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CHAPTER 4

Cathode-Ray Tubes

Electron Motion

Cathode-Ray Tubes

Oscilloscopes

The discovery of cathode rays a century ago marked the beginning of the electronic era, and the invention of the multielectrode vacuum tube brought electronics into our daily living. As we learned to build efficient devices for generating and controlling streams of electrons, these electron tubes were applied in communication, entertainment, industrial control, and instrumentation. In many of these applications, the tube has been replaced by the semiconductor devices emphasized in the remainder of this book. However, sophisticated versions of the basic cathode-ray tube are widely used in oscilloscopes, data display devices, television cameras and receivers, and radar scanners.

In this chapter we study some of the physical principles that underlie the operation of electronic devices. First we examine the behavior of electrons in a vacuum and derive the equations for motion in electric and magnetic fields. Then we consider electron emission, acceleration, and deflection in a cathode-ray tube. Finally, we see how cathode-ray oscilloscopes can be used for precise observations over a wide range of conditions.

ELECTRON MOTION

The model of the electron as a negatively charged particle of finite mass but negligible size is satisfactory for many purposes. Based on many careful measurements, the accepted values for charge and mass of the electron are

$$e = 1.602 \times 10^{-19}\ \text{C} \cong 1.6 \times 10^{-19}\ \text{C}$$

$$m = 9.109 \times 10^{-31}\ \text{kg} \cong 9.1 \times 10^{-31}\ \text{kg}$$

In contrast, the hydrogen ion, which carries a positive charge of the same magnitude, has a mass approximately 1836 times as great. If the mass, charge, and initial velocity are known, the motion of individual electrons and ions in electric and magnetic fields can be predicted using Newton's laws of mechanics.

Motion in a Uniform Electric Field

A uniform electric field of strength $\mathcal{E}$ is established between the parallel conducting plates of Fig. 4.1a by applying a potential difference or voltage. By definition (Eq. 1-5)

$$\mathcal{E} = -\frac{dv}{dl} = -\frac{V_b - V_a}{L}\ \text{volts/meter} \qquad (4\text{-}1)$$

if the spacing is small compared to the dimensions of the plates.

By definition (Eq. 1-4), the electric field strength is the force per unit positive charge. Therefore, the force in newtons on a charge q in coulombs is

$$\mathbf{f} = q\mathcal{E} \qquad (4\text{-}2)$$

In Fig. 4.1b, $V_b > V_a$, $V_b - V_a$ is a positive quantity, and the electric field is negative (directed in the $-x$ direction). Considering the energy dw gained by a charge q moving a distance dl against the force of the electric field (Eq. 1-3), the voltage of point b with respect to point a is

$$V_{ba} = \frac{1}{q}\int_a^b dw = \frac{1}{q}\int_a^b f\,dl = \frac{1}{q}\int_a^b q(-\mathcal{E})\,dl = -\int_a^b \mathcal{E}\,dl \qquad (4\text{-}3)$$

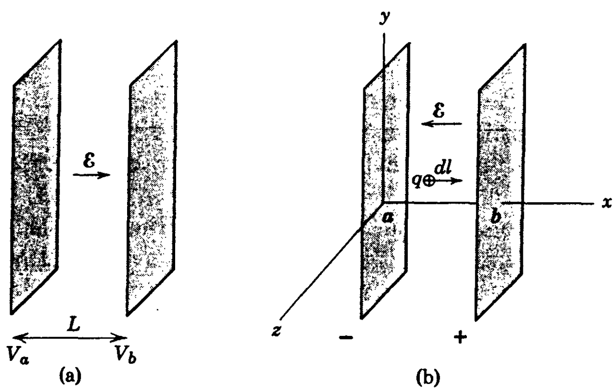


Figure 4.1 An electric charge in a uniform electric field.

In words, the voltage between any two points in an electric field is the line integral of the electric field strength. Equation 4-3 is the corollary of Eq. 4-1.

In general, an electron of charge $-e$ in an electric field $\mathcal{E}$ experiences a force

$$f = (-e)(\mathcal{E}) = -e\,\mathcal{E} = ma$$

and an acceleration

$$a = \frac{f}{m} = -\frac{e\mathcal{E}}{m} \tag{4-4}$$

We see that an electron in a uniform electric field moves with a constant acceleration. We expect the resulting motion to be similar to that of a freely falling mass in the earth's gravitational field. Where u is velocity and x is displacement, the equations of motion in a field $\mathcal{E}_x$ are

$$u_x = \int_0^t a_x\,dt = a_x t + U_O = -\frac{e\,\mathcal{E}_x}{m}t + U_O \tag{4-5}$$

$$x = \int_0^t u_x\,dt = \frac{a_x t^2}{2} + U_O t + X_O = -\frac{e\,\mathcal{E}_x}{2m}t^2 + U_O t + X_O \tag{4-6}$$

Application of these equations is illustrated in Example 1.

EXAMPLE 1

A voltage V_D is applied to an electron deflector consisting of two horizontal plates of length L separated a distance d, as in Fig. 4.2. An electron with initial velocity U_O in the positive x direction is introduced at the origin. Determine the path of the electron and the vertical displacement at the time it leaves the region between the plates.

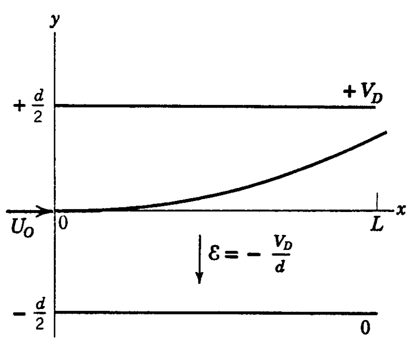


Figure 4.2 Calculation of electron deflection in a uniform electric field.

Assuming no electric field in the x direction and a uniform electric field $\mathcal{E}_y = -V_D/d$, the accelerations are

$$a_x = 0 \quad \text{and} \quad a_y = -\frac{e\,\mathcal{E}_y}{m} = \frac{eV_D}{md}$$

There is no acceleration in the x direction and the electron moves with constant velocity to the right. There is a constant upward acceleration and the electron gains a vertical component of velocity. The path is determined (Eq. 4-6) by

$$x = U_O t \quad \text{and} \quad y = -\frac{e\,\mathcal{E}_y}{2m}t^2 = \frac{eV_D}{2md}t^2$$

Eliminating t,

$$y = \frac{eV_D}{2mdU_O^2}x^2 \tag{4-7}$$

or the electron follows a parabolic path.

At the edge of the field, $x = L$ and the vertical displacement is

$$y_L = \frac{eV_D}{2mdU_O^2}L^2$$

If V_D exceeds a certain value, displacement y exceeds $d/2$ and the electron strikes the upper plate.

Energy Gained by an Accelerated Electron

When an electron is accelerated by an electric field it gains kinetic energy at the expense of potential energy, just as does a freely falling mass. Since voltage is energy per unit charge, the potential energy "lost" by an electron in "falling" from point a to point b is, in joules,

$$PE = W = q(V_a - V_b) = -e(V_a - V_b) = eV_{ba} \tag{4-8}$$

where V_{ba} is the potential of b with respect to a.

The kinetic energy gained, evidenced by an increase in velocity, is just equal to the potential energy lost, or

$$KE = \tfrac{1}{2}mu_b^2 - \tfrac{1}{2}mu_a^2 = PE = eV_{ba} \tag{4-9}$$

This important equation indicates that the kinetic energy gained by an electron in an electric field is determined only by the voltage difference between the initial and final points; it is independent of the path followed and the electric field configuration. (We assume that the field does not change with *time*.)

Frequently we are interested in the behavior resulting from a change in the energy of a single electron. Expressed in joules, these energies are very small; a more convenient unit is suggested by Eq. 4-8. An *electron volt* is the potential energy lost by 1 electron falling through a potential difference of 1 volt. By Eq. 4-8,

$$1\ \text{eV} = (1.6 \times 10^{-19}\ \text{C})(1\ \text{V}) = 1.6 \times 10^{-19}\ \text{J} \tag{4-10}$$

For example, the energy required to remove an electron from a hydrogen atom is about 13.6 eV. The energy imparted to an electron in a linear accelerator may be as high as 24 BeV (24 billion electron volts).

For the special case of an electron starting from rest ($u_a = 0$) and accelerated through a voltage V, Eq. 4-9 can be solved for $u_b = u$ to yield

$$u = \sqrt{2(e/m)V} = 5.93 \times 10^5 \sqrt{V}\ \text{m/s} \tag{4-11}$$

In deriving Eq. 4-11 we assume that mass m is a constant; this is true only if the velocity is small compared to the velocity of light, $c \cong 3 \times 10^8$ m/s.

EXAMPLE 2

Find the velocity reached by an electron accelerated through a voltage of 3600 V.

Assuming that the resulting velocity u is small compared to the velocity of light c, Eq. 4-11 applies and

$$u = 5.93 \times 10^5\sqrt{3600} = 3.56 \times 10^7\ \text{m/s}$$

In this case,

$$\frac{u}{c} = \frac{3.56 \times 10^7}{3 \times 10^8} \cong 0.12$$

At the velocity of Example 2 the increase in mass is appreciable, and the actual velocity reached is about 0.5% lower than that predicted. For voltages above 4 or 5 kV, a more precise expression should be used. (See Problem 1.)

Motion in a Uniform Magnetic Field

One way of defining the strength of a magnetic field (Eq. 1-6) is in terms of the force exerted on a unit charge moving with unit velocity normal to the field. In general, the force in newtons is

$$\mathbf{f} = q\mathbf{u} \times \mathbf{B} \tag{4-12}$$

where q is charge in coulombs and $\mathbf{B}$ is magnetic flux density in teslas (or webers/meter^2). The vector cross product is defined by the right-hand screw rule illustrated in Fig. 4.3. Rotation from the direction of $\mathbf{u}$ to the direction of $\mathbf{B}$ advances

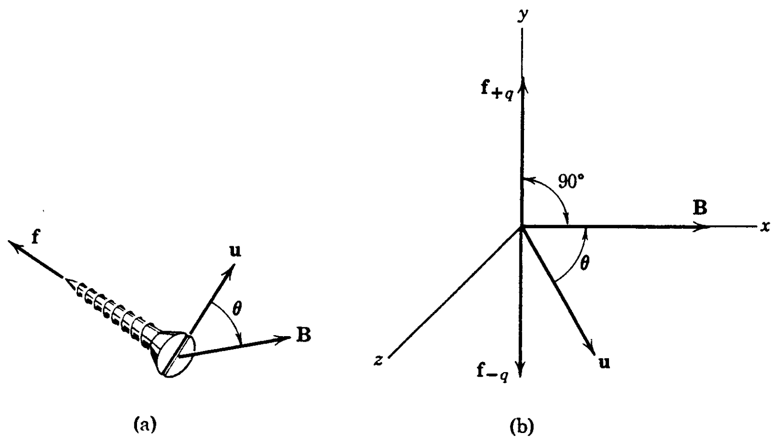


Figure 4.3 The right-hand screw rule defining the vector cross product.

the screw in the direction of $\mathbf{f}$. The magnitude of the force is $quB \sin \theta$ and the direction is always normal to the plane of $\mathbf{u}$ and $\mathbf{B}$.

Equation 4-12 is consistent with three observable facts:

1. A charged particle at rest in a magnetic field experiences no force ($u = 0$).
2. A charged particle moving parallel with the magnetic flux experiences no force ($\theta = 0$).
3. A charged particle moving with a component of velocity normal to the magnetic flux experiences a force that is normal to u and therefore the magnitude of velocity (or speed) is unchanged.

From the third statement we conclude that no work is done by a magnetic field on a charged particle and its kinetic energy is unchanged.

Figure 4.4 shows an electron entering a finite region of uniform flux density. For an electron ($q = -e$) moving in the plane of the paper ($\theta = 90°$), the force is in the direction shown with a magnitude

$$f = eU_0B \tag{4-13}$$

Applying the right-hand rule, we see that the initial force is downward and, therefore, the acceleration is downward and the path is deflected as shown. A particle

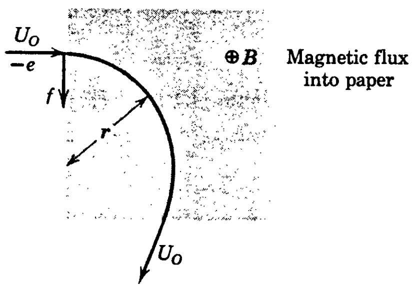


Figure 4.4 Electron motion in a uniform magnetic field.

moving with constant speed and constant normal acceleration follows a circular path. The centrifugal force due to circular motion must be just equal to the centripetal force due to the magnetic field, or

$$m\frac{U_O^2}{r} = eU_OB$$

and the radius of the circular path is

$$r = \frac{mU_O}{eB} \tag{4-14}$$

The dependence of radius on the mass of the charged particle is the principle underlying the *mass spectrograph* for analyzing unknown substances.

Motion in Combined $\mathcal{E}$ and B Fields

In the general case, both electric and magnetic fields are present and exert forces on a moving charge. The total force is

$$\mathbf{f} = q(\mathcal{E} + \mathbf{u} \times \mathbf{B}) \tag{4-15}$$

The special cases previously described can be derived from this general relation. If both $\mathcal{E}$ and B are present, the resulting motion depends on the relative orientation of $\mathcal{E}$ and B, and also on the initial velocities. An interesting case is that in which an electron starts from rest in a region where $\mathcal{E}$ and B are mutually perpendicular. The electron is accelerated in the direction of $-\mathcal{E}$, but as soon as it is in motion there is a reaction with B and the path starts to curve. Curving around, the electron is soon traveling in the $+\mathcal{E}$ direction and experiences a decelerating force that brings it to rest. Once the electron is at rest, the cycle starts over; the resulting path is called a *cycloid* and resembles the path of a point on a wheel as it rolls along a line.

If the fields are not uniform, mathematical analysis is usually quite difficult. A case of practical importance is the "electron lens" used in electron microscopes and for electric-field focusing of the electron beam in the cathode-ray tube.

CATHODE-RAY TUBES

The television picture tube and the precision electron display tube used in oscilloscopes are modern versions of the evacuated tubes used by Crookes and Thomson to study cathode rays. The electrons constituting a "cathode ray" have little mass or inertia, and therefore they can follow rapid variations; their ratio of charge to mass is high, so they are easily deflected and controlled. The energy of high-velocity electrons is readily converted into visible light; therefore, their motion is easily observed. For these reasons, the C-R tube or CRT is a unique information processing device; from our standpoint, it is also an ingenious application of the principles of electron motion and electron emission.

Electron Emission

The Bohr model of the atom is satisfactory for describing how free electrons can be obtained in space. As you may recall, starting with the hydrogen atom consisting of a single proton and a single orbital electron, models of more complex atoms are built up by adding protons and neutrons to the nucleus and electrons in orbital groups or shells. In a systematic way, shells are filled and new shells started. The chemical properties of an element are determined by the *valence* electrons in the outer shell. Good electrical conductors like copper and silver have one highly mobile electron in the outer shell.

Only certain orbits are allowed, and atoms are stable only when the orbital electrons have certain discrete energy levels. Transfer of an electron from an orbit corresponding to energy W_1 to an orbit corresponding to a lower energy W_2 results in the radiation of a *quantum* of electromagnetic energy of frequency f given by

$$W_1 - W_2 = hf \tag{4-16}$$

where h is Planck's constant $= 6.626 \times 10^{-34}$ J·s.

The energy possessed by an orbital electron consists of the kinetic energy of motion in the orbit and the potential energy of position with respect to the positive ion representing all the rest of the neutral atom. If other atoms are close (as in a solid), the energy of an electron is affected by the charge distribution of the neighboring atoms. In a crystalline solid, there is an orderly arrangement of atoms and the permissible electron energies are grouped into *energy bands*. Between the permissible bands there may be ranges of energy called *forbidden bands*.

For an electron to exist in space it must possess the energy corresponding to motion from its normal orbit out to an infinite distance; the energy required to move an electron against the attractive force of the net positive charge left behind is the *surface barrier energy* W_B. Within a metal at absolute zero termperature, electrons possess energies varying from zero to a maximum value W_M. The minimum amount of work that must be done on an electron before it is able to escape from the surface of a metal is the *work function* W_W where

$$W_W = W_B - W_M \tag{4-17}$$

For copper, $W_W = 4.1$ eV, while for cesium $W_W = 1.8$ eV.

The energy required for electron emission may be obtained in various ways. The beta rays given off spontaneously by *radioactive* materials (along with alpha and gamma rays) are emitted electrons. In *photoelectric* emission, the energy of a quantum of electromagnetic energy is absorbed by an electron. In *high-field* emission, the potential energy of an intense electric field causes emission. In *secondary* emission, a fast-moving electron transfers its kinetic energy to one or more electrons in a solid surface. All these processes have possible applications, but the most widely used process is *thermionic* emission in which thermal energy is added by heating a solid conductor.

The temperature of an object is a measure of the kinetic energy stored in the motion of the constituent molecules, atoms, and electrons. The energies of the individual constituents vary widely, but an average energy corresponding to temperature T can be expressed as kT, where $k = 8.62 \times 10^{-5}$ eV/K is the Boltzmann constant. At a temperature above absolute zero, the distribution of electron energy in a metal is modified and some electrons possess energies appreciably above W_M. Statistical analysis shows that the probability of an electron receiving sufficient energy to be emitted is proportional to $e^{-W_W/kT}$. At high temperatures, many electrons possess energies greater than W_B and emission current densities of the order of 1 A/cm^2 are practical. Commercial cathodes make use of special materials that combine low work function with high melting point.

CRT Components

The essential components of a CRT are shown in Fig. 4.5; an *electron gun* produces a focused beam of electrons, a *deflection system* determines the direction of the beam, and a *fluorescent screen* converts the energy of the beam into visible light.

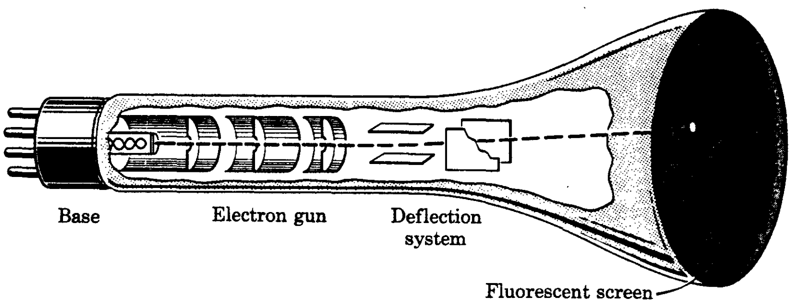


Figure 4.5 The essential components of a cathode-ray tube.

Electron Gun. Electrons are emitted from the hot cathode (Fig. 4.6a) and pass through a small hole in the cylindrical control electrode; a negative voltage (with respect to the cathode) on this electrode tends to repel the electrons and, therefore, the voltage applied controls the intensity of the beam. Electrons passing the control electrode experience an accelerating force due to the electric field established by the positive voltages V_F and V_A on the focusing and accelerating anodes. The space between these anodes constitutes an electron lens (Fig. 4.6b); the electric flux lines and equipotential lines resulting from the voltage difference $V_A - V_F$ provide a precise focusing effect. A diverging electron is accelerated forward by the field, and at the same time it receives an inward component of velocity that brings it back to the axis of the beam at the screen.

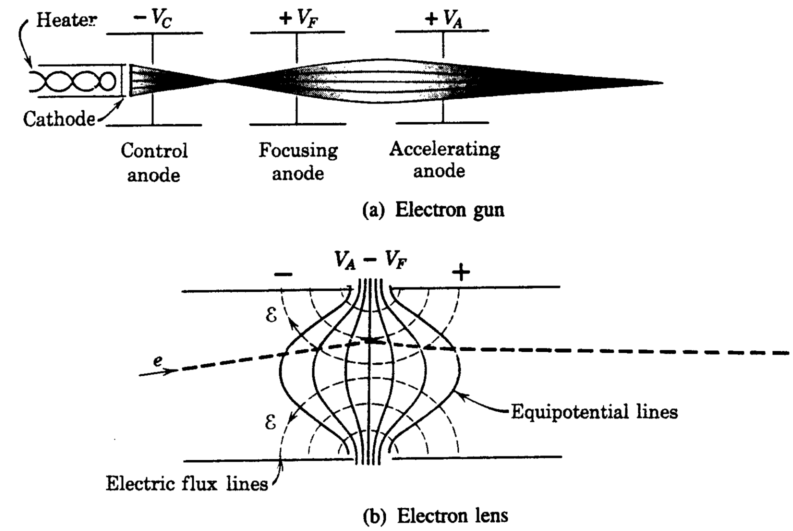


Figure 4.6 An elementary electron gun with electric-field focusing.

Deflection System. In a television picture tube, the beam is moved across the screen 15,750 times per second, creating a picture consisting of 525 horizontal lines of varying intensity. The deflection of the beam may be achieved by a magnetic field or an electric field. Figure 4.7 shows the beam produced in the electron gun entering the vertical deflection plates of an electric-field system.

We can calculate the beam deflection at the screen by using our knowledge of electron motion. For the coordinate system of Fig. 4.7, Eq. 4-11 gives an axial velocity

$$U_z = \sqrt{2eV_A/m} \tag{4-18}$$

where V_A is the total accelerating potential. Within the deflecting field, the parabolic path (Eq. 4-7) is defined by

$$y = \frac{eV_D}{2mdU_z^2}z^2 = kz^2 \tag{4-19}$$

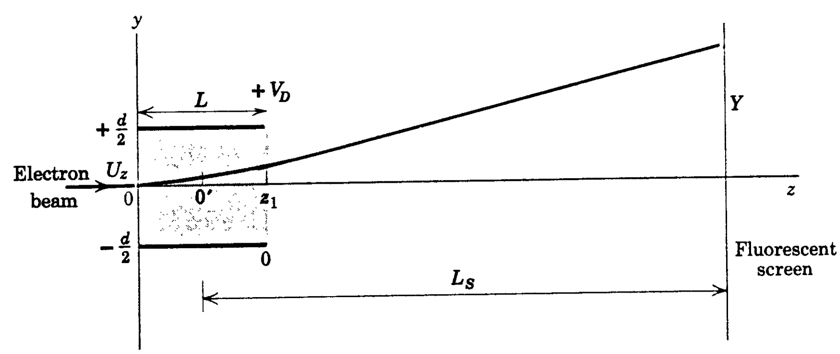


Figure 4.7 An electric-field vertical deflection system.

The slope of the beam emerging from the deflecting field at $z = z_1 = L$ is

$$\frac{dy}{dz} = 2kz = 2kL \tag{4-20}$$

The equation of the straight-line path followed by the beam to the screen is

$$y - y_1 = \frac{dy}{dz}(z - z_1) = 2kL(z - z_1)$$

where $z_1 = L$ and $y_1 = kz_1^2 = kL^2$. Substituting these values and solving,

$$y = 2kL(z - L) + kL^2 = 2kL\left(z - \frac{L}{2}\right) \tag{4-21}$$

Since for $y = 0$, $z = L/2$, Eq. 4-21 leads to the conclusion that the electron beam appears to follow a straight-line path from a virtual source at 0′. At the screen, $z = L_S + L/2$ and (by Eqs. 4-19 and 4-18) the deflection is

$$Y = 2kLL_S = \frac{eV_D LL_S}{mdU_z^2} = \frac{eV_D LL_S}{md} \cdot \frac{m}{2eV_A} = \frac{LL_S}{2dV_A} V_D \tag{4-22}$$

or the vertical deflection at the screen is directly proportional to V_D, the voltage applied to the vertical deflecting plates. A second set of plates provides horizontal deflection.

EXAMPLE 3

Determine the *deflection sensitivity* in centimeters of deflection per volt of signal for a CRT in which $L = 2$ cm, $L_S = 30$ cm, $d = 0.5$ cm, and the total accelerating voltage is 2 kV.

From Eq. 4-22, the deflection sensitivity is

$$\frac{Y}{V_D} = \frac{LL_S}{2dV_A} = \frac{0.02 \times 0.3}{2 \times 0.005 \times 2000}$$

$$= 0.0003 \text{ mV} = 0.03 \text{ cm/V}$$

To obtain a reasonable deflection, say 3 cm, a voltage of 100 V would be necessary. In a practical CRO, amplifiers are provided to obtain reasonable deflections with input signals of less than 0.1 V.

Fluorescent Screen. Part of the kinetic energy of the electron beam is converted into luminous energy at the screen. Absorption of kinetic energy results in an immediate *fluorescence* and a subsequent *phosphorescence*. The choice of screen material depends on the application. For laboratory oscilloscopes, a medium persistence phosphor with output concentrated in the green region is desirable; the eye is sensitive to green and the persistence provides a steady image of a repeated pattern. For color television tubes, short persistence phosphors that emit radiation at various wavelengths are available. For radar screens, a very long persistence is desirable.

OSCILLOSCOPES

The CRT provides a controlled spot of light whose x and y deflections are directly proportional to the voltages on the horizontal and vertical deflecting plates. The cathode-ray oscilloscope (CRO), consisting of the tube and appropriate auxiliary apparatus, enables us to "see" the complex waveforms that are critical in the performance of electronic circuits.

The block diagram of Fig. 4.8 indicates the essential components of a CRO. A signal applied to the "Vert" terminal causes a proportional vertical deflection of the spot; the calibration of the *vertical amplifier* can be checked against an internal calibrating signal. The *attenuator* precisely divides large input voltages. An external *intensity control* varies the accelerating potentials in the electron gun, and a *focus control* determines the potentials on the focusing electrodes.

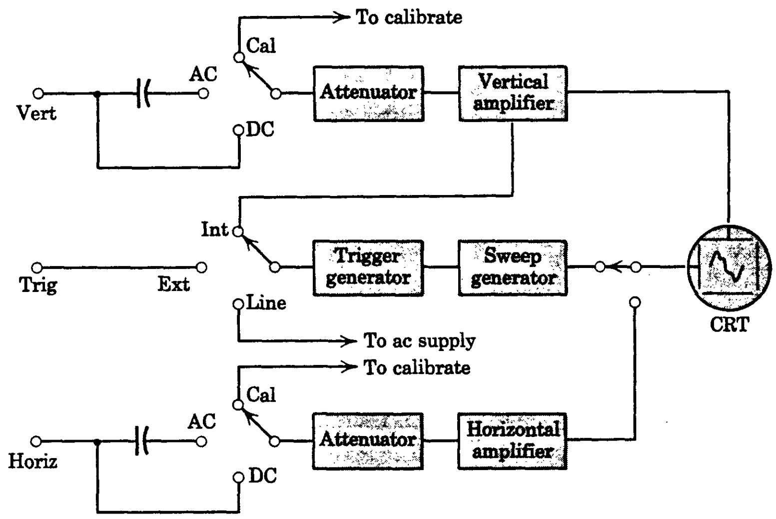


Figure 4.8 Basic components of a cathode-ray oscilloscope.

The *sweep generator* causes a horizontal deflection of the spot proportional to time; it is *triggered* to start at the left of the screen at a particular instant on the internal vertical signal ("Int"), an external signal ("Ext"), or the ac supply ("Line"). Instead of the sweep generator, the *horizontal amplifier* can be used to cause a deflection proportional to a signal at the "Horiz" terminal.

The particular point on the triggering waveform that initiates the sawtooth sweep is determined by setting the *slope* and *level* controls. In Fig. 4.9, the input to the vertical amplifier provides the triggering waveform. The level control determines the instantaneous voltage level at which a trigger pulse is produced by the trigger generator. With the slope switch in the "+" position, triggering occurs only on a positive slope portion of the triggering waveform. By proper adjustment of these two controls, it is possible to initiate the sweep consistently at almost any point in the triggering waveform. For example, if slope is set to "−" and level is set to "0," the sweep will begin when the triggering signal goes down through zero.

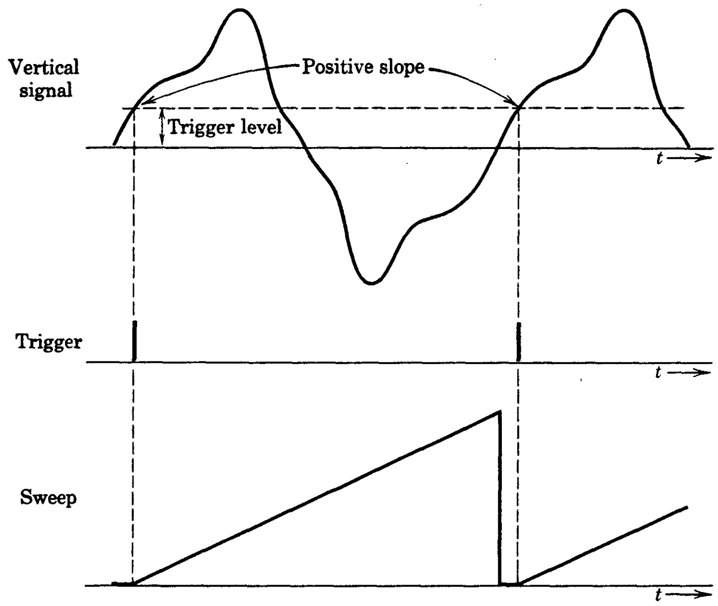


Figure 4.9 Trigger operation.

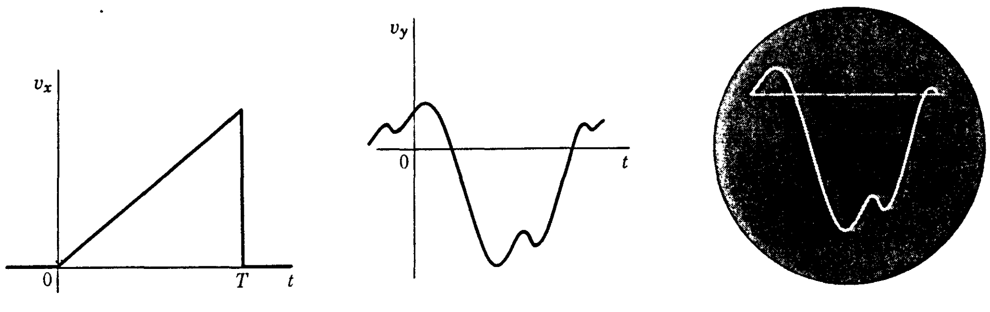
CRO Applications

In its practical form, with controls conveniently arranged for precise measurements over a wide range of test conditions, the CRO is the most versatile laboratory instrument.

Voltage Measurement. An illuminated scale dividing the screen into 1-cm divisions permits use of the CRO as a voltmeter. With the vertical amplifier sensitivity set at 0.1 V/cm, say, a displacement of 2.5 cm indicates a voltage of 0.25 V. Currents can be determined by measuring the voltage across a known resistance.

Time Measurement. A calibrated sweep generator permits time measurement. With the sweep generator set at 5 ms/cm, two events separated on the screen by 2 cm are separated in time by 10 ms or 0.01 s. By measuring the period of a wave, the frequency can be determined by calculation.

Waveform Display. A special property of the CRO is its ability to display high-frequency or short-duration waveforms. If voltages varying with time are applied to vertical (y) and horizontal (x) input terminals, a pattern is traced out on the screen; if the voltages are periodic and one period is an exact multiple of the other, a stationary pattern can be obtained. A sawtooth wave from the sweep generator (Fig. 4.10a) applied to the x-deflection plates provides an x-axis deflection directly proportional to time. If a signal voltage wave is applied to the vertical-deflection plates, the projection of the beam on the y-axis is directly proportional to the amplitude of this signal. If both voltages are applied simultaneously, the pattern displayed on the screen is the signal as a function of time (Fig. 4.10c). A *blanking circuit* turns off the electron beam at the end of the sweep so the return trace is not visible. Nonrepetitive voltages are made



(a) Sawtooth sweep (b) Signal waveform (c) Oscilloscope pattern

Figure 4.10 Display of a repeated waveform on an oscilloscope.

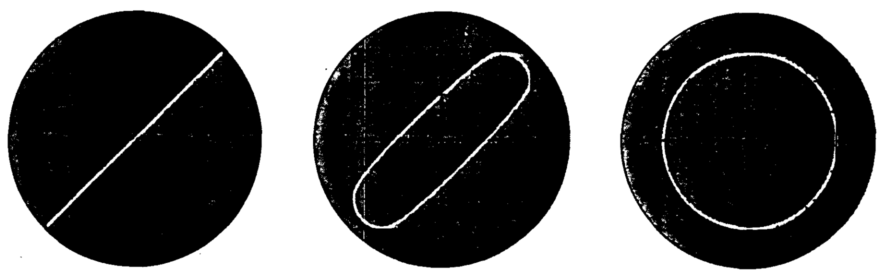
more visible by using a long persistence fluorescent material, or a high-speed camera can be used for a permanent record. New CROs provide digital waveform storage.

***X-Y* Plotting.** The relation between two periodic variables can be displayed by applying a voltage proportional to x to the horizontal amplifier and one proportional to y to the vertical amplifier. The characteristics of diodes or transistors are quickly displayed in this way. The hysteresis loop of a magnetic material can be displayed by connecting the induced voltage (proportional to B) to the vertical amplifier and an iR drop (proportional to H) to the horizontal amplifier. Since the two amplifiers usually have a common internal ground, some care is necessary in arranging the circuits.

Phase-Difference Measurement. If two sinusoids of the same frequency are connected to the X and Y terminals, the phase difference is revealed by the resulting pattern (Fig. 4.11). For applied voltages $v_x = V_x \cos \omega t$ and $v_y = V_y \cos(\omega t + \theta)$, it can be shown that the phase difference is

$$\theta = \sin^{-1}\frac{A}{B} \qquad (4\text{-}23)$$

where A is the y deflection when the x deflection is zero and B is the maximum y deflection. The parameters of the circuit are useful in determining whether the angle is leading or lagging.



(a) 0° (b) 30° (c) 90°

Figure 4.11 Phase difference as revealed by CRO patterns.

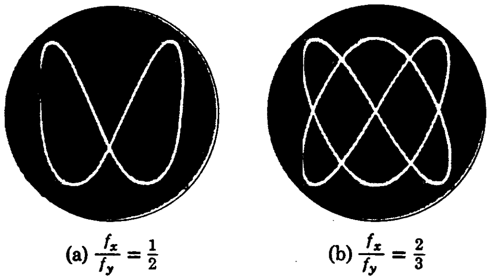


Figure 4.12 Lissajous figures for comparing the frequencies of two sinusoids.

Frequency Comparison. When the frequency of the sinusoid applied to one input is an exact multiple of the frequency of the other input, a stationary pattern is obtained. For a 1:1 ratio, the so-called Lissajous patterns are similar to those in Fig. 4.11. For the 1:2 and 2:3 ratios, the Lissajous figures might be as shown in Fig. 4.12. For a stationary pattern, the ratio of the frequencies is exactly equal to the ratio of the numbers of tangencies to the enclosing rectangle. Patterns can be predicted by plotting x and y deflections from the two signals at corresponding instants of time.

Square-Wave Testing. The unique convenience of a CRO is illustrated in the measurement technique called *square-wave testing*. Any periodic wave can be represented by a Fourier series of sinusoids. As indicated in Fig. 4.13b, the sum of the first three odd harmonics (with appropriate amplitude and phase) begins to approximate a square wave; additional higher harmonics would increase the slope of the leading edge and smooth off the top. If a square-wave input to a device under test results in an output resembling Fig. 4.13c, it can be shown that low-frequency components have been attenuated and shifted forward in phase. If the output resembles Fig. 4.13d, it can be shown that high-frequency components have been attenuated and shifted backward in phase. The frequency response of an amplifier, for example, can be quickly determined by varying the frequency of the square-wave input until these distortions appear; the useful range of the amplifier is bounded by the frequencies at which low-frequency and high-frequency defects appear. As another example, two devices giving similar responses to a square-wave input can be expected to respond similarly to other waveforms.

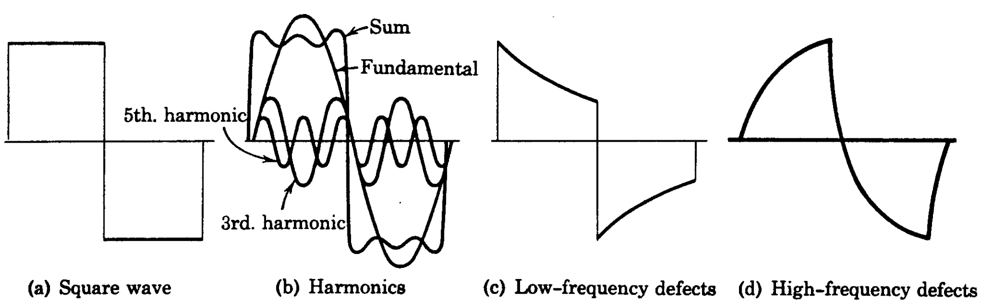


Figure 4.13 Square-wave testing with a CRO.

CRO Features

Every year sees new advances in CRO design as the instrument manufacturers strive to meet new needs of the laboratory and the field by taking advantage of newly developed devices and techniques. Among the features offered by modern CROs are:

Differential Inputs. Each amplifier channel has two terminals in addition to the ground terminal. By amplifying only the difference between two signals, any common signal such as hum is rejected. (See p. 439.)

Optional Probes. To improve the input characteristics of a CRO or to expand its functions, the input stage can be built into a small unit (connected to the CRO by a shielded cable) that can be placed at the point of measurement. *Voltage probes* insert impedances in series with the CRO input to increase the effective input impedance. *Current probes* use transformer action or the Hall effect to convert a current into a proportional voltage for measurement or display.

Dual Channels. The waveforms of two different signals can be displayed simultaneously by connecting the signals alternately to the vertical deflection system. The more expensive *dual beam* feature requires separate electron guns and deflection systems but permits greater display flexibility.

Delayed and Expanded Sweep. An auxiliary delayed sweep with a faster sweep speed displays a magnified version of a selected small portion of a waveform.

Storage. If the input signal is a single, nonrepetitive event, the image can be *stored* by using a long persistence phosphor. By placing a storage mesh directly behind the phosphor and controlling the rate at which the charge pattern leaks off the mesh, a *variable persistence* is obtained.

Sampling. To display signals at frequencies beyond the limits of the CRO components, very short *sample* readings are taken on successive recurrences of the waveform. Each amplitude sample is taken at a slightly later instant on the waveform and the resulting dots appear as a continuous display. With this sampling technique, 18-GHz signals can be displayed.

Digital Readout. The addition of a built-in *microprocessor* provides direct digital readout of time interval, frequency, or voltage in addition to the conventional CRO display. The operator sets two markers to indicate a horizontal or vertical displacement on the waveform; the microprocessor, a computer-on-a-chip, interrogates the function switches and the scale switches, calculates the desired variable, and converts it to digital form for display by light-emitting diodes.

Programmability. The oscilloscope is preeminent in displaying information; the digital computer is preeminent in processing information. The new digitizing oscilloscope is an open-ended instrument that accepts either preprogrammed or user-programmable instructions and into which the user enters his or her choice of parameters, functions, frequency ranges, data reduction operations, and display characteristics by means of a single keyboard just as we enter numbers in a hand-held calculator. It can store, display, measure, and analyze complex signal waveforms.

SUMMARY

- Individual charged particles in electric and magnetic fields obey Newton's laws. For electrons (of primary interest here),

$$\mathbf{f} = -e(\mathcal{E} + \mathbf{u} \times \mathbf{B})$$

- In any electric field, KE gained = PE lost = $W_{ab} = eV_{ab}$.
  For an electron starting from rest (and for $V < 4$ kV),

$$u = \sqrt{2(e/m)V} = 5.93 \times 10^5\sqrt{V} \text{ m/s}$$

  In a uniform electric field where $\mathcal{E}_x = \mathcal{E}$, $f_x = -e\mathcal{E}$, and

$$a_x = -\frac{e\mathcal{E}}{m} \qquad u_x = -\frac{e\mathcal{E}}{m}t + U_O \qquad x = -\frac{e\mathcal{E}}{2m}t^2 + U_Ot + X_O$$

- In any magnetic field, **f** is normal to **u** and no work is done.
  In a uniform magnetic field, the path is circular with $r = mU_O/eB$.
- Electron emission from a solid requires the addition of energy equal to the work function; this energy can be obtained in various ways.
  The probability of an electron possessing energy eV_T varies as $e^{-eV_T/kT}$.
- A cathode-ray tube consists of an electron gun producing a focused beam, a magnetic or electric deflection system, and a fluorescent screen for visual display.
  For deflecting voltage V_D, the deflection is

$$Y = \frac{LL_s}{2dV_A}V_D$$

- A cathode-ray oscilloscope includes display tube, intensity and focus controls, amplifiers and attenuators, sweep generator, and triggering circuit.
  A cathode-ray oscilloscope measures voltage and time, displays waveforms, and compares phase and frequency.

REVIEW QUESTIONS

1. What quantities are analogous in the equations of motion of a mass in a gravitational field and motion of an electron in an electric field?
2. Define the following terms: electron-volt, electron gun, vector cross product, work function, and virtual cathode.
3. Justify the statement that "no work is done on a charged particle by a steady magnetic field."
4. Describe the motion of an electron starting from rest in a region of parallel electric and magnetic fields.
5. Sketch the pattern expected on a CRT screen when the deflection voltages are $v_x = V \sin \omega t$ and $v_y = V \cos 2\omega t$.
6. What is the effect on a TV picture of varying the voltage on the control anode (Fig. 4.6)? On the accelerating anode?
7. Sketch a magnetic deflection system for a CRT.
8. Explain qualitatively the process of thermionic emission.
9. How can a CRO be used to measure voltage? Frequency? Phase?
10. Explain the operation of CRO sweep, trigger, and blanking circuits.

EXERCISES

1. An electron is accelerated from rest by a potential of 200 V applied across a 5-cm distance under vacuum. Calculate the final velocity and the time required for transit. Repeat for a hydrogen ion. Express the energy gained by each particle in electron-volts.
2. In Fig. 4.14, an electron is introduced at point P in an evacuated space. It is accelerated from rest toward anode 1 at voltage V_1 and passes through a small hole at point b. In terms of the given quantities and the properties of an electron:
   (a) What is the acceleration at point a?
   (b) What is the velocity at point b?
   (c) What voltage V_2 would just bring the electron to rest at point c?

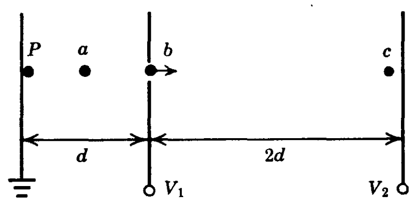


Figure 4.14

3. In a region where $\mathcal{E}$ = 200 V/m, an electron is accelerated from rest. Predict the velocity, kinetic energy gained, displacement, and potential energy lost after 2 ns. Compare the PE lost ($e\Delta V$) and the KE gained ($\frac{1}{2}mu^2$).
4. An electron is to be accelerated from rest to a velocity $u = 12 \times 10^6$ m/s after traveling a distance of 5 cm.
   (a) Specify the electric field required and estimate the time required.
   (b) Repeat part (a) for a hydrogen ion.
   (c) Express the energy gained by each particle in joules.
5. For the electron of Exercise 3, derive expressions for velocity as a function of time and as a function of distance.
6. An electron with an energy of 200 eV is projected at an angle of 45° into the region between two parallel plates carrying a voltage V and separated a distance d = 5 cm. (See Fig. 4.15.)
   (a) If V = −200 V, determine where the electron will strike.
   (b) Determine the voltage V at which the electron will just graze the upper plate.

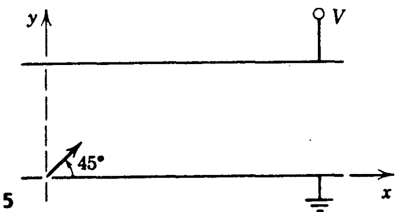


Figure 4.15

7. If the polarity of the upper plate in Fig. 4.15 is reversed so that V = +200 V, determine where the electron will strike.
8. Two electrons, e_1 traveling at velocity u and e_2 traveling at velocity $2u$, enter an intense magnetic field directed out of the paper (Fig. 4.16). Sketch the paths of the electrons.

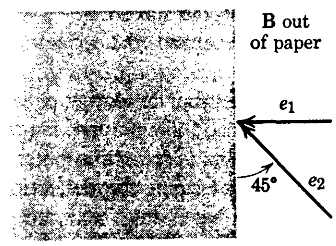


Figure 4.16

9. An electron with a velocity of 30 km/s is injected at right angles to a uniform magnetic field where B = 0.02 T into the paper.
   (a) Sketch the path of the electron.
   (b) Predict the radius of the path and the time required to traverse a semicircle.
10. In a *mass spectrograph*, isotopes of charge e and unknown mass are accelerated through voltage V and injected into a transverse magnetic field B. Derive an expression for the mass m in terms of the radius r of the circle described by the particle.
11. In a cyclotron (see any physics book), hydrogen ions are accelerated by an electric field, curved around by a magnetic field, and accelerated again. If B is limited to 2 T, what cyclotron diameter is required for a velocity half that of light?
12. An electron moves in an electric field $\mathcal{E}$ = 100 V/m with a velocity of 10^6 m/s normal to the earth's magnetic field ($B = 5 \times 10^{-5}$ T). Compare the electric and magnetic forces with that due to the earth's gravitational field. Is it justifiable to neglect gravitational forces in practical problems?

13. The work function of copper is 4.1 eV and the melting point is 1356 K. Thorium has a work function of 3.5 eV and melts at 2120 K. Compare the factor of $e^{-W_W/kT}$ for these two metals at 90% of their respective melting points and decide which is more likely to be used as a cathode.

14. The Richardson-Dushman equation indicates that cathode emission current is proportional to $T^2 e^{-W_W/kT}$. For a special cathode coating with a work function of 1 eV, the emission current density at 1100 K is 200 mA/cm^2. Estimate the emission current densities at 800 and 1200 K.

15. In the CRT deflection system of Fig. 4.7, the accelerating voltage is 1000 V, the deflecting plates are 2.5 cm long and 1 cm apart, and the distance to the screen is 40 cm.
   (a) Determine the velocity of the electrons striking the screen.
   (b) Determine the deflecting voltage required for a deflection of 4 cm.

16. The vertical deflecting plates of a CRT are 2 cm long and 0.5 cm apart; length L_S in Fig. 4.7 is 50 cm. The acceleration potential is 5 kV.
   (a) Determine the deflection sensitivity.
   (b) What is the maximum allowable deflection voltage?
   (c) For a brighter picture, the acceleration potential is increased to 10 kV. What is the new deflection sensitivity?

17. A CRO with a 10 × 10-cm display has vertical amplifier settings of 0.1, 0.2, and 0.5 V/cm and horizontal sweep settings of 1, 2, and 5 ms/cm. Assuming the sweep starts at $t = 0$, select appropriate settings and sketch the pattern observed when a voltage $v = 2 \sin 60\pi t$ V is applied to the vertical input.

18. Repeat Exercise 17 for an applied voltage $v = 0.5 \cos 1000t$ V.

19. On the CRO of Exercise 17, determine the frequency of a square-wave signal that occupies:
   (a) 2.5 cm per cycle at a sweep setting of 5 ms/cm.
   (b) 4.0 cm for 10 cycles at a sweep setting of 1 ms/cm.

20. On a CRO vertical and horizontal amplifiers are set at 1 V/cm. Sketch the pattern observed when the voltages applied to the vertical and horizontal inputs are:
   (a) $v_v = 5 \cos 400t$ and $v_h = 5 \cos 100t$ V.
   (b) $v_v = 5 \cos 200t$ and $v_h = 5 \cos (700t - \pi/2)$ V.

21. Design a circuit for displaying the v-i characteristic of a diode on a CRO. Assume that the horizontal and vertical amplifiers have a common internal ground.

22. The CRO of Exercise 21 has an 8 × 8-cm screen. Anticipating an I-V characteristic similar to that shown in Fig. 5.8, specify the significant CRO settings.

PROBLEMS

1. As Einstein pointed out, the actual mass m of a moving particle is $m = m_O/\sqrt{1 - u^2/c^2}$ where m_O is the rest mass, u is velocity, and c is the velocity of light. Since energy and mass are equivalent, the potential energy lost in falling through a voltage V must correspond to an increase in mass $m - m_O$.
   (a) Equate the potential energy lost to the equivalent energy gained and calculate the voltage required to accelerate an electron to 99% of the speed of light.
   (b) Calculate the electron velocity for $V =$ 24 kV, a typical value in a television picture tube, and determine the percentage of error in Eq. 4-11.

2. The magnetic deflecting *yoke* of a TV picture tube provides a field $B = 0.002$ T over an axial length $l = 2$ cm. The *gun* provides electrons with a velocity $U_Z = 10^8$ m/s. Determine:
   (a) The radius of curvature of the electron path in the magnetic field.
   (b) The approximate angle at which electrons leave the deflecting field.
   (c) The distance from yoke to screen for a 3-cm *positive* deflection.
   (d) A general expression for deflection Y in terms of acceleration potential V_a and compare with Eq. 4-22. (Assume small angular deflections where $Y/L = \tan \alpha \cong \alpha$.)

3. How long is an electron in the deflecting region of the CRT of Exercise 16? If the deflecting voltage should not change more than 10% while deflection is taking place, approximately what frequency limit is placed on this deflection system?
4. The useful range of an amplifier is bounded by frequencies f_1 and f_2 at which low-frequency and high-frequency defects occur.
   (a) In a certain amplifier, signals at $3f_1$ are amplified linearly, but at f_1 sinusoidal components are reduced to 70% of their relative value and shifted forward 45° in phase. For a square-wave input of frequency f_1, draw the fundamental and third harmonic components in the output and compare their sum to Fig. 4.13c.
   (b) In the same amplifier, signals at $f_2/3$ are amplified linearly, but at f_2 sinusoidal components are reduced to 70% of their relative value and shifted backward 45° in phase. For a square-wave input at frequency $f_2/3$, repeat part (a) and compare to Fig. 4.13d.