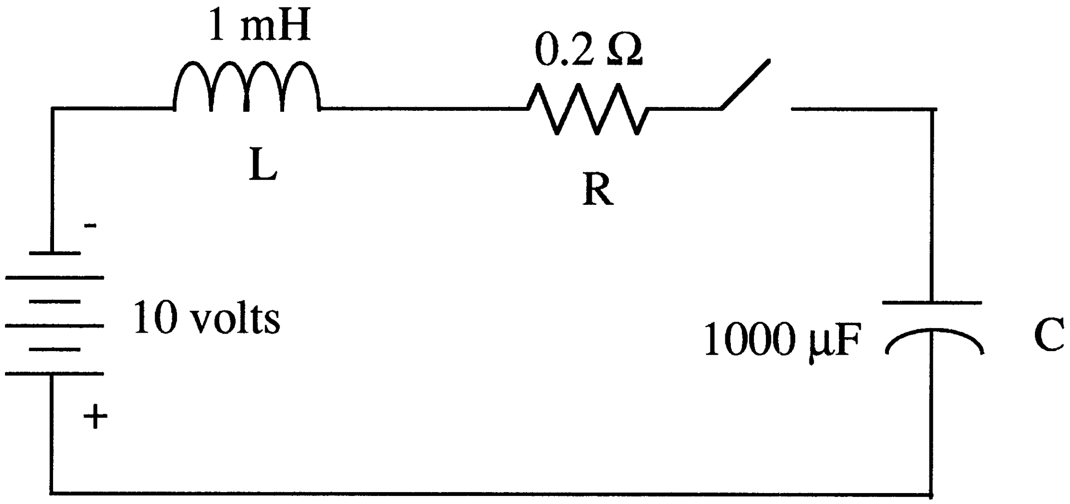
INS TCP/IP PrintServer 20

Print engine name:	PrintServer 20
Print engine version:	17
Printer firmware version:	32
Server Adobe PostScript version:	48.3
Server software version:	V2.0
Server network node:	crawford
Server name:	crawford
Server job number:	28
Client software version:	WRL-1.0
Client network node:	util
Client name:	flm
Client job name:	prob_soln.ps.732547382
Submitted at:	Fri Mar 19 08:28:20 1993
Printed at:	Fri Mar 19 08:28:20 1993

flm@util
prob_soln.ps.732547382

digital

Solve for the current in the following circuit using Laplace transform techniques.



Use KVL to write the equation for i. I chose to have the current go in a counterclockwise direction to coincide with current flow from the positive terminal of the battery.

The problem with the example done in class was that the voltage across the capacitor did not obey the passive sign convention. If you go around the above circuit counterclockwise you should get the equation:

$+V_c + Ri + L\frac{di}{dt} - 10u(t) = 0$

where

$V_c = \frac{1}{C}\int_0^t i(x)dx$

Substituting this expression for V_C into the above equation:

$\frac{1}{C}\int_0^t i(x)dx + Ri + L\frac{di}{dt} - 10u(t) = 0$

Taking the Laplace transform of the above equation gives:

$\frac{1}{C}\frac{I(s)}{s} + RI(s) + L(sI(s) - i(0^-)) - \frac{10}{s} = 0$

which was obtained using the Laplace transform relationships for the derivative and integral of a function, i.e.

$i(t) \leftrightarrow I(s)$

$\int_0^t i(x)dx \leftrightarrow \frac{I(s)}{s}$

$\frac{di}{dt} \leftrightarrow sI(s) - i(0^-)$

Since $i(0^-)=0$ this reduces to:

$\frac{I(s)}{sC} + RI(s) + LsI(s) - \frac{10}{s} = 0$

Solving for I(s):

$$I(s) = \frac{\frac{10}{s}}{sL + R + \frac{1}{sC}} = \frac{\frac{10}{L}}{s^2 + s\frac{R}{L} + \frac{1}{LC}} = \frac{\frac{10}{0.001}}{s^2 + s\frac{0.2}{0.001} + \frac{1}{0.001(1000\times10^{-6})}} = \frac{10{,}000}{s^2 + 200s + 10^6}$$

You can factor the denominator using the quadratic formula

$$s = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-200 \pm \sqrt{(200)^2-4(1)(10^6)}}{2(1)} = -100 \pm j995$$

and, after substituting into the above expression for I(s):

$$I(s) = \frac{10{,}000}{(s+100+j995)(s+100-j995)}$$

Expanding by partial fractions gives:

$$I(s) = \frac{10{,}000}{(s+100+j995)(s+100-j995)} = \frac{A}{(s+100+j995)} + \frac{B}{(s+100-j995)}$$

Equating the coefficients of s:

s^1: A + B = 0

s^0: (100 – j995)A + (100 + j995)B = 10000

which can be solved to give A = +j2.51 and B = -j2.51. The final expression for I(s) is:

$$I(s) = \frac{+j5.03}{(s+100+j995)} + \frac{-j5.03}{(s+100-j995)}$$

Inverse Laplace transforming using the relationship

$$e^{-at}u(t) \leftrightarrow \frac{1}{s+a}$$

yields

$$i(t) = j5.03e^{-(100+j995)t}u(t) - j5.03e^{-(100-j995)t}u(t) = -j5.03e^{-100t}\left[e^{-j995t} - e^{+j995t}\right]u(t)$$

which can be further simplified using the Euler identities

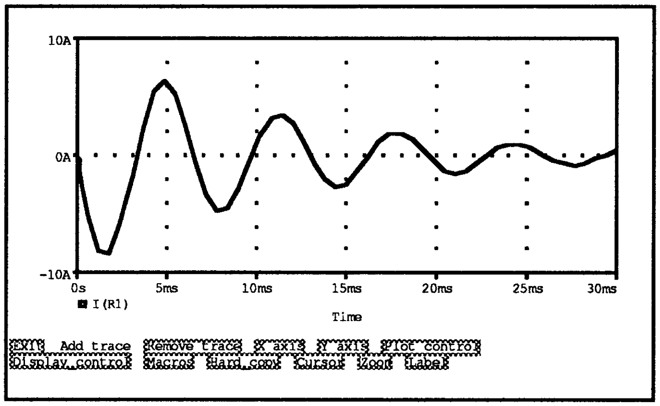
$e^{j\theta} - e^{-j\theta} = 2j\sin\theta$ and $e^{j\theta} + e^{-j\theta} = 2\cos\theta$

to give the final result for i(t), i.e.

$$i(t) = -j5.03e^{-100t}[2j\sin(995t)]u(t) = +10.06e^{-100t}\sin(995t)u(t)$$

A PSpice simulation of the circuit was done:

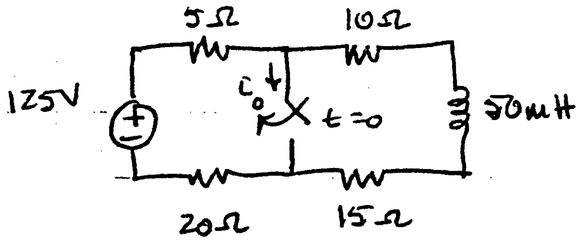
```
Vs      0    1    PWL 0,0 1u,10
L1      1    2    1m
R1      2    3    0.2
C1      3    0    1000u
.TRAN        30m 30m
.PROBE
.END
```



Note that the simulation result is the negative of the Laplace transform result because PSpice measures the current through the resistor using node 2 as the positive node whereas we used node 3 as the positive node for R in our calculations. Furthermore, I did not use an actual unit step but only an approximation to one using the PWL (piece-wise linear) source command.

①

Problem 8.3 - Natural Response of an Inductor



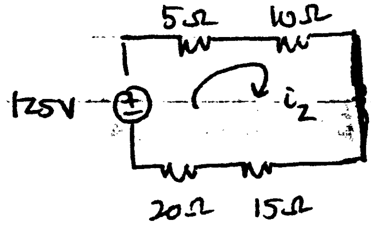
(ckt 1)

Recitation Solution (for a, b)

a) (1) After a long time, an inductor in a circuit with a DC voltage becomes a short

(2) the current across an inductor cannot change instantaneously. ($i_L(0^-) = i_L(0^+)$)

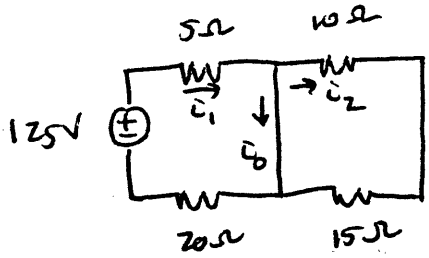
So the circuit before the switch is thrown looks like this



(CKT 2)

The current in this circuit is $i_2 = \frac{125V}{50\Omega} = 2.5A.$ $= i_L(0^-)$

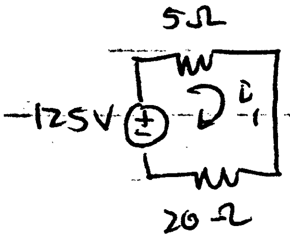
When the switch is closed, a short will appear ~~acros~~ across the middle of the circuit and it will look as follows:



(CKT3)

for time $t=0^+$. What was previously termed i_2 will flow on the right hand side of the circuit and this will be subtracted from i_1 to obtain t_o ($i_o = i_1 - t_2$).

i_1 is the current on the left side of the circuit and is defined by the following circuit



$$i_1 = \frac{125V}{25\Omega} = 5A$$

So $i(0+) = i_1 - i_2 = 5A - 2.5A = 2.5A$.

the current on the inductor side of the circuit will expontial die out so the remaining current will simply be i_1. So $i_o(\infty) = i_1 = 5A$.

③

b) For part b), in order to find the current $i(t)$ I used the natural response charge equation for an RL circuit. It is

$$i(t) = I_0 e^{-t/\tau}$$

Where $\tau = \frac{L}{R}$. It is derived in section 8.1 of the text. Using this equation:

$$\tau = \frac{L}{R} = \frac{50 \times 10^{-3}}{25} = 2ms$$

(R = resistance in inductive portion of the circuit)

$i_2 = 2.5 e^{-t/2ms} = 2.5 e^{-500t}$ A $\quad t \geq 0$

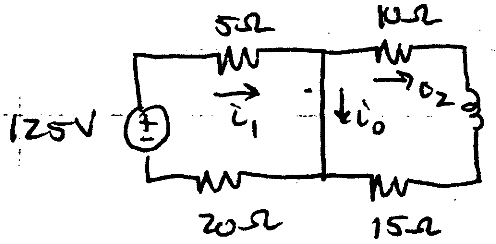
$i_1 = 5A \quad t \geq 0$ (no inductor here)

$i_0 = i_1 - i_2 = 5 - 2.5 e^{-500t} \quad t \geq 0$

Dr Gruber's Solution (for a, b)

Dr Gruber used the same analysis to find $i_2 = i_L(0^-)$. $i_2 = \frac{125}{50} - 2.5$ A from CKT 2. ($i_2 = i_L(0^-)$)

His analysis after the switch closed is slightly different, but gives the same result. The circuit he used is as follows



Looking at the left side of the circuit, i_1 has to equal 5A by KVL

$$125V = (5+20)i_1 \qquad i_1 = 5A.$$

So

$$i_0 + i_2 = i_1 = 5A$$

(1) $$i_0 = 5 - i_2$$

Now do KVL on the right side

$$\underset{\text{(voltage across short)}}{0} = 25 i_2 + L\frac{di_2}{dt}$$

$$i_2(0^-) = 2.5$$

Now he assumes that $i_2 = Ie^{st}$ so

$$0 = 25Ie^{st} + 50\times10^{-3}\, sIe^{st}$$

$$s = \frac{-25}{50\times10^{-3}} = -500$$

$\therefore\ i_2 = Ie^{-500t}$ @ $t=0$, $i_2 = 2.5$

$$\underline{2.5 = I}$$

$$i_2 = 2.5e^{-500t}$$

and $i_0^{(t)} = 5 - i_2 = 5 - 2.5e^{-500t}$

$$\underline{i_0(0) = 5 - 2.5e^{0} = 2.5\ A}$$

$$i_0(\infty) = 5 - 2.5e^{-\infty} = 5\ A$$

c) Part c is the same for both.

In how many μseconds will $i_0(t) = 3A$?

$$3 = 5 - 2.5e^{-500t}$$

$$-2 = -2.5e^{-500t}$$

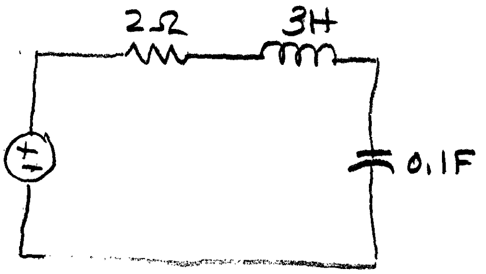
$$\ln\ 0.8 = \ln\ e^{-500t}$$

$$-500t = \ln 0.8$$

$$t = 446.28\ \mu s.$$

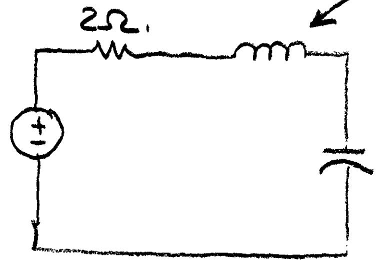
Example

$v(t) = 60e^{-2t}\cos(4t+10°)$



$$\Downarrow$$

identify $s = -2+j4$.

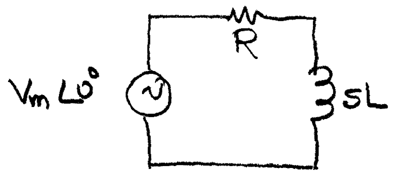


$sL = (-2+j4)3 = -6+j12$

$v_s = 60\angle 10°$

$$\frac{1}{sC} = \frac{1}{(-2+j4)(0.1)} = -1-j2$$

12-5 Frequency response as a function of σ



$$I = \frac{V_m\angle 0°}{R+sL}$$

what happens when $\omega \to 0$, $s = \sigma + j0$

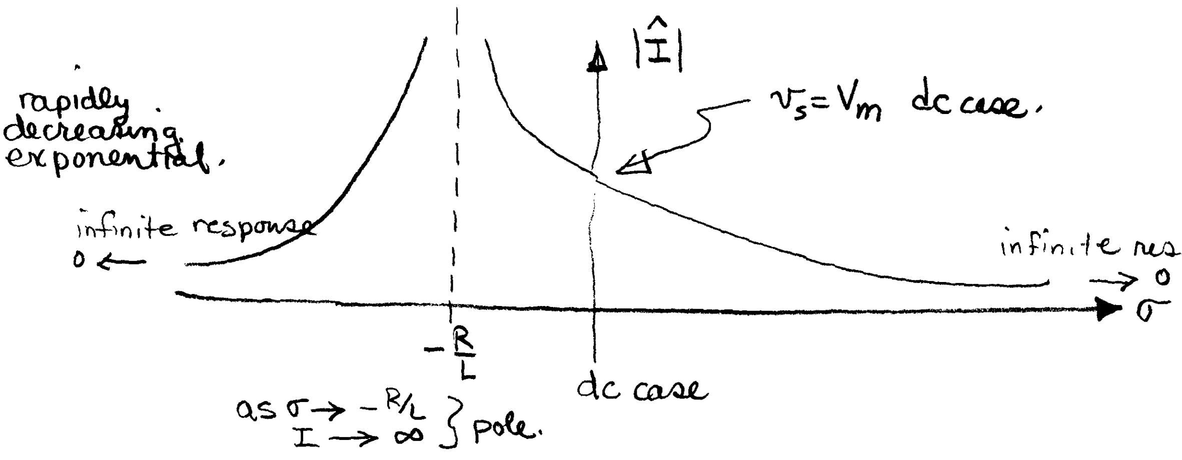
sources of form $v_s = V_m e^{\sigma t}$

$$\hat{I} = \frac{V_m}{R+\sigma L} = \frac{V_m}{L}\,\frac{1}{\sigma + R/L}$$

form $\frac{1}{s+a}$

inverse transforming

$$i(t) = \frac{V_m}{L}\,e^{-\frac{R}{L}t}$$



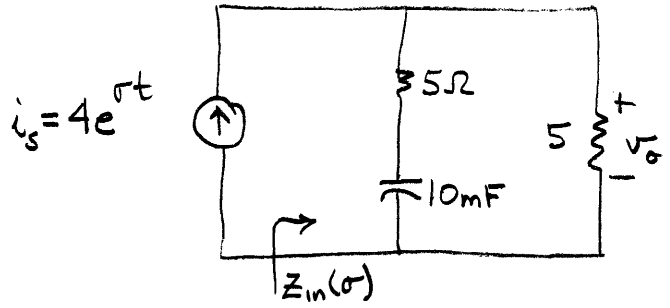
Example: desire $i(t) = 1e^{\sigma t}$ at $s = \sigma + j0$

$$1 = \frac{V_m}{\sigma+1}$$

$$V_m = \sigma + 1$$

This current will produce a current of 1A amplitude

Example:



calculate input impedance

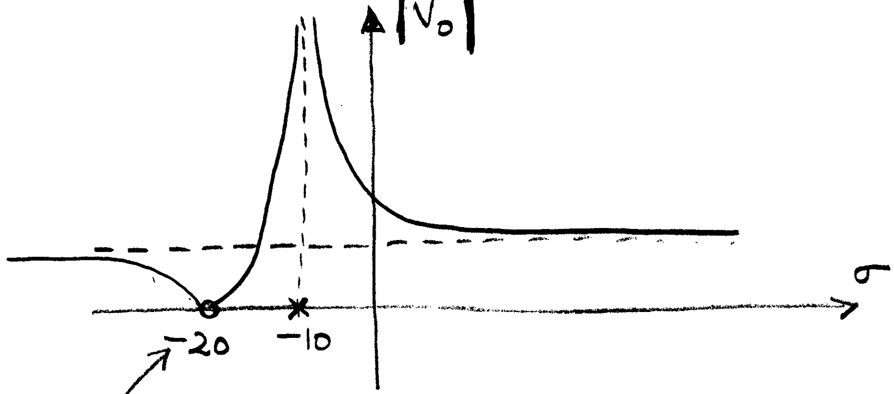
$$Z_{in}(\sigma) = 5 \| \left(5 + \frac{1}{sC}\right) = 5 \| \left(5 + \frac{1}{\sigma .01}\right) = \frac{5(5+.01\sigma)}{5+5+.01\sigma}$$

$$Z_{in}(\sigma) = 2.5\,\frac{\sigma+20}{\sigma+10}$$

in the frequency domain

$$\hat{V}_o = I_s Z_{in} = (4)\,2.5\,\frac{\sigma+20}{\sigma+10} = 10\,\boxed{\frac{\sigma+20}{\sigma+10}}$$

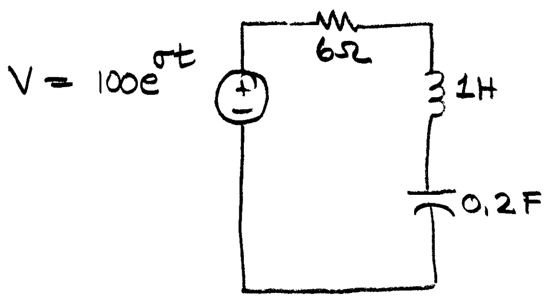
nice form for response



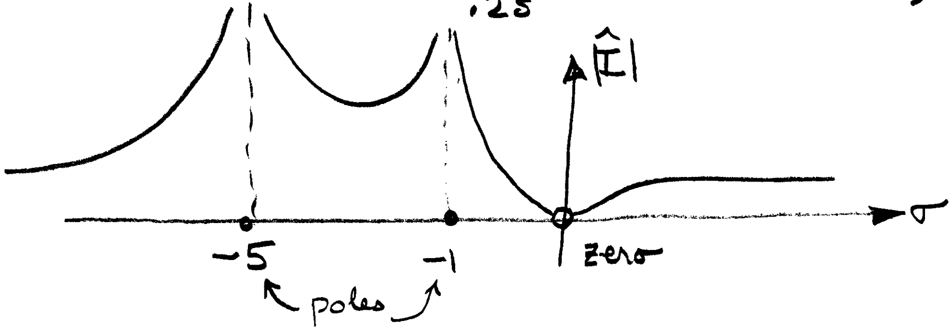
no output at zero

time domain voltage. $10\,\dfrac{\sigma+20}{\sigma+10}\,e^{\sigma t}$

d



$$I = \frac{V}{Z} = \frac{100}{6 + s + \frac{1}{.2s}} = 100\,\frac{\sigma}{(\sigma+1)(\sigma+5)}$$



forced response for $\sigma = -3$ is $\hat{I} = 100\,\frac{-3}{(-3+1)(-3+5)} = 100\,\frac{-3}{(-2)(+2)} = +75A$

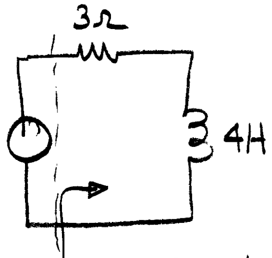
$$i(t) = 75e^{-3t}$$

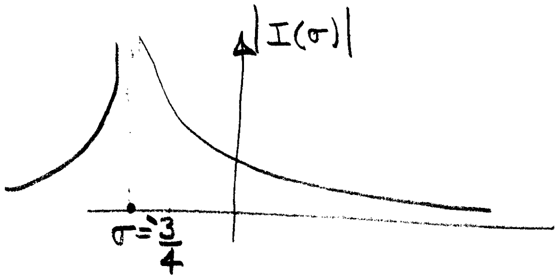
this is forced response.

natural response when no forcing function

12-6 The complex frequency plane.

let's consider what happens when both σ, ω are non-zero





$z(s) = 3 + 4s$

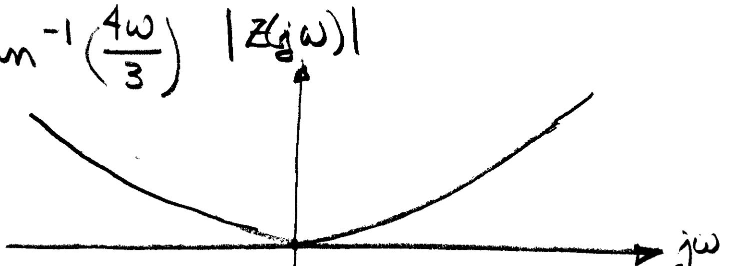
plot response as function of σ for ω=0 $\quad Z(\sigma) = 3 + 4\sigma \qquad I(s) = \dfrac{V(s)}{3+4\sigma}$

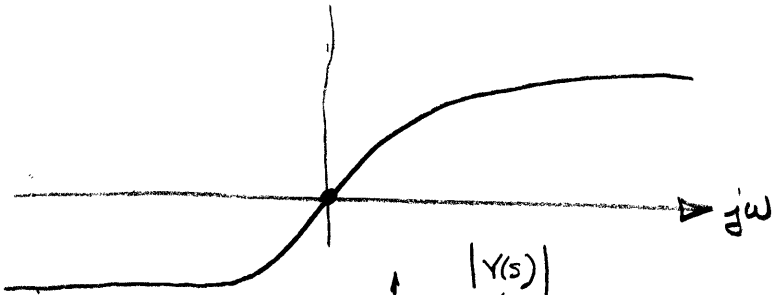
pole at $\sigma = -\frac{3}{4}$

plot response as function of jω for σ=0 $\quad Z(j\omega) = 3 + j4\omega$

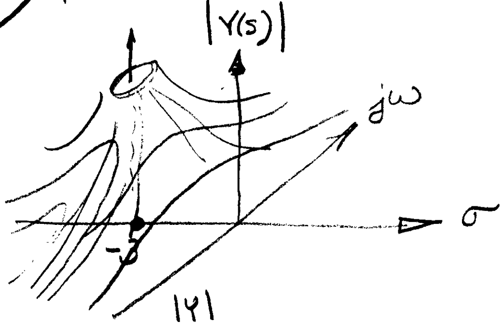
$$|Z(j\omega)| = \sqrt{9+16\omega^2}$$

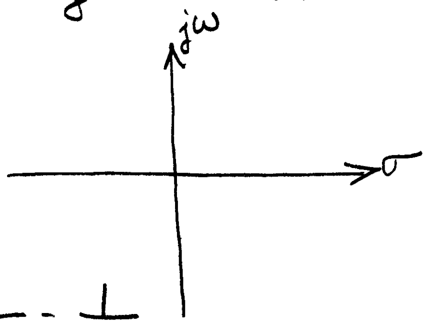
$$\angle Z(j\omega) = \tan^{-1}\left(\frac{4\omega}{3}\right)$$





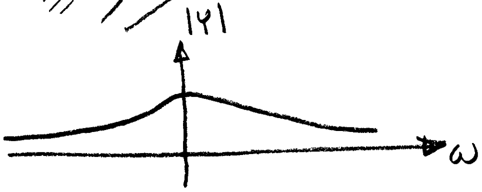
consider plotting in two dimensions

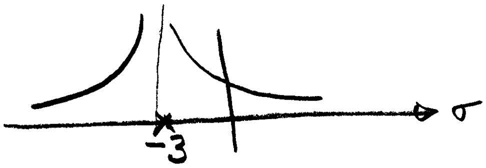




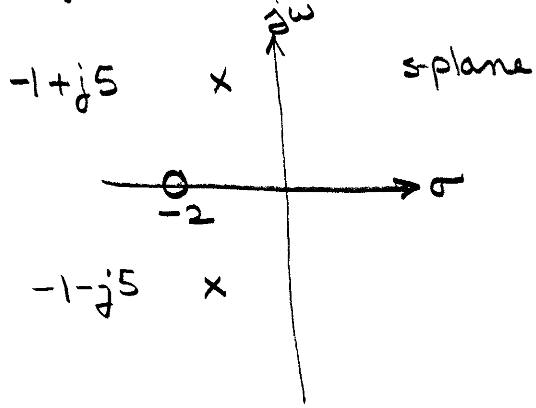
$$Y(s) = \frac{1}{Z(s)} = \frac{1}{s+3}$$

$$|Y(s)| = \frac{1}{\sqrt{(\sigma+3)^2+\omega^2}}$$

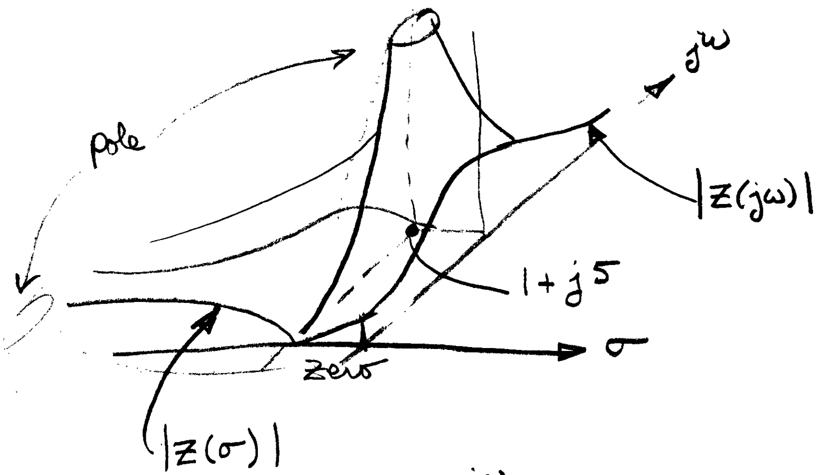


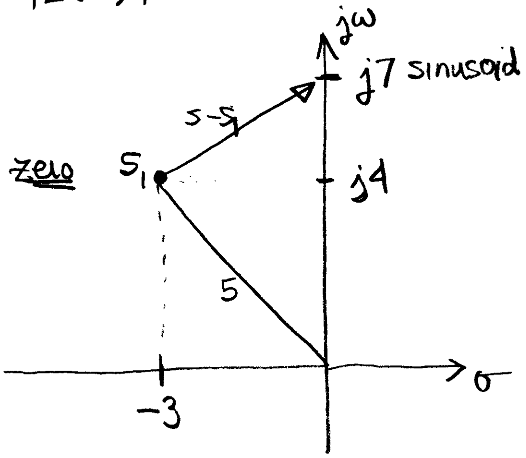


suppose you are given



$$Z(s) = k\,\frac{s+2}{(s+1-j5)(s+1+j5)} = k\,\frac{s+2}{s^2+2s+26}$$

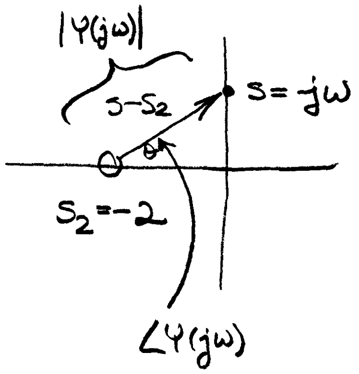


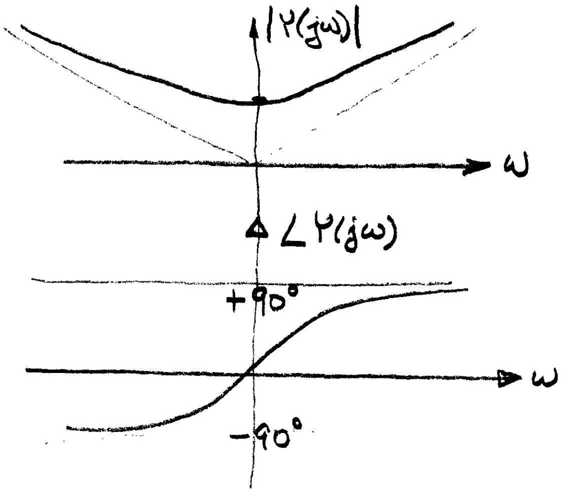


$s_1 = -3 + j4 = 5\angle 126.9^\circ$

suppose $Y(s) = s+2$

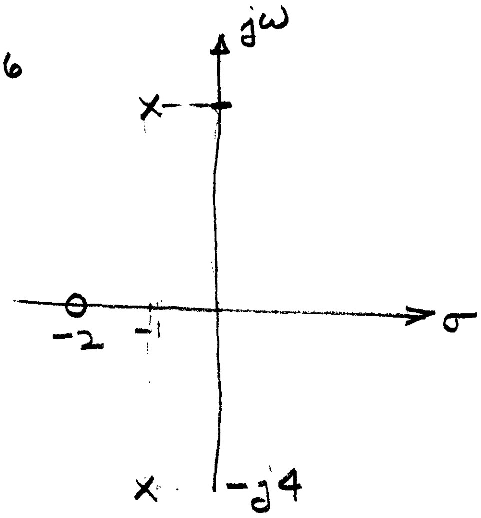
$I(s) = \frac{V(s)}{Z(s)} = Y(s)\,V(s).$





h

Problem 16
p. 374.



$$H(s) = k\,\frac{s+2}{(s+1-j4)(s+1+j4)}$$

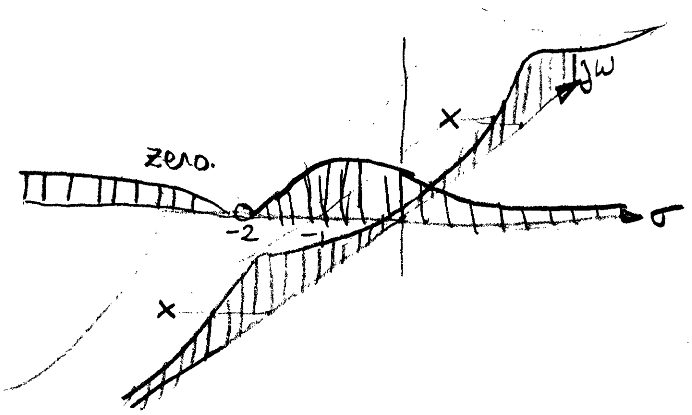
$$= k\,\frac{s+2}{s^2+2s+17}$$

$(s+a)(s+a^*)$

$s^2+(a+a^*)+|a|^2$

$(1-j4)(1+j4) = 1-j4+j4+16$

sketch H(s) vs. σ $\omega=0$
vs. ω $\sigma=0$.

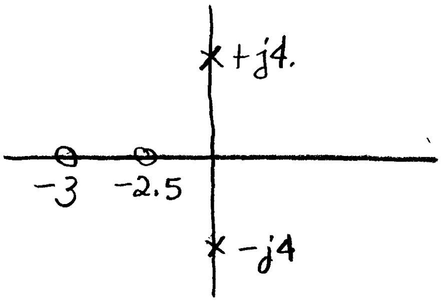


problem 17

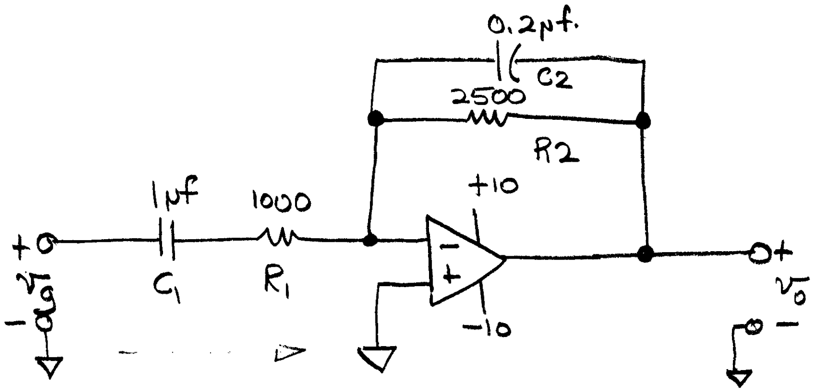
$$H(s)=\frac{10s^2+55s+75}{s^2+16} = \frac{(s\quad)(s\quad)}{(s+j4)(s-j4)}.$$

$$\frac{-55\pm\sqrt{(55)^2-4(10)(75)}}{2(10)} = \frac{-55\pm5}{20} = -3, -2.5$$

$s^2=-16$
$s=\pm j4.$



16.51



$$\frac{1}{sC} = \frac{1}{s(.2\times10^{-6})}$$
$$= \frac{5\times10^{6}}{s}$$

/i

no energy stored initially

$V_g = 6000\boxed{t}u(t)$

find (a) $V_o(t)$

(b) v_o

(c) how long it takes to saturate

(d) ——— v_g.

KCL

$$\frac{V_g - 0}{Z_i} - \frac{V_o - 0}{Z_f} = 0 \qquad V_o = -\frac{Z_f}{Z_i}V_g$$

$$Z_f = \frac{(2500)\left(\frac{5\times10^6}{s}\right)}{2500 + \frac{5\times10^6}{s}} = \frac{\frac{12.5\times10^9}{s}}{2500+\frac{5\times10^6}{s}} = \frac{12.5\times10^9}{5\times10^6 + 2500s}$$

$$Z_f = \frac{5\times10^6}{s+2000}$$

$$Z_i = \frac{1}{s(1\times10^{-6})} + 1000 = \frac{10^6}{s} + 1000 = \frac{10^6 + 1000s}{s} = 10^3\left(\frac{s+1000}{s}\right)$$

$$\int_0^t 6000\,dx \longrightarrow \frac{6000}{s}$$

$$6000t\ u(t) \longrightarrow \frac{6000}{s^2}$$

$$V_g = \frac{6000}{s^2}$$

$$V_o(s) = -\frac{\frac{5\times10^6}{s+2000}}{10^3(s+1000)}\frac{6000}{s^2} = \frac{(5\times10^6)(6000)\quad\frac{s}{s^2}}{10^3(s+1000)(s+2000)}$$

$$V_o(s) = \frac{-30\times10^{6^{s}}}{s(s+1000)(s+2000)}$$

(b) $V_o(s) = \frac{A}{s} + \frac{B}{s+1000} + \frac{C}{s+2000} = \frac{-30\times10^6}{s(s+1000)(s+2000)}.$

$$A(s+1000)(s+2000) + Bs(s+2000) + Cs(s+1000) = -30\times10^6$$

$$A(s^2+3000s+2\times10^6) + B(s^2+2000s) + C(s^2+1000s) = -30\times10^6$$

s^2: $A+B+C=0$

s^1: $3000A + 2000B + 1000C = 0$ $\qquad$ $3A+2B+C=0.$

s^0: $2\times10^6 A = -30\times10^6$ $\therefore$ $A=-15$

$$\begin{array}{r} \overline{B}+\overline{C}=15 \\ 2B+C=45 \\ \hline B=30 \end{array}$$

$C = 15 - B = 15-30 = -15.$

$$V_o(s) = \frac{-15}{s} + \frac{30}{s+1000} - \frac{+15}{s+2000}$$

$$v_o(t) = -15u(t) + 30e^{-1000t}u(t) - 15e^{-2000t}u(t)$$

(c) op-amp saturates when it reaches ± 10 volts.

at $t=0^+$ $\quad -15+30-15=0$

$t\to\infty$ $\quad v_o(t)\to -15.$

let $x = e^{-1000t}$

$-15+30x-15x^2 = -10$

$15x^2 - 30x + 5 = 0.$

$$x = \frac{30\pm\sqrt{(30)^2-4(15)(5)}}{2(15)} = \frac{30\pm24.5}{30} = 1\pm.816$$

$= 1.816,\ \boxed{.184}$ ← this is the one.

$.184 = e^{-1000t}$

$-1.7 = \ln .184 = -1000t$

$1.7\text{mS} = t$ $\qquad$ 1.7mS to saturate k

[d] let $v_g = mt$ $\qquad V_g = \frac{m}{s^2}$

$$V_o = \frac{-5\times10^3 s}{(s+1000)(s+2000)} \cdot \frac{m}{s^2} = -\frac{5000\ m}{s(s+1000)(s+2000)}$$

final value less than −10 volts.

initial value

$$\lim_{s\to\infty} sF(s) = \lim_{t\to 0^+} f(t)$$

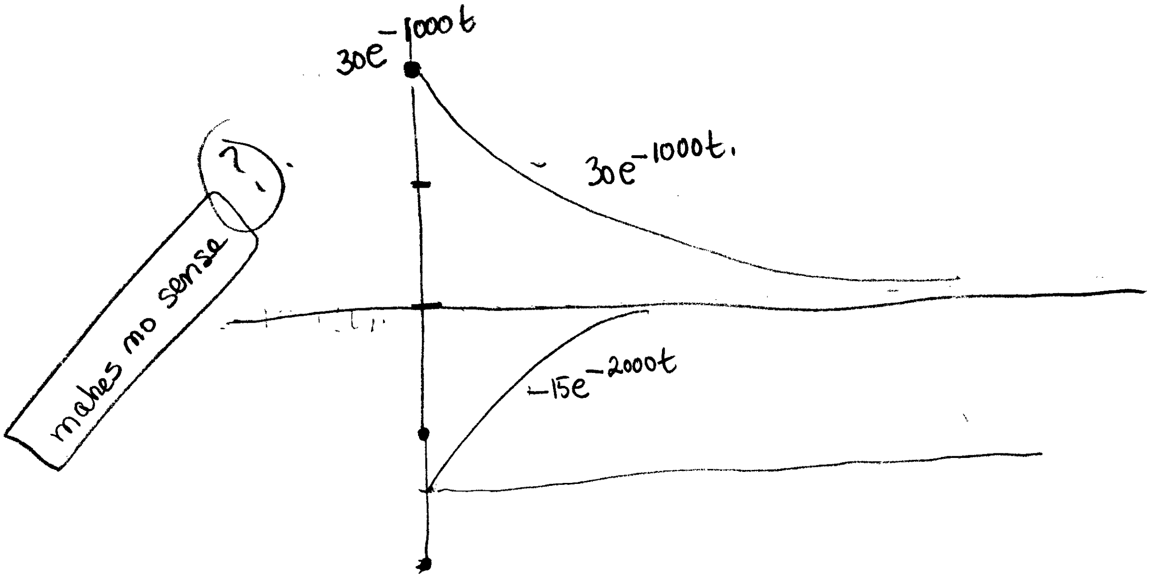
final value

$$\lim_{s\to 0} sF(s) = \lim_{t\to\infty} f(t)$$

$$\lim_{s\to 0} \frac{-5000m\,\cancel{s}}{\cancel{s}(s+1000)(s+2000)} = \frac{-5000m}{2\times10^6} = -\frac{5m}{2000}$$

$$-\frac{5m}{2000} = -10$$

$$m = 4000\ V_{s}.$$



3/9/93

finish up chapter 15

15.5 Operational Transforms $\mathcal{L}[f(t)] = \int_{0^-}^{\infty} f(t)e^{-st}dt$

if $\mathcal{L}[f(t] = F(s)$ then $\mathcal{L}[Kf(t)] = KF(s)$

if $\mathcal{L}[f_1(t)] = F_1(s)$ $\mathcal{L}[f_1(t)+f_2(t)] = F_1(s) + F_2(s)$.
and $\mathcal{L}[f_2(t)] = F_2(s)$

if $\mathcal{L}[f(t)] = F(s)$
$\mathcal{L}\left[\frac{df(t)}{dt}\right] = sF(s) - f(0^-)$

if $\mathcal{L}[f(t)] = F(s)$. $\mathcal{L}\left[\int_{0^-}^{t} f(x)\,dx\right] = \frac{F(s)}{s}$.

translation in the time domain

what is $\mathcal{L}[f(t-a)\,u(t-a)]$?

$$\mathcal{L}[f(t-a)u(t-a)] = \int_{0^-}^{\infty} u(t-a)\,f(t-a)\,e^{-st}\,dt$$

$$= \int_{a}^{\infty} f(t-a)e^{-st}dt.$$

let $x = t-a$ $t = x+a$. $dt = dx$.

$$= \int_{0}^{\infty} f(x)\,e^{-s(x+a)}\,dx$$

$$= e^{-as}\int_{0}^{\infty} f(x)\,e^{-sx}\,dx = e^{-as}F(s).$$

Shift in time → multiplication in s

works backwards.

$$F(s+a) \longrightarrow \mathcal{L}[e^{-at}f(t)]$$

scale changing

$$\mathcal{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right).$$

B

initial and final value theorems.

$$\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s).$$ initial value

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s).$$ final value

only if $f(\infty)$ exists, i.e. all poles in left-half of s-plane.

HW: 15.19, 15.30, 15.32.

Example

15.23 a) $$F(s) = \frac{8s^2+37s+32}{(s+1)(s+2)(s+4)}.$$

$s^2+3s+2.$

all poles in LHP.

$$sF(s) = \frac{8s^3+37s^2+32s.}{s^3+7s^2+14s+8}$$

initial value $$\lim_{s\to\infty} sF(s) = 8.$$

final value $$\lim_{s\to 0} sF(s) = 0.$$

Example: 15:30

$$I_0(s) = \frac{V_{dc}/L}{s^2+(R/L)s+\left(\frac{1}{LC}\right)}$$

initial value $$\lim_{s\to\infty} sI_0(s) = \lim_{s\to\infty}\frac{sV_{dc}/L}{s^2+(R/L)s+\frac{1}{LC}} \to 0.$$

final value $$\lim_{s\to 0} sI_0(s) = \lim_{s\to 0}\frac{sV_{dc}/L}{s^2+\left(\frac{R}{L}\right)s+\frac{1}{LC}} = 0.$$

C.

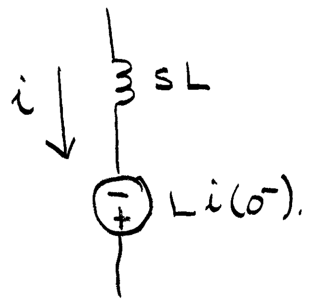
Ch. 16

16.1
inductors and capacitors in the s-domain

for an inductor $v = L\frac{di}{dt}$

$$V(s) = L\left[sI - i(0^-)\right]$$

$$= sL\ I - i(0^-)L$$

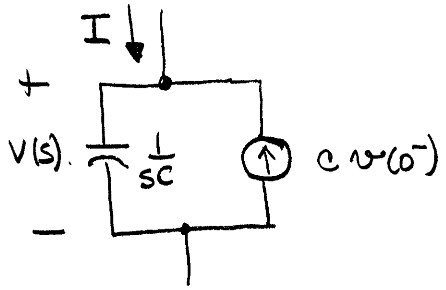


for a capacitor

$$i = C\frac{dv}{dt}$$

$$I(s) = C\left[sV(s) - v(0^-)\right].$$

$$= sC\ V(s) - Cv(0^-)$$



no $\delta(t)$

no Thevenin.

16.6 poles & zeros.

16.14, 16.18 make before break switches

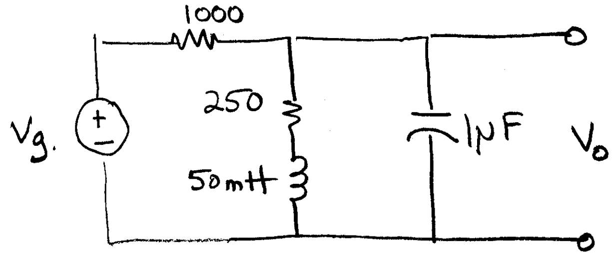
16.31, 16.34 initial & final value

16.52, 16.53 op amps.

E

Example 17.2.

$V_g = 50t\,u(t)$



use KCL
$$\frac{V_g - V_o}{1000} + \frac{V_o}{250 + .05s} + \frac{V_o s}{10^6} = 0.$$

$$V_o = \frac{1000(s+5000)V_g}{s^2 + 6000s + 25E6} \longleftarrow V_g(s) = \frac{50}{s^2}.$$

$$H(s) = \frac{V_o}{V_g} = \frac{1000\ (s+5000)}{s^2 + 6000s + 25E6}.$$

poles: $-3000 \pm j4000$

zeros: -5000

by partial fractions

$$V_o(s) = \frac{K_1}{s + 3000 - j4000} + \frac{K_1^*}{s + 3000 + j4000} + \frac{K_s}{s^2} + \frac{K_3}{s.}$$

$K_1 = 5\sqrt{5} \times 10^{-4} \angle 79.7°$

$K_2 = 10$

$K_3 = -4 \times 10^{-4}.$

time domain

$$v_o(t) = \Big[\underbrace{10\sqrt{5} \times 10^{-4}\, e^{-3000t} \cos(4000t + 79.7°)}_{\text{transient term determined by poles of } H(s).} + \underbrace{10t - 4 \times 10^{-4}}_{\text{steady state.}}\Big]u(t)$$

see book p. 692.

D

chapter 17 The transfer function

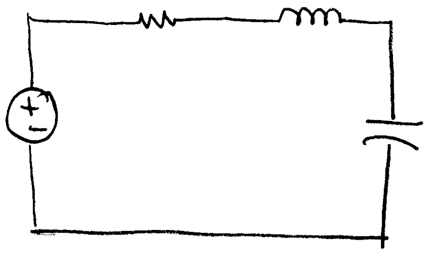
review before convolution

17.1 superposition linear, lumped-parameter systems.

17.2 transfer function

s domain ratio of $\dfrac{\text{output (response)}}{\text{input (source)}}$

series RLC circuit



$$H(s) = \frac{Y(s)}{X(s)} \begin{array}{l} \leftarrow \text{output.} \\ \leftarrow \text{input} \end{array}$$

we got $I(s) = \dfrac{10000}{s^2+200s+10^6}$

but $V(s) = 10/s$.

$$H(s) = \frac{\dfrac{10\,000}{s^2+200s+10^6} \cdot \dfrac{s}{10}}{\dfrac{10}{s} \cdot \dfrac{s}{10}}$$

$$= \frac{1000\,s}{s^2+200s+10^6}$$

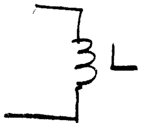
$$\therefore \quad Y(s) = \underbrace{H(s)\,X(s)}_{\text{rational functions of } s}.$$

poles of H(s) → transient component of total response

poles of X(s) → steady state solution

Hayt 12-4 Z(s) and Y(s)

Let's consider only the inductor:



Let $v(t) = V_m e^{\sigma t} \cos(\omega t + \theta)$

$$= \mathrm{Re}\left\{ V_m e^{\sigma t} e^{j(\omega t + \theta)} \right\}.$$

$$= \mathrm{Re}\left\{ V_m e^{j\theta}\, e^{j\omega t + \sigma t} \right\}.$$

$$v(t) = \mathrm{Re}\left\{ \hat{V}_m e^{(\sigma + j\omega) t} \right\}$$

In a similar fashion

$$i(t) = \mathrm{Re}\left\{ \hat{I} e^{(\sigma + j\omega)} \right\}$$

$$v(t) = L \frac{di}{dt}$$

$$\mathrm{Re}\left\{ \hat{V}_m e^{(\sigma + j\omega) t} \right\} = L\, \mathrm{Re}\left\{ (\sigma + j\omega)\, \hat{I} e^{(\sigma + j\omega) t} \right\}$$

in the complex domain

$$\hat{V}_m e^{(\sigma + j\omega) t} = L(\sigma + j\omega)\, \hat{I} e^{(\sigma + j\omega) t}$$

$s \triangleq \sigma + j\omega$

$$\hat{V}_m e^{st} = sL\, \hat{I} e^{st}$$

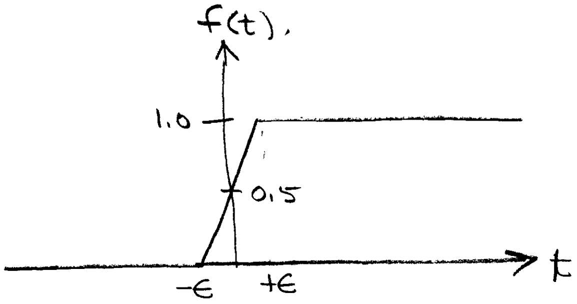
$$\frac{\hat{V}_m}{\hat{I}} = sL$$ complex impedance. of an inductor

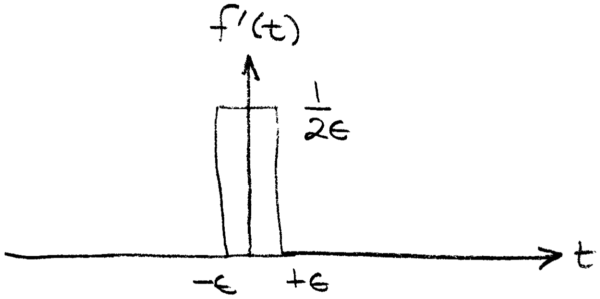
without detail

	R	L	C
Z(s)	R	sL	$\frac{1}{sC}$

3/11/93

Impulse function (607, 608, 609).
Impulsive sources (660, 666).





Dirac delta function

$$\lim_{\epsilon \to 0} f'(0) = \delta(t)$$

Area under curve is always 1.

$$\text{i.e.} \int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

There are any number of mathematical definitions, e.g.

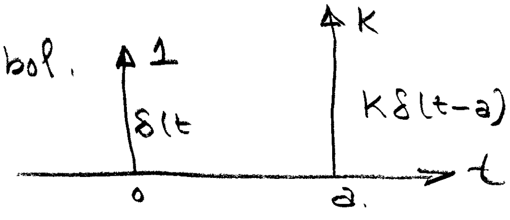
$$f(t) = \frac{K}{2\epsilon} e^{-\frac{|t|}{\epsilon}}$$

The one most commonly used is

$$\int_{-\infty}^{\infty} K\delta(t)\, dt = K$$

$$\delta(t) = 0 \text{ for } t \neq 0.$$

graphical symbol.



sifting property of the impulse function

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-a) = \int_{a-\epsilon}^{a+\epsilon} f(t)\,\delta(t-a)\,dt$$

since $\delta(t-a)=0$ except at $t=a$

$$= \int_{a-\epsilon}^{a+\epsilon} \underset{\uparrow}{f(a)}\,\delta(t-a)dt = f(a)\int_{a-\epsilon}^{a+\epsilon} \delta(t-a)\,dt$$

$f(t) \approx f(a)$ since $\delta(t-a)$ is zero everywhere else.

$$= f(a).$$

What is the Laplace transform of $\delta(t)$?

$$\mathcal{L}\{\delta(t)\} = \int_{0^-}^{\infty} \delta(t)\,e^{-st}\,dt = \int_{0^-}^{\infty} \delta(t)\,dt = 1$$

since $e^{-st} = 1$ for $s=0$.

You can also use the Laplace transform to define the integral and derivative of $\delta(t)$.

$$\mathcal{L}\left\{\frac{d\,\delta(t)}{dt}\right\} = s\,\underset{\uparrow}{1} \quad \text{no initial conditions} \quad = s$$

transform of $\delta(t)$

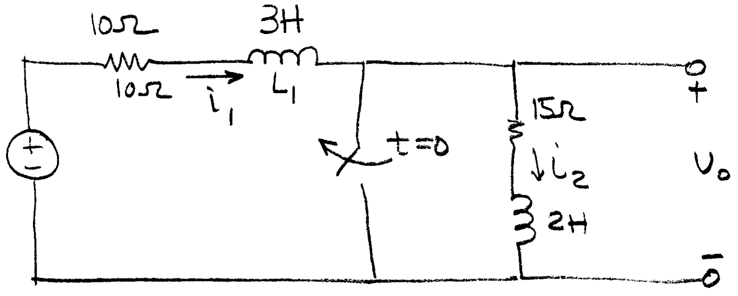
this is the transform of a doublet.

$$\mathcal{L}\left\{\int_0^t \delta(x)\,dx\right\} = \frac{1}{s} \leftarrow \text{transform of } \delta(t)$$

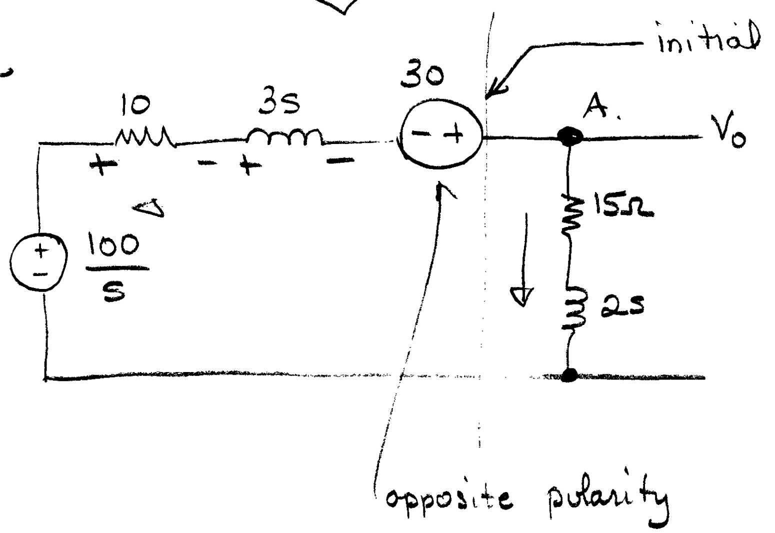
which is the transform of $u(t)$.

16.4 The Impulse function in circuit analysis

modeling



s-domain



$$v = L\frac{di}{dt}$$

$$V = L\left[sI - i(0^-)\right]$$

$$= sLI - \underbrace{Li(0^-)}_{\text{a voltage}}$$

once you understand the above circuit

~~Use~~ KCL at node A.

$$\frac{V_o}{2s+15} + \frac{(V_o - 30) - \frac{100}{s}}{10 + 3s} = 0.$$

solving for $V_o = \frac{40(s+7.5)}{s(s+5)} + \frac{12(s+7.5)}{s+5}$ } bigger than 1 divide out.

$$s+5\,\overline{)\,s+7.5}^{\;1\times 12}$$
$$\underline{s+5}$$
$$2.5\times 12 = 30$$

partial fraction exp.

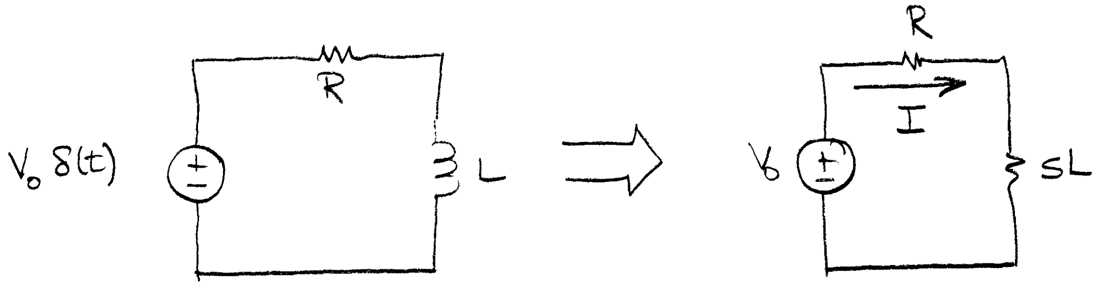
$$= \frac{60}{s} - \frac{20}{s+5} + 12 + \boxed{\frac{30}{s+5}}$$

$$= 60u(t) + 10e^{-5t}u(t) + 12\delta(t)$$

$$\frac{1}{s+a} \rightarrow e^{-at}$$

constant currents creates this impulse

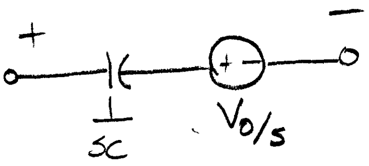
impulsive driving sources

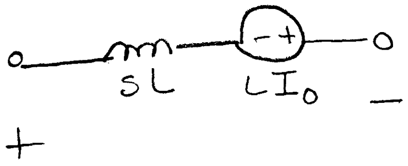


$$I = \frac{V_0}{R+sL} = \frac{V_0/L}{s+R/L}$$

inverse transforming $i(t) = \frac{V_0}{L}\, e^{-\frac{R}{L}t}\, u(t)$.

You must watch out for initial conditions on inductors <u>and</u> capacitors

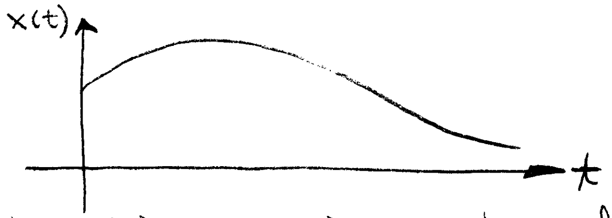




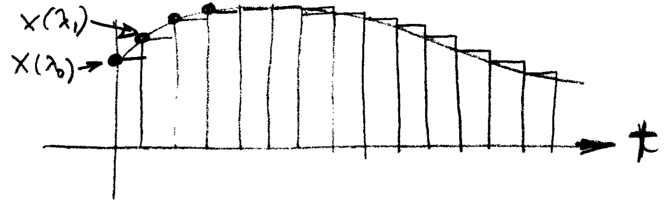
17.4 The transfer function and the convolution integral

Assumptions: linear
time-invariant

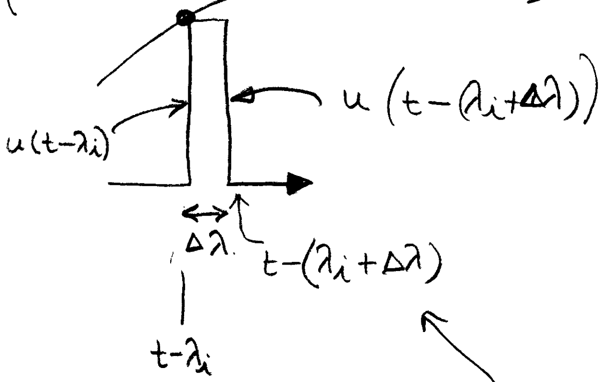
will use superposition



approximate x(t) as a series of rectangular pulses of uniform width $\Delta\lambda$



$$x_i(t) = x(\lambda_i)\left\{u(t-\lambda_i) - u(t-(\lambda_i+\Delta\lambda))\right\}$$

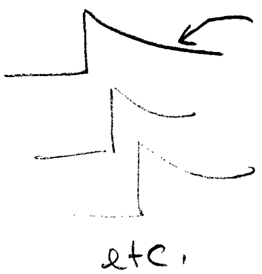


what happens as $\Delta\lambda \to 0$. and we write as an impulse of area $f(t-\lambda)\Delta\lambda$

i.e. $x(t) = x(\lambda_0)\Delta\lambda\,\delta(t-\lambda_0) + x(\lambda_1)\Delta\lambda\,\delta(t-\lambda_1) + \ldots$

response of each one ~~to~~ a system is $h(t)$,

i.e.



$$\underbrace{x(\lambda_0)\Delta\lambda}_{\text{magnitude}}\ \underbrace{h(t-\lambda_0)}_{\text{response}}$$

etc.

using linearity and superposition

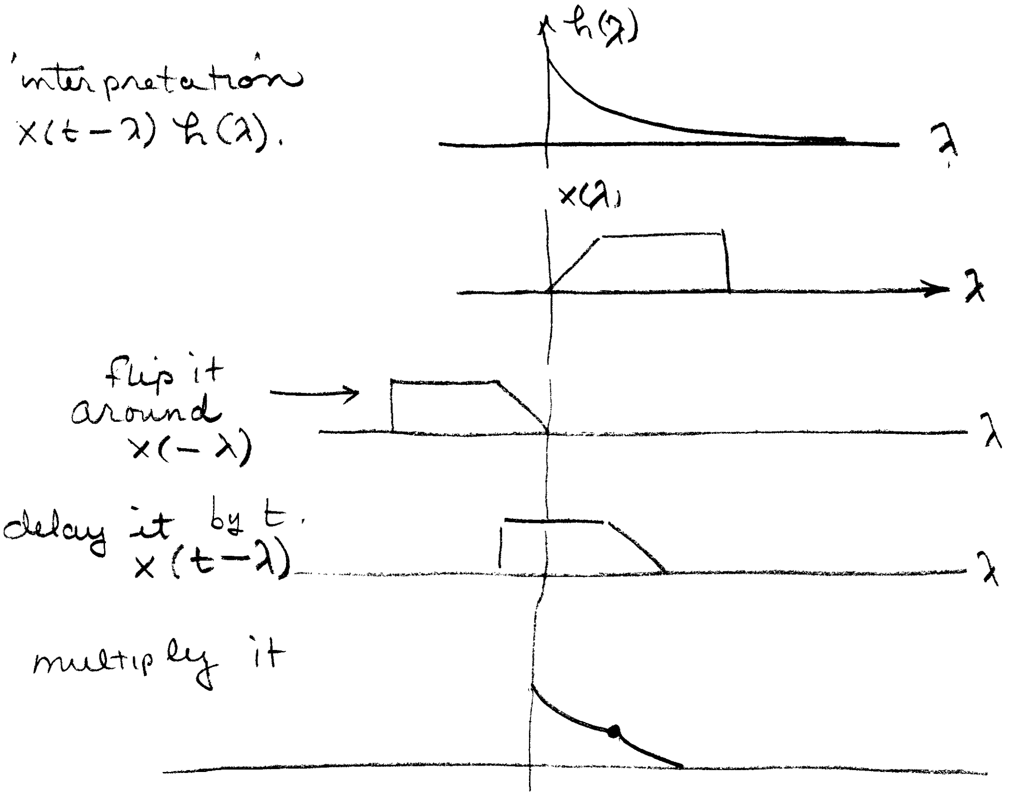
$$y(t) = \sum_{i=0}^{\infty} x(\lambda_i)\Delta\lambda\, h(t-\lambda_i) \longrightarrow \int_0^{\infty} x(\lambda)h(t-\lambda)\,d\lambda$$

This is the convolution integral

$$y(t) = \int_0^\infty x(\lambda)\, h(t-\lambda)\, d\lambda = \int_0^\infty x(t-\lambda)\, h(\lambda)\, d\lambda$$

$$= x(t) * h(t).$$

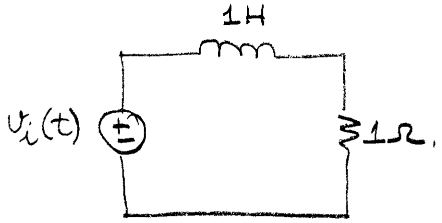
graphical interpretation of $x(t-\lambda)\, h(\lambda)$.

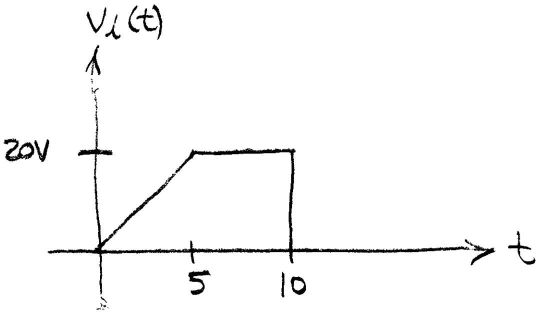


integrate it

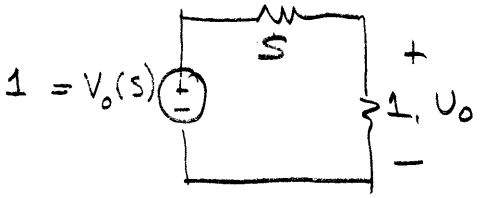
Example 17.3

use convolution to find $v_o(t)$





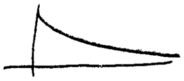
in the s-domain



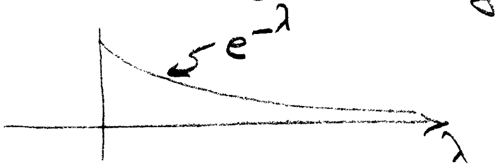
What is unit impulse response? Let source = $\delta(t)$, then $V(s)=1$.

$$V_o(s) = \underbrace{\frac{1}{s+1}}_{i(s)} \cdot \underbrace{1}_{1\Omega} = \frac{1}{s+1}$$

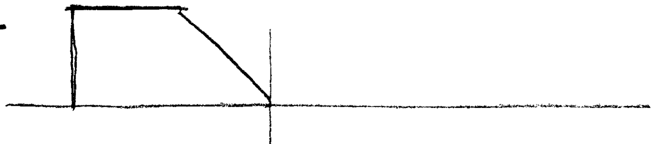
The impulse response is then $\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t}u(t)$.



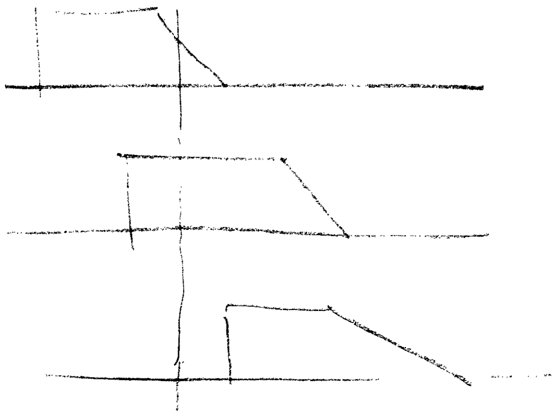
Let's try to figure out the response graphically



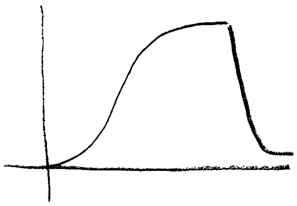
"fold" flip the excitation



now shift and multiply.

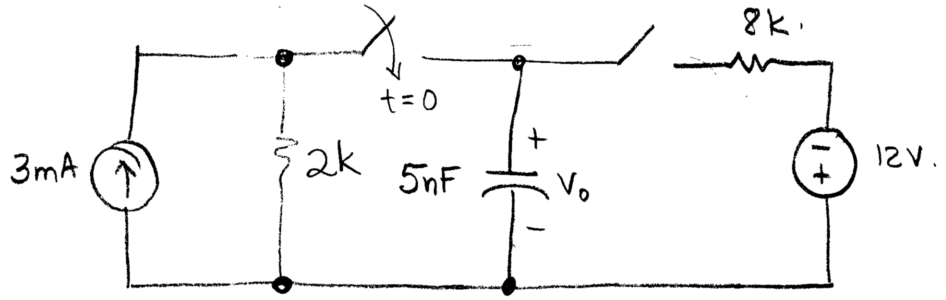


plot integrals

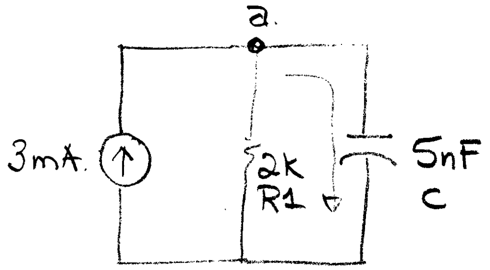


some transient problems.

8.58



at $t=0$.



$i = C\frac{dv}{dt}$.

KCL@ a

$$+3mA - \frac{V_c}{\underset{R1}{\boxed{2K}}} - C\frac{dv_c}{dt} = 0.$$

$$C\frac{dv_c}{dt} + \frac{V_c}{R1} = 3mA.$$

$$\frac{dv_c}{dt} + \frac{V_c}{R1C} = \frac{3mA}{C}.$$

steady state: $V_c = (3mA)(R1) = (3mA)(2k) = 10V$

transient:

$R1C = (2\times10^3)(5\times10^{-9})$
$= 10\times10^{-6}$

$$\frac{dv_c}{dt} + \frac{v_c}{10\times10^{-6}} = 0.$$

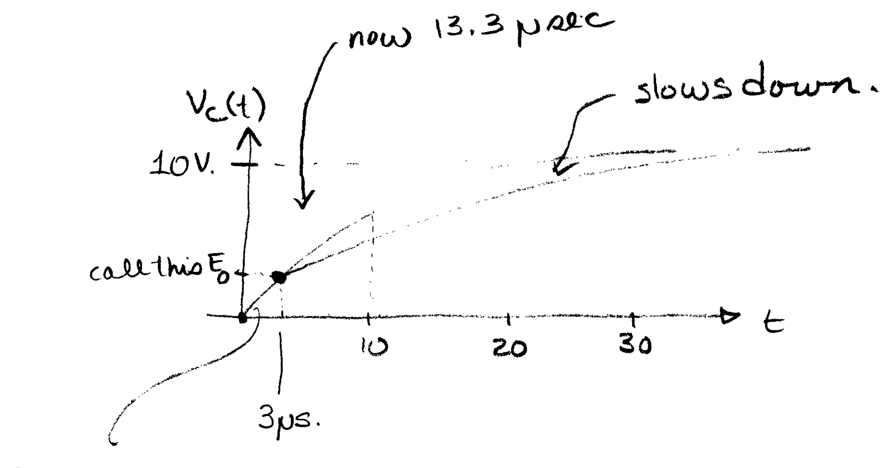
let $V_c = Ae^{st}$.

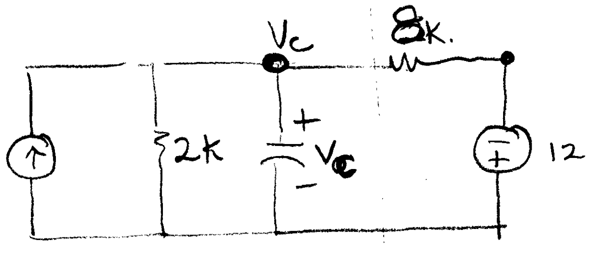
$$\cancel{A}se^{\cancel{st}} + \frac{\cancel{Ae^{st}}}{10\times10^{-6}} = 0.$$

$$s = -\frac{1}{10\times10^{-6}}.$$

$$V_c(t) = Ae^{-\frac{t}{10\times10^{-6}}} + 10V.$$

at $t=0$ $v_c(0)=0$ $A=-10$ $V_c(t) = 10e^{-\frac{t}{10\times10^{-6}}} + 10.$ volts.





$$R_2C = (8\times10^3)(5\times10^{-9})$$
$$= 40\times10^{-6}.$$

still KCL but V_o has an ititial condition E_0.

$$+3mA - \frac{V_c}{R1} - C\frac{dV_c}{dt} + \frac{V_c-(-12)}{R2} = 0.$$

$$3mA - \frac{V_c}{R_1} - C\frac{dV_c}{dt} + \frac{V_c}{R_2} + \frac{12}{R_2} = 0.$$

$$\frac{12V}{8K}$$

$$3mA + \frac{12}{R_2} = C\frac{dV_c}{dt} + V_c\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{dV_c}{dt} + V_c\left(\frac{1}{R_1C}-\frac{1}{R_2C}\right) = \frac{3mA+\frac{12}{R2}}{5nF} = \frac{3mA+1.5mA}{5nF}$$

$$\frac{dV_c}{dt} + V_c\left(\frac{1}{10}-\frac{1}{40}\right)\frac{1}{10^{-6}} = 0.9\times10^6$$

$$\frac{4.5}{5}\times\frac{10^{-3}}{\cancel{\times10^{-9}}}\frac{10^9}{10^9}$$

$$\frac{4.5}{5}\times10^6.$$

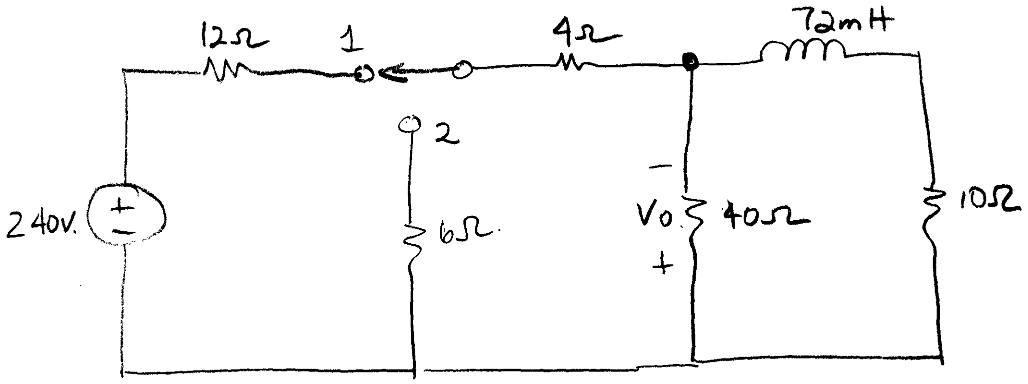
<u>dc</u> Final value

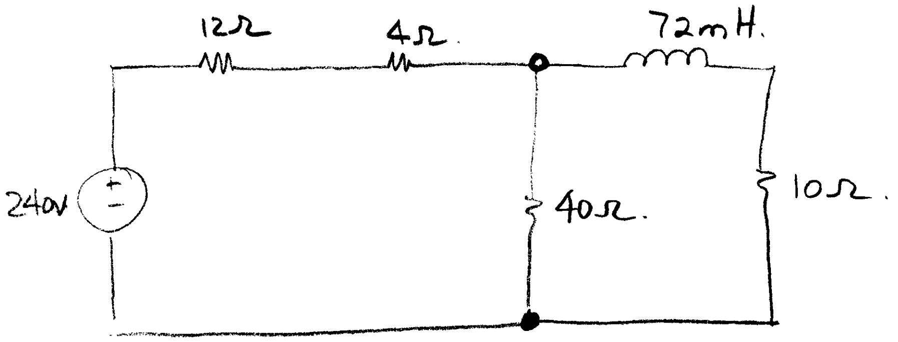
$$V_c = \frac{0.9\times\cancel{10^6}}{\frac{1}{10}-\frac{1}{40}\times\cancel{10^6}}$$

$$= \frac{0.9}{0.1-.025} = \frac{0.9}{.075} = 12\text{volts}$$

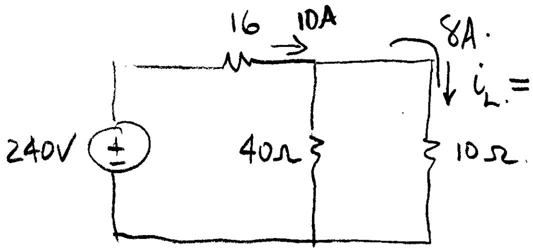
8.6.

3.

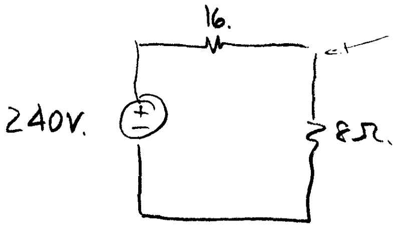




do the simple way.



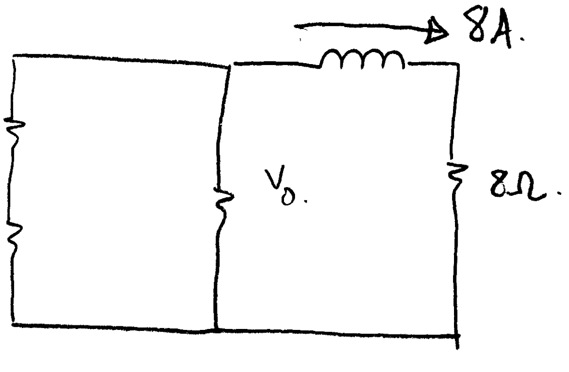
$$i_L = \frac{40}{40+10} \cdot 10 = \frac{40}{50} 10 = 8A.$$



$$\frac{40 \cdot 10}{40+10}$$

$$i = \frac{240V}{16+8} = 10A.$$

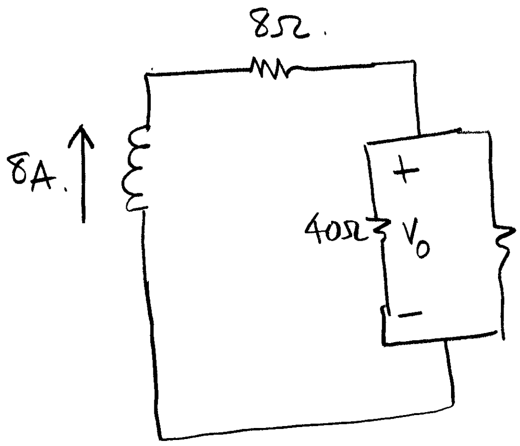
$$\frac{40\cancel{0}}{5\cancel{0}} = 8\Omega.$$

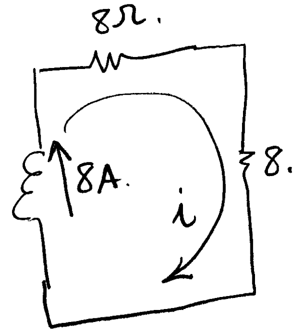


4

$\frac{40 \cdot 10}{40+10}$

8





toget $V_o(t)$ find $i(t)$.

do KVL

$$L\frac{di}{dt} + iR_1 + iR_2 = 0.$$

$$L\frac{di}{dt} + i\,8\Omega = 0.$$

$$\frac{di}{dt} + i\,\frac{8\Omega}{L} = 0.$$

$$\frac{di}{dt} + i\,\frac{1}{L/R} = 0.$$

$$\frac{L}{R} = \frac{72\times10^{-3}}{16} = 4.5\,mS.$$

$$\frac{di}{dt} + \frac{i}{4.5msec.} = 0.$$

looks like.

$$Ae^{st}.$$

$$s\cancel{Ae^{st}} + \frac{\cancel{Ae^{st}}}{4.5msec} = 0$$

$$S = -\frac{1}{4.5msec}$$

$$i(t) = Ae^{-\frac{t}{4.5msec}}. \quad A = 8.$$

$$i(t) = 8e^{-t/4.5m.}$$

$$V_o(t) = 8i(t)$$

$$= 64e^{-t/4.5m.}$$

EE 243 - 3/12/93

Quiz #1 review

Natural Response of R-LC circuits (pp 347 -358)

SERIES

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int^{t} i\,dt = V_0(t)$$ (no initial charge on C)

let's take the L transform of this equation

$$I(s)\left[R + Ls + \frac{1}{Cs}\right] = \cancel{0}\,V_o(s)$$

$$I(s) = \frac{s/L\,V_o(s)}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

impulse

We set $V_o(s) = 1$, then

if $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$ we can have a response even without a source (after impulse is gone). This is called the characteristic equation for this circuit.

$$s = -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

where I've written this assuming $\frac{1}{LC} > \left(\frac{R}{2L}\right)^2$

~~Then [illegible]~~

we can also use the form

$$\mathcal{L}^{-1}\left\{\frac{s+\alpha}{(s+\alpha)^2 + \beta^2}\right\} = e^{-\alpha t}\cos\beta t$$

$$I(s) = \frac{s/L}{s^2 + (R/L)s + \frac{1}{LC}} = \frac{\frac{1}{L}(s+\alpha)}{s^2 + 2\alpha s + \alpha^2 + \beta^2} - \frac{\frac{1}{L}\alpha}{(s+\alpha)^2 + \beta^2}$$

where $\alpha = \frac{R}{2L}$, $\beta^2 = \frac{1}{LC} - \left(\frac{R}{2L}\right)^2$

$$i(t) = \frac{1}{L}\left[e^{-\alpha t}\cos\beta t - \frac{\alpha}{\beta}e^{-\alpha t}\sin\beta t\right] \quad t \geq 0$$

Impulse response at series RLC circuit.

inverse xform example

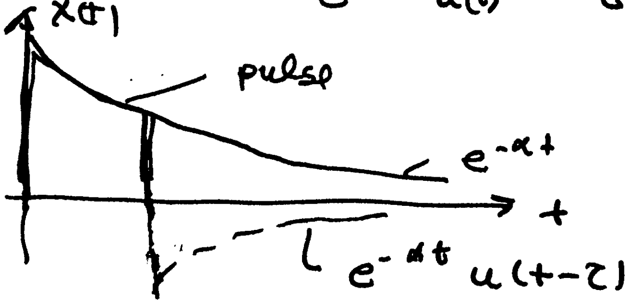
$$X(s) = \frac{1 - e^{-(s+\alpha)T}}{s+\alpha} = \frac{1}{s+\alpha} - \frac{e^{-\alpha T}}{s+\alpha} e^{-sT}$$

Recall the xform pair

$$y(t-T) \Leftrightarrow Y(s) e^{-sT}$$

hence $x(t) = e^{-\alpha t} u(t) - e^{-\alpha T} e^{-\alpha(t-T)} u(t-T)$

$$= e^{-\alpha t} u(t) - e^{-\alpha t} u(t-T)$$



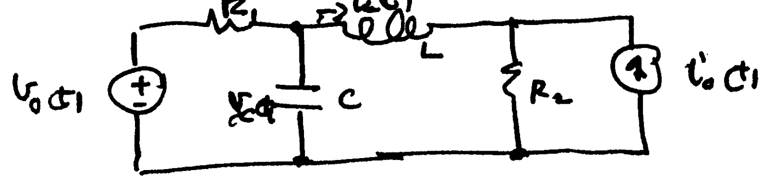
what is the region of convergence?

since $X(s)$ has no poles, it is the entire s-plane - this is a result of the fact that $\int_0^\infty x(t) e^{-st} dt$ is finite for any s since $x(t)$ exists only over a finite range.

The s-domain response of LTI circuits have a simple structure. The structure like $\frac{1}{s+\alpha}$ is called a single-order pole at s=-

Understanding the nature of this structure in more detail will help you to <u>design</u> complex systems - such as control systems, sensors ~~and other~~ etc

Let's look at a two-source circuit



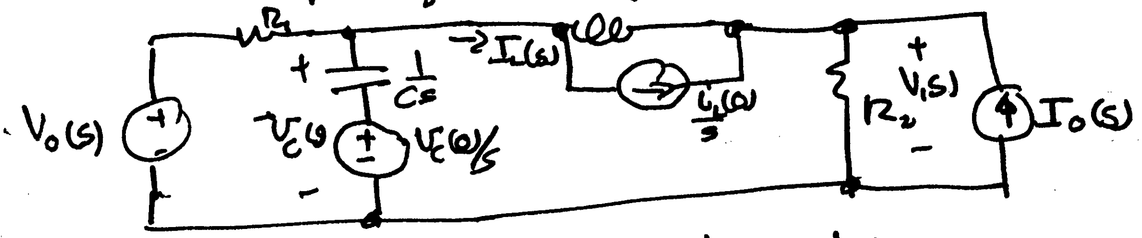
$v_0(t) = 3$ $t > 0$

$i_0(t) = 4e^{-t}$ $t > 0$

$v_C(0) = -1$

$R_1 = 0.5$, $R_2 = 1$, $L = 1H$, $C = 0.5F$ $i_L(0) = 2$

in the frequency domain



$$V_1(s) = \frac{R_2\left(s^2 + s\frac{1}{R_1C} + \frac{1}{LC}\right)}{s^2 + s\left(\frac{1}{R_1C} + \frac{R_2}{L}\right) + \frac{1}{LC}\left(1 + \frac{R_2}{R_1}\right)} I_0(s)$$

$$+ \frac{\frac{R_2}{R_1} \cdot \frac{1}{LC}}{s^2 + s\left(\frac{1}{R_1C} + \frac{R_2}{L}\right) + \frac{1}{LC}\left(1 + \frac{R_2}{R_1}\right)} V_0(s)$$

$$+ \frac{s\frac{R_2}{L}}{s^2 + s\left(\frac{1}{R_1C} + \frac{R_2}{L}\right) + \frac{1}{LC}\left(1 + \frac{R_2}{R_1}\right)} \cdot v_C(0)/s$$

$$+ \frac{s^2 R_2 + s\frac{R_2}{R_1}C}{s^2 + s\left(\frac{1}{R_1C} + \frac{R_2}{L}\right) + \frac{1}{LC}\left(1 + \frac{R_2}{R_1}\right)} i_L(0)/s$$

We note that each source has the same frequency domain denominator.

We can generalize this result for any LTI system

$$Y(s) = \sum_{m=1}^{M} H_{em}(s) X_m(s) + \sum_{n=1}^{N} H_{in}(s) \frac{\lambda_n(0)}{s}$$

where $Y(s)$ is the $\mathcal{L}$-transform of the ~~desir~~ object wes

$X_m(s)$ is the $\mathcal{L}$-transform of the m^{th} independent source

243 - 3/12/93 ④

· $\frac{\lambda_n(0)}{s}$ is the effect of initial condx.

$H_{em}(s)$, $H_{in}(s)$ give the relationship between sources and/or initial conditions with the output.

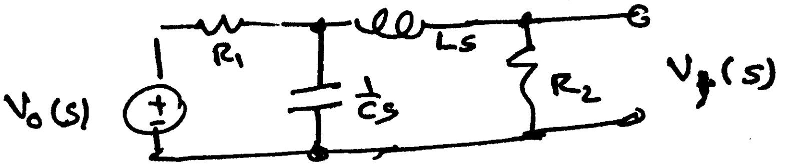
$$\text{The} \quad \sum_{n=1}^{N} H_{in}(s)\frac{\lambda_n(0)}{s} \Rightarrow \text{Zero input response (ZIR)}$$

Not a function of the sources called natural response.

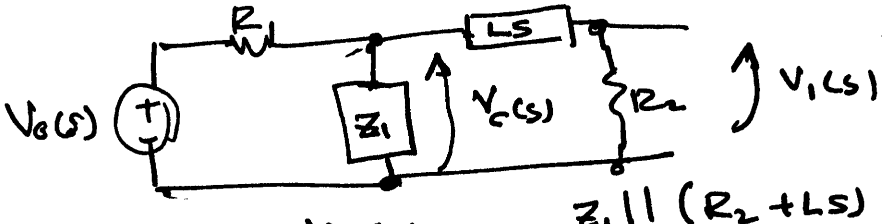
$$\sum_{m=1}^{M} H_{em}\,\alpha_m(s) \Rightarrow \text{Zero state response (ZSR)}$$

The system function is defined by the s-domain ratio of the desired output to ~~a~~ an input.

Note the ZSR for $\frac{V_1(s)}{V_0(s)}$ is derived from



$$V_1(s) = \frac{\left(\frac{R_2}{Ls+R_2}\right)\left[\frac{\frac{1}{Cs}(R_2+Ls)}{\left(\frac{1}{Cs}+R_2+Ls\right)}\right]}{\left[R_1 + \frac{\frac{1}{Cs}(R_2+Ls)}{\frac{1}{Cs}+R_2+Ls}\right]} V_0(s)$$



$$V_c(s) = \frac{Z_1 \| (R_2+Ls)}{R_1 + Z_1\|(R_2+Ls)}$$

$$V_1(s) = \left(\frac{R_2}{R_2+Ls}\right)V_c(s)$$

$$\frac{V_1(s)}{V_0(s)} = \frac{\frac{R_2}{R_1LC}}{s^2+s\left(\frac{1}{R_1C}+\frac{R_2}{L}\right)+\frac{1}{LC}\left(1+\frac{R_2}{R_1}\right)}$$

Hem: [illegible]

Dorf 14.8 from last time

$$y(t) = \int_0^t h(\tau) f(t-\tau)\, d\tau$$

basically the output is the sum of the responses to a continuous stream of impulses which form the input function

(This is 17.22 of text)

$$y(t) = h(t) * f(t)$$

usually more useful in s-domain

$$Y(s) = H(s)\, F(s).$$

we've actually used this the way we defined H(s).

i.e. $H(s) = \dfrac{Y(s)}{F(s)}.$

Let's prove these are equivalent

$$H(s) = \int_0^\infty h(\tau) e^{-s\tau}\, d\tau = \mathcal{L}[h(t)].$$

multiplying both sides by F(s).

$$H(s)\, F(s) = \int_0^\infty h(\tau)\, \underline{\underline{e^{-s\tau}\, F(s)}}\, d\tau$$

Remember that

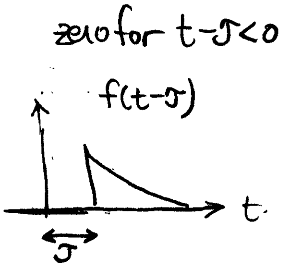
$$\mathcal{L}[f(t-\tau)] = e^{-s\tau} F(s)$$

substituting

$$H(s)\, F(s) = \int_0^\infty h(\tau)\, \mathcal{L}[f(t-\tau)]\, d\tau$$

delayed by τ.

$$= \int_0^\infty h(\tau) \left[\int_0^\infty f(t-\tau) e^{-st}\, dt \right] d\tau.$$



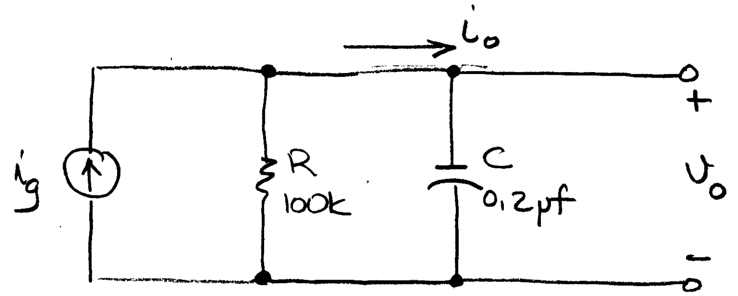
Now reverse order of integration

$$= \int_0^\infty e^{-st} \left[\int_0^t h(\tau) f(t-\tau)\, d\tau \right] dt$$

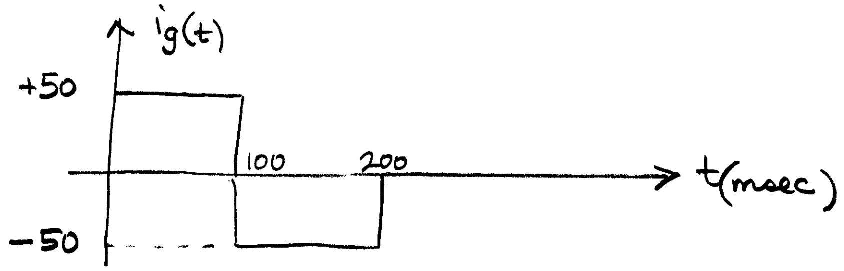
zero for t−τ<0 or τ>t

$$= \mathcal{L}\left[\int_0^t h(\tau) f(t-\tau)\, d\tau \right]$$ ∎

Problem 17- 19



find v_o if i_g is as given



[a] analyzing the circuit.

$$I_g(s) = \frac{V_o}{R} + \frac{V_o}{\frac{1}{sC}} = \frac{V_o}{10^5} + \frac{V_o s}{\frac{1}{.2\times10^{-6}}} = \frac{V_o}{10^5} + \frac{V_o s}{50\times10^5}$$

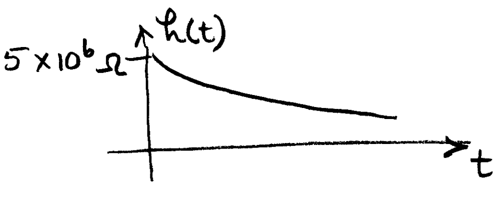
$$I_g(s) = \frac{V_o}{10^5}\left(1+\frac{s}{50}\right) = \frac{V_o}{5\times10^6}(50+s)$$

Solving for the transfer function

$$\overset{\text{output}\,\longrightarrow}{H(s)} = \frac{V_o}{I_g} = \frac{5\times10^6}{s+50}\ \Omega.$$

Inverse transforming

$$h(t) = 5\times10^6 e^{-50t} u(t)$$



Let's do solution graphically.

$$v_o = \int_0^t h(\tau) f(t-\tau)\, d\tau$$

flip in λ and delay.

f(t−λ)

Now multiply and integrate

graphically you can see there are three regions of the answer.

$0 \le t \le 0.1\,s.$

$0.1 \le t \le 0.2\,s.$

$0.2 \le t \le \infty$

These answers are

$0 \le t \le 0.1\,s \qquad v_o = 5(1-e^{-50t})\,u(t)$

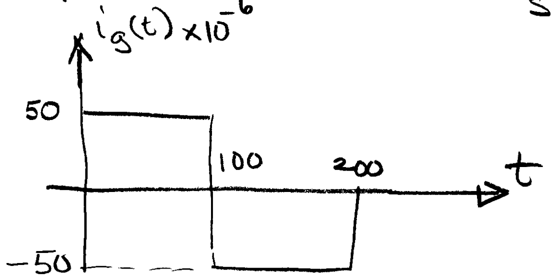
$0.1 \le t \le 0.2\,s \qquad v_o = \left[10e^{-50(t-0.1)} - 5e^{-50t} - 5\right]u(t)$

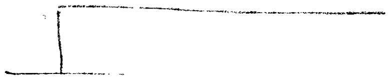
$0.2 \le t \le \infty \qquad v_o = \left[10e^{-50(t-0.1)} - 5e^{-50(t-0.2)} - 5e^{-50t}\right]u($

Let's do this neatly using convolution in s-domain.

We already know $h(\lambda) = 5\times10^{6}\,e^{-50\lambda}\,u(\lambda)$

Laplace transforming $H(s) = \dfrac{5\times10^{6}}{s+50}$





$$i_g(t) = \left[50\,u(t) - 100\,u(t-0.1) + 50\,u(t-0.2)\right]\mu A.$$

the Laplace transform of $i_g(t)$ $\quad I_g(s) = 50\frac{1}{s} - 100\frac{1}{s}e^{-0.1s} + 50\frac{1}{s}e^{-0.2s}.$

the response of the circuit is $\quad V(s) = H(s)\,I_g(s) = \left[250\times10^{6}\dfrac{1}{s(s+50)} - 500\times10^{6}\dfrac{e^{-0.1s}}{s(s+50)} + \dfrac{250\times10^{6}\,e^{-0.2s}}{s(s+50)}\right]10^{-}$

inverse transforming

$$\frac{1}{s(s+50)} = \frac{a}{s} + \frac{b}{s+50} = \frac{as + 50a + bs}{s(s+50)}$$

$$\therefore \quad 50a = 1 \qquad a = \frac{1}{50} = .02$$

$$a + b = 0 \qquad b = -.02.$$

$$V(s) = \Big\{ 250 \times 10^6 \; \frac{1}{50}\left[\frac{1}{s} - \frac{1}{s+50}\right] - 500 \times 10^6 \; \frac{e^{-0.1s}}{50}\left[\frac{1}{s} - \frac{1}{s+50}\right].$$

$$+ \; 250 \times 10^6 \; \frac{e^{-0.2s}}{50}\left[\frac{1}{s} - \frac{1}{s+50}\right]. \Big\} 10^{-6}$$

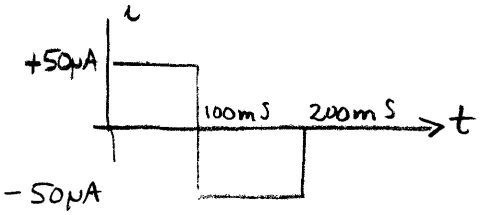
$$V(s) = 5\left[\frac{1}{s} - \frac{1}{s+50}\right] - 10\left[\frac{1}{s} - \frac{1}{s+50}\right]e^{-0.1s} + 5\left[\frac{1}{s} - \frac{1}{s+50}\right]e^{-0.2s}$$

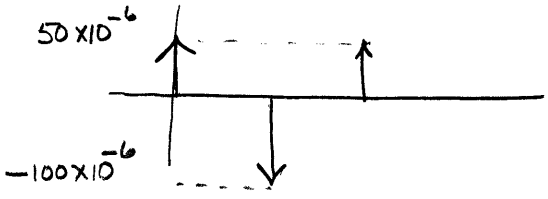
$$v(t) = \underline{5u(t) - 5e^{-50t}u(t)} - \underline{10u(t-0.1) + 10e^{-50(t-0.1)}u(t-0.1)}$$

$$\underline{+ 5u(t-0.2) - 5e^{-50(t-0.2)}u(t-0.2).} \quad ▨$$

Let's try it the other way.

What's the derivative of $i_g(t)$?





What is the system response to this function?

if $h(\lambda) = 5\times10^{6} e^{-50\lambda} u(\lambda)$.

Then
$$o(t) = (50\times10^{-6})(5\times10^{6} e^{-50t} u(t)) + (-100\times10^{-6})(5\times10^{6} e^{-50(t-0.1)} u(t-0.1)) + (50\times10^{-6})(5\times10^{6} e^{-50(t-0.2)} u(t-0.2)).$$

$$o(t) = 250\, e^{-50t} u(t) - 500\, e^{-50(t-0.1)} u(t-0.1) + 250\, e^{-50(t-0.2)} u(t-0.2).$$

The integral of this is the actual system response

$$\int_0^t 250\, e^{-50\lambda} u(\lambda)\, d\lambda = 250 \left. \frac{e^{-50\lambda}}{-50} \right|_0^t = -5e^{-50\lambda}\Big|_0^t$$

$$= -5e^{-50t} + 5.$$

$$-\int_0^t 500\, e^{-50(\lambda-0.1)} u(\lambda-0.1)\, d\lambda = -\int_{-0.1}^{t-0.1} 500\, e^{-50\gamma} u(\gamma)\, d\gamma.$$

$\gamma = \lambda - 0.1$

$$= -\int_0^{t-0.1} 500\, e^{-50\gamma} u(\gamma)\, d\gamma = -500 \left. \frac{e^{-50\gamma}}{-50} \right|_0^{t-0.1} = 10\, e^{-50\gamma}\Big|_0^{t-0.1}$$

$$= 10e^{-50(t-0.1)} - 10$$

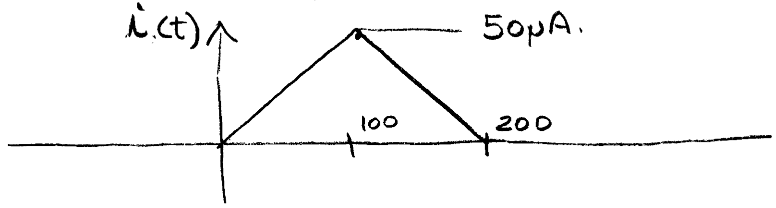
$$\int_0^t 250\, e^{-50(\lambda-0.2)} u(\lambda-0.2)\, d\lambda = \int_{-0.2}^{t-0.2} 250\, e^{-50\gamma} u(\gamma)\, d\gamma$$

$$\gamma = \lambda - 0.2$$

$$= 250 \left. \frac{e^{-50\gamma}}{-50} \right|_0^{t-0.2} = -5e^{-(t-0.2)} + 5$$

$$\int o(t)\, dt = \left(5 - 5e^{-50t}\right)u(t) + \left(10e^{-50(t-0.1)} - 10\right)u(t-0.1)$$
$$+ \left(5 - 5e^{-(t-0.2)}\right)u(t-0.2) \quad \blacksquare$$

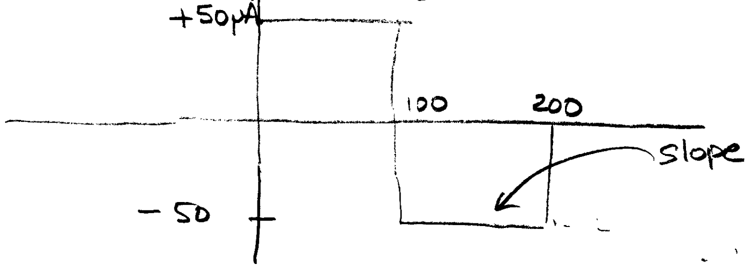
How about the response to



its derivative is $i_1'(t)$

slope = $\frac{50\times10^{-6}\text{A}}{100\times10^{-3}\text{s}} = 0.5\times10^{-3}$ A/s.

$=+5\times10^{-4}$ A/s.



slope = -5×10^{-4} A/s.

The response to $\pm50\times10^{-6}$ of the same time in a rectangular pulse is then

$$v(t) = 5u(t) - 5e^{-50t}u(t) - 10u(t-0.1) + 10e^{-50(t-0.1)}$$
$$+ 5u(t-0.2) - 5e^{-50(t-0.2)}u(t-0.2).$$

Since systems are linear

$$\text{if} \qquad i_1'(t) \longrightarrow v(t)$$
$$i_1(t) = \int i(\tau)d\tau \longrightarrow \int v(t).$$

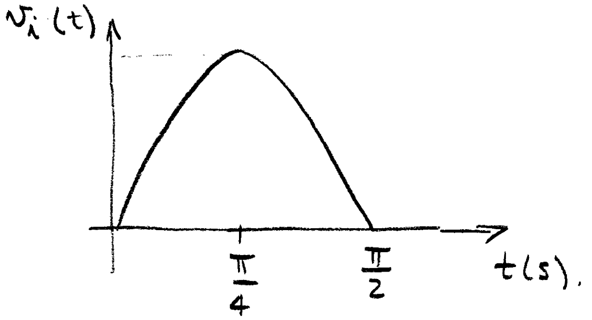
response to integral is integral of response.

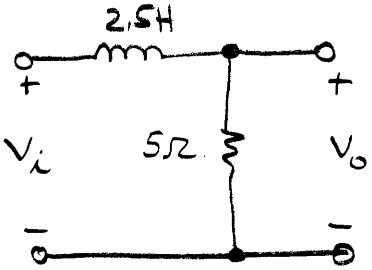
$$\int_0^t \left[5 - 5e^{-50\lambda}\right] u(\lambda)\, d\lambda$$

$$= 5\lambda\Big|_0^t - \frac{5e^{-50\lambda}}{-50}\Big|_0^t = 5t + 0.1e^{-50\lambda}\Big|_0^t$$

$$= \left[5t + 0.1e^{-50t} - 0.1\right] u(t).$$

Problem 17.17.





the transfer function

$$H(s) = \frac{V_o}{V_i} = \frac{5}{5+2.5s} = \frac{2}{s+2}$$

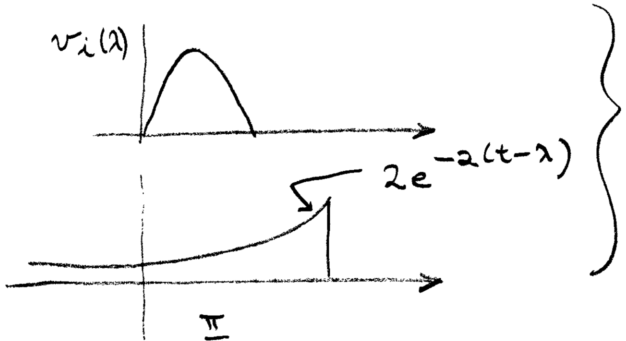
inverse transforming

$$h(\lambda) = 2e^{-2\lambda}$$

period: $\frac{T}{2} = \frac{\pi}{2}$ $\therefore$ $T = \pi$ or $f = \frac{1}{\pi}$ $\qquad \omega = 2\pi f = 2\pi\frac{1}{\pi} = 2.$

$$v_i(\lambda) = 20\sin 2\lambda\,[u(\lambda) - u(\lambda - \tfrac{\pi}{2})]$$

This you just have to grind out:



for $t \geq \frac{\pi}{2}.$

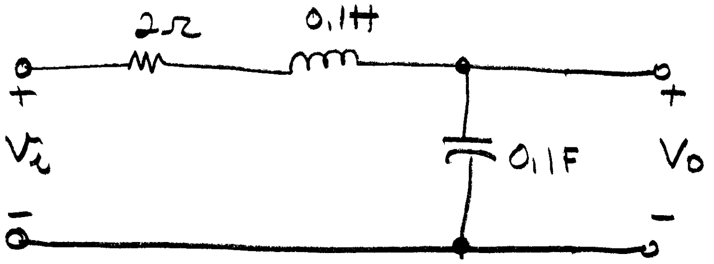
$$v_o = \int v_i(\lambda)h(t-\lambda)d\lambda = \int_0^{\frac{\pi}{2}} \underbrace{2e^{-2t}e^{2\lambda}}_{h(t-\lambda)}\ \underbrace{20\sin 2\lambda}_{v_i(\lambda)}\,d\lambda = 40e^{-2t}\int_0^{\frac{\pi}{2}} e^{2\lambda}\sin 2\lambda\, d\lambda$$

$$= 40e^{-2t}\left[\frac{e^{2\lambda}}{8}(2\sin 2\lambda - 2\cos 2\lambda)\right]_0^{\frac{\pi}{2}} = 10e^{-2t}\left[e^{\pi}(\overset{0}{\cancel{\sin\pi}} - \overset{-1}{\cancel{\cos\pi}}) - e^{0}(\overset{0}{\cancel{\sin 0}} - \overset{1}{\cancel{\cos 0}})\right]$$

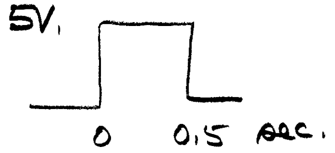
$$= 10e^{-2t}\left[-e^{\pi} + 1\right].$$

at $t = 2.2$ sec. $\qquad v_o(2.2\text{sec}) = 10e^{-2(2.2)}\left[1 - e^{\pi}\right] \cong 2.96\text{V}.$

17.15



$$v_i(t) = 5\left[u(t) - u(t-0.5)\right]$$



$$H(s) = \frac{1/sC}{R + sL + 1/sC} = \frac{1/C}{sR + s^2L + \frac{1}{C}} = \frac{1/C}{s^2L + Rs + \frac{1}{C}} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\frac{1}{LC} = \frac{1}{(0.1)(0.1)} = 100$$

$$\frac{R}{L} = \frac{2}{0.1} = 20$$

$$H(s) = \frac{100}{s^2 + 20s + 100} = \frac{100}{(s+10)^2}.$$

$$te^{-at} \longleftrightarrow \frac{1}{(s+a)^2}.$$

$$h(\lambda) = 100\lambda e^{-10t} u(t)$$

ⓐ do graphically

$$v_i(t) \longleftrightarrow \frac{5}{s} - \frac{5}{s}e^{-0.5s} = \frac{5}{s}\left(1 - e^{-0.5s}\right)$$

$$V_o(s) = H(s)\overbrace{V_i(s)}$$

$$= \frac{5}{s}\left(1 - e^{-0.5s}\right)\left(\frac{100}{(s+10)^2}\right) = 500\,\frac{1}{s(s+10)^2}\left(1 - e^{-0.5s}\right).$$

~~$\frac{1}{s(s+10)^2} = \frac{a}{s} + \frac{b}{(s+10)^2} = \frac{a(s+10)^2 + bs}{s(s+10)^2}$~~

~~$a(s^2+20s+100) + bs = 1$~~

~~$as^2 = 0$~~

~~$20as + bs = 0$~~

~~$100a = 1$~~

~~$\frac{1}{s(s+10)^2} = \frac{a}{s} + \frac{bs}{(s+10)^2} = \frac{a(s+10)^2 + bs^2}{s(s+10)^2}$~~

~~$a(s^2+20s+100) + bs^2 = 1$~~

~~$as^2 + bs^2 = 0$~~

~~$20as = 0$~~

$$\frac{1}{s(s+10)^2} = \frac{a}{s} + \frac{b}{s+10} + \frac{c}{(s+10)^2}$$

$$= \frac{a(s+10)^2 + bs(s+10) + cs}{s(s+10)^2}$$

$$\underline{a}(\underline{s^2}+20s+\underline{100}) + \underline{b}(\underline{s^2}+10s) + cs$$

s^2: $a + b = 0$

s^1: $20a + 10b + c = 0$

s^0: $100a = 1$

$a = \frac{1}{100}$

$b = -\frac{1}{100}$

$20\left(\frac{1}{100}\right) + 10\left(-\frac{1}{100}\right) + c = 0$

$10\left(\frac{1}{5} - \frac{1}{10} + c = 0\right)$

$2 - 1 + 10c = 0$

$10c = -1$

$c = -1/10$

$$V_o(s) = \frac{.01}{s} - \frac{.01}{s+10} - \frac{0.1}{(s+10)^2}$$

$$v_o(t) = .01\,u(t) - .01\,e^{-10t}u(t) - 0.1\,t\,e^{-10t}u(t),$$

ANNOUNCEMENTS

Quiz Average = 63 median = 65

Quiz Makeup draw poker - 2 problems to people w zero
must turn in old exam for this

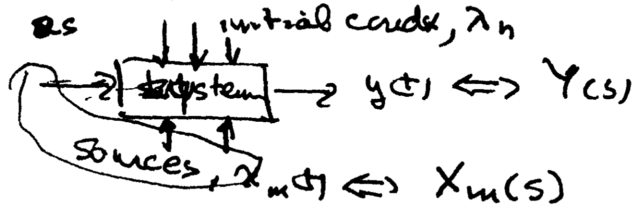
when ? Wed ~~am~~ 9 pm

one lab makeup permitted.

Correction equation on bottom ps lab 5 $\Omega_{ss} = \frac{V_1}{R_a(1 \quad \overline{R_a B / k^2})}$

SYSTEM FUNCTIONS

we saw that we are able to write the output of a system in terms of its Laplace transform as

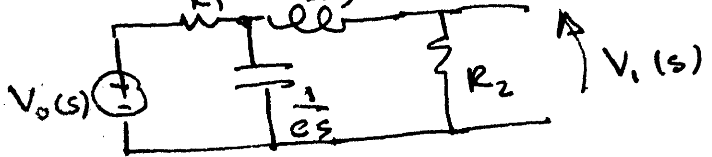


$$Y(s) = \underbrace{\sum_{m=1}^{M} H_{em}(s) X_m(s)}_{} + \sum_{n=1}^{N} H_{in}(s) \underbrace{\frac{\lambda_n(0)}{s}}$$

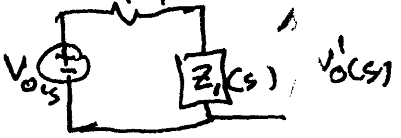
due to inputs
called ZSR

due to initial state i.e. of current in inductors voltages on capacit
called ZIR

$H_{em}(s)$ ~~is~~ is the system function for the m^{th} source

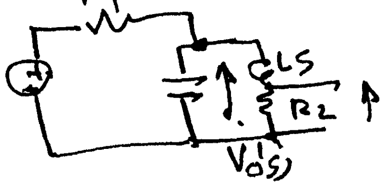


$\Rightarrow$



$$V_1(s) = V_o \times \frac{\left(\frac{1}{Cs}\right)(Ls + R_2)/(R_2 + Ls + \frac{1}{Cs})}{R_1 + \frac{1/Cs}{}}$$

$$Z_1(s) = \frac{\frac{1}{Cs} \| (R_2 + Ls)}{R_2 + Ls + \frac{1}{Cs}}$$

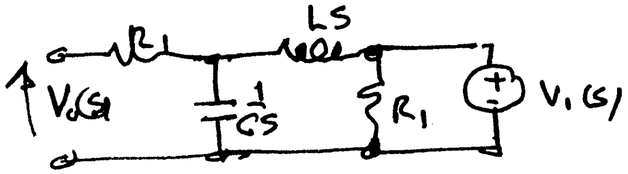


$$V_o'(s) \times \frac{R_2}{R_2 + Ls}$$

dimensional check

$$H_1(s) = \frac{V_1(s)}{V_o(s)} = \frac{\frac{R_2}{R_1 LC}}{s^2 + s\left(\frac{1}{R_1 C} + \frac{R_2}{L}\right) + \frac{1}{LC}\left(1 + \frac{R_2}{R_1}\right)}$$

Now let's excite the system from the right

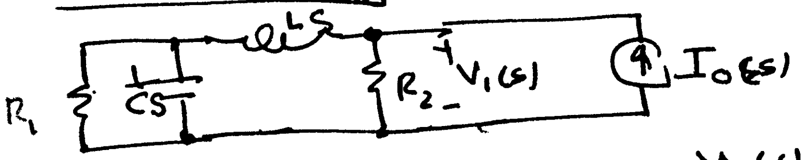


$$V_o(s) = \frac{\left(\frac{1}{Cs}\right)V_1(s)}{Ls + \frac{1}{Cs}} = \frac{\frac{1}{LC}}{s^2 + \frac{1}{LC}} \times V_1(s)$$

$$\frac{V_o(s)}{V_1(s)} = H_2(s) = \frac{\frac{1}{LC}}{s^2 + \frac{1}{LC}} \quad \text{which is not} \quad H_1(s)$$

So identifying the output & input is absolutely important to obtain the correct system function.

<u>Another example</u>



we wish to find $\frac{V_1(s)}{I_o(s)}$ called the <u>driving point</u> impedance:

$$V_1(s) = R_2 \,||\, \underbrace{\left(Ls + \frac{1}{Cs} || R_1\right)}\; I_o(s)$$

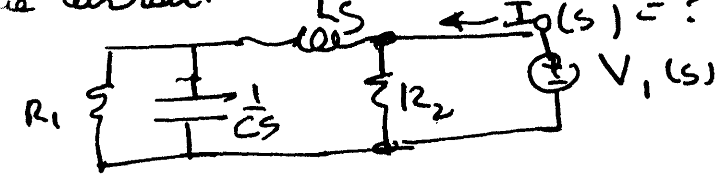
$$\searrow$$

$$Ls + \left(\frac{R_1/Cs}{R_1 + \frac{1}{Cs}} \right) = Ls + \frac{\frac{1}{C}}{s + \frac{1}{R}}$$

$$\frac{Ls\left(s + \frac{1}{R_1C}\right) + \frac{1}{C}}{s + \frac{1}{R_1C}} = \frac{s^2 + s/R_1C +}{\frac{1}{L}\left(s + \frac{1}{R_1C}\right.}$$

$$H_3(s) = V_1(s)/I_o(s) = \frac{R_2\left(s^2 + \frac{s}{R_1C} + \frac{1}{LC}\right)}{s^2 + s\left(\frac{1}{R_1C} + \frac{R_2}{L}\right) + \frac{1}{LC}\left(1 + \frac{R_2}{R_1}\right)}$$

Now let's drive the circuit with a voltage & measure the current



$$I_o(s) = \frac{V_1(s)}{R_2} + \frac{V_1(s)}{Ls + \frac{R_1/Cs}{R_1 + \frac{1}{Cs}}}$$

$$= V_1(s) \times \frac{s^2 + s\left(\frac{1}{R_1C} + \frac{R_2}{L}\right) + \frac{1}{LC}\left(1 + \frac{R_2}{R_1}\right)}{R_2\left(s^2 + \frac{s}{R_1C} + \frac{1}{LC}\right)}$$

the driving-point admittance, $H_4(s) = \frac{I_o(s)}{V_1(s)}$

$H_4(s) = \frac{1}{H_3(s)}$ the reciprocal of the driving-point impedance.

You will recall that when we were dealing with the sinusoidal steady state the impedances were $j\omega L$, $\frac{1}{j\omega C}$. The Laplace transform domain has them as Ls, $\frac{1}{Cs}$ so the only mathematical difference is the replacement of s with $j\omega$.

If the input to an LTI network is a complex exponential $Xe^{j\omega t}$ and the steady-state output is $Ye^{j\omega t}$ then

$$Y/X = H(s)\Big|_{s=j\omega}$$

so we can use the system function to arrive at the response to the s-s-s- input because the assumption is that the ZIR due to starting transients have died out.

we have seen that the ~~transient~~ response to $X(s)$ is

$Y(s) = H(s)X(s)$ in general we found the time response by performing a partial fraction expansion of $Y(s)$ to get it into the form of a known transformed time function

$$Y(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0} = \frac{N(s)}{D(s)}$$

a, b are real.

we can always write the N(s) & D(s) in terms of their roots, s_i

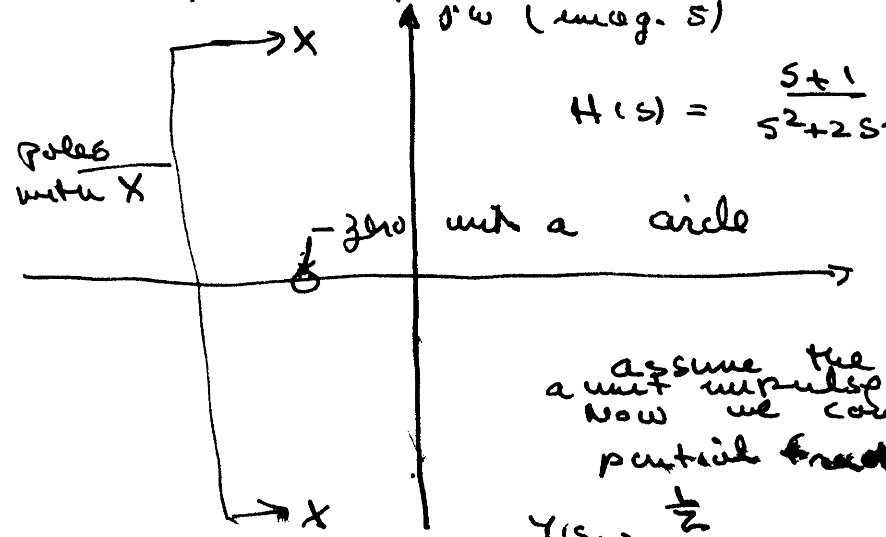
$$N(s) = a_n(s - s_{z1})(s - s_{z2})(s - s_{z3}) \cdots (s - s_{zn})$$

$$D(s) = b_n(s - s_{p1})(s - s_{p2}) \cdots (s - s_{pm})$$

called pole, called zero of $Y(s)$

if the root s_{pi}, (or s_{zi}) is complex then there must ~~have~~ be a complex conjugate of it t

We can characterize the system function by the locations of its poles and zeroes in the complex s-plane



$s = \sigma + j\omega$

$$H(s) = \frac{s+1}{s^2+2s+3} = \frac{s+1}{(s+1+j\sqrt{2})(s+1-j}$$

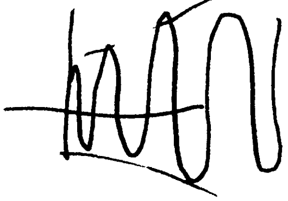
assume the system is excited by a unit impulse. Now we could ~~do~~ a partial ~~fract~~ion expansion

$$Y(s) = \frac{\frac{1}{2}}{s+1-j\sqrt{2}} + \frac{\frac{1}{2}}{s+1+j\sqrt{2}}$$

$$y(t) = \frac{1}{2}e^{-t}e^{j\sqrt{2}t} + \frac{1}{2}e^{-t}e^{-j\sqrt{2}t}$$

if the pole were in the rhp what happen

Yes, we would have had $e^{+t}\left[\frac{e^{j\sqrt{2}t}+e^{-j\sqrt{2}t}}{2}\right]$ as a response - called unstable.



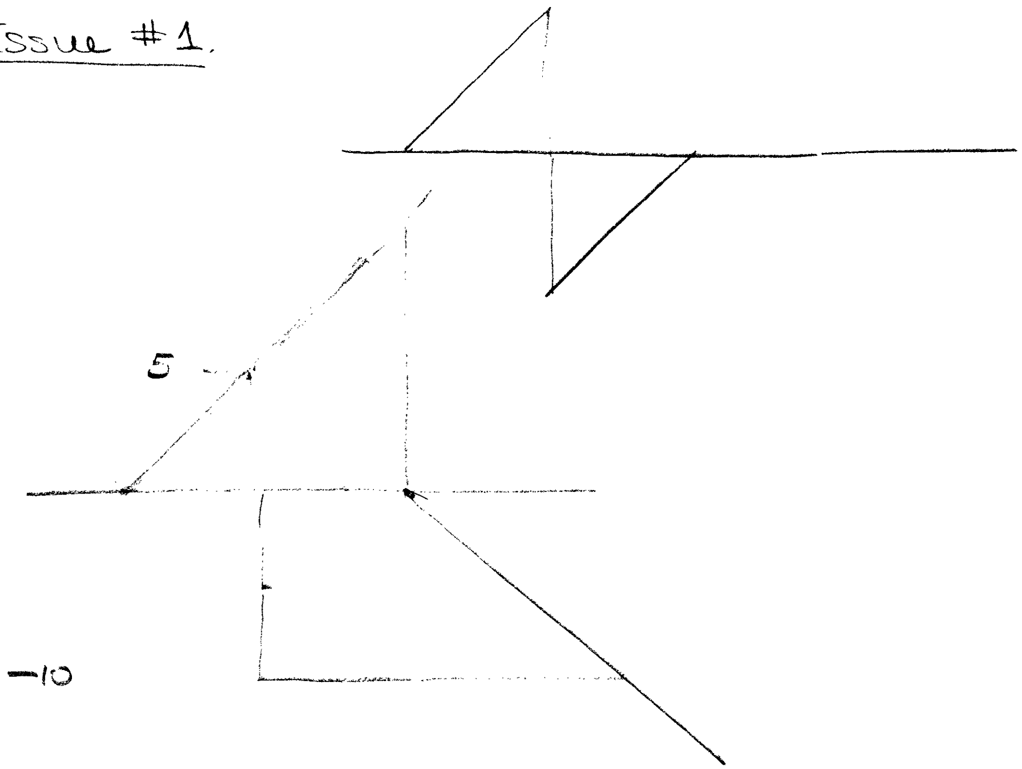
$(s-1-j\sqrt{2})(s-1+j\sqrt{2})$

$D(s) = (s-1)^2 + 2 = s^2 - 2s + 1 + 2 = s^2 - 2s + 3$

is unstable

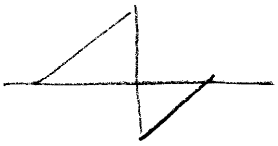
The poles must not lie in the right-half plane for the system to be stable!

Issue #1.



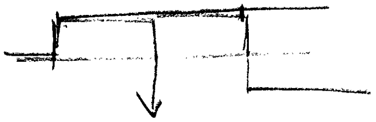
appropriate functional representation

$$5t\,u(t) - 10u(t-1) - 5t\,u(t-2).$$



Derivatives.

$$5u(t) - 10\delta(t-1) - 5u(t-2).$$



Integrals not really useful.

Laplace transform techniques

complex frequency s

special cases dc $s = 0$

exponential $s = \sigma$

sinusoidal $s = j\omega$

exponential sinusoid $s = \sigma + j\omega$

we defined the <u>one-sided</u> Laplace transform

$v(t) \longleftrightarrow V(s)$

$$V(s) = \int_{0^-}^{\infty} e^{-st} v(t)\, dt$$

$$v(t) = \frac{1}{2\pi j} \int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} e^{st} V(s)\, ds.$$

two-sided or bilateral Laplace transform

$$V(s) = \int_{-\infty}^{\infty} e^{-st} v(t)\, dt$$

	sinusoidal steady state	laplace
R:	R.	R
C:	$\frac{1}{j\omega C}$	$\frac{1}{sC}$
L:	$j\omega L$	sL.

Suppose the input to a LTI is $A\cos(\omega t+\phi)$

We know that $Y(s) = H(s)\,X(s)$.

If $x(t) = A\cos(\omega t+\phi)\,u(t)$

$$X(s) = \frac{A(s\cos\phi - \omega\sin\phi)}{s^2+\omega^2}$$

$$Y(s) = H(s)\,\frac{A(s\cos\phi-\omega\sin\phi)}{s^2+\omega^2} \qquad \frac{1}{s-j\omega}$$

$$Y(s) = \underbrace{\frac{K_1}{s-j\omega} + \frac{K_1^*}{s+j\omega}}_{\text{steady state response.}} + \underbrace{\sum \text{terms generated by poles of } H(s).}_{\text{should all lie in left-half plane and will die out} \to \text{natural response.}}$$

so, what is K_1? $K_1 = \left.\frac{H(s)\,A(s\cos\phi-\omega\sin\phi)}{s+j\omega}\right|_{s=j\omega}$

$$K_1 = \tfrac{1}{2}\,H(j\omega)\,Ae^{j\phi}$$

complex $|H(j\omega)|e^{j\theta(\omega)}$.

This is an amazing result. It says that we can determine the transfer function and then evaluate it for $s=j\omega$.

inverse transform to get

$$y_{ss}(t) = A\,|H(j\omega)|\cos[\omega t+\phi \;+\theta(\omega)].$$

EE&P 243 3/26/93

Last time we were examining a ~~wa~~ way to exhibit the nature of a system function, $\cancel{H}(s) = \frac{N(s)}{D(s)}$

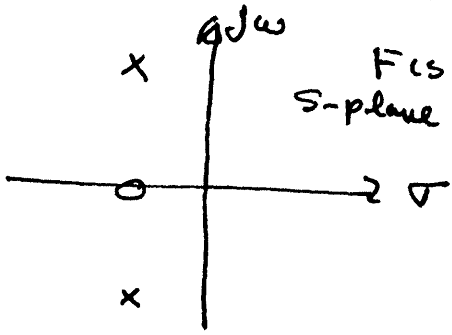
N(s) and D(s) may be written in terms of ~~the~~ their root.

$N(s) = a_n(s - s_{z_1})(s - s_{z_2}) \cdots (s - s_{z_n})$

$D(s) = b_m(s - s_{p_1})(s - s_{p_2}) \cdots (s - s_{p_m})$

And when $s = s_{z_j}$ $F(s) = 0$ - zero

or when $s = s_{p_k}$ $F(s) = \infty$ - pole



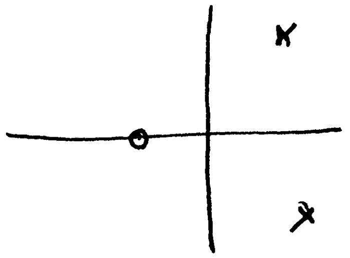
S-plane

$F(s) = \frac{s+1}{s^2+2s+1} = \frac{(s+1)}{(s+1-j\sqrt{2})(s+1+j\sqrt{2}}$

the system is s<u>tabl</u>e

$(f(t) \rightarrow 0$ as $t \rightarrow \infty)$

if poles are in l.h.p.



for $F(s) = \frac{s+1}{s^2-2s+1} = \frac{s+1}{(s-1+j\sqrt{2})(s-1-j}$

impulse response is.

$f(t) = e^{t} \cos\sqrt{2}t \qquad t \geq 0$

r.h.p roots indicate an unstable system.

<u>Note:</u> roots are either real or appear as a complex-conjugate pair

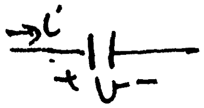
Let's take a moment for a short review of the methods we've encountered in this course

① Kirchoff's laws - KCL, KVL

- applies to time varying systems - as well as s-s.
- terminal relations $I = f(V)$ or $V = g(I)$

inductor $v = L\frac{di}{dt}$, $i = \frac{1}{L}\int^{t} v\,d\lambda + i(0)$

initial cond. tion



Capacitor $i = C\frac{dv}{dt}$, $v = \frac{1}{C}\int_0^t i(\lambda)d\lambda + v(0)$

resistor $v = iR$, $i = \frac{1}{R}v = Gv$

The sinusoidal Steady State

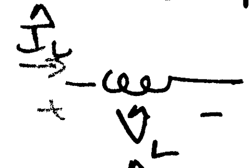
Signals such as $\hat{I} e^{j\omega t}$ represent sinusoids i.e. $Re\,\hat{I} e^{j\omega t} = i(t)$, $v(t) = Re\,\hat{V} e^{j}$

The equations with $Z_L = j\omega L$

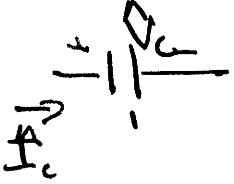
$Z_c = \frac{1}{j\omega c}$

$R = R$

represent the relationship between the complex magnitudes of voltages and curren



$$\hat{V}_L = j\omega L \hat{I}_L$$ magnitudes which multiply e^{j}



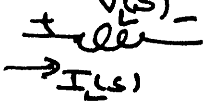
$$\hat{V}_c = \frac{1}{j\omega c}\hat{I}_c$$, the true (time voltage) is $v(t) = Re(\hat{V}_c e^{j}$

WHY DO WE NOT HAVE $i_L(0)$ or $v_c(0)$? (initial condx)

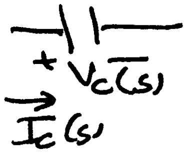
Transient problems - sources start at t=

$$I(s) = \int_0^\infty i(t) e^{-st} dt$$ ← Laplace xform

we have $V_L(s) = Ls\, I_L(s)$



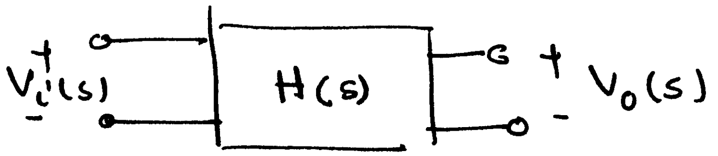
$V_c(s) = \left(\frac{1}{Cs}\right) I_c(s)$



The cute thing we see is that we can switch from a transient problem to a s-s-s-s problem by $s \to j\omega$. But these are two different problems with the same circuit.

So if we want to find the s-s-s-s- equivalent

of



we simply set $s = j\omega$, find $V_o(j\omega) = V_i(j$
$\times H(j$

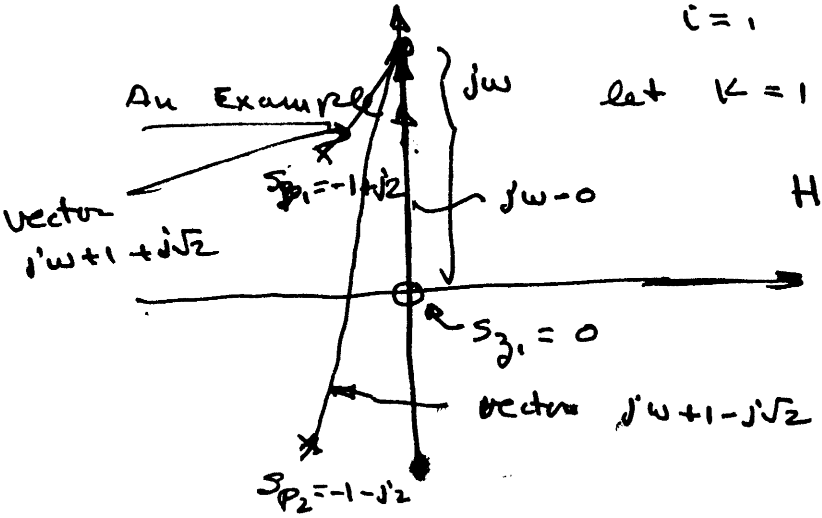
then $v_o(t) = Re\{V_o(j\omega)\, e^{j\omega t}\}$

Now let's look at $H(j\omega)$ knowing $H(s)$

$$H(s)\Big|_{s=j\omega} = k\, \frac{(s - s_{z_1})(s - s_{z_2}) \cdots (s - s_{z_n})}{(s - s_{p_1})(s - s_{p_2}) \cdots (s - s_{p_m})}\Bigg|_{s=j\omega}$$

$$|H(j\omega)| = k\, \frac{\prod_{i=1}^{n} |j\omega - s_{z_i}|}{\prod_{j=1}^{m} |j\omega - s_{p_j}|}$$

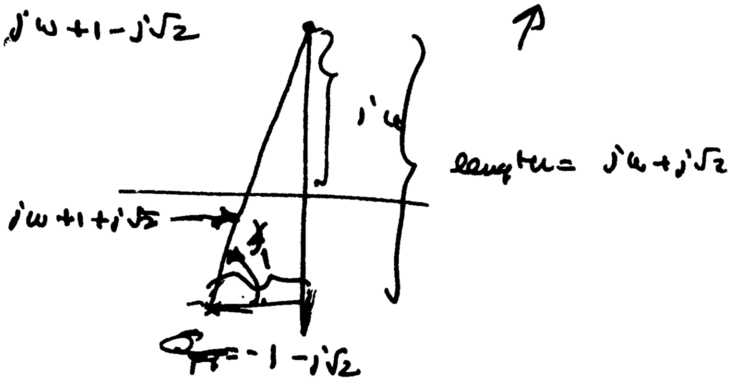
and $$\angle H(j\omega) = \sum_{i=1}^{n} \angle(j\omega - s_{z_i}) - \sum_{j=1}^{m} \angle(j\omega - s_{p_i})$$



let $K = 1$

$$H(s) = \frac{(s - 0)}{(s + 1 + j2)(s + 1 - j2)}$$

$$H(j\omega) = \frac{j\omega - 0}{(j\omega + 1 + j\sqrt{2})(j\omega + 1 -}$$

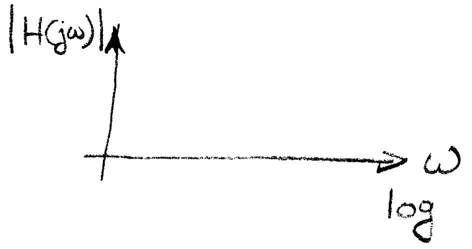


17.6. Bode diagrams

recall that the sinusoidal steady state response of a circuit is its transfer function evaluated at $s = j\omega$.

Plots of $|H(j\omega)|$ and $\angle H(j\omega) = \theta(\omega)$ give a good idea of how the circuit will behave when the poles and zeros are reasonably separated.

Bode plot



① starting point
$$H(s) = \frac{K(s+z_1)}{s(s+p_1)}$$

② evaluate at $s = j\omega$
$$H(j\omega) = \frac{K(j\omega+z_1)}{j\omega(j\omega+p_1)}$$

③ Put into standard form
$$H(j\omega) = \frac{K z_1 \left(1 + j\frac{\omega}{z_1}\right)}{j\omega\, p_1 \left(1 + j\frac{\omega}{p_1}\right)} = \overset{K_o}{\boxed{\frac{K z_1}{p_1}}} \frac{1 + j\omega/z_1}{j\omega\left(1 + j\omega/p_1\right)}$$

④ go to polar form.
$$H(j\omega) = K_o \frac{\left|1 + j\frac{\omega}{z_1}\right| \angle \psi_1}{|\omega| \angle 90^\circ \left\{\left|1 + j\frac{\omega}{p_1}\right| \angle \beta_1\right\}}$$
where $\psi_1 = \mathrm{Tan}^{-1}\left(\frac{\omega}{z_1}\right)$

$\beta_1 = \mathrm{Tan}^{-1}\left(\frac{\omega}{p_1}\right)$

$$\Rightarrow \quad |H(j\omega)| = K_o \frac{\left|1 + j\frac{\omega}{z_1}\right|}{\omega\left|1 + j\frac{\omega}{p_1}\right|} \qquad \theta(\omega) = \psi_1 - 90^\circ - \beta_1$$

Plot $|H(j\omega)| = K_0 \dfrac{|1+j\frac{\omega}{z_1}|}{\omega\,|1+j\frac{\omega}{P_1}|}$

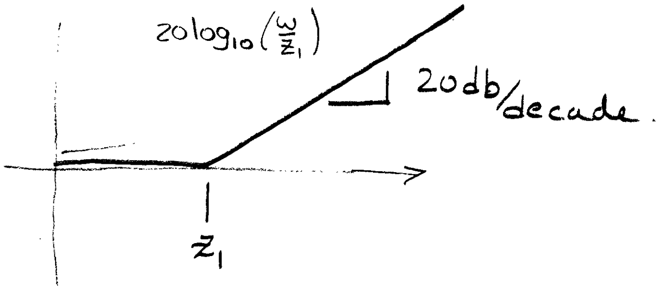
do in logs.

$$A_{db} = 20\log_{10} |H(j\omega)|$$

$$A_{db} = 20\log_{10} \frac{K_0 |1+j\frac{\omega}{z_1}|}{\omega\,|1+j\frac{\omega}{p_1}|}$$

$$= 20\log_{10} K_0 + 20\log_{10}|1+j\tfrac{\omega}{z_1}| - 20\log_{10}\omega - 20\log_{10}|1+j\tfrac{\omega}{P_1}|$$

Plot each term separately and combine

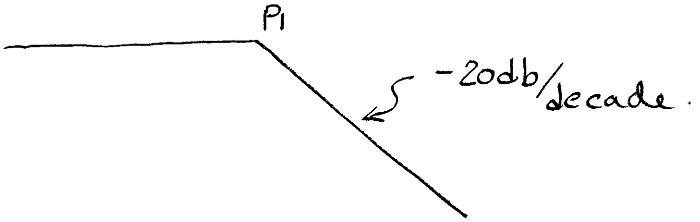


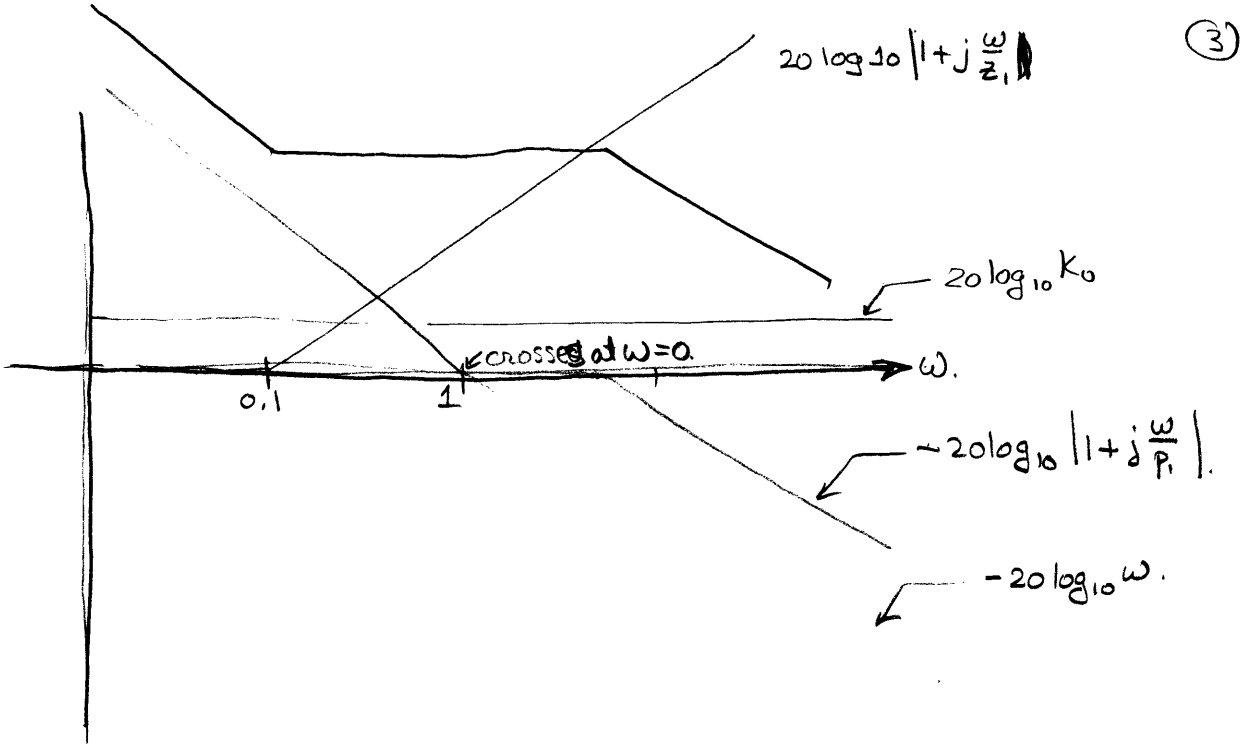
decade = power of 10.

for $\frac{\omega}{z_1} \ll 1$ $\quad 1+j\frac{\omega}{z_1} \approx 1$ and $\log(1) = 0$.

for $\frac{\omega}{z_1} \gg 1$ $\quad 1+j\frac{\omega}{z_1} \approx j\frac{\omega}{z_1}$ and $\log|j\frac{\omega}{z_1}| = \log\frac{\omega}{z_1}$

$20\log_{10}(1+j\frac{\omega}{P_1})$ looks about the same.





$$A_{db} = 20\log_{10}\frac{K_0\left|1+j\frac{\omega}{z_1}\right|}{\omega\left|1+j\frac{\omega}{p_1}\right|}$$

$$= 20\log_{10}K_0 + 20\log_{10}\left|1+j\frac{\omega}{z_1}\right| - 20\log_{10}\omega - 20\log_{10}\left|1+j\frac{\omega}{p_1}\right|.$$

Reading Assignment: 17.6, 17.7, (17.8)

Possible Problems:
17.27 (17.3)
17.28
17.29
17.30
17.31
$\boxed{17.34}$

17.32 Xtra Credit
16.46 modified

PE review exam

- Bode diagrams provide information on the frequency response of a system. You normally plot magnitude in decibels and angle in degrees versus the logarithm of frequency. You use decibels because they are logarithmic and allow the product of terms to be represented as a sum.

$$A_{I|db} = 20 \log \left|\frac{I_2}{I_1}\right|$$

$$A_{V|db} = 20 \log \left|\frac{V_2}{V_1}\right|$$

In general,

$$A_{db} = 20 \log |A(j\omega)|$$

Im general,

$$A(j\omega) = A\frac{(j\omega + z_1)(j\omega + z_2)\cdots(j\omega + z_k)}{(j\omega)^n(j\omega + p_1)(j\omega + p_2)\cdots(j\omega + p_m)}.$$

Rewrite $j\omega + z_i = z_i\left(1 + j\frac{\omega}{z_i}\right)$

$j\omega + p_j = p_j\left(1 + j\frac{\omega}{p_j}\right)$

$$A(j\omega)\Big|_{db} = 20\log k + \sum_{i=1}^{K} 20\log\left|1 + j\frac{\omega}{z_i}\right| - 20\,n\log\omega - \sum_{j=1}^{m} 20\log\left|1 + j\frac{\omega}{p_j}\right|$$

$$K = A\frac{\prod_{i=1}^{k}(z_i)}{\prod_{j=1}^{m}(p_j)}$$

$$\angle A(j\omega) = \underset{\substack{\uparrow \\ \text{from } \frac{1}{(j\omega)^n}}}{-90n} + \sum_{i=1}^{k}\tan^{-1}\left(\frac{\omega}{z_i}\right) - \sum_{j=1}^{m}\tan^{-1}\left(\frac{\omega}{p_j}\right)$$

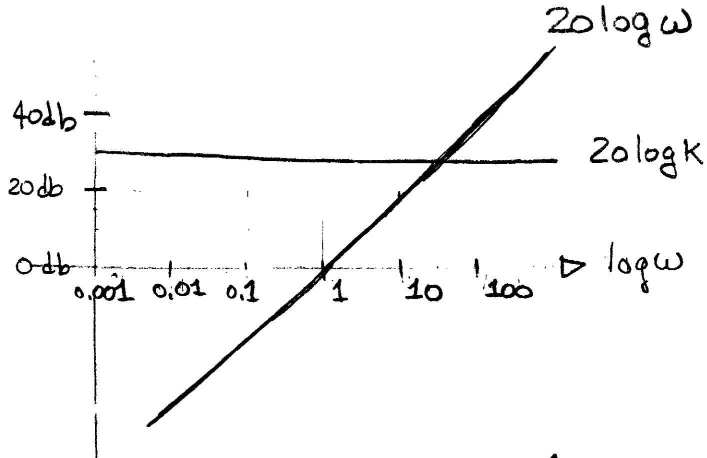
$Z + j\omega$

$\tan^{-1}\left(\frac{\omega}{Z}\right)$

likewise

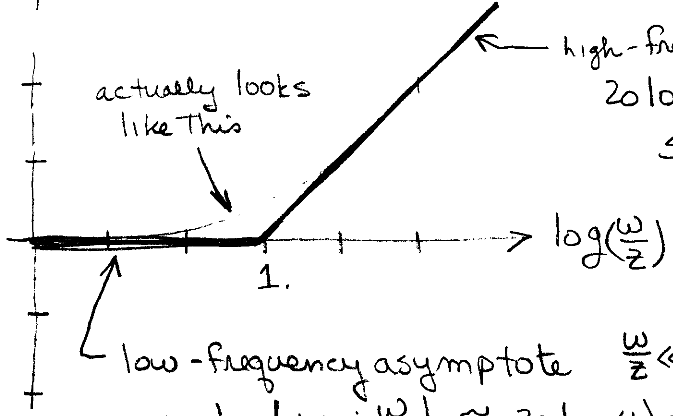
basic forms:

ω, k



$20\log K$ is a straight line

$1 + j\frac{\omega}{z}$



high-frequency asymptote $\frac{\omega}{z} \gg 1$

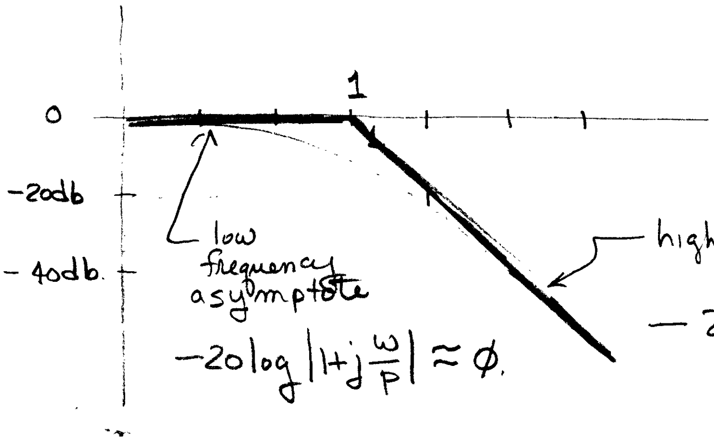
$20\log|1 + j\frac{\omega}{z}| \approx 20\log\frac{\omega}{z}$

Slope is 20db/decade.

low-frequency asymptote $\frac{\omega}{z} \ll 1$

$20\log|1 + j\frac{\omega}{z}| \approx 20\log(1) = 0.$

$\frac{1}{1 + j\frac{\omega}{p}}$



$-20\log|1 + j\frac{\omega}{p}| \approx \phi.$

high-frequency $\frac{\omega}{p} \gg 1$

$-20\log|1 + j\frac{\omega}{p}| \approx -20\log\frac{\omega}{p}.$

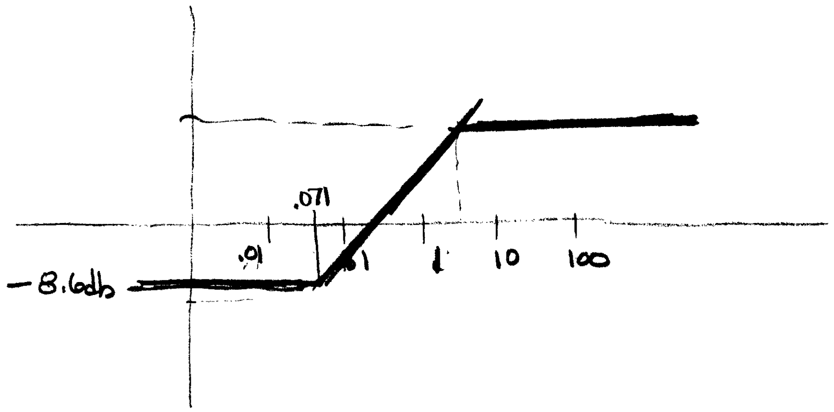
$$G_1(j\omega) = K \frac{\left(1 + j\frac{\omega}{z}\right)}{\left(1 + j\frac{\omega}{p}\right)}$$ zero at $\omega = z$

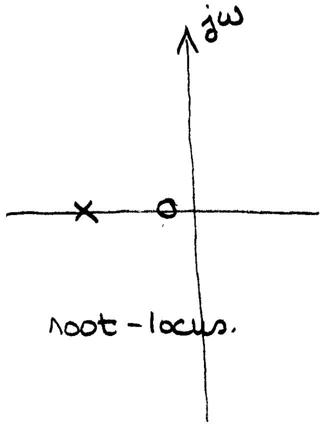
pole at $\omega = p$.

called the gain K

use logs. $G_1(j\omega)\Big|_{db} = 20 \log |K| + 20 \log \left|1 + j\frac{\omega}{z}\right| - 20 \log \left|1 + j\frac{\omega}{p}\right|$

I. suppose $z < p$. $K = 0.37$ $z = .071$ $p = 4.6$





$$20 \log |24| + 20 \log \left|1 + j\frac{\omega}{.071}\right| - 20 \log \left|1 + j\frac{\omega}{4.6}\right|$$

for ω small only $20 \log |0.37| \cong -8.6$db.

since $\frac{1}{z} >> \frac{1}{p}$ the zero comes in first. adding

$20 \log \left|1 + j\frac{\omega}{z}\right|$ corner frequency at $\omega = z$.

continues at + 20db/decade until effects of pole come in.

one pole goes at −20db/decade canceling out zero.

upper frequency limit

$$K \frac{j\frac{\omega}{z}}{j\frac{\omega}{p}} = K\frac{p}{z} = (.37)\frac{4.6}{(.071)} = 24$$

$20 \log 24 = 27.6$ db.

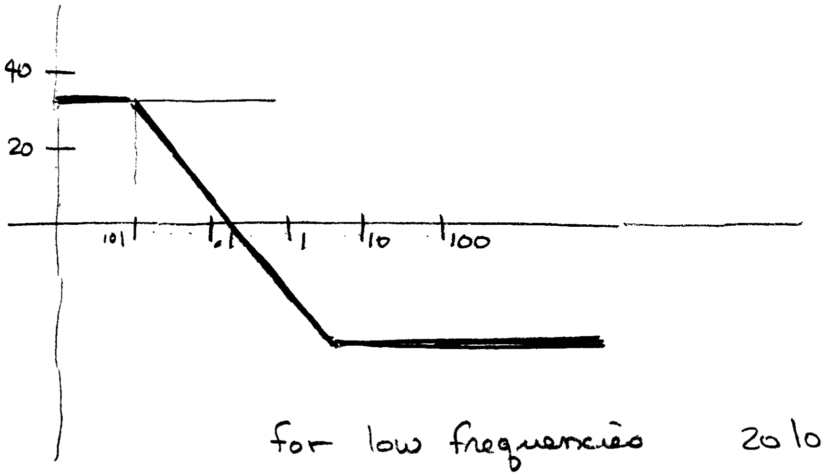
same transfer function

$$G_1(j\omega)\Big|_{db} = 20\log|K| + 20\log\left|1 + j\frac{\omega}{z}\right| - 20\log\left|1 + j\frac{\omega}{p}\right|$$

$K = 40 \qquad z = 5 \qquad p = .02$

case II. suppose $p < z$

$$G_1(j\omega)\Big|_{db} = 20\log|40| + 20\log\left|1 + j\frac{\omega}{5}\right| - 20\log\left|1 + j\frac{\omega}{.02}\right|$$



for low frequencies $20\log|40| = 32$

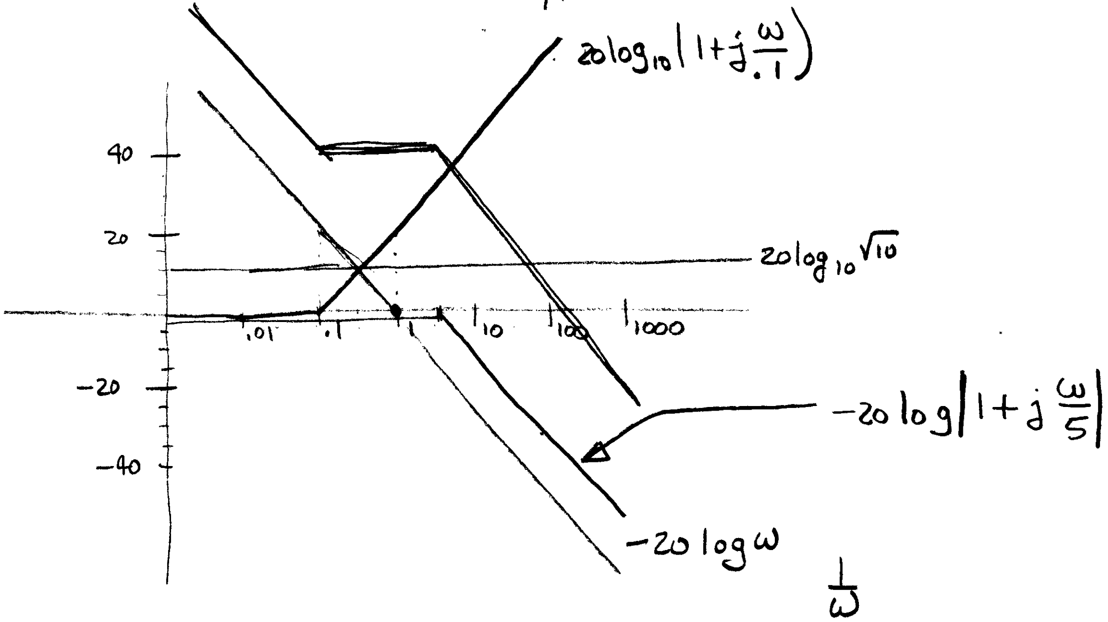
as ω increases pole at $\omega = .02$ comes in.
− 20 db / decade

as ω continues to increase zero at
$\omega = 5$ comes in
+ 20 db / decade which cancels
out pole

more functions

$$A_{db} = 20 \log_{10} \frac{K_0 \left| 1 + j \frac{\omega}{z_1} \right|}{\omega \left| 1 + j \frac{\omega}{p_1} \right|}$$

$K_0 = \sqrt{10}$ $z_1 = 0.1$
$p_1 = 5$



what does sum look like?

17.7 complex poles & zeros.

$$H(s) = \frac{K}{(s+\alpha-j\beta)(s+\alpha+j\beta)}$$

re-write in standard form

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$s^2 + s\alpha + \cancel{sj\beta} + s\alpha + \alpha^2 + \cancel{j\alpha\beta} - \cancel{j\beta s} - \cancel{j\beta\alpha} + \beta^2$$

$$s^2 + 2s\alpha + \alpha^2 + \beta^2$$

by inspection $\omega_n^2 = \alpha^2 + \beta^2$

$$\zeta\omega_n = \alpha$$

ω_n corner frequency

zeta $\rightarrow$ ζ damping factor

if $\zeta < 1$ roots are complex

$\zeta \geq 1$ $\underbrace{(s+p_1)(s+p_2)}_{\text{this is what we have done.}}$

for $\zeta < 1$. put into standard form

$$H(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{K}{\omega_n^2}\,\frac{1}{1 + 2\zeta\frac{s}{\omega_n} + \frac{s^2}{\omega_n^2}}$$

$$H(j\omega) = \frac{K_0}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j\left(\frac{2\zeta\omega}{\omega_n}\right)}$$ where $K_0 = \frac{K}{\omega_n^2}$

do normalized variable $u = \omega/\omega_n$ $H(j\omega) = \frac{K_0}{1 - u^2 + j2\zeta u}$

put into polar view

$$H(j\omega) = \frac{K_0}{|(1-u^2) + j2\zeta u| \angle \beta_1}$$ where $\beta_1 = \frac{2\zeta u}{1-u^2}$

amplitude plot

$$A_{db} = 20\log_{10} K_o - 20\log_{10}\left|(1-u^2)+j2\zeta u\right|$$

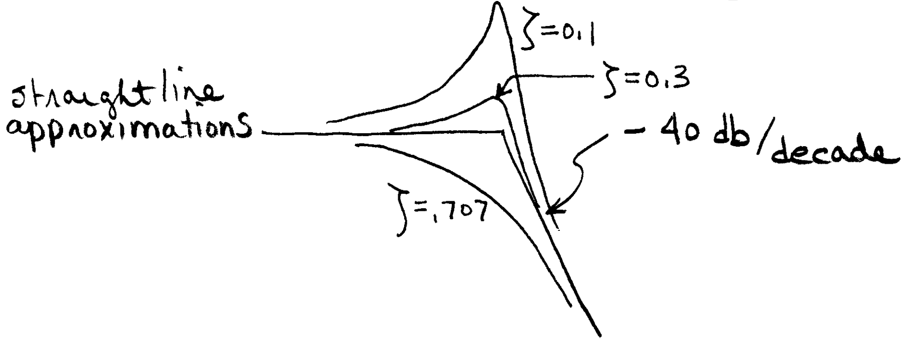
$$-20\log\left|(1-u^2)+j2\zeta u\right| = -20\log_{10}\sqrt{(1-u^2)^2+4\zeta^2u^2}$$

$$= -20\,\frac{1}{2}\log_{10}\left(u^4-2u^2+1+4\zeta^2u^2\right)$$

$$A_{db} = -10\log_{10}\left(u^4+2u^2(2\zeta^2-1)+1\right)$$

as $u \longrightarrow 0$ $\quad A_{db} \rightarrow -10\log_{10}(1) = 0$

as $u \rightarrow \infty$ $\quad A_{db} \rightarrow -10\log_{10} u^4 = -40\log_{10} u$



ζ controls the behavior near the corner frequency.

non-trivial example. $H(s) = \frac{100\,(s+1)}{s^2 + 8s + 100}$

put into standard form $$H(s) = \frac{s+1}{\underbrace{1 + \left(\frac{s}{10}\right)^2 + 0.8\left(\frac{s}{10}\right)}_{\text{should be in form } \frac{s}{\omega_n} \Rightarrow \omega_n = 10}}$$ ← after dividing by 100

$$H(j\omega) = \frac{|1 + j\omega| \angle \psi_1}{\left|1 - \left(\frac{\omega}{10}\right)^2 + j0.8\left(\frac{\omega}{10}\right)\right| \angle \beta_1}$$

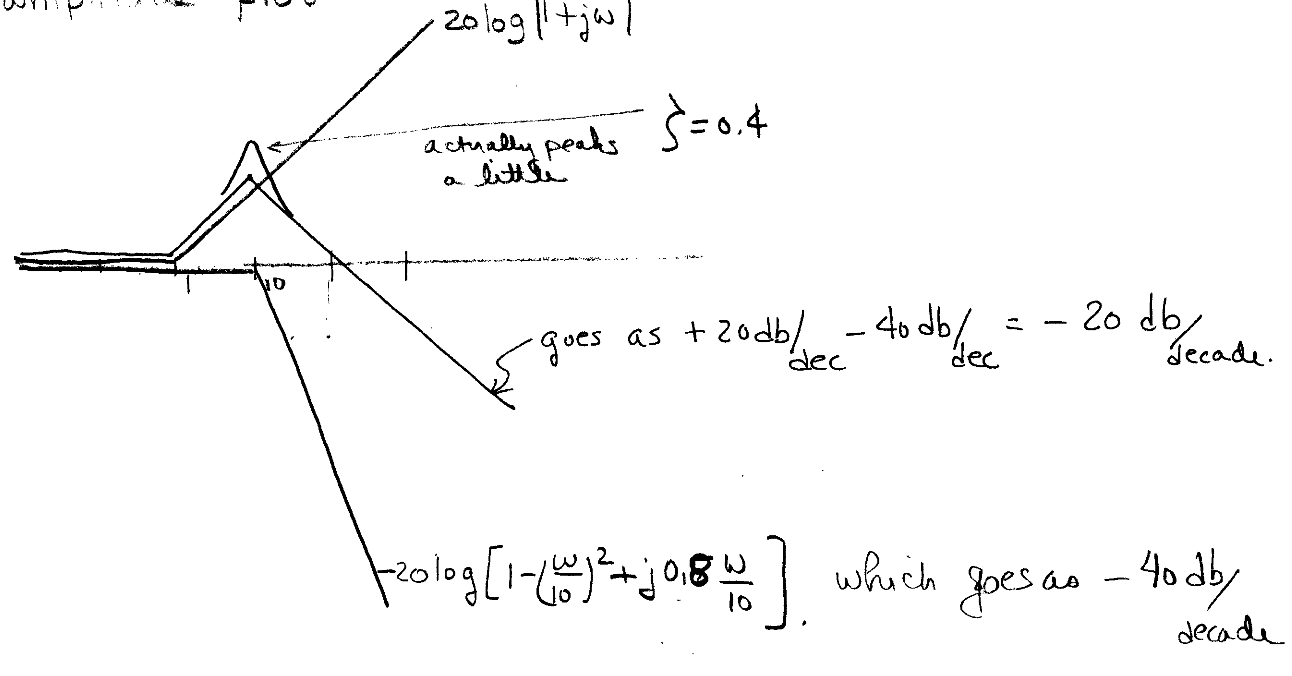
$\psi_1 = \mathrm{Tan}^{-1}(\omega)$

$\beta_1 = \mathrm{Tan}^{-1}\left[\frac{0.8\,\frac{\omega}{10}}{1 - \left(\frac{\omega}{10}\right)^2}\right]$

$$A_{db} = 20\log_{10}|1 + j\omega| - 20\log_{10}\left|1 - \left(\frac{\omega}{10}\right)^2 + j0.8\left(\frac{\omega}{10}\right)\right|$$

$$\theta(\omega) = \psi_1 - \beta_1 = \tan^{-1}\omega - \tan^{-1}\left(\frac{.8\,\frac{\omega}{10}}{1 - \left(\frac{\omega}{10}\right)^2}\right)$$

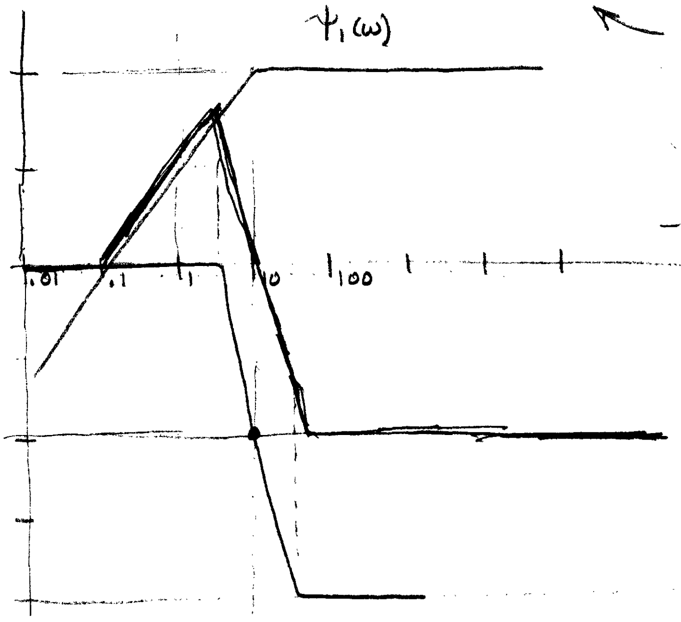
basic amplitude plot

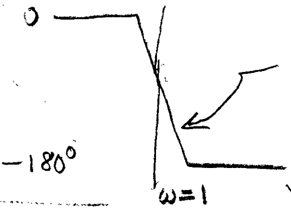


phase plot.

$$\psi_1 = \tan^{-1}(\omega) - \tan^{-1}\left[\frac{0.8\left(\frac{\omega}{10}\right)}{1-\left(\frac{\omega}{10}\right)^2}\right].$$

this we just have to learn





slope depends on ζ
larger ζ
larger slope.

for denominator position

Here is a tough one

$$H(s) = \frac{5\,(1+.1s)}{s(1+.5s)\left[1+.6\left(\frac{s}{50}\right)+\left(\frac{s}{50}\right)^2\right]}$$

$$\frac{5\left(1+j\frac{\omega}{10}\right)}{(j\omega)\left(1+j\frac{\omega}{2}\right)\left[1+.6\left(\frac{j\omega}{50}\right)+\left(\frac{j\omega}{50}\right)^2\right]}$$

① constant gain $K=5$

② pole at origin

③ pole at $s=-2$

④ zero at $s=-10$

⑤ complex poles $s=-15\pm$

put into standard form

$$\left[1-\left(\frac{\omega}{50}\right)^2+j\,0.6\left(\frac{\omega}{50}\right)\right]$$

$\omega_n = 50$

$\zeta = .6$

$2\zeta u$

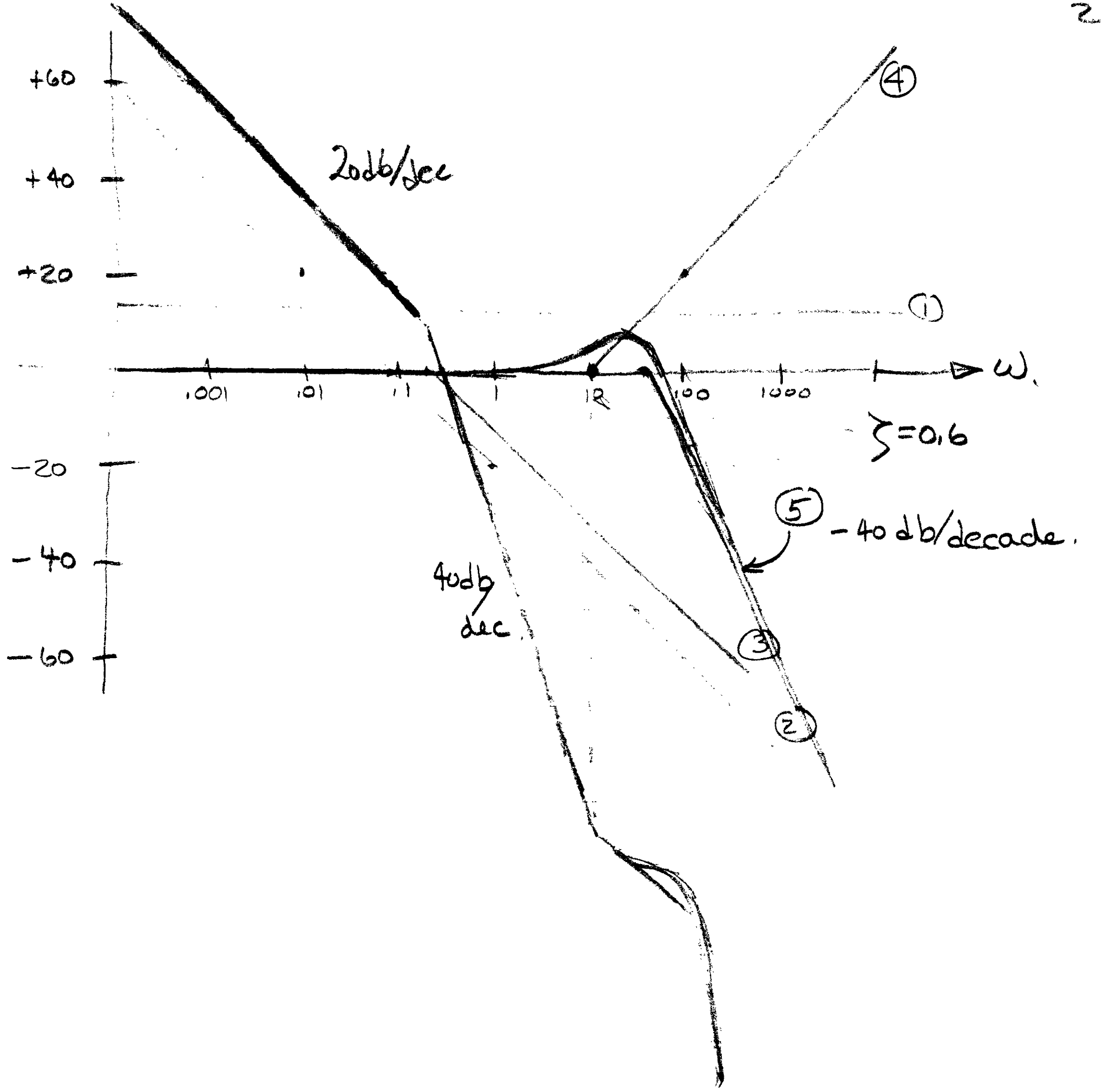
$2(.3)$

$$(50)^2\left[\left(\frac{s}{50}\right)^2+.6\left(\frac{s}{50}\right)+1\right]$$

$$s^2+30s+2500$$

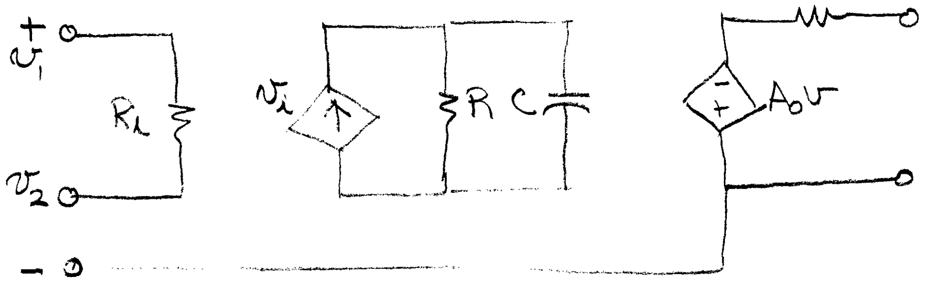
$$s=\frac{-30\pm\sqrt{900-4(2500)}}{2}$$

$$s=\frac{-30\pm\sqrt{9100}}{2}$$



Frequency response of op-amp circuits

Gain of an op-amp varies with frequency.



$$A(j\omega) = \frac{A_0}{1 + j\frac{\omega}{\omega_1}}$$

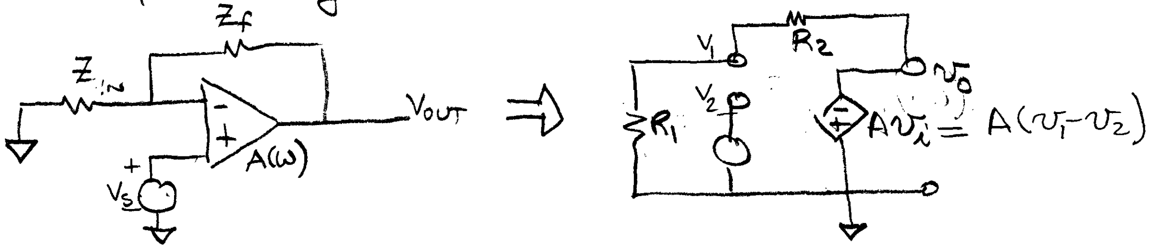
ω_1 break frequency. usually less than 100

$$\omega_1 = \frac{1}{RC}$$

A_0 dc gain $> 10^4$.

How about a non-inverting op-amp?

We have previously derived the dc gain as.



$$v_o = -A v_i$$

$$v_i = v_1 - v_2 = v_1 - v_s$$

use divider

$$v_1 = \overset{\equiv k_1}{\boxed{\frac{R_1}{R_1+R_2}}}\, v_o$$

$$v_i = k v_o - v_s.$$

$$-\frac{v_o}{A} = k v_o - v_s$$

$$v_o = -A(k v_o - v_s)$$

$$v_o = \frac{+A v_s}{1 + Ak} \quad \text{or} \quad \frac{v_o}{v_s} = \frac{A}{1 + kA}$$

$$\therefore \quad \frac{v_o}{v_s} = \frac{A}{1+Ak}.$$

normally this would become $\Rightarrow \frac{A}{Ak} = \frac{1}{k}$

$$\text{if} \quad A(j\omega) = \frac{A_0}{1+j\,\omega/\omega_1}$$

where $\omega_1 = 10$ rad/sec.

$A_0 = 10^5$

$R_2 = 100k$

$R_1 = 10k.$

$k = 0.1$

$$\frac{V_o(s)}{V_s(s)} = \frac{\frac{A_0}{1+j(\frac{\omega}{\omega_1})}}{1+k\frac{A_0}{1+j\frac{\omega}{\omega_1}}} = \frac{A_0}{1+j(\frac{\omega}{\omega_1})+kA_0}$$

$$= \frac{A_0}{1+kA_0 + j\frac{\omega}{\omega_1}} \; \frac{\frac{1}{1+kA_0}}{\frac{1}{1+kA_0}} = \frac{\boxed{\frac{A_0}{1+kA_0}}}{1+j\frac{\omega}{\omega_1}\boxed{\frac{1}{1+kA_0}}}$$

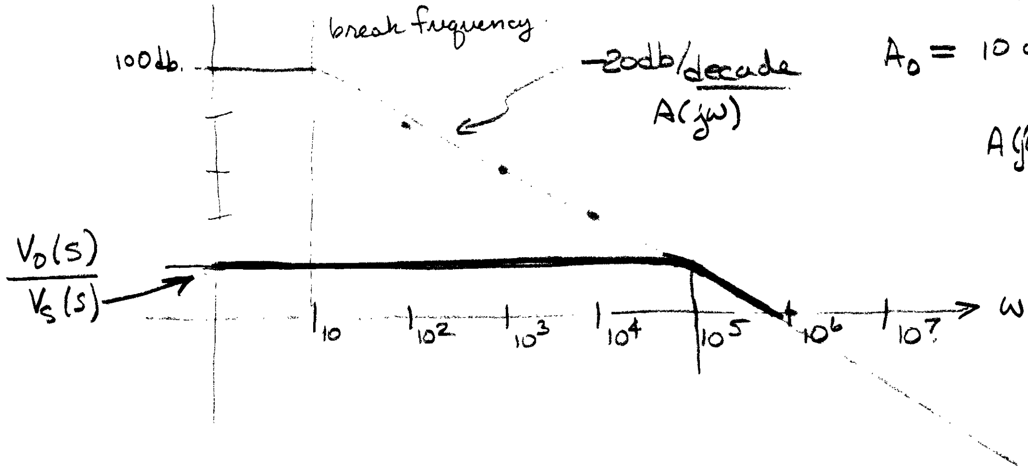
$\boxed{\frac{A_0}{1+kA_0}} \Rightarrow A_c$ total gain at dc.

$\boxed{\frac{1}{1+kA_0}} \Rightarrow A_2$

$$= \frac{A_c}{1+j\frac{\omega}{\omega_2}\frac{1}{A}}$$

← single pole

-20 db/dec.



$A_0 = 10\,0000$

$$A(j\omega) = \frac{A_0}{1+j\frac{\omega}{\omega_1}}$$

$20 \log 10^5 = 100$db.

$$\frac{V_o(s)}{V_s(s)} = \frac{10}{1+j\frac{\omega}{10}\underset{\wedge}{(10^{-4})}} = \frac{10}{1+j\frac{\omega}{10^5}}$$

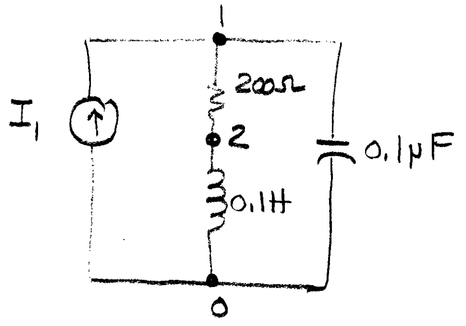
$$A_c = \frac{A_0}{1+.1A_0} \approx 10.$$

20 log

$$\frac{1}{1+.1\,10^5} \approx 10^{-4}.$$

\# of points

.AC <sweep type> <n> <start f> <end freq>.



```
Resonant circuit
I1  0 1  AC  0.1
R   1 2  200
L   2 0  0.1
C   1 0  0.1U
.AC  LIN  100  1E2  1E4.
.PLOT  AC  VM(1,0)
.PROBE
.END.
```

linear frequency.

VDB(1,0) plot |V| in dB.

VP (1,0) plot phase V

Interesting PROBE FEATURES.

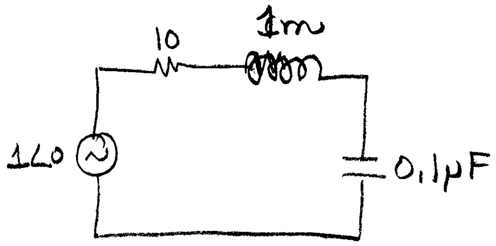
Plot_control
Add_plot
Exit
} adds a second plot.

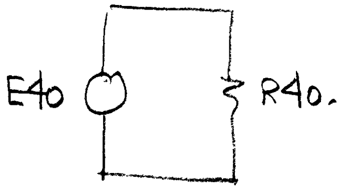
Importing data

E name +node -node FREQ {expression} = {freq, magdB, phase}.

Comparison —

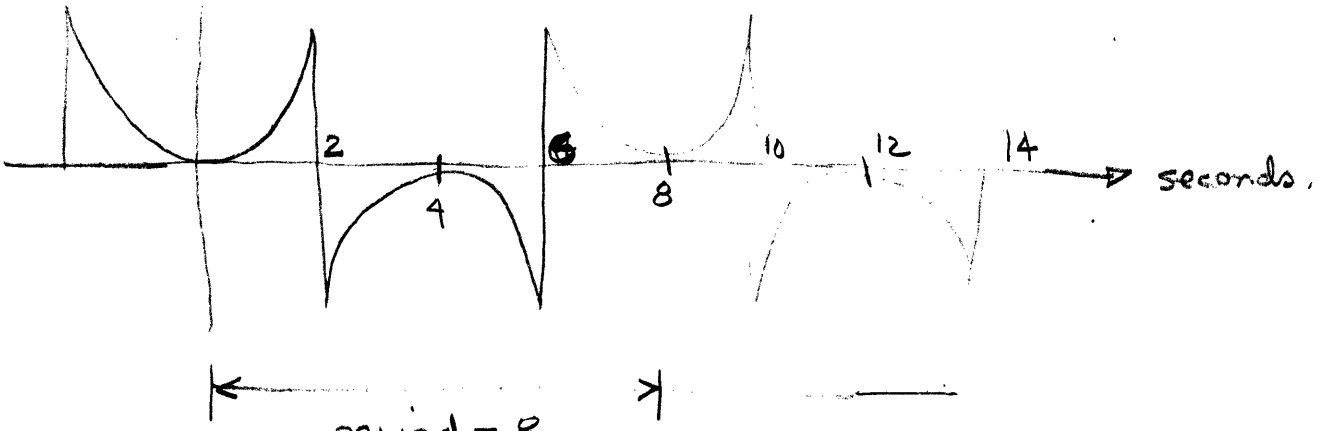
```
VS   1  0  AC  1
R12  1  2  10
L23  2  3  1m
C30  3  0  0.1μ
E40  4  0   FREQ { V(1) }  =  (
+     10085    2.384    -7.31
+     10511    4.601    -0.74
                 :
+     24637   -2.360   -178.67
+     25824   -2.610   -174.36 }
R40   4   0   1.
.AC   DEC   20   10k  25k.
.PROBE
.END.
```





A

Problem 18.12.



by inspection even symmetry. $f(t) = f(-t)$

$\Rightarrow$ all sine terms are zero

quarter wave symmetry.

half-wave { shift by $\frac{1}{2}$ cycle (4.)
{ and invert

+ symmetry around <u>midpoint</u> of positive half cycles.
& negative

if function is ~~even~~ $\rightarrow$ $a_v = 0$

dc term

$a_k = 0$ for k even

$a_k = \frac{8}{T}\int_0^{T/4} f(t) \cos k\omega_0 t \, dt$, k odd

$b_k = 0$ since even function

basically always do dc and odd/even terms.

dc. $a_v = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt = \frac{1}{T}\int_0^8 f(t)\,dt$

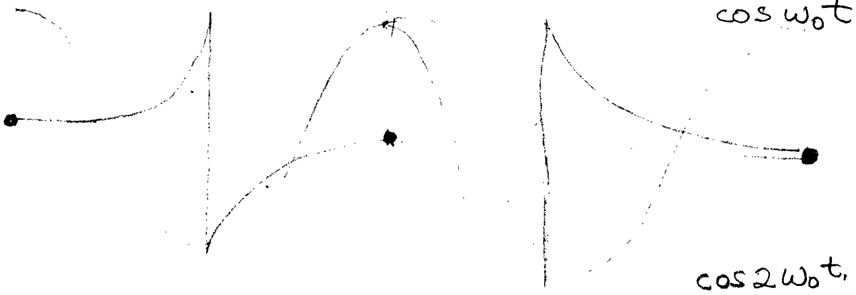
by inspection this integral is zero.

sine terms by inspection are zero.

how about cosine terms

$$a_k = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos k\omega_0 t \, dt.$$

$$= \frac{2}{8}\int_{0}^{8} f(t)\cos k\omega_0 t \, dt.$$



$\cos \omega_0 t,\quad k=1.$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$$

$\cos 2\omega_0 t,$

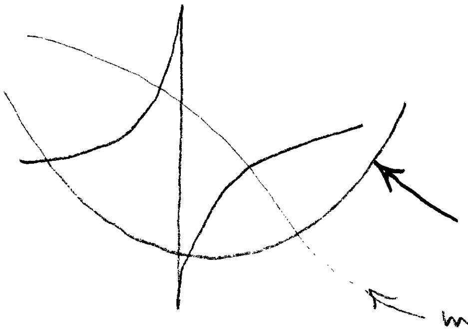
$$\cos \omega_0 t = \cos \frac{\pi}{4} t.$$

$$\cos 2\omega_0 t = \cos \frac{2\pi}{4} t$$

$$= \cos \frac{\pi}{2} t.$$

quarter wave
symmetry about midpoint
If even k even = 0,

examining



18.29 $$v_g = 50 \sum_{n=1,3,5\ldots}^{\infty} \frac{\pi^2 n^2 - 8}{n^3} (-1)^{n+1} \cos n\omega_0 t \, V.$$

$\underline{I}$

period = 0.2π seconds.

response at $\omega = 250$ rad/sec.

$$\omega_0 = \frac{2\pi}{T} = \frac{2\cancel{\pi}}{.2\cancel{\pi}} = \frac{20}{2} = 10.$$

look at 25th harmonic.

$$V_{g,25} = 50 \frac{\pi^2(625) - 8}{(25)^3} (-1)^{26} \cos 250t \text{ volts.}$$

$$= 19.71 \cos 250t$$

put this into circuit.

c

$$a_k = \frac{1}{4}\int_0^8 f(t)\cos\left(k\frac{\pi}{4}t\right)dt$$

$$= \frac{1}{4}\Bigg[\int_0^2 t^2\cos\left(k\frac{\pi}{4}t\right)dt + \int_2^4 (t-4)^2\cos\left(k\frac{\pi}{4}t\right)dt$$

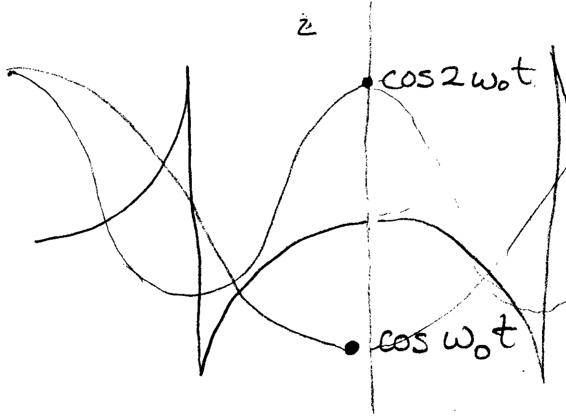
$$+ \int_4^6 (t-4)^2\cos\left(k\frac{\pi}{4}t\right)dt + \int_6^8 (t-8)^2\cos\left(k\frac{\pi}{4}t\right)dt$$

$$= \frac{1}{4}\Bigg[\int_0^2 t^2\cos\left(k\frac{\pi}{4}t\right)dt + \int_2^6 (t-4)^2\cos k\frac{\pi}{4}t\,dt + \int_6^8 (t-8)\cos\frac{k\pi}{4}t\,dt$$

symmetric
function [redacted] about
[illegible] [redacted]
cosine is also

but according to previous picture its not this integral but each half i.e.

$$a_k = \frac{1}{4}\left[\int_0^2 t^2\cos\left(k\frac{\pi}{4}t\right)dt + \int_2^4 (t-4)^2\cos\left(k\frac{\pi}{4}t\right)dt, + \text{other terms}\right].$$



by ~~inspection~~ all even terms will have integrals of ~~zero~~ leaving only odd terms

by inspection all even terms go to zero.
and the integrals for the odd terms are all the same

D

$$a_k = 4 \times \frac{1}{4}\int_0^2 t^2 \cos\left(k\frac{\pi}{4}t\right)\,dt$$

$$= \int_0^2 t^2 \cos\left(k\frac{\pi}{4}t\right)\,dt.$$

look integrals up.

$$= 4\left\{ \frac{2\textcircled{t}}{k^2\left(\frac{\pi}{4}\right)^2}\cos k\frac{\pi}{4}t + \frac{k^2\left(\frac{\pi}{4}\right)^2 t^2 - 2}{k^3\left(\frac{\pi}{4}\right)^3}\sin k\frac{\pi}{4}t \right\}\Bigg|_0^2$$

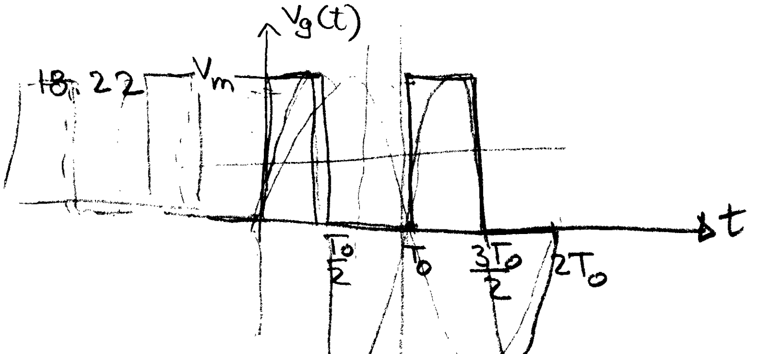
$$= 4\left\{ \frac{2}{k^2\left(\frac{\pi}{4}\right)^2}\left[\cancel{2}\cancelto{0}{\cos k\frac{\pi}{2}} - \cancelto{0}{\cos 0}\right] + \frac{k^2\left(\frac{\pi}{4}\right)^2\overset{4}{t^2}-2}{k^3\left(\frac{\pi}{4}\right)^3}\left[\sin k\frac{\pi}{2} - \cancelto{0}{0}\right]\right\}$$

for all k.

$$= 4\left[\frac{k^2\left(\frac{\pi}{4}\right)^2 4-2}{k^3\left(\frac{\pi}{4}\right)^3}\right]\sin\left(k\frac{\pi}{2}\right) = 4\left[\frac{\frac{k^2\pi^2}{4}-2}{k^3\frac{\pi^3}{64.}}\sin\left(k\frac{\pi}{2}\right)\right]$$

$$= \frac{k^2\pi^2 - 8}{k^3\pi^3}\cdot 64\sin\left(k\frac{\pi}{2}\right)$$ ∎

/E



half-wave

half - wave

$f(t) = -f(t - \frac{T}{2})$

I don't think this is half-wave symmetric

what is the Fourier series for $V_g(t)$

function is odd ⟶ sine terms
no cosine terms
expect dc term

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2T_0} = \frac{\pi}{T_0}$$

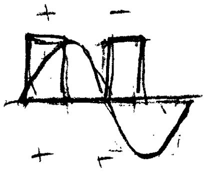
$$a_v = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt$$

$$= \frac{1}{2T_0}\int_0^{2T_0} f(t)\,dt = \frac{1}{2\cancel{T_0}}\,\cancel{2} \times V_m \frac{\cancel{T_0}}{\cancel{2}} = \frac{V_m}{2}.$$

how about other terms   sine only

$$a_K = 0$$

$$b_K = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin(k\omega_0 t)\,dt = \frac{2}{2T_0}\int_0^{2T_0} f(t)\sin(k\frac{\pi}{T_0}t)\,dt.$$

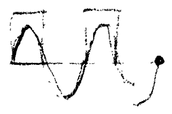


$\sin|, \frac{\pi t}{T_0}$ 2π $\frac{\pi}{T_0}, 2T_0$

first term will add

second term will cancel $b_2 =$

only even terms



$\sin 2\pi\frac{t}{t_0}$ $\frac{2\pi}{T_0} \cdot T_0$

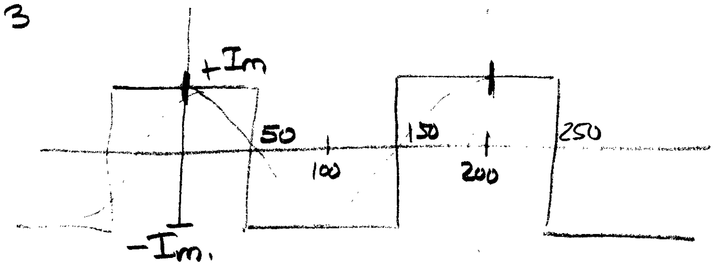
$$2\times\frac{1}{T_0}\int_0^{2T_0}\sin\left(k\frac{\pi t}{T_0}\right)dt = \frac{2}{\cancel{T_0}}\left.\frac{-V_m\cos\left(\frac{k\pi t}{T_0}\right)}{\left(\frac{k\pi}{\cancel{T_0}}\right)}\right|_0^{2T_0}$$

$$= -\frac{2V_m}{k\pi}\cos\left(\frac{k\pi t}{T_0}\right)\Bigg|_0^{2T_0} = -\frac{2V_m}{k\pi}\left[\cos k\overset{0}{\cancel{\pi}}2 - 1\right]$$

$$= +\frac{2V_m}{k\pi}$$

$$\therefore\ v_g(t) = \frac{V_m}{2} + \frac{2V_m}{\pi}\sum{}' \frac{1}{k}\sin k\omega_0 t$$

18.23 $I_m = 105\pi$ mA. F



$\omega_0 = \frac{2\pi}{T}$

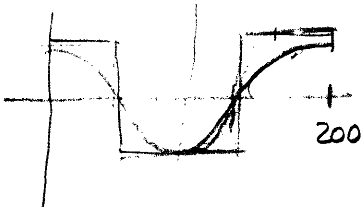
no dc term $v_g = 0$

has even symmetry: cosines only. $a_k \neq 0$, $b_k = 0$.

$$a_k = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos k\omega_0 t\, dt$$

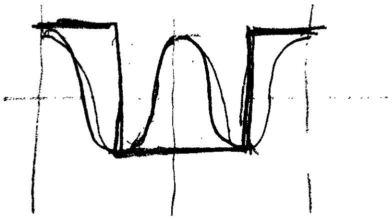
$$= \frac{2}{200\times10^{-6}}\int_0^{200\times10^{-6}} f(t)\cos k\omega_0 t.$$

k=1. $\cos \omega_0 t$

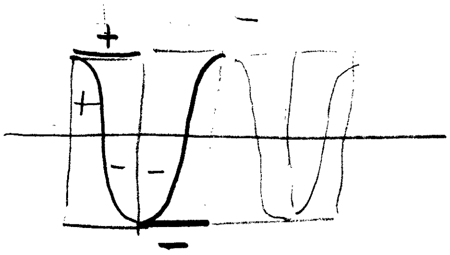


all add

k=2 $\cos 2\omega_0 t$



all cancel



overall each sub-integral is positive

So all odd.

$$a_k = 4\times\frac{2}{200\mu}\int_0^{50\mu} I_m\cos\left(\frac{k\cdot 2\pi}{200\mu}t\right)dt = \frac{8}{200\mu} I_m \frac{\sin\left(\frac{k2\pi t}{200\mu}\right)}{\frac{k2\pi}{200\mu}}\Bigg|_0^{50\mu}$$

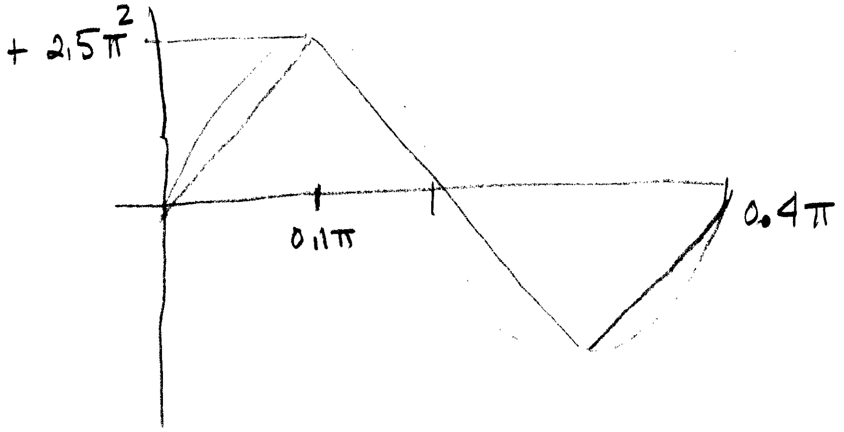
$$= \frac{\frac{8 I_m}{\cancel{200\mu}}\sin\left(\frac{k2\pi\cdot\cancel{50\mu}}{4\,\cancel{200\mu}}\right)}{\frac{k2\pi}{\cancel{200\mu}}} = \frac{8 I_m}{k2\pi}\sin\left(\frac{k\pi}{2}\right)$$

if k is odd good!

$$\boxed{a_k = \frac{4 I_m}{k\pi}(-1)^{k+1}}$$

18.28

G



find $i_g(t)$ by inspection odd function ⇒ no cosines
no dc.
sines only.

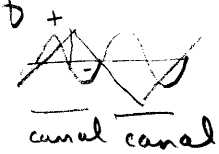
$a_v = 0$
$a_k = 0$

$$b_k = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin k\omega_0 t\,dt \qquad \omega_0 = \frac{2\pi}{0.4\pi}$$

$$= \frac{2}{T}\int_0^{T} f(t)\sin\left(k\frac{2\pi}{T}t\right)dt$$

k=1 perfect match 4x odd's only.

k=2



0 no even

H

$$b_k = 4\times\frac{2}{T}\int_0^{T/4}\frac{2.5\pi^2}{T/4}\,t\sin\left(k\frac{2\pi t}{T}\right)dt$$

$$= \frac{80\pi^2}{T^2}\int_0^{T/4} t\sin\left(\frac{k2\pi t}{T}\right)dt = \frac{80\pi^2}{T^2}\int_0^{T/4} t\,d\left[-\frac{\cos\left(\frac{k2\pi t}{T}\right)}{\frac{k2\pi}{T}}\right]$$

$$= \frac{80\pi^2}{T^2}\left[-t\cos\frac{k2\pi t}{T}\Big|_0^{T/4} + \int_0^{T/4}\cos\left(\frac{k2\pi t}{T}\right)dt\right]\frac{1}{k2\pi/_4}$$

$$= \frac{80\pi^2}{T^2}\left[\left[-\frac{T}{4}\overset{\nearrow 0}{\cancel{\cos\left(k\overset{k\pi}{\cancel{\frac{2\pi}{2}}}\frac{\cancel{T}}{\cancel{T}\,4}\right)}} - \overset{\nearrow 1}{\cancel{0\cos(0)}}\right] + \frac{\sin\left(\frac{k2\pi t}{T}\right)}{\frac{k2\pi}{T}}\Bigg|_0^{\frac{T}{4}}\right]\frac{1}{\frac{2\pi k}{T}}$$

$$= \frac{80\pi^2}{T^2}\left[\quad - \quad \frac{\sin k\cancel{\frac{2\pi}{T}}\cancel{\frac{T}{4}}_2}{\left(k\frac{2\pi}{T}\right)^2}\right]$$

$$= \frac{\overset{20}{\cancel{80}}\cancel{\pi^2}}{\cancel{T^2}}\;\frac{\cancel{T^2}}{k^2\cancel{4}\cancel{\pi^2}}\sin\left(k\frac{\pi}{2}\right) = \boxed{\frac{20}{k^2}(-1)^{k+1}}$$

$$i_g(t) = \sum_{k=1,3,5}^{\infty}\frac{20}{k^2}\sin\left(k\frac{2\pi t}{T}\right)$$

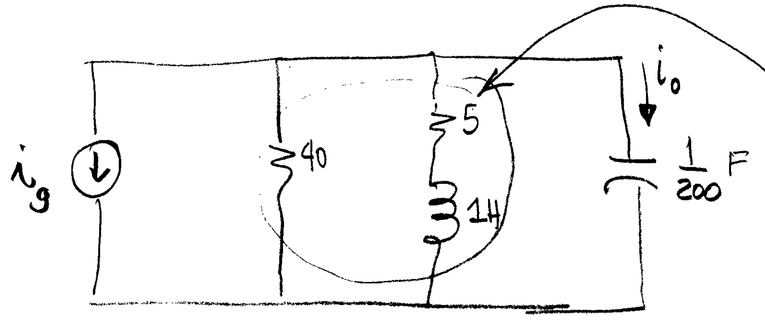
$$\frac{10}{.4} = \frac{100}{4} = 25$$

fifth harmonic

$$i_5(t) = \frac{20}{25}\sin\left(\frac{5\cdot 2\cancel{\pi}\,t}{0.4\cancel{\pi}}\right) = \frac{4}{5}\sin 25t$$

$$i_5(t) = 0.8\sin 25t$$

$$5\omega_0 = 5\,\frac{2\cancel{\pi}}{0.4\cancel{\pi}} = \frac{10}{.4} = 25$$

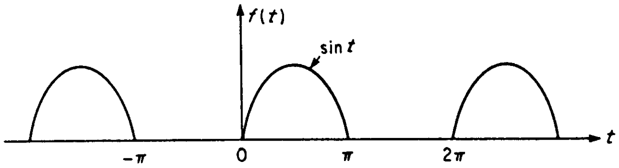


$$Z = \frac{40\,(5+j25)}{40+5+j25}$$

$$= \frac{\overset{40}{\cancel{200}}(1+j5)}{\cancel{5}\,(9+j5)} = 40\,\frac{1+j5}{9+j5}$$

Find the Fourier series of the half-wave rectified sinusoid as shown in Fig. 1.

Fig. 1



Solution: The function f(t) can be represented over one period as

$$f(t) = \begin{cases} \sin t & 0 \le t < \pi \\ 0 & \pi \le t < 2\pi \end{cases}$$

Since there is no half-wave symmetry we can expect a d.c. value. Hence,

$$a_o = \frac{1}{\tau}\int_o^{\tau} f(t)dt = \frac{1}{2\pi}\int_o^{\pi} \sin t\ dt + \frac{1}{2\pi}\int_{\pi}^{2\pi} 0\ dt$$

$$a_o = \frac{1}{2\pi}[-\cos t]_o^{\pi} = \frac{1}{2\pi}[1 + 1] = \frac{1}{\pi}\ .$$

However we cannot say that the function has odd or even symmetry. Therefore, we must determine a_n and b_n.

The Fourier representation of a periodic function is

$$a_o + \sum_{n=1}^{\infty} (a_n \cos n\omega_o t + b_n \sin n\omega_o t)$$

where

$$a_n = \frac{2}{\tau}\int_o^{\tau} f(t) \cos n\omega_o t\ dt$$

and

$$b_n = \frac{2}{\tau}\int_o^{\tau} f(t) \sin n\omega_o t\ dt.$$

First we find

$$b_n = \frac{2}{2\pi}\int_o^{\pi} \sin(t) \sin(nt)\ dt; \qquad \omega_o = 1$$

by the trigonometric function-product relationship

$$\sin t \sin nt = \frac{1}{2}\cos(t-nt) - \frac{1}{2}\cos\ (t+nt).$$

We obtain

$$b_n = \frac{1}{\pi}\int_o^{\pi} [\frac{1}{2}\cos t(1-n) - \frac{1}{2}\cos t\ (1+n)]\ dt$$

$$b_n = \frac{1}{\pi}\ [\frac{1}{2(1-n)} \sin t(1-n) - \frac{1}{2(1+n)} \sin t(1+n)]_o^{\pi}$$

$$b_n = \frac{1}{\pi}\left[\frac{1}{2(1-n)}\sin(\pi-n\pi) - \frac{1}{2(1+n)}\sin(\pi+n\pi) - 0 + 0\right]$$

$$b_n = \frac{1}{2\pi}\left(\frac{1}{1-n}\sin(\pi-n\pi) - \frac{1}{1+n}\sin(\pi+n\pi)\right).$$

When we substitute any positive value for n into the above expression we obtain $b_n = 0$; for $n = 1$ we obtain the undetermined form $\frac{0}{0}$ for the first term. Using L'Hospital's rule we can evaluate $\frac{1}{1-n}\sin(\pi-n\pi)$ for $n = 1$ as follows:

Let $$g(n) = \frac{g_1(n)}{g_2(n)} = \frac{\sin(\pi-n\pi)}{1-n}$$

where $g_1(n) = \sin(\pi-n\pi)$ and $g_2(n) = 1-n$.

By L'Hospital's rule we can evaluate

$$G(n) = \frac{g_1'(n)}{g_2'(n)} = \frac{-\pi\cos(\pi-n\pi)}{-1}\ .$$

If $G(1)$ exists then $g(1) = G(1)$. Hence.

$$g(1) = \pi\cos(\pi-\pi) = \pi \qquad \text{and}$$

$$b_1 = \frac{1}{2\pi}(\pi) = \frac{1}{2}\ .$$

The only sine component in the Fourier series is $\frac{1}{2}\sin t$. We proceed to find a_n:

$$a_n = \frac{1}{\pi}\int_0^{\pi}\sin(t)\cos(nt)\,dt$$

$$a_n = \frac{1}{\pi}\int_0^{\pi}\left[\frac{1}{2}\sin t(1+n) + \frac{1}{2}\sin t(1-n)\right]dt$$

$$a_n = -\frac{1}{2\pi}\left[\frac{1}{1+n}\cos t(1+n) + \frac{1}{1-n}\cos t(1-n)\right]_0^{\pi}$$

$$a_n = -\frac{1}{2\pi}\left[\frac{1}{1+n}\cos(\pi+n\pi) + \frac{1}{1-n}\cos(\pi-n\pi) - \left(\frac{1}{1+n} + \frac{1}{1-n}\right)\right].$$

When $n = 1$ we obtain

$$a_1 = -\frac{1}{2\pi}\left[\frac{1}{2} + \frac{1}{0} - \left(\frac{1}{2} + \frac{1}{0}\right)\right]$$

where the two indeterminate terms are both equal to zero by L'Hospital's rule, giving $a_1 = 0$.

When n = 2 we obtain

$$a_2 = -\frac{1}{2\pi}\left[-\frac{1}{3} + 1 - \frac{1}{3} + 1\right] = -\frac{2}{3\pi} .$$

When n = 3 we again find

$$a_3 = -\frac{1}{2\pi}\left[\frac{1}{4} - \frac{1}{3} - \frac{1}{4} + \frac{1}{3}\right] = 0.$$

Continuing in this way we find that for all odd harmonics ($n = 1,3,5,\ldots$) a_n is zero and all even harmonics ($n = 2,4,6\ldots$)

$$a_n = -\frac{1}{2\pi}\left[-\frac{1}{1+n} - \frac{1}{1-n} - \frac{1}{1+n} - \frac{1}{1-n}\right]$$

$$a_n = -\frac{1}{2\pi}\left[-\frac{2}{1+n} - \frac{2}{1-n}\right] = \frac{1}{2\pi}\left[\frac{2}{1+n} + \frac{2}{1-n}\right]$$

$$a_n = \frac{1}{2\pi}\left[\frac{2(1-n)+2(1+n)}{n^2-1}\right] = \frac{2}{\pi}\frac{1}{1-n^2} .$$

Hence, we can express the half-wave rectified sine wave as

$$f(t) = \frac{1}{\pi} + \frac{1}{2}\sin t + \frac{2}{\pi}\sum_{n=2,4,6\ldots}\frac{1}{1-n^2}\cos nt.$$